# Reputation and Sovereign Default

Manuel Amador and Christopher Phelan

LEAF Conference on Credibility, Reputation, and Time Consistency
Santa Barbara, California
November 15, 2019

University of Minnesota
Federal Reserve Bank of Minneapolis
NBER

# Some characteristics of sovereign debt markets

1. Some countries are "serial defaulters".
2. Recent defaulters sustain less debt: "Debt intolerance".
3. Some countries do eventually "graduate".
4. Default hard to predict and weakly associated with fundamentals.
5. Countries with positive trade deficits do sometimes default.
6. Higher haircuts associated with lower subsequent bond prices.

## Some characteristics of sovereign debt markets

1. Some countries are "serial defaulters".
2. Recent defaulters sustain less debt: "Debt intolerance".
3. Some countries do eventually "graduate".
4. Default hard to predict and weakly associated with fundamentals.
5. Countries with positive trade deficits do sometimes default.
6. Higher haircuts associated with lower subsequent bond prices.

In this paper:

Consistent with a *reputational theory* of sovereign debt repayment.

# Model's main ingredient

- Two government types: "commitment" and "opportunistic".
    - *Opportunistic* type can repudiate the debt (default).
    - *Commitment* type cannot default.

# Model's main ingredient

- Two government types: "commitment" and "opportunistic".
  - *Opportunistic* type can repudiate the debt (default).
  - *Commitment* type cannot default.
- Exogenous switches of which type of government in power.
  - Market does not observe switches, infers type from actions.
- **Reputation**: Market belief that a country's government is a *commitment type*.

# Results: Features of any Markov equilibria

- Serial Default:
  - Recent defaulters more likely to default.
- Debt Intolerance:
  - Recent defaulters pay higher interest rates and ...
    ... have lowest debt levels.
- Graduation:
  - Finite amount of time since last default $T$ where if a country achieves highest reputation, and pays lowest interest rate.
- Countries default even when they have positive trade deficits.
- Default is random.

## Extensions

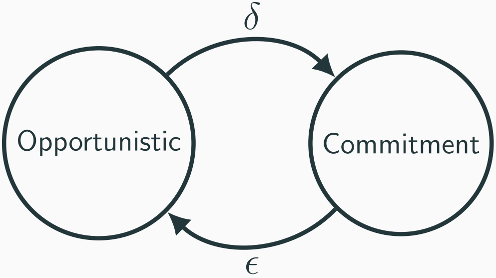
- Partial defaults have smaller effects on yields than full default.
- Aggregate shocks – asymmetric equilibrium responses.

# Environment

- Time continuous and infinite.
- Small open economy with constant endowment flow $y$.

# Environment: Players

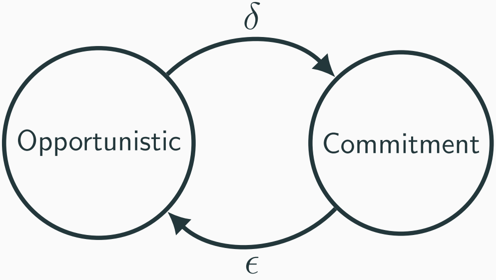
- Countable list of potential *governments* with alternating types.




- $\rho_0 = 0$: initial market belief that govt is commitment type.

# Environment: Players

- Countable list of potential *governments* with alternating types.




  - $\rho_0 = 0$: initial market belief that govt is commitment type.

- Risk-neutral foreign *lenders* who discount at rate $i$.

# Environment: Strategies

- Governments sell lenders long-term *bonds*.
- A *bond* sold at date $t$: promise to pay coupon

$$(i+\lambda)e^{-\lambda(s-t)} \text{ for all } s \geq t$$

  - Note that default-free price is one:

$$\int_t^{\infty} \underbrace{(i+\lambda)e^{-\lambda(s-t)}}_{\text{coupon}} e^{-i(s-t)} ds = 1.$$

- Start with no debt $b_0 = 0$ (for now).

