# Innovation and the Elasticity of Trade Volumes to Tariff Reductions*

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**Abstract**

I study the implications of endogenous productivity choices ("innovation") on the effects of trade liberalization. I find that a model with innovation generates an elasticity of trade volumes to tariff reductions that is fifty percent larger than models without innovation, and consistent in magnitude to empirical estimates. To show this, I develop a new model of international trade with innovation, and calibrate it to data on Canada and the United States before the Free Trade Agreement. Feeding into the calibrated model the tariff drop that resulted from the agreement, the increase in the trade volumes is similar to that observed in the data. Without innovation, the change in trade volumes is considerably lower, and similar in magnitude to what existing models without innovation have found.

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# 1 Introduction

A classic question in international trade concerns the effect of tariff reductions on trade volumes. Empirical evidence indicates that this effect is large: for example, studies of the Free Trade Agreement between Canada and the United States conclude that a one percentage point drop in tariffs leads to an increase in trade volumes of around ten percentage points. While this empirical fact is well documented, the literature has yet to produce an empirically reasonable model that can generate effects of this magnitude.

In this paper I develop a new model of trade and assess its ability to generate responses in trade volumes to reductions in tariffs that are consistent with those found in the data. The key novel feature of my model is that firms make a costly decision that determines their productivity. I call this decision innovation. When I calibrate my model to data on Canada and the United States, I find that it can account for the entire increase in trade volumes observed during the free trade agreement from 1988 to 1996.

The notion that adding an innovation decision increases the response of trade volumes is intuitive. This results from two basic observations. First, the incentive to innovate depends critically on market size, since resources devoted to innovation represent an up front cost. Second, decreases in tariffs lead to increased demand for imports, and thereby to a larger market size for exporters. It follows that a reduction in tariffs increases exporter innovation. This drives exporters to lower their prices, which in turn increases trade beyond the increase in a model without innovation. A corollary of this is that the incentives to increase innovation are largest among those firms that adjust along the extensive margin, that is, firms that start to export when the tariff is reduced. The reason is that these firms face the largest increases in demand.

The result that the incentive to increase innovation is largest among new exporters is consistent with the empirical evidence. This has been documented by Lileeva (2008) and Lileeva and Trefler (2009) for Canada; De Loecker (2007) and Kostevc and Damijan (2008) for Slovenia; Van Biesebroeck (2005) for Sub-Saharan Africa; Eslava, Haltiwanger, Kluger and Kluger (2009) for Colombia; Bustos (2009) for Argentina during the Mercosur; and Aw, Roberts and Xu (2008) for Taiwan.

The model that I develop features two countries of possibly different size. Each country has a tradable and a nontradable sector. The tradable sector consists of a continuum of firms that produce distinct varieties of goods with heterogeneous production functions. To export, firms must incur a fixed export cost, as in Melitz (2003). The novelty in my model is that firms can increase their productivity through costly innovation. The cost of innovation is an up front cost (i.e., it does not depend on units sold), and therefore firms with larger sales have larger incentives to innovate. As a result, the equilibrium in the model is such that only the most productive firms export, and these innovate more than non exporters.

I calibrate the model to study the effects of the reductions in tariffs during the Canada-U.S. Free Trade Agreement. A key element of the calibration is the response of firm productivity to innovation expenses. The larger this response is, the larger the response of trade volumes to reductions in tariffs. I calibrate my model so that the increase in industry productivity following the adoption of the Free Trade Agreement matches the increase in Canadian industry productivity found in Trefler (2004).

My main finding is that a 1 percentage point drop in tariffs increases imports by 9.6 percentage points. This is well within the range of empirical estimates for the elasticity of trade volumes to tariff reductions during the Canada-U.S. Free Trade Agreement provided by Head and Ries (2001). To my knowledge, I am the first to account for the entire reaction of trade volumes within an empirically plausible model of international trade.[1] The most successful contribution in this respect so far has been Ruhl (2008), who builds a Melitz type model that accounts for about two thirds of the observed increase in trade volumes. The reason why I can account for the entire observed increase is that I also model the reaction of innovation to a reduction in tariffs. If I close the innovation channel down, my model generates the same increase in trade volumes as Ruhl.

Productivity gains from trade are highly asymmetric in the model. While the productivity gain in Canada is calibrated to match the observed increase of 5%, the model predicts that the productivity gain in the U.S. (which is not targeted) is just 0.1%. This is consistent with the fact that empirical studies find no significant effects of trade liberalization on firm productivity in the U.S. (see Bernard and Jensen, 1999). The reason is that the reduction in

[1] See Kehoe (2003) for a summary of other attempts to account for the changes in trade volumes during the Canada-U.S. Free Trade Agreement.

tariffs increases the market size in the small country by much more than in the large country.

My model has interesting implications that go beyond the analysis of the Canada-U.S. Free Trade Agreement. In particular, in an exercise similar to one proposed by Yi (2003), I show that the model can capture reasonably well the behavior of U.S. trade volumes from the early 1960s to the late 1990s. Yi concludes that this is challenging for models of the Krugman (1980) variety or of the Backus, Kehoe and Kydland (1994) variety. If I feed the observed reduction in tariffs between 1962 and 1999 into my model, it accounts for two thirds of the observed increase in trade volumes in the United States.

Another implication of my model is that the productivity gains from trade depend critically on the costs of innovation. Productivity gains from trade will be lower in countries where innovation costs are higher. Countries with heavily regulated labor markets, or with poor enforcement of property rights, will likely have high costs of innovation. There is a large literature that argues that developing countries have highly regulated markets (see Heckman and Pagés, 2004) and poor enforcement of property rights (see Djankov et. al. 2002). Thus, developing economies are likely to have higher costs of innovation, and therefore lower productivity gains from trade. This is consistent with Clerides, Lach and Tybout (1998), who find no gains from trade in Colombia, Mexico, and Morocco, and Havrylyshyn (1990), who surveys the literature for developing economies and concludes that there is small evidence of productivity gains from trade.

My work falls into a growing literature on innovation and international trade. The paper that is closest to mine is Constantini and Melitz (2007), who describe the transition from a high tariff steady state to a low tariff steady state when firms can innovate. A key contribution of my work compared to this paper is to show that a carefully calibrated model of trade and innovation can account for the observed reaction of trade volumes to reductions in tariffs. The work of Atkeson and Burstein (2009) is also closely related to mine. They develop a dynamic trade model with innovation and conclude that innovation has fairly small effects on aggregate productivity. I show that while the implications of innovation on aggregate productivity are similar to what Atkeson and Burstein find, the implications of innovation on the productivity of new exporters are nonetheless large, thereby generating large effects on trade volumes. Lastly, Yeaple (2005) and Ederington and McCalman (2008) are related studies on innovation and trade liberalization, but the focus of their work is mainly theoretical

whereas the focus of my work is mainly quantitative.

The outline of the paper is as follows. Section 2 describes the model, defines the equilibrium and describes the properties of the equilibrium. Section 3 describes the most salient features of the Free Trade Agreement between Canada and the United States. Section 4 calibrates the model. Section 5 presents the results and establishes the importance of the innovation channel. Section 6 studies the effects of U.S. tariffs on trade volumes from the 1960s to the 1990s. Section 7 introduces two extensions to my model. Section 8 discusses how innovation can address some empirical observations that have not been accounted for. Section 9 concludes.

# 2 Model and Equilibrium

The environment is static. There are two countries, indexed by $i = 1, 2$. The only factor of production is labor. Country $i$ is populated by a measure $N_i$ of identical individuals, each endowed with one unit of labor.

There are two sectors in each country, a tradable and a non tradable sector. The tradable sector in country $i$ is comprised by a continuum of differentiated goods $\omega \in \Omega_i$. The set $\Omega_i$ has measure $M_i$. These goods can be sold domestically or exported, but any given good can only be produced in one country, so that $\Omega_1 \cap \Omega_2 = \emptyset$, as in Melitz (2003). There is a single non tradable good produced and sold in both countries.

Monopolists produce and sell each tradable good. A good $\omega \in \Omega_i \subset \mathbb{R}$ is associated with a technology parameter $\theta \in \Theta \subset \mathbb{R}_{++}$, where the set $\Theta$ is the same in both countries. Without loss of generality, I assume there exist measurable functions[2] $\theta : \Omega_1 \cup \Omega_2 \to \Theta, i = 1, 2$ that map names $\omega \in \Omega_i$ into technology parameters $\theta \in \Theta$. The technology to produce good $\omega \in \Omega_i$ is

$$y(\omega) = A(\theta(\omega), z)n$$

where $y(\omega)$ is output of good $\omega$, productivity $A(\theta, z) = \theta z$, $n$ is labor services, and $z \geq 0$ is

[2]These assumptions are required to apply a change of variables theorem and focus on the space $\Theta$ rather than $\Omega_i$'s.

innovation level.

These firms make decisions in two stages.[3] In the first stage, firms make innovation and exporting decisions. In the second stage, they set prices and quantities to maximize profits, taking the decisions on innovation and exporting as given. Thus, the problem in stage 2 is a standard monopolistic competition problem, as in Dixit-Stiglitz (1977).

Innovation is the main departure from standard trade models. To set innovation to level $z$, a firms must incur $c(z)$ units of labor, where

$$c(z) = z^{\alpha}, \alpha \geq 0$$

Firms export by incurring a fixed export cost $\kappa$, as in Melitz (2003). This is in units of labor. Let $x(\omega) = 1$ denote the decision of monopolist $\omega$ in country $i$ to export, $x(\omega) = 0$ otherwise. Additionally, there are tariffs collected on goods traded. I assume that these are paid by the consumer, and describe them in detail later on.

The non tradable sector in country $i$ is perfectly competitive. A representative firm produces this good, labeled $S_i$, with technology

$$S_i = N_{s_i}$$

where $N_{s_i}$ is labor units.

Preferences of a consumer in country $i$ are defined over tradable goods produced domestically, tradable goods imported, and the domestic non tradable good. As in Dixit-Stiglitz (1977) models, tradable goods are aggregated through a CES function. Let $q_i(\omega)$ be the quantity of good $\omega$ consumed in country $i$ by each consumer. The CES aggregator over tradable goods is, for $i \neq j$

$$C_i = \left[ \int_{\omega \in \Omega_i} q_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega + \int_{\omega \in \Omega_j} q_i(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}}$$

where $\sigma \in (1, 1+\alpha)$ is the elasticity of substitution between tradable goods. The joint restriction between $\sigma$ and $\alpha$ guarantees that the monopolist's profit maximizing problem is

[3] Whether there are one or two stages does not change the model. The problem becomes more intuitive when there are two stages, due to the familiarity of the problem in stage 2.

well defined.

