Learning, Knowledge Diffusion and the Gains from Globalization

Kunal Dasgupta *

First draft : November 2008

This draft : November 2009

Abstract

We develop a dynamic, general equilibrium model to understand how multinationals affect host countries through knowledge diffusion. Workers learn from their managers and knowledge diffusion takes place through worker mobility. We identify two forces that determine wages: the labor demand effect and the learning effect. The former tends to raise wages while the latter tends to reduce it. We show that in a model without learning, an integrated steady-state equilibrium, in which incumbent host country managers operate alongside multinationals, can never be a Pareto improvement for the host country. In contrast, we present a novel mechanism through which a Pareto improvement occurs in the presence of learning dynamics. We study how integration affects the life time earnings of agents and the degree of inequality in the host country, as well as, analyze the pattern of multinational activity. In the quantitative section of the paper, we calibrate our model to fit key moments from the U.S. wage distribution and quantify gains from integration. Our estimates suggest that learning produces welfare gains that range from 2% for the middle-income countries to 43% for the low-income countries.

*University of Toronto. E-mail : *kunal.dasgupta@utoronto.ca* I am indebted to Esteban Rossi-Hansberg, without whose guidance and support this paper would never have been possible. I am also grateful to Gene Grossman and Marc Melitz for their valuable advice and encouragement. I would also like to thank Pol Antràs, David Atkin, Thomas Chaney, Cecilia Fieler, Luis Garicano, Stefania Garetto, Marc Muendler, Miklos Koren, Per Krusell, Jordi Mondria, Alexander Monge-Naranjo, Stephen Redding, Xianwen Shi, Colin Stewart, Daniel Trefler, Vinayak Tripathi, Jonathan Vogel and seminar participants in various institutions for their comments and suggestions. Financial support from the International Economics Section, Princeton University is greatly appreciated.

1 Introduction

An important asset that a Multinational Enterprise (MNE) brings to a foreign market is superior knowledge. This is a widely held belief not only among academics (See Kogut and Zander, 1993), but among policymakers in many host countries as well. The latter also tend to believe that a part of this knowledge spills over to the economy of the host country. Blomström and Kokko (1998) point out that most of the host countries have liberalized their Foreign Direct Investment (FDI) regimes since the early 1980s; many of them have been actively trying to woo FDI since then. Naturally, a substantial body of literature that tries to understand how this knowledge diffuses to domestically owned firms and its consequences, has developed (Lipsey and Sjöholm, 2005).

The theoretical literature has studied various channels for knowledge diffusion.[1] A channel that has received relatively little attention is diffusion through worker mobility. [2] MNEs provide domestic workers with access to a vast pool of advanced knowledge. By learning from MNEs, the workers increase their productive capabilities; some of them even go on to start their own firms. Giarratana et al. (2004) look at the spin-offs from MNEs that were created in India after the country liberalized in 1991. Based on interviews conducted with the founders of some of these spin-offs, Giarratana et al. (2004) conclude that the founders bring a high-level of technological expertise from the MNEs to the new firms. The flow of knowledge, however, does not stop there. Workers of successful spin-offs, in turn, leave to start their own ventures. [3]

Furthermore, the educational system in many developing countries is not geared towards meeting the needs of the industry. As noted in the UNCTAD (1992), *"A wide gap exists in many developing countries between the demand for and the supply of knowledgeable and skilled indigenous managerial talent. The gap represents a major constraint to the development of entrepreneurial activities and the utilization of foreign direct investment."* In these countries, foreign MNEs provide the domestic workers with the opportunity to acquire marketable skills.

[1]Rodríguez-Clare (1996) focuses on the impact of multinationals on developing countries through the generation of backward and forward linkages. In Markusen and Venables (1999), the effect on host country firms depends on the relative strengths of competition and linkage effects.

[2]There is evidence that MNEs provide more and better training to its workers compared to domestic firms (Gershenberg, 1987). Görg et al. (2007) find that workers who are trained in subsidiaries of multinational firms have a steeper wage gradient compared to workers who receive training in local firms. They take this as evidence that foreign subsidiaries provide more effective training to workers.

[3]The classic example is the Bangladeshi garments industry. Easterly (2001) talks about how a single Korean company, Daewoo Corporation, was instrumental in turning the Bangladeshi garments industry into a $2 billion export-oriented industry. In 1979, Daewoo entered into an agreement with a Bangladeshi company, Desh Garment Ltd., whereby Daewoo agreed to train some of Desh's workers in return for royalties and sales commissions. During the 1980s, almost all of the trained workers left Desh to start their own garment export firms."

In China, for example, many potential managers perceive the MNEs as schools where they can train themselves; many of them leave to start their own business, once they have the required expertise.[4]

Knowledge acquisition by a worker not only affects his life time earnings, but also the productivity and profitability of firms that hire him. As workers learn, the entire knowledge distribution of the host country changes, which, in turn, has aggregate implications. In this paper, we present a model that allows us to shed light on the impact of MNE entry on welfare and earning dynamics at the individual level, as well as, on inequality and the pattern of multinational activity at the aggregate level.

We develop a dynamic, general equilibrium model, with heterogeneous agents. Our model adds learning to a framework that is similar to Antràs et al. (2006). In our model, production is carried out in firms by workers, supervised by a manager. Agents make two decisions: (1) whether to be a worker or a manager, and (2) whom to match with, i.e., a manager chooses his workers while a worker chooses for whom to work. The production technology exhibits complementarity in knowledge.[5] Agents are endowed with knowledge at birth, but can acquire more by working in firms. We assume that workers learn on the job. Learning is stochastic, with expected learning being an increasing function of both the worker's and the manager's knowledge. Thus, there is also 'complementarity' in learning.

We have an overlapping generations model, where a newborn agent draws his knowledge from an exogenously given distribution. Agents also face a constant probability of death every period. Apart from the exogenous entry and exit of agents, there is endogenous movement of agents within the knowledge distribution due to learning. We initially focus on a closed country. We show that complementarity in the production and learning technologies leads to positive assortative matching (PAM), whereby more knowledgeable workers team with more knowledgeable managers to produce and learn. The equilibrium is characterized by a threshold level of knowledge, such that every agent below the threshold is a worker while those above are managers. The combination of PAM and learning, however, implies that every agent who starts his life as a worker, works for better and better managers until he himself becomes a manager, provided that he survives long enough. Therefore, agents move up along the knowledge ladder and their earnings increase over their lifetimes. Birth, learning and death determine the

[4]This could partly explain the high turnover rates that the county has witnessed in recent years. The Economist reports that employee turnover rates have gone up from 8.3% in 2001 to 11.3% in 2004. See "China's people problem", *The Economist*, 14th April, 2005.

[5]This means that the marginal productivity of an agent is enhanced if he matches with more knowledgeable agents.

evolution of the knowledge distribution.

Next, we look at the situation where the Home country (or the host country) integrates with a Foreign country (or the source country), the only difference between the two countries being their newborn knowledge distribution.[6] In particular, the Foreign newborn distribution dominates the Home newborn distribution in the sense of first-order stochastic dominance. This is a way to capture the assumption that the Foreign country has relatively more knowledgeable agents compared to the Home country. Integration, in our model, means that managers are able to form international teams.[7]

In order to understand the impact of integration on the welfare of Home agents, we first consider a version of the model where agents do not learn. Integration leads to a re-adjustment among Home managers. The least knowledgeable among them exit; among those who persist as managers, some are matched with less knowledgeable workers compared to autarky. Under this situation, we show that some agents are necessarily worse off compared to autarky. Introducing learning complicates the analysis because the knowledge distribution becomes endogenous. In this setting, we identify two effects that determine wages. Integration increases the competition for workers, which tends to raise wages. We call this the labor demand effect. But there is another effect. The entry of MNEs creates the possibility for the workers to be matched with more knowledgeable managers. By working for the MNEs, workers can learn more and earn more than under autarky. The result that MNEs hire more knowledgeable workers, however, implies that the less knowledgeable workers can expect to work for the MNEs in the future only if they learn from the less knowledgeable Home managers. Thus, learning creates a rent. The managers extract a part of this rent by paying lower wages and thereby internalize the knowledge "spillover". We call this the learning effect. We show that if agents learn fast enough, this effect dominates and the wage schedule shifts down by enough to makes the incumbent managers better off. The workers are better off too, because the increase in their continuation value outweighs the reduction in current wage. We believe that this is a novel mechanism through which integration can lead to Pareto improvement.[8]

We analyze the model in more detail in the numerical section. The model allows us to study the evolution of earnings over the lifetime of individuals. By improving the matches, integration increases the amount of knowledge that agents can acquire in each period. This

[6]Throughout the paper, we use the terms "globalization", "integration" and "opening up" interchangeably.

[7]In this paper, MNEs are synonymous with international production teams. We abstract from the issue related to the boundaries of international firms. For some recent papers which deal with this issue, see Antràs (2003),Antràs and Helpman (2004) and Grossman and Helpman (2003).

[8]See Fosfuri et al. (2001) and Glass and Saggi (2002) for game-theoretic treatments of knowledge diffusion through worker mobility.

raises the gradient of the lifetime earnings function. For a slow learning rate, integration also reduces consumption inequality. Inequality, however, rises if agents learn fast enough. In that case, integration amplifies the initial inequality in the Home country. We also analyze the pattern of multinational activity. First, we show that domestic firms and MNEs coexist. Second, the MNEs are, on average, more productive than domestic firms.

In the quantitative section, we calibrate the closed-economy model to match key moments of the U.S. wage distribution. Then we ask the following question : how much do the developing countries gain by moving from autarky to frictionless integration with the average developed country? We focus on bilateral integration rather than multilateral integration, because we want to examine how the host country benefits from integrating with a country which has a relatively greater endowment of knowledgeable agents. Using parameter values obtained from the calibration, we find welfare gains that range from two percent for the richest middle-income countries to almost forty-three percent for the poorest countries. Most of these gains can be attributed to learning, rather than efficient allocation of managerial talent across countries.

The two papers closest to ours are Monge-Naranjo (2007) and Beaudry and Francois (2010). Monge-Naranjo (2007) also studies the impact of MNEs on the domestic accumulation of skills.[9] Monge-Naranjo develops a two period overlapping generations model where young agents can either be workers or potential managers. The latter can acquire knowledge and become managers when they are old. When an economy opens up, foreign entrepreneurs, who have higher knowledge than their domestic counterparts, relocate and carry out production with local workers. Beaudry and Francois (2010) are motivated by the question, why do firms in many developing countries fail to adopt the latest technology, in spite of its wide availability. Beaudry and Francois claim that tacit managerial skills are important for development, and these skills are transferred through learning by observing within firms. In their model, workers accumulate managerial skills during their tenure and then use these skills to manage other workers. As in our model, firms offer a bundle of wage and knowledge to workers who trade current wage with the continuation value. Despite the similar structures, the above models differ from our model in one important respect - these models do not involve matching. As we shall see, positive assortative matching among workers and managers is crucial in generating Pareto gains in our model. [10]

[9]On the empirical side, Poole (2006) uses matched employer-employee data from Brazil to investigate whether knowledge spillovers occur through worker mobility. She finds that (1) higher-skilled former MNE workers are better able to convey knowledge while higher-skilled incumbent domestic workers are better able to absorb knowledge, and (2) incumbent production workers learn more from former MNE workers or managers.

[10]For quantitative models that compute static welfare gains associated with multinational production see Ramondo (2008), Garetto (2008) and Burstein and Monge-Naranjo (2009). Rodríguez-Clare (2007) develops a model

The remainder of the paper proceeds as follows. In Section 2, we describe the model while in Section 3, we study the properties of a stationary equilibrium. In Section 4, we analyze how integration affects welfare in the host country. We study a numerical example in Section 5 and use it to further characterize the equilibrium. Section 6 discusses the calibration and the quantitative results. Section 7 concludes. All the proofs are in the Appendix.

2 The Model

Our model introduces learning and dynamics to a framework that is similar to Antràs et al. (2006), henceforth AGR. Time is discrete.

2.1 Preferences and Endowments

We have an overlapping generations framework. There is a continuum of heterogeneous agents with different levels of their knowledge. Knowledge is embodied in an agent, but can be acquired through interactions, i.e., an agent can learn from others. Knowledge includes any quality that enhances productivity, for example, management skills and experience.[11] One can think of knowledge as some composite of different attributes that affects an agent's productive capability.[12] A newborn agent draws his knowledge from an exogenously given distribution $\Phi(k)$ with support $[\underline{k}, \overline{k}]$ ($\phi(k)$ being the corresponding density). Agents also die every period with a constant probability δ and are replaced by newborns. Let us denote the actual distribution of knowledge at time t by $\Psi_t(k)$, with $\psi_t(k)$ being the corresponding density. Agents are risk neutral.

2.2 Production

Production of a single, non-storable good is carried out in firms. We call this good GDP. A firm comprises of a manager and production workers. Production workers do routine jobs and each production worker combines with the manager to produce $f(y)$ units of output, where y is the knowledge of the manager. Thus, *"f(y) captures the indivisibility of management-type decisions and implies a scale economy because it improves productivity of all the workers in*

of trade and diffusion where growth is caused by technological progress. Unlike our model, however, diffusion of ideas is an exogenous process.

[11] In the standard Mincerian wage equation, the right-hand side consists of education, as well as, experience. In this paper, knowledge encompasses both, along with other unobservables.

[12] For a trade model where agents have two attributes, see Ohnsorge and Trefler (2007).

the firm, irrespective of their numbers" as in Rosen (1982) p. 314. Notice that the productivity of workers in a firm run by a manager with knowledge y is simply $f(y)$. The manager pays wages to the workers and is the residual claimant of the output.[13]

There is a technological restriction to the number of workers a manager can hire. The span of control of a manager depends only on the knowledge of the workers he hires. The span of control is given by $n(x;\beta)$, where x is the knowledge of the worker (See Garicano, 2000, for a micro-foundation of such a technology).[14] β measures the sensitivity of the span of control with respect to x, with $\frac{\partial n}{\partial \beta} > 0$. Henceforth, we suppress the dependence of n on β and introduce it only when necessary. Apart from the knowledge of the worker and the manager, output also depends on local conditions like government policies, infrastructure, political stability, etc., which is captured by σ.[15] A firm faces the σ of the country in which it is producing. Total output of a firm is then given by

$$q = \sigma f(y) n(x) \tag{1}$$

We make the following assumptions regarding production technology:

ASSUMPTION 1a : *f is continuous, strictly increasing and weakly convex in y; n is continuous, strictly increasing and strictly concave in x. Furthermore, $\frac{\partial}{\partial y}[\frac{f'(y)}{f(y)}] \leq 0$.*

ASSUMPTION 1b : $\frac{f'(\underline{k})}{f(\underline{k})} > \frac{n'(\underline{k})}{n(\underline{k})}$.