# Environment: Strategies

- Commitment types cannot *default*.
- Opportunistic types can *default* — *default* defined as
  - No more coupon payments made.
  - Debt stock $b$ set to zero.
- Strategic choices at any given time are
  - Commitment government: how much to borrow.
  - Opportunistic government: how much to borrow, **and** with what probability to default.
  - Lenders: how much to lend.

# Markov Strategies

- All strategies are a function of *payoff relevant* state variables:
    - current level of debt, $b$
    - market belief, $\rho$, that govt is commitment type
- Claim: time since last default, $\tau$, is sufficient as a state.

# Markov Strategies:

- Will look for objects as function of time since last default
    - $b(\tau)$ (evolution of the level of debt),
    - $\rho(\tau)$ (evolution of the country's reputation),
    - $q(\tau)$ (evolution of the price of the country's bonds).
    - $\{F_\tau\}_{\tau \geq 0}$ (evolution of the opportunistic type's default behavior).

  which satisfy to-be-specified equilibrium conditions.

# Markov Strategies: Further Assumptions.

**Commitment Government:**

- Commitment type assumed completely *non-strategic*.
- Can't default, so no default decision to be made.
- Follows a given net borrowing rule

$$b'(\tau) = H(b, q)$$

  - Buys tractability.
  - *Avoids informed type having rich choice space*.

# Markov Strategies: Further Assumptions.

**Commitment Government:**

- Commitment type assumed completely *non-strategic*.
- Can't default, so no default decision to be made.
- Follows a given net borrowing rule

$$b'(\tau) = H(b, q)$$

  - Buys tractability.
  - *Avoids informed type having rich choice space*.

- Check later if optimal for it to follow $H$ if by deviating, foreign lenders assume it is an opportunistic type.

## Markov Strategies: Further Assumptions.

**Commitment Government Borrowing:**

- $H(b, q) : [0, \frac{y}{i+\lambda}] \times [0, 1] \rightarrow \mathbb{R}$
- $H(b, q)$ assumed continuous, differentiable, decreasing in $b$, and increasing in $q$.
- (bounded debt): $H(0, 0) \geq 0$ and $H(\frac{y}{i+\lambda}, 1) \leq 0$.
- (impatience relative to outside lenders): $H(0, \frac{i+\lambda}{i+\lambda+\delta+\epsilon}) > 0$.

# Markov Strategies: Further Assumptions.

- Implies consumption rule

$$C(b,q) \equiv y - \underbrace{(i+\lambda)b}_{\text{coupon payments}} + q\underbrace{(H(b,q)+\lambda b)}_{\text{bond sales}}.$$

- Boundedness and a (coming) break-even condition for *lenders* will imply government budget constraint.

# Markov Strategies: Further Assumptions.

**Opportunistic Government:**

- Can default. (Set $b = 0$.)
- If it doesn't default, **follows borrowing rule of commitment type**. (No reason to reveal type without defaulting).
- Only strategic decision for this type is its default decision.

# Markov Strategies

**Opportunistic Government:**

- **Collection** of functions $\{F_\tau\}_{\tau=0}^{\infty}$
- $F_\tau$: **cumulative distribution function** of future default for an opportunistic government in power at date $\tau$.
- That is, $1 - F_\tau(s)$ is probability of *not* defaulting between $\tau$ and $s$, conditional on remaining continuously in power.

*This form allows us to be as agnostic as possible on how (or whether) an opportunistic government mixes. (For instance, whether to default with positive probability for specific $\tau$, or to instead choose a Poisson arrival rate).*

# Markov Strategies

**Why do we need a collection of CDFs and not just $F_0$?**

- $F_\tau$ specifies default behavior of opportunistic government born at time $\tau$.
  - Needs to be specified or implied for all $\tau$.