Following Helpman and Itskhoki (2007), the utility function of a country $i$ consumer is

$$U(C_i, s_i) = \gamma \log C_i + s_i \tag{1}$$

where $s_i$ is the quantity of the non tradable good consumed by an individual in country $i$.

Consumers maximize (1) subject to the budget constraint. I next describe the budget constraint. Income is determined by wage earnings and the consumer's share of profits from the firms. Denote the wage rate in country $i$ by $w_i$. I set $w_1 = 1$ as the numeraire. Let $\pi_i(\omega)$ denote the profits of monopolist $\omega$. The consumers own domestic firms in equal shares, so that profits are divided equally among all domestic consumers.

The expenditure side consists of payments for domestic tradable goods, imports, and the non tradable good. Note that, in equilibrium, the price of the non tradable good in country $i$ is $w_i$. Therefore, I do not introduce additional notation for this price. Denote by $p_i(\omega)$ the price of a good $\omega$. I show later on that in equilibrium, the producer sets the same price for its exports and domestic sales, and therefore $p_i(\omega)$ is not indexed by the market in which the good is sold.

A consumer in country $i$ that imports a good from country $j$ pays a tariff $\tau_i$ on this good.[4] This tariff is the same across all country $i$ imports. The amount paid in country $i$ per unit of good $\omega$ imported from country $j$ is $(1+\tau_i)p_j(\omega)$. Tariffs are paid to a domestic government. The government rebates these revenues lump sum back to the consumers. Let $G_i$ denote total rebates in country $i$, and $g_i = \frac{G_i}{N_i}$ the rebates to each individual. Government budget balance in country $i$ is therefore

$$N_i \tau_i \int_{\Omega_j} x(\omega) p(\omega) q_i(\omega) d\omega = G_i$$

[4] A common assumption in the literature is that tariffs are iceberg costs. That is, to consume $q$ units, an importer must purchase $(1+\tau)q$ units, where $\tau$ units are lost in transit. I do not make this assumption, and in section 7 I provide arguments justifying this choice.

The budget constraint for a country $i$ individual is[5]

$$\int_{\Omega_i} p(\omega)q_i(\omega)d\omega + (1+\tau_i)\int_{\Omega_j} p(\omega)q_i(\omega)d\omega + w_i(s_i - g_i) = w_i + \frac{\int_{\Omega_i}\pi_i(\omega)d\omega}{N_i} \tag{2}$$

Notice that the budget constraint, together with government budget balance, imply that trade balances, that is, for $i \neq j$,

$$N_j \int_{\Omega_i} x(\omega)p(\omega)q_j(\omega)d\omega = N_i \int_{\Omega_j} x(\omega)p(\omega)q_i(\omega)d\omega$$

Feasibility conditions require the following for each country. Total labor supply must be equal to total labor used for the production of the tradable and the non tradable goods. Thus, in country $i$:

$$N_i = \int_{\Omega_i} \left[n(\omega) + c(z(\omega)) + x(\omega)\kappa\right] d\omega + N_{s_i}$$

The output of each tradable good must be equal to its consumption. In country $i$, for $i \neq j$, for all $\omega \in \Omega_i$

$$\theta_i(\omega)z(\omega)n(\omega) = N_i q_i(\omega) + x(\omega)N_j q_j(\omega)$$

Output of the non tradable good must be equal to the quantities consumed and the quantities used in the tradable sector for innovation and exporting.

$$S_i = N_i s_i$$

As in Dixit Stiglitz models, I introduce a price index for the aggregate tradable good $C_i$. This is the minimum cost to purchase one unit of $C_i$, and is given by

$$P_i = \left[\int_{\Omega_i} p(\omega)^{1-\sigma}d\omega + (1+\tau_i)^{1-\sigma}\int_{\Omega_j} p(\omega)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}} \tag{3}$$

Finally I introduce the demand functions for each tradable good in country $i$. These functions map prices into the quantity of each $\theta$ type good that maximizes utility subject to the budget

[5] To solve for the equilibrium, I impose sufficient conditions so that $s_i - g_i > 0$.

constraint. These are

$$q_i(p(\omega); w_i, P_i), \text{ for } \omega \in \Omega_i \qquad q_i((1+\tau_i)p(\omega); w_i, P_i) \text{ for } \omega \in \Omega_j$$

Demands are functions of the prices of all goods[6]. The first argument is the price of the own good, the second is the price of the non tradable, and the third summarizes the price of all tradable goods available, i.e., imported and produced domestically.

Monopolists take these demand functions as given and maximize profits. I assume the problem of profits maximization is solved sequentially. In stage 1, firms choose innovation levels and export status. In stage 2, firms take their productivity and export status as given to maximize "variable" profits, $\pi^v(\omega, z, x)$. This problem is

$$\pi_i(\omega) = \max_{z \geq 0, x \in \{0,1\}} \pi^v(\omega, z, x) - w_i z^\alpha - x w_i \kappa \tag{4}$$

$$\begin{aligned} \textit{where} \qquad & \pi^v(\omega, z, x) = \max_{p_d, p_x, Q_d, Q_x, n} p_d Q_d + x p_x Q_x - w_i n \\ & s.t. \qquad Q_d = N_i q_i(p; w_i, P_i) \\ & \qquad\quad Q_x = N_j q_i((1+\tau_i)p; w_i, P_i) \\ & \qquad\quad Q_x + Q_d = \theta(\omega) z n \\ & \qquad\quad Q_d \geq 0, Q_x \geq 0, n \geq 0 \end{aligned}$$

## 2.1 Equilibrium Definition

An equilibrium is a list of monopolists' decisions $x(\omega), z(\omega), p(\omega)$, profits $\pi(\omega)$ for all $\omega$, demand functions $q_i(p(\omega); w_i, P_i)$, for $\omega \in \Omega_i, q_i((1+\tau_i)p(\omega); w_i, P_i)$, for $\omega \in \Omega_j$, allocations $n_{s_i}, g_i, S_i$, and prices $w_i, P_i$ for $i, j \in \{1, 2\}, i \neq j$ such that $(i)$ consumers maximize (1) subject to (2); $(ii)$ $P_i$ satisfies equation (3); $(iii)$ monopolists maximize profits subject to the demand functions from the consumers; $(iv)$ non tradable firms maximize profits taking prices as given; $(v)$ markets clear; and $(vi)$ government balances budget.

[6]Typically, they are also functions of income. Given that quasilinear utility function, there is no income effect on the demand for tradables.

## 2.2 Change of Variables

Recall that each good $\omega$ is associated with a technology parameter $\theta$. In equilibrium, two firms in the same country with the same $\theta$ make the same decisions. In this sense, it is convenient to perform a change of variables to concentrate on the space of $\theta$'s and country of origin rather than the space of $\omega$'s.

To do this, define $x_i(\theta)$ as the export decision of a type $\theta$ monopolist in country $i$, and define $z_i(\theta), p_i(\theta)$, and $\pi_i(\theta)$ similarly. Next let $q_{di}(p_i; w_i, P_i)$ be the demand in country $i$ for a tradable good produced domestically, and $q_{mi}((1+\tau_i)p_j; w_i, P_i)$ the demand in country $i$ for an imported good.

Finally, I need to keep track of the measure of firms of each type produced and imported in each country. Let $f(\theta)$ denote the distribution of firms in each country (I assume firms are distributed in the same way in each country).

The list of variables that define the equilibrium are monopolists' decisions $x_i(\theta), z_i(\theta), p_i(\theta)$, profits $\pi_i(\theta)$, demand functions $q_{di}(p_i; w_i, P_i), q_{mi}(p_j; w_i, P_i)$, allocations $n_{s_i}, g_i, S_i$, and prices $w_i, P_i$ for $i, j \in \{1, 2\}, i \neq j$ where

$$P_i = \left[ \int p_i(\theta)^{1-\sigma} M_i f(\theta) d\theta + (1+\tau_i)^{1-\sigma} \int p_j(\theta)^{1-\sigma} M_j f(\theta) d\theta \right]^{\frac{1}{1-\sigma}} \tag{5}$$

Recall that $M_i$ denotes the measure of goods in country $i$. Equation (5) is derived from equation (3) by applying a change of variables theorem as in Billingsley (1995).

## 2.3 Equilibrium Properties

In this section I describe firm behavior in equilibrium. I show that $(i)$ firms set price equal to a constant mark-up over marginal cost; $(ii)$ exporting follows a cut-off rule; and $(iii)$ innovation is increasing in $\theta$.

Firm choices are a solution to the profit maximizing problem, taking the demand functions for their products as given. From the solution to the consumer problem, these demand functions

are

$$q_{di}(p_i; w_i, P_i) = \gamma w_i P_i^{\sigma-1} p_i^{-\sigma}$$
$$q_{mi}(p_j; w_i, P_i) = \gamma w_i P_i^{\sigma-1} \left((1+\tau_i)p_j\right)^{-\sigma}$$

They feature a constant elasticity of demand $\sigma$ on own price. $P_i$ summarizes the relevant information on the price of all other tradable goods available for consumption (domestic and imported). An increase in the price of another tradable good in country $i$ increases the price index $P_i$, and this increases the demand for each tradable good, since $\sigma > 1$. This is intuitive, since the assumption $\sigma > 1$ implies that tradable goods are relative substitutes for each other.

Given these demand functions, monopolists solve their problem in two stages. In the first one, they choose a level of innovation and export status. In the second stage, they choose prices and quantities to maximize profits given their innovation and export decisions.

I start by describing the second stage. The second stage problem is common to any problem of monopolistic competition. Given the constant elasticity of substitution, price is a constant mark-up over marginal cost. The maximizing prices for a type $\theta$ monopolist in country $i$ that exports are $p_{id}(\theta) = p_{ix}(\theta) = p_i(\theta)$, and for a non exporter, $p_{id}(\theta) = p_i(\theta)$, where

$$p_i(\theta) = \underbrace{\left(\frac{\sigma}{\sigma-1}\right)}_{\text{Mark-up}} \underbrace{\frac{w_i}{\theta z_i(\theta)}}_{\text{Mg. Cost}}$$

Define $\pi_i^v(\theta; z, x)$ as the variable profits of a type $\theta$ in country $i$ with innovation level $z$ and export status $x$. Given the mark-up rule, $\pi_i^v(\theta, z, x)$ is increasing in $\theta$, $z$ and $x$

$$\pi_i^v(\theta, z, 1) = (K_{di} + K_{xi}) z^{\sigma-1} \theta^{\sigma-1} \tag{6}$$
$$\pi_i^v(\theta, z, 0) = K_{di} z^{\sigma-1} \theta^{\sigma-1} \tag{7}$$

where $K_{di} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} N_i w_i^{2-\sigma} \gamma P_i^{\sigma-1}$ and $K_{xi} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} N_j w_j^{1-\sigma} \gamma P_j^{\sigma-1} (1+\tau_j)^{-\sigma} w_i$.