Assumptions 1a and 1b together imply that output is more sensitive to the knowledge of the manager relative to that of the worker. As we shall see, this results in the more knowledgeable agents becoming managers in equilibrium. Assumption 1b also says that for a given knowledge distribution, there should be sufficient asymmetry between the manager and the worker's contribution to output.[16] This is a technical condition required for the existence and uniqueness of

[13]Here, as in Monge-Naranjo (2007), we assume that there is no difference between the managers and entrepreneurs. For a model which make this distinction, see Holmes and Schmitz (1990).

[14]Managers could potentially choose different types of workers. Measure consistency, however, implies that a manager can never be matched with an interval of workers. Given the technology, it can also be shown that in equilibrium, the manager would not want to hire more than one type of worker (See AGR).

[15]Our assumption that labor is the only factor of production is without loss of generality. We can always introduce capital. The cost of capital usually has three components - sunk cost, fixed cost and variable cost. In the absence of uncertainty in production and credit market imperfections, the first two do not really have any effect. So we just normalize those to zero. As for variable capital requirement, we think of it as being subsumed in $f(y)$.

[16]To see this, note that the output elasticity of the manager's knowledge is $\frac{\partial q}{\partial y}\frac{y}{q} = \frac{f'(y)y}{f(y)}$, while that of the worker is $\frac{n'(x)x}{n(x)}$. Assumption 1b says that $\frac{f'(y)}{f(y)}$ is non-increasing in y while Assumption 1a implies that the same

equilibrium.

2.3 Learning

Agents also learn in firms.[17] Since the seminal work of Gary Becker (Becker, 1962), economists have been studying on-the-job training. In this paper, we abstract from formal training provided by firms and instead focus on the knowledge that workers acquire in the process of production. We follow Jovanovic and Rob (1989) in defining our learning technology. Within each firm, a worker learns from the manager.[18] Learning is stochastic and depends both on the knowledge of the manager and the worker. The randomness in learning does not necessarily reflect any randomness in the knowledge transfer process but rather, is a simplistic way of modeling the heterogeneity in the capacity to absorb knowledge. A worker with knowledge x at time t has knowledge x' at time $t+1$. The learning distribution is given by $L(x'|x, y)$, $x' \in [x, y]$. For all $h(x')$ increasing in x', we make the following assumptions about the learning technology:

ASSUMPTION 2a : $\int h(x')dL(x'|x_1, y) > \int h(x')dL(x'|x_2, y)$ *if* $x_1 > x_2$.

ASSUMPTION 2b : $\int h(x')dL(x'|x, y_1) > \int h(x')dL(x'|x, y_2)$ *if* $y_1 > y_2$.

The above conditions are the familiar ones for first-order stochastic dominance. These conditions imply that expected learning is increasing in the knowledge of both the workers and the managers. Although the learning distribution is taken as given, we can derive it from a micro-founded model where learning requires effort and workers optimally choose how much effort to allocate.

3 Equilibrium

Agents are price-takers. There are two prices in the economy. First, the managers hire workers and pay a price for their marginal product. Second, the workers learn from the managers and

is true for $\frac{n'(x)}{n(x)}$. Moreover, $\frac{f'(\overline{k})}{f(\overline{k})} > \frac{f'(\overline{k})}{f(\overline{k})}\frac{f'(\underline{k})}{f'(\overline{k})} > \frac{n'(\underline{k})}{n(\underline{k})}$. This inequality, combined with the previous observation, implies that $\frac{f'(k)k}{f(k)} > \frac{n'(k)k}{n(k)}$ $\forall k$.

[17]As pointed out by Rosen (1972) p. 326, *"education is not produced only in schools and learning does not cease after graduation.......Rather learning and work are complementary..........In fact, learning in the workplace is extremely widespread and characterizes almost all labor market activities."*

[18]Unlike Jovanovic and Rob (1989), learning is one-sided. Assuming that managers also learn from workers could be an interesting extension and would be one channel through which growth can be introduced in this model.

pay a price for the acquired knowledge. It is inconsequential whether there are two transactions within the firm or whether the managers simply pay the wage net of the rent (See Rosen, 1972). What matters is the net payment to workers $w_t(k)$; we simply call it wage. Note that the wage is a function of the knowledge of the worker, but *not* the manager. We shall return to this issue shortly.

The absence of aggregate uncertainty in the model, combined with a large number of agents, implies that the evolution of the knowledge distribution is deterministic. Since, in equilibrium, $w_t(k)$ is a function of only the knowledge distribution, the evolution of $w_t(k)$ is also deterministic. Therefore, given an initial distribution of knowledge $\Psi_0(k)$, a competitive equilibrium is characterized by a deterministic sequence $\{\Psi_t(k), w_t(k)\}$. Given a sequence of wage functions $\{w_t\}_{t=0}^{\infty}$, the manager's problem is defined recursively as

$$V_M(y, w_t) = \max_x \sigma f(y)n(x) - w_t(x)n(x) + (1-\delta)\max[V_W(y, w_{t+1}), V_M(y, w_{t+1})]. \quad (2)$$

where $V_W(y, w_t)$ is the value function of an agent with knowledge y, if he chooses to be a worker while $V_M(y, w_t)$ is the value function if, instead, he chooses to be a manager.[19] The value of a manager depends on the current distribution $\Psi_t(k)$ through the net wage schedule w_t. The second term on the right allows for the possibility that an agent, who is a manager at time t, might choose to be a worker at time $t+1$. $V_W(y, w_t)$ is given by

$$V_W(x, w_t) = w_t(x) + (1-\delta)\int_x^{m_t(x)} \max[V_W(x', w_{t+1}), V_M(x', w_{t+1})]dL(x'|x, m_t(x)). \quad (3)$$

where $m_t(x)$ is the knowledge of the manager who hires a worker with knowledge x in equilibrium. The term within the integral denotes the expected value of the worker if he works for a manager with knowledge $m_t(x)$. Depending on how much he learns, the worker might become a manager or continue as a worker at $t+1$. As before, this decision will depend not only on the worker's own knowledge but also on the wage function. Let the sets of workers be denoted by W_i, $i = 1, ...S$, where S is determined in equilibrium, and let $\overline{W} = \{W_1, ...W_S\}$. Similarly, let the sets of managers be denoted by M_i, with $\overline{M} = \{M_1, ...M_S\}$. Then the labor market-clearing condition can be written as

[19] Notice the absence of time discounting in the above formulation. This is because, a positive probability of death acts as a discount factor.

$$\int_{\overline{W}} \psi_t(k)dk = \int_{\overline{M}} n(m^{-1}(k))\psi_t(k)dk \tag{4}$$

The left-hand side denotes the total supply of workers. The right-hand side denotes the total demand for workers, where $n(m^{-1}(k)$ is the number of workers demanded by a manager with knowledge k.

Since workers learn, their knowledge increases over time. But agents also die every period with probability δ and are replaced by newborns who draw knowledge from the exogenous distribution $\Phi(k)$. Birth, learning and death implies a rule for the evolution of the knowledge distribution $\Psi_t(k)$:

$$\Psi_{t+1}(k) = \delta\Phi(k) + (1-\delta)\int_{\underline{k}}^{k}\int_{s}^{k} dL(s'|s, m_t(s))d\Psi_t(s) \text{ for all } k \in [\underline{k}, \overline{k}] \tag{5}$$

The first term on the right-hand side denotes the fraction of agents who are born in period $t+1$ with knowledge less than k. The second term denotes the agents who remain below k in period $t+1$, despite learning from their managers in period t. There is an alternative way of looking at the evolution. Let A be any Borel set of $[\underline{k}, \overline{k}]$. Then the transition function for the knowledge distribution satisfies, for every $k \in [\underline{k}, \overline{k}]$,

$$P_t(k, A) = \begin{cases} (1-\delta)\int_A dL(s|k, m_t(k)) + \delta\int_A d\Phi(s) & \text{if } k \in \overline{W} \\ \delta\int_A d\Phi(s) & \text{if } k \in \overline{M} \end{cases}$$

Equation (5) implies that Ψ_{t+1} is determined by how individuals acquire knowledge in period t, and the acquisition of knowledge by individuals is determined only by who they match with at time t, which in turn depends only on Ψ_t. Therefore, Ψ_{t+1} is a function of Ψ_t.

Definition 1. *A competitive equilibrium of this economy consists of the following objects:*
(i) Value functions, $V_W(k) : \overline{W} \to \mathbb{R}$ *and* $V_M(k) : \overline{M} \to \mathbb{R}$*;*
(ii) Current earnings, $w_t(k) : \overline{W} \to \mathbb{R}$ *and* $\pi_t(k) : \overline{M} \to \mathbb{R}$*;*
(iii) Matching function, $m_t(k) : \overline{W} \to \overline{M}$*;*
(iv) Occupational structure, $\overline{W}$ *and* $\overline{M}$
such that,
(a) $V_W(k)$ *and* $V_M(k)$ *satisfy the worker's and manager's problems respectively;*
(b) $m_t(k)$ *is the corresponding policy function;*

(c) labor market clears;

(d) the knowledge distribution evolves according to equation (5).

It might seem natural to write the wage as $w_t(x, y)$ given that (a) the same manager can produce different levels of output by hiring different types of workers and (b) the same worker can acquire different levels of knowledge by working for different managers. (a) suggests that the price for labor should be specific to a worker-manager pair while (b) suggests that the same should be true for the price of knowledge. In order to understand why $w_t(x)$ only has the knowledge of the worker as its argument, let us look at the underlying mechanism that determines the wage function.

First, consider an economy without learning. In this economy, every agent with knowledge y, in the role of a manager, offers a wage schedule $\widetilde{w}_{t,NL}(x, y)$ (NL denotes no learning). This wage schedule is such that y *is indifferent between hiring any* x. Denote the corresponding profit of y as $\widetilde{\pi}_t(y)$. The following lemma establishes some properties of $\widetilde{w}_{t,NL}(x, y)$.

Lemma 1. $\widetilde{w}_{t,NL}(x, y)$ *is continuous and increasing in* x *for all* y.

At the same time, each agent, in the role of a worker, faces a series of wages offered by the potential managers. If $\widetilde{\pi}_t(x) > \max_y \widetilde{w}_{t,NL}(x, y)$, agent x becomes a manager; otherwise, he becomes a worker and works for $y(x)$ where $y(x) = \arg\max \widetilde{w}_{t,NL}(x, y)$. The occupation of agents is endogenously determined. Equilibrium is attained when every agent is employed and maximizes utility. The equilibrium wage schedule is given by $\widetilde{w}_{t,NL}(x, y(x)) \equiv w_{t,NL}(x)$, i.e., $w_{t,NL}(x)$ is the upper envelope of the individual $\widetilde{w}_{t,NL}(x, y)$ schedules. From lemma (1), it then follows that $w_{t,NL}(x)$ is continuous and increasing in x.[20]

Now, consider what happens when we introduce learning. The worker acquires knowledge from the manager, which raises the former's continuation value. Thus, learning creates a rent and the manager tries to extract a part of this rent by paying a wage that is lower than what he

[20]To see why this is an equilibrium, consider an arbitrary worker with knowledge x^*. By the definition of $w_{t,NL}(x)$, he maximizes his utility by getting $w_{NL}(x^*)$. Let the manager who offers this wage be denoted by y^*. To see why y^* is also maximizing his utility, note that

$$\begin{aligned}
\pi(y^*) &= f(y^*)n(x^*) - w_{t,NL}(x^*)n(x^*) \\
&= f(y^*)n(x^*) - \widetilde{w}_{t,NL}(x^*, y^*)n(x^*) \\
&= f(y^*)n(x) - \widetilde{w}_{t,NL}(x, y^*)n(x) \quad \forall x \\
&\geq f(y^*)n(x) - w_{t,NL}(x)n(x)
\end{aligned}$$

where the third line follows from the definition of $\widetilde{w}_{t,NL}(.)$ and the last line follows from the fact that $w_{t,NL}(.)$ is the upper envelope of $\widetilde{w}_{t,NL}(.)$. Therefore, y^* maximizes his utility by hiring x^*. Since, x^* was chosen arbitrarily, the result follows.

would have paid in the absence of learning. Denoting $\widetilde{w}_t(x,y)$ as the individual wage schedule under learning, $\widetilde{w}_t(x,y) = \widetilde{w}_{t,NL}(x,y) - C(x,y)$. Since the wage schedule offered by the manager is such that he is indifferent across workers, the entire wage schedule must shift down.[21] Each manager now offers a pair $\{\widetilde{w}_t(x,y), k(x,y)\}$ where $k(x,y)$ is the expected knowledge that a worker x acquires by working for manager y.[22] Therefore, each manager effectively offers a $\widetilde{V}_W(x,y,\widetilde{w}_t(x,y))$ schedule since the current wage and expected future knowledge level determines the present value of the workers. The equilibrium $V_W(x,w_t)$ schedule is the upper envelope of the individual $\widetilde{V}_W(x,y,\widetilde{w}_t(x,y))$ schedules. The $\widetilde{w}_t(.)$ and $\widetilde{V}_W(.)$ schedules are closely related, as shown in the next lemma.

Lemma 2. *If y^* is the solution to $\arg\max_y \widetilde{w}(x,y)$, then it also solves $\arg\max_y \widetilde{V}_W(x,y)$.*

The above lemma says that the manager who offers x the highest wage, also maximizes the present value of earnings of x. In other words, if worker x maximizes the profit of manager y, then it must be the case that manager y also maximizes the welfare of worker x. Consequently, we can simply focus on the manager's static optimization problem to solve for the matching function $m_t(x)$. The following lemma characterizes $m_t(x)$.