# Markov Strategies

**Why do we need a collection of CDFs and not just $F_0$?**

- $F_\tau$ specifies default behavior of opportunistic government born at time $\tau$.
  - Needs to be specified or implied for all $\tau$.

- **Markov restriction**: government born at time $\tau$ acts like government at time zero who didn't default for $\tau$ units of time, or

$$1 - F_\tau(s) = (1 - F_\tau(m^-))(1 - F_m(s)) \text{ for all } 0 < \tau \leq m \leq s$$

# Markov Strategies

**Why do we need a collection of CDFs and not just $F_0$?**

- $F_\tau$ specifies default behavior of opportunistic government born at time $\tau$.
  - Needs to be specified or implied for all $\tau$.

- **Markov restriction**: government born at time $\tau$ acts like government at time zero who didn't default for $\tau$ units of time, or

$$1 - F_\tau(s) = (1 - F_\tau(m^-))(1 - F_m(s)) \text{ for all } 0 < \tau \leq m \leq s$$

- $F_0$ pins down $F_\tau$ for all $\tau$ such that $F_0(\tau) < 1$.
- But, it puts no restrictions on $F_\tau$ where $F_0(\tau) = 1$.
- Hence, we need the family.

# Payoffs

**Commitment Government:**

- No need to have preferences (for now) since not strategic.

**Opportunistic Government:**

- $u(c)$, discounted by $r$, as long as it is alive. (Opportunistic government doesn't care what happens to country after a type switch.)
- preview: $u(c)$ and $r$ won't matter at all. Same outcomes as long as more is preferred to less and now is preferred to later.

# Markov Perfect Equilibrium Definition

A collection $(b(\tau), \rho(\tau), q(\tau), \{F_\tau\}_{\tau \geq 0})$ is a *Markov Perfect Equilibrium* if

1. Expected discounted flow of coupon payments of every bond equals its price, $q(\tau)$. (**Lender optimization**)
2. Market beliefs are rational.
   - $\rho(0) = 0$ (or after a default, beliefs revert to certainty of the opportunistic type) and
   - *Conditional on no default*, beliefs follow **Bayes' rule**.
3. Debt evolution follows: $b'(\tau) = H(b(\tau), q(\tau))$ with $b(0) = 0$.
4. Opportunistic government at all times $\tau$ optimizes with respect to its default strategy, $F_\tau$.
5. $\{F_\tau\}_{\tau \geq 0}$ is Markov.

## Beliefs: Bayes' rule

**Recursive version:**

- If $F_\tau(\tau) > 0$, reputation jumps if no default, from $\rho(\tau^-)$ to $\rho(\tau)$, where

$$\rho(\tau) = \frac{\rho(\tau^-)}{\rho(\tau^-) + (1 - \rho(\tau^-))(1 - F_\tau(\tau))}$$
$$= \frac{\text{Prob commitment and no default at } \tau}{\text{Prob no default at } \tau}.$$

- If $F_\tau(\tau) = 0$, reputation evolves smoothly with no default:

$$\rho'(\tau) = (1 - \rho(\tau))\epsilon + \rho(\tau)((1 - \rho(\tau))F'_\tau(\tau) - \delta).$$

# Lender's optimization and prices

- $q_o(\tau)$: valuation conditional on opportunistic type in power
- $q_c(\tau)$: valuation conditional on commitment type in power

$$q(\tau) = \rho(\tau)q^c(\tau) + (1-\rho(\tau))q^o(\tau)$$
$$q^c(\tau) = \mathbb{E}_t\left[\int_\tau^t (i+\lambda)e^{-(i+\lambda)(s-\tau)}ds + e^{-(i+\lambda)(t-\tau)}q^o(t)\right]$$

- $q^o(\tau)$ more complicated expectation.
- However, if $F_\tau(\tau) = 0$, then $q(\tau)$ obeys the following differential equation:

$$\underbrace{\left[i+\lambda+(1-\rho(\tau))F'_\tau(\tau)\right]}_{\text{effective discount rate}} q(\tau) = \underbrace{(i+\lambda)}_{\text{coupon}} + \underbrace{q'(\tau)}_{\text{capital gain}}.$$

# Taking stock

- We have now defined all of the elements and conditions necessary to construct a Markov equilibrium.
- What's next?