In the first stage, firms choose innovation and export status to maximize total profits. Innovation choices are as follows. From the first order conditions, optimal expenditures on

innovation are proportional to $\pi_i^v(\theta, z, x)$. Let $z_{xi}(\theta)$ be the maximizing level of innovation for a type $\theta$ exporter in country $i$ and define $z_{di}(\theta)$ similarly. The first order conditions set

$$w_i z_{xi}(\theta)^\alpha = \frac{\sigma - 1}{\alpha} \tilde{\pi}_i(\theta, z_{xi}(\theta), 1)$$
$$w_i z_{di}(\theta)^\alpha = \frac{\sigma - 1}{\alpha} \tilde{\pi}_i(\theta, z_{di}(\theta), 0)$$

This implies that innovation in increasing in $\theta$ and in export status. That is, for all $\theta$,

$$z'_{xi}(\theta) > 0 \tag{8}$$
$$z'_{di}(\theta) > 0 \tag{9}$$
$$z_{xi}(\theta) > z_{di}(\theta) \tag{10}$$

Next, I show that the export decision is determined by a cut-off rule, as in Melitz (2003). Firms decide whether to export or not by comparing the profits from begin an exporter with the profits from not begin an exporter. The cut-off result follows from a single crossing argument. Specifically, type $\theta$ in county $i$ exports if and only if the following holds[7]

$$\pi_i^v(\theta, z_{xi}(\theta), 1) - w_i z_{xi}(\theta)^\alpha - [\pi_i^v(\theta, z_{di}(\theta), 0)) - w_i z_{di}(\theta)^\alpha] \geq w_i \kappa \Leftrightarrow$$
$$\left(\frac{\alpha + 1 - \sigma}{\alpha}\right) (\pi_i^v(\theta, z_{xi}(\theta), 1) - \pi_i^v(\theta, z_{di}(\theta), 0)) \geq w_i \kappa \tag{11}$$

I show that (11) can hold with equal sign for at most one $\theta$. Differentiating (6) and (7) with respect to $\theta$ and considering that $z_{xi}(\theta) > z_{di}(\theta)$ shows that

$$\frac{\partial \pi_i^v(\theta, z_{xi}(\theta), 1)}{\partial \theta} > \frac{\partial \pi_i^v(\theta, z_{di}(\theta), 0)}{\partial \theta}$$

Therefore, the left hand side of expression (11) is strictly increasing in $\theta$. Thus, if type $\theta^*$ chooses to export, so do every type $\theta \geq \theta^*$.

Finally, the export cost shifts the exporter profits downwards. If $\Theta$ is a convex set, and there

[7] I adopt the convention that indifferent firms choose to export. Since there is only one type which is indifferent, and the measure of that type is zero, there is no loss of generality. This assumption is convenient because it means innovation is a function of $\theta$. Otherwise, it is a correspondence that is not single valued at $\hat{\theta}_i$. The Berge theorem guarantees that this correspondence is upper hemi continuous.

exist $\underline{\theta} \in \Theta$ and $\bar{\theta} \in \Theta$ such that

$$\left(\frac{\alpha+1-\sigma}{\alpha}\right)\pi_i^v(\underline{\theta}, z_{xi}(\underline{\theta}), 1) < w_i\kappa < \left(\frac{\alpha+1-\sigma}{\alpha}\right)\left[\pi_i^v(\bar{\theta}, z_{xi}(\bar{\theta}), 1) - \tilde{\pi}_i(\bar{\theta}, z_{di}(\bar{\theta}), 0)\right]$$

then there is exactly one $\theta$ such that (11) holds with equality[8], and therefore the single crossing property holds. This is illustrated in figure 2. The cut-off level is $\theta_1$ in the figure.

Thus, conditions (8) through (10), plus the export cut-off rule, imply that innovation is a strictly increasing function of $\theta$ with a discontinuous "jump" at the export cut-off $\hat{\theta}_i$. Figure 3 illustrates these innovation rules.

# 3 The Canada-U.S. Free Trade Agreement

I calibrate the model to evaluate the effects of the reductions in tariffs in the Free Trade Agreement between Canada and the United States. The choice of this episode is based on (*i*) we have good, reliable, tariff data; and (*ii*) several studies have focused on this episode, providing a good understanding of the effect of the reduction in tariffs on trade volumes and productivity. The advantage of understanding the effects of tariffs on trade volumes is that it provides a good empirical benchmark to compare the results of the model with. Also, the estimates on the effects on productivity are a useful target for the calibration of the model.

I next describe briefly the essence of the agreement, and its empirical effects on aggregate trade volumes and productivity. The agreement eliminated tariffs between the United States and Canada over a course of ten years. It was entered into in January 1, 1989. Broadly speaking, the agreement divided all goods into three categories. The tariffs of the first group would be eliminated immediately, the second group would eliminate them proportionally over 5 years, and the third would eliminate them proportionally over 10 years.

Several papers have focused on the empirical effects of this agreement. In particular, I focus especially on the estimates two of these: Trefler (2004), and Head and Ries (2001). One characteristic that they share is that both end the analysis in 1996, two years before the

[8]This is an application of the Brower fixed point theorem, since the Berge theorem guarantees that the left hand side of expression 11 is continuous. Uniqueness comes from strict monotonicity.

agreement was officially through. This is not a problem, since by 1996 most of the effects of the agreement had already taken place. To put this in perspective, in 1988, the average tariff was close to 6%. One in 4 tariffs was over 10%. By 1996, the average tariffs were close to 1%, and no tariff was over 5%.

Head and Ries estimate the effect of the reductions in tariffs on trade volumes. The authors identify the effect of tariff reductions by isolating it from other factors such as non tariff trade barriers. Their measure of trade volume is the ratio of bilateral manufacturing exports to total manufacturing output minus total manufacturing exports. In this way, the authors abstract from trade with other countries. For example, the trade volume in Canada in year $t$ in manufacturing industry $i$ is

$$m_{it} = \frac{\text{exports to US}_{it}}{\text{total output}_{it} - \text{total exports}_{it}}$$

From the measure of trade volumes per industry per year per country, the authors build a measure of trade volumes per year per industry by taking the geometric average between the country measures. Then they perform a regression with this measure as the dependent variable and industry-year tariffs as the independent variable, with other controls. The industry-year tariffs are a trade weighted average of the tariffs in each country. They provide two estimates of the elasticity of trade volumes to tariffs: 7.88 and 11.41. Clausing (2001) performs a similar exercises where she estimates this elasticity between 8.9 and 9.6.

Canadian productivity increased during the agreement. Trefler (2004) estimates the effect of the agreement on industry productivity in Canada. He works with several models, and his basic findings are robust to all the different specifications. He concludes that the decrease in tariffs increased the average productivity across Canadian industries by between 5% and 8.3%. Productivity is measured as value added per production worker, in 1992 constant prices. Potentially, this increase could be due to the reallocation of resources from least productive to more productive firms. However, it is unlikely that this reallocation has big effects on measured productivity. Kehoe and Ruhl (2008) show that a reallocation of resources has small, if any, effects on real productivity when measured in constant or chain weighted prices. Furthermore, I show in the context of my model in the appendix that the reallocation of resources has no effects on productivity in constant or chain weighted prices with no

innovation. Finally, Trefler also estimates the effect of productivity at the firm level, using a sample of firms that were operating in every year from 1980 through 1996, and finds evidence of similar increases in productivity, suggesting that there were considerable effects at the firm level.

To further explore the effect of the agreement on firm behavior, Lileeva and Trefler (2009) study the productivity and innovation patterns of firms that entered the export market during the agreement years. The innovation study is based on an innovation survey in Canada. Their findings suggest that firms entering the export market increased their innovation and their labor productivity relative to non exporters, or firms that were exporting before 1989. Given the lack of data on capital, the authors question whether the increases in labor productivity are due to these new exporters becoming larger, and therefore hiring more capital services. If this were the case, and the total factor productivity had stayed constant, then the price of these goods relative to other goods should not change, and consequently the domestic market share of these firms should remain unaltered. However, they find that the market share of these firms increased, suggesting that total factor productivity increased.

# 4 Calibration

I match the tradable sectors to the manufacturing sectors in the data, as suggested in Herrendorf and Valentinyi (2007). Country 1 is the U.S., and country 2 is Canada.

First, I choose a functional form for the exogenous distribution of firms $f(\theta)$ and the set $\Theta$. I assume $\Theta = [1, \bar{\theta}]$, and the $f(\theta)$ is a truncated Pareto density function, as in Ruhl (2008) and Leal (2009), among others. This function is

$$f(\theta) = \frac{\eta}{1 - \bar{\theta}^{-\eta}} \theta^{\eta - 1}$$

where $\theta \in [1, \bar{\theta}]$ and $\eta > 0$ is a curvature parameter. A consequence of this functional form is that the distribution of exporters according to employees in equilibrium is Pareto shaped. This is consistent with the findings of Luttmer (2007) for U.S. manufacturing large firms.

The measure of firms in the U.S. is normalized to 1 and the measure of firms in Canada is

$M$, which I calibrate next.

The elasticity of substitution $\sigma$ is key, since the reaction of trade volumes to tariff reductions is very sensitive to this parameter. I choose a parameter $\sigma$ consistent with empirical estimates. Broda and Weinstein (2006) provide the most detailed study of import elasticities, in which they estimate over 30,000 elasticities. The medians reported vary from 2.2 to 3.1, depending on the level of aggregation.[9] This study makes adjustments for the changing number of imported goods, as first suggested in Feenstra (1994). Other studies such as Reinert and Roland-Holst (1992) and Blonigen and Wilson (1999) do not make this adjustment, and estimate elasticities at the industry level, with average elasticities close to 1, and a maximum of 3.5. Ruhl (2008) shows that $\sigma = 2$ is consistent with these studies. This is also consistent with Broda and Weinstein, so I set $\sigma = 2$.

