

Home Production with Time to Consume

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Abstract

We build a home production model where consumers choose how to spend their off-market time using market consumption purchases. Heterogeneity in the labor intensities of different home production activities governs the degree to which income changes or relative price changes affect the composition of market consumption expenditure. We demonstrate that failing to account for time use complementarities with market purchases implies that the value of the skills of homemakers engaging in home production is zero. In a quantitative exercise on aggregate expenditure data, we use the model to estimate the degree to which relative price changes versus wage growth have contributed to the rise in the services share of U.S. expenditure since 1948. Our findings suggest that structural change is mostly driven by supply-side factors affecting relative prices rather than consumers having increasing preferences for services as averages wages rise. Further, we demonstrate that our non-homothetic preference structure based on home production admits an aggregate representation. Robustness tests show that empirical estimates on aggregate expenditure data are not significantly affected by aggregation bias.

Keywords: home production, time use, structural change, services, goods

JEL Classification: D13, E21, O33

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1 Introduction

All consumption takes time. One does not simultaneously purchase and consume a product. It takes time to procure the product and then to subsequently mold it into its final consumable form. Gary Becker recognized this in his original home production paper, “A Theory of the Allocation of Time” (Becker 1965). We focus on several previously un-discussed theoretical implications of Beckerian home production, then apply a tractable home production model to a quantitative exercise on aggregate United States (U.S.) consumption expenditure data in order to estimate the degree to which wage growth and relative price effects have contributed to the rise in the services share of consumption expenditure since 1948. From hereon we will refer to this rise as *structural change*, the dynamics of which are well-studied with most authors in agreement that some kind of non-homothetic preference structure is required to reconcile this phenomenon in data. Up to this point income effects are generally theorized to result from consumers’ need to consume a basic subsistence level of consumption, modeled with variations of Stone-Geary preferences (Geary 1950; Stone 1954).¹ In this paper we provide an alternative micro-foundational explanation for why expenditure shares may vary in income and relative prices. As in Becker (1965), we suppose households derive utility only from final commodities produced in the home using some combination of market purchases and time. Using this model we demonstrate that the responsiveness of the composition of household expenditure to income and relative price changes depends on differentials in the labor intensities of different home production activities. Additionally, we demonstrate that failing to allow for non-zero time to consume market purchases implies that the skills of household members provide no additional value added. Such implications are extreme, suggesting that, for example, the time one spends cooking a meal at home has no value. Taking our model to aggregate expenditure data, we argue that structural change appears mostly driven by supply-side factors affecting relative prices, not consumers adjusting their expenditure patterns as a response to long run wage growth.

Thus far in the structural change literature, consumers’ need to achieve a basic subsistence level of consumption is the most common microfoundational explanation for the existence of apparent income effects in the data. Depending on the definition of the commodity space, whether only goods and services are modeled or agricultural goods and manufactured goods are separately considered, the estimated subsistence level of consumption varies considerably, depending on both the time frame and the country over which the data is collected. While the argument that subsistence consumption is indeed the reason for the existence of income effects is cogent in developing countries where agricultural output is a considerable fraction of total economic output, the story is not as intuitive in explaining the more recent rise of the services expenditure share in advanced economies. Indeed, Buera and Kaboski (2009) note that estimates of Stone-Geary

¹For structural change analyses that rely on subsistence preferences see Kongsamut, Rebelo, and Xie (2001), Buera and Kaboski (2009), Matsuyama (2009), Herrendorf, Rogerson, and Valentinyi (2013), Uy, Yi, and Zhang (2013), and Bridgman, Duernecker, and Herrendorf (2018). Alternatively, Boppart (2014) uses a form of “price independent generalized linearity” (PIGL) preferences described by Muellbauer (1975, 1976) that admit aggregation across households.

subsistence consumption levels for the U.S. depend on the length of the time series employed, with income effects resulting from subsistence preferences unable to simultaneously explain the early-nineteenth century transition of the U.S. economy from agriculture toward manufacturing and the mid-twentieth century transition from manufacturing to services. This puzzle suggests the need to employ an alternative and more comprehensive model of household decision-making with the flexibility to capture both economic transitions. As a proposed alternative, we explicitly model the in-home tradeoffs faced by households between spending time engaging in home production, market consumption, and market work. Rather than suppose consumers must purchase a certain amount of market consumption to survive, in the event of pure market breakdown where no trades are made our model allows consumers to produce 100% of final consumption in the household. At this extreme, the model could theoretically describe the tradeoffs faced by an early American frontier family that goes months or years without civilized contact, forced to *subsist* for themselves without the ability to trade or barter for market goods and services. At the other extreme households may spend most of their time working so that they can simply purchase market services and rarely have to engage in time-consuming off-market tasks involving the manipulation of purchased market goods into final consumption. In our formulation households buy market purchases so that they may be used, though how households allocate their time and available market resources depends on the degree of substitutability between different market purchases and time spent devoted to different off-market tasks. These within-household substitution effects depend on both the gross substitutability between the outputs of home production and how labor intensive those various home production processes are.

The theoretical model we describe generates quantitative results regarding structural change that are robust to the inclusion of different degrees of consumer durable asset service flows in the overall goods series. When only non-durable goods are used, income effects appear relatively stronger. In our empirical exercises, we show that in the long run, after counterfactually fixing relative prices at those observed in 1948, income gains do not alone affect changes in the relative consumption basket. Rather, these changes appear almost totally driven by changes in relative prices. Thus, using our home production framework, supply-side factors appear to be the primary drivers of structural change, with income effects playing only a minor role. For example, Ngai and Pissarides (2007) propose that structural transformation is primarily driven by differentials in sectoral productivities which lead to capital deepening and force labor to shift to the less productive sector, driving up market prices in that sector (services) relative to the more productive one (goods). Acemoglu and Guerrieri (2008) achieve similar results with sectoral differences in factor shares for production inputs, leading to differential rates of capital accumulation. Buera and Kaboski (2012), on the other hand, show that the relative rate of structural change depends on the productivity advantage of high-skilled workers in predominantly service industries versus their low-skilled counterparts working in manufacturing. Autor and Dorn (2013) tell a story with implications that contradict the premises but not the results in Buera and Kaboski (2012): automation in manufacturing has driven low-skilled workers to low-wage service industry jobs. As

consumer preferences have evolved to favor variety over specialization, new services are created, but these jobs are occupied mostly by low-skilled workers. An implication of Autor and Dorn (2013) is that the decline in the relative price of goods to services would be driven almost exclusively by total factor productivity differentials between sectors, since they contend that relatively high levels of low-skilled workers in services versus manufacturing negatively affects the relative productivity of the two sectors.

While our empirical results justify continued exploration of the degree to which supply-side phenomena may be driving structural change, we focus explicitly in this paper on the decision process of households engaging in home production. Our analyses are thus partial equilibrium in nature and operate under the assumption that each household in the economy is a price-taker and does not take into consideration how his actions on the market will affect general equilibrium outcomes. This assumption is common in partial equilibrium structural change literature, as demonstrated in Herrendorf, Rogerson, and Valentinyi (2013). This paper thus proceeds as follows. First, in Section 2 we describe our version of a Beckerian model of home production and document the model's implications for home production value-added, income and relative price effects, and household labor supply decisions. Then in Section 3 we present empirical facts, regression equations, discuss identifying assumptions, derive sufficient conditions by which the model can be estimated using aggregate data, and present estimation and counterfactual results. In Section 4 we conclude.

2 Theory of the Household

Consider a consumer who purchases a real quantity of groceries q at price p which he intends to use to make dinner for his family. The household derives final utility in the sense of Becker (1965) from consuming this dinner prepared and cooked by our master homemaker using recently purchased market goods — the groceries — as inputs. Perhaps he also uses knives, mixing bowls, cutting boards, the stove or microwave. Denote these durable items owned by the household which are used to achieve the production of dinner as a . Finally, the master homemaker uses his time n both to shop for q and to prepare the dinner. If our homemaker is indeed a gastronomic maestro, the amount of time he uses will likely exceed that of the microwaving novice. Along both extremes, time is required nonetheless. Let c denote dinner, the final consumption commodity, produced using market goods q , durable possessions a , and time n according to some general production process f , which is such that $c = f(q, a, n)$. This formulation differs from those of Gronau (1977), Graham and Green (1984), Benhabib, Rogerson, and Wright (1991), Rupert, Rogerson, and Wright (1995), Ingram, Kocherlakota, and Savin (1997), Bridgman, Duernecker, and Herrendorf (2018), and Boerma and Karabarbounis (2019) who all dispense with the idea that market goods *and* time are combined together in some home production function to produce a final commodity. Notable exceptions are McGrattan, Rogerson, and Wright (1993) and Gomme, Kydland, and Rupert (2001) which allow for durable capital to be combined with time toward the produc-

tion of home consumption commodities, but additional non-durable market goods are still absent from their formulations. Micro-data analyses, like those in Aguiar and Hurst (2005, 2007) and Aguiar, Hurst, and Karabarbounis (2013), provide evidence that consumer expenditure and off-market time use decisions are interdependent, suggesting home production models should take such complementarities into consideration.

When thinking about how households use market goods, it is anecdotally evident that complementarities exist between the consumption of these purchased goods and non-work time. In models with only one consumption commodity and elastic labor supply, complementarities between leisure and consumption are explicitly considered. However, this is usually not the case in models where consumers derive utility from multiple consumption commodities. Yet, time use and consumption complementarities exist because households derive final utility not simply from purchasing a product at a store, but rather as the outcome of what they do with that product after it is purchased. This fundamental observation lies at the core of Gary Becker's original argument — that time and all market purchases, not just durables, are combined to produce the final commodity from which the household derives its final utility. Even when using market services, like for example hiring a home cleaning service, consumers must at least briefly spend time finding the maid and explaining to the hired hand what cleaning needs to be done. While this amount of time is less than the amount of time required to clean the house oneself, the amount of time is nevertheless *not* zero. Time use complementarities are more readily apparent when thinking about the inputs required in such a process as making a meal. Meanwhile, this model of household behavior challenges the conventional assumption made in modern macroeconomic models to classify as leisure all time spent outside of a formal, income-producing job.² The Beckerian approach, rather, is to model all off-market time as spent engaging in some home production activity, one of which could be leisure.

To illustrate the importance of time use and market purchase complementarities, consider someone who purchases a market good, like say a boat, and does not spend any time using it. Can we truly say that the boat is providing value to this consumer? He works overtime nights and weekends to pay for a boat that he never uses. It is well-established common knowledge that boats depreciate in value very quickly, so surely this consumer does not view the purchase as a capital investment. He must make time in his schedule to use the boat and derive utility from it. He thus faces an implicit tradeoff between working more to earn more money to pay for the boat and all the maintenance it requires versus spending time actually using the boat and deriving utility from it. If he knows he does not have the time to both supply labor to pay for the boat and enjoy the boat once he has it, he will not make the decision to purchase the boat.³

The implicit returns a household derives from using a product one period may be different than the returns derived from using the same product in another period. Perhaps the household's time is more valuable due to wage increases, so the time they once spent cooking an intricate

²An exception here is the analysis in Aguiar, Hurst, and Karabarbounis (2013) which distinguishes between off-market leisure time and off-market non-leisure time.

³Or at least he shouldn't.

meal requiring one hour of preparation may be more valuably spent on another task and so the household is more willing to order take out. But wages do not just have to change to affect the implicit returns of home production. Holding wages and all other prices fixed, suppose our master homemaker takes a cooking class and learns some new techniques in the kitchen he is now itching to try out. The same meal he cooked using the same ingredients in a previous period now tastes better due to the culinary knowledge he acquired. The productivity of his meal preparation has thus improved because he is now able to produce a higher quality product within the home in the same amount of time, purchasing the same ingredients at the same market price.

In this manner, our work here fits in with a broader discussion regarding the aggregate impacts of changes to in-home activities. In his book “The Rise and Fall of American Growth,” Robert Gordon contends that one reason United States GDP growth rates in the post war era were so consistently high was because new household consumer products, such as dishwashers, washing machines, air conditioners, etc., were being produced and purchased at mass rates (Gordon 2016). Gordon contends that adoption of such household durables freed up off-market time of homemakers, reducing the amount of time they spent doing chores. In this context there exists a fundamental distinction between market demand and off-market time use that is more commodity-specific and task-specific than a mere consumption/leisure tradeoff. Inspired by Gordon (2016) we incorporate such a distinction into our model while allowing for the ability of households to perform certain tasks to evolve exogenously over time.

Modeling explicit complementarities between time use and market goods purchases assumes three anecdotal ideas relating to consumer behavior: 1) a consumer cannot enjoy a market good and derive final utility from its consumption if he cannot spend time using it; 2) there appear to exist no market purchase for which the former is not true; 3) the return consumers derive from using market purchases in home production processes depends on the exogenous, time-varying ability of the consumer to engage in transformation of market goods to final goods. Our model is flexible enough to capture all limiting conditions, where time is valued on its own, absent market purchase complementarities, and vice-versa. The remainder of the theoretical model exposition proceeds as follows. In Section 2.1 we construct a stylized model of household decisions which captures fundamental aspects of the theory described here. In Section 2.3, we perform comparative statics in a two good economy in order to illustrate the tradeoffs faced by households in simultaneously choosing how much to purchase and how to use those purchases.

2.1 General Model of the Household

We will develop the model at the household level in order to eventually arrive at a proposition which shows, under certain conditions, that our relative demand representation aggregates. Time is discrete and indexed by t . Each period a household derives utility from the consumption of I final goods c_{ith} each produced under the home production process f_{ith} , which takes as inputs a J_i

dimensional vector of market goods \mathbf{q}_{ith} and time n_{ith} .⁴ Final consumption is such that

$$c_{ith} = f_{ith}(\mathbf{q}_{ith}, n_{ith}) \quad \forall i, t, h \quad (1)$$

We dispense with modeling fixed durable assets as inputs. Assume instead that some components of market inputs contain the service flows from durables, which ultimately are what the household uses when engaging in home production. In our quantitative exercises we will construct the data series so that the value of service flows from durable goods held by households is included in the value of market goods, as if the household rents durables from some firm “producing” new service flows each period. Price indices will be adjusted to account for this via procedures described in Appendix A.1.

Let \mathbf{q}_{th} be a J dimensional vector whose components are each of the commodities on the market place. Let \mathbf{P}_t be a J dimensional vector of market prices and \mathbf{x}_{th} the associated expenditure vector, the components of which are such that $P_{jt}q_{jth} = x_{jth}$. Given we are building a household’s problem from the ground up, we need to arrive at a preference representation that admits aggregation across the commodity space. To do this, Assumptions 1 and 2 are necessary.

Assumption 1. There is no joint production using market goods. That is, if q_{jth} is a component of \mathbf{q}_{ith} then q_{jth} cannot be a component of $\mathbf{q}_{i'th}$ for any $i' \neq i$.

Assumption 2. All input resources are used up in the production of final consumption commodities. That is $\forall j \in \{1, \dots, J\}, \exists i \in \{1, \dots, I\}$ such that q_{jth} is a component of \mathbf{q}_{ith} .

Assumptions 1 and 2 together imply that $J = \sum_{i=1}^I J_i$.⁵

Each period households derive flow utility $u(\mathbf{c}_{th})$ from consumption of the I dimensional vector of final goods \mathbf{c}_{th} .⁶

Assumption 3. $u(\mathbf{c}_{th})$ is separable across components of \mathbf{c}_{th} . That is, for all $k \neq i$ and $k \neq j$, the marginal rate of substitution between final consumption derived from processes i and j is independent of consumption in process k .

The separability imposed by Assumption 3 will allow us to invoke an aggregation theorem to collapse the market commodity space into indices such that the production process for each final good takes one and only one market good as input. This is described in Lemma 1 and its proof, which requires specification of equilibrium conditions. Before moving on to that, let us finish characterizing the household’s choices.

⁴All vectors are column vectors and denoted using bold font.

⁵Assumption 1 allows us to avoid the parameter identification issues in home production models with joint production described in a back and forth between Bill Barnett, Robert Pollak, and Michael Wachter in the 1970s (Pollak and Wachter 1975; Barnett 1977). Special thanks to Javier Birchenall for pointing this out.

⁶These “goods” are the outputs of home production activities. In the spirit of Becker (1965), a single component of \mathbf{c} might capture the total enjoyment one feels both from spending time cooking *and* time eating a meal. We also refer to the components of \mathbf{c}_{th} as distinct household “activities.”

Let \bar{n} denote the total time available to the household, and assume that all households face the same time constraints. Households earn wages w_{th} from supplying labor l_{th} on the open market place. They also choose n_{ith} which is time spent engaging in home production activity i . Let \mathbf{n}_{th} denote the I -dimensional vector describing time spent on home activities. Total time allocated to market and home activities must satisfy:⁷

$$l_{th} + \sum_{i=1}^I n_{ith} \leq \bar{n} \quad (2)$$

Let y_{th} represent effective cash-on-hand, which is a function of total available income and the efficiency-value of time. Given this information, we can write the household budget constraint:⁸

$$\sum_{j=1}^J P_{jth} q_{jth} + w_{th} \sum_{i=1}^I n_{ith} \leq y_{th} \quad (5)$$

Note that our analysis here focusses on the intratemporal tradeoffs households face with regards to market purchases and off-market time use decisions. For this reason, our analyses will operate on the static equilibrium conditions describing the period t marginal rates of substitution for purchasing different market commodities and allocating time toward their utilization. For this reason, we can dispense with specifying the household's full problem in dynamic terms, since we are not concerned with consumption-smoothing motives behind investment decisions, only how household's allocate their cash-on-hand across different consumption commodities each period. Under these restrictions a household's problem can be framed

$$\max_{\mathbf{q}_{ith}, \mathbf{n}_{th}} \sum_{t=0}^{\infty} \beta^t u(c_{th}) \quad (6)$$

$$\text{subject to } \sum_{j=1}^J P_{jth} q_{jth} + w_{th} \sum_{i=1}^I n_{ith} \leq y_{th} \quad (7)$$

$$c_{ith} = f_{ith}(\mathbf{q}_{ith}, \mathbf{n}_{ith}) \quad \forall i, t, h \quad (8)$$

We will not explicitly specify a parameterization for $f_{ith}(\mathbf{q}_{ith}, \mathbf{n}_{ith})$. The reason for this is that our modeling assumptions allow us to invoke an aggregation theorem over commodities to make

⁷Note that each home production process is associated with multiple market inputs but only one time input. Therefore, we have separate indices for market inputs used in each home production activity, but the time use vector is I dimensional.

⁸Specifically, since market expenditure must satisfy

$$\sum_{j=1}^J P_{jth} q_{jth} \leq w_{th} l_{th} + R_t k_{th} - k_{t+1,h} \quad (3)$$

where $R_t k_{th} - k_{t+1}$ is capital income net of savings. Substituting (2) for l_{th} , it is clear that effective cash-on-hand is

$$y_{th} = w_{th} \bar{n} + R_t k_{th} - k_{t+1} \quad (4)$$

the analysis more compact.⁹ This is described in Lemma 1.

Lemma 1. Assume each household is a utility maximizer. Under Assumptions 1, 2, and 3, and under Theorem 1 of Green (1964) attributed to Leontief (1947), we can restrict our analysis to

$$\tilde{u}_{th}(q_{1th}, \dots, q_{ith}, \dots, q_{Ith}, n_{1th}, \dots, n_{Ith}) \quad (9)$$

where q_{ith} is some index that describes the grouping of market goods $\{q_{i1th}, \dots, q_{ijith}, \dots, q_{iJith}\}$.

Proof. See Appendix C.1. □

The intuition behind Lemma 1 is that Assumptions 1 and 3 guarantee that the intratemporal marginal rate of substitution for two goods used in the same home production process is independent of other goods not used in that process. The econometrician can then use an index of his choice to form a single composite good q_{ith} which describes the market value of the entire vector of goods \mathbf{q}_{ith} used in production process f_{ith} .¹⁰ Thus rather than specifying a functional form for $f_{ith}(\mathbf{q}_{ith}, n_{ith})$ from here on our analysis operates on $c_{ith} = \tilde{f}_{ith}(q_{ith}, n_{ith})$, the commodity-aggregated home production function for final good c_{ith} .¹¹

Assumption 4. The aggregated home production function $\tilde{f}_{ith}(q_{ith}, n_{ith})$ is strictly increasing, quasi-concave, and homogeneous of degree one.

Assumption 4 is used in our equilibrium results to characterize the value added to household market purchases from engaging in home production activities. Composing $u(c_{th})$ with each $\tilde{f}_{ith}(q_{ith}, n_{ith})$ gives us \tilde{u}_{th} :

$$u(c_{th}) = u(\tilde{f}_{1th}(q_{1th}, n_{1th}), \dots, \tilde{f}_{ith}(q_{ith}, n_{ith}), \dots, \tilde{f}_{Ith}(q_{Ith}, n_{Ith})) = \tilde{u}_{th}(\mathbf{q}_{th}, \mathbf{n}_{th}) \quad (10)$$

Unlike $u(c_{th})$, composed utility depends on the possibly time-varying home production process, so we index it with both t and h to account for this. Each home production process is associated with time and one market input, which implies that there are the same number of market goods as final goods, i.e. $J = I$, in the commodity-aggregated problem.

2.1.1 Household Equilibrium Conditions

Let μ_{th} denote the period t marginal utility of wealth, i.e. the Lagrange multiplier on the budget constraint. Each period household choices of market consumption \mathbf{q}_t and off-market time use \mathbf{n}_t

⁹Thanks to Laurence Ales for pointing this result out.

¹⁰Scalar q_{ith} is the composite good. Again, bold font is reserved for vectors.

¹¹We admit our usage of the word “aggregated” is slightly abusive throughout this paper. To be clear, \tilde{f}_{ith} is an “aggregated” home production function in the sense that it takes the composite q_{ith} as an input, where q_{ith} is the sum over the quantities of all commodities in its class. We will also use the word “aggregate” to describe total expenditure and consumption in the entire United States economy for specific commodity classes, i.e. “goods” and “services.”

must satisfy the budget constraint plus the following conditions:

$$\frac{\partial u}{\partial c_{ith}} \frac{\partial \tilde{f}_{ith}}{\partial q_{ith}} = P_{it} \mu_{th} \quad \forall i, t, h \quad (11)$$

$$\frac{\partial u}{\partial c_{ith}} \frac{\partial \tilde{f}_{ith}}{\partial n_{ith}} = w_{th} \mu_{th} \quad \forall i, t, h \quad (12)$$

For each final activity c_{ith} we can combine the equilibrium conditions for the marginal utilities of q_{ith} and n_{ith} to arrive at an expression describing the marginal rate of technical substitution between time and market inputs for process i :

$$\frac{\partial \tilde{f}_{ith}}{\partial q_{ith}} \bigg/ \frac{\partial \tilde{f}_{ith}}{\partial n_{ith}} = \frac{P_{it}}{w_{th}} \quad (13)$$

Of interest are the tradeoffs faced by consumers when engaging in market purchases. Consider the following expression describing the marginal rate of substitution between different final activities c_{jth} and c_{ith} :

$$\frac{\partial u}{\partial c_{jth}} \bigg/ \frac{\partial u}{\partial c_{ith}} = \frac{P_{jt}}{P_{it}} \frac{\frac{\partial \tilde{f}_{ith}}{\partial q_{ith}}}{\frac{\partial \tilde{f}_{jth}}{\partial q_{jth}}} \quad (14)$$

The two terms on the right hand side of (14) are the shadow prices, with respect to the internal household marketplace, of consuming c_{ith} and c_{jth} . If market the price P_{it} is in dollar units, then the shadow price of c_{ith} is equal to dollar-value of market inputs per unit of output from process \tilde{f}_{ith} .

Lemma 2. The shadow price of activities c_{ith} associated with the consumption of q_{ith} is equal exactly to P_{it} if and only if $\frac{\partial \tilde{f}_{ith}}{\partial q_{ith}} = 1$.

Proof. See Appendix C.1. □

When the marginal product of market inputs is unity the production function is perfectly linear in q_{ith} . Thus, increases in market inputs do not result in diminishing marginal activities. That is, the amount of activity associated with the consumption of q_{ith} constantly increases at the same rate across all levels of market inputs. Let us consider why diminishing marginal returns may make more sense. First, holding time n_{ith} fixed, adding another unit of market inputs q_{ith} will conceivably lead to a smaller increase in final consumption output due to the fact that with more market purchases and the same amount of time, the amount of time consumers have to use each specific input decreases, perhaps leading to unusable purchases, i.e. waste. Similarly, holding q_{ith} fixed and increasing n_{ith} , consumers can devote more time toward using each market purchase, wasting time perhaps on frivolous tasks using such purchases after the totality of their usefulness has been reached. Lemma 2 thus shows that the marginal value of final consumption is only

exactly equal to the market price of inputs in the absence of home production frictions inducing diminishing returns.

It is often common in economic analyses to think of the household as ultimately a consumer rather than a producer. Truly, the household engages in both production and consumption tasks, using its time to manipulate market purchases into a final consumable item, like for example a meal. At each step along a supply chain the value of the new outputs created using inputs is at least the sum of the values of the inputs used. Thus we should expect that if time and market inputs are used to produce an output in the home, additional value added should ensue as in all other steps along the supply chain. The explicit market costs of home production are simply the cost of market purchases. Proposition 1 states that the value added from home production, above and beyond explicit costs, is the market value of time used in the home production process. Unlike in production at the firm level, the laborer and the ultimate end-user of output are necessarily the same. Thus, the market value of the time the consumer spends engaging in specific home production tasks directly quantifies the additional value his efforts provide him, since he faces the opportunity cost of not working on the market and earning more income which he could use to purchase additional market inputs.¹²

Proposition 1. For each i , the value added in the production of final good c_{ith} is equal to $w_{th}n_{ith}$, the market value of time spent on task i .