Lemma 3. *$m_t(x)$ is continuous and strictly increasing in x.*

Lemma 3 says that the equilibrium is characterized by Positive Assortative Matching or PAM.[23] Although PAM imposes some structure on the equilibrium allocation, the occupational structure is still too general (Recall that S, the number of sets of workers and managers, could potentially take any value). We claim that in equilibrium, there exists a threshold k_t^* such that all agents with knowledge less than k_t^* are workers, while those with knowledge above k_t^* are managers. That is, we claim that $S = 1$. Under threshold matching, the equation (4) reduces to

$$\int_{\underline{k}}^{k} d\Psi_t(s) = \int_{k^*}^{m_t(k)} n(m_t^{-1}(s))d\Psi_t(s) \quad \forall k \leq k_t^* \tag{6}$$

Note that the labor market-clearing condition is not standard. The left hand side denotes the supply of workers in the interval $[\underline{k}, k]$, while the right hand side denotes the demand for workers coming from managers in the interval $[m_t(\underline{k}), m_t(k)]$. Measure consistency requires

[21] Note that the manager does not directly gain when his workers learn. Rather, he gains because learning creates a rent.

[22] This is similar to Boyd and Prescott (1987), where the old workers in a firm offer the young a package of current consumption and future expertise.

[23] This is standard feature of production functions that exhibit complementarity.

that these two values be equal for every k. This follows from lemma 3, since the workers hired by managers with knowledge in $[m_t(\underline{k}), m_t(k)]$, must have knowledge in $[\underline{k}, k]$. Differentiating equation (6) with respect to k yields

$$m_t'(k) = \frac{\psi_t(k)}{n(k)\psi_t(m_t(k))} \tag{7}$$

This differential equation, along with the boundary conditions $m_t(\underline{k}) = k_t^*$ and $m_t(k_t^*) = \overline{k}$, allows us to solve for the matching function. As the following lemma shows, given $\Psi_t(k)$, the threshold k_t^* and consequently, the matching function are uniquely determined.

Lemma 4. *For a given a $\Psi_t(k)$, k_t^* exists and is unique.*

The above discussion suggests that the solution of the model has a block recursive structure - matches can be determined completely once we know the knowledge distribution. We do not need to know the wage schedule in order to determine the matches; rather, once the matches are determined, wages adjust so as to support the matches that emerge. Of course, this does not mean that how agents match does not depend on wages. In this economy, wages (and profits) not only determine the remuneration of the agents but they also play an allocative role (Sattinger (1993)). But for the purpose of solving the model, we can derive the matching function without any information on the wage function. PAM also allows us to derive some properties of the wage and the value functions, as shown in the following lemma.

Lemma 5. *In equilibrium, (i) $w_t(k)$ is continuous and increasing in k, and (ii) $V_W(k)$ and $V_M(k)$ exist, are continuous and increasing in k.*

This completes our characterization of the competitive equilibrium. The following proposition provides for the existence and uniqueness of the equilibrium.

Proposition 1. *There exists a δ^*, such that $\forall \delta > \delta^*$, a threshold equilibrium exists and it is unique. Moreover, this equilibrium is efficient.*

Finally, recall that equation (5) defines Ψ_{t+1} as a function of Ψ_t. We seek a fixed point of Ψ_t, i.e., an invariant knowledge distribution Ψ^*. As the following proposition shows, such a fixed point exists and is unique.

Proposition 2. *A unique, invariant knowledge distribution Ψ^* exists and any initial distribution Ψ_0 weakly converges to Ψ^*.*

Therefore, in the long run, the knowledge distribution converges to Ψ^*, with threshold k^*. Agents, who are born with knowledge above k^*, become managers instantaneously. Since managers do not learn, these agents are stuck with the level of knowledge they are born with. On the other hand, agents, who are born with knowledge below k^*, start their lives as workers. These agents learn in every period and move up, till they eventually cross the threshold and become managers themselves. For these agents, the lifetime earnings profiles are positively sloped. Proposition 2 implies that we can perform comparative statics on the invariant distribution. For the remainder of the paper, we shall restrict most of our attention to the stationary equilibrium of the model, although we do consider transitional dynamics briefly in Section 5.

4 Analytical Results

In this section, we present some analytical results of our model. To simplify exposition, we make the following assumption about the learning technology:

ASSUMPTION 2c : *If, in period t, a worker with knowledge x works for a manager with knowledge y, then in period $t+1$, the worker has knowledge x with probability θ and knowledge y with probability $1-\theta$.*

Thus, learning is an all-or-nothing proposition for the worker. In the next section, we relax this assumption and work with a more general learning technology. Notice that, in spite of the learning distribution having just two points, it still satisfies assumptions 2a and 2b.[24]

4.1 Autarky

We begin by examining the equilibrium under autarky. Recall that the density function for the newborn distribution is given by $\phi(k)$. The learning technology, along with the newborn distribution, implicitly defines the invariant distribution and allows us to solve for the threshold k^*.

Proposition 3. *k^* is defined implicitly by the following equation*

[24] To see this, note that for any $h(.), h' > 0$, the expected value of $h(x)$ is $\theta h(x) + (1-\theta)h(y)$. This expression is increasing in both x and y.

$$\int_{\underline{k}}^{k^*} \frac{\phi(k)}{n(k;\beta)} dk = \int_{k^*}^{\overline{k}} \phi(k) dk + (1-\theta)(\frac{1-\delta}{\delta}).$$

Moreover, k^* has the following properties :

$$\frac{\partial k^*}{\partial \theta} < 0; \frac{\partial k^*}{\partial \delta} < 0; \frac{\partial k^*}{\partial \beta} > 0.$$

Proposition 3 sheds light on how the distribution changes as the rate of learning increases. δ, being the probability of death in a period, proxies for the length of a time period. A lower δ, holding θ unchanged, implies that agents are acquiring the same expected knowledge over a smaller interval of time. On the other hand, a lower θ, holding δ unchanged, implies that agents are acquiring more expected knowledge over the same interval of time. Both these cases translate into faster learning for the agents. An increase in the rate of learning makes the knowledge distribution negatively-skewed, as more and more mass shifts to the upper tail. Consequently, labor market-clearing requires that the threshold shift to the right. The threshold also rises with an increase in the span of control (higher β). Intuitively, a greater span of control of each manager implies that fewer managers are required to employ the workers.

Recall that a worker with knowledge k produces $f(m(k))$ units of output. Hence, total output produced in this economy is given by

$$Y = \int_{\underline{k}}^{k^*} f(m(k)) d\Psi(k) \tag{8}$$

Total welfare is given by

$$W = \int_{\underline{k}}^{k^*} V_W(k) d\Psi(k) + \int_{k^*}^{\overline{k}} V_M(k) d\Psi(k) \tag{9}$$

In this model, individual welfare equals the present value of consumption (or income, since the good is non-storable) because agents are risk-neutral.

4.2 Integration

Integration, in the context of our model, means that managers from one country can hire workers in another country,i.e., integration leads to the creation of MNEs. The managerial input is rival and as a result, managers can not operate plants in both countries.[25] The motive behind the formation of MNEs is exploiting differences in factor prices.[26] In this paper, we focus on full integration, i.e. we assume that MNEs are formed costlessly. In particular, we assume away any cost that might be associated with opening a plant in another country. We do acknowledge that these costs are important, but the introduction of such costs increases the complexity of the model without any gain in insight.

Let us introduce some notation. Define the subscripts $i = \{A, I\}$, $j = \{H, F\}$, where A and I stand for autarky and integration respectively, while H and F stand for Home and Foreign respectively. The Home newborn distribution is denoted by $\Phi_H(k)$ with support $[\underline{k}, \overline{k}_H]$, while the Foreign newborn distribution is $\Phi_F(k)$, with $[\underline{k}, \overline{k}_F]$ being the corresponding support. We assume that $\overline{k}_F > \overline{k}_H$ and that $\Phi_F(k)$ first-order stochastic dominates $\Phi_H(k)$. The latter assumption reflects the relative abundance of more knowledgeable agents in the Foreign country. We also assume that $\sigma_F \geq \sigma_H$, where σ_H and σ_F denote the Home and Foreign country-specific productivity respectively. The steady-state knowledge distributions are indexed by i and j. So, for example, $\Psi_{A,H}(k)$ is the Home steady-state knowledge distribution under autarky. Let P_H and P_F denote the population in the two countries. With integration, the fundamental change is in the distribution of newborns, which is given by

$$\Phi_I(k) = \begin{cases} \frac{P_H}{P_H+P_F}\Phi_H(k) + \frac{P_F}{P_H+P_F}\Phi_F(k) & \text{for } k \in [\underline{k}, \overline{k}_H] \\ \frac{P_H}{P_H+P_F} + \frac{P_F}{P_H+P_F}\Phi_F(k) & \text{for } k \in [\overline{k}_H, \overline{k}_F] \end{cases} \tag{10}$$

$\Phi_I(k)$, combined with the learning technology, determines the integrated knowledge distribution $\Psi_I(k)$. The new threshold, k_I^*, would typically be different from k_A^*, the autarky threshold. Before deriving the relation between the thresholds under autarky and integration, let us state the following lemma.

Lemma 6. *If a knowledge distribution G first-order stochastic dominates another distribution H, then $k_G^* > k_H^*$, where k_G^* and k_H^* are the thresholds under G and H respectively.*

Equation (10), along with the assumption that $\Phi_F(k)$ first-order stochastic dominates $\Phi_H(k)$, implies that $\Phi_I(k)$ first-order stochastic dominates $\Phi_H(k)$. In the benchmark case of no-

[25] Whether managers travel from the source-country to the host-country or not, however, is irrelevant.

[26] This motive for establishing subsidiaries in other countries is the same as in Helpman (1984).

learning, the knowledge distributions in both the countries coincide with the newborn distributions. Consequently, under no-learning, $k_I^* > k_A^*$ (this follows directly from Lemma 6). With learning, however, the knowledge distributions are no longer exogenous. Still, we can derive a relation between $k_{A,H}^*$ and k_I^*, as shown in the following proposition.

Proposition 4. *$k_I^* > k_A^*$, where k_I^* is defined implicitly by the following equation*

$$\int_{\underline{k}}^{k_I^*} \frac{p_H\phi_H(k) + p_F\phi_F(k)}{n(k;\beta)} dk = \int_{k_I^*}^{\overline{k}_F} (p_H\phi_H(k) + p_F\phi_F(k))dk + (1-\theta)(\frac{1-\delta}{\delta})$$

where $p_H = \frac{P_H}{P_H+P_F}$ and $p_F = \frac{P_F}{P_H+P_F}$.

The range of knowledge for the Home workers expands under integration. The agents with knowledge in $[k_A^*, k_I^*]$ switch from being managers to workers. The entry of highly knowledgeable managers from the Foreign country raises the opportunity cost of being a manager for a Home agent. An incumbent Home manager weighs the cost of becoming a worker for a MNE (forgone current profits) against the benefit (higher expected profits in the future). For the managers in $[k_A^*, k_I^*]$, benefits outweigh costs and consequently they switch.

Although Proposition 4 indicates the direction of change for the threshold, it says nothing about its magnitude. In particular, the following two scenarios are possible:

Case I ($k_I^* > \overline{k}_H$) : In this case, every agent born in the Home country starts his life as a worker. The support of $\Psi_{I,H}(k)$ is $[\underline{k}, m(\overline{k}_H)]$, despite the fact that, the Home newborn distribution $\Phi_H(k)$ still has the smaller support.[27] Though theoretically an interesting case, this situation is quite extreme because it implies that integration results in the destruction of all incumbent firms (managers), who are replaced by a new class of bigger and more productive firms.

Case II ($k_I^* < \overline{k}_H$) : In this case, the support of $\Psi_{I,H}(k)$ is $[\underline{k}, \overline{k}_F]$. This case is characterized by the birth of a new class of Home firms (with knowledge in $[\overline{k}_H, \overline{k}_F]$), who are on par with the Foreign MNEs in terms of size and productivity. But unlike Case I, a set of incumbent Home managers with knowledge in $[k_I^*, \overline{k}_H]$ continues to operate in the integrated economy.

Whether we are in Case I or Case II depends on the parameters of the model. As long as $\overline{k}_F$

[27] It is not the case that every Home agent is a worker. There are Home managers in $[k_I^*, m(\overline{k}_H)]$. This, however, means that the Home managers in the integrated economy have knowledge greater than $\overline{k}_H$.

is not too different from $\overline{k}_H$, there will be some incumbent managers in the Home country.[28] Intuitively, a large gap between $\overline{k}_F$ and $\overline{k}_H$ implies that following integration, the Home agents have an opportunity to work for very knowledgeable managers. This is also true for the incumbent Home managers, who would rather work in Foreign MNEs, learn and become much better managers in the future than remain managers with low levels of knowledge.

Irrespective of which case we are in, integration affects the matching of agents. An immediate implication of Proposition 4 is that $m_I(\underline{k}) > m_A(\underline{k})$, where $m_A(.)$ and $m_I(.)$ are the matching functions under autarky and integration respectively.[29] Therefore, the least knowledgeable worker in the Home country, and by continuity, a set of less knowledgeable workers, is matched with better managers. This is formally stated in the following proposition:

Theorem 1. *A positive measure of Home workers have a better match in the integrated equilibrium compared to autarky.*

On the other hand, some of the Home managers are now matched with less able workers. Since the output of firms depends positively on workers' knowledge, the output of some of the Home firms under integration are necessarily lower than in autarky.

Corollary 1. *Under integration, the output of a positive measure of Home firms goes down.*

Note that since the output produced by a worker depends only on the knowledge of the manager he is matched with, the productivity of a firm, as measured by the value-added per worker, does not change.

Total Home output (GDP) produced in the integrated equilibrium is given by

$$Y = \int_{\underline{k}}^{k_I^*} f(m_I(k)) d\Psi_{I,H}(k) P_H \tag{11}$$

This is different from Gross National Income (GNI), which is given by

$$GNI = \int_{\underline{k}}^{k_I^*} w_I(k) d\Psi_{I,H}(k) P_H + \int_{k_I^*}^{\overline{k}_F} \pi_I(k) d\Psi_{I,H}(k) P_H \tag{12}$$

The difference between the two arises from the fact that, in an integrated equilibrium, a part of the Home output goes to the Foreign country as profits of Foreign MNEs, while some of the

[28] Note that for a given $\overline{k}_H$, there exists a k' such that $\overline{k}_F < k'$ implies that $k_I^* < \overline{k}_H$. This follows from the result that k_I^* is monotone increasing in $\overline{k}_F$, and $k_I^* < \overline{k}_H$ when $\overline{k}_F = \overline{k}_H$.