The weight $\gamma$ is set so that the ratio of tradable output to total output is of 15% in U.S. While I do not calibrate this share in Canada, this implies that the share in Canada is very close to that of U.S., consistent with the data. I normalize $N_{US} = 1$, and set $L_{CA} = 0.11$ to match the ratio of population in Canada and U.S. in 1988.

The remaining parameters are calibrated jointly. $\eta$ and $\bar{\theta}$ are calibrated to minimize the sum of squared difference between the employee size distribution of manufacturing firms in the U.S. data and model in 1992.[10] These data come from the 1992 Census of Manufactures published by the U.S. Department of Commerce. The census provides the share of firms with employees less than $H_i$, for given $H_i$'s. A problem that arises in this calibration is that due to the discrete jump between exporters and non exporters, the equilibrium distribution might have no firms with exactly $H_i$ employees. To solve this, when I minimize the difference between the employee size distribution in the data and model, I find the maximum number of employees in each bin so that the fraction of firms in that bin is the same in the model and the data.

[9] I report medians instead of averages because the authors restrict the elasticities to be larger than 1, which biases in the average elasticity, but the median does not change.

[10] I do not have data for 1988, so I use the closest available, which is 1992. This would be a problem if the distribution of firms in the U.S. in the model would have big changes from tariff drops. Turns out this hardly changes.

I solve

$$\min_{\eta,\bar{\theta}} \sum_{i=2}^{7} \left( \frac{H_m(\theta_i)}{H_m(\theta_1)} - \frac{H_d(\theta_i)}{H_d(\theta_1)} \right)^2$$

where $H_d(\theta_i)$ is labor in the firm $\theta_i$ in the data and $H_m(\theta_i)$ is the equivalent in the model. Figure 1 shows the proportion of firms in the data and model according to the employee bins in the data. These are also the bins in Ruhl (2008). Recall that in equilibrium, the model might feature no firm with exactly $H_i$ employees. Thus, to compare the model distribution with the data, I add (or subtract) employees to each bin in the model, and extrapolate linearly the share of firms. The resulting distribution is plotted in figure 1.

The fixed export cost $\kappa$ and the measure of firms in Canada $M$ are calibrated to match export shares in each country. I define a measure of trade volumes as in Head and Ries (2001). Denote the trade volume of country $i$ by $m_i$, then:

$$m_i = \frac{\text{Country } i \text{ Exports}}{\text{Country } i \text{ Total Tradable Output } - \text{Country } i \text{ Exports}}$$

Trade balances in the equilibrium in the model. In the data, trade does not balance, but the imbalance is small. The numerator I choose is therefore the average between exports from U.S. to Canada and from Canada to U.S. The trade volumes in 1988 are 3.5% in U.S. and 41.5% in Canada. The data are from the OECD STAN Database for Structural Analysis.

The last parameter to calibrate is $\alpha$. This is the elasticity of productivity to resources devoted to innovation. Ideally, I would use an estimate of this elasticity, but I found no such estimates. However, it turns out that $\alpha$ determines the productivity gains from trade liberalization, for which there are estimates. Trefler (2004) finds that the average Canadian labor productivity increased by between 5% and 8.3% due to the tariff reductions in the U.S.-Canada Free Trade Agreement between 1988 and 1996. Therefore, I set $\alpha$ such that the gain in labor productivity in Canada equals 5%, to be conservative.[11] I measure the increase in productivity in the model using chain weighted prices.[12]

[11] I also provide the results of targeting the upper bound, 8.3%.

[12] Trefler uses 1992 prices, which I do not compute. The chain weighted increase is a geometric average of the productivity increase using 1988 and 1996 prices. See Appendix A for details.

I use Trefler's (2004) tariffs. Trefler provides tariffs for bilateral trade per industry per year between 1980 and 1996. I set tariffs before trade liberalization to the average across industries in 1988 and equal to the average in 1996 for the after liberalization tariffs.

Notice that the last year of tariff drops was 1998. However, Trefler's analysis stops in 1996. Since I use Trefler's tariffs, I take 1996 as the last year of tariff drops.[13] The calibrated parameters are in Table 1.

# 5 Findings

I proceed as follows. First, I compute the equilibrium in the model using the calibrated parameters in table 1, with 1988 tariffs. This determines an initial trade volume in each country $m_{i0}$. Next, I keep all parameters fixed except the tariff rates, which I replace with their values for 1996. This results in a second trade volume $m_{i1}$. The trade volumes are computed as in Head and Ries (2001) and are explained in section 3. The trade elasticity I report is the percentage point change in trade volumes per percentage point change in tariffs.

## 5.1 Equilibrium Pre-Liberalization

Figure 2 shows the profits of firms in the United States. Profits in Canada are similar. These are an increasing, continuous function of type, with a kink at the cut-off type $\hat{\theta}_i$. Figure 4 shows the innovation rules and the extensive margin for U.S. and Canada with 1988 tariffs.

Innovation is higher for firms in U.S. This follows from the fact that the domestic market in U.S. is much larger than in Canada. The "jump" in innovation from non exporters to exporters is larger in Canada than in U.S., since the foreign market is more significant for Canadian exporters.

[13] Tariffs in 1996 are small but positive. I also computed the equilibrium with zero tariffs in the equilibrium post liberalization and the results are very similar.

## 5.2 Reducing Tariffs

I refer to variables in the equilibria with high tariffs as the before liberalization variables, and with low tariffs as after liberalization. Therefore, even when the model is static, new exporters are firm types that do not export under the high tariff regime, but do so when tariffs are low. Similarly, a firm increases innovation if its level of innovation is higher with lower tariffs than with high tariffs.

The reduction in tariffs has two main effects: it increases the measure of exporters (extensive margin), and it increases the sales volume of all exporters (intensive margin). This drives exporters to increase innovation, since its cost can be spread over a larger sales volume. This increase in innovation is greatest among new exporters, given that these firms face the largest increases in sales volumes.

Non exporters in each country decrease their innovation. This is because the price $P_i$ in country $i$ drops, and reduces domestic domestic demand for all firms. There are four factors that contribute to this. First, the tariff falls in each country. Second, the prices of all goods that are exported drop. Third, the price of imported goods drop. And fourth, the measure of imported goods $\mu_{mi}$ increases. From equation (3), all these factors contribute to lowering the index price $P_i$ in each country. While exporters can overcompensate this decrease in domestic demand with the increase in foreign demand, and therefore increase innovation, non exporters cannot, subsequently decreasing innovation.

Figures 5 and 6 show the change in innovation in each country. Quantitatively, only the extensive margin changes in U.S. The changes are larger in Canada. This is natural since the changes introduced by the Free Trade Agreement had a much larger effect in Canada than in the United States. The reason is that Canada is small relative to the U.S., and therefore the increase in demand for Canadian firms is more important than for U.S. firms.

The resulting trade elasticity is 9.6. This is within the empirical estimates found by Head and Ries (2001), that are between 7.88 and 11.41. In addition, Clausing (2001), estimates this elasticity between 8.9 and 9.6. The effects on productivity are highly asymmetric. While the increase in Canadian productivity is calibrated to match 5%, the increase in productivity in U.S. is much smaller, of 0.14%. This is because the expansion in Canadian firms is much

larger than the expansion in U.S. firms. The small effect on U.S. industries is consistent with the findings in Bernard and Jensen (1999), who argue that there is no evidence of productivity gains from trade among U.S. manufacturers.

Recall that Trefler estimates the increase in industry productivity between 5% and 8.3%, and I chose 5% to be conservative. Targeting 8.3% increases the elasticity to 10.5, which is still within the empirical range.

I next comparing this elasticity with the results of a model with no innovation. This implies taking the limit as $\alpha \to \infty$, and the increase in measured productivity is zero.[14] The elasticity is equal to 6.4, which implies that innovation amplifies the response of trade volumes to tariffs by 50 percent. Also, the elasticity in the model without innovation is below the empirical estimates.

The elasticity in the model without innovation is the same as the elasticity that results from lowering tariffs in Ruhl (2008), which is noteworthy given the models are very different. Both models share the main mechanism for increases in trade volumes given tariff drops and calibrate to very similar datasets. However, Ruhl studies business cycle properties of temporary fluctuations in exchange rates as well as the effects of tariff drops. Thus, his model is dynamic and requires several additional elements that I abstract for, such as firm death and birth. The fact that both studies find the same trade elasticities suggests that the additional elements in Ruhl are not important for the change in trade volumes from changes in tariffs.

## 5.3 Sensitivity Analysis

In this section I explore how the response of trade volumes to tariffs changes when two parameters change. First I change the elasticity of substitution parameter $\sigma$, and then the elasticity of productivity to resources devoted to innovation, $\alpha$. The sensitivity analysis is based on two types of exercises. The first is the standard type. It implies changing one parameter while keeping the rest of the parameters fixed, and documenting the effects of

[14] Productivity in constant or chain weighted prices does not change (see appendix A). Kehoe and Ruhl (2008), and Gibson (2006) elaborate more on this issue.

this change along relevant dimensions. The problem with this exercise is that, given the joint calibration, the model with changed parameters misses the targets discussed in section 4. Missing some of this targets is costly, leading to corner solutions with no exporters, for example. Thus, the margin of change in some of these parameters, especially $\sigma$, is very limited.

To address this issue, I perform a second, non standard, kind of sensitivity analysis. I change the value of a parameter such as $\sigma$ and calibrate the remaining parameters to match the targets in section 4.

The result of these two exercises have two different interpretations. The first exercise shows how the relevant parameter influences the results of the model along certain dimensions. The second exercise shows how the initial choice of targets affects the results. For example, my benchmark choice for $\alpha$ was set to match an increase in Canadian productivity of 5 percent. Changing $\alpha$, and recalibrating the model shows the results if I had chosen a different target.

Table 2 reports the changes in trade volumes, productivity, and in the extensive margin under different choices of $\sigma$. Except for the elasticity, these changes are the changes in Canada. The changes for the U.S. are smaller in magnitude, but in the same direction. Column 1 reports the statistics using the benchmark calibration, column 2 reduces $\sigma$ by one percent and leaves all other parameters unchanged, and column 3 sets $\sigma = 1.75$ and calibrates all other parameters to match the targets in section 4.