Proof. See Appendix C.1. □

The proof of Proposition 1 requires two applications of Euler’s theorem for homogeneous functions and is left for inspection in the attached appendix. Corollary 1 to Proposition 1 demonstrates why

Corollary 1. If $c_{ith} = q_{ith}$, so that consumers derive utility directly from market purchases, then home production provides no additional value to the household.

Corollary 1 may seem obvious: of course there can be no value added from a process that never happens. But what this says is that if we fail to account for how households spend their off-market time using market purchases, then we are essentially saying that engaging in home production provides no additional value to the household beyond the value of the market purchases themselves. This is an extreme statement that says a meal cooked and prepared in the household is only as valuable as the sum of all the market commodities used to prepare it. Under such a modeling assumption, the intrinsic skills of the homemaker contribute nothing to the value of the final meal. This result thus demonstrates how splitting the off-market time allocation decision into a vector of decisions over activities while also allowing for time use complementarities with market

¹²Our home production value added measure is very similar to that in Bridgman, Duernacker, and Herrendorf (2018), except that their measure accounts for the opportunity costs of leasing household-held capital to firms. Since we model household-held capital as durable consumption service flows which provide utility and contribute directly to home production, our measure of value added is just slightly different, though as in Bridgman, Duernacker, and Herrendorf (2018) we use the replacement wage approach to value off-market time.

purchases provides a mechanism for quantifying and capturing the value of engaging in home production.

2.2 Parameterization & Relative Demand

From here on, our analyses will operate on equilibrium conditions involving fully parameterized utility and home production functions. We choose a constant elasticity of substitution (CES) form for $u(\mathbf{c}_{th})$:

$$u(\mathbf{c}_{th}) = \left(\sum_{i=1}^I \theta_i c_{ith}^\rho \right)^{\frac{1}{\rho}} \quad (15)$$

ρ parameterizes the intratemporal elasticity of substitution which is $\frac{1}{1-\rho}$. Let z_{ith} describe the total factor productivity of process \tilde{f}_{ith} . This term captures several things: 1) a household's exogenously evolving ability to spend time using market commodity i in order to produce the final consumption commodity; 2) the intrinsic value to a household of quality gains to q_{ith} ; 3) folk knowledge possessed by the household as to how best accomplish the production of c_{ith} . We specify a Cobb-Douglas form for the composite-commodity aggregate home production functions $\tilde{f}_{ith}(q_{ith}, n_{ith})$:

$$\tilde{f}_{ith}(q_{ith}, n_{ith}) = z_{ith} q_{ith}^{\omega_i} n_{ith}^{1-\omega_i} \quad (16)$$

ω_i is the output elasticity of market goods in process i and is assumed to be interior to the unit interval. Further ω_i is assumed the same for all households, so that final consumption heterogeneity across households results only from variation in productivities and market wages. Combining (15) and (16), the composite-commodity aggregated flow utility function $\tilde{u}_{th}(\mathbf{q}_{th}, \mathbf{n}_{th})$ is:

$$\tilde{u}_{th}(\mathbf{q}_{th}, \mathbf{n}_{th}) = \left(\sum_{i=1}^I z_{ith} q_{ith}^{\rho \omega_i} n_{ith}^{\rho - \rho \omega_i} \right)^{\frac{1}{\rho}} \quad (17)$$

The parameterized first-order conditions are:

$$\frac{\partial \tilde{u}_{th}}{\partial q_{ith}} = \tilde{u}_{th}(\mathbf{q}_{th}, \mathbf{n}_{th})^{1-\rho} \omega_i \theta_i z_{ith}^\rho q_{ith}^{\rho \omega_i} n_{ith}^{\rho - \rho \omega_i} \left(\frac{1}{q_{ith}} \right) = P_{it} \mu_t \quad (18)$$

$$\frac{\partial \tilde{u}_{th}}{\partial n_{ith}} = \tilde{u}_{th}(\mathbf{q}_{th}, \mathbf{n}_{th})^{1-\rho} (1 - \omega_i) \theta_i z_{ith}^\rho q_{ith}^{\rho \omega_i} n_{ith}^{\rho - \rho \omega_i} \left(\frac{1}{n_{ith}} \right) = w_{th} \mu_t \quad (19)$$

Using the above marginal utilities, we can derive conditions describing the equilibrium relationship between time use and market inputs for a single production process. After taking first-order conditions and doing some algebra, the infra-marginal rate of substitution between time and market goods for process i is linear due to the homogeneity of degree one assumption:

$$\frac{n_{ith} \omega_i}{q_{ith} (1 - \omega_i)} = \frac{P_{it}}{w_{th}} \quad (20)$$

Using (20), we can write the equilibrium choice of n_{ith} as an implicit function of q_{ith} and vice-versa:

$$n_{ith}(q_{ith}) = \left(\frac{1 - \omega_i}{\omega_i} \right) \left(\frac{P_{it}}{w_{th}} \right) q_{ith} \quad (21)$$

$$q_{ith}(n_{ith}) = \left(\frac{\omega_i}{1 - \omega_i} \right) \left(\frac{w_{th}}{P_{it}} \right) n_{ith} \quad (22)$$

These functions will be used to marginalize out n_{ith} and arrive at relative demand representations we can eventually estimate using only market expenditure data. Proposition 2 provides us with this relative demand representation.

Proposition 2. Under Lemma 1, CES utility for final consumption, and Cobb-Douglas aggregated home production, the relative demand for market good j to market good i can be written

$$\left(\frac{q_{jth}}{q_{ith}} \right) = \left[\frac{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}}{\theta_j \omega_j [(1 - \omega_j)/\omega_j]^{(1-\omega_j)\rho}} \right]^{\frac{1}{\rho-1}} P_{jt}^{\frac{1-\rho+\rho\omega_j}{\rho-1}} P_{it}^{\frac{1-\rho+\rho\omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i-\omega_j)}{\rho-1}} \left[\frac{z_{ith}}{z_{jth}} \right]^{\frac{\rho}{\rho-1}} \quad (23)$$

Proof. See Appendix C.1. □

Relative market consumption is thus a power function of prices, wages, and unobserved relative productivities. Proposition 2 is the starting point toward deriving a system of reduced-form linear estimating equations which map directly back to the structural equilibrium conditions.

Corollary 2. The same procedure can be applied as in Proposition 2 to express the time devoted toward production process j relative to process i as follows:

$$\left(\frac{n_{jth}}{n_{ith}} \right) = \left[\frac{\theta_i (1 - \omega_i) [\omega_i / (1 - \omega_i)]^{\rho\omega_i}}{\theta_j (1 - \omega_j) [\omega_j / (1 - \omega_j)]^{\rho\omega_j}} \right]^{\frac{1}{\rho-1}} P_{jt}^{\frac{\rho\omega_j}{\rho-1}} P_{it}^{\frac{\rho\omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i-\omega_j)}{\rho-1}} \left[\frac{z_{ith}}{z_{jth}} \right]^{\frac{\rho}{\rho-1}} \quad (24)$$

Proof. See Appendix C.1. □

If indeed the researcher possesses information about time use in home production, Corollary 2 provides additional degrees of freedom for estimation or calibration.

2.3 Comparative Statics

To illustrate the important micro-foundational theoretical implications of our formulation, we engage in several comparative statics in this partial equilibrium environment with households taking prices as given. We dispense with time t and household h subscripts for simplicity. The household uses its fixed cash on hand \bar{y} each period to purchase two market commodities q_1 and q_2 at prices P_1 and P_2 , which enter into home production functions, along with time n_1 and n_2 , to produce final commodities c_1 and c_2 according to a time-independent Cobb-Douglas specification as in (16). Assume households know market prices and home productivities z_1 and z_2 and take

them as given. The exercises can generally be grouped into two camps. First, we consider a household operating in a static environment, inelastically supplying a fixed amount of labor \bar{l} for which it receives wages w . With inelastic labor supply, we consider several equilibrium tradeoffs which depend on the relative price of market goods $\frac{p_1}{p_2}$ and the relative productivity of home production $\frac{z_1}{z_2}$. For simplicity, suppose $\theta_1 = \theta_2$.¹³ Assuming inelastic labor supply, we will undergo two broad exercises. First, we look at relative productivity and relative price effects on relative market purchases and relative time use. Second, we consider how changes in wages impact relative market purchases and relative time use through the model. Finally, we relax the inelastic labor supply assumption and examine how l varies in relative prices under different home production time use intensities.

For the first exercise, we will set $\omega_1 = \omega_2 = \omega$ so that differentials in relative factor inputs to different home production processes are entirely driven by price and productivity differentials. This strong assumption will allow us to conduct our indifference-curve analysis on clean, closed-form expressions, which we do in Section 2.3.1. Note that when $\omega_1 = \omega_2$, $\tilde{u}(\mathbf{q}, \mathbf{n})$ is homogeneous of degree one in \mathbf{q} , so relative market consumption will not change as wages change. This can be readily verified by confirming that the coefficient on wages in (23), $\frac{\rho(\omega_2 - \omega_1)}{\rho - 1}$, is 0 when $\omega_1 = \omega_2$, so that relative consumption is independent of wages under this parameterization. However, for the $\omega_1 \neq \omega_2$ case, $\tilde{u}(\mathbf{q}, \mathbf{n})$ is non-homothetic in \mathbf{q} . That is $\tilde{u}(a \cdot \mathbf{q}, \mathbf{n}) \neq a \tilde{u}(\mathbf{q}, \mathbf{n})$ even after using (21) to substitute out \mathbf{n} . Note that $\tilde{u}(\mathbf{q}, \mathbf{n})$ is still homogeneous of degree one in *all* arguments. However, there is a limit to how much \mathbf{n} can be reasonably scaled, with amplification of home time use automatically taking away from available market time. If labor is supplied inelastically, as would be the case for a household whose members are contractually obligated to work a certain number of hours per week, then assessing the homogeneity of $\tilde{u}(\mathbf{q}, \mathbf{n})$ in \mathbf{n} is not meaningful. Thus, in Section 2.3.2, we relax the assumption that output elasticities across processes are equal in order to analyze how income changes induce relative market purchase changes under different elasticities of substitution for final consumption. Finally, in Section 2.3.3 we relax the assumption that labor is inelastically supplied and demonstrate that on the intensive margin household labor supply is independent of market prices if home production time use intensities are identical, $\omega_1 = \omega_2$. We then show that when $\omega_1 \neq \omega_2$ both the sign and magnitude of household labor supply responses to relative price changes depend on the degree of final consumption substitutability ρ and which market commodity q_i is associated with the more time intensive production process.

2.3.1 Consumption and Off-Market Time Use Tradeoffs Under Identical Factor Intensities:

$$\omega_1 = \omega_2 = \omega$$

Fix cash-on-hand at $y = \bar{y}$. Consider the following expressions for the marginal rates of substitution for market inputs and time use which we arrive at after composing first order conditions

¹³ Assuming otherwise does not change the qualitative nature of our results.

with the static equivalents of (21) and (22) respectively:

$$MRS(q_1, q_2) = \left(\frac{z_1}{z_2}\right)^\rho \left(\frac{q_1}{q_2}\right)^{\rho-1} \left(\frac{P_1}{P_2}\right)^{\rho(1-\omega)} \quad (25)$$

$$MRS(n_1, n_2) = \left(\frac{z_1}{z_2}\right)^\rho \left(\frac{n_1}{n_2}\right)^{\rho-1} \left(\frac{P_2}{P_1}\right)^{\rho\omega} \quad (26)$$

Proposition 3 summarizes the effects of changes in $\frac{z_1}{z_2}$ on household equilibrium choices.

Proposition 3. Fix $P_1 = P_2 = 1$ and z_2 . Consider the following cases separately:

- i. If final goods are substitutes so that $\rho \in (0, 1)$, then an increase (decrease) in z_1 is welfare improving and results in an increase (decrease) in equilibrium $\frac{q_1}{q_2}$ and an increase (decrease) in $\frac{n_1}{n_2}$.
- ii. If final goods are complements so that $\rho \in (-\infty, 0)$, then an increase (decrease) in z_1 is welfare improving and results in a decrease (increase) in equilibrium $\frac{q_1}{q_2}$ and a decrease (increase) in $\frac{n_1}{n_2}$.

Proof. See Appendix C.1. □

Figures 1 and 2 demonstrate how changes in the curvature of the utility function driven by relative changes in productivities affect equilibrium outcomes as described in Proposition 3. Our indifference curve analyses in this section operate on two-dimensional slices of the four-dimensional utility surface. One slice looks at the tradeoff consumers face between choosing q_1 and q_2 while the other looks at the time use tradeoff with respect to choosing n_1 and n_2 .¹⁴ Note that for $\rho \in (0, 1)$, the slopes of the quasi-indifference curves become more negative as z_1 increases, holding all other variables fixed. For $\rho < 0$, the opposite occurs. In the former case, consumers now get more out of every q_1 and n_1 input to the home production process behind c_1 . Since c_1 and c_2 are substitutes, it makes sense to buy more q_1 and use more n_1 since relative increases in z_1 ensure c_1 can be produced more efficiently than c_2 . Essentially consumers substitute both market purchases and their time allocation toward the more productive process. When $\rho < 0$, complementarities induce consumers to forego additional purchases of q_1 and spend less time n_1 on process c_1 in order to devote more resources to produce c_2 , thus catching up to the efficiency gains of process c_1 . Regardless of the value of ρ , increasing z_1 while holding z_2 fixed is absolutely welfare improving .

¹⁴We call the partial indifference curves presented in these plots “quasi-indifference curves.”

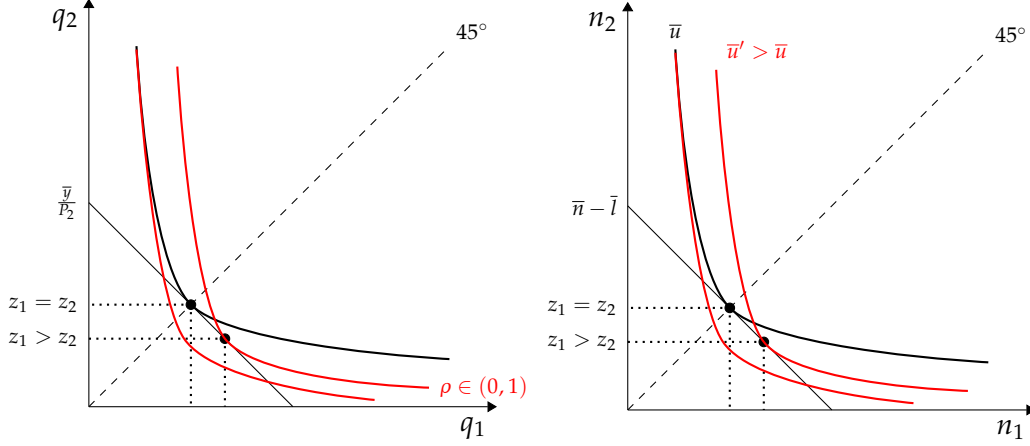


Figure 1: ($0 < \rho < 1$) — Consider equilibrium changes resulting from increases in z_1 relative to z_2 when $0 < \rho < 1$, i.e. final goods are substitutes. Here, $P_1 = P_2 = 1$. Holding q_1 and n_1 fixed at every utility level, notice that q_2 and n_2 decrease as a result of increases in $\frac{z_1}{z_2}$. Increases to z_1 must be welfare improving because the slopes of the quasi-indifference curves that pass through every (q_1, q_2) and (n_1, n_2) pair steepen, becoming more negative, meaning the consumer now receives utility $\bar{u}' > \bar{u}$. The formal argument for the welfare improvement can be found in the proof to Proposition 3 in Appendix C.1.

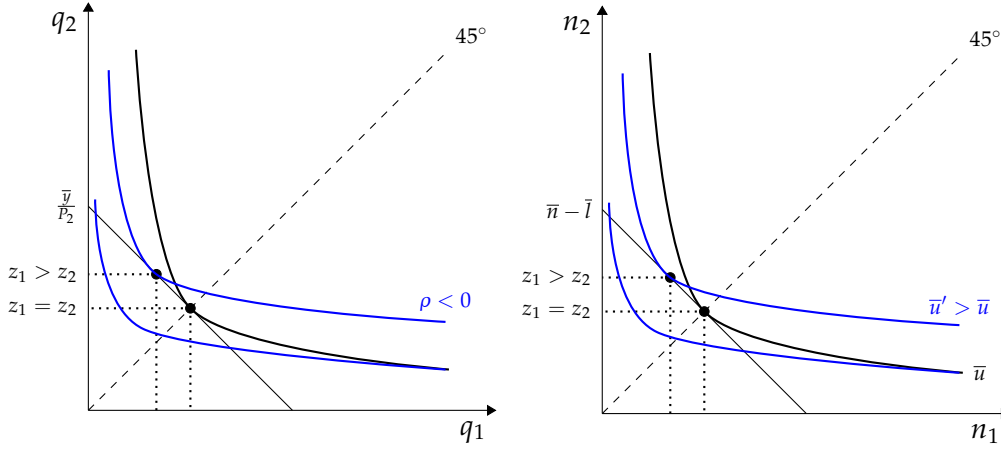


Figure 2: ($\rho < 0$) — Consider equilibrium changes resulting from increases in z_1 relative to z_2 when $\rho < 0$, i.e. final goods are complements. Again, $P_1 = P_2 = 1$. Holding q_1 and n_1 fixed at every utility level, notice that q_2 and n_2 increase as a result of increases in $\frac{z_1}{z_2}$. Increases to z_1 must be welfare improving because the slopes of the quasi-indifference curves that pass through every (q_1, q_2) and (n_1, n_2) pair flatten, becoming less negative, meaning consumers can now afford a new bundle at a higher utility level.

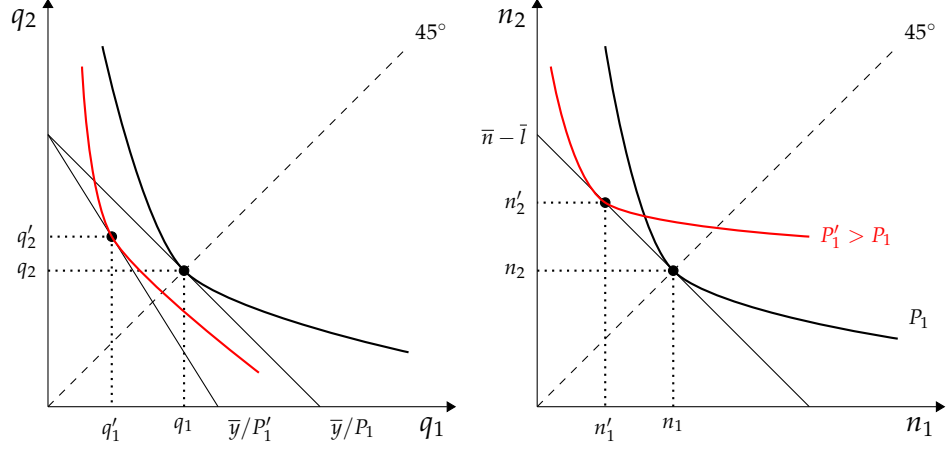


Figure 3: ($0 < \rho < 1$) — The plot demonstrates how equilibrium outcomes change when $\frac{P_1}{P_2}$ increases relative to baseline unit relative prices, i.e. $P_1 = P_2$. A change in $\frac{P_1}{P_2}$ when final consumption commodities are perfect substitutes induces positive co-movement of $\frac{q_1}{q_2}$ and $\frac{n_1}{n_2}$. The proof of Proposition 4 details the exact mathematical mechanisms causing this phenomenon.

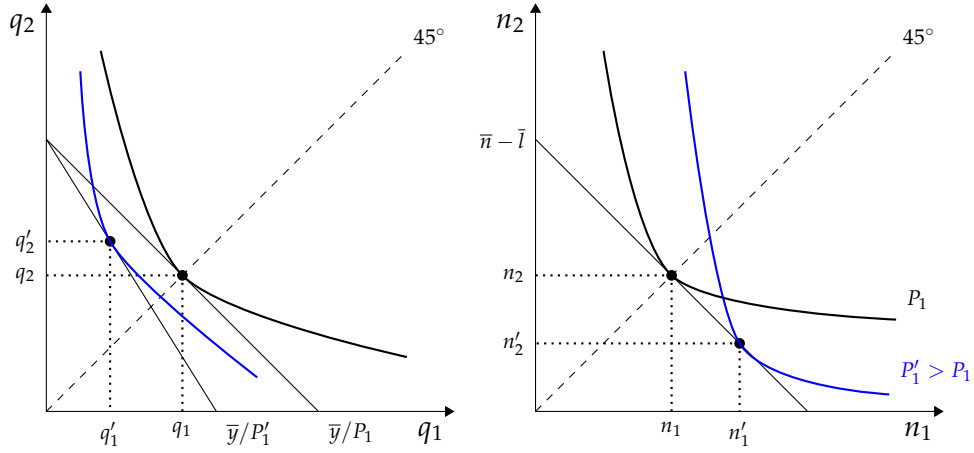


Figure 4: ($\rho < 0$) — The plot demonstrates how equilibrium outcomes change when $\frac{P_1}{P_2}$ increases relative to baseline unit relative prices, i.e. $P_1 = P_2$. A change in $\frac{P_1}{P_2}$ when final consumption commodities are perfect substitutes induces negative co-movement of $\frac{q_1}{q_2}$ and $\frac{n_1}{n_2}$.

Analyzing how relative market purchases and home time use respond to relative market price changes may perhaps be more interesting to readers, especially econometricians, given prices are what we observe, not productivities. Proposition 4 demonstrates that for fixed relative productivities with P_2 as numeraire, the elasticity of substitution for final consumption dictates the sign of co-movements in $\frac{q_1}{q_2}$ and $\frac{n_1}{n_2}$ as a result of changes to P_1 .

Proposition 4. Fix $z_1 = z_2 = 1$ and P_2 . Consider the following cases separately:

- i. If final goods are substitutes so that $\rho \in (0, 1)$, then an increase (decrease) in $\frac{P_1}{P_2}$ leads to a decrease (increase) in equilibrium $\frac{q_1}{q_2}$ and a decrease (increase) in equilibrium $\frac{n_1}{n_2}$.
- ii. If final goods are complements so that $\rho \in (-\infty, 0)$, then an increase (decrease) in $\frac{P_1}{P_2}$ leads to a decrease (increase) in equilibrium $\frac{q_1}{q_2}$ and an increase (decrease) in $\frac{n_1}{n_2}$.

Proof. See Appendix C.1. □

Turning now to relative price effects, when final goods are substitutes the ratios move together. When they are complements, increases in P_1 lead to decreases in $\frac{q_1}{q_2}$, but consumers offset the decline in q_1 by shifting time toward the production of c_1 , so $\frac{n_1}{n_2}$ increases. How can market consumption and time use ratios move in different directions? Note first that price changes induce shifts in both the quasi-indifference curve that shows the tradeoff between consumption of q_1 and q_2 and the budget constraint. Yet, with respect to time use, only the quasi-indifference curves shift. When $\rho \in (0, 1)$, we show in the proof to Proposition 4 that $\frac{\rho\omega}{\rho-1}$, the coefficient on relative prices $\frac{P_1}{P_2}$ in the relative time use equation, is negative. Consumers thus substitute their resources away from the process associated with the market good whose prices are increasing. Yet when $\rho < 0$, $\frac{\rho\omega}{\rho-1} > 0$, and consumers devote less market resources to process c_1 but relatively more time. In this case complementarities within the household, specifically complementarities between the outputs of different home production processes, dominate the traditional substitution effect induced by raising relative prices. In this way, when final activities are complements, consumers can insure themselves against adverse price shocks by substituting time for market purchases in the activity for which market prices increased. These results demonstrate the complexity of various substitution effects when the time-allocation vector is split up among different tasks in the manner we impose.

2.3.2 Wage Effects Under Differing Factor Intensities: $\omega_1 \neq \omega_2$

One of the main contributions of our framework is that it provides a possible micro-foundational explanation for why income effects appear in the data: they fundamentally depend on how consumers spend their off-market time engaging in home production activities using different market commodities. We theorize that non-homotheticities are generated by differences in the labor intensity of in-home activities along with the relative freedom by which consumers can divert their resources toward other activities (i.e. the substitution elasticity between activities). We now show that these effects induce non-linear Engel curves in wages when factor shares are different. There

is no reason to believe that similar effects would not be generated if the CES or Cobb-Douglas assumptions were relaxed.

Utility functions that generate linear Engel curves are homothetic, and homotheticity implies that expenditure shares in market goods are constant regardless of income. Holding prices fixed, this implies that $\frac{q_1}{q_2}$ is constant in wages w which is clearly only true if $\omega_1 = \omega_2$. Thus, how relative market consumption varies in w depends on the sign of the coefficient $\frac{\rho(\omega_2 - \omega_1)}{\rho - 1}$ which itself depends on whether final goods are complements or substitutes and which production process is more time intensive. Proposition 5 summarizes this.