[29] To see this, note that $m_A(\underline{k}) = k_A^*$ and $m_I(\underline{k}) = k_I^*$. Proposition 4 then gives the result.

Home firms may become multinationals and earn profits from their operations in the Foreign country. Finally, aggregate welfare in the Home country is given by

$$W = \int_{\underline{k}}^{k^*} V_W(k) d\Psi_{I,H}(k) P_H + \int_{k^*}^{\overline{k}_F} V_M(k) d\Psi_{I,H}(k) P_H \tag{13}$$

To sum up, with integration, the threshold of the knowledge distribution shifts to the right. This necessarily means that some of the Home workers are hired by more knowledgeable managers. These workers also learn more compared to autarky. At the same time, some of the incumbent firms suffer a decline in output.

4.3 Change in Welfare

Integration changes individual, as well as, aggregate welfare of the Home country. We focus our attention on the case where there are surviving Home managers, i.e., Case II.[30] In order to understand how learning affects welfare, first we look at the benchmark case of no learning. This is similar to the static framework presented in AGR. A key result that emerges from AGR is that integration raises aggregate consumption, and with risk-neutral agents, the aggregate welfare of the Home economy. What about individual welfare? In the previous section, we showed that the output produced by the less knowledgeable Home managers goes down under integration.[31] The actual change in profits and welfare, however, depends on the wages they pay, which would be different from those under autarky. Of course, as wages change, the welfare of the workers change too.

Theorem 2. *In the absence of learning, an integrated steady-state equilibrium with incumbent Home firms can never be a Pareto improvement relative to the autarky steady-state equilibrium in the Home country.*

In the absence of learning, integration creates winners and losers. The identity of the winners and losers, though, will depend on the specific parameter values. If we think of workers and managers as two separate factors of production, Theorem 2 essentially gives us a Heckscher-Ohlin like result.[32]

[30]The reason for this is the following : If $\overline{k}_F$ is very different from $\overline{k}_H$, then irrespective of whether agents learn or not, every Home agent is better off working for the more knowledgeable Foreign managers. Thus we get Pareto improvement, but the Home firms disappear completely.

[31]This is true for both the learning and no-learning case.

[32]To be technically correct, we have infinitely many factors.

Does Proposition 5 continue to hold when we introduce learning? In order to prove otherwise, we have to show that every agent in the Home economy is strictly better off under integration. Corollary 1 implies that some of the Home managers earn lower revenue compared to autarky.[33] Hence, for these managers to be better-off under integration, the wage bill has to go down more than revenue.

In this model, there are two forces that determine wages. First, there is a *labor demand effect*. The entry of Foreign managers increases the demand for Home workers. At the same time, integration increases competition faced by the Home workers from their Foreign counterparts. As shown by AGR, (1) if the two countries are not too similar and, (2) if the span of control is not too small, the labor demand effect raises the wages of all Home workers.[34]

Second, there is a *learning effect*. A worker, in this model, can be hired by any manager with a positive probability. Working for a more knowledgeable manager means higher expected learning and consequently, higher earnings. Hence, the entry of highly knowledgeable Foreign managers raises the continuation value of the Home workers. PAM, however, implies that the most knowledgeable managers hire only the most knowledgeable workers. Therefore, the less knowledgeable workers can work for the MNEs only if they learn and acquire enough knowledge from their current managers, some of whom are the incumbent Home managers. A positive value of learning implies that workers are willing to pay in order to learn. Thus learning creates a rent. This allows the managers to compress the wage. The workers accept this wage reduction because they expect to be compensated in the future. So the learning effect tends to lower the wage schedule.

The final impact on wages depends on the relative strengths of the two effects. The following Proposition shows the condition under which the Home economy realizes Pareto gains.

Theorem 3. *The integrated steady-state equilibrium is a Pareto improvement for the Home country if*

$$(1-\theta)(\frac{1-\delta}{\delta}) > \frac{2f(\overline{k}_H)[n(\overline{k}_H)-n(\underline{k})]+n(\underline{k})[\mu(\overline{k}_H)f(\overline{k}_H)-\mu(\underline{k})f(\underline{k})]}{\mu(\underline{k})[f(\overline{k}_F)+f(\underline{k})]n(\underline{k})-\mu(\overline{k}_H)[n(\underline{k})+n(\overline{k}_H)]f(\overline{k}_H)}$$

where $\mu(\underline{k}) = \frac{n(\underline{k})}{1+n(\underline{k})}$ *and* $\mu(\overline{k}_H) = \frac{n(\overline{k}_H)}{1+n(\overline{k}_H)}$.

Let us denote by Ω, the set of pairs of θ and δ that satisfy the above condition. The left-hand side of the above expression is positive by definition. The numerator of the fraction on the

[33] Since there is only one good, output equals revenue.

[34] For a similar effect in a trade model, see Melitz (2003).

right-hand side is positive too.[35] For Ω to be non-empty, the denominator has to be positive. We can show that, the denominator is an increasing function of $\frac{f'(k)}{f(k)} - \frac{n'(k)}{n(k)}$, i.e., the degree of asymmetry between the manager's and the worker's contribution to output. Intuitively, the greater is this asymmetry, the greater is the increase in the worker's earning when he becomes a manager; and the greater is the wage cut that the worker is willing to accept in order to learn.

Theorem 3 also sheds light on how welfare changes as the rate of learning changes. Assuming that the right-hand side of the expression in Theorem 3 is positive, the inequality is not satisfied for low enough δ (or θ). In the limiting case of no-learning, $\delta = 1$ (or $\theta = 1$), the left-hand side is equal to zero. As δ (or θ) falls, the left-hand side starts to increase and at some point, exceeds the right-hand side. According to Corollary 1, some of the incumbent firms produce less under integration relative to autarky. For these firms to be better off, they must be paying lower wages to the workers. Corollary 2 follows naturally.

Corollary 2. *If all Home agents gain from integration, Home workers must earn a lower wage compared to autarky.*

In our model, if a MNE has more knowledge than an incumbent Home firm, PAM implies that the workers in the MNE are more knowledgeable than the ones in the Home firm. PAM also implies that after working for the (more knowledgeable) MNE, a worker never works for the (less knowledgeable) Home firm. Therefore, there is no flow of knowledge from the MNE to the Home firm. Despite this, the incumbent firm could be better off if learning is fast enough.[36] Of course, some of the former MNE workers set up their own firms and these managers directly benefit from the superior knowledge of MNEs.[37]

5 Numerical Results

To explore the model more fully, we resort to numerical analysis in this section. We study how the equilibrium allocation changes with the rate of learning, and whether the welfare results from the previous section go through for a more general learning technology. We go on to

[35] Since $\frac{n(k)}{1+n(k)}$ is increasing in k.

[36] The traditional view regarding knowledge spillover is that workers with experience in MNEs are hired by domestic managers. These workers bring with them knowledge regarding better technology and management practices and this raises the productivity of the domestic firms. See, for example, Barba Navaretti and Venables (2004).

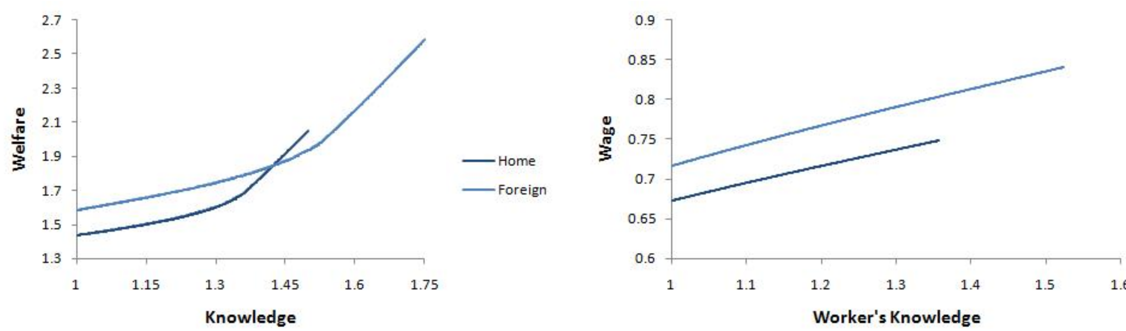
[37] This effect is similar to Monge-Naranjo (2007) where the transfer of skills from MNEs materialize in a new sector of firms, not in the pre-existing sector of firms.

analyze the impact of integration on the evolution of individual earnings, inequality and the pattern of multinational activity. Throughout, our focus is on the Home economy.

The only change from the last section is in the learning technology. We assume that a worker with knowledge x, and working for a manager with knowledge y, draws his knowledge in the next period from a distribution which is uniform on $[x, y]$. The production function is given by $f(y) = y^{\alpha}$, $n(x; \beta) = x^{\beta}$. Finally, we assume that the distribution of newborns is a truncated exponential in $[1, \overline{k}]$ with parameter λ. By setting $\underline{k} = 1$, the size of the smallest firm is implicitly set to two (one manager and one worker). The following figures are drawn for $\alpha = 1$, $\beta = 0.5$, $\lambda = 1$, $\sigma_H = \sigma_F = 1$, $\overline{k}_H = 1.5$, $\overline{k}_F = 1.75$.

5.1 Earnings and Welfare

Figure 1 compares the Home and Foreign economies under autarky. Figure 1a shows the welfare of an agent *as a function of his knowledge at birth*. The only difference between the two countries is in the distribution of newborns. In particular, the Foreign country has a larger knowledge support. This translates into relatively greater endowment of more knowledgeable agents in the Foreign country.[38]



(a) *Welfare under autarky* **(b)** *Wages under autarky*

Figure 1: *Home and Foreign economy under autarky*

The less knowledgeable agents are relatively scarce in the Foreign country and hence, are better off compared to their Home counterparts. But the most knowledgeable agents at Home are better off compared to their foreign counterparts. The relative abundance of less knowledgeable agents at Home translates into a Home wage schedule that lies below the Foreign

[38] If both Φ_H and Φ_F are truncated exponentials with the same parameter and $\overline{k}_H < \overline{k}_F$, then Φ_F first-order stochastic dominates Φ_H.

wage schedule. Thus, labor is cheap at Home and this motivates the formation of MNEs as the two countries integrate. In the previous section, we had compared the steady-state under two regimes: autarky and integration. To compute the welfare gains from the integration policy, however, one must compare the autarky steady-state equilibrium with the integrated equilibrium in the period just after the policy is put in place. This requires solving the entire transitional path.

5.1.1 Transition

With the policy in place, there is no immediate change in the knowledge distribution. Consequently, the matches do not change. As soon as the policy is implemented, however, the agents' expectations about the future knowledge distributions change. Figure 2 shows the transition of the integrated knowledge distribution from the time the Home country integrates until the new steady-state is reached. The initial distribution is simply the sum of the Home and Foreign steady-state distributions. As shown in Figure 2, there are three discontinuities in the initial distribution. The first one occurs at k_A^*.[39] The second one occurs at $\overline{k}_H$, because no Home agents are born to the right of $\overline{k}_H$. The third discontinuity occurs at the Foreign threshold. In the new steady-state, the discontinuities occur at k_I^* and $\overline{k}_H$.

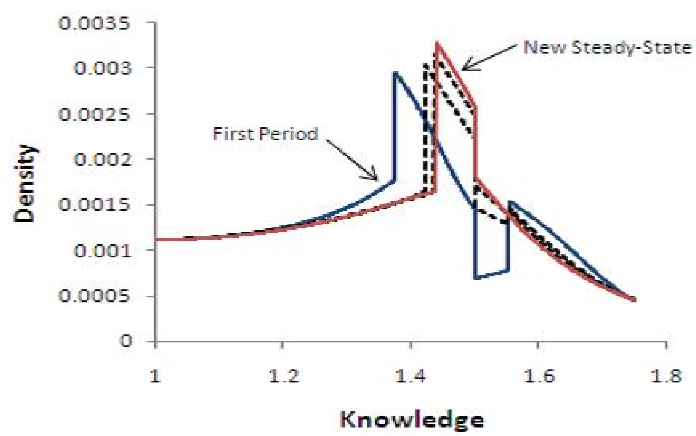


Figure 2: *Evolution of the knowledge distribution*

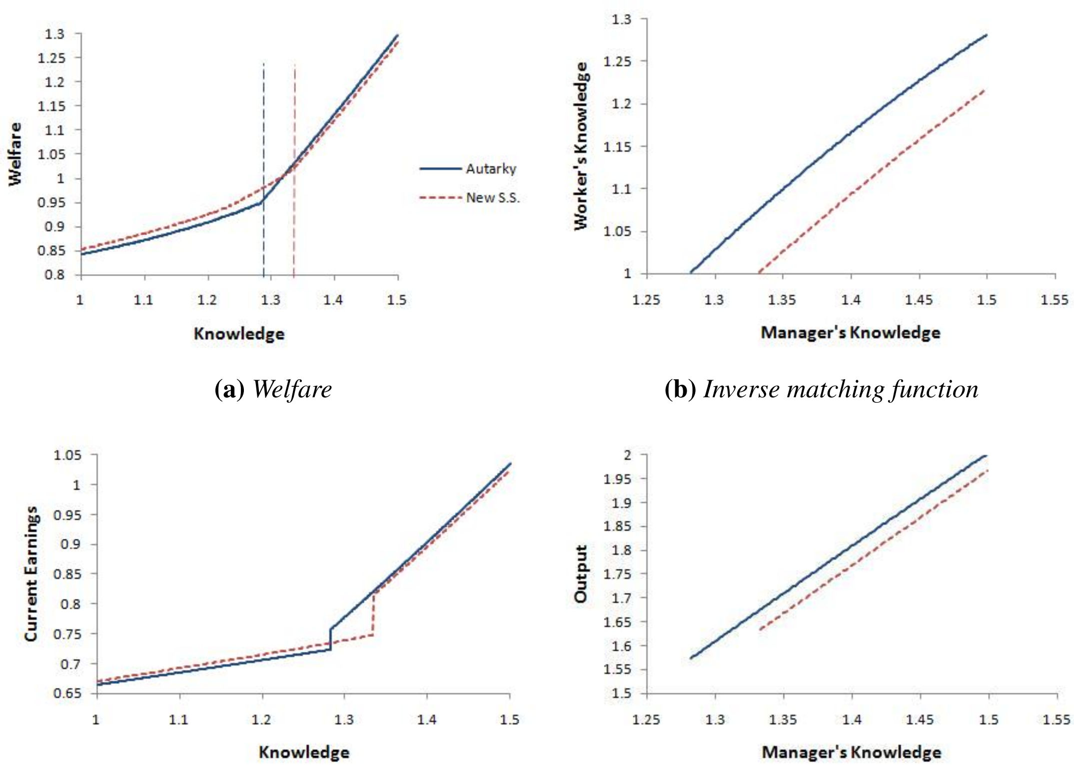
The agents have rational expectations and know exactly how the distribution will evolve. Accordingly, they know what the wages and profits will be at each period during the transition.