# Taking stock

- We have now defined all of the elements and conditions necessary to construct a Markov equilibrium.
- What's next?
  - Mechanically construct a candidate equilibrium, taking as given one arbitrary initial condition (the initial bond price, $q(0)$).
  - Show candidate is an equilibrium **if and only if** all constructed time paths are *continuous*.
  - Argue there exists $q(0)$ such that our construction generates continuous paths.
  - Show there are no other Markov equilibria. (**All** Markov equilibria have to be able to be constructed using our method).

# Constructing a candidate equilibrium

**We guess the following:**

- **Constant** *on path* consumption for the opportunistic type.
  - Before date $T$, constant consumption:

    $$c(\tau) = c^{\star} > y \text{ for all } \tau < T$$

    with $c^{\star} = c(0) = y + q(0)H(0, q(0))$.
  - After date $T$, $c(\tau) \leq c^{\star}$ for $\tau \geq T$.

- For $\tau \geq T$, $F_{\tau}(\tau) = 1$. (certain immediate default).
- Positive but **smooth** default behavior before $T$:

$$F_{\tau}(\tau) = 0 \text{ for all } \tau < T$$

$$F'_{\tau}(\tau) > 0 \text{ for all } \tau < T.$$

# Constructing a candidate equilibrium

**Given $q(0)$, everything else is pinned down.**

- Two ordinary differential equations in $b$ and $q$.

$$b'(\tau) = H(b(\tau), q(\tau))$$
$$q'(\tau) = \frac{-C_b(b(\tau), q(\tau))}{C_q(b(\tau), q(\tau))} H(b(\tau), q(\tau)).$$

- Two starting conditions: $b(0) = 0$ and guessed $q(0)$.
- $b'(\tau) > 0$ (given $c^\star > y$) and thus $q'(\tau) > 0$.
- Roll out $b(\tau)$ and $q(\tau)$ that solve above ODEs.
- Define $T$ as the date where $q(T) = \frac{i+\lambda}{i+\lambda+\delta} < 1$.
- These functions are the *candidate* $b(\tau)$ and $q(\tau)$ for $\tau < T$.

# Constructing a candidate equilibrium

- From the ODE for prices for $\tau < T$:

$$\left[i + \lambda + \underbrace{(1 - \rho(\tau))F'_\tau(\tau)}_{x(\tau)}\right] q(\tau) = (i + \lambda) + q'(\tau).$$

  - Solve for the unconditional default rates, $x(\tau)$.

- Use $\rho(0) = 0$ and Bayes' rule:

$$\rho'(\tau) = (1 - \rho(\tau))\epsilon + \rho(\tau)(x(\tau) - \delta).$$

  to solve for the evolution of beliefs.

- Given, $(1 - \rho(\tau))F'_\tau(\tau)$ and $\rho(\tau)$ we get $F'_\tau(\tau)$.
- Use Markov restriction to determine $\{F_\tau\}_{\tau \leq T}$.

## Constructing a candidate equilibrium

- This gives us everything for $\tau < T$:

$$b(\tau), q(\tau), \rho(\tau), \{F_\tau\}$$

- For $\tau \geq T$, set:

$$F_\tau(\tau) = 1$$
$$q(\tau) = \frac{i+\lambda}{i+\lambda+\delta}$$
$$\rho(\tau) = 1.$$

and get $b(\tau)$ for $\tau \geq T$ from $b'(\tau) = H(b(\tau), \frac{i+\lambda}{i+\lambda+\delta})$ with starting condition $b(T)$.

- This pins down all equilibrium objects as functions of $q(0)$.