Proposition 5. Consider the implications of two separate cases and their corresponding sub-cases:

- i. Suppose $\frac{\rho(\omega_2 - \omega_1)}{\rho - 1} < 0$ so that $\frac{q_1}{q_2}$ is decreasing in w then one and only one of the following must hold:
 - a. $\rho < 0$ and $\omega_2 < \omega_1$
 - b. $\rho \in (0, 1)$ and $\omega_2 > \omega_1$
- ii. Suppose $\frac{\rho(\omega_2 - \omega_1)}{\rho - 1} > 0$ so that $\frac{q_1}{q_2}$ is increasing in w then one and only one of the following must hold:
 - a. $\rho < 0$ and $\omega_2 > \omega_1$
 - b. $\rho \in (0, 1)$ and $\omega_2 < \omega_1$

Proof. See Appendix C.1. □

The proof of Proposition 5 is fairly trivial. The main takeaways are as follows. If $\rho < 0$, so that final goods are complements, then consumption and time use shift toward the more time-intensive task as w increases. If $\rho \in (0, 1)$, so that final goods are substitutes, then consumption and time use shift toward the less time-intensive task as w increases. Consider case (i) primarily, so that relative market purchases of q_1 to q_2 fall as wages rise. If w is rising and final goods are complements, so that $\rho < 0$, then consumers scale up purchases of q_2 at a faster rate than q_1 since they can take advantage of q_1 's relatively higher factor intensity, ω_1 .¹⁵ They thus want relatively *more* of the factor input associated with the relatively more time-intensive process — in this case q_2 . n_2 would increase relative to n_1 as well in this case. For case (ii.a.) the same phenomenon occurs, except q_1 is relatively more time-intensive, so relative consumption of q_1 to q_2 increases. Returning to case (i), suppose now $\rho \in (0, 1)$, so that final goods are substitutes. Then consumers scale up purchases of the good associated with the *less* time-intensive process faster — in this case, q_2 . Put another way, they want more of the good associated with the more goods-intensive process. In case (ii.b.) the argument remains the same except consumers want relatively more of q_1 and n_1 as w rises.

¹⁵Note that q_1 is indeed a “good,” so consumption is increasing in w just more slowly than that of q_2 .

Let us place this into anecdotal context. Consider the tradeoff faced by a consumer who is choosing whether to buy more cleaning supplies in order to clean his house or pay someone to do it for him. The former activity is more time-intensive than the latter which is more dependent on market resources — the services provided by the hired maid. Suppose the consumer receives a wage increase. If $\rho < 0$ so that the outputs from these activities are complements, consumers will choose to purchase more cleaning supplies rather than more cleaning services as a result of a wage increase. On the other hand, if $\rho \in (0, 1)$ so that the outputs from these activities are substitutes, consumers will hire more cleaning services as a result of a wage increase.

We remind the reader that the results here hold under the assumption that prices remain fixed. If prices and real wages are simultaneously changing the value of ρ along with the absolute difference $|\omega_2 - \omega_1|$ will determine whether relative price effects or wage effects dominate. Indeed, we find in our quantitative exercises that $\rho < 0$ and services are less time-intensive, yet the wage effect plays only a minor role driving long-run structural change compared to relative price changes.

2.3.3 Labor Supply Dependencies on Relative Price Changes

In this section we relax the assumption that labor supply is fixed to analyze how consumers adjust their labor supply as a response to changes in the relative price of market goods. Note that in our model there are multiple forces weighing on equilibrium labor supply decisions. Given consumers have multiple choices with respect to how to spend their off-market time, each of which are complimentary with a separate market purchase commodity, changes in the prices of market purchases can impact both the equilibrium distribution of off-market time and labor supply on the intensive margin. These tradeoffs will depend on the underlying time intensities of home production processes, ω_1 and ω_2 , as well as the gross substitutability (or gross complementarity) of final consumption ρ . As before, assume effective cash on hand \bar{y} is fixed and $\theta_1 = \theta_2$. We show in Proposition 6 that if consumers are adjusting their labor supply in response to price changes then the underlying labor intensities of the two home production processes must be different.

Proposition 6. Fix $z_1 = z_2 = 1$. If household labor supply is non-constant in prices, i.e. $\frac{\partial l}{\partial P_i} \neq 0$ for all $i \in \{1, 2\}$, then $\omega_1 \neq \omega_2$.

Proof. See Appendix C.1. □

Note that Proposition 6 *does not* say that l is non-constant in prices if and only if the input elasticities are equal. In fact, when $\omega_1 \neq \omega_2$, the relationship between labor and prices can be non-monotonic for certain parameter combinations.

To illustrate how the intensive margin of labor l depends on prices P_1 and P_2 , input elasticities ω_1 and ω_2 , and the gross substitutability of final consumption ρ , we derive the equilibrium labor supply function $l(P_1, P_2, \omega_1, \omega_2, w)$ and plot it for different values of relative prices P_1/P_2 under different parameterizations.¹⁶ We allow P_1/P_2 to vary from 0.1 to 10, which is accomplished by

¹⁶A detailed derivation of $l(P_1, P_2, \omega_1, \omega_2, w)$ can be found in Appendix C.2.

setting $P_2 = 1$ and varying P_1 . We also fix $w = 1$ and $\bar{n} = 24$, while letting $\bar{y} = w\bar{n} + \text{save}$ where $\text{save} = 0.1 \bar{y}$, giving a rounded value for cash on hand of 26.66667. We choose several combinations of ω_1 , ω_2 , and ρ presenting l and n_2 supply and demand functions side by side below in four different figures.

Interactions between home production time intensities governed by ω_1 and ω_2 and gross substitutability governed by ρ lead to some non-monotonic relationships in prices for labor and off-market time use functions. We begin our discussion focussing on cases where $\rho \in (0, 1)$ so that the final outputs of home production are substitutes. Figure 5 demonstrates that if the market price associated with the market input for the less time intensive process increases, and home production outputs are substitutes, then off-market time dramatically shifts toward the less time intensive process and labor supply falls just as dramatically. Yet this phenomenon does not appear to be symmetric. When the price of the market commodity associated with the more time intensive process increases, consumers do not substitute time away from this process as quickly, in fact increasing time devoted toward this process if the final outputs of home production are only mildly substitutable. This can be seen by noting in Figure 6b that n_2 is non-monotonic in changes to P_1 for $0 < \rho \leq 0.6$, so that if P_1 is big enough n_2 and l both fall and consumers spend more time on n_1 . Note that q_1 is falling in P_1 , but when $0 < \rho \leq 0.6$ it appears that home production time use and market purchase complementarities dominate final commodity substitution effects. As $\rho \rightarrow 1$ the substitution effect over final consumption becomes stronger until it is strong enough to induce increases in n_2 and thus corresponding decreases in both n_1 (and q_1), which is evident via the purple line in Figure 6b.

In Figures 7 and 8, we consider supply of l and off-market time allocation when the outputs of home production are complements, $\rho < 0$. Notice that price sensitivity is the same for l as when $\rho \in (0, 1)$: in Figure 7a, l is sensitive to price changes affecting the market commodity associated with the less time intensive process just as in Figure 5a. However, when $\rho \in (0, 1)$ labor supply generally declines in price increases since consumers must allocate more time to the time intensive process ($i = 2$) to make up for relative increases in q_2 to q_1 , while when $\rho < 0$ labor supply increases as P_1 rises since consumers need to allocate more market resources toward q_1 due to both the gross complementarities and home production complementarities. Further, in Figure 7a notice that n_2 falls faster than l rises when P_1 increases past $P_1 \approx P_2 = 1$. This demonstrates that for high enough relative prices the increase in P_1 leads to both increases in labor to fund more expensive market purchases and increases in n_1 relative to n_2 , consistent with Proposition 4. Turning to the case where $i = 1$ is more time intensive, labor supply is more insensitive to variation in P_1 than when $i = 2$ is more time intensive, as is seen in Figure 8a. This is similar to the flatness of household labor supply when $\rho \in (0, 1)$ observed in Figure 6a.

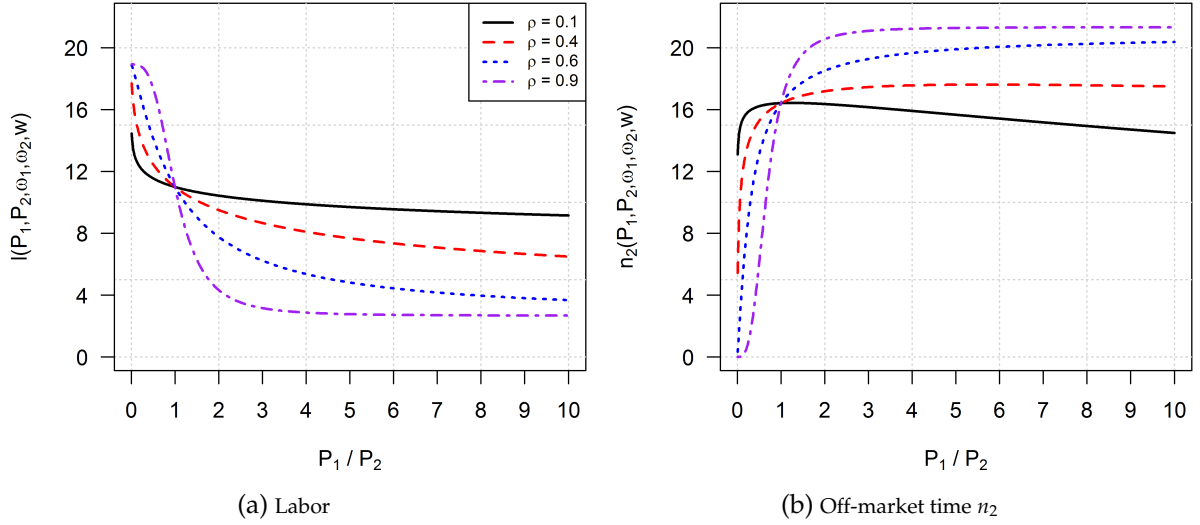


Figure 5: ($0 < \rho < 1$ and $\omega_1 > \omega_2$) — From left to right, we present labor supply l and the off-market time use for final consumption activity $i = 2$, n_2 . We set $\omega_1 = 0.8$ and $\omega_2 = 0.2$, so that process $i = 2$ is more time intensive. Notice that increasing P_1 relative to numeraire P_2 causes l to fall. As $\rho \rightarrow 1$, the n_2 policy function transitions from being non-monotonic to strictly increasing in P_1/P_2 , so that as home production outputs become more substitutable, the strength of this substitutability dominates complementarities between market purchases and time use.

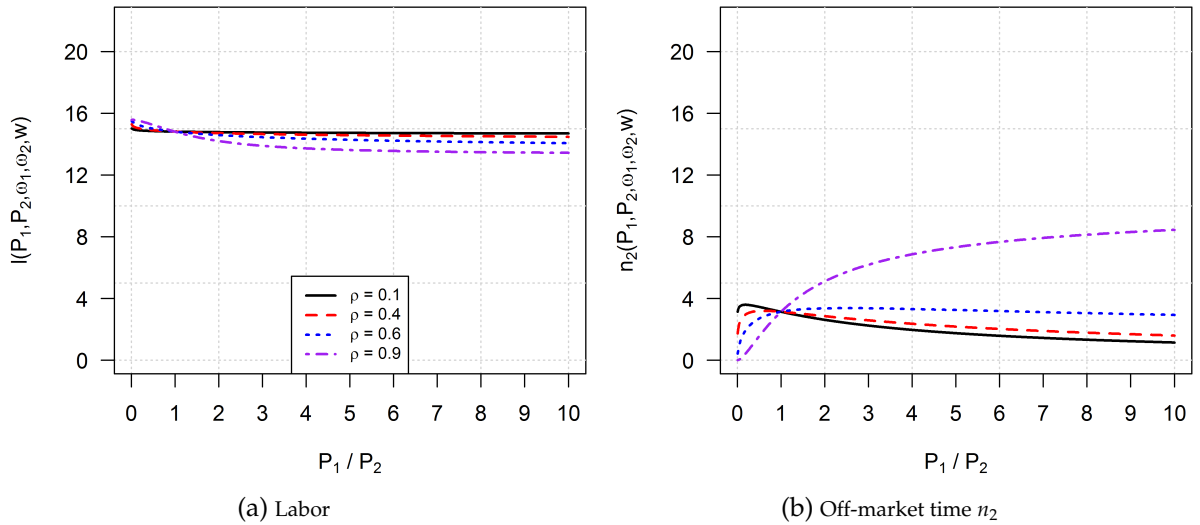


Figure 6: ($0 < \rho < 1$ and $\omega_1 < \omega_2$) — This time we set $\omega_1 = 0.4$ and $\omega_2 = 0.6$, so that process $i = 1$ is more time intensive. As $\rho \rightarrow 1$, approaching linear preferences for final consumption, n_2 begins to increase in P_1/P_2 to the point where for $\rho = 0.9$, the n_2 policy function appears monotonic in P_1/P_2 . For big enough ρ consumers substitute toward the less time-intensive process, though if ρ is small enough they compensate for declines in q_1 by spending more time on process $i = 1$.

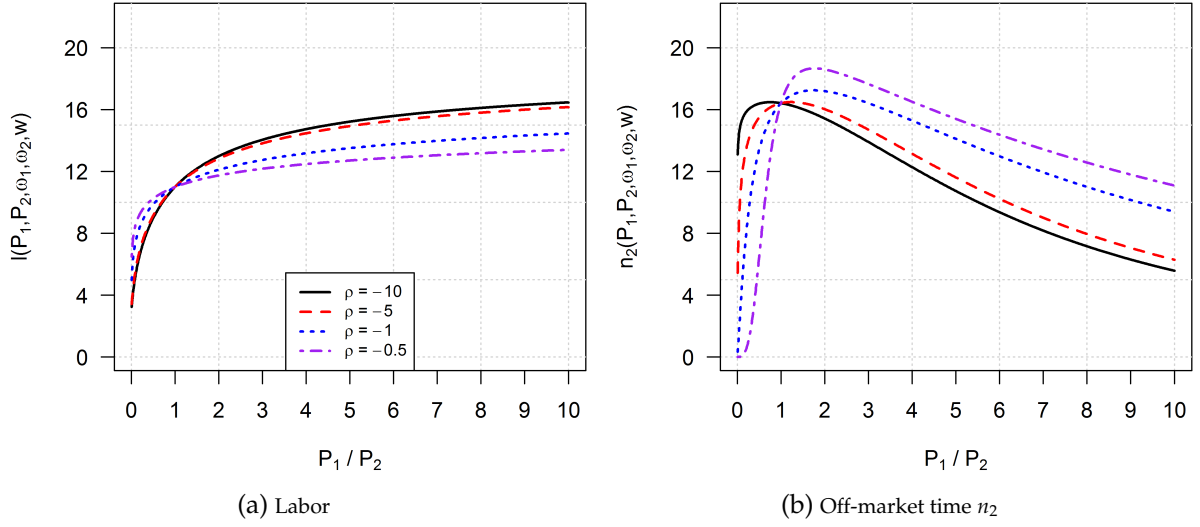


Figure 7: ($\rho < 0$ and $\omega_1 > \omega_2$) — Now final activities are complements. As in Figure 5 we set $\omega_1 = 0.8$ and $\omega_2 = 0.2$, so that process $i = 2$ is more time intensive. Labor varies in P_1/P_2 in the opposite way than when $\rho \in (0, 1)$. Further, n_2 declines in P_1/P_2 faster than l rises, so that the remaining time moves to process $i = 1$, the task for which the market input experienced a price increase, consistent with Proposition 4.

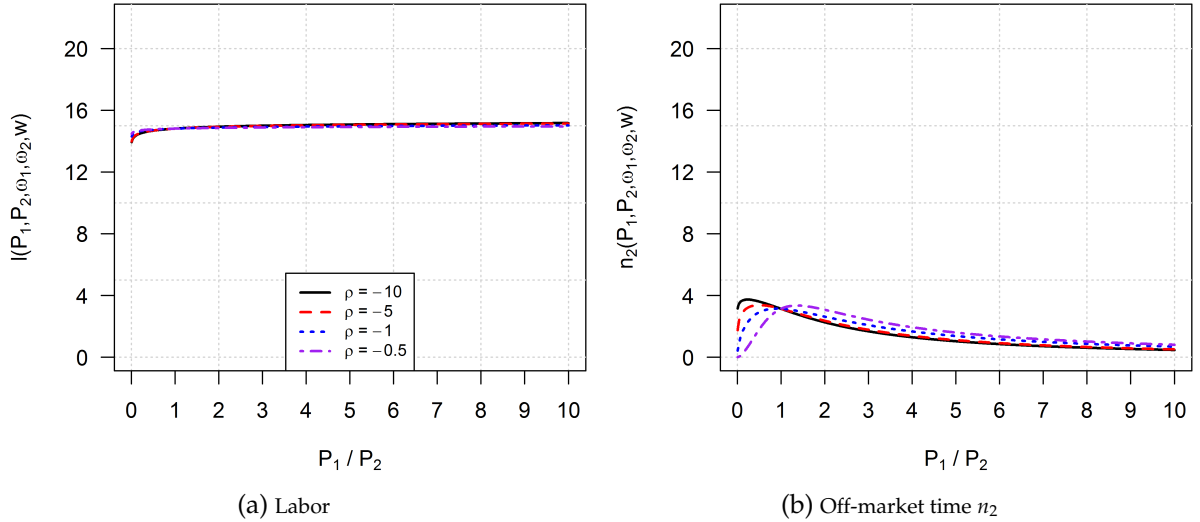


Figure 8: ($\rho < 0$ and $\omega_1 < \omega_2$) — As in Figure 6 we set $\omega_1 = 0.4$ and $\omega_2 = 0.6$, so that now process $i = 1$ is more time intensive. Labor supply is flat in prices again and n_2 exhibits very small variation, similar to when $\rho \in (0, 1)$. Notice that, though when $\rho \in (0, 1)$ there appears to be a limiting condition in which increases in P_1 lead to changes in the behavior of n_2 , this does not occur when $\rho < 0$.

3 Quantitative Applications

To understand the empirical implications of our theoretical results, we engage in a quantitative exercise using the model to estimate the degree to which relative price or wage changes have contributed to the long-run increase in the services share of U.S. consumption expenditure. Prior to engaging in this empirical exercise, we first examine several long-run trends in U.S. consumption expenditure. We then define the estimating equations and establish the assumptions needed to identify whether the final, household activities associated with goods and services are substitutes or complements and which processes are more time-intensive. Of note, we show that under certain assumptions regarding the relative household productivities of goods to services, our equilibrium relative demand representation in Proposition 2 admits aggregation over households. Under these aggregation assumptions, we estimate the model using a Cochrane-Orcutt procedure on linear, reduced-form estimating equations. With our estimated coefficients, we engage in counterfactual exercises fixing relative prices and wages separately in order to understand which channel has most affected long-run structural change.

3.1 Empirical Regularities

Our quantitative exercises operate on several well-established long-run trends in U.S. economic activity from 1948-2018: the decline in the aggregate nominal consumption value of goods to services X_{gt}/X_{st} , changes in the aggregate relative quantity indices of goods and services $\tilde{Q}_{gt}/\tilde{Q}_{st}$, and the decline in aggregate relative goods to services prices P_{gt}/P_{st} .^{17,18} Both the signs and magnitudes of these changes depend on the degree to which we account for the presence of consumer durables in the various goods series — X_{gt} , \tilde{Q}_{gt} , and P_{gt} . Since durable flows are a non-trivial part of aggregate goods consumption, the failure to properly account for how consumers use accumulated durables in their everyday activities can lead to different estimates as to what degree wage and relative price effects have contributed to structural change. We contend that different market commodities are associated with different tradeoffs between consumption and off-market time use, and these differences are the fundamental causes of income and relative price effects. Since these tradeoffs result from home production complementarities, we must naturally examine data accounting for the value of the entire stock of durables, not just new durables investment. This is

¹⁷Throughout the quantitative analysis we will distinguish between aggregate quantity indices and actual quantities by denoting quantity indices with a tilde. Actual quantities of goods and services are unknown, so for empirical exercises involving quantity changes, we must use quantity indices.

¹⁸We should note that previous drafts of this paper naively used American Time Use Survey (ATUS) diary data from 2003-2018 to construct separate data series for total off-market time spent using goods and services, with the goal to jointly estimate the relative demand and relative time use equations. Upon closer examination of the structure of the ATUS survey, however, we began to notice some glaring omissions and subsequently removed unreliable estimates that relied on this data. Appendix A.3 provides an explanation of how the survey is constructed and why it appears particularly biased against how consumers use new services. Due to this likely measurement error and lacking any consistent way to correct for the bias against services in ATUS, our estimates are conducted solely using market expenditure data. Recall that the model we build in Section 2 can be conveniently collapsed, with time use variables marginalized out by combining first order conditions, in order to facilitate estimation solely using expenditure data.

because accumulated durable consumption assets, like kitchen appliances for example, contribute to home production output. Assuming the nominal value of the service flows of durables is equal to the aggregate resale value of all durables presently in utilization, the main goods expenditure series we construct will be the sum of non-durable expenditure and the nominal value of all consumer durables. Goods prices will be adjusted to accommodate this new series we construct, the details of which are described further in Appendix A.1.

The degree to which the U.S. has undergone structural change from goods to services dominated consumption depends to an extent on what underlying products and activities actually comprise the goods and services expenditure series. The sensitivity of measures of structural change can be illustrated by separately examining the long run ratios of goods to services expenditure, goods to services chain-weighted 2012 quantities, and goods to services chain-weighted 2012 prices when the goods data series account for consumer durables in varying degrees. These three different aggregate data series are presented in Figure 9, where we plot relative goods to services expenditure, relative quantity indices, and relative price indices separately depending on the degree to which we account for consumer durables. Note that in each of the plots, the services series are taken directly from the National Income and Product Accounts (NIPA) for services consumption expenditure, services chain-weighted 2012 quantity index, and services chain-weighted 2012 price index series. Meanwhile, the goods series in each plot are constructed so as to include the relevant data series associated with non-durable consumption in addition to one of the following: 1) the entire stock of consumer durable assets not including residential housing (solid black line), 2) only investment in new durable assets (dotted red line), 3) no measure of durables at all. Data on the nominal stock of durable assets is taken from the Bureau of Economic Analysis' (BEA) Fixed Asset Tables, while investment in durables comes from NIPA measures of durable consumption expenditure. For details on how our consumption and expenditure data series and their corresponding price and quantity indices are constructed, see Appendix A.1. We follow Bernanke (1985), McGrattan, Rogerson, and Wright (1993), and Gomme, Kydland, and Rupert (2001) in constructing a data series of goods expenditure using the durables stock in order to account for the presence of service flows from consumer durables in households' home production activities. In the model, we will not explicitly separate durables and non-durables, so that households can be thought to be purchasing the service flows of durables on the market, hence the need to adjust aggregate goods prices accordingly. This allows us to avoid having to estimate more than one simultaneous demand equation and generates ready comparisons to the literature examining the forces driving the evolution of the U.S. economy from manufacturing to services domination.

Upon first glance, failure to include durables service flows can lead to biased estimates of the degree to which the value of final consumption in the U.S. has changed over the last half century. In Figure 9a notice that the decline in the nominal value of consumption goods, including the durables stock, relative to services is 64.2% (black line) versus a 75.5% decline when durables are totally left out (dashed blue line). Meanwhile, Figure 9c shows that when the value of the full durables stock is included, relative market-equivalent prices were over 3.5 times higher in 1948

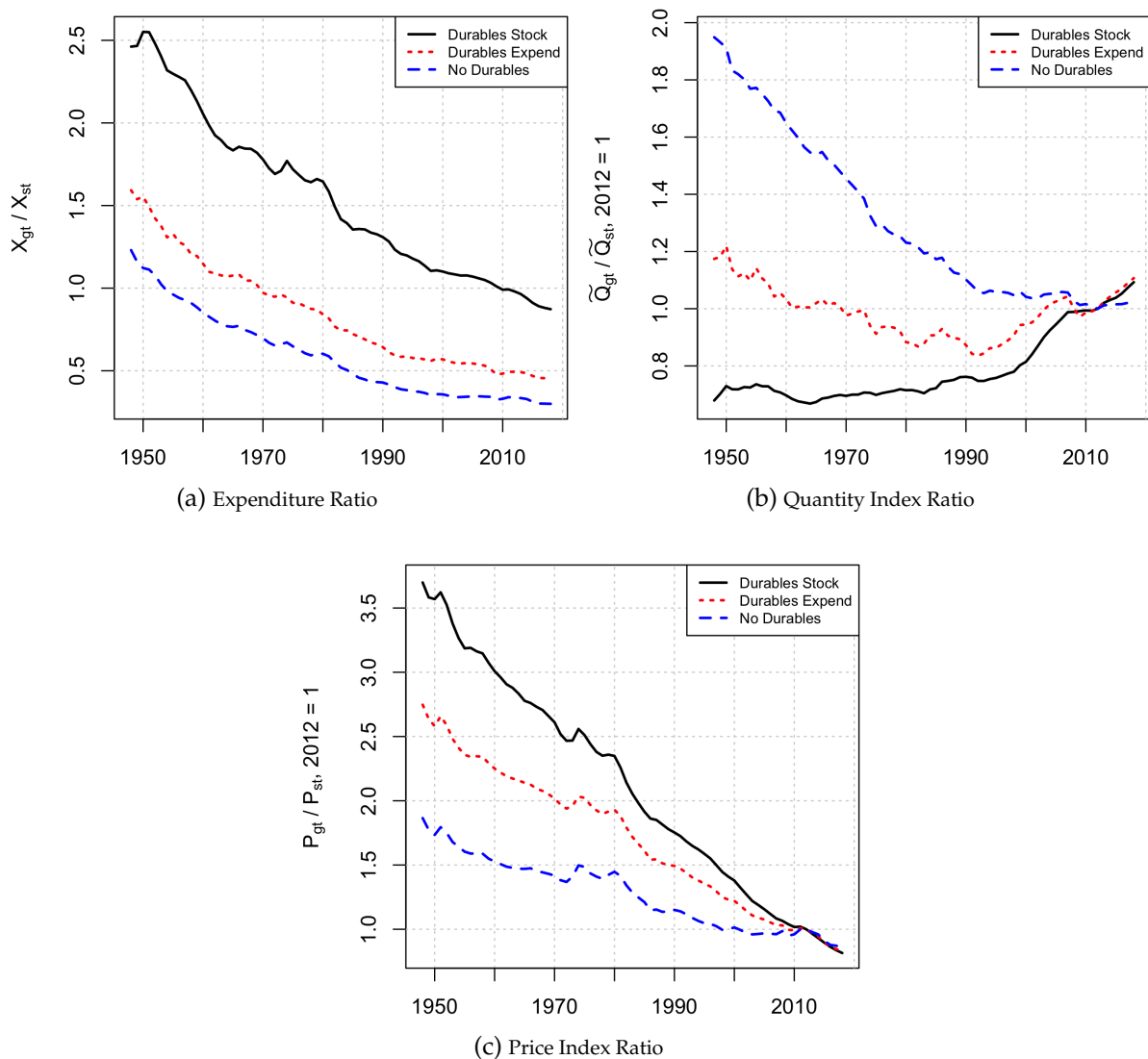


Figure 9: Clockwise from top left, we present the ratio of the aggregate nominal value of final goods to services consumption (a), the ratio of goods to services chain-weighted quantity indices with 2012 = 1 (b), and the relative chain-weighted price of goods to services where 2012 = 1 (c). In each plot, we show three data series each constructed to include different measures of consumer durables. The “Durables Stock” plots (solid black line) include the entire stock of existing consumer durables in the goods series. The “Durables Expend” plots (dotted red line) include only new investment in durables. The “No Durables” plots (dashed blue line) only include non-durables in the goods series. All series are annual, 1948-2018.

compared to 2018, but only 1.8 times higher when leaving out durables. When including the full stock of durables, the decline in the nominal ratio appears at first glance to be driven more by strong relative price declines, since relative 2012 chain-weighted quantities of goods to services actually have increased over the last half century (black line in Figure 9b).