[39]There is always a discontinuity at the threshold. In a small interval to the left of the threshold (where all agents are workers) there is both an inflow of workers and an outflow of workers. But in a small interval just to the right of the threshold, there is only inflow and no outflow (since managers do not learn).

This, in turn, allows them to compute their welfare in every period. Having solved the transition, we go on to study the integrated equilibrium for different learning rates. Recall from Section 4, that keeping the learning distribution unchanged, this implies choosing different values for δ.

5.1.2 Slow learning ($\delta = 0.8$)

In Figure 2, we compare the two steady-states: autarky and integration. In the steady-state under integration (New S.S.), the welfare of individuals who are born with less knowledge is higher, while the welfare of those born with high levels knowledge is lower, as shown in Figure 3a. Figure 3b indicates that the incumbent managers have a worse match; every incumbent manager produces less under integration. This is confirmed in Figure 3d. Therefore revenues are lower. But the effect on profits, which determines the managers' welfare, also depends on the wage bill.



(a) *Welfare* **(b)** *Inverse matching function*

(c) *Current earnings* **(d)** *Output*

Figure 3: *Effect of Integration when learning is slow*

The discussion in the previous section suggests that, the effect of integration on wages depends on the relative strength of the labor demand effect and the learning effect. When agents learn slow, the former effect dominates and the wage schedule shifts up, thereby lowering the profits, and welfare, of incumbent managers. This is shown in Figure 3c.[40] Notice that we restrict our attention to the agents who are born in $[\underline{k}, \overline{k}_H]$. Although, in the new steady state, there are Home agents with knowledge in $[\overline{k}_H, \overline{k}_F]$, but at the time of birth, these agents still draw their knowledge from the Home newborn distribution, which does not change with integration. Home agents attain knowledge in $[\overline{k}_H, \overline{k}_F]$ *through learning, not through birth.*

5.1.3 Fast learning ($\delta = 0.5$)

Results are different when agents learn at a faster rate. This is displayed in Figure 4. Now, the learning effect dominates the labor demand effect, thereby lowering the wage schedule. Although the output of the incumbent Home managers fall (Figure 4d) due to a worsening of their matches (Figure 4b), the wage bill decreases by so much, that it outweighs the fall in revenue, resulting in higher profits. This makes the home managers better-off. The less knowledgeable agents are better-off too, as the increase in their continuation value outweighs the decline in wages. This is true both in the new steady-state, as well as, in the period following the policy implementation.[41]

5.1.4 Discussion

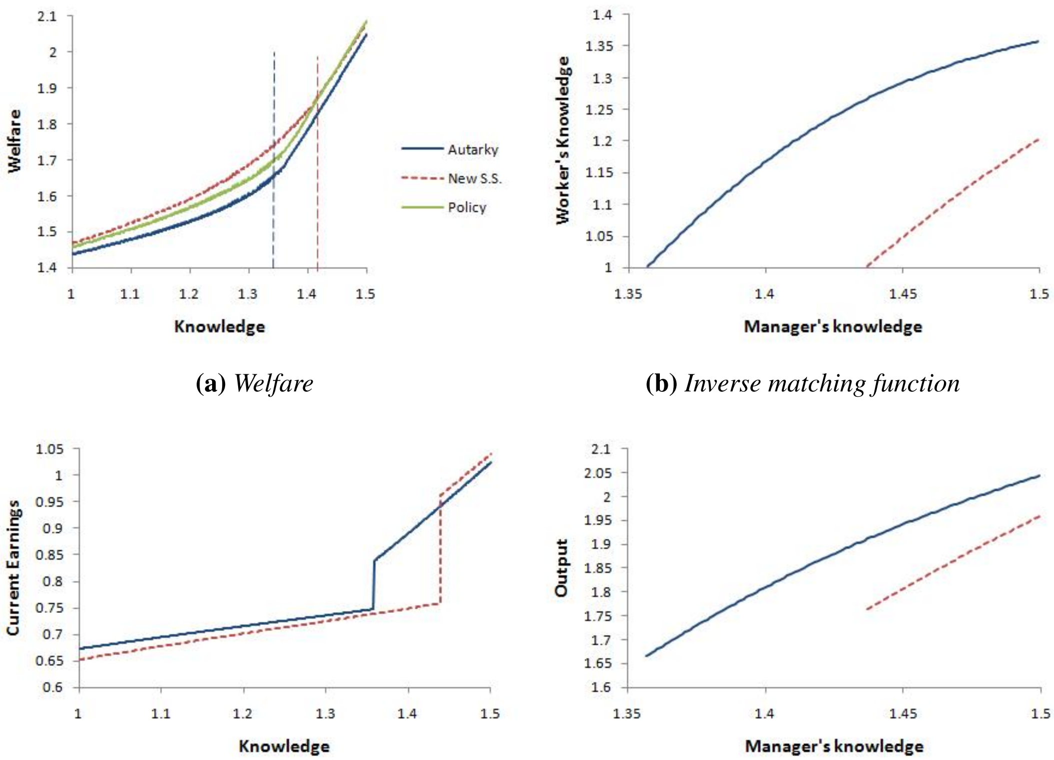
The above plots suggest that the incumbent firms experience a decline in output, irrespective of the rate of learning. According to Aitken and Harrison (1999), FDI was accompanied by a decline in the productivity of domestically owned firms in Venezuela. This decline, the authors report, is due to a contraction in output of domestic firms due to the "market stealing effects" of foreign firms.[42] In our model, the output reduction is a natural consequence of complementarity in production and learning. Under integration, the most knowledgeable workers are hired by the MNEs leaving less knowledgeable workers to work for incumbent domestic firms. We call this the "worker stealing effect". Despite this effect, the Home managers are actually better off if learning is fast enough.

Evidence regarding the impact of multinational production on wages has been mixed. Aitken et al. (1996) report that in Mexico and Venezuela, the wage spillover to domestic firms is neg-

[40] It can be shown that for $\delta = 0.8$, not only are there losers in the new steady-state, but the policy itself creates losers.

[41] It can be shown that the evolution of the value function during the transition is monotonic. This implies that for $\delta = 0.5$, agents are better-off compared to autarky at *each period* during the transition.

[42] With fixed costs of production, foreign firms with lower marginal costs can expand their output at the expense of domestic firms.



(a) *Welfare*

(b) *Inverse matching function*

(c) *Current earnings*

(d) *Output*

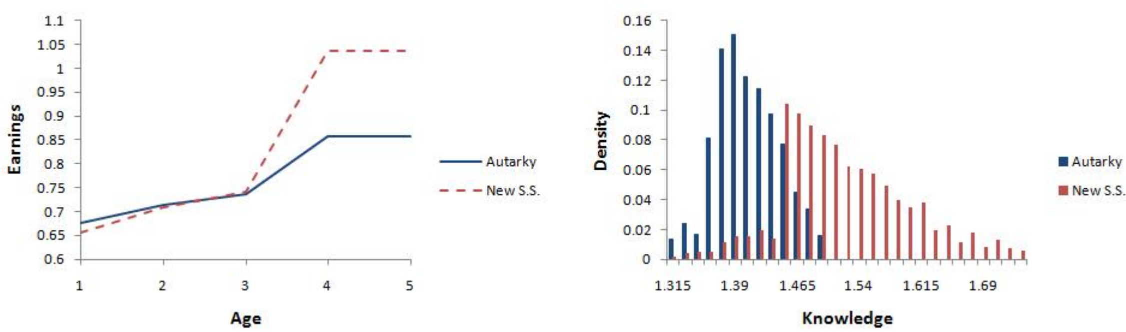
Figure 4: *Effect of Integration when learning is fast*

ative and significant. On the other hand, Lipsey and Sjöholm (2004) find significant positive wage spillovers to domestic firms in Indonesia. In the previous section, we saw that if learning is fast enough, the wage schedule shifts down. This result, however, is not inconsistent with the finding of positive wage spillovers. In the above mentioned studies, the reported wage is the average of wages paid by all domestic firms. The average wage depends not only on the level of the wage schedule but also on the distribution of workers. With integration, as workers get matched with better managers and learn more, the mass of the distribution shifts to the right. This is confirmed in Figure 2. Therefore, a lowering of the wage schedule and a higher average wage can go hand in hand.

The numerical results of this section confirm the analytical results obtained in the last section. When learning is slow, integration creates winners and losers. In the above example, the more knowledgeable agents in the host country lose. But if agents learn fast enough, integration can make every agent better-off. In this case, there is a decline in the output of the incumbent firms, as well as, the wages of the workers. Therefore, a change in current wages or output could be misleading when it comes to assessing welfare gains from integration.

5.2 Earnings Dynamics

Although the economy as a whole does not growing in the steady-state, individual earnings grow over the lifetime. Figure 5a plots the earnings path of the median worker for $\delta = 0.5$. The figure is drawn under the assumption that the actual knowledge he acquires every period is the expected knowledge that an agent with his level of knowledge would acquire. In the figure, the agent works for the first three periods and manages from the fourth period onwards.[43] Under integration, a lower wage in the first two periods is more than compensated by the increase in future profits. The lifetime earnings schedule under integration is also steeper than that under autarky.



(a) *Evolution of individual earnings* **(b)** *Distribution of knowledge at death*

Figure 5: *Dynamics of earnings and knowledge*

Figure 5b provides an explanation for the greater jump in future profits. It shows the distribution of knowledge of the median worker at the time of his death.[44] Following integration, the distribution shifts to the right. On average, the agent becomes a more knowledgeable manager compared to autarky and hence, his expected profits are higher.

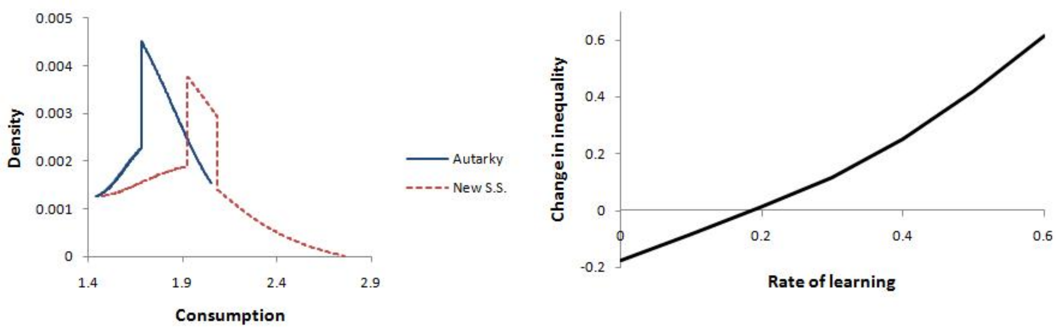
5.3 Inequality

By generating a non-degenerate consumption distribution, the model also allows us to examine the effect of integration on inequality. This is illustrated in Figure 6. δ is set to 0.5 as

[43]Note that once the agent becomes a manager, his earnings do not change because he stops learning.

[44]Given the parameter values, the probability of the agent living for more than 5 periods is extremely small. Here, we assume that the agent dies after 5 periods.

before. Figure 6a plots the Home consumption distributions under autarky and integration. With integration, the distribution stretches out, as mass shifts to the upper tail (The maximum consumption under autarky is 2.05, while that under integration is 2.77). Figure 6a plots the percentage change in the gini coefficient due to integration, as a function of δ. For $\delta = 0.5$, inequality rises by about 40%. But this rise in inequality is not a general phenomenon. For higher values of δ, integration actually leads to a reduction in inequality. Moreover, there is a monotonic relation between inequality and the rate of learning.



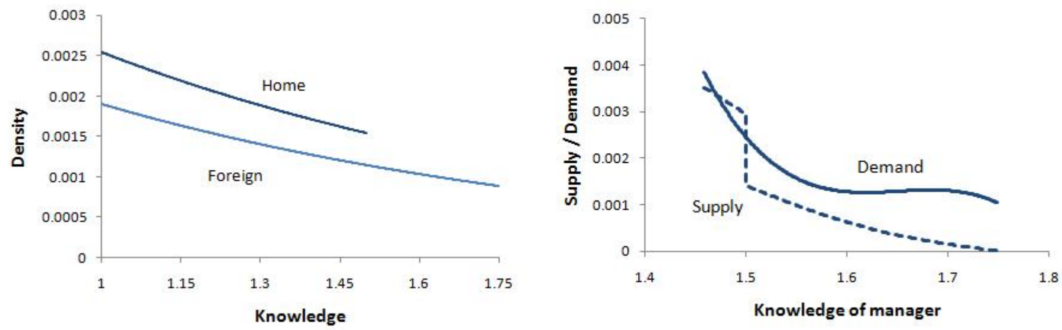
(a) *Consumption distributions* **(b)** *Change in gini coefficient*

Figure 6: *Consumption Inequality*

When agents learn fast enough, integration gives an advantage to the those who are born as the most knowledgeable workers. They work for the most knowledgeable Foreign managers, learn a lot, and in turn, become knowledgeable managers in the future. Agents who are born with very little knowledge continue to be matched with the less knowledgeable incumbent Home managers and accordingly, learn less - learning amplifies the initial inequality in the economy.