A lower elasticity of substitution implies that the response of the intensive margin is lower, since demands are less elastic. This accounts for the lower aggregate trade elasticity when $\sigma$ is smaller. Also, the productivity increase is smaller. Notice however that the change in the extensive margin is larger when $\sigma$ is smaller. This is consistent with Chaney (2008). He argues that tariffs have larger effects on the extensive margin when $\sigma$ is small. The reason is the following. When tariffs fall, less productive firms enter the export market. When the elasticity of substitution is high, competition is high, and therefore a lower productivity is a severe disadvantage. Instead, when this elasticity is low, firms are protected from competition, and more firms choose to enter the export market. Notice that the amplification effects of innovation are less when $\sigma$ is low. This is because when the elasticity of substitution is low, then the incentives to innovate of the new exporters are lower.

Table 3 reports the changes when the parameter $\alpha$ changes. Column 2 reduces $\alpha$ by one percent, and column 3 sets $\alpha = 1.75$, about 10 percent lower than the benchmark calibration. A reduction in $\alpha$ means that innovation is cheaper. Consequently, the increase in the trade volumes and productivity is larger. The changes in the extensive margin are smaller when $\alpha$ is small. Intuitively, the effects of cheaper innovation are similar to the effects of more elastic demands. Inelastic demands reduce the returns from innovating, since a drop in price has small effects, and therefore the incentives to innovate are small. Cheaper innovation increases these incentives by reducing the cost to lower the marginal cost, and hence the price. Therefore, the effect is similar as the effect of increasing $\sigma$. Notice that the amplification effect is larger when innovation is cheaper. This result is intuitive. Since the effects of cheaper innovation are similar to the effects of a larger $\sigma$, the amplification effects are increasing in $\sigma$.

# 6 Revisiting Yi's Calculations

The previous analysis focused on the effects of the reduction in tariffs between two countries. This is one of the two most common exercises in the trade literature. The second common exercise studies the time series effects of tariffs on trade volumes. Traditionally, these papers assume there are two symmetric countries, and study the effects of lowering tariffs over a long period of time. This was the approach in Yi (2003), which led him to conclude that models of the Krugman (1980) type and of the Backus et al. (1994) type fail when accounting for the effects of tariffs on trade volumes. In this section, I perform an exercise to study the time series effect of the reductions in tariffs in U.S., as in Yi. In contrast to his findings, I find that the drop in tariffs in U.S. during the same period of time can account for two thirds of the increase in trade volumes.

The exercise performed by Yi is the following. He builds a Krugman type model with two symmetric countries, and calibrates it to U.S. data in 1962. Then he drops tariffs to the level in 1999, and computes the change in trade volume as a function of the elasticity of substitution between varieties. He concludes that these models need to assume an elasticity parameter between 12 and 13 to generate an increase in trade volumes of the magnitude in the data, which is much higher than the empirical estimates. He reaches similar conclusions

when reproducing this exercise with an international business cycle model as in Backus, Kehoe and Kydland (1994). This exercise is subject to a wide variety of critiques, which mainly imply studying the historic effect of tariffs on U.S. trade volumes using a model with two symmetric countries. Notwithstanding, it has generally been successful in showing that these models perform poorly from a quantitative perspective.

I next reproduce this exercise with my model. First I describe the data. Tariffs in U.S. roughly decreased from 14% in 1962 to 3% in 1999 (Yi, 2003). Trade volumes (manufacturing exports relative to manufacturing output minus manufacturing exports) in 1963 are 4.3% and 13.8% in 1997[15], an increase of 220%. Data come from the Benchmark Input Output Tables published by the Bureau of Economic Analysis.

To study the symmetric country case, I change parameters as follows. I set $N_1 = N_2 = M = 1$, and calibrate the export cost to match the initial trade volumes. The rest of the parameters do not change from section 4. The resulting increase in trade volumes in the model is of 140%, about 2/3 of the actual increase in the data. In the model with no innovation, the increase is half as much, equal to 1/3 of that in the data. In Krugman type models, the increase is one tenth of the increase in the data.

Thus, the model with innovation can account for a much larger share of the increase in trade volumes than the Krugman model or the model with no innovation. It is important to note that we should not expect the drop in tariffs to account for all the increase in trade volumes. Other factors, including changes in non tariff barriers and transportation costs, are also responsible. Many of the GATT Rounds for trade liberalization concentrated more on non tariff barriers than on tariffs. For example, the Uruguay Round, from 1986 through 1993, concentrated on issues such as farming subsidies and intellectual property rights. With respect to transport costs, Bridgman (2008) finds that energy prices, and therefore costs of transportation, played an important role in the increase in trade in the 1980s.

[15] There were no tables computed for 1962 and 1999, so I use the closest available years: 1963 and 1997.

# 7 Extensions

I explore two extensions of the model. The first one introduces an exit decision. Recall that the tariff drop reduces the profits of non exporters in equilibrium. In the presence of a fixed cost of production, low productivity non exporters might choose to exit, which can potentially have important consequences on the trade elasticity and the productivity increase. Given the measure for trade volumes, the output of low productivity non exporters appears in the denominator. Thus, if these firms exit, the denominator becomes smaller, and the change in trade volumes increases.

The second extension explores the role of free entry. So far, I have assumed a fixed, exogenous measure of firms in each country. Atkeson and Burstein (2009) find that free entry has important consequences on the increase in industry productivity from tariff drops. Thus, I study the effects of a free entry condition on the results.

## 7.1 Firm Exit

In Melitz models, a drop in tariffs encourages the exit of low productivity firms. This is because there is a fixed cost of production, and firms incur it only when their profits can cover it. Notice that my model includes a selection effect: following a reduction in trade barriers, non exporters become smaller relative to exporters. However, one could argue that the effects of selection are sharper with firm exit.

First I point out that empirically, the evidence suggests that there was exit during the Free Trade Agreement between Canada and the United States. Lileeva (2008) finds that the tariff reductions boosted the exit of moderately productive non exporters in Canada. Thus, this could potentially be a costly abstraction of my model. In this section, I show that quantitatively this is not important.

A problem that arises is that in a static model, there can be no exit. Therefore, I interpret exit as follows. As in the benchmark model, there is a pool of monopolists with the knowledge to produce differentiated tradable goods. In this section, I assume that a fixed cost $\kappa_F$ in

units of labor is required for production. That is, the production function of a good $\omega$ is

$$y(\omega) = \begin{cases} A(\theta(\omega), z)n - \kappa_F \text{ if } n > 0 \\ 0 \text{ otherwise} \end{cases}$$

If, for a given type, the profits from operating are positive, then the firm incurs the fixed operation cost and engages in production activities. Otherwise, it does not produce, and I refer to this as firm exit.

I calibrate the parameters common to the benchmark model as in section 4. The new parameter is the fixed production cost. It turns out that the value of this parameter has no effect on the equilibrium trade elasticity or productivity gain. The reason is the following. In equilibrium, exit follows a cut-off rule, as in Melitz (2003). Firm types above a certain type choose to produce, while the rest do not. The introduction of the fixed cost of operation shifts the distribution of firm types, with no effects on the trade elasticity or productivity gains. Therefore, the value of the fixed cost of operation is not important, although the existence of it is. This is because the exit cutoff changes when tariffs change.

There is one additional element to consider. For the fixed cost to be operative, a firm with the lowest possible draw ($\theta = 1$), must choose not to produce. One way to do this is by setting the fixed cost of operation high enough. The problem with doing this is that if this is in terms of the non tradable good, operating firms might require more units of the non tradable good than what can feasibly be produced, and the equilibrium might not exist. To get around this problem, I can scale up or down the profits of firms by changing the innovation cost function to $c(z) = \delta z^{\alpha}$. It turns out that a change in $\delta$ has no effect on the productivity gains from trade or the response of trade volumes. Thus, I set $\delta$ so that the profits are low enough such that firms with $\theta = 1$ choose not to produce given the fixed cost of operation, and the solution is feasible.

The fixed production cost has very small quantitative effects. The elasticity of substitution increases by 6%, from 9.6 to 10.3. I also study the predictions of a model with no innovation ($\alpha = 0$) with exit. There is no study of the trade elasticity in such a model[16]. The resulting elasticity is below the empirical estimates. Without innovation, exit increases the elasticity

[16]Ruhl (2008) does not model exit.

by 8%, from 6.4 to 6.9. Note that the amplification effect of innovation with an exit decision is also close to 50 percent. Thus, introducing firm exit does not change quantitatively the results even with no innovation.

Notice that the change without innovation is larger. This is because the model with innovation and no firm exit already includes a selection effect from a drop in trade barriers. The model with no innovation does not.

## 7.2 Free Entry

Atkeson and Burstein (2009) ("AB") study the role of innovation on productivity gains from trade. Their general conclusion is that, while trade liberalization may have important reallocation of innovation from non exporters to exporters, the aggregate effects are quantitatively small, if any. An important element in their conclusions is the role of free entry, which I do not assume. Therefore, in this section I introduce a free entry assumption to the model and compare my results to theirs.

I first decribe AB's findings. The first set of results they provide are analytic. These are based on three models, and in all of these cases they prove that to a first order degree approximation, innovation has no effect on the productivity gains from trade at the aggregate level. A key element driving this result is that, in equilibrium, the optimal allocation of labor between the tradable sector and the innovation sector does not change with tariffs. The reason is that a reduction in tariffs increases the demand for the innovation good and labor in the tradable sector. In many cases the authors study, the increase is proportionally the same. Since the total supply of labor is fixed, the adjustment is through prices. Thus, intuitively, a change in trade costs has no effect on the aggregate production of the innovation good, and therefore aggregate innovation and aggregate productivity stay constant.

The first model they study is a standard Melitz (2003) model with no innovation at the firm level. The second is a Krugman (1980) model (the fixed export cost is zero, so every firm exports) extended with firm heterogeneity and the possibility of innovation, and the third is a model closer to mine, except that the measure of exporters is exogenously determined and therefore does not change with tariffs. That is, firms can choose their innovation levels, but

cannot choose whether to become exporters or not.

Next I describe how the third model works, since this is the closest to my model. Old exporters increase innovation, while non exporters reduce it. To a first order degree approximation, these effects cancel out. Since there are no new exporters, aggregate innovation does not change. The changes in my model are in the same direction. In the quantitative section I have shown that firms that export under both tariffs increase their innovation, while firms that never export decrease it, consistent with AB. However, the behavior of the firms that enter the export market only when tariffs are low is key, and AB close this channel. Innovation of firms in the extensive margin increases considerably, and this accounts for most changes in aggregate innovation.