Looking at the dashed blue lines where durables are excluded, note that relative expenditure,

relative quantities, and relative prices are all simultaneously falling, suggesting that something akin to income effects are generally outweighing relative price effects in long run structural transformation, a conclusion consistent with work in Kongsamut, Rebelo, and Xie (2001), Buera and Kaboski (2009), Herrendorf, Rogerson, and Valentinyi (2013), and Boppart (2014). This is because one would expect relative prices to move opposite relative quantities if relative price effects were significant. Either way, even after ignoring the solid black line which includes the full measure of the stock of durables, a natural question to ask is, why might such income effects be driving this trend? If income effects driving structural change do indeed result from consumers substituting market purchases away from time-intensive home production tasks toward services, then the entire stock of consumer durables must be included in any goods data series used for quantitative analysis. This is because consumers use durables frequently in home production activities. Thus, inference on structural change using final expenditure cannot be complete without properly accounting for the value of this stock of assets and the ever-changing ways in which consumers use them.

While different empirical assessments of structural change have found that income effects inherent in consumer preferences are important, the definition of what constitutes an income effect has not been consistently deployed. The model in Boppart (2014) accounts for income effects by estimating the expenditure elasticity of demand for goods simultaneously while instrumenting for household income with a fixed-effects regression using a cross-section of consumption expenditure survey (CEX) data. Thus, Boppart (2014) quantifies a pure, classical income effect that can explain both dynamic and cross-sectional variation in expenditure shares. Herrendorf, Rogerson, and Valentinyi (2013), meanwhile, uses aggregate data to counterfactually simulate the degree to which aggregate expenditure shares change when relative prices evolve as observed, but total expenditure remains constant. Since total consumption expenditure is historically a constant share of aggregate output, the income effect documented in Herrendorf, Rogerson, and Valentinyi (2013) can be read as an *aggregate income* effect. The strength of the income effect in Herrendorf, Rogerson, and Valentinyi (2013) is stronger than that in Boppart (2014), though the latter accounts for possible cross-sectional household preference heterogeneity. Herrendorf, Rogerson, and Valentinyi (2013) affirm the strength of the aggregate income effect by comparing the fit of a homothetic model with a model containing non-homotheticities via Stone-Geary preferences.

Empirically, we examine how changes in relative prices and wages affect long-run expenditure and consumption for goods relative to services. Thus, the “income effect” we estimate is actually a wage effect since we do not account for changes in capital income impacting consumption. Our results and those in the literature are not necessarily comparable one-to-one, since the labor share of income has fallen in the U.S. from approximately 65% in 1948 to just over 58% in 2016.¹⁹ Nonetheless, over the period 1948-2018, aggregate wages equal to the sum of total labor compensation (employees’ hourly wages plus proprietors’ income) divided by total hours worked has increased. This can be seen in Figure 10a, where the separate chain-weighted price deflators

¹⁹See Giandrea and Sprague (2017).

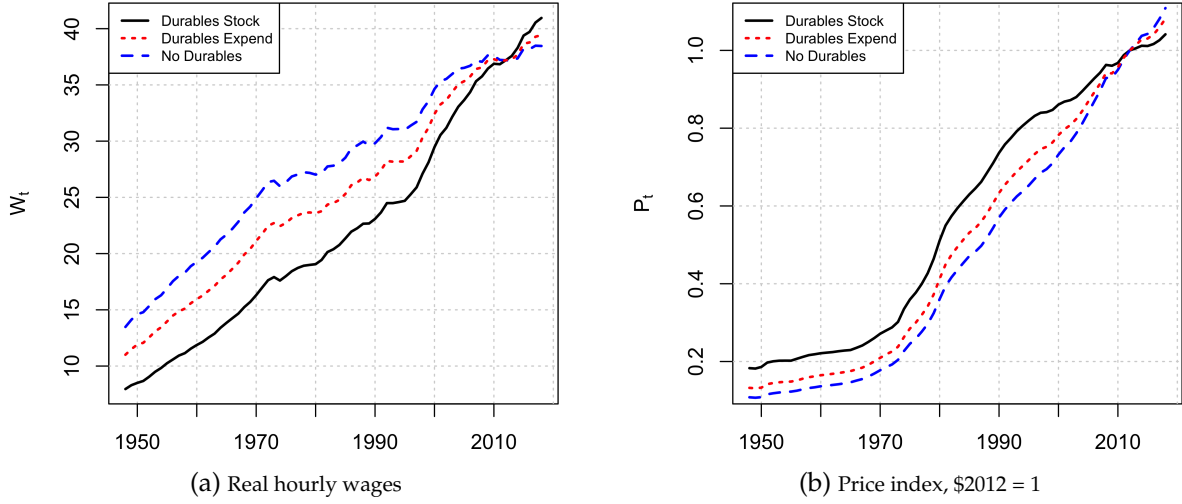


Figure 10: We plot long run real hourly wages W_t in 2012 chained-dollars in panel (a) and the corresponding chain-weighted price index we use to deflate the nominal wage series in panel (b). Notice that when the stock of durable goods is not totally included in the construction of the chain-weighted index (dashed blue line and dotted red line), imputed real wages have grown at a slower rate.

corresponding to the different price indices for the different aggregate baskets of goods are presented alongside in Figure 10b.²⁰ The chain-weighted aggregate price indices, accounting for both changes to the composition of the consumption basket and the overall level of consumption over time, are different, accounting for the changing nominal value of either the durables stock, just durables expenditure, or no durables at all. The differences in real wage measurements are solely due to the differences in these price level measurements. With our model we exploit the convenient log-linear structure of relative demand to examine wage effects on the composition of aggregate consumption along with price changes. Given the decline in the labor share of income, wage effects will not necessarily be correlated one-to-one with income effects, but instead provide a proxy for us to understand how income effects operate in a structural model of consumption expenditure accounting for both durables utilization and time use complementarities with market purchases.

3.2 Identifying Assumptions & Parameter Restrictions

The estimation procedure outlined here operates on household relative demand for market purchases described in Proposition 2 for the U.S. economy. Note that, given we are in possession of chain-weighted quantity indices normalized so that 2012 = 1, direct estimation of relative demand in (23) would yield estimates of β_{0i} that depend on the base year chosen for the construction of \tilde{Q}_{jt} , for all j . Instead, we multiply both sides of the relative demand equation in Proposition 2 by the fraction $\frac{P_{jt}}{P_{it}}$ and then take logs of both sides to get a reduced-form equation describing log expen-

²⁰See Appendix A.1 for how chain-weighted aggregate prices are constructed and Appendix A.2 for the specific data series used to construct real aggregate hourly wages.

diture ratios $\ln \left(\frac{x_{jth}}{x_{ith}} \right)$. All of our estimations operate on this expenditure equilibrium condition modified from (23), using Cochrane-Orcutt linear time series estimation when the relative productivity residuals are found to be serially correlated (Cochrane and Orcutt 1949). In Appendix B we also estimate regressions using ratios of chain-weighted quantity indices in order to assess the robustness of our price and wage elasticity estimates, finding no significant deviations from the results we present in Section 3.4. Note that all equations are defined relative to some base commodity group, which we denote $i = 1$. It does not matter how or which commodity is selected: it only changes the interpretation of the relative home productivity residuals. After taking logs we can write

$$\ln \left(\frac{x_{1th}}{x_{ith}} \right) = \beta_{0i} + \beta_1 \ln P_{1t} + \beta_{2i} \ln P_{it} + \beta_{3i} \ln w_{th} + \xi_{ith} \quad (27)$$

$$\text{where } \beta_{0i} = \left(\frac{1}{\rho - 1} \right) \ln \left[\frac{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}}{\theta_1 \omega_1 [(1 - \omega_1)/\omega_1]^{(1-\omega_1)\rho}} \right] \quad (28)$$

$$\beta_1 = \frac{\rho \omega_1}{\rho - 1} \quad (29)$$

$$\beta_{2i} = \frac{\rho \omega_i}{1 - \rho} \quad (30)$$

$$\beta_{3i} = \frac{\rho(\omega_i - \omega_1)}{\rho - 1} \quad (31)$$

$$\xi_{ith} = \left(\frac{\rho}{\rho - 1} \right) \ln \left[\frac{z_{ith}}{z_{1th}} \right] \quad (32)$$

If we have I commodity groups, equilibrium relative choices are exactly describe by $I - 1$ equations of (27). Note that in the event $I > 2$, the possibility of simultaneous equations endogeneity bias occurs, since ξ_{ith} and $\xi_{i'th}$ for $i \neq i'$ and $i, i' > 1$ are each functions of z_{1th} . Thus, in the event a modeler desires to estimate the given system for more than two classes of market commodities, some set of instruments for in-home productivities is required. In Section 3.4 our actual estimation operates on a two commodity partition of U.S. so this problem will be avoided.

For each $i \geq 2$, the relative home productivities ξ_{ith} can be parameterized as evolving according to an AR(1) process $\xi_{ith} = \phi_i \xi_{i,t-1,h} + v_{ith}$ where v_{ith} are *iid* white noise. We will first estimate (27) under varying degrees of reduced-form parameter restrictions to get the time series of structural residuals $\{\xi_{ith}\}_{\forall i,t,h}$. Then, we use these residuals to estimate ϕ_i . In the second stage, we use the estimates of the autoregressive coefficients to difference the data and back out estimates for the reduced-form β 's using the procedure outlined in Kadiyala (1968).

The values of β_1 , β_{2i} , and β_{3i} for all $i \geq 2$ are the elasticities of relative expenditure x_{1th}/x_{ith} and time use n_{1th}/n_{ith} with respect to P_{1t} , P_{it} , and w_{th} . Since we multiplied both sides of (23) by the price ratio, it follows that the elasticities of relative real quantities demanded with respect to P_{1t} and P_{it} are $\beta_1 - 1$ and $\beta_{2i} + 1$ respectively. All of these elasticities depend on the underlying elasticity of substitution between final goods $\frac{1}{1-\rho}$ and the market consumption and time use factor intensities in home production ω_i . In Proposition 7, we specify restrictions on these values which

are a result of the model structure and confirm classical economic intuition regarding the relationships between demand and price. Note that if $\rho = 0$, so that utility over final consumption is Cobb-Douglas, then the first order conditions collapse and relative market expenditure and time use are a constant proportion of relative prices. In such a case, relative productivities are irrelevant to equilibrium outcomes, which can be seen by inspecting the relative demand equations in Proposition 2. Further, when $\rho = 0$ relative market expenditure is constant over time, a condition which data and previous work in Boppart (2014) suggest we can safely reject when the commodity-space contains just goods and services. Therefore, we ignore the case where $\rho = 0$ and instead focus on two cases: i) final goods c_{th} are substitutes and ii) final goods c_{th} are complements.

Proposition 7. Assume $\omega_i \in (0, 1)$ for all i and suppose $\rho \neq 0$. For all structurally-valid values of β_1 and β_{2i} , $\beta_{3i} = -\beta_1 - \beta_{2i}$. Further, one of the following reduced-form restrictions must hold:

- i. Home activities are substitutes so that $\rho \in (0, 1)$ and:

$$\begin{aligned}\beta_1 &\in (-\infty, 0) \\ \beta_{2i} &\in (0, \infty) \\ \beta_{3i} &\in (-\infty, \infty)\end{aligned}\tag{33}$$

- ii. Home activities are complements so that $\rho \in (-\infty, 0)$ and:

$$\begin{aligned}\beta_1 &\in (0, 1) \\ \beta_{2i} &\in (-1, 0) \\ \beta_{3i} &\in (-1, 1)\end{aligned}\tag{34}$$

For completeness, $\beta_{0i} \in (-\infty, \infty)$.

Proof. See Appendix C.1. □

The sign restrictions on β_1 thru β_{3i} are natural. Recall from earlier discussion that β_1 is the elasticity of relative expenditure with respect to the base commodity price P_{1t} , β_{2i} is the elasticity with respect to the price of commodity $i \geq 2$, P_{it} , and β_{3i} is the elasticity with respect to wages w_{th} . When the final consumption commodities produced are all substitutes, the relative quantity demanded elasticities $\beta_1 - 1$ and $\beta_{2i} + 1$ adjust as one would expect: a 1% increase in the price of good $i \geq 2$, for example, yields a $> 1\%$ increase in the relative demand of good 1 to good i and vice-versa. When final consumption commodities are complements, price responsiveness is inelastic. The wage elasticity is restricted directly by the values of the price elasticities, the limiting conditions of which explain the corresponding restrictions on β_{3i} . When β_{3i} is negative, two things could be happening. First, if final in-home commodities are substitutes, then wage increases drive consumers to substitute consumption toward market inputs associated with the less time-intensive process, in this example $i \geq 2$, c_{ith} , consistent with Proposition 5. Second,

if final in-home goods are complements, then the production process associated with final good $i \geq 2$ must be more time-intensive, so wage increases induce consumers to buy more market inputs for this process, freeing up their off-market time. When $\beta_{3i} > 0$ the same logic applies, just replace $i \geq 2$ with the commodity featured in the numerator of the left-hand side variables, $i = 1$.

Our estimation operates on reduced-form relative expenditure elasticities because we cannot exactly identify point-wise estimates of the underlying structural parameters ρ , θ_i , and ω_i for any i . Thus, Proposition 7 provides set-identification restrictions that ensure our reduced-form elasticity estimates are consistent with the underlying restrictions on structural parameters. Based on estimates of β_1 and β_{2i} 's for all $i \geq 2$, we can identify whether final consumption activities are substitutes or complements, restricting either $\beta_{2i} \in (-1, 0)$ or $\beta_{2i} \in (0, \infty)$ for every $i \geq 2$. Having identified the sign of ρ from the price elasticities, the sign of the restrictions $\beta_{3i} = -\beta_1 - \beta_{2i}$ identifies the sign of $(\omega_i - \omega_1)$, providing information on which market commodity is associated with a more time-intensive home-production process.

Note that we cannot identify absolute total factor productivities, but ξ_{ith} provides us with an expression to identify the sign of log-relative productivities, allowing inference as to which home production process features greater total factor productivity in any given period. Again, after identifying the sign of ρ from the price elasticities we can identify the sign of the coefficient $\frac{\rho}{\rho-1}$ which is negative if $\rho \in (0, 1)$ and positive otherwise. For example, if $\rho \in (0, 1)$ and $\xi_{ith} < 0$, then we can infer that $\ln \left[\frac{z_{ith}}{z_{1th}} \right] > 0$ and $z_{ith} > z_{1th}$, so returns to process i are higher than those for process $i = 1$ in period t . If $\xi_{ith} < 0$ but $\rho < 0$, then we would conclude process $i = 1$ is associated with relatively higher productivity.

3.3 Conditions for Estimation with Aggregate Data

In Sections 3.4.1 and 3.4.3 our estimation routine operates on aggregate U.S. expenditure plus household durables stock data from the BEA. We must ensure that our relative demand equation aggregates so that we can estimate model parameters on a stand-in household. If the relative demand equation indeed does aggregate then it represents true summation over household equilibrium decision outcomes under the assumption our model preference structure accurately captures and explains the decisions of individual households. This would then ensure that our estimates are not systematically biased due to the endogeneity of prices, under the additional assumption that all households are price takers facing the decision structure we outline. We are not the first to extend a demand representation derived from a non-homothetic preference structure to a representative agent environment by simply showing that summation over all households preserves the structure of select equilibrium conditions of the decision problem: Herrendorf, Rogerson, and Valentinyi (2013) do the same. Proposition 8 outlines a sufficient condition in our model that ensures we can replace x_{ith} with aggregate expenditure X_{it} in (27) for all $i \geq 1$.²¹

Proposition 8. Let W_t be aggregated real labor income per hour worked, and let $\frac{Z_{it}}{Z_{1t}}$ be the rela-

²¹Aggregates, from here on, will be denoted with capital letters.

tive aggregated total factor home productivity. If $\frac{W_t}{w_{th}} = \left(\frac{z_{ith}}{z_{1th}}\right)^{\frac{1}{\omega_i - \omega_1}} / \left(\frac{Z_{it}}{Z_{1t}}\right)^{\frac{1}{\omega_i - \omega_1}}$, then the relative expenditure function admits aggregation.

Proof. See Appendix C.1 □

Proposition 8 requires the assumption that the ratio of household h 's wages relative to the national average is inversely proportional to the ratio of household h 's relative in-home productivity on task i to aggregate in-home productivity on that same task. Let us consider the implications of this assumption for a moment. Suppose household h earns an hourly wage below the national average, so that $w_{th} < W_t$. Further, suppose output elasticities are such that $\omega_i > \omega_1$. Then our assumption implies $\frac{z_{ith}}{z_{1th}} > \frac{Z_{it}}{Z_{1t}}$, so that household h is relatively more productive at process i (or relatively less productive at process $i = 1$) than the national average. Conversely if $\omega_i < \omega_1$ then household h is relatively more productive at process $i = 1$ than the national average. Thus, this assumption says that for more time-intensive tasks, poorer households are relatively less productive than wealthier ones. This assumption is just an extension of a feature of our model wherein the marginal product of off-market time in a home production process is just the household's market wage. Naturally, if the values of off-market and market time are the same then as long as labor and off-market time are efficiently allocated, we should expect richer households to be relatively more efficient at generating final consumption with their off-market time. Note that the assumption underlying Proposition 7 *does not* say that richer households are absolutely *less* productive than poorer ones at less time intensive tasks. Rather, the assumption is just a statement about differences in relatives — that is, relative in-home productivities.

Generally speaking in data, Proposition 8 is difficult to test. Theoretically, if we could estimate reduced-form relative productivities at the household level ξ_{ith} , then we could compute the degree to which aggregation error biases our elasticity estimates and subsequently correct for this bias, by re-running the regression accounting for this error. However, this would require knowledge of the value of durable stocks owned by each household, not just their total consumption expenditure. Lacking this data, we will estimate and correct for the aggregation bias as follows. Define ϵ_{it} as average period t , commodity i aggregation bias. Denoting aggregate reduced-form productivity residuals as Ξ_{it} then empirically we seek to substitute out $w_{th}^{\beta_{3i}} e^{\xi_{ith}}$ with $W_t^{\beta_{3i}} e^{\Xi_{it}} e^{\epsilon_{it}}$ where

$$\epsilon_{it} = \mathbb{E}_{it} \left\{ \beta_{3i} (\ln w_{th} - \ln W_t) + \xi_{ith} - \Xi_{it} \right\} \quad (35)$$

with expectations taken over households. The aggregation-bias corrected relative expenditure equation for estimation is then

$$\ln \left(\frac{X_{1t}}{X_{it}} \right) = \beta_{0i} + \beta_1 \ln P_{1t} + \beta_{2i} \ln P_{it} + \beta_{3i} \mathbb{E}_t \ln w_{th} + \mathbb{E}_{it} \xi_{ith} + \Xi_{it} \quad (36)$$

Since $\mathbb{E}_{it} \xi_{ith}$ and Ξ_{it} cannot be separately identified, we will then treat them as a combined, single residual.

If household-level wage data and appropriate weights can be acquired then equation (36) can be used to check the degree to which long-run parameter estimates using aggregate average wages from NIPA, computed by dividing total nominal labor consumption by total hours, are subject to aggregation bias.²² Note the degree to which our model fails to accurately aggregate consumer preferences depends on the underlying true elasticity of relative expenditure with respect to wages, β_{3i} . If we possessed time series of household-level data that incorporated stocks of durable goods into the goods series, we could estimate this parameter using a household-level fixed effects regression. We could then take estimates of $\hat{\beta}_{3i}$ obtained on household-level data, plug them into an aggregated version of equation (27), compute $\hat{\Xi}_{it}$, then compute an unbiased and consistent estimate of \hat{e}_{it} using estimates of $\hat{\beta}_{3i}$, $\hat{\xi}_{ith}$, and $\hat{\Xi}_{it}$. But we are aware of no good measure of household-level durables stock data to perform such an estimation. Thus, only in possession of cross sections of household level wage data from the BLS, we are left with estimating equation (36) as a robustness check to understand the degree to which aggregation bias may be affecting our parameter estimates and thus our counterfactual conclusions when using a measure of average aggregate wages W_t . The results of this robustness test are discussed in Section 3.4.3.

3.4 Estimation Results

Our estimation results operate on an economy where the commodity space consists simply of goods g and services s . From here on we call our commodity index $i \in \{g, s\}$. We perform two separate estimation exercises, first looking at only long run X_{gt}/X_{st} from 1948-2018, then for robustness regressing 2003-2018 X_{gt}/X_{st} both with and without correcting for aggregation bias. The results generally reveal several things. We estimate that final commodities consumed, C_{gt} and C_{st} , are complements, i.e. $\rho < 0$. Parameter estimates and counterfactual simulations demonstrate that long run structural change in relative demand is driven mostly by changes in relative prices, not aggregate wage effects. In fact, when the restriction $\beta_3 = -\beta_1 - \beta_2$ is enforced and either the stock of durables or durables investment is included in the goods series, wage growth has very little impact on changes to relative demand. Third, when we estimate the model on a shortened time series featuring 2003-2018 consumption expenditure data and correct for aggregation bias in our measure of aggregate average wages, we find that our results are robust, though slight aggregation bias exists. Nonetheless, we cannot reject null hypotheses that coefficient estimates before and after correcting for aggregation bias are the same, leading us to conclude our estimates are not significantly affected by our aggregation assumptions.

3.4.1 Long Run Structural Change: 1948-2018, X_{gt}/X_{st}

Our first exercise involves estimating (27) for U.S. final consumption expenditure data from 1948-2018 in order to understand how real wage increases and relative price changes have contributed to the long run structural transformation of the composition of U.S. consumption from goods

²²For how we construct aggregate W_t , see Appendix A.2.

dominated to services dominated. To accomplish this, we first estimate reduced-form parameters and assess whether or not these estimates satisfy the restrictions imposed by Proposition 7. The estimations can be divided into three camps, each dependent on how consumer durables are incorporated into the goods time series. First, the “Durables Stock” models are estimations on data where X_{gt} is the sum of non-durable expenditure and the current dollar value of the entire stock of durable goods. Prices reflect this composition.²³ In the “Durables Expend” estimations X_{gt} corresponds to the sum of non-durable and durable *expenditure*; that is, in this model consumers do not derive productive returns from using the entire stock of durables, just new durable purchases. Finally, in the “No Durables” estimations we take the goods series to just represent non-durable consumption expenditure and prices. For each dataset, we run two separate Cochrane-Orcutt regressions — a regression where β_3 is a free parameter and a restricted one where $\beta_3 = -\beta_1 - \beta_2$ as discussed in Proposition 7.

Second stage reduced-form regression results after correcting for residual autocorrelation are presented in Table 1, along with the results of the AR(1) estimation on first-stage residuals. Coefficient standard errors are in parentheses. For each of the datasets, accounting for durables consumption in varying ways, the unrestricted regression results feature the goods price elasticity $\beta_1 \in (0, 1)$ and the services price elasticity $\beta_2 \in (-1, 0)$ suggesting the elasticity of substitution for the two final home production activities is less than one, and these activities are imperfect complements with $\rho \in (-\infty, 0)$. Note that this does not say that market goods and services themselves are complements in the consumer’s consumption basket, just that the activities associated with their consumption are complements. Similar results are attained in the restricted regressions, though the restricted regression for the “No Durables” dataset features parameter estimates which violate the structural restrictions imposed by Proposition 7. Note that all of the results presented here operate on relative nominal expenditure or the nominal value of final goods utilization when the durables stock is included. For robustness, we performed the same estimates and counterfactual experiments when ratios of quantity indices are left-hand side variables in Appendix B. While the reduced-form parameter restrictions for relative quantity equations are slightly different than those described in Proposition 7, after correcting for this, estimated price and wage elasticities are practically identical to those estimated using nominal ratios.