5.4 Pattern of MNE activity

As mentioned in Section 4, integration leads to the creation of a new class of Home managers, who are as productive as their counterparts in the Foreign country. This can be seen in Figure 7. Figure 7a shows the newborn distribution of the two countries. Under autarky, the support of the Home knowledge distribution coincides with that of the Home newborn distribution and hence, the knowledge of the best manager is bounded above by $\overline{k}_H$. Figure 7b plots the supply and demand for managers at Home in the integrated steady-state equilibrium. The supply of managers is simply the part of the knowledge distribution that lies above the threshold. Recall that the upper bound of the Home newborn distribution is 1.5. Hence, there is a discrete drop in the density of newborn agents to the right of 1.5, which explains the discontinuity at 1.5. The demand for managers is obtained by looking at the number of workers of each type and the demand for manager per worker.[45]



(a) *Newborn distributions* **(b)** *Supply and demand for managers*

Figure 7: *MNE activity at Home*

Figure 7b suggests that, in the integrated steady-state equilibrium, there are Home managers who are as knowledgeable as their Foreign counterparts. At the same time, the supply of Home managers is not sufficient to meet the demand. In the new equilibrium, some of the Foreign managers hire Home workers and hence, Home firms and Foreign MNEs operate together.[46] Figure 7b also throws light on the pattern of multinational activity. The supply of Home managers falls short of demand, and there are *almost* no Home MNEs in this equilibrium.[47] Moreover, most of the MNEs operating at Home are the best Foreign firms. Thus, the MNEs, on average, are bigger and more productive than the Home firms. PAM implies that a worker in a MNE, on average, is more knowledgeable than a worker in a Home firm. Therefore, the former employees of MNEs are also more productive managers.

In a survey of firms in Ghana, Görg and Strobl (2005) investigate whether knowledge spillovers occur through worker mobility. They combine information on whether or not the owner of a domestic firm had previous experience in a multinational with information on firm-level productivity. They show that firms which are run by owners who worked for foreign

[45]The demand for managers per worker is simply the reciprocal of the span of control. This is a special feature of the span of control depending only on the knowledge of the worker.

[46]See Markusen and Venables (1999) for the case where FDI leads to the development of local industry, but is driven out as the industry develops enough.

[47]Here we follow AGR in assuming that, a manager will hire workers in the other country only if he strictly prefers doing so.

multinationals in the same industry immediately prior to opening their firm, are more productive than other domestic firms. Using data on Danish firms, Malchow-Mø ller et al. (2007) show that previous experience in foreign-owned firms increases a worker's current wage. Both pieces of evidence are consistent with our model.

6 Gains from Globalization

In this section, we quantitatively measure the gains from bilateral integration. In order to quantify the gains, we need the values for the technology parameters α and β, the parameter of the exponential distribution λ, the probability of exit δ, and the upper bound $\overline{k}$ for the countries that integrate. The most difficult parameter to obtain is δ. Recall from the discussion in Section 4, that inherent in the value of δ is the length of a time period. Hence, we estimate δ in two parts. First, we calibrate the length of a period under the assumption that the learning technology is uniform. Next, we combine this with the exit rate for firms (managers in our model) to get δ.

We estimate the length of a time period as follows. In an influential paper, Topel (1991) showed that ten years of current job seniority raises the wage of the typical male worker in the United States by over 25 percent. This is an estimate of what the typical worker would lose if his job were to end exogenously. One can consider this as evidence that workers accumulate firm-specific knowledge and it gives an indication of the speed with which workers learn.[48] We show in the Appendix that this translates into the length of time being 17 years.

The probability of exit for firms varies widely in the U.S. economy. Larger and older firms have a smaller probability of exit compared to smaller and new firms (See Dunne et al., 1989). Here we assume that all firms face the same probability of exit. Dunne et al. (1988) report an average exit rate that varies between 0.3 and 0.39 between each pair of census years over the period 1963-82,[49] where the gap between consecutive census years is 5 years. We take the average exit rate in 5 years to be 0.34. Hence δ, which is the probability of exit of a firm over 17 years, turns out to be approximately 0.7.

Parameter	δ	α	β	λ
Value	0.7	1.9	0.55	1.4

Table 1: *Values of the parameters*

[48] To be technically correct, the knowledge acquired by agents in our model is general.

[49] This excludes the smallest firms.

We choose the three parameters α, β, λ to match three key moments of the U.S. wage distribution. A measure of inequality, that is quite popular in the literature, is the ratio of the 90th percentile to the 10th percentile (Juhn et al., 1993). We ignore the observations at the tails and instead choose a different measure : the ratio of the 75th percentile to the 25th percentile. This ratio was around 2.5 in 2007 (Source : U.S. Bureau of Labor Statistics). This ratio is a measure of dispersion of the wage distribution. We also match the ratio of the mean to the median, which can be interpreted as a measure of skewness. In 2007, this value was around 1.25. The third moment that we match is the ratio of the median wage of managers to the overall median wage. The corresponding value in 2007 was about 2.6. The implied values of the parameters are shown in Table 1.

We assume that the parameter values in Table 1 are common to all countries. This means that there are other country-specific factors that generate different levels of output and factor prices. The country-specific parameters in this model are $\overline{k}$ and σ. We interpret $\overline{k}$ (and hence the support of the knowledge distribution) as being determined by the education policies of the government and the general research environment while σ is a measure of business risk. These two, combined with technology, result in final output. In order to calibrate these two parameters for the different countries, we proceed as follows.

Groups	Basis of classification[50]	Average p.c. GDP	Average $\overline{k}$	σ
High-income	Above 60%	1 (normalized)	2	0.7
Middle-income (high)	40%-60%	0.64	1.85	0.5
Middle-income (low)	20%-40%	0.35	1.5	0.38
Low-income (high)	10%-20%	0.19	1.21	0.29
Low-income (low)	Below 10%	0.07	1.05	0.18

Table 2: *Country specific parameters*

We divide the countries into groups based on their autarky per capita GDP relative to the U.S. Since those figures are not available, we choose the relative per capita GDP of these countries in 1970.[51] The figures for per capita GDP have been obtained from the Penn World Tables. We assume that every country within an income group faces the same σ. We also assume that

[51]Our choice of 1970 as the base year is due to the fact that the Bretton Woods collapsed in 1971. During the entire post-war period, until the fall of the Bretton Woods, most of the countries had a system of fixed exchange rates. Consequently, there was a lot of capital control in place which limited cross border investment, both in financial assets and FDI (Irwin, 2002). This can be judged from the fact that FDI inflows in 1970-73 was around 1.5% of what it was in 1998-2001 (UNCTAD). Of course, world GDP has also been growing during this time. But the growth in real FDI inflows at 17.7% per year far outstripped the corresponding growth in real GDP of 2.5% per year during the period 1985-99.

the richest country in an income group has the same $\overline{k}$ (and hence the same distribution) as the average country belonging to the income group just above it. The difference in income between these two countries arises due to a difference in σ. Therefore, once we know the average $\overline{k}$ of an income group, we can figure out the σ for the next income group. Within an income group, the difference in income arises only due to a difference in $\overline{k}$. Once we know the $\overline{k}$ of the richest country in a group, we can compute the average $\overline{k}$ for that group. We repeat this procedure for every group. Our classification scheme, and σ and $\overline{k}$ of the average country in each income group, is shown in Table 2.

Assumption 1b imposes some restriction on the support of the knowledge distribution. For the High-income countries, we fix $\overline{k}$ at 2. We also normalize the average per capita GDP of the High-income countries to 1. This gives $\sigma = 0.7$ for this group. Following the procedure outlined above, we back out the $\overline{k}$ and σ for the other income groups.

Once we have all the parameter values, we perform the following counter-factual exercise. We let the average country from each of the income groups integrate with the average High-income country. By looking at the resulting GDP and welfare in the host country and comparing it with the autarky levels, we get an estimate of the gains from integration.[52] These gains are displayed in Table 3.

	Percentage change in			Percentage change
	GNI	GDP	Welfare	due to learning
Middle-income (high)	1.6%	10.9%	2%	98%
Middle-income (low)	5.7%	57.1%	13.2%	88%
Low-income (high)	10.2%	157.9%	32.1%	83%
Low-income (low)	22.2%	300%	42.9%	74%

Table 3: *Welfare gains due to integration*

Under autarky, GDP and Gross National Income (GNI) are the same. That is not true anymore under integration. Table 3 suggests that the gain in per capita GDP is much larger than the gain in per capita GNI. Under integration, the workers in the host country are matched with better managers. Since output per worker depends on the knowledge of the manager he is matched with, integration has a significant impact on output. The effect on GNI is much more muted. As the analysis in the last section suggests, higher wages are associated with lower profits and vice versa. The positive effect on welfare, however, is much more pronounced.

[52]The form of multinational activity that we consider in this model corresponds to vertical FDI, which is more prevalent among developed and developing countries.

Under integration, the Home agents learn from more knowledgeable managers and this raises the present value of their income, even when current wages go down. These welfare gains range from a low 2% for the relatively rich Middle-income countries to almost 43% for the poorest countries of the world.

The gains from integration can be decomposed into two parts. There are gains from more efficient allocation of managers to workers. These gains are purely static in nature. On top of this, there are dynamic gains because learning changes the knowledge distribution. The last column in Table 3 shows the percentage of gains that can be attributed to learning. This percentage is higher for the richer developing countries. The Middle-income countries are more similar to the High-income countries in terms of their knowledge distribution. As a result, the gains from re-allocation are quite small.

7 Conclusion

In this paper, we develop a dynamic, general equilibrium model to study the effects of knowledge diffusion from MNEs on host countries. In our model, agents are heterogeneous in terms of the knowledge they possess. Every period, an agent chooses his occupation (worker or manager) and with whom he wants to work. We assume that workers learn on the job. Both the production and the learning technology exhibit complementarity. We show that the complementarity of the production and learning technologies results in positive assortative matching (PAM). In the stationary equilibrium, every agent born above a knowledge threshold is a manager. Agents born with knowledge less than the threshold, work for more knowledgeable managers and learn. Some of them eventually become managers.

We allow the Home country to integrate with a Foreign country, where the Foreign country has relatively more knowledgeable agents. By integration, we mean that managers are able to form international teams. First, we consider a version of the model where agents do not learn. Integration leads to a re-adjustment among host country managers whereby the least knowledgeable among them exit, whereas among those who persist as managers, some are now matched with less knowledgeable workers. Under this situation, we show that integration can never generate Pareto gains for the Home country.

In the presence of learning, there are two effects that determine wages: the labor demand effect and the learning effect. The former tends to raise the wage schedule while the latter tends to lower it. We show that if learning is fast enough, the learning effect dominates and the wage schedule shifts down enough so as to make the incumbent managers better-off. At the same time, the continuation value of the workers outweighs the decline in their current wages, thereby making them better-off. We believe that this is a novel mechanism through which integration, in the presence of learning, can lead to Pareto improvement for the Home country.

In the quantitative section, we calibrate the closed-economy model to match key moments of the U.S. wage distribution. Then we perform the following counter-factual: How much do the developing countries gain by moving from autarky to frictionless integration with the average developed country? Using parameter values obtained from the calibration, we find welfare gains that range from two percent for the richest middle-income countries to almost forty-three percent for the poorest countries.

To conclude, we do not have a good understanding of, for example, how knowledge evolves and the distribution of skills change as a country gradually integrates with the rest of the world. We believe that our paper is a step in that direction. Our quantitative analysis, although performed under some strong assumptions about the knowledge distribution of newborns, suggests that the dynamic gains from integration are non-negligible, especially for the poorest countries of the world. We believe that opening up not only leads to a re-allocation of agents across occupations, but by opening up new opportunities, influences the educational attainment that determines the newborn distribution. We leave this examination for future work.

References

B. Aitken, A. Harrison, and R. E. Lipsey. Wages and foreign ownership a comparative study of Mexico, Venezuela, and the United States. *Journal of International Economics*, 40(3-4):345 – 371, 1996.

B. J. Aitken and A. E. Harrison. Do domestic firms benefit from direct foreign investment? Evidence from Venezuela. *The American Economic Review*, 89(3):605–618, 1999.

P. Antràs. Firms, contracts, and trade structure. *The Quarterly Journal of Economics*, 118(4): 1375–1418, 2003.

P. Antràs and E. Helpman. Global sourcing. *Journal of Political Economy*, 112(3):552–580, 2004.

P. Antràs, L. Garicano, and E. Rossi-Hansberg. Offshoring in a knowledge economy. *The Quarterly Journal of Economics*, 121(1), 2006.

G. Barba Navaretti and A. Venables. *Multinational Firms in the World Economy*. Princeton University Press, 2004.

P. Beaudry and P. Francois. Managerial skill acquisition and the theory of economic development. Forthcoming review of economic studies, 2010.

G. S. Becker. Investment in human capital: A theoretical analysis. *Journal of Political Economy*, 70(s5):9, 1962.

M. Blomström and A. Kokko. Multinational corporations and spillovers. *Journal of Economic Surveys*, 12(3):247–77, 1998.

J. H. Boyd and E. C. Prescott. Dynamic coalitions: Engines of growth. *The American Economic Review*, 77(2):63–67, 1987.

A. T. Burstein and A. Monge-Naranjo. Foreign know-how, firm control, and the income of developing countries. *The Quarterly Journal of Economics*, 124(1):149–195, 2009.

T. Dunne, M. J. Roberts, and L. Samuelson. Patterns of firm entry and exit in U.S. manufacturing industries. *The RAND Journal of Economics*, 19(4):495–515, 1988.

T. Dunne, M. J. Roberts, and L. Samuelson. The growth and failure of U.S. manufacturing plants. *The Quarterly Journal of Economics*, 104(4):671–698, 1989.

W. Easterly. *The Elusive Quest for Growth: Economists' Adventures and Misadventures in the Tropics*. The MIT Press, 2001.

A. Fosfuri, M. Motta, and T. Rnde. Foreign direct investment and spillovers through workers' mobility. *Journal of International Economics*, 53(1):205 – 222, 2001.

S. Garetto. Input sourcing and multinational production. mimeo, University of Chicago, 2008.

L. Garicano. Hierarchies and the organization of knowledge in production. *Journal of Political Economy*, 108(5):874–904, 2000.

I. Gershenberg. The training and spread of managerial know-how, a comparative analysis of multinational and other firms in kenya. *World Development*, 15(7):931–939, 1987.

M. Giarratana, A. Pagano, and S. Torrisi. The role of multinational firms in the evolution of the software industry in India, Ireland and Israel. In A. Arora and A. Gambardella, editors, *From Underdogs to Tigers: Bridging the Gap*, pages 207–235. Oxford University Press, New York, 2004.

A. J. Glass and K. Saggi. Multinational firms and technology transfer. *The Scandinavian Journal of Economics*, 104(4):495–513, 2002.