# Is the candidate actually an equilibrium?

**Checking equilibrium conditions**

- $b'(\tau) = H(b(\tau), q(\tau))$ for all $\tau$ by construction.
- Beliefs follow Bayes' rule by construction.
- Markov restriction on $\{F_\tau\}_{\tau \geq 0}$ holds by construction.
- Is the opportunistic type optimizing?

# Is the candidate actually an equilibrium?

**Is the opportunistic type optimizing?**

**Result**: $c(\tau) \leq c^\star$ for $\tau > T$

- $b'(T) > 0$ as $c^\star > y$ (the country is accumulating debt at $T$)
- As $q$ is constant after $T$, $b'(\tau) \geq 0$ for all $\tau \geq T$.
- $c(\tau) = C(b(\tau), q(\tau))$ is weakly decreasing for $\tau \geq T$, as $b(\tau)$ weakly increasing and $q$ constant.

Hence, the opportunistic government is optimizing:

- It is either indifferent (for $\tau \leq T$) or strictly prefers it (when $\tau > T$)

$\Rightarrow$ (There is no equilibrium value to reputation)

# Is the candidate actually an equilibrium?

**Checking prices**

- The (final) equilibrium condition on prices does **NOT** hold by construction.
- We show it does **if and only if** constructed $\rho(\tau)$ is such that:

$$\rho(\tau) \text{ is continuous at } T$$

This is not necessarily true for arbitrary $q(0)$.
Need to guess the right one.

# Some other things

**Result**

Any Markov equilibrium must satisfy our construction.

# Some other things

**Result**

Any Markov equilibrium must satisfy our construction.

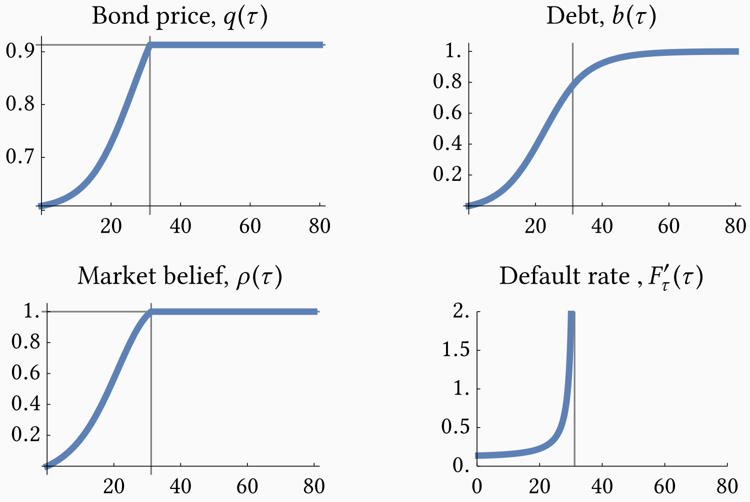
- Nowhere in our construction are any parameters associated with the preferences of the optimizing type.
- In equilibrium, this type faces no variation in its consumption (either across time or states)

# An Example

- $y = 1$.
- $\epsilon = .01$, $\delta = .02$. (long run chance of being commitment type, $\frac{\epsilon}{\epsilon+\delta} = \frac{1}{3}$.)
- $i = .01$, $\lambda = .2$ (five period debt).
- Functional form of $H(b, q)$ falls out of log utility maximizer who discounts at .15 and can borrow at the yield associated with $q$, except debt limit changed to $y$ from $\frac{y}{i}$:

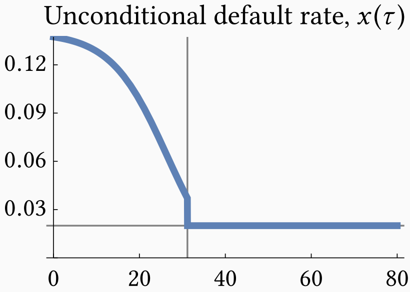
$$H(b, q) = \left( .15 - \left( \frac{i + \lambda}{q} - \lambda \right) \right) (y - b) .$$

# An Example



Bond price, $q(\tau)$
0.9
0.8
0.7
20
40
60
80
Debt, $b(\tau)$
1.
0.8
0.6
0.4
0.2
20
40
60
80
Market belief, $\rho(\tau)$
1.
0.8
0.6
0.4
0.2
20
40
60
80
Default rate , $F'_\tau(\tau)$
2.
1.5
1.
0.5
0.
0
20
40
60
80

# An Example




Even though conditional default probability *increases* with $\tau$ ...