²³See Appendix A.1 for how chain-weighted price indices for goods are constructed when durables are present.

Table 1: Two-Stage Cochrane-Orcutt Regression of (27) with Aggregate $\ln(X_{gt}/X_{st})$, 1948-2018

Parameter	Second Stage Results					
	Durables Stock ^a		Durables Expend ^b		No Durables ^c	
	(UR) ^d	(R) ^e	(UR) ^d	(R) ^e	(UR) ^d	(R) ^e
$\beta_0^\#$	0.456 (0.321)	-0.204 (0.183)	0.502 (0.380)	-1.725 (0.232)	0.230 (0.247)	-2.086 (0.140)
β_1	0.436 (0.086)	0.661 (0.045)	0.149 (0.104)	0.659 (0.086)	0.712 (0.062)	1.114 (0.069)
$\beta_2^\#$	-0.501 (0.082)	-0.705 (0.041)	-0.386 (0.092)	-0.938 (0.051)	-0.899 (0.062)	-1.399 (0.055)
$\beta_3^\#$	-0.134 (0.089)		-0.334 (0.106)		-0.359 (0.069)	
Observations	71	71	71	71	71	71
R ²	0.959	0.900	0.952	0.880	0.988	0.962
Residual Std. Error	0.013 (df = 67)	0.013 (df = 68)	0.018 (df = 67)	0.022 (df = 68)	0.010 (df = 67)	0.015 (df = 68)
F Statistic	390.710 (df = 4; 67)	203.377 (df = 3; 68)	331.770 (df = 4; 67)	166.649 (df = 3; 68)	1,340.827 (df = 4; 67)	579.827 (df = 3; 68)
Parameter	AR(1) on First Stage Residuals					
	Durables Stock ^a		Durables Expend ^b		No Durables ^c	
	(UR) ^d	(R) ^e	(UR) ^d	(R) ^e	(UR) ^d	(R) ^e
ϕ	0.899 (0.053)	0.951 (0.037)	0.851 (0.063)	0.900 (0.053)	0.904 (0.051)	0.920 (0.047)
Observations	70	70	70	70	70	70
Residual Std. Error	0.014	0.017	0.019	0.032	0.011	0.025

[#] Since we model $I = 2$ commodities, we ignore i -subscripts on β_0 , β_2 , and β_3 .

^a Goods expenditure and prices account for value of total durables stock.

^b Goods expenditure and prices only account for value of new durables purchases.

^c No measure of durables is included in goods expenditure or prices.

^d "UR" denotes an unrestricted regression in which β_3 is a free parameter that we estimate.

^e "R" denotes a restricted regression in which we take $\beta_3 = -\beta_1 - \beta_2$ as in Proposition 7.

Under our estimates and the critical assumption that the structural model accurately describes households' decision processes, we find $-\beta_1 > \beta_2$ implying that $\omega_g < \omega_s$. That is, in the home production process associated with using market services the output elasticity for time use is relatively lower than that in the home production process associated with using market goods, so that producing one unit of the final commodity associated with market goods requires relatively more time on the part of the consumer. This makes sense when one compares, for example, the labor intensity of producing a home-cooked meal versus purchasing take out. Spending more

time preparing a meal yields relatively higher returns to meal quality versus spending more time eating out. Rather, to increase meal quality eating out, you often have to go to a fancier restaurant, thus spending more on the market commodity itself, Q_{st} . Sure, you may spend slightly more time waiting for food at a fancier restaurant, but what our results say is that the increase in productive output of that additional unit of time is less than the increase associated with using market services to produce final consumption.

At first glance, our reduced-form parameter estimates lend suspicion to the contention that income effects are the primary cause of long run changes in the composition of household expenditure. In both restricted regressions that feature some measure of durables expenditure, $\beta_3 = -\beta_1 - \beta_2 > 0$. Thus, holding prices fixed, our estimates predict that increases in wages should lead to increases in the relative expenditure of goods to services, the opposite of the trend we observe. Instead, it appears that the negative relationship between relative expenditure and the services price must be driving the downward trend, lending credence to theories that relative price effects are important. Given households are price takers in our model and assuming aggregation under Proposition 8 holds, this suggests that the primary forces driving structural change may be supply-side factors causing relative market price changes independent of consumer demand, such as, for example, sector-biased technical progress (Ngai and Pissarides 2007), sectoral differences in factor shares leading to differential rates of capital accumulation (Acemoglu and Guerrieri 2008), or skill-biased technical change (Buera and Kaboski 2012; Autor and Dorn 2013).

3.4.2 Counterfactual Simulations

We run three separate counterfactual simulations of expenditure ratios. First, we fix relative prices at their 1948 value, then for the latter two exercises we fix wages at their 1948 level and adjust relative prices holding the final value of the entire consumption basket fixed to that observed in 1948.

Our first counterfactual exercise targets the wage effect, independent of relative price changes. We answer the following question: what would relative aggregate expenditure be in 2018 if consumer wages were the same as actually observed in 2018, but relative prices had remained at their 1948 value? To answer this we fix relative prices but still allow for the counterfactual price levels to grow at the same rate as the aggregate chain-weighted price level P_t that we use to deflate wages,²⁴ in order to ensure that the real aggregate consumption value of labor hours W_t remains unchanged. After computing counterfactual price levels, we use our elasticity and productivity estimates to compute counterfactual log expenditure ratios:²⁵

$$\ln \left(\frac{\tilde{X}_{gt}}{\tilde{X}_{st}} \right) = \beta_0 + \beta_1 \ln \tilde{P}_{gt} + \beta_2 \ln \tilde{P}_{st} + \beta_3 \ln W_t + \Xi_t \quad (37)$$

²⁴The details of the construction of P_t are described in Appendix A.1.

²⁵All counterfactual values will feature tildes.

The goal is to assess the percentage difference in ultimate counterfactual ratios $\frac{\tilde{X}_{g,2018}}{\tilde{X}_{s,2018}}$ versus observed ratios $\frac{X_{g,2018}}{X_{s,2018}}$ in order to understand how important relative price changes have been to changes in relative expenditure.²⁶ The simulated series are presented for each dataset in Figure 11.

In our second and third counterfactual exercises, we target the relative price effect independent of wage growth. We answer the opposite question: what would relative aggregate expenditure be in 2018, if relative prices had evolved to their 2018 value but purchasing power, as measured by average wages, remained fixed at 1948 levels? To accomplish this, we fix wages at $\tilde{W}_t = W_{1948}$ effectively zeroing out the income effect, and then adjust relative prices in two ways, running two separate simulations. First, we fix the counterfactual level of goods prices such that $\tilde{P}_{gt} = P_{g,1948}$ and compute counterfactual $\tilde{P}_{st} = P_{g,1948} / \left(\frac{P_{g,2018}}{P_{s,2018}} \right)$. Next we run the same simulation but do the opposite, fixing $\tilde{P}_{st} = P_{s,1948}$ and computing $\tilde{P}_{gt} = P_{s,1948} \left(\frac{P_{g,2018}}{P_{s,2018}} \right)$, so the absolute price level of goods falls relative to its 1948 value. We then simulate

$$\ln \left(\frac{\tilde{X}_{gt}}{\tilde{X}_{st}} \right) = \beta_0 + \beta_1 \ln \tilde{P}_{gt} + \beta_2 \ln \tilde{P}_{st} + \beta_3 \ln \tilde{W}_t + \Xi_t \quad (39)$$

The goal is to again assess the percentage difference in 2018 counterfactual and observed expenditure ratios in order to understand if zeroing out wage growth would cause the relative expenditure time series to look any different. Both of these simulations are presented for each dataset in Figures 12 and 13.

Table 2: Percent Deviations of 2018 Counterfactual Expenditure Ratios from Data

	$100 \left(\frac{\tilde{X}_{g,2018}/\tilde{X}_{s,2018} - X_{g,2018}/X_{s,2018}}{X_{g,2018}/X_{s,2018}} \right)$					
	<u>Durables Stock</u>		<u>Durables Expend</u>		<u>No Durables</u>	
	(UR)	(R)	(UR)	(R)	(UR)	(R)
$\left(\frac{\tilde{P}_{g,2018}}{\tilde{P}_{s,2018}} \right) = \left(\frac{P_{g,1948}}{P_{s,1948}} \right)^a$	104.547	182.380	34.815	153.583	82.128	155.000
$\tilde{W}_{2018} = W_{1948} \text{ \& } \tilde{P}_{s,2018} \text{ Adjusted}^b$	35.332	-2.355	115.424	3.825	107.497	25.303
$\tilde{W}_{2018} = W_{1948} \text{ \& } \tilde{P}_{g,2018} \text{ Adjusted}^c$	49.318	4.351	186.057	45.112	139.670	56.015

^a Zeroing-out relative price changes.

^b Wage effect counterfactual, adjusting services prices only.

^c Wage effect counterfactual, adjusting goods prices only.

²⁶Specifically we compute

$$100 \left(\frac{\tilde{X}_{g,2018}/\tilde{X}_{s,2018} - X_{g,2018}/X_{s,2018}}{X_{g,2018}/X_{s,2018}} \right) \quad (38)$$

In each of Figures 11, 12, and 13 all three long run relative expenditure series (black) are plotted against their counterfactual series under both unrestricted (orange) and restricted (green) parameterizations. Looking first at wage effects in a vacuum when relative price effects are eliminated, Figure 11 shows that we should expect long run nominal ratios to fall at slower rates than observed in the data. If the downward trend in X_{gt}/X_{st} observed since 1948 were driven more by wage growth than relative price changes, we would expect the counterfactual series, in this case, to exhibit greater long run declines, not less. Notice further that the relative price effect appears strongest in Figure 11a when the full nominal value of the stock of durables is included in the goods series. This is exhibited in more detail by comparing the percent differences between 2018 counterfactual nominal ratios and those in the data presented in Table 2. To understand the strength of wage and relative price effects, readers should compare the rows of each column of Table 2 independently. The first row describes predicted 2018 deviations from observed data for counterfactual time series when only wage growth is at play, so that relative prices are fixed. Rows 2 and 3 describe the two different counterfactual simulations targeting relative price effects, by holding wage growth fixed. Notice that in the restricted regression for the “Durables Stock” dataset the 2018 counterfactual nominal goods to services ratio is 182.38% higher than that observed in the data, while wage growth independent of relative price changes appears to play only a minimal role. In Figures 12a and 13a, holding real wages fixed as in rows 2 and 3 at their 1948 value while adjusting \tilde{P}_{st} first then \tilde{P}_{gt} to achieve relative price parity with that observed in 2018 leads to practically no observed counterfactual change in the nominal ratio under the restricted regression when $\beta_3 = -\beta_1 - \beta_2$, accounting for the presence of durables service flows. Again, if wage growth were driving this change, we would expect the counterfactual series in this case to exhibit *less* long run declines. Under our model restrictions, if we do not control for durables service flows, notice that wage effects become slightly stronger, though relative price effects still dominate. However, failing to impose the restriction $\beta_3 = -\beta_1 - \beta_2$ and failing to control for the presence of durable service flows causes wage effects to dominate, as previously documented in Boppart (2014) (see “UR” columns for “Durables Expend” and “No Durables”). Looking at the unrestricted column under “Durables Stock” counterfactuals, notice that the relative price effect still dominates, though it is not as strong. This leads us to conclude that, while imposing our model restrictions do lead to relatively weaker wage effects, failing to account for durables service flows leads to substantially more biased inference. Thus changes in relative prices, not wage growth, appear most responsible for the evolution of the composition of U.S. consumption expenditure from 1948-2018.

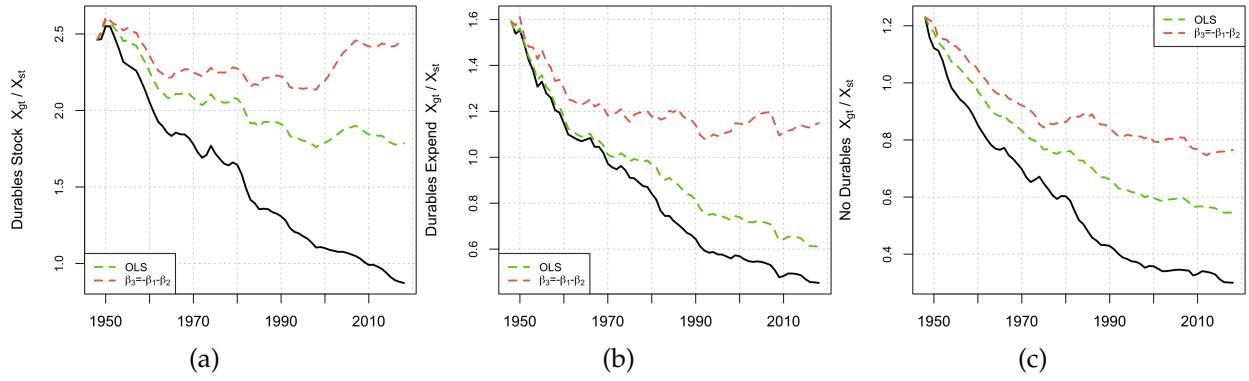


Figure 11: Constant relative prices

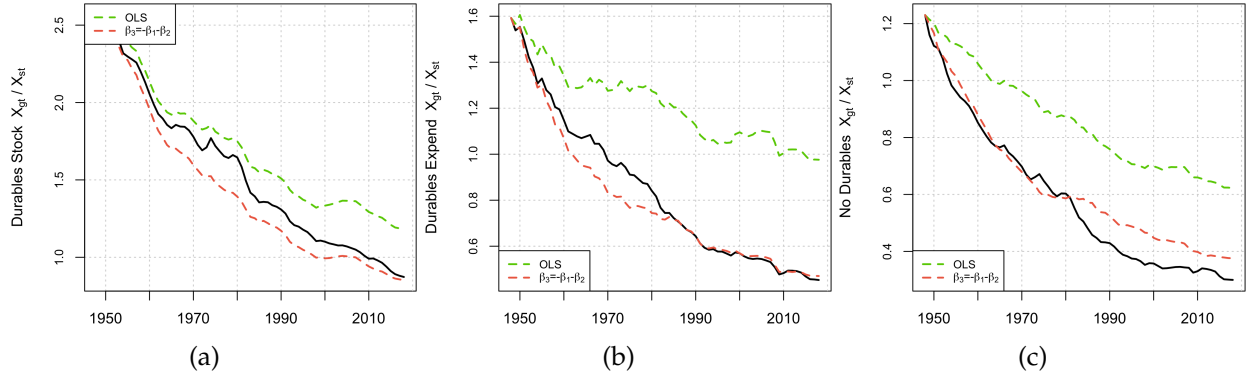


Figure 12: No wage growth, adjusting \tilde{P}_{st}

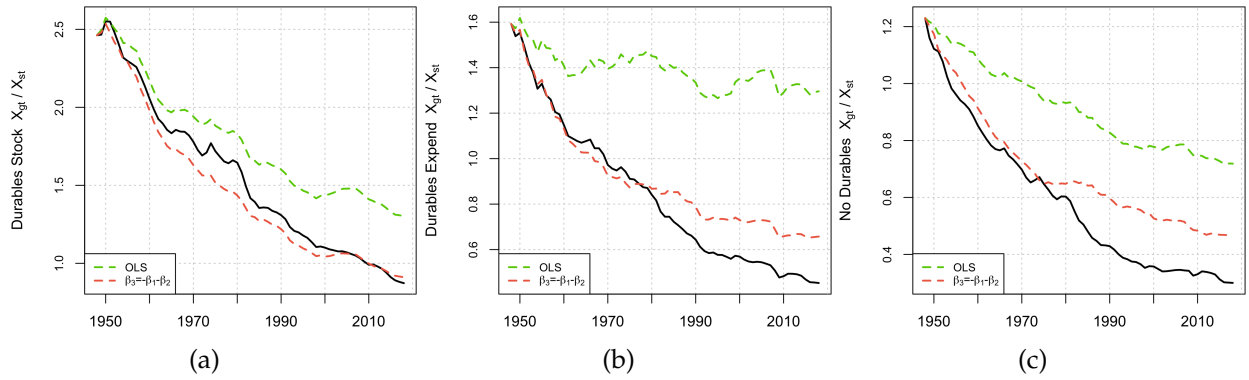


Figure 13: No wage growth, adjusting \tilde{P}_{gt}

3.4.3 Checking For Aggregation Bias: 2003-2018, X_{gt}/X_{st}

To understand the degree to which our results may be subject to aggregation bias, we compared the results of two additional regressions using only 2003-2018 expenditure and price data to be consistent with the length of the ATUS time series. First, we estimated an aggregated version of

(27) using W_t computed as described in Appendix A.2. Then we estimated equation (36) computing $\mathbb{E}_t \ln w_{th}$ as a weighted average of respondent wages from the ATUS. We then compared the price and wage elasticities in both regressions under the null hypothesis that the parameters are the same. The results suggest that our elasticity estimates are robust to potential aggregation bias.

Table 5 shows the absolute Z-scores with corresponding p -values underneath in parentheses for tests on hypotheses that the parameter estimates do not significantly change when correcting for aggregation bias. The regression results without correcting for aggregation bias are presented in Table 3 with the corrected results in Table 4. Note that we cannot reject the null hypotheses that reduced-form price and wage parameter estimates are the same between the two regressions for any conventionally-used level of significance (i.e. 0.10 or lower). This suggests that aggregation bias may not be significantly affecting our results. Further, inspection of the estimated elasticity values shows that aggregation bias correction does not lead to values contradicting our inference on underlying structural parameters. That is, elasticity estimates still suggest $\rho < 0$. Also, estimates of β_3 are more inelastic than the price elasticities, again suggesting that relative price effects dominate the impacts of wage growth. We also examined the sign of ϵ_t over time, noting that for all but the unrestricted “Durables Expend” regression, $\epsilon_t > 0$. Regardless of the specific regression, ϵ_t is also consistently signed over time and stable. Upon inspection of our estimates in Tables 3 and 4, bias correction appears to affect intercept estimates only, systematically shifting the regression surface. This is a product of the fact that for each regression, the bias over the period 2003-2018 is fairly stable, neither growing, declining, nor oscillating. Plots of ϵ_t for the different regression are featured in Figure 14.

Table 3: Two-Stage Cochrane-Orcutt Regressions of (27), 2003-2018

<i>Second Stage Results, $\ln(X_{gt}/X_{st})$</i>						
<i>Parameter</i>	<u>Durables Stock</u>		<u>Durables Expend</u>		<u>No Durables</u>	
	(UR)	(R)	(UR)	(R)	(UR)	(R)
β_0	-1.027 (1.230)	0.665 (0.839)	5.796 (2.909)	-0.740 (0.911)	0.031 (1.554)	-2.362 (0.409)
β_1	0.922 (0.181)	0.622 (0.076)	-0.012 (0.226)	0.660 (0.196)	0.757 (0.101)	0.872 (0.088)
β_2	-0.706 (0.215)	-0.433 (0.163)	0.076 (0.320)	-0.675 (0.125)	-0.731 (0.324)	-1.229 (0.109)
β_3	0.281 (0.340)		-1.797 (0.804)		-0.305 (0.430)	
Observations	16	16	16	16	16	16
R ²	0.974	0.966	0.928	0.997	1.000	1.000
Residual Std. Error	0.008 (df = 12)	0.008 (df = 13)	0.020 (df = 12)	0.018 (df = 13)	0.009 (df = 12)	0.010 (df = 13)
F Statistic	114.433 (df = 4; 12)	123.738 (df = 3; 13)	51.585 (df = 4; 12)	1,493.415 (df = 3; 13)	21,919.690 (df = 4; 12)	20,498.610 (df = 3; 13)
<i>AR(1) on First Stage Residuals, $\ln(X_{gt}/X_{st})$</i>						
<i>Parameter</i>	<u>Durables Stock</u>		<u>Durables Expend</u>		<u>No Durables</u>	
	(UR)	(R)	(UR) [#]	(R)	(UR)	(R)
ϕ	0.523 (0.228)	0.546 (0.224)	— —	0.604 (0.213)	0.401 (0.245)	0.488 (0.233)
Observations	15	15	15	15	15	15
Residual Std. Error	0.008	0.009	—	0.019	0.009	0.010

[#] No autocorrelation detected.

Table 4: Two-Stage Cochrane-Orcutt Regressions of (36), 2003-2018 w/ Bias Correction

<i>Second Stage Results, $\ln(X_{gt}/X_{st})$</i>						
<i>Parameter</i>	<u>Durables Stock</u>		<u>Durables Expend</u>		<u>No Durables</u>	
	(UR)	(R)	(UR)	(R)	(UR)	(R)
β_0	0.207 (0.283)	0.358 (0.204)	1.213 (0.727)	0.346 (0.917)	-0.210 (0.332)	-1.386 (0.105)
β_1	0.767 (0.193)	0.626 (0.041)	-0.241 (0.261)	0.715 (0.110)	0.586 (0.089)	0.893 (0.088)
β_2	-0.495 (0.057)	-0.482 (0.053)	-0.349 (0.121)	-0.551 (0.145)	-0.808 (0.071)	-1.016 (0.081)
β_3	-0.084 (0.109)		-0.743 (0.282)		-0.335 (0.128)	
Observations	16	16	16	16	16	16
R ²	0.974	0.974	0.936	0.996	0.977	1.000
Residual Std. Error	0.008 (df = 12)	0.008 (df = 13)	0.019 (df = 12)	0.017 (df = 13)	0.009 (df = 12)	0.011 (df = 13)
F Statistic	113.598 (df = 4; 12)	163.072 (df = 3; 13)	58.024 (df = 4; 12)	1,168.760 (df = 3; 13)	169.132 (df = 4; 12)	22,368.830 (df = 3; 13)
<i>AR(1) on First Stage Residuals, $\ln(X_{gt}/X_{st})$</i>						
<i>Parameter</i>	<u>Durables Stock</u>		<u>Durables Expend</u>		<u>No Durables</u>	
	(UR)	(R)	(UR) [#]	(R)	(UR) [#]	(R)
ϕ	0.523 (0.228)	0.503 (0.231)	— —	0.683 (0.195)	— —	0.412 (0.243)
Observations	15	15	15	15	15	15
Residual Std. Error	0.008	0.008	—	0.018	—	0.009

[#] No autocorrelation detected.

Table 5: Hypothesis Tests on Elasticity Differences

Parameter	$ Z $ scores (p -values)					
	Durables Stock		Durables Expend		No Durables	
	(UR)	(R)	(UR)	(R)	(UR)	(R)
β_1	0.586 (0.279)	0.051 (0.480)	0.664 (0.253)	0.248 (0.402)	1.272 (0.102)	0.171 (0.432)
β_2	0.948 (0.172)	0.286 (0.387)	1.242 (0.107)	0.646 (0.259)	0.232 (0.408)	1.575 (0.058)
β_3	1.023 (0.153)		1.237 (0.108)		0.067 (0.473)	

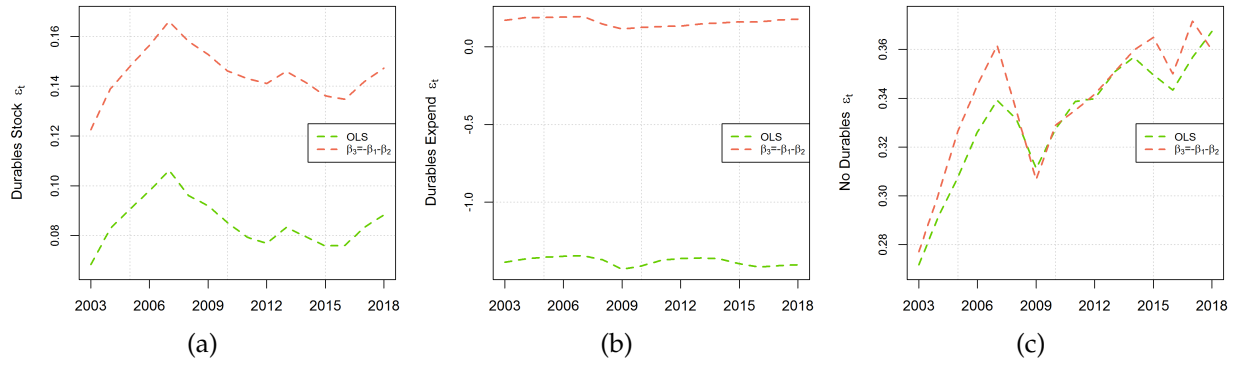


Figure 14: Aggregation error.

4 Conclusion

Whether you are talking about boats, beds, or restaurant meals, households cannot derive utility from a market good unless they can allocate time to consume it. This idea is consistent with early home production models. In this paper, we formalize the concept by allowing households to explicitly choose time to spend consuming individual market goods. We use the resulting estimation of the structural model to understand how accounting for home production affects inference as to the causes of the structural transformation of market demand in the U.S. economy. We show that after controlling for both how consumers derive utility from the service flows of durable goods and how consumers spend their off-market time in home production activities, relative price effects appear to dominate the impact of long run wage growth, calling into question some results in the literature suggesting income effects are strongest.

Future work on macroeconomic trends resulting from household preferences should be careful to consider the importance of home production motivations. Further, since households value and derive utility from the entire stock of durable assets they own, economists should consider incor-

porating the value of consumer durables into consumption series, lest estimation of underlying utility parameters be biased. Finally, we contend that Beckerian home production models, which feature implicit complementarities between time use and market consumption, are behaviorally reasonable. Considering models where off-market time is split into various tasks in order to better understand how household preferences drive various economic phenomena could lead to future results that call into question other long-held conclusions in macroeconomics.