Next AB analyze more cases, but from a numerical perspective. In particular, they find that in some cases, innovation can have aggregate effects on productivity, although these are likely to be quantitatively small. In the version of their model that is most similar to mine, in which firms can innovate and make export decisions, lower innovation costs increase the effects of a reduction in tariffs on aggregate productivity. Quantitatively, a percentage point drop in tariffs can increase aggregate innovation by up to 0.2 percentage points.

To introduce a free entry assumption, I modify the model as follows. There is an unbounded pool of firms in country $i$, that draw a parameter $\theta$ from the distribution $f(\theta)$ by incurring a fixed entry cost in terms of labor. A firm enters as long as the expected payoff from entry is larger than the fixed cost. Thus, in equilibrium, average variable profits equal the entry cost. After incurring this cost, all the uncertainty disappears, and the model works as before.

A problem with this model is that I cannot match trade volumes in each country with a common fixed entry cost *and* a common export cost. Since the main goal of this section is to show the effects of introducing a free entry condition, I solve the problem by allowing this cost to differ across countries.

To keep this extension as close as possible to the benchmark model, I fix the entry cost so that the equilibrium with high tariffs is the same as the equilibrium without entry costs. In other words, I set the entry cost in country $i$ equal to average profits before paying the entry cost in country $i$. I compare the equilibria in the model with high and low tariffs keeping the entry cost constant.

The results hardly change. First, I have no problem in matching the productivity increase target. Second, the trade elasticity is very similar to the model with no free entry, decreasing from 9.6 to 9.5. In the model with no innovation, the elasticity drops from 6.4 to 6.2. Thus, the amplification from innovation with an entry decision is close to 50 percent.

Next, I argue that the results of my model are in line with AB's. To do this, I first highlight some differences between AB's work and mine. Then I show that once these differences are accounted for, the effect of tariffs on aggregate productivity are similar in both models.

A first difference is the way the productivity increase is measured. As explained in section 4, I measure the increase in real productivity as in the data, that is, I measure the current price productivity in both periods, before and after the drops in tariffs, and then deflate this by the appropriate price level. In AB, productivity is the ratio of final output ($C$) to labor in the production of tradables. This assumes that the statistician can observe real output changes, which in the data he cannot.

In terms of the model, an important difference is related to the trade costs. The authors assume that these costs are of the iceberg type, that is, an exporter must ship $(1+\tau)q$ units for the importer to consume $q$ units. This assumption undermines the changes introduced by a reduction in trade costs. To see this, I first describe the effects of a drop in trade costs when tariffs are as in my model, and then compare this to the iceberg cost specification.

A drop in tariffs in my model increases the sales volume of exporters via an increase in demand. Thus, exporters increase production, and therefore profits (since price is a mark-up over marginal cost, profits are proportional to units sold).

If trade costs are of the iceberg type, there is an additional effect. The exporter produces less units that are lost in transport, which were also marked up, so the increase in production and profits given the change in tariff is lower with iceberg costs.

To illustrate this, consider a standard model with no innovation. The export revenues of a firm that charges price $p$ and exports a quantity $y$ are $py$. Suppose there is a measure 1 of consumers importing the good, and the demand is given by $q = K(1+\tau)^{-\sigma}p^{-\sigma}$, where $K$ is

a constant. When trade costs are as in my model, $y = q$, so firm revenue is

$$py = K(1+\tau)^{-\sigma}p^{1-\sigma}$$

The elasticity of revenues to a drop in tariffs is $\sigma$. If trade costs are of the iceberg type, then $y = (1+\tau)q$, and firm revenue is

$$py = K(1+\tau)^{1-\sigma}p^{1-\sigma}$$

In this case, the elasticity of revenues to a drop in tariffs is $\sigma - 1$, which is lower than the elasticity without the iceberg assumption.

Therefore the overall changes introduced by trade liberalization are smaller when trade costs are iceberg costs. Thus, the different assumptions regarding trade costs are likely to generate larger changes in my model.

One last important difference between the models is the existence of a non tradable sector in mine. Most of the authors' results are driven by the equilibrium condition that specifies that the elasticity of the allocation of labor across sectors with respect to changes in trade costs is small. This is because the demand for labor for the production of tradable goods and the demand for innovation both increase with a reduction in tariffs, and both cannot increase simultaneously without a third sector. Balistreri et al. (2009) show that adding a non tradable sector has important consequences on aggregate changes, since the labor in the production of tradable goods and in the production of the innovation good can increase simultaneously. This is a second modeling difference that accounts for higher productivity changes in my model.

Next, I measure the increase in productivity in my model as AB do in theirs. Since the authors assume two symmetric countries, I focus on the exercise of section 6, that assumes two symmetric countries. I find that a percentage point drop in tariffs increases aggregate innovation by 0.4 percentage points. This is close to their increase of 0.2, especially when considering the modeling assumption differences that are likely to provide higher changes in my model.

This exercise shows that my results are consistent with AB. Moreover, it shows that even

when the effect of innovation on aggregate productivity may be low, the effect on the elasticity of trade volumes is large.

# 8 Discussion

Next, I argue that innovation can help us understand why productivity gains from trade differ across countries. Empirically, productivity gains from trade are large in small industrialized countries. For example, Trefler (2004) finds gains of up to 15% in the industries in Canada which faced the largest reduction in U.S. tariffs during the Free Trade Agreement. De Loecker (2007) finds firm gains from trade liberalization in Slovenia close to 50%.

The evidence is less conclusive for developing economies. While Van Biesebroeck (2005) finds gains from trade among African countries, Havrylyshyn (1990) concludes that there is no evidence of gains from trade among developing countries based on a survey of studies, and Clerides, Lach and Tybout (1998) do not find evidence of productivity gains from trade among exporting firms in Colombia, Mexico and Morocco. Furthermore, Eslava et al. (2009) find productivity gains from trade in Colombia during a different period of trade liberalization that the one studied by Clerides et al.

My model suggests that the different findings can at least in part be attributed to differences in innovation costs. Where innovation costs are higher, productivity gains from trade are likely to be lower, and in developing countries these costs are likely to be higher. The reason is that institutions and policies are generally an important determinant of innovation costs. In developing countries, high labor market regulation, corruption, and low enforcement of intellectual property rights tend to increase the costs of innovation. Botero et al. (2004) find that the labor market is more heavily regulated in developing economies. Heckman and Pagés (2004) find that Latin American labor markets are more heavily regulated. Among developing economies, corruption is usually higher, and the enforcement of intellectual property rights lower (Djankov et al. (2002), and Chandima Dedigama (2009)).

This explanation complements that of Kambourov (2009), who argues that highly regulated labor markets reduce the productivity gains from trade by making the reallocation of workers to more productive sectors slower. Innovation provides a second channel through which

developing countries should expect lower gains from trade.

# 9 Conclusion

This paper develops a model of international trade with a costly productivity decision, using a framework based on Melitz (2003). Recently, the international trade literature has made extensive use of Melitz type models to model trade patterns. While these models have been concentrating in accounting for aspects more related to the industrial organization literature, such as firm entry, exit, export, and growth, they have not successfully accounted for the effects of trade liberalization. This lack of success led Arkolakis, Demidova, Klenow and Rodríguez Clare (2008) to the conclusion that the only attraction of these models is their power to qualitatively account for many micro level observations, such as exporters being relative larger than non exporters, and entry and exit patterns into the economy and into the export market.

I go beyond the qualitative accounting of microeconomic observations. My work represents the first successful attempt to quantitatively account for the effect of reductions in tariffs on *aggregate* trade volumes. This success should encourage the use of models of the Melitz type with innovation to study international trade at the macroeconomic level.

The main result is that introducing innovation to a model of international trade amplifies the effect of trade liberalization on trade volumes considerably, by about 50 percent. Moreover, only with innovation can the model generate a reaction of trade volumes to tariff drops of the magnitude observed in the data during the Canada-U.S. Free Trade Agreement.

A conclusion that should be drawn from this study is the need to introduce innovation into models of international trade. For example, is a model of trade and innovation consistent with the real business cycle fluctuations? It has been widely documented that the fluctuations in trade volumes are much smaller when the real exchange rates fluctuate temporarily than when tariffs change permanently. Can a model with innovation account for this difference? A second question is, what accounts for the different productivity gains from trade across countries? In this work I suggest that size matters, and so do costs of innovation.

To answer these questions, the literature must develop more models of international trade with innovation. I address the first question in Rubini (2009), where I develop a model to study the different reactions of trade volumes to short-run exchange rate volatility and long-run tariff reductions with innovation. To answer the second question, we need more detailed studies of the costs and returns from innovation across countries. This last point has become increasingly important given the increasing participation of developing countries such as China, India and Brazil in world trade.

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Table 1: Calibration

| Parameter | Target | Value Innov. | Value No Innov. |
|---|---|---|---|
| $\alpha$ | CA Prod. Increase = 5% | 1.90 | $\infty$ |
| $\sigma$ | Estimated demand elast. for imports | 2.00 | 2.00 |
| $\eta$ | U.S. distribution of firms | 2.13 | 1.01 |
| $\bar{\theta}$ | U.S. distribution of firms | 21.72 | 664.32 |
| $\gamma$ | Share of manufacturing output | 0.30 | 0.30 |
| $\kappa$ | CA and U.S. trade volumes | 5.55 | 3.16 |
| $N_1$ | Normalization | 1.00 | 1.00 |
| $N_2$ | Pop. CA rel to U.S. | 0.11 | 0.11 |
| $M$ | CA and U.S. trade volumes | 0.38 | 0.20 |
| $\tau_{10}$ | Avg. U.S. tariffs for CA imports 1988 | 4.00% | 4.00% |
| $\tau_{20}$ | Avg. CA tariffs for U.S. imports 1988 | 8.2% | 8.20% |
| $\tau_{11}$ | Avg. U.S. tariffs for CA imports 1996 | 0.81% | 0.81% |
| $\tau_{21}$ | Avg. CA tariffs for U.S. imports 1996 | 1.45% | 1.45% |

Table 2: Different values of $\sigma$

| | $\sigma = 2.00$<br>Benchmark | $\sigma = 1.98$<br>Other parameters fixed | $\sigma = 1.75$<br>Calibrate other parameters |
|---|---|---|---|
| Elasticity<br>$\Delta$ productivity<br>$\Delta\hat{\theta}$ | 9.63<br>5.00%<br>-3.55% | 9.27<br>4.51%<br>-4.01% | 8.02<br>5.00%<br>-4.91% |
| Elasticity no innovation | 6.37 | 6.20 | 5.51 |
| Amplification | 51.18% | 49.52% | 45.55% |