H. Görg and E. Strobl. Spillovers from foreign firms through worker mobility: An empirical investigation. *Scandinavian Journal of Economics*, 107(4), 2005.

H. Görg, E. Strobl, and F. Walsh. Why do foreign-owned firms pay more? The role of on-the-job training. *Review of World Economics*, 143(3):464–482, 2007.

G. M. Grossman and E. Helpman. Outsourcing versus fdi in industry equilibrium. *Journal of the European Economic Association*, 1(2/3):317–327, 2003.

E. Helpman. A simple theory of international trade with multinational corporations. *The Journal of Political Economy*, 92(3):451–471, 1984.

T. J. Holmes and J. Schmitz, James A. A theory of entrepreneurship and its application to the study of business transfers. *The Journal of Political Economy*, 98(2):265–294, 1990.

D. Irwin. *Free Trade under Fire*. Princeton University Press, 2002.

B. Jovanovic and R. Rob. The growth and diffusion of knowledge. *The Review of Economic Studies*, 56(4):569–582, 1989.

C. Juhn, K. M. Murphy, and B. Pierce. Wage inequality and the rise in returns to skill. *Journal of Political Economy*, 101(3):410, 1993.

B. Kogut and U. Zander. Knowledge of the firm and the evolutionary theory of the multinational corporation. *Journal of International Business Studies*, 24(4):625–645, 1993.

R. E. Lipsey and F. Sjöholm. Fdi and wage spillovers in Indonesian manufacturing. *Review of World Economics*, 140(2):287–310, 2004.

R. E. Lipsey and F. Sjöholm. The impact of inward FDI on host countries: Why such different answers? In T. H. Moran, E. M. Graham, and M. Blomstrm, editors, *Does foreign direct investment promote development?*, pages 23–44. Peterson Institute, 2005.

N. Malchow-Mø ller, J. R. Markusen, and B. Schjerning. Foreign firms, domestic wages. Working paper, NBER, 2007.

J. R. Markusen and A. J. Venables. Foreign direct investment as a catalyst for industrial development. *European Economic Review*, 43(2):335 – 356, 1999.

M. J. Melitz. The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica*, 71(6):1695–1725, 2003.

A. Monge-Naranjo. Foreign firms, domestic entrepreneurial skills and development. mimeo, Northwestern University, 2007.

F. Ohnsorge and D. Trefler. Sorting it out: International trade with heterogeneous workers. *Journal of Political Economy*, 115(5):868–892, 2007.

J. Poole. Multinational spillovers through worker turnover. mimeo, University of San Diego, 2006.

N. Ramondo. Size, geography and multinational production. mimeo, University of Texas at Austin, 2008.

A. Rodríguez-Clare. Multinationals, linkages, and economic development. *American Economic Review*, 86(4):852–73, 1996.

A. Rodríguez-Clare. Trade, diffusion and the gains from openness. Working paper, NBER, 2007.

S. Rosen. Learning and experience in the labor market. *The Journal of Human Resources*, 7 (3):326–342, 1972.

S. Rosen. Authority, control, and the distribution of earnings. *The Bell Journal of Economics*, 13(2):311–323, 1982.

M. Sattinger. Assignment models of the distribution of earnings. *Journal of Economic Literature*, 31(2):831–880, 1993.

R. Topel. Specific capital, mobility, and wages: Wages rise with job seniority. *Journal of Political Economy*, 99(1):145, 1991.

UNCTAD. *World Investment Report 1992 : Transnational Corporations as Engines of Growth*. United Nations, New York, 1992.

Appendix A

Proof of Lemma 1: Since y is indifferent along $\widetilde{w}_{t,NL}(x,y)$, for any x_1 and x_2 we must have $\sigma f(y)n(x_1) - \widetilde{w}_{t,NL}(x_1,y)n(x_1) = \sigma f(y)n(x_2) - \widetilde{w}_{t,NL}(x_2,y)n(x_2)$. Letting $x_2 = x_1 + h$ and re-arranging, we have $\widetilde{w}_{t,NL}(x_1+h,y)n(x_1+h) - \widetilde{w}_{t,NL}(x_1,)n(x_1) = \sigma f(y)n(x_1+h) - \sigma f(y)n(x_1)$. Using Taylor series approximation of $n(x_1+h)$ around h (small), we have $n(x_1+h) = n(x_1) + n'(x_1)h + o_2$. Replacing this in the above equation, we have $[\widetilde{w}_{t,NL}(x_1+h,y) - \widetilde{w}_{t,NL}(x_1,y)]n(x_1) + \widetilde{w}_{t,NL}(x_1+h,y)n'(x_1)h = \sigma f(y)n(x_1+h) - \sigma f(y)n(x_1)$. Dividing by h and taking the limit as $h \to 0$, we get $\widetilde{w}'_{t,NL}(x_1,y)n(x_1) + \widetilde{w}_{t,NL}(x_1,y)n'(x_1) = \sigma f(y)n'(x_1)$. Re-arranging, we have $\widetilde{w}'_{t,NL}(x_1) = \frac{n'(x_1)(\sigma f(y) - \widetilde{w}_{t,NL}(x_1,y))}{n(x_1)}$. Since $\sigma f(y) - \widetilde{w}_{t,NL}(x_1,y) \geq 0$ (in equilibrium, profits must be non-negative) and $n'(x_1) > 0$, it follows that $\widetilde{w}'_{t,NL}(x_1) \geq 0$. Moreover, since the derivative is well-defined, it must be the case that $\widetilde{w}_{t,NL}(x_1)$ is also continuous at x_1. Since x_1 was chosen randomly, the result follows.

Proof of Lemma 2: Let us denote the continuation value of x, when he works for y, by $C(x,y)$. Continuity of the value functions implies that $C(x,y)$ is continuous too. We shall prove this lemma by contradiction. Suppose $\exists\, x'$ and y' such that $V(x') = \widetilde{V}_W(x',y')$ but $\widetilde{w}(x',y') < \max_y \widetilde{w}(x',y) = \widetilde{w}(x',\overline{y})$ (say). Then, given the properties of $\widetilde{w}(x,y)$, we can have either of the following situations - (a) Suppose $y' < \overline{y}$. At $x = x'$, $\widetilde{w}(x',y') < \widetilde{w}(x',\overline{y})$. Moreover, the learning technology implies that the continuation value of x' is higher if he works for $\overline{y}$ rather than y'. But then, $\widetilde{V}_W(x',y') < \widetilde{V}_W(x',\overline{y})$. Hence, $V(x') \neq \widetilde{V}_W(x',y')$ and we get a contradiction. (b) Suppose $y' > \overline{y}$. Then either $\exists\, x'' < x'$ and $y'' < \overline{y}$ such that $w(x'') = \widetilde{w}(x'',y'') < \widetilde{w}(x'',\overline{y})$ in which case we are back to case (a). Or, $\exists\, x'' < x'$ such that there is a discontinuity in $w(x)$ at $x = x''$. In particular, $\underset{\epsilon\to 0}{lim} w(x''-\epsilon) = \underset{\epsilon\to 0}{lim} \widetilde{w}(x''-\epsilon,\overline{y}) > \widetilde{w}(x'',y') = w(x'')$. Since $V(x)$ must be continuous at x'', $\underset{\epsilon\to 0}{lim} C(x''-\epsilon,\overline{y}) < C(x'',y')$. Now, consider the sequence $x \to x''$, $(x < x'')$. Continuity of $\widetilde{w}(.)$ implies that $\underset{\epsilon\to 0}{lim} \widetilde{w}(x''-\epsilon,y') = \widetilde{w}(x'',y')$ and $\underset{\epsilon\to 0}{lim} \widetilde{w}(x''-\epsilon,\overline{y}) = \widetilde{w}(x'',\overline{y})$. Similarly, continuity of $C(.)$ implies that $\underset{\epsilon\to 0}{lim} C(x''-\epsilon,y') = C(x'',y')$ and $\underset{\epsilon\to 0}{lim} C(x''-\epsilon,\overline{y}) = C(x'',\overline{y})$. Now $\underset{\epsilon\to 0}{lim} \widetilde{V}_W(x''-\epsilon,y') = \underset{\epsilon\to 0}{lim} \widetilde{w}(x''-\epsilon,y') + \underset{\epsilon\to 0}{lim} C(x''-\epsilon,y') = \widetilde{w}(x'',y') + C(x'',y') > \widetilde{w}(x'',\overline{y}) + C(x'',\overline{y}) = \underset{\epsilon\to 0}{lim} \widetilde{w}(x''-\epsilon,\overline{y}) + \underset{\epsilon\to 0}{lim} C(x''-\epsilon,\overline{y}) = \underset{\epsilon\to 0}{lim} \widetilde{V}_W(x''-\epsilon,\overline{y})$. But then, as $\epsilon \to 0$, $argmax \widetilde{V}_W(x''-\epsilon,y) \neq \overline{y}$ and we get a contradiction. The reverse implication can be proved in a similar fashion.

Proof of Lemma 3: The manager chooses his workers in order to maximize his current profits given by $\pi_t(y) = \sigma f(y)n(x) - w_t(x)n(x)$. The first-order condition (FOC) for the manager's problem is $[\sigma f(y) - w_t(x)]n'(x) - w'_t(x)n(x) = 0$. Re-arranging, we have $w'_t(x) = \frac{(\sigma f(y) - w_t(x))n'(x)}{n(x)}$. Totally differentiating the FOC,

$$-[(\sigma f(y) - w_t(x))n''(x) - 2w'_t(x)n'(x) - w''_t(x)n(x)] = \sigma f'(y)n'(x)\frac{dy}{dx}.$$

Profit-maximization implies that the LHS is positive.[53] So is $f'(y)n'(x)$. Therefore, $\frac{dy}{dx} > 0$, i.e., more knowledgeable workers will work for more knowledgeable managers. Hence we have Positive Assortative Matching (PAM).

[53]Profit-maximization implies that the second-order condition is satisfied : $(\sigma f(y) - w_t(x))n''(x) - 2w'_t(x)n'(x) - w''_t(x)n(x) < 0$.

Proof of Lemma 4: Equilibrium in the labour market implies that

$$\int_{\underline{k}}^{k_t^*} \psi(s)ds = \int_{k_t^*}^{\overline{k}} n(m_t^{-1}(s))\psi(s)ds$$

where the LHS is the supply of workers while the RHS is the demand for workers. Define

$$\mathcal{L}(k_t^*) = \int_{\underline{k}}^{k_t^*} \psi(s)ds - \int_{k_t^*}^{\overline{k}} n(m_t^{-1}(s))\psi(s)ds$$

Now, $\mathcal{L}(\underline{k}) = -\int_{\underline{k}}^{\overline{k}} n(m_t^{-1}(s))\psi(s)ds < 0$, while $\mathcal{L}(\overline{k}) = \int_{\underline{k}}^{\overline{k}} \psi(s)ds > 0$. Moreover, $\frac{\partial L(k_t^*)}{\partial k_t^*} = [1 + n(m_t^{-1}(k_t^*)]\psi(k_t^*) > 0$. Hence, by the Intermediate Value Theorem, $\exists$ a unique k_t^* such that $\mathcal{L}(k_t^*) = 0$.

Proof of Lemma 5: (i) From the manager's optimization, we have $w_t'(x) = \frac{(\sigma f(y) - w_t(x))n'(x)}{n(x)}$. Profit-maximization implies that the numerator is positive. Otherwise, the manager can always become a worker and have positive earnings. Thus, $w_t'(x) > 0$. Furthermore, the RHS of the above expression is defined everywhere. Therefore, $w_t(x)$ is continuous.

(ii)The value function for the manager is given by

$$V_M(k, w_t) = \max_x\{\sigma f(k)n(x) - w_t(x)n(x)\} + (1-\delta)\max[V_W(k, w_{t+1}), V_M(k, w_{t+1})]$$

The value function for the worker is given by

$$V_W(k, w_t) = w_t(k) + (1-\delta)\int_k^{m_t(k)} \max[V_W(k', w_{t+1}), V_M(k', w_{t+1})]dL(k'|k, m_t(k))$$

Define the vector function $V = [V_M(k, w_t)\ V_W(k, w_t)]'$. Then $\max\{V_W, V_M\} = \max\{[1\ 0]V, [0\ 1]V\}$. Also, define $\alpha = [\max_x\{\sigma f(k)n(x) - w_t(x)n(x)\}\ w_t(k)]'$. Then we have the following equation:

$$\begin{aligned} V &= \alpha + (1-\delta)\begin{bmatrix} \max\{[1\ 0]V, [0\ 1]V\} \\ \int\limits_k^{m_t(k)} \max\{[1\ 0]V, [0\ 1]V\}dL \end{bmatrix} \\ &= T(V) \end{aligned}$$

It can be established, using Blackwell's Sufficiency Conditions, that the operator T is a contraction in the space of continuous vector functions with norm $max[\sup_k |V_M(k)|, \sup_k |V_W(k)|]$. Therefore, a fixed point of V exists and is unique.