A Data Appendix

In this section of the appendix, we both describe the procedures used to construct the datasets featured in our analyses and discuss why time use data from the American Time Use Survey (ATUS) may be biased against how consumers use new services. Any additional questions a reader may have that are unanswered in this appendix should be referred to the corresponding author. Dataset construction and estimation code in Stata and R are available upon request.

A.1 Consumption Expenditure Data Appendix

Consumption expenditure data series are taken from the Bureau of Economic Analysis' (BEA) National Income and Product Account (NIPA) tables. Specifically, we take the non-durable goods and services nominal expenditure series from NIPA Table 1.1.5. To construct real data series, we download the chain type price indices from NIPA Table 1.1.4. and chain type quantity indices from NIPA Table 1.1.3. for non-durable goods and services. Since goods consumption includes durables, in order to account for the fact that consumers enjoy service flows from durable expenditure over more than one period we turn to BEA Fixed Asset Table 1.1. which gives the current dollar value of the nominal capital stock, including consumer durables. BEA Fixed Asset Table 1.2. provides a corresponding quantity index. From each of these, we use only the "Consumer durable goods" series. Since this section describes construction of aggregate expenditure series, household indices h are suppressed.

Now in possession of data series for nominal expenditure of non-durable goods and services, the nominal value of the stock of consumer durables and corresponding price and quantity indices where available, we can construct our aggregate "goods" and "services" consumption series in real chained 2012 dollars. Note that construction of the real services consumption series requires no additional steps beyond a standard deflationary procedure dividing the nominal services expenditure series from Table 1.1.5. by the chain type price index from Table 1.1.4. The units of this series should be read as "the real value of services consumption expenditure in 2012 chained dollars."²⁷ Since "goods" consumption is the sum of non-durable consumption and the consumption of service flows from the net stock of durable assets, we follow the procedure outlined in Online Appendix C of Herrendorf, Rogerson, and Valentinyi (2013) and discussed in Whelan (2002) to construct a measure of real goods consumption in units of 2012 chained dollars. Unfortunately we cannot simply sum expenditure of non-durables and durables and divide this number by the sum of 2012 chain-weighted real consumption since chain-weighted series are generally not additive (Whelan 2000, 2002). Instead we require an aggregate "goods" price index that accounts for changing relative prices of non-durables and the price associated with the stock of all durables. Note that the price index the BEA uses to construct the quantity index associated with the stock of consumer durables from BEA Fixed Asset Table 1.2. is the same 2012 chained dollar index for

²⁷See Whelan (2000, 2002) for further discussion of the units of this series.

durables expenditure presented in NIPA Table 1.1.4.²⁸ Using this durables expenditure price index, we can construct a total “goods” quantity index that accounts for both aggregate non-durable consumption and service flows derived from the entire stock of consumer durables following Herrendorf, Rogerson, and Valentinyi (2013).

Let \tilde{Q}_{gt} be a chain-weighted “goods” quantity index. Let the subscript nd denote non-durables and d durables. Let X_{it} denote current dollar expenditure, P_{it} be a chain-weighted price such that $P_{i,2012} = 100$, and Q_{it} be the real value of consumption in 2012 chained dollars for all $i \in \{nd, d\}$. Note that $Q_{it} = \frac{X_{it}}{P_{it}}$. Set the 2012 base year aggregate quantity index, $\tilde{Q}_{g,2012} = 1$. We compute

$$\tilde{Q}_{gt} = \tilde{Q}_{g,t-1} \sqrt{\frac{(\sum_i P_{i,t-1} Q_{it})(\sum_i X_{it})}{(\sum_i X_{i,t-1})(\sum_i P_{it} Q_{i,t-1})}} \quad \forall t > 2012 \quad \text{where } i \in \{nd, d\} \quad (\text{A.1})$$

$$\tilde{Q}_{gt} = \tilde{Q}_{g,t+1} / \sqrt{\frac{(\sum_i P_{it} Q_{i,t+1})(\sum_i X_{i,t+1})}{(\sum_i X_{it})(\sum_i P_{i,t+1} Q_{it})}} \quad \forall t < 2012 \quad \text{where } i \in \{nd, d\} \quad (\text{A.2})$$

Aggregate goods consumption in chained 2012 dollars is then

$$Q_{gt} = \tilde{Q}_{gt} \sum_{i \in \{nd, d\}} X_{i,2012} \quad (\text{A.3})$$

Finally, the aggregate chain-weighted goods price index with $P_{g,2012} = 1$ is just

$$P_{gt} = \frac{\sum_{i \in \{nd, d\}} X_{it}}{Q_{gt}} \quad (\text{A.4})$$

In addition to a composite goods price index, we also need an aggregate consumption price index that accounts for relative changes in the value of both services and goods over time in order to properly place aggregate wages in the same 2012 chain-weighted units as consumption prices. To construct this series, repeat the above procedure except this time sum over $i \in \{s, nd, d\}$ to get \tilde{Q}_t , an aggregate consumption quantity index in 2012 chain-weighted units. From there, an aggregate consumption price index in 2012 chain-weighted units can be easily derived by first computing real 2012 chain-weighted aggregate consumption $Q_t = \tilde{Q}_t \sum_{i \in \{s, nd, d\}} X_{i,2012}$, then using that value to get an aggregate price index $P_t = \frac{\sum_{i \in \{s, nd, d\}} X_{it}}{Q_t}$.

A.2 Wage Data Appendix

We use two separate sources to construct the aggregate wage series for representative agent models and the separate wage series for different income levels in the heterogeneous agents model. For

²⁸Note that the BEA only presents current dollar value X_{dt} and 2012 chain-weighted quantity indices \tilde{Q}_{dt} for durable stocks in the fixed asset tables, not prices. Nonetheless, it can be confirmed that the price index associated with the stock of durables is the same as that associated with the flow of durables expenditure by performing the following procedure. First, compute the 2012 chain-weighted real value of durables $Q_{dt} = \frac{\tilde{Q}_{dt} X_{d,2012}}{100}$, since $\tilde{Q}_{d,2012} = 100$. Then compute $P_{dt} = \frac{X_{dt}}{Q_{dt}}$ and compare this series to the price index for durables expenditure in NIPA Table 1.1.4. They are the same.

aggregate total nominal wages, we turn to NIPA Table 2.1.1. from the BEA, specifically “Compensation of employees.” We then divide this number by non-seasonally adjusted annual total hours worked by full-time and part-time employees from NIPA Tables 6.9B., 6.9C, and 6.9D., “Hours worked by full-time and part-time employees.” This series is available from 1948-2018, thus we truncate all of our analyses to expenditure and price data from this period only. When we analyze the time use patterns of heterogeneous agents, we take weighted average wages from ATUS conditional upon observing positive labor earnings for households in different income quintiles.

To arrive at a measure of real wages in chained 2012 dollars that is consistent with our measures of consumption expenditure, we construct a chain-weighted aggregate consumption price index with base year 2012 using the procedure as described in Appendix A.1. Let P_t describe the price level of aggregate consumption and let W_t describe real wages. For each year we compute

$$W_t = \left(\frac{\text{Total Nominal Wages}_t}{\text{Total Hours}_t} \right) \frac{1}{P_t} \quad (\text{A.5})$$

This gives a measure of wages in chain-weighted 2012 consumption-equivalent dollars. Figure 10a shows imputed real wages have grown at a slower rate when the value of the stock of consumer durables is not factored into the aggregate price index used to deflate nominal wages. This is because long run inflation appears overstated without accounting for durables which is evident in Figure 10b.

A.3 Measurement Error in Time Use Data

Upon first glance, the structure of time use tasks recorded in the American Time Use Survey (ATUS) diary survey permits a convenient aggregation to goods and services. The problem, however, is that the structure of the survey, specifically the way tasks are coded, biases time use against market services. This is because the survey fails to accurately categorize consumers’ time spent using new services, like smartphone apps and other mobile internet services. First, to understand how one could think that the survey could be used to categorize time spent using goods and services separately, consider the following example. The survey encodes time spent engaged in “Interior cleaning” separately from time spent engaged “Using interior cleaning services.” In the former, presumably the consumer is utilizing soaps, mops, brooms, etc. he has purchased on the market to clean, while in the latter he is supervising a maid. Spending time “Interior cleaning” is thus complementary with purchasing or using stocks of cleaning products (goods). Meanwhile, spending time “Using interior cleaning services” is complementary with paying for a cleaning service. The structure of this survey thus appears, on the surface, to lend itself to neat aggregation in parallel with the way consumption expenditure is aggregated in the NIPA tables. Yet when examining the amount of time consumers spend engaging with interpersonal communication technology and telephones, the survey’s bias against new services reveals itself.

The ATUS runs from 2003-2018 and the time use categorization in the diary survey has not been significantly updated over this time to record the amount of time respondents spend using

internet-based communication technologies. For example, the survey explicitly records how much time a respondent spent on “Telephone calls to/from ...” but features no mechanism to capture how much time consumers spend engaging in mobile communication via texting or social media. The survey does ask respondents how much time they spent engaged in “personal e-mail and messages,” but only for purposes of “household management.” In the classification of time use activities, there is no mention of the word “internet.” “Computer use” features twice in the survey: once as “Computer use for leisure (exc. Games)” and another time as “Computer use” under “Volunteer Activities: Administrative & Support Activities.” The survey does not directly ask consumers how much time they spent browsing the internet or the amount of time they spent engaged with virtual applications on their smartphones. Yet, trends in smartphone utilization suggest that consumers may, on average, now spend more time engaged with media over their smartphones than by watching television.²⁹ To attempt to empirically assess how measures of consumer utilization of communication services derived from ATUS data may be downwardly biased, Figure 15 shows how the measured absolute average time spent using the telephone and engaged in communication activities with others has fallen from an average of approximately 72 minutes per day in 2003 to just less than 65 minutes per day in 2018. Meanwhile, the average U.S. adult spent over 3 hours per day engaged with a mobile electronic device in 2018 according to a leading marketing researcher.³⁰ This discrepancy suggests that there is profound mis-measurement in ATUS as to the amount of time consumers spend engaging with communication services.

B Estimation Results Using Quantity Indices

To test the robustness of our estimation results using expenditure ratios, we perform the same log-linear regressions using chain-weighted aggregate quantity index ratios where $\frac{\tilde{Q}_{g,2012}}{\tilde{Q}_{s,2012}} = 1$. We estimate the following regression for all three datasets using both unrestricted OLS and also enforcing the restriction that $\beta_8 = -\beta_6 - \beta_7$, which is consistent with Proposition 5:

$$\ln \left(\frac{\tilde{Q}_{gt}}{\tilde{Q}_{st}} \right) = \beta_5 + \beta_6 \ln P_{gt} + \beta_7 \ln P_{st} + \beta_8 \ln W_{th} + \Lambda_{th} \quad (\text{B.1})$$

In general $\beta_5 \neq \beta_0$ since the base-year normalization would make β_5 dependent on the initial level of $\frac{\tilde{Q}_{g,2012}}{\tilde{Q}_{s,2012}}$. Thus, theoretically in this regression we cannot identify β_0 . Further, in general $\Lambda_{th} \neq \Xi_{th}$, the aggregate relative home productivity after aggregation of (27). However, the elasticities are identified, and it should be the case that $\beta_6 = \beta_1 - 1$, $\beta_7 = \beta_2 + 1$, and $\beta_8 = \beta_3$. Regression results for (B.1) from 1948-2018 without time use are presented in Table 6. In Table 7 we test the coefficient restrictions on both our long run nominal value and quantity index regressions. Notice that there is no statistically significant difference between the elasticity estimates, suggesting that any counterfactual results we run on quantity index ratios should lead to the same inference as

²⁹See <https://www.emarketer.com/content/average-us-time-spent-with-mobile-in-2019-has-increased>.

³⁰Again, see <https://www.emarketer.com/content/average-us-time-spent-with-mobile-in-2019-has-increased>.



Figure 15: This is a time series of weighted average aggregates of ATUS telephone utilization and communication time. The exact diary variables aggregated here are t120101 and t120301 plus t160101 thru t169989.

counterfactuals on nominal ratios.

Indeed that is what we find. Figures 17a and 18a show that income effects appear to have little leverage on long run quantity ratios under our model-imposed regression restrictions. In Figure 16a long run relative price changes lead to approximately 37.8% higher ratios of goods to services (restricted regression) than if relative prices had remained fixed at their 1948 value. For the case where we look only at non-durable quantities, if relative prices had not fallen, relative consumption would be higher in 2018 by about 18.2% (see Figure 16c). Table 8 is the quantity index ratio equivalent of Table 2. The fact that counterfactual aggregate quantity ratios fall as a result of holding relative prices fixed again shows the significance of the substitution effect, confirming the results in the main text.

Table 6: Two-Stage Cochrane-Orcutt Regression on Aggregate $\ln(\tilde{Q}_{gt}/\tilde{Q}_{st})$, 1948-2018

Parameter	Second Stage Results					
	Durables Stock ^a		Durables Expend ^b		No Durables ^c	
	(UR) ^d	(R) ^e	(UR) ^d	(R) ^e	(UR) ^d	(R) ^e
β_5	0.476 (0.321)	-0.184 (0.183)	1.208 (0.380)	-1.020 (0.232)	1.313 (0.247)	-1.003 (0.140)
β_6	-0.564 (0.086)	-0.339 (0.045)	-0.851 (0.104)	-0.342 (0.086)	-0.288 (0.062)	0.114 (0.069)
β_7	0.499 (0.082)	0.295 (0.041)	0.614 (0.092)	0.062 (0.051)	0.101 (0.062)	-0.399 (0.055)
β_8	-0.141 (0.088)		-0.340 (0.105)		-0.367 (0.068)	
Observations	71	71	71	71	71	71
R ²	0.868	0.690	0.536	0.238	0.944	0.832
Residual Std. Error	0.013 (df = 67)	0.013 (df = 68)	0.018 (df = 67)	0.022 (df = 68)	0.010 (df = 67)	0.015 (df = 68)
F Statistic	109.907 (df = 4; 67)	50.560 (df = 3; 68)	19.381 (df = 4; 67)	7.099 (df = 3; 68)	279.960 (df = 4; 67)	112.475 (df = 3; 68)

Significance levels: $p < 0.1$; $p < 0.05$; $p < 0.01$

^a Goods quantities and prices account for value of total durables stock.

^b Goods quantities and prices only account for value of new durables purchases.

^c No measure of durables is included in goods quantities or prices.

^d "UR" denotes an unrestricted regression in which β_8 is a free parameter that we estimate.

^e "R" denotes a restricted regression in which we take $\beta_8 = -\beta_6 - \beta_7$.

Table 7: Hypothesis Tests on Parameter Restrictions

H_0	$ Z $ scores (p -values)					
	Durables Stock		Durables Expend		No Durables	
	(UR)	(R)	(UR)	(R)	(UR)	(R)
$\beta_6 - \beta_1 + 1 = 0^\#$	0.001 (0.499)	0.002 (0.499)	0.001 (0.500)	0.001 (0.499)	0.002 (0.499)	0.002 (0.499)
$\beta_7 - \beta_2 - 1 = 0$	0.001 (0.500)	0.0004 (0.500)	0.0001 (0.500)	0.001 (0.500)	0.001 (0.500)	0.001 (0.499)
$\beta_8 - \beta_3 = 0$	0.0003 (0.500)		0.001 (0.500)		0.001 (0.500)	

[#] For example, $Z = \frac{\beta_6 - \beta_1 + 1}{\sqrt{SE_{\beta_6}^2 + SE_{\beta_1}^2}}$.

Table 8: Percent Deviations of 2018 Counterfactual Quantity Ratios from Data

	$100 \left(\frac{\tilde{Q}_{g,2018}/\tilde{Q}_{s,2018} - \tilde{Q}_{g,2018}/\tilde{Q}_{s,2018}}{\tilde{Q}_{g,2018}/\tilde{Q}_{s,2018}} \right)$					
	Durables Stock		Durables Expend		No Durables	
	(UR)	(R)	(UR)	(R)	(UR)	(R)
$\left(\frac{\tilde{P}_{g,2018}}{\tilde{P}_{s,2018}} \right) = \left(\frac{P_{g,1948}}{P_{s,1948}} \right)^a$	-54.925	-37.769	-59.310	-23.470	-15.567	18.215
$\tilde{W}_{2018} = W_{1948} \ \& \ \tilde{P}_{s,2018} \text{ Adjusted}^b$	35.337	-2.953	115.414	3.827	107.504	25.308
$\tilde{W}_{2018} = W_{1948} \ \& \ \tilde{P}_{g,2018} \text{ Adjusted}^c$	49.344	4.359	186.072	45.131	139.687	56.027

^a Analyzing the substitution effect by zeroing-out relative price changes.

^b Income effect counterfactual, adjusting services prices only.

^c Income effect counterfactual, adjusting goods prices only.

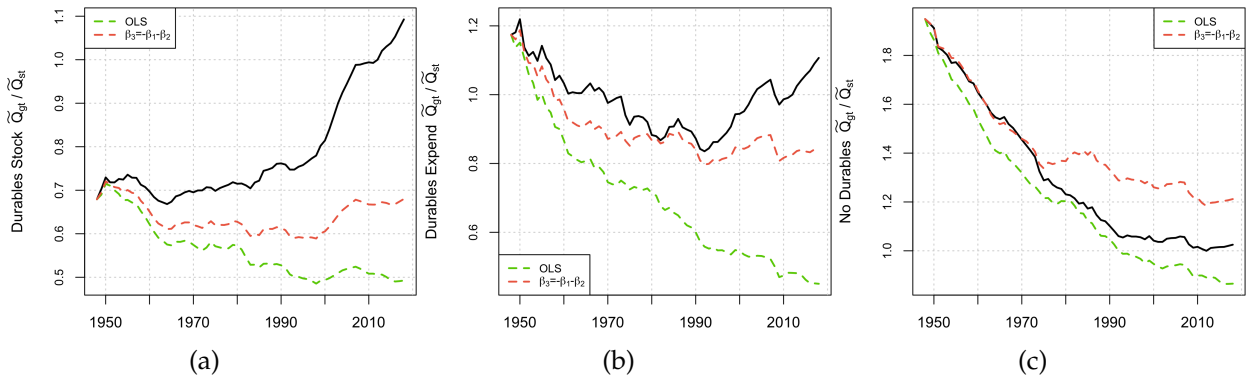


Figure 16: No substitution effect

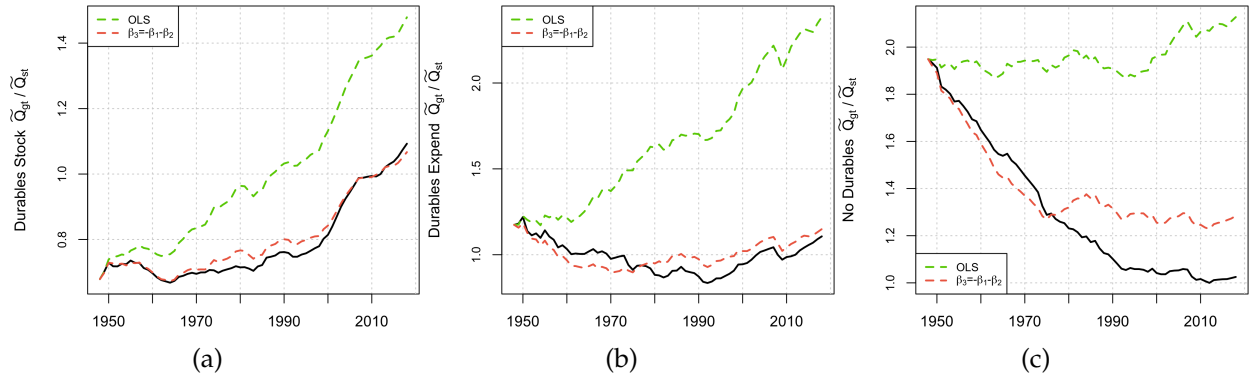


Figure 17: Income effect only, adjusting \tilde{P}_{st}

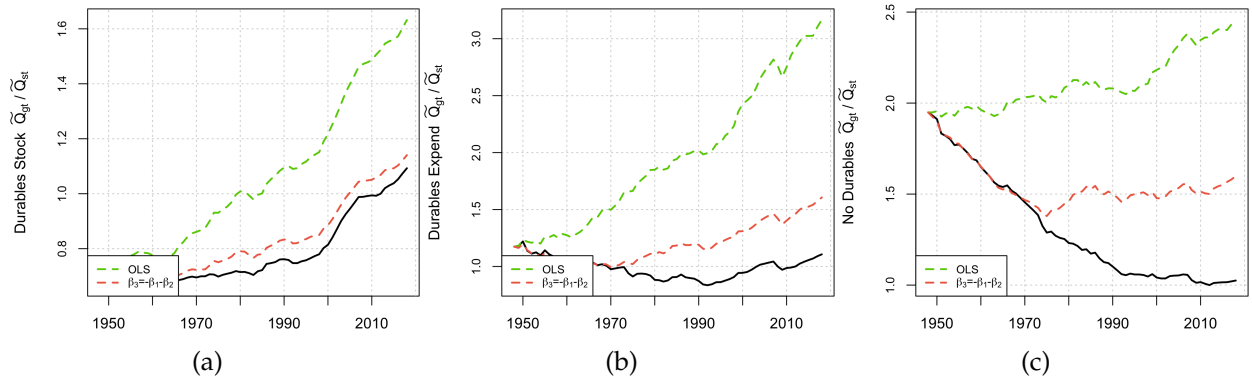


Figure 18: Income effect only, adjusting \tilde{P}_{gt}

C Mathematical Appendix

C.1 Proofs

Lemma 1. Assume each household is a utility maximizer. Under Assumptions 1, 2, and 3, and under Theorem 1 of Green (1964) attributed to Leontief (1947), we can restrict our analysis to

$$\tilde{u}_{th}(q_{1th}, \dots, q_{ith}, \dots, q_{lth}, n_{1th}, \dots, n_{lth}) \quad (\text{C.1})$$

where q_{ith} is some index that describes the grouping of market goods $\{q_{i1th}, \dots, q_{ijith}, \dots, q_{ilth}\}$.

Proof. For notational simplicity, denote the marginal final utility for the consumption of market good q_{ijit} by $MU_t(q_{ijit})$ which is

$$MU_t(q_{ijit}) = \frac{\partial u}{\partial c_{it}} \frac{\partial f_{it}}{\partial q_{ijit}} \quad (\text{C.2})$$

Let j_i and j'_i index two distinct components of q_{it} . Theorem 1 of Green (1964) states that the indices comprising grouped market goods must be constructed so that

$$\frac{\partial}{\partial q_{i'j'_it}} \left(\frac{MU_t(q_{ijit})}{MU_t(q_{ij'_it})} \right) = 0 \quad \forall i' \neq i \quad (\text{C.3})$$

where j'_i indexes a component of $q_{i't}$. Note that under Assumption 3 we can write

$$\left(\frac{MU_t(q_{ijit})}{MU_t(q_{ij'_it})} \right) = \frac{\frac{\partial u}{\partial c_{it}} \frac{\partial f_{it}}{\partial q_{ijit}}}{\frac{\partial u}{\partial c_{it}} \frac{\partial f_{it}}{\partial q_{ij'_it}}} \quad (\text{C.4})$$

$$= \frac{\frac{\partial f_{it}}{\partial q_{ijit}}}{\frac{\partial f_{it}}{\partial q_{ij'_it}}} \quad (\text{C.5})$$

(C.5) only depends on goods in q_{it} . By Assumption 2, (C.3) thus holds. Together under Assumption 1, Theorem 1 of Green (1964) is satisfied. \square

Lemma 2. The shadow price of activities c_{ith} associated with the consumption of q_{ith} is equal exactly to P_{it} if and only if $\frac{\partial \tilde{f}_{ith}}{\partial q_{ith}} = 1$.

Proof. (\Leftarrow) Suppose $\frac{\partial \tilde{f}_{ith}}{\partial q_{ith}} = 1$, then clearly $\frac{\partial u}{\partial c_{ith}} = P_{it} \mu_{th}$ from (11).

(\Rightarrow) By contrapositive, suppose $\frac{\partial \tilde{f}_{ith}}{\partial q_{ith}} \neq 1$. Let \tilde{P}_{it} be a candidate shadow price. We will show this cannot equal P_{it} . Note that

$$\frac{\partial u}{\partial c_{ith}} = \tilde{P}_{it} \mu_{th} \quad (\text{C.6})$$

$$\text{and } \frac{\partial u}{\partial c_{ith}} = \frac{P_{it}}{\frac{\partial \tilde{f}_{ith}}{\partial q_{ith}}} \mu_{th} \quad (\text{C.7})$$

$$\Rightarrow \tilde{P}_{it} = \frac{P_{it}}{\frac{\partial \tilde{f}_{ith}}{\partial q_{ith}}} \neq P_{it} \quad (\text{C.8})$$

□

Proposition 1. For each i , the value added in the production of final good c_{ith} is equal to $w_{ith}n_{ith}$, the market value of time spent on task i .