Table 3: Different values of $\alpha$

| | $\alpha = 1.90$<br>Benchmark | $\alpha = 1.88$<br>Other parameters fixed | $\alpha = 1.75$<br>Calibrate other parameters |
|---|---|---|---|
| Elasticity<br>$\Delta$ productivity<br>$\Delta\hat{\theta}$ | 9.63<br>5%<br>-3.55% | 9.96<br>5.3%<br>-3.41% | 10.4<br>7.88%<br>-3.03% |
| Elasticity no innovation | 6.37 | 6.37 | 6.37 |
| Amplification | 51.18% | 56.36% | 63.27% |

Figure 1: Fit of the Size Distribution of Firms

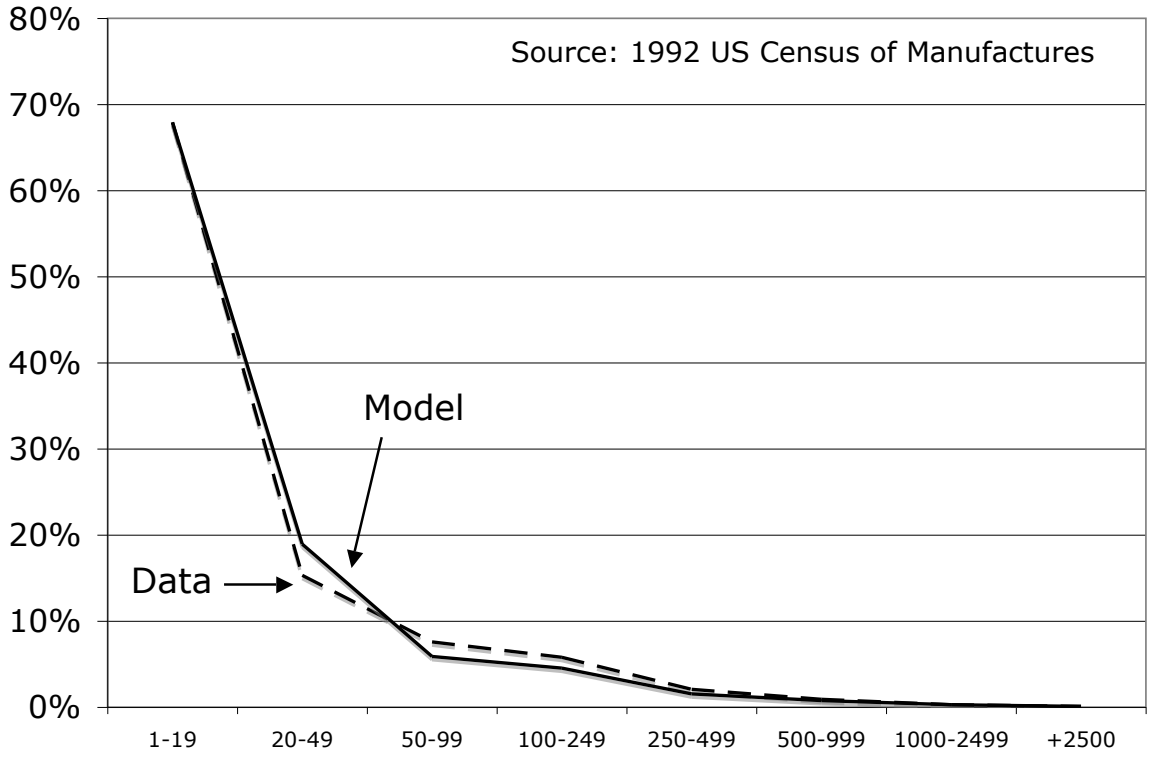


Figure 2: Exporter and Non exporter Profits

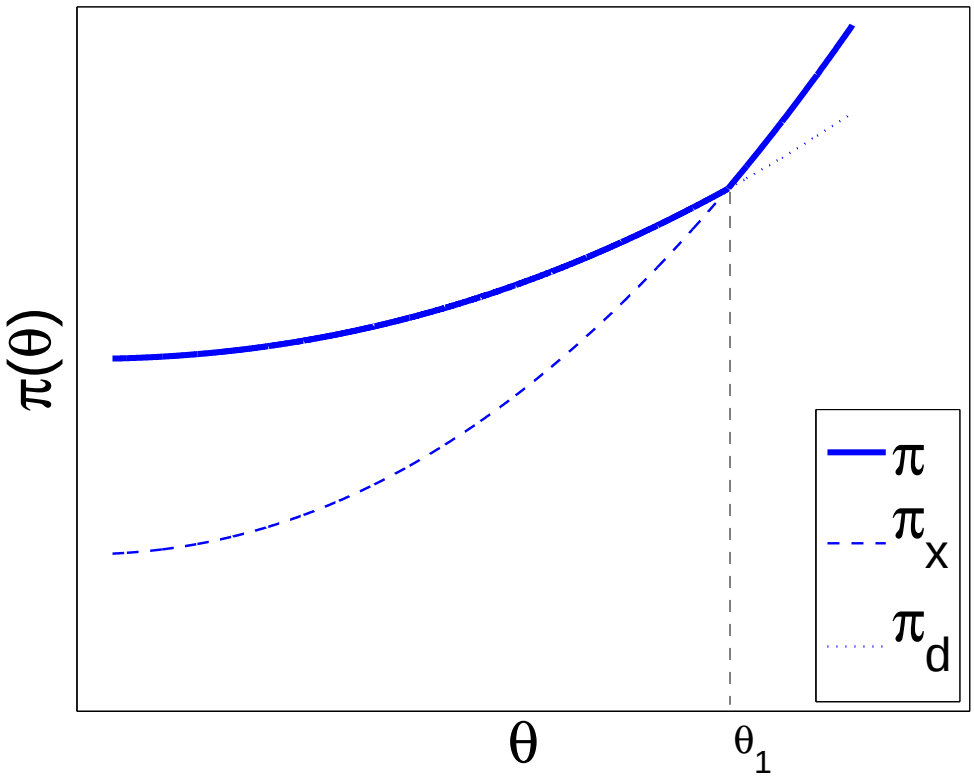


Figure 3: Innovation Rules

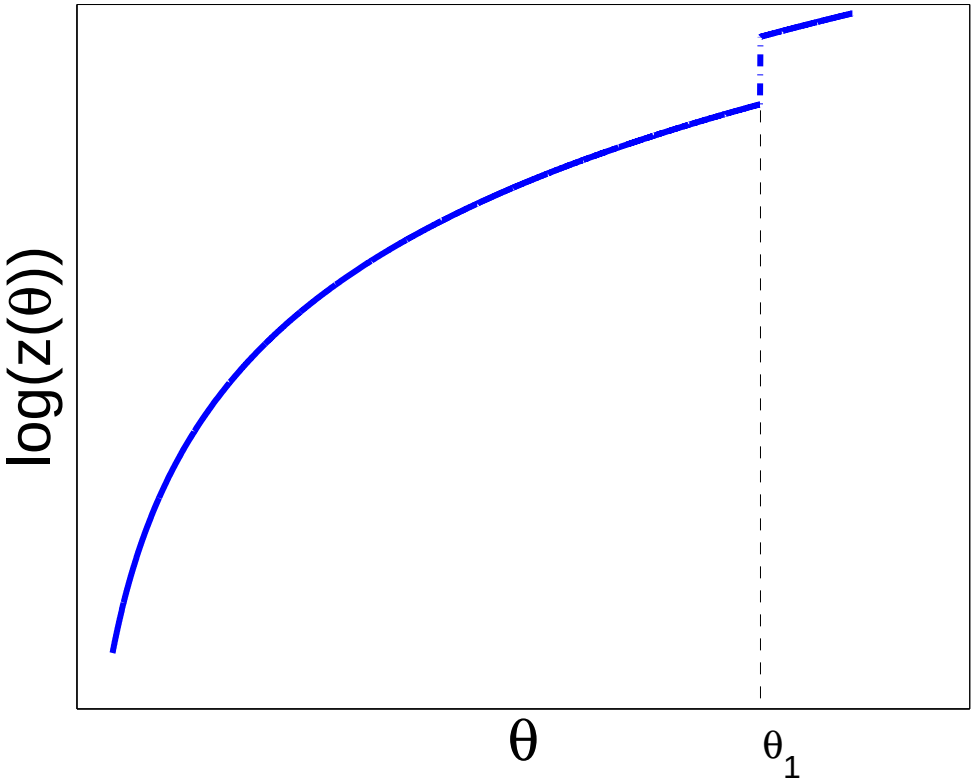


Figure 4: Innovation Before Tariff Drops

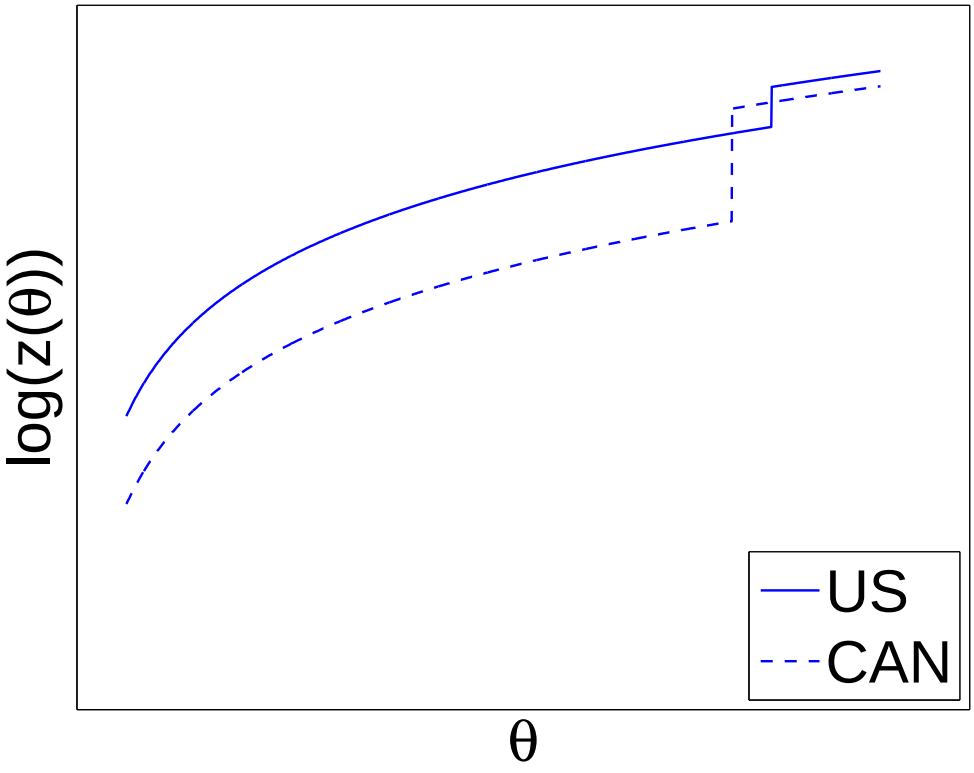


Figure 5: US Innovation

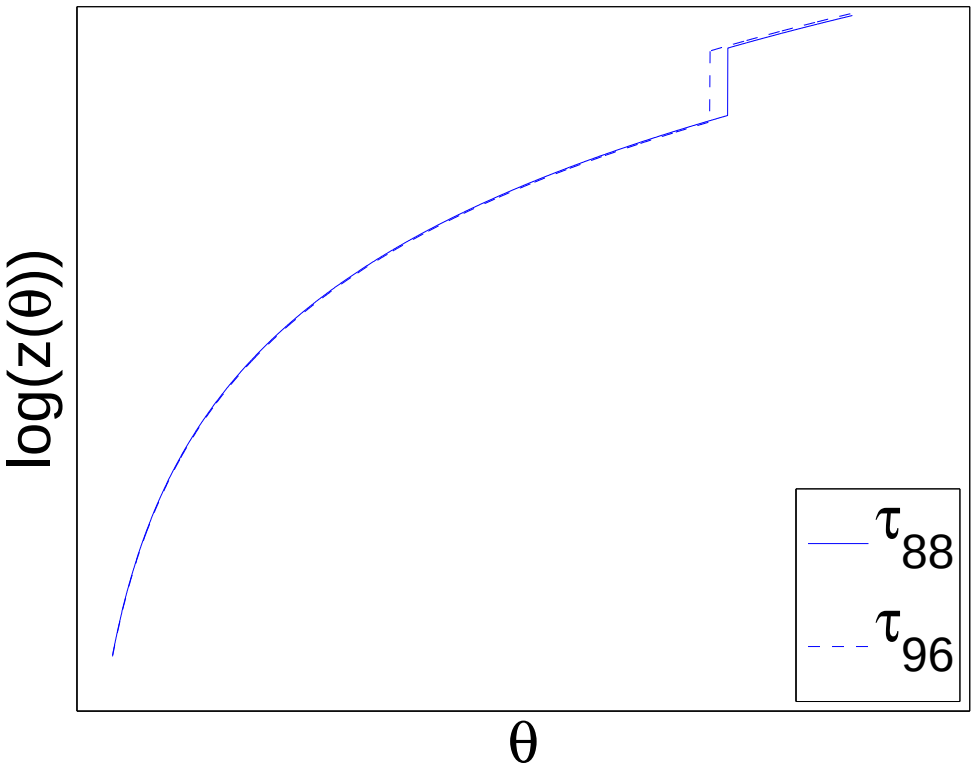
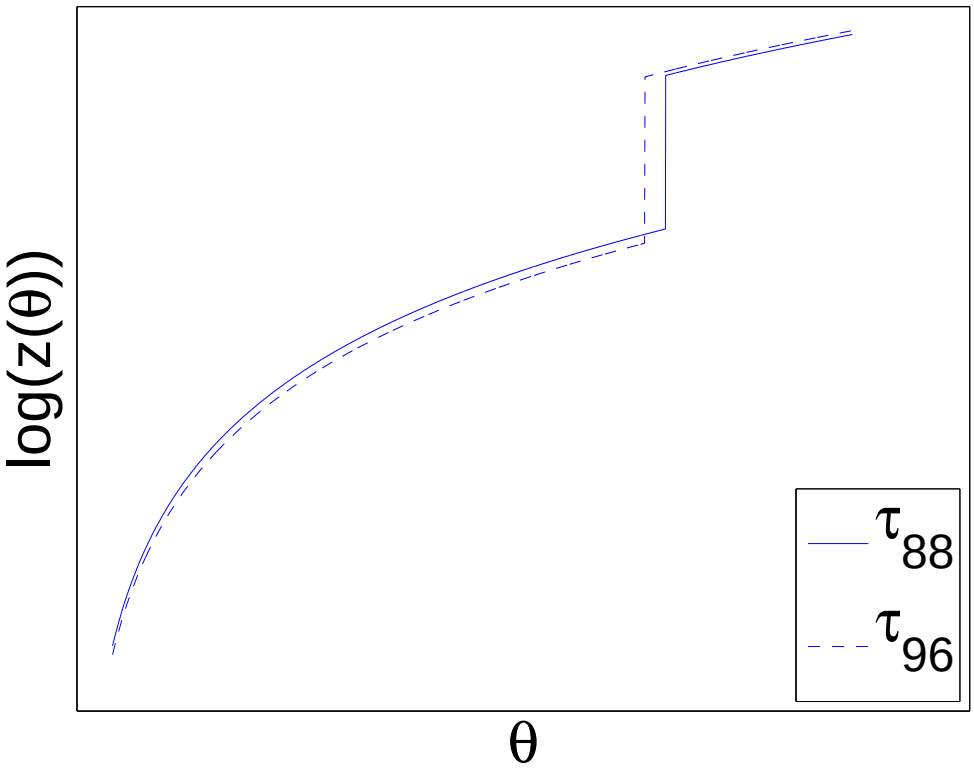


Figure 6: CA Innovation



# APPENDIX

## A Measuring Productivity Increase in the Model

I measure the increase in productivity using chain weighted prices. In the data, real productivity is measured by calculating current productivity, and deflating it by some price level. The price level is built by choosing a bundle off goods, and computing the value of that bundle using current and base period prices.

Denote by period 0 variables those variables in the equilibrium with high tariffs and period 1 variables the low tariff variables. I deflate the productivity increase in current prices by two price deflators, one that uses period 0 quantities and one that uses period 1 quantities. The final increase in productivity is a geometric average of the increase using both indices, that is, a chain weighted productivity increase.

The increase in current price productivity in country $i$ is

$$\Delta Prod_i = \frac{\dfrac{\int_{\Omega_i} p_1(\omega)Q_1(\omega)d\omega}{\int_{\Omega_i} h_1(\omega)d\omega}}{\dfrac{\int_{\Omega_i} p_0(\omega)Q_0(\omega)d\omega}{\int_{\Omega_i} h_0(\omega)d\omega}} = \frac{w_1}{w_0} \tag{A-1}$$

where $h_t(\omega)$ is labor used in production of good $\omega$ in period $t$ and $Q_t(\omega) = Q_{1t}(\omega) + Q_{2t}(\omega)$ is total output of good $\omega$ in period $t$.

I deflate $\Delta Prod_i$ using two price indeces, one using period 0 quantities (Laspeyres price deflator), and a second one using period 1 quantities (Paasche price deflator). These deflators are