... unconditional default probability *falls* with $\tau$.

# An Example

The lessons from the example are general:

- Serial Default: recent defaulters have a higher probability of defaulting.
- Debt Intolerance: recent defaulters have less debt and higher yields.
- Graduation date: there exists a finite time $T$ after which reputation is unchanged.
- Default is random: opportunistic type always mixes.

## An Example

### Other things

- $T = 31$.
- Once graduated, the expected time to default is $\frac{1}{\delta} \approx 50$.
- The average length of time to graduate after a default is

$$T + \int_0^T t \times x(t) e^{\int_t^T x(s) ds} dt \approx 200.$$

- The probability of not defaulting for $T$ years after a default is

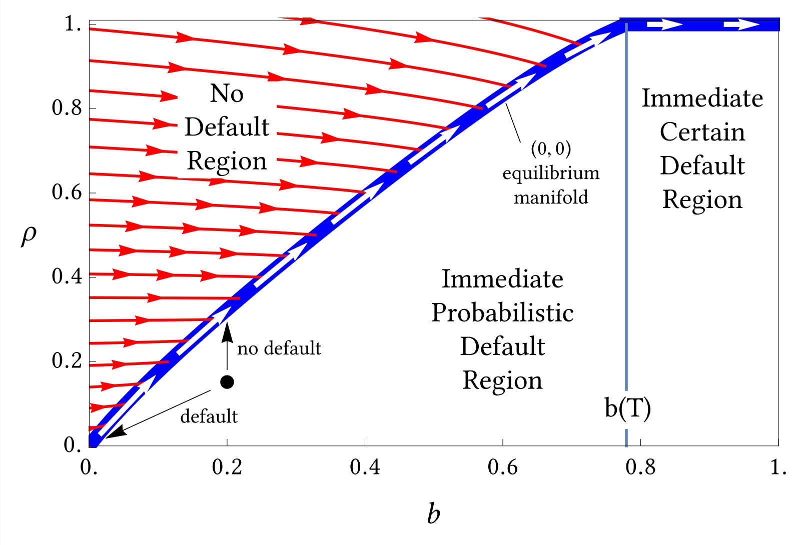
$$e^{-\int_0^T x(t) dt} \approx 10\%.$$

# General $(b_0, \rho_0)$: A Key Figure

What if we start from $(b_0, \rho_0) \neq (0, 0)$?

# General $(b_0, \rho_0)$: A Key Figure

What if we start from $(b_0, \rho_0) \neq (0, 0)$?



# General $(b_0, \rho_0)$: Key Figure.

- Solving for off manifold $(b_0, \rho_0)$ allow for
  - Checking if following $H(b, q)$ is optimal for both types.
  - Probability zero permanent shocks to endowment flow.
  - Extension to Partial Default.

# Optimality of following $H(b, q)$

**Opportunistic**

- $\rho$ jumps to 0 if it deviates from $H$. Either doesn't change value ($\rho$ below line) or lowers it ($\rho$ above line).

**Commitment**

- Again, $\rho$ jumps to 0 if it deviates from $H$. Since strategy calls for immediate certain default, same as not being able to borrow.
- Best deviation is to not borrow and pay down $b$ for a time. We show not profitable if commitment type discounts enough.