Proof. While this proof holds for our explicit Cobb-Douglas parameterization of the home production process $\tilde{f}_{ith}(q_{ith}, n_{ith})$, we provide the proof using a general function $f_{ith}(q_{ith}, n_{ith})$ with the following properties: the function is homogeneous of degree one in all arguments, strictly increasing, and strictly concave. From Becker (1965), let \bar{p}_{ith} be market inputs per unit of output c_{ith} and let \bar{w}_{ith} be off-market time per unit of c_{ith} . Then we have

$$q_{ith} = \bar{p}_{ith}c_{ith} \quad (\text{C.9})$$

$$n_{ith} = \bar{w}_{ith}c_{ith} \quad (\text{C.10})$$

which implies that implicit prices can be written

$$\bar{p}_{ith} = \frac{q_{ith}}{c_{ith}} = \frac{\frac{\partial f_{ith}}{\partial q_{ith}} \frac{q_{ith}}{c_{ith}}}{\frac{\partial f_{ith}}{\partial q_{ith}}} \quad (\text{C.11})$$

$$\bar{w}_{ith} = \frac{n_{ith}}{c_{ith}} = \frac{\frac{\partial f_{ith}}{\partial n_{ith}} \frac{n_{ith}}{c_{ith}}}{\frac{\partial f_{ith}}{\partial n_{ith}}} \quad (\text{C.12})$$

Now under the homogeneity of degree one assumption, we can apply Euler's Homogeneous Function Theorem to write:

$$c_{ith} = f_{ith}(q_{ith}, n_{ith}) = \frac{\partial f_{ith}}{\partial q_{ith}} q_{ith} + \frac{\partial f_{ith}}{\partial n_{ith}} n_{ith} \quad (\text{C.13})$$

$$\Rightarrow 1 = \frac{\partial f_{ith}}{\partial q_{ith}} \frac{q_{ith}}{c_{ith}} + \frac{\partial f_{ith}}{\partial n_{ith}} \frac{n_{ith}}{c_{ith}} \quad (\text{C.14})$$

Note that the terms on the right hand side of (C.14) are the output elasticities. Denote this value as ω_{ith} , where in the context of this proof the output elasticity is not necessarily time independent. Since $\frac{\partial f_{ith}}{\partial q_{ith}}, \frac{\partial f_{ith}}{\partial n_{ith}} > 0$ and q_{ith}, n_{ith} , and $c_{ith} > 0$, then all terms on the right hand side must be between $(0, 1)$ implying $\omega_{ith} \in (0, 1)$. Thus we can write implicit prices in (C.11) and (C.12) as

$$\bar{p}_{ith} = \frac{\omega_{ith}}{\frac{\partial f_{ith}}{\partial q_{ith}}} \quad (\text{C.15})$$

$$\bar{w}_{ith} = \frac{1 - \omega_{ith}}{\frac{\partial f_{ith}}{\partial n_{ith}}} \quad (C.16)$$

Now consider a version of the budget constraint:

$$\sum_{i=1}^I P_{it} q_{ith} \leq w_t \left(\bar{n} - \sum_{i=1}^I n_{it} \right) + R_t k_{th} - k_{t+1,h} \quad (C.17)$$

Multiply both sides of (C.9) by the market price P_{it} and both sides of (C.9) by the market wage w_{th} then substitute into the budget constraint to get:

$$\sum_{i=1}^I (P_{it} \bar{p}_{it} + w_{th} \bar{w}_{ith}) c_{ith} \leq w_{th} \bar{n} + R_t k_{th} - k_{t+1,h} \quad (C.18)$$

Following Becker (1965) the price of final consumption ψ_{ith} is

$$\psi_{ith} = P_{it} \bar{p}_{ith} + w_{th} \bar{w}_{ith} \quad (C.19)$$

Now consider the nominal value of final consumption $\psi_{ith} c_{ith}$. Using (C.15) and (C.16) we can write this as:

$$\psi_{ith} c_{ith} = \left(\frac{P_{it} \omega_{ith}}{\frac{\partial f_{ith}}{\partial q_{ith}}} + \frac{w_{th} (1 - \omega_{ith})}{\frac{\partial f_{ith}}{\partial n_{ith}}} \right) c_{ith} \quad (C.20)$$

Applying Euler's theorem as before:

$$\psi_{ith} c_{ith} = \left(\frac{P_{it} \omega_{ith}}{\frac{\partial f_{ith}}{\partial q_{ith}}} + \frac{w_{th} (1 - \omega_{ith})}{\frac{\partial f_{ith}}{\partial n_{ith}}} \right) \left(\frac{\partial f_{ith}}{\partial q_{ith}} q_{ith} + \frac{\partial f_{ith}}{\partial n_{ith}} n_{ith} \right) \quad (C.21)$$

$$= q_{ith} P_{it} \omega_{ith} + \frac{\partial f_{ith}}{\partial n_{ith}} n_{ith} P_{it} \omega_{ith} + \frac{\partial f_{ith}}{\partial q_{ith}} q_{ith} w_{th} (1 - \omega_{ith}) + n_{ith} w_{th} (1 - \omega_{ith}) \quad (C.22)$$

But note that $\frac{\partial f_{ith}}{\partial n_{ith} \frac{\partial f_{ith}}{\partial q_{ith}}}$ is just the marginal rate of technical substitution which must be

$$\frac{\frac{\partial f_{ith}}{\partial n_{ith}}}{\frac{\partial f_{ith}}{\partial q_{ith}}} = \frac{w_{th}}{P_{it}} \quad (C.23)$$

Thus (C.22) collapses to $\psi_{ith} c_{ith} = P_{it} q_{ith} + w_{th} n_{ith}$. Then the value added from home production is just the value of the home productive process less the value of market inputs purchased, i.e.:

$$\psi_{ith} c_{ith} - P_{it} q_{ith} = w_{th} n_{ith} \quad (C.24)$$

□

Proposition 2. Under Lemma 1, CES utility for final consumption, and Cobb-Douglas aggregated home production, the relative demand for market good j to market good i can be written

$$\left(\frac{q_{jth}}{q_{ith}}\right) = \left[\frac{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}}{\theta_j \omega_j [(1 - \omega_j)/\omega_j]^{(1-\omega_j)\rho}} \right]^{\frac{1}{\rho-1}} P_{jt}^{\frac{1-\rho+\rho\omega_j}{\rho-1}} P_{it}^{\frac{1-\rho+\rho\omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i-\omega_j)}{\rho-1}} \left[\frac{z_{ith}}{z_{jth}} \right]^{\frac{\rho}{\rho-1}} \quad (C.25)$$

Proof. Start with the marginal rate of substitution for goods i and j :

$$\frac{\frac{\partial \tilde{u}_{th}}{\partial q_{ith}}}{\frac{\partial \tilde{u}_{th}}{\partial q_{jth}}} = \frac{P_{it}}{P_{jt}} \quad (C.26)$$

$$\Rightarrow \frac{\tilde{u}_{th}(q_{th}, n_{th})^{1-\rho} \omega_i \theta_i z_{ith}^{\rho} q_{ith}^{\rho\omega_i} n_{ith}^{\rho-\rho\omega_i} \left(\frac{1}{q_{ith}}\right)}{\tilde{u}_{th}(q_{th}, n_{th})^{1-\rho} \omega_j \theta_j z_{jth}^{\rho} q_{jth}^{\rho\omega_j} n_{jth}^{\rho-\rho\omega_j} \left(\frac{1}{q_{jth}}\right)} = \frac{P_{it}}{P_{jt}} \quad (C.27)$$

Substitute the implicit function $n_{ith}(q_{ith})$ expressed in (21) for each process then cancel like terms to get

$$\frac{\omega_i \theta_i z_{ith}^{\rho} q_{ith}^{\rho\omega_i} (P_{it}(1 - \omega_i)/(w_{th}\omega_i)q_{ith})^{\rho-\rho\omega_i} \left(\frac{1}{q_{ith}}\right)}{\omega_j \theta_j z_{jth}^{\rho} q_{jth}^{\rho\omega_j} (P_{jt}(1 - \omega_j)/(w_{th}\omega_j)q_{jth})^{\rho-\rho\omega_j} \left(\frac{1}{q_{jth}}\right)} = \frac{P_{it}}{P_{jt}} \quad (C.28)$$

$$\Rightarrow \frac{\omega_i \theta_i z_{ith}^{\rho} (P_{it}(1 - \omega_i)/(w_{th}\omega_i))^{\rho-\rho\omega_i} q_{ith}^{\rho-1}}{\omega_j \theta_j z_{jth}^{\rho} (P_{jt}(1 - \omega_j)/(w_{th}\omega_j))^{\rho-\rho\omega_j} q_{jth}^{\rho-1}} = \frac{P_{it}}{P_{jt}} \quad (C.29)$$

Collect like terms and move everything but quantities to the right side:

$$\left(\frac{q_{jth}}{q_{ith}}\right)^{\rho-1} = \left[\frac{\theta_j \omega_j [(1 - \omega_j)/\omega_j]^{(1-\omega_j)\rho}}{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}} \right] P_{jt}^{\rho-\rho\omega_j-1} P_{it}^{1-\rho+\rho\omega_i} w_{th}^{\rho(\omega_i-\omega_j)} \left[\frac{z_{jth}}{z_{ith}} \right]^{\rho} \quad (C.30)$$

Rewrite the left side so that q_{jth} is on top, then raise both sides to $\frac{1}{1-\rho}$ power to get

$$\left(\frac{q_{jth}}{q_{ith}}\right) = \left[\frac{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}}{\theta_j \omega_j [(1 - \omega_j)/\omega_j]^{(1-\omega_j)\rho}} \right]^{\frac{1}{\rho-1}} P_{jt}^{\frac{1-\rho+\rho\omega_j}{\rho-1}} P_{it}^{\frac{1-\rho+\rho\omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i-\omega_j)}{\rho-1}} \left[\frac{z_{ith}}{z_{jth}} \right]^{\frac{\rho}{\rho-1}} \quad (C.31)$$

□

Corollary 2. The same procedure can be applied as in Proposition 2 to express the time devoted toward production process j relative to process i as follows:

$$\left(\frac{n_{jth}}{n_{ith}}\right) = \left[\frac{\theta_i (1 - \omega_i) [\omega_i/(1 - \omega_i)]^{\rho\omega_i}}{\theta_j (1 - \omega_j) [\omega_j/(1 - \omega_j)]^{\rho\omega_j}} \right]^{\frac{1}{\rho-1}} P_{jt}^{\frac{\rho\omega_j}{\rho-1}} P_{it}^{\frac{\rho\omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i-\omega_j)}{\rho-1}} \left[\frac{z_{ith}}{z_{jth}} \right]^{\frac{\rho}{\rho-1}} \quad (C.32)$$

Proof. Start with the marginal rate of substitution for time use between processes i and j :

$$\frac{\frac{\partial \tilde{u}_{th}}{\partial n_{ith}}}{\frac{\partial \tilde{u}_{th}}{\partial n_{jth}}} = 1 \quad (\text{C.33})$$

$$\Rightarrow \tilde{u}_{th}(\mathbf{q}_{th}, \mathbf{n}_{th})^{1-\rho} (1-\omega_i) \theta_i z_{ith}^\rho q_{ith}^{\rho\omega_i} n_{ith}^{\rho-\rho\omega_i} \left(\frac{1}{n_{ith}} \right) = \tilde{u}_{th}(\mathbf{q}_{th}, \mathbf{n}_{th})^{1-\rho} (1-\omega_j) \theta_j z_{jth}^\rho q_{jth}^{\rho\omega_j} n_{jth}^{\rho-\rho\omega_j} \left(\frac{1}{n_{jth}} \right) \quad (\text{C.34})$$

Substitute (22) for each good and cancel like terms to get

$$(1-\omega_i) \theta_i z_{ith}^\rho (w_{th} \omega_i / [P_{it}(1-\omega_i)] n_{ith})^{\rho\omega_i} n_{ith}^{\rho-\rho\omega_i} \left(\frac{1}{n_{ith}} \right) = (1-\omega_j) \theta_j z_{jth}^\rho (w_{th} \omega_j / [P_{jt}(1-\omega_j)] n_{jth})^{\rho\omega_j} n_{jth}^{\rho-\rho\omega_j} \left(\frac{1}{n_{jth}} \right) \quad (\text{C.35})$$

Collect like terms and isolate relative time use on the left hand side:

$$\left(\frac{n_{ith}}{n_{jth}} \right)^{\rho-1} = \left[\frac{\theta_j (1-\omega_j) [\omega_j / (1-\omega_j)]^{\rho\omega_j}}{\theta_i (1-\omega_i) [\omega_i / (1-\omega_i)]^{\rho\omega_i}} \right] P_{jt}^{-\rho\omega_j} P_{it}^{\rho\omega_i} w_{th}^{\rho(\omega_j-\omega_i)} \left[\frac{z_{jth}}{z_{ith}} \right]^\rho \quad (\text{C.36})$$

Rewrite the left side so that n_{jth} is on top, then raise both sides to $\frac{1}{1-\rho}$ power to get

$$\left(\frac{n_{jth}}{n_{ith}} \right) = \left[\frac{\theta_i (1-\omega_i) [\omega_i / (1-\omega_i)]^{\rho\omega_i}}{\theta_j (1-\omega_j) [\omega_j / (1-\omega_j)]^{\rho\omega_j}} \right]^{\frac{1}{\rho-1}} P_{jt}^{\frac{\rho\omega_j}{\rho-1}} P_{it}^{\frac{\rho\omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i-\omega_j)}{\rho-1}} \left[\frac{z_{ith}}{z_{jth}} \right]^{\frac{\rho}{\rho-1}} \quad (\text{C.37})$$

□

Proposition 3. Fix $P_1 = P_2 = 1$ and z_2 . Consider the following cases separately:

- i. If final goods are substitutes so that $\rho \in (0, 1)$, then an increase (decrease) in z_1 is welfare improving and results in an increase (decrease) in equilibrium $\frac{q_1}{q_2}$ and an increase (decrease) in $\frac{n_1}{n_2}$.
- ii. If final goods are complements so that $\rho \in (-\infty, 0)$, then an increase (decrease) in z_1 is welfare improving and results in a decrease (increase) in equilibrium $\frac{q_1}{q_2}$ and a decrease (increase) in $\frac{n_1}{n_2}$.

Proof. We present the proofs for the two sub-statements of the proposition together since they each rely on the quasi-Hicksian relative demand functions:

$$q_2(q_1) = \left(\frac{\bar{u} - \theta_1 z_1^\rho q_1^\rho \left(\frac{1-\omega}{\omega} \right)^{\rho-\rho\omega} P_1^{\rho-\rho\omega}}{\theta_2 z_2^\rho \left(\frac{1-\omega}{\omega} \right)^{\rho-\rho\omega} P_2^{\rho-\rho\omega}} \right)^{\frac{1}{\rho}} \quad (\text{C.38})$$

$$n_2(n_1) = \left(\frac{\bar{u} - \theta_1 z_1^\rho n_1^\rho \left(\frac{\omega}{1-\omega} \right)^{\rho\omega} P_1^{\rho\omega-1}}{\theta_2 z_2^\rho \left(\frac{\omega}{1-\omega} \right)^{\rho\omega} P_2^{\rho\omega-1}} \right)^{\frac{1}{\rho}} \quad (\text{C.39})$$

In (C.38) time use has been concentrated out using (21) and in (C.39) market purchases have been concentrated out using (22).

To understand how relative demand changes for any given utility level \bar{u} due to a change in $\frac{z_1}{z_2}$, suppose z_2 , q_1 , and \bar{u} are fixed, along with prices and elasticities. Partially differentiating (C.38) in z_1 we get:

$$\frac{\partial q_2(q_1)}{\partial z_1} = -q_2(q_1)^{\frac{1-\rho}{\rho}} \left(\frac{P_1}{P_2} \right)^{\rho-\rho\omega} \left(\frac{1}{z_2} \right)^\rho q_1^\rho z_1^{\rho-1} < 0 \quad \forall \rho \quad (\text{C.40})$$

Thus for all possible ρ we are considering, an upward adjustment in z_1 leads to a downward adjustment in q_2 relative to q_1 at every possible utility level. An identical argument holds for $\frac{\partial n_2(n_1)}{\partial z_1}$. Thus, changes in z_1 affect the curvature of the indifference curves. Specifically, as a result of changes in z_1 , relative market purchases and time use either fall dramatically or very little which is readily apparent following from the limiting conditions

$$\lim_{q_1 \rightarrow 0} \frac{\partial q_2(q_1)}{\partial z_1} \bigg|_{\rho \in (0,1)} = 0 \quad (\text{C.41})$$

$$\lim_{q_1 \rightarrow 0} \frac{\partial q_2(q_1)}{\partial z_1} \bigg|_{\rho < 0} = -\infty \quad (\text{C.42})$$

$$\lim_{q_1 \rightarrow \infty} \frac{\partial q_2(q_1)}{\partial z_1} \bigg|_{\rho \in (0,1)} = -\infty \quad (\text{C.43})$$

$$\lim_{q_1 \rightarrow \infty} \frac{\partial q_2(q_1)}{\partial z_1} \bigg|_{\rho < 0} = 0 \quad (\text{C.44})$$

The same limiting outcomes occur for $\frac{\partial n_2(n_1)}{\partial z_1}$. This implies that for every $\frac{q_1}{q_2}$ pair, holding q_1 fixed the associated value of q_2 declines, causing the indifference curves for every \bar{u} to shift downward.

The sign of equilibrium changes to $\frac{q_1}{q_2}$ and $\frac{n_1}{n_2}$ depends on how the slope of the indifference curves change as a result of changes to z_1 . Note that the composite utility function is homothetic, implying linear expansion paths. This ensures that equilibrium outcomes move in accordance with those in Figures 1 and 2. \square

Proposition 4. Fix $z_1 = z_2 = 1$ and P_2 . Consider the following cases separately:

- i. If final goods are substitutes so that $\rho \in (0, 1)$, then an increase (decrease) in $\frac{P_1}{P_2}$ leads to a decrease (increase) in equilibrium $\frac{q_1}{q_2}$ and a decrease (increase) in equilibrium $\frac{n_1}{n_2}$.
- ii. If final goods are complements so that $\rho \in (-\infty, 0)$, then an increase (decrease) in $\frac{P_1}{P_2}$ leads to a decrease (increase) in equilibrium $\frac{q_1}{q_2}$ and an increase (decrease) in $\frac{n_1}{n_2}$.

Proof. Suppose $P_2 = 1$. Consider the two cases separately, and note we need only prove monotonicity:

- i. $\rho \in (0, 1)$ — If P_1 increases then the budget constraint shifts inward, as in Figure 3. Refer now to the marginal rate of substitution conditions in (25) and (26) and consider equilibrium choices which must satisfy

$$\frac{q_1}{q_2} = \left(\frac{P_1}{P_2} \right)^{\frac{1-\rho+\rho\omega}{\rho-1}} \quad (\text{C.45})$$

$$\frac{n_1}{n_2} = \left(\frac{P_1}{P_2} \right)^{\frac{\rho\omega}{\rho-1}} \quad (\text{C.46})$$

$\forall \rho, \omega \in (0, 1)$, $\frac{1-\rho+\rho\omega}{\rho-1} < 0$ and $\frac{\rho\omega}{\rho-1} < 0$. Thus as P_1 rises equilibrium $\frac{q_1}{q_2}$ and $\frac{n_1}{n_2}$ fall.

- ii. $\rho < 0$ — Note that for $\rho < 0$, $\frac{1-\rho+\rho\omega}{\rho-1} < 0$ as long as $\rho < \frac{1}{1-\omega}$ which holds $\forall \omega \in (0, 1)$. But now the sign of $\frac{\rho\omega}{\rho-1}$ has changed and $\frac{\rho\omega}{\rho-1} > 0$. Thus $\frac{q_1}{q_2}$ and $\frac{n_1}{n_2}$ co-move negatively.

□

Proposition 5. Consider the implications of two separate cases and their corresponding sub-cases:

- i. Suppose $\frac{\rho(\omega_2-\omega_1)}{\rho-1} < 0$ so that $\frac{q_1}{q_2}$ is decreasing in w then one and only one of the following must hold:
 - a. $\rho < 0$ and $\omega_2 < \omega_1$
 - b. $\rho \in (0, 1)$ and $\omega_2 > \omega_1$
- ii. Suppose $\frac{\rho(\omega_2-\omega_1)}{\rho-1} > 0$ so that $\frac{q_1}{q_2}$ is increasing in w then one and only one of the following must hold:
 - a. $\rho < 0$ and $\omega_2 > \omega_1$
 - b. $\rho \in (0, 1)$ and $\omega_2 < \omega_1$

Proof. We will prove each case separately:

- i. Suppose $\frac{\rho(\omega_2-\omega_1)}{\rho-1} < 0$. Clearly, for all feasible $\rho < 1$ the denominator is less than 0. Therefore, we must have $\rho(\omega_2 - \omega_1) > 0$. This happens when $\rho < 0$ and $\omega_2 < \omega_1$ or when $\rho \in (0, 1)$ and $\omega_2 > \omega_1$. The converse clearly holds as well.
- ii. Suppose $\frac{\rho(\omega_2-\omega_1)}{\rho-1} > 0$. Clearly, for all feasible $\rho < 1$ the denominator is less than 0. Therefore, we must have $\rho(\omega_2 - \omega_1) < 0$. This happens when $\rho < 0$ and $\omega_2 > \omega_1$ or when $\rho \in (0, 1)$ and $\omega_2 < \omega_1$. The converse, again, clearly holds.

□

Proposition 6. Fix $z_1 = z_2 = 1$. If household labor supply is non-constant in prices, i.e. $\frac{\partial l}{\partial P_i} \neq 0$ for all $i \in \{1, 2\}$, then $\omega_1 \neq \omega_2$.

Proof. We will prove this using the contrapositive argument that if $\omega_1 = \omega_2$ then $\frac{\partial l}{\partial P_i} = 0$ for all $i \in \{1, 2\}$. Suppose $\omega_1 = \omega_2 = \omega$. Ignore variation in wages w for this exercise. Note that, from (26) we can have the implicit function $n_1(n_2) = \left(\frac{P_1}{P_2}\right)^{\frac{\rho\omega}{\rho-1}} n_2$ which we can substitute into the time allocation constraint $l + n_1 + n_2 = \bar{n}$ to get the implicit function

$$l(P_1, P_2) = \bar{n} - n_2(P_1, P_2) \left[1 + \left(\frac{P_1}{P_2}\right)^{\frac{\rho\omega}{\rho-1}} \right] \quad (\text{C.47})$$

Given we are assuming w is fixed, we can use (22) and the equilibrium ratio of $\frac{q_1}{q_2}$ given by setting (25) equal to $\frac{P_1}{P_2}$ to get an expression for q_1 as a function of n_2 :

$$\begin{aligned} q_2(n_2) &= \left(\frac{\omega}{1-\omega}\right) \left(\frac{w}{P_2}\right) n_2 \\ q_1(q_2) &= \left(\frac{P_1}{P_2}\right)^{\frac{1-\rho+\rho\omega}{\rho-1}} q_2 \\ \Rightarrow q_1(q_2(n_2)) &= \left(\frac{P_1}{P_2}\right)^{\frac{1-\rho+\rho\omega}{\rho-1}} \left(\frac{\omega}{1-\omega}\right) \left(\frac{w}{P_2}\right) n_2 \end{aligned} \quad (\text{C.48})$$

Now, we can substitute q_1 , q_2 , and n_1 out of the budget constraint to get n_2 as a function of prices

$$\begin{aligned} P_1 q_1 + P_2 q_2 + w n_1 + w n_2 &= \bar{y} \\ \Rightarrow \left(\frac{P_1}{P_2}\right)^{\frac{1-\rho+\rho\omega}{\rho-1}} \left(\frac{\omega}{1-\omega}\right) \left(\frac{P_1 w}{P_2}\right) n_2 + \left(\frac{\omega}{1-\omega}\right) w n_2 + w \left(\frac{P_1}{P_2}\right)^{\frac{\rho\omega}{\rho-1}} n_2 + w n_2 &= \bar{y} \\ \Rightarrow n_2(P_1, P_2) &= \frac{(1-\omega)\bar{y}}{w \left[1 + \left(\frac{P_1}{P_2}\right)^{\frac{\rho\omega}{\rho-1}} \right]} \end{aligned} \quad (\text{C.49})$$

Returning to (C.47) we can use the above expression for $n_2(P_1, P_2)$ to get

$$l(P_1, P_2) = \bar{n} - \frac{(1-\omega)\bar{y}}{w} \quad (\text{C.50})$$

where $\frac{\partial l}{\partial P_1} = 0$ and $\frac{\partial l}{\partial P_2} = 0$ for fixed \bar{y} . □

Proposition 7. Assume $\omega_i \in (0, 1)$ for all i and suppose $\rho \neq 0$. For all structurally-valid values of β_1 and β_{2i} , $\beta_{3i} = -\beta_1 - \beta_{2i}$. Further, one of the following reduced-form restrictions must hold:

i. Home activities are substitutes so that $\rho \in (0, 1)$ and:

$$\begin{aligned}\beta_1 &\in (-\infty, 0) \\ \beta_{2i} &\in (0, \infty) \\ \beta_{3i} &\in (-\infty, \infty)\end{aligned}\tag{C.51}$$

ii. Home activities are complements so that $\rho \in (-\infty, 0)$ and:

$$\begin{aligned}\beta_1 &\in (0, 1) \\ \beta_{2i} &\in (-1, 0) \\ \beta_{3i} &\in (-1, 1)\end{aligned}\tag{C.52}$$

For completeness, $\beta_{0i} \in (-\infty, \infty)$.