$$P_L = \frac{\int_{\hat{\Omega}_i} p_1(\omega)Q_0(\omega)d\omega}{\int_{\hat{\Omega}_i} p_0(\omega)Q_0(\omega)d\omega}$$
$$P_P = \frac{\int_{\hat{\Omega}_i} p_1(\omega)Q_1(\omega)d\omega}{\int_{\hat{\Omega}_i} p_0(\omega)Q_1(\omega)d\omega}$$

where $\hat{\Omega}$ is the set of goods in the bundle chosen to calculate the price deflator.

## A.1 No Innovation

The set of goods is constant across periods, so $\hat{\Omega}_i = \Omega_i$. The model with no innovation has no increases in real productivity from lower tariffs. In this case, $P_L = P_P = \frac{w_1}{w_0}$, so that the deflated $\Delta Prod$ equals 1. This follows from, for $t = 0, 1$, $Q_t(\omega) = K_t p_t(\omega)^{-\sigma}$, where $K_t$ is a constant that does not depend on $\omega$, and $p_t(\omega) = \frac{\sigma}{\sigma-1}\frac{w_t}{\theta(\omega)}$.

## A.2 Base Model

As in the case with no innovation, in the base model $\hat{\Omega}_i = \Omega_i$. Next I show that in this case, deflating the current price productivity increase amounts to computing the productivity increase in constant prices. First I deflate the productivity increase given by equation (A-1) using the Laspeyres price index.

$$\Delta Prod_{iL} = \frac{\frac{\int_{\Omega_i} p_1(\omega)Q_1(\omega)d\omega}{\int_{\Omega_i} h_1(\omega)d\omega}}{\frac{\int_{\Omega_i} p_0(\omega)Q_0(\omega)d\omega}{\int_{\Omega_i} h_0(\omega)d\omega}} \div P_L = \frac{\frac{\int_{\Omega_i} p_1(\omega)Q_1(\omega)d\omega}{\int_{\Omega_i} h_1(\omega)d\omega}}{\frac{\int_{\Omega_i} p_1(\omega)Q_0(\omega)d\omega}{\int_{\Omega_i} h_0(\omega)d\omega}}$$

Similarly, I find the productivity increase using the Paasche price deflator

$$\Delta Prod_{iP} = \frac{\frac{\int_{\Omega_i} p_1(\omega)Q_1(\omega)d\omega}{\int_{\Omega_i} h_1(\omega)d\omega}}{\frac{\int_{\Omega_i} p_0(\omega)Q_0(\omega)d\omega}{\int_{\Omega_i} h_0(\omega)d\omega}} \div P_P = \frac{\frac{\int_{\Omega_i} p_0(\omega)Q_1(\omega)d\omega}{\int_{\Omega_i} h_1(\omega)d\omega}}{\frac{\int_{\Omega_i} p_0(\omega)Q_0(\omega)d\omega}{\int h_0(\omega)d\omega}}$$

The chain weighted price productivity increase is the geometric average between these two productivity increases

$$\Delta Prod_i = \sqrt{\Delta Prod_{iP} \times \Delta Prod_{iL}}$$

## A.3 Extensions

In the extensions in section 7, the set of goods produced in each country changes, so I must take a stance on what the bundle of goods used to compute the price deflator is. I choose the goods that are produced in both periods as the bundle.