# Aggregate (MIT) Shocks

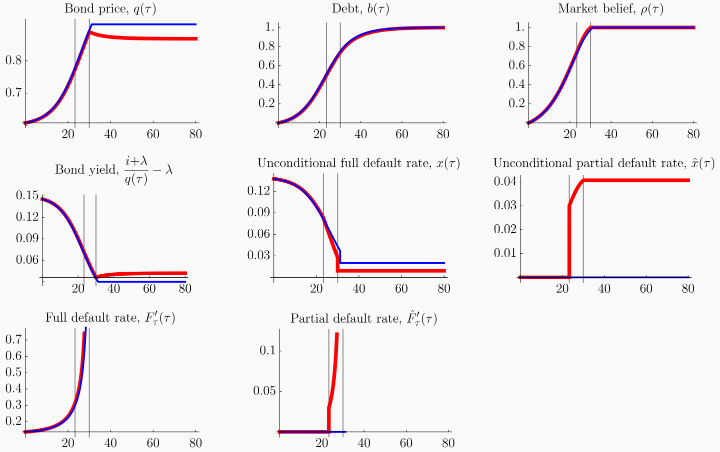
- Country receives one-time probability 0 **good** shock. ($y$ goes permanently up.)
  - $b$ and $\rho$ unaffected, but manifold moves **down**.
  - Equilibrium then calls for zero probability of default for a while.
- Country receives one-time probability 0 **bad** shock. ($y$ goes permanently down.)
  - $b$ and $\rho$ again unaffected, but manifold moves **up**.
  - Equilibrium then calls immediate probabilistic default. Bigger the shock, higher the probability.
  - Lack of predictability of reaction to bad shock.
- Asymmetry between reaction to good and bad shocks.

## Partial Default

- Suppose $b^* > 0$ such that if $b > b^*$, **commitment** type is forced to partially default so that $b$ is reset to $b^*$ (and each bondholder gets coupon payments proportionally reduced) with constant exogenous arrival rate $\theta > 0$.
- Opportunistic type can mimic this as well.
- Opportunistic type now chooses $F_\tau(s)$ (full default behavior) and $\hat{F}_\tau(s)$ (partial default behavior).

# Partial Default

- Equilibrium looks almost the same - consumption constant.
- Full default resets $\tau$ to zero. Partial default resets $\tau$ to the amount of time it takes to get from zero to $b^*$.
- Probability of partial default chosen in equilibrium so that if it happens, reputation drops exactly as much as it needs to in order to stay on manifold.
    - Can't have too little probability of opportunistic type partially defaulting, or else reputation too high after a partial default (and thus incentive to immediately partially default.)
    - Can't have too high probability of opportunistic type partially defaulting, relative to fully, or else reputation too low after a partial default — off manifold equilibrium calls for immediate probabilistic full default




- 3% yearly risk of exogenous partial default.
- Bond price falls more for full default than partial default.
- Could have had multiple $b_n^*$. Just set clock back further for smaller $b_n^*$. Bigger default means more of a bond price drop.

# Conclusion

- Tractable sovereign debt model:
    - Borrower's reputation and its interaction with default generate dynamics of debt and asset prices consistent with several facts.
- A government that defaults loses its reputation, and it takes periods of borrowing and not defaulting to restore it.
- During these periods, bond prices are low and default frequencies are high, as in the data.
- Relative to countries that have not recently defaulted, debt levels are low.
    - Countries with low debt levels face relatively high interest rates, a phenomenon referred to as "debt intolerance."

# Conclusion

- In model, a country can "graduate" into the set of "debt-tolerant" countries by not defaulting for a sufficiently long period of time.
- In the data, default is less than fully predictable and somewhat untied to fundamentals.
- Recent work uses this an argument for introducing features that lead to multiple equilibria. In our environment, such an outcome arises naturally.
- Equilibrium default in our model is necessarily random, both in our baseline model and in our consideration of when a country is hit by a bad shock.
- Such randomness is a fundamental ingredient for the dynamics of learning and reputation.