Proof. First, note that

$$\omega_1 = \frac{\beta_1(\rho - 1)}{\rho}\tag{C.53}$$

$$\omega_i = \frac{\beta_{2i}(1 - \rho)}{\rho}\tag{C.54}$$

$$\Rightarrow \beta_{3i} = \frac{\beta_{2i}(1 - \rho) - \beta_1(\rho - 1)}{\rho - 1}\tag{C.55}$$

$$= -\beta_{2i} - \beta_1\tag{C.56}$$

Now, for β_1 and β_{2i} consider the two cases separately:

i. Home activities are substitutes so that $\rho \in (0, 1)$. Start with $\beta_1 = \frac{\rho\omega_1}{\rho-1}$. Assume ρ is interior.

$$\lim_{\omega_1 \rightarrow +0} \frac{\rho\omega_1}{\rho-1} = 0 \quad \text{and} \quad \lim_{\omega_1 \rightarrow -1} \frac{\rho\omega_1}{\rho-1} = \frac{1}{\rho-1}\tag{C.57}$$

Now take ρ to 0 and 1, holding ω_1 interior and fixed. As $\rho \rightarrow -1$, $\frac{1}{\rho-1} \rightarrow -\infty$. So

$$\begin{aligned}\lim_{\rho \rightarrow +0} \frac{\rho\omega_1}{\rho-1} &= 0 \\ \text{and } \lim_{\rho \rightarrow -1} \frac{\rho\omega_1}{\rho-1} &= -\infty\end{aligned}\tag{C.58}$$

For $\rho \in (0, 1)$, these cases are exhaustive. Thus $\beta_1 \in (-\infty, 0)$.

Consider $\beta_{2i} = \frac{\rho\omega_i}{1-\rho}$. Again, assume ρ is interior. By applying the same logic as in (C.57) and (C.58) and reversing the sign, $\beta_{2i} \rightarrow 0$ as $\omega_i \rightarrow +0$, $\beta_{2i} \rightarrow \frac{1}{1-\rho}$ as $\omega_i \rightarrow -1$, $\beta_{2i} \rightarrow 0$ as $\rho \rightarrow +0$, and $\beta_{2i} \rightarrow \infty$ as $\rho \rightarrow -1$ with $\frac{1}{1-\rho} \rightarrow \infty$ as $\rho \rightarrow -1$. Thus $\beta_{2i} \in (0, \infty)$.

Now consider $\beta_3 = \frac{\rho(\omega_i - \omega_1)}{\rho - 1}$. First, assume ω_i and ω_1 are interior and send ρ to its endpoints:

$$\lim_{\rho \rightarrow +0} \frac{\rho(\omega_i - \omega_1)}{\rho - 1} = 0 \quad \text{and} \quad \lim_{\rho \rightarrow -1} \frac{\rho(\omega_i - \omega_1)}{\rho - 1} = -\infty \quad (\text{C.59})$$

Now assume ω_1 is interior. Note that

$$\lim_{\rho \rightarrow -1} \lim_{\omega_i \rightarrow +0} \frac{\rho(\omega_i - \omega_1)}{\rho - 1} = -\infty \quad \text{and} \quad \lim_{\rho \rightarrow -1} \lim_{\omega_i \rightarrow +0} \frac{\rho(\omega_i - \omega_1)}{\rho - 1} = \infty \quad (\text{C.60})$$

The same procedure can be applied assuming ω_i is interior and sending ω_g to its endpoints, flipping the sign. It follows that $\beta_{3i} \in (-\infty, \infty)$ if $\rho \in (0, 1)$.

- ii. Home activities are complements so that $\rho \in (-\infty, 0)$. Again, start with $\beta_1 = \frac{\rho\omega_1}{\rho - 1}$. Consider the double limits:

$$\lim_{\rho \rightarrow -0} \lim_{\omega_1 \rightarrow +0} \frac{\rho\omega_1}{\rho - 1} = \lim_{\rho \rightarrow -0} \lim_{\omega_1 \rightarrow -1} \frac{\rho\omega_1}{\rho - 1} = 0 \quad (\text{C.61})$$

Taking ρ to $-\infty$, note that

$$\lim_{\omega_1 \rightarrow +0} \lim_{\rho \rightarrow -\infty} \frac{\rho\omega_1}{\rho - 1} = \frac{\infty}{\infty} \quad (\text{C.62})$$

$$\Rightarrow \text{By L'Hôpital's Rule: } \lim_{\omega_1 \rightarrow +0} \lim_{\rho \rightarrow -\infty} (\omega_1) = 0$$

$$\text{and } \lim_{\rho \rightarrow -\infty} \lim_{\omega_1 \rightarrow -1} \frac{\rho\omega_1}{\rho - 1} = 1 \quad (\text{C.63})$$

Thus, $\beta_1 \in (0, 1)$.

Turning to β_{2i} , the logic is the same except the signs are flipped, so $\beta_{2i} \in (-1, 0)$.

Consider $\beta_{3i} = \frac{\rho(\omega_i - \omega_1)}{\rho - 1}$. First, assume ω_i and ω_1 are interior. Note that as $\rho \rightarrow -0$, $\beta_{3i} \rightarrow 0$. Consider the following

$$\lim_{\rho \rightarrow -\infty} \frac{\rho(\omega_i - \omega_1)}{\rho - 1} = \frac{\infty}{\infty} \quad (\text{C.64})$$

$$\Rightarrow \text{By L'Hôpital's Rule: } \lim_{\rho \rightarrow -\infty} (\omega_i - \omega_1) = (\omega_i - \omega_1)$$

Now send ω_i and ω_1 to their endpoints, using the result of applying L'Hôpital's Rule to the limit as $\rho \rightarrow -\infty$. Clearly since $\omega_i \in (0, 1)$, $\forall i \geq 1$, then $(\omega_i - \omega_1) \in (-1, 1)$. Thus $\beta_{3i} \in (-1, 1)$ when $\rho \in (-\infty, 0)$.

For completeness, let us now show that β_{0i} can take any real value, regardless of ρ . We require $\tilde{\theta}_i > 0$. Recall

$$\beta_{0i} = \left(\frac{1}{\rho - 1} \right) \ln \left[\frac{\tilde{\theta}_i \omega_i [(1 - \omega_i)/\omega_i]^{(1 - \omega_i)\rho}}{\omega_1 [(1 - \omega_1)/\omega_1]^{(1 - \omega_1)\rho}} \right] \quad (\text{C.65})$$

Using the additive property of logarithms:

$$\beta_{0i} = \left(\frac{1}{\rho - 1} \right) \left[\ln \tilde{\theta}_i + \ln \omega_i - \ln \omega_1 + (1 - \omega_i) \rho \left(\ln(1 - \omega_i) - \ln \omega_i \right) - (1 - \omega_1) \rho \left(\ln(1 - \omega_1) - \ln \omega_1 \right) \right] \quad (\text{C.66})$$

Suppose $\tilde{\theta}_i$ is finite and ω_i and ω_1 are interior. As $\rho \rightarrow 0$ from both sides, β_{0i} can be any positive or negative real number depending on the values of $\tilde{\theta}_i$, ω_i , and ω_1 . As $\rho \rightarrow -1$, $\beta_{0i} \rightarrow -\infty$. Now consider the case where $\tilde{\theta}_i$, ω_i , and ω_1 are finite and $\rho \rightarrow -\infty$:

$$\lim_{\rho \rightarrow -\infty} \left(\frac{1}{\rho - 1} \right) \left[\ln \tilde{\theta}_i + \ln \omega_i - \ln \omega_1 + (1 - \omega_i) \rho \left(\ln(1 - \omega_i) - \ln \omega_i \right) - (1 - \omega_1) \rho \left(\ln(1 - \omega_1) - \ln \omega_1 \right) \right] = \pm \frac{\infty}{\infty} \quad (\text{C.67})$$

where the sign of the limit depends on whether ω_i and ω_1 are less than or greater than $\frac{1}{2}$. Applying L'Hôpital's Rule:

$$\begin{aligned} & \lim_{\rho \rightarrow -\infty} \left[(1 - \omega_i) \left(\ln(1 - \omega_i) - \ln \omega_i \right) - (1 - \omega_1) \left(\ln(1 - \omega_1) - \ln \omega_1 \right) \right] \\ &= \lim_{\rho \rightarrow -\infty} (1 - \omega_i) \left(\ln(1 - \omega_i) - \ln \omega_i \right) - \lim_{\rho \rightarrow -\infty} (1 - \omega_1) \left(\ln(1 - \omega_1) - \ln \omega_1 \right) \end{aligned} \quad (\text{C.68})$$

Now consider the double limits:

$$\lim_{\omega_i \rightarrow +0} \lim_{\rho \rightarrow -\infty} (1 - \omega_i) \left(\ln(1 - \omega_i) - \ln \omega_i \right) = -\infty \quad \forall i \geq 1 \quad (\text{C.69})$$

$$\begin{aligned} & \lim_{\omega_i \rightarrow -1} \lim_{\rho \rightarrow -\infty} (1 - \omega_i) \left(\ln(1 - \omega_i) - \ln \omega_i \right) \\ &= \lim_{\omega_i \rightarrow -1} \lim_{\rho \rightarrow -\infty} \frac{\ln(1 - \omega_i) - \ln \omega_i}{\frac{1}{1 - \omega_i}} = \frac{-\infty}{\infty} \\ \Rightarrow \text{By L'Hôpital's Rule} & \lim_{\omega_i \rightarrow -1} \lim_{\rho \rightarrow -\infty} \frac{-\frac{1}{1 - \omega_i} - \frac{1}{\omega_i}}{\frac{1}{(1 - \omega_i)^2}} \\ &= \lim_{\omega_i \rightarrow -1} \lim_{\rho \rightarrow -\infty} \frac{\omega_i - 1}{\omega_i} = 0 \quad \forall i \geq 1 \end{aligned} \quad (\text{C.70})$$

Depending on whether ω_1 or ω_i goes to 0, (C.69) implies that $\beta_{0i} \rightarrow \pm\infty$. Sending $\omega_i \rightarrow 1$ sends the corresponding term to 0, so this limiting condition does not affect the bounds for β_{0i} . The cases here are enough to show $\beta_{0i} \in (-\infty, \infty)$ for any feasible ρ .

Finally, $\beta_{4i} \in (0, \infty)$ since $0 < (1 - \omega_i)/\omega_i < \infty$ always, and

$$\lim_{\omega_i \rightarrow +0} (1 - \omega_i)/\omega_i = \infty \quad (\text{C.71})$$

$$\lim_{\omega_i \rightarrow -1} (1 - \omega_i)/\omega_i = 0 \quad (\text{C.72})$$

Thus both the numerator and denominator of $\beta_{4i} = \frac{(1-\omega_i)/\omega_i}{(1-\omega_1)/\omega_1}$ independently range over the positive real line, so $\beta_{4i} \in (0, \infty)$. □

Proposition 8. Let W_t be aggregated real labor income per hour worked, and let $\frac{Z_{it}}{Z_{1t}}$ be the relative aggregated total factor home productivity. If $\frac{W_t}{w_{th}} = \left(\frac{z_{ith}}{z_{1th}}\right)^{\frac{1}{\omega_i - \omega_1}} / \left(\frac{Z_{it}}{Z_{1t}}\right)^{\frac{1}{\omega_i - \omega_1}}$, then the relative expenditure function admits aggregation.

Proof. Start with household h 's relative real demand representation from Proposition 1, letting the numerator good be denoted as commodity $i = 1$. Multiply both sides by $\frac{P_{1t}}{P_{it}}$ and write

$$\left(\frac{x_{1th}}{x_{ith}}\right) = \left[\frac{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}}{\theta_1 \omega_1 [(1 - \omega_1)/\omega_1]^{(1-\omega_1)\rho}}\right]^{\frac{1}{\rho-1}} P_{1t}^{\frac{\rho \omega_1}{\rho-1}} P_{it}^{\frac{\rho \omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i - \omega_1)}{\rho-1}} \left[\frac{z_{ith}}{z_{1th}}\right]^{\frac{\rho}{\rho-1}} \quad (\text{C.73})$$

$$\Leftrightarrow x_{1th} = x_{ith} \left[\frac{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}}{\theta_1 \omega_1 [(1 - \omega_1)/\omega_1]^{(1-\omega_1)\rho}}\right]^{\frac{1}{\rho-1}} P_{1t}^{\frac{\rho \omega_1}{\rho-1}} P_{it}^{\frac{\rho \omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i - \omega_1)}{\rho-1}} \left[\frac{z_{ith}}{z_{1th}}\right]^{\frac{\rho}{\rho-1}} \quad (\text{C.74})$$

Summing both sides of (C.74) over households

$$\sum_h x_{1th} = \sum_h x_{ith} \left[\frac{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}}{\theta_1 \omega_1 [(1 - \omega_1)/\omega_1]^{(1-\omega_1)\rho}}\right]^{\frac{1}{\rho-1}} P_{1t}^{\frac{\rho \omega_1}{\rho-1}} P_{it}^{\frac{\rho \omega_i}{1-\rho}} w_{th}^{\frac{\rho(\omega_i - \omega_1)}{\rho-1}} \left[\frac{z_{ith}}{z_{1th}}\right]^{\frac{\rho}{\rho-1}} \quad (\text{C.75})$$

Given $\frac{W_t}{w_{th}} = \left(\frac{z_{ith}}{z_{1th}}\right)^{\frac{1}{\omega_i - \omega_1}} / \left(\frac{Z_{it}}{Z_{1t}}\right)^{\frac{1}{\omega_i - \omega_1}}$, we are assuming that household wages as a fraction of the average wage are inversely proportionate to household relative productivities and aggregate total factor home productivities:

$$w_{th}^{\frac{\rho(\omega_i - \omega_1)}{\rho-1}} \left[\frac{z_{ith}}{z_{1th}}\right]^{\frac{\rho}{\rho-1}} = W_t^{\frac{\rho(\omega_i - \omega_1)}{\rho-1}} \left[\frac{Z_{it}}{Z_{1t}}\right]^{\frac{\rho}{\rho-1}} \quad (\text{C.76})$$

This assumption is needed in order to make a substitution on the right hand side of (C.75) that eliminates the dependency of wages and relative productivities on households. For every h sub out (C.76) and factor then divide both sides by $\sum_h x_{ith} = X_{it}$ to write:

$$\left(\frac{X_{1t}}{X_{it}}\right) = \left[\frac{\theta_i \omega_i [(1 - \omega_i)/\omega_i]^{(1-\omega_i)\rho}}{\theta_1 \omega_1 [(1 - \omega_1)/\omega_1]^{(1-\omega_1)\rho}}\right]^{\frac{1}{\rho-1}} P_{1t}^{\frac{\rho \omega_1}{\rho-1}} P_{it}^{\frac{\rho \omega_i}{1-\rho}} W_t^{\frac{\rho(\omega_i - \omega_1)}{\rho-1}} \left[\frac{Z_{it}}{Z_{1t}}\right]^{\frac{\rho}{\rho-1}} \quad (\text{C.77})$$

which is relative aggregate expenditure expressed only as a function of prices, aggregate average wages, and aggregate in-home productivities. \square

C.2 Labor Supply Dependency on Prices

Here, we derive an implicit expression for l as a function of prices, wages, and elasticities in the two-good economy we use for comparative statics. Recall that cash on hand \bar{y} is fixed, $z_1 = z_2 = 1$, and $\theta_1 = \theta_2$. First, start with the relative time use expression from (24), multiply both sides by n_2 , and define the function $\phi^1(P_1, P_2, \omega_1, \omega_2, w)$ to get an implicit expression for n_1 :

$$n_1(n_2) = \underbrace{\left[\frac{(1 - \omega_2)[\omega_2/(1 - \omega_2)]^{\rho\omega_2}}{(1 - \omega_1)[\omega_1/(1 - \omega_1)]^{\rho\omega_1}} \right]^{\frac{1}{\rho-1}} P_1^{\frac{\rho\omega_1}{\rho-1}} P_2^{\frac{\rho\omega_2}{1-\rho}} w^{\frac{\rho(\omega_2 - \omega_1)}{\rho-1}}}_{\phi^1(P_1, P_2, \omega_1, \omega_2, w)} n_2 \quad (\text{C.78})$$

Continuing with the relative consumption expression from (23), multiply both sides by q_2 , and define the function $\phi^2(P_1, P_2, \omega_1, \omega_2, w)$ to get an implicit expression for q_1 :

$$q_1(q_2) = \underbrace{\left[\frac{\omega_2[(1 - \omega_2)/\omega_2]^{(1-\omega_2)\rho}}{\omega_1[(1 - \omega_1)/\omega_1]^{(1-\omega_1)\rho}} \right]^{\frac{1}{\rho-1}} P_1^{\frac{1-\rho+\rho\omega_1}{\rho-1}} P_2^{\frac{1-\rho+\rho\omega_2}{1-\rho}} w^{\frac{\rho(\omega_2 - \omega_1)}{\rho-1}}}_{\phi^2(P_1, P_2, \omega_1, \omega_2, w)} q_2 \quad (\text{C.79})$$

Using $q_2(n_2) = \left(\frac{\omega_2}{1-\omega_2}\right)\left(\frac{w}{P_2}\right)n_2$ we can write q_1 as an implicit function of n_2 as previously:

$$q_1(q_2(n_2)) = \phi^2(P_1, P_2, \omega_1, \omega_2, w) \left(\frac{\omega_2}{1 - \omega_2}\right) \left(\frac{w}{P_2}\right) n_2 \quad (\text{C.80})$$

We can now rewrite the budget constraint to get $n_2(P_1, P_2, \omega_1, \omega_2)$:

$$\begin{aligned} & \phi^2(P_1, P_2, \omega_1, \omega_2, w) \left(\frac{\omega_2}{1 - \omega_2}\right) \left(\frac{P_1 w}{P_2}\right) n_2 + \left(\frac{\omega_2}{1 - \omega_2}\right) w n_2 + w \phi^1(P_1, P_2, \omega_1, \omega_2, w) n_2 + w n_2 = \bar{y} \\ \Rightarrow \quad n_2(P_1, P_2, \omega_1, \omega_2, w) &= \frac{(1 - \omega_2) \bar{y}}{w \left[1 + \phi^1(P_1, P_2, \omega_1, \omega_2, w) + \omega_2 \left(\frac{P_1}{P_2}\right) \phi^2(P_1, P_2, \omega_1, \omega_2, w) \right]} \end{aligned} \quad (\text{C.81})$$

Finally, with the time use constraint we can write $l(P_1, P_2, \omega_1, \omega_2)$ using the objects we just derived:

$$l(P_1, P_2, \omega_1, \omega_2, w) = \bar{n} - \phi^1(P_1, P_2, \omega_1, \omega_2, w) n_2(P_1, P_2, \omega_1, \omega_2, w) - n_2(P_1, P_2, \omega_1, \omega_2, w) \quad (\text{C.82})$$

For completeness, note that we require $0 \leq l \leq \bar{n}$, a restriction we impose in our numerical simulations in Section 2.3.3.

References

- Acemoglu, Daron, and Veronica Guerrieri. 2008. "Capital Deepening and Nonbalanced Economic Growth". *Journal of Political Economy* 116 (3). (Cit. on pp. 2, 35).
- Aguiar, Mark, and Erik Hurst. 2005. "Consumption versus Expenditure". *Journal of Political Economy* 113 (5). (Cit. on p. 4).
- . 2007. "Life-Cycle Prices and Production". *The American Economic Review* 97 (5). (Cit. on p. 4).
- Aguiar, Mark, Erik Hurst, and Loukas Karabarbounis. 2013. "Time Use During the Great Recession". *The American Economic Review* 103 (5). (Cit. on p. 4).
- Autor, David, and David Dorn. 2013. "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market". *The American Economic Review* 103 (5): 1553–1597. (Cit. on pp. 2, 3, 35).
- Barnett, William. 1977. "Pollak and Wachter on the Household Production Function Approach". *Journal of Political Economy* 85 (5). (Cit. on p. 6).
- Becker, Gary. 1965. "A Theory of the Allocation of Time". *The Economic Journal* 75 (299). (Cit. on pp. 1, 3, 6, 53, 54).
- Benhabib, Jess, Richard Rogerson, and Randall Wright. 1991. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations". *Journal of Political Economy* 99 (6). (Cit. on p. 3).
- Bernanke, Ben. 1985. "Adjustment Costs, Durables, and Aggregate Consumption". *Journal of Monetary Economics* 15:41–68. (Cit. on p. 24).
- Boerma, Job, and Loukas Karabarbounis. 2019. "Inferring Inequality with Home Production". *Working Paper*. (Cit. on p. 3).
- Boppart, Timo. 2014. "Structural Change and the Kaldor Facts in a Growth Model with Relative Price Effects and Non-Gorman Preferences". *Econometrica* 82 (6). (Cit. on pp. 1, 26, 29, 37).
- Bridgman, Benjamin, George Duernecker, and Berthold Herrendorf. 2018. "Structural transformation, marketization, and household production around the world". *Journal of Development Economics* 133:102–126. (Cit. on pp. 1, 3, 10).
- Buera, Francisco, and Joseph Kaboski. 2009. "Can Traditional Theories of Structural Change Fit the Data?" *Journal of the European Economic Association* 7 (2-3): 469–477. (Cit. on pp. 1, 26).
- . 2012. "The Rise of the Service Economy". *The American Economic Review* 102 (6). (Cit. on pp. 2, 35).
- Cochrane, Donald, and Guy Orcutt. 1949. "Application of Least Squares Regression to Relationships Containing Auto-Correlated Error Terms". *Journal of the American Statistical Association* 44 (245): 32–61. (Cit. on p. 28).

- Geary, Roy. 1950. "A Note on 'A Constant-Utility Index of the Cost of Living'". *Review of Economic Studies* 18 (2): 65–66. (Cit. on p. 1).
- Giandrea, Michael, and Shawn Sprague. 2017. "Estimating the U.S. Labor Share". *Monthly Labor Review*, U.S. Bureau of Labor Statistics. <https://doi.org/10.21916/mlr.2017.7>. (Cit. on p. 26).
- Gomme, Paul, Finn Kydland, and Peter Rupert. 2001. "Home Production Meets Time to Build". *Journal of Political Economy* 109 (5). (Cit. on pp. 3, 24).
- Gordon, Robert. 2016. *The Rise and Fall of American Growth*. Princeton University Press. (Cit. on p. 5).
- Graham, John, and Carole Green. 1984. "Estimating the Parameters of a Household Production Function with Joint Products". *The Review of Economics and Statistics* 66 (2). (Cit. on p. 3).
- Green, H.A. John. 1964. *The Concept of Functional Separability*. (Cit. on pp. 8, 52).
- Gronau, Reuben. 1977. "Leisure, Home Production, and Work – the Theory of the Allocation of Time Revisited". *Journal of Political Economy* 85 (6). (Cit. on p. 3).
- Herrendorf, Berthold, Richard Rogerson, and Akos Valentinyi. 2013. "Two Perspectives on Preferences and Structural Transformation". *American Economic Review* 103 (7). (Cit. on pp. 1, 3, 26, 30, 44, 45).
- Ingram, Beth, Narayana Kocherlakota, and N.E. Savin. 1997. "Using theory for measurement: An analysis of the cyclical behavior of home production". *Journal of Monetary Economics* 40:435–456. (Cit. on p. 3).
- Kadiyala, Koteswara Rao. 1968. "A Transformation Used to Circumvent the Problem of Autocorrelation". *Econometrica* 36 (1): 93–96. (Cit. on p. 28).
- Kongsamut, Piyabha, Sergio Rebelo, and Danyang Xie. 2001. "Beyond Balanced Growth". *Review of Economic Studies* 68:869–882. (Cit. on pp. 1, 26).
- Leontief, Wassily. 1947. "Introduction to a Theory of the Internal Structure of Functional Relationships". *Econometrica* 15 (4). (Cit. on pp. 8, 52).
- Matsuyama, Kiminori. 2009. "Structural Change in an Interdependent World: A Global View of Manufacturing Decline". *Journal of the European Economic Association* 7 (2-3): 478–486. (Cit. on p. 1).
- McGrattan, Ellen, Richard Rogerson, and Randall Wright. 1993. "Household Production and Taxation in the Stochastic Growth Model". *Federal Reserve Bank of Minneapolis: Staff Report #166*. (Cit. on pp. 3, 24).
- Muellbauer, John. 1975. *Review of Economic Studies* 62], Number = 4, Pages = 526-542, Title = Aggregation, Income Distribution, and Consumer Demand. (Cit. on p. 1).
- . 1976. "Community Preferences and the Representative Consumer". *Econometrica* 44 (5): 979–999. (Cit. on p. 1).

- Ngai, Rachel, and Christopher Pissarides. 2007. "Structural Change in a Multisector Model of Growth". *American Economic Review* 97 (1). (Cit. on pp. 2, 35).
- Pollak, Robert, and Michael Wachter. 1975. "The Relevance of the Household Production Function and Its Implications for the Allocation of Time". *Journal of Political Economy* 83 (2). (Cit. on p. 6).
- Rupert, Peter, Richard Rogerson, and Randall Wright. 1995. "Estimating Substitution Elasticities in Household Production Models". *Economic Theory* 6 (1). (Cit. on p. 3).
- Stone, Richard. 1954. "Linear Expenditure Systems and Demand Analysis: An Application to the Pattern of British Demand". *The Economic Journal* 64 (255). (Cit. on p. 1).
- Uy, Timothy, Kei-Mu Yi, and Jing Zhang. 2013. "Structural change in an open economy". *Journal of Monetary Economics* 60:667–682. (Cit. on p. 1).
- Whelan, Karl. 2000. "A Guide to the Use of Chain Aggregated NIPA Data". *Federal Reserve Board Working Paper*. (Cit. on p. 44).
- . 2002. "A Guide to U.S. Chain Aggregated NIPA Data". *Review of Income and Wealth* 48 (2): 217–233. (Cit. on p. 44).