Proof of Proposition 1: First, let us derive the equilibrium conditions for a threshold equilibrium. Since k_t^* is indifferent between being a worker and a manager, we must have $V_W(k_t^*, w_t) = V_M(k_t^*, w_t)$. Furthermore, for k_t^* to be the threshold, it must be the case that $\overline{k}$ cannot hire $k_t^* + \epsilon$ and be strictly better-off. If $k_t^* + \epsilon$ is a manager, he earns $V_M(k_t^* + \epsilon)$. In order to hire $k_t^* + \epsilon$, the manager has to pay him a wage such that he is just indifferent between being a manager and a worker. Let this wage be ω. ω

should satisfy

$$\omega + (1-\delta)\int V_M(k)dL(k|, k_t^* + \epsilon, \overline{k}) = V_M(k_t^* + \epsilon)$$

Therefore, period profit of $\overline{k}$ if he hires $k_t^* + \epsilon$ is given by

$$\begin{aligned}
\pi_{k_t^*+\epsilon}(\overline{k}) &= (\sigma f(\overline{k}) - \omega)n(k_t^* + \epsilon) \\
&= \sigma f(\overline{k})n(k_t^* + \epsilon) - n(k_t^* + \epsilon)(V_M(k_t^* + \epsilon) - (1-\delta)\int V_M(k)dL(k|, k_t^* + \epsilon, \overline{k}))
\end{aligned}$$

For k_t^* to be a threshold equilibrium, it must be the case that $\underset{\epsilon\to 0}{lim}\frac{\partial \pi_{k_t^*+\epsilon}(\overline{k})}{\partial \epsilon} \leq 0$ Now,

$$\begin{aligned}
\lim_{\epsilon\to 0}\frac{\partial \pi_{k_t^*+\epsilon}(\overline{k})}{\partial \epsilon} &= \sigma f(\overline{k})n'(k_t^*) - n(k_t^*)(V_M'(k_t^*) - (1-\delta)\frac{\partial}{\partial k_t^*}\int V_M(k)dL(k|, k_t^*, \overline{k})) \\
&\quad - n'(k_t^*)V_M(k_t^*) - (1-\delta)\int V_M(k)dL(k|, k^*, \overline{k})
\end{aligned}$$

From the manager's profit-maximizing problem, we have

$$\sigma f(\overline{k})n'(k_t^*) = w_t'(k_t^*)n(k_t^*) + w_t(k_t^*)n'(k_t^*)$$

Also, for a worker with knowledge k_t^*,

$$V_W(k_t^*) = w_t(k_t^*) + (1-\delta)\int V_M(k)dL(k|k_t^*, \overline{k})$$

Since k_t^* is the threshold, $\max[V_W(k), V_M(k)] = V_M(k)\ \forall k \geq k_t^*$. Differentiating w.r.t. k_t^*,

$$V_W'(k_t^*) = w_t'(k_t^*) + (1-\delta)\frac{\partial}{\partial k_t^*}\int V_M(k)dL(k|k_t^*, \overline{k})$$

Replacing in the expression for $\lim_{\epsilon\to 0}\frac{\partial \pi_{k_t^*+\epsilon}(\overline{k})}{\partial \epsilon}$ and using the fact that $V_W(k_t^*) = V_M(k_t^*)$, we have

$$\begin{aligned}
\lim_{\epsilon\to 0}\frac{\partial \pi_{k_t^*+\epsilon}(\overline{k})}{\partial \epsilon} &= [w_t'(k_t^*) - V_M'(k_t^*) - V_W'(k_t^*) + w_t'(k_t^*)]n(k_t^*) \\
&= V_W'(k_t^*) - V_M'(k_t^*)
\end{aligned}$$

where we use the fact that $V_W(k_t*, w_t) = V_M(k_t^*, w_t)$. Hence, $\underset{\epsilon\to 0}{lim}\frac{\partial \pi_{k_t^*+\epsilon}(\overline{k})}{\partial \epsilon} < 0$ implies that

$$V_W'(k_t^*) < V_M'(k_t^*)$$

The above condition needs to be satisfied for k_t^* to be the equilibrium threshold. We shall prove this proposition in a slightly different way. First, we shall prove the existence of the threshold equilibrium, assuming that the equilibrium is unique. Then we shall show that the sufficient condition for existence is also sufficient for uniqueness.

By assuming uniqueness, we are basically assuming that the set of workers and managers has to be connected in equilibrium (See AGR). Given that there exists a unique market-clearing threshold k_t^*, we check whether the threshold satisfies the equilibrium condition $V_W'(k_t^*) < V_M'(k_t^*)$. Dropping the time subscript, we have

$$V_M(k^*) = \frac{1}{\delta}(\sigma f(k^*) - w(\underline{k}))n(\underline{k})$$

Using the Envelope Theorem,

$$V_M'(k^*) = \frac{1}{\delta}\sigma f'(k^*)n(\underline{k})$$

Also,

$$\begin{aligned} V_W'(k^*) &= w'(k^*) + (1-\delta)\frac{\partial}{\partial k^*}(\int V_M(k)dL(k|k^*, \overline{k})) \\ &= \frac{(\sigma f(\overline{k}) - w(k^*))n'(k^*)}{n(k^*)} + (1-\delta)\frac{\partial}{\partial k^*}(\int V_M(k)dL(k|k^*, \overline{k})) \end{aligned}$$

where the second line follows from the manager's profit-maximization condition. Therefore, for k^* to be an equilibrium, it must be the case that

$$\frac{(\sigma f(\overline{k}) - w(k^*))n'(k^*)}{n(k^*)} + (1-\delta)\frac{\partial}{\partial k^*}(\int V_M(k)dL(k|k^*, \overline{k})) \leq \frac{1}{\delta}\sigma f'(k^*)n(\underline{k})$$

If $\delta = 1$, this condition reduces to

$$\frac{(\sigma f(\overline{k}) - w(k^*))n'(k^*)}{n(k^*)} \leq \sigma f'(k^*)n(\underline{k})$$

Since $w(k^*) > 0$, for the above inequality to hold, we need to find the conditions under which $\frac{f(\overline{k})n'(k^*)}{n(k^*)} \leq f'(k^*)n(\underline{k})$, or $f(\overline{k})n'(k^*) \leq f'(k^*)n(\underline{k})$, since $n(k^*) \geq 1$.

But $f(\overline{k})n'(k^*) \leq f(\overline{k})n'(\underline{k})$ ($\because n''(.) \leq 0$) and $f'(k^*)n(\underline{k}) \geq f'(\underline{k})n(\underline{k})$ (because $n''(.) \geq 0$). Hence, it follows that

$$f(\overline{k})n'(k^*) \leq f(\overline{k})n'(\underline{k}) \leq f'(\underline{k})n(\underline{k}) \leq f'(k^*)n(\underline{k})$$

where the inequality in the middle follows from Assumption 3. Thus for $\delta = 1$, the condition on technology is sufficient for an equilibrium. But when $\delta \neq 1$, we need to determine the magnitude of $\frac{\partial}{\partial k^*}(\int V_M(k)dL(k|k^*, \overline{k}))$, since this term is positive by assumption on the learning technology. This term is endogenous and it depends on the invariant distribution, which in turn is determined by the learning distribution. This term is bounded above, since the domain is compact. Hence by the Least Upper Bound Property, the supremum exists. Let

$$\zeta = \sup\{\frac{\partial}{\partial k^*}(\int V_M(k)dL(k|k^*, \overline{k}))\}$$

Define δ^* as the value of δ that satisfies

$$f(\overline{k})n'(\underline{k}) + (1-\delta^*)\zeta = f'(\underline{k})n(\underline{k})$$

This can be re-written as

$$\frac{n'(\underline{k})}{n(\underline{k})} + (1-\delta^*)\frac{\zeta}{f(\overline{k})n(\underline{k})} = \frac{f'(\underline{k})}{f(\overline{k})}$$

The fact that $\frac{n'(\underline{k})}{n(\underline{k})} < \frac{f'(\underline{k})}{f(\overline{k})}$ implies that $\delta^* < 1$. Hence $\forall \delta \in [\delta^*, 1]$, we have

$$f(\overline{k})n'(\underline{k}) + (1-\delta)\zeta \leq f'(\underline{k})n(\underline{k})$$

Thus,

$$\begin{aligned} f(\overline{k})n'(k^*) + (1-\delta)\frac{\partial}{\partial k^*}(\int V_M(k)dL(k|k^*,\overline{k})) &\leq f(\overline{k})n'(\underline{k}) + (1-\delta)\zeta \\ &\leq f'(\underline{k})n(\underline{k}) \\ &\leq f'(k^*)n(\underline{k}) \end{aligned}$$

$\delta \leq 1$ implies that $f'(k^*)n(\underline{k}) \leq \frac{1}{\delta}f'(k^*)n(\underline{k})$. Therefore,

$$f(\overline{k})n'(k^*) + (1-\delta)\frac{\partial}{\partial k^*}(\int V_M(k)dL(k|k^*,\overline{k})) \leq \frac{1}{\delta}f'(k^*)n(\underline{k})$$

This completes our proof about the existence of equilibrium. As mentioned before, showing uniqueness entails showing that the set of workers and managers is connected. Suppose not. WLOG let us assume that the knowledge distribution has the following partition - $([\underline{k}, k_1], [k_1, k_2], [k_2, k_3], [k_3, k_4])$. Workers in $[\underline{k}, k_1]$ work for managers in $[k_1, k_2]$ while workers in $[k_2, k_3]$ work for managers in $[k_3, k_4]$. For this to be an equilibrium, it must be the case that k_2 must be indifferent between being a worker and a manager. In other words, a deviation involving k_3 hiring $k_2 - \epsilon$ should not make both k_3 and $k_2 - \epsilon$ better off. Using a similar logic as developed above, one can show that that the condition for equilibrium is $V'_W(k_2) > V'_M(k_2)$. One can then show that if $\frac{n'(\underline{k})}{n(\underline{k})} < \frac{f'(\underline{k})}{f(\overline{k})}$, then for δ high enough, this condition will always be violated. Therefore, an allocation with disconnected sets of workers and managers can never be sustained as an equilibrium implying that the only equilibrium is the threshold equilibrium.

To prove efficiency, we follow Legros and Newman (2002). Let us define $\lambda(k)$ as the value that the social planner attaches to knowledge k. In this set-up, when a manager with knowledge y and workers with knowledge x get together, they produce $f(y)n(x)+n(x)[\lambda(E(x'|x,y)-\lambda(x)]$, where $E(x'|x,y)$ is the expected knowledge that workers acquire. Let us consider the no-learning case first. In the absence of learning, the total value produced is simply $f(y)n(x)$. It is simple to show that complementarity implies that the planner uses Positive Assortative Matching. We need to show that the planner maximizes value from threshold matching. We prove this by contradiction. Suppose there is segregation, i.e., the set of workers and managers is disconnected. In this case, one can always find $k_1 < k_2 < k_2 + \epsilon < k_3$ such that k_1 works for k_2 and $k_2 + \epsilon$ works for $k_3(\epsilon)$, i.e., k_2 is a threshold. Since the planner is maximizing value, this implies that the planner can not do better my switching the team members,i.e., it must not be the case that

$$f(k_2+\epsilon)n(k_1) + f(k_3(\epsilon))n(k_2) > f(k_2)n(k_1) + f(k_3(\epsilon))n(k_2+\epsilon)$$

Re-arranging, we have

$$n(k_1)(f(k_2+\epsilon)-f(k_2)) > f(k_3(\epsilon))(n(k_2+\epsilon)-n(k_2))$$

Dividing by ϵ and taking limit as $\epsilon \to 0$, we get

$$n(k_1)f'(k_2) > f(k_3(0))n'(k_2)$$

In the above inequality, the $LHS = n(k_1)f'(k_2) > n(\underline{k})f'(k_2) \geq n(\underline{k})f'(\underline{k})$ and the $RHS = f(k_3(0))n'(k_2) < f(\overline{k})n'(k_2) \leq f(\overline{k})n'(k_1)$. Since $n(\underline{k})f'(\underline{k}) > f(\overline{k})n'(\underline{k})$, the above inequality holds. But then we get a contradiction. Hence the social planner maximizes value by threshold matching. Now we introduce learning. Larger is δ, smaller is the value of learning. Since the above inequality holds for $\delta = 1$, by continuity, it will also hold for δ large enough. Consequently, for δ large enough, threshold matching is also efficient.

Proof of Proposition 2: Suppose P is monotone, has the Feller property and satisfies a mixing condition. Then P has a unique, invariant probability measure Ψ^* (Stokey, Lucas with Prescott, 1989). Define the operator T as

$$(Tf)(k) = \int f(k')P(k,dk'), \qquad \text{all } k \in [\underline{k},\overline{k}]$$

where $f : [\underline{k},\overline{k}] \to \mathbb{R}$ is a bounded function. If f is non-decreasing, then the first-order stochastic dominance property of the learning distribution implies that Tf is also non-decreasing. (Monotone Property) It is straight-forward to verify that if f is bounded and continuous, then the same holds for Tf, i.e., $T : C(k) \to C(k)$ (Feller Property). The mixing condition requires that $\exists c \in [\underline{k},\overline{k}]$, $\epsilon > 0$ and $N \geq 1$ such that $P^N(\underline{k},[c,\overline{k}]) \geq \epsilon$ and $P^N([\underline{k},c],\overline{k}]) \geq \epsilon$. Choose $k' \in [\underline{k},\overline{k}]$. Define $\epsilon_1 = \int_{[k',\overline{k}]} d\Psi_N(s)$ and $\epsilon_2 = \int_{[\underline{k},k']} d\Psi_N(s)$. By the assumption on $\Psi_N(.)$, we know that both these objects are greater than 0. Choose $\epsilon = \delta \min\{\epsilon_1,\epsilon_2\}$ and $N = 1$. Then $P(\underline{k},[k',\overline{k}]) \geq \epsilon$ and $P([\underline{k},k'],\overline{k}]) \geq \epsilon$. Therefore all the conditions for the existence and uniqueness of the invariant distribution are satisfied.

Proof of Proposition 3: Let the number of people being born every period be normalized to 1. Cohort t at time t are all newborns. All agents in $[\underline{k},k_A^*]$ are workers. The measure of these agents is $\int_{\underline{k}}^{k_A^*} \phi_H(k)dk$. A worker with knowledge k demands $\frac{1}{n(k)}$ managers. Therefore, the total demand for managers by cohort t workers is $\int_{\underline{k}}^{k_A^*} \frac{\phi_H(k)}{n(k;\beta)}dk$. The supply of cohort t managers is simply the measure of agents in $[k_A^*,\overline{k}_H]$. This is given by $\int_{k_A^*}^{\overline{k}_H} \phi_H(k)dk$. Let us consider the distribution of cohort $t-1$ agents at time t. A fraction $1-\delta$ of every type of agent in $[\underline{k},k_A^*]$ survive in period t. Out of the ones that survive, a fraction π of every type of agent do not learn and remain where they are. Hence, the total demand for managers by cohort $t-1$ workers is $\int_{\underline{k}}^{k_A^*} \frac{\pi(1-\delta)\phi_H(k)}{n(k;\beta)}dk$. Similarly, a fraction $1-\delta$ of the cohort $t-1$ managers in $[k_A^*,\overline{k}]$ survive in period t. These agents do not learn. Moreover, $(1-\theta)(1-\delta)\int_{\underline{k}}^{k_A^*} \phi_H(k)dk$ agents move into this interval from $[\underline{k},k_A^*]$. They are the cohort $t-1$ agents who were workers in period $t-1$ but become managers in period t. Therefore, the supply of cohort $t-1$ managers is $(1-\delta)[\int_{k_A^*}^{\overline{k}_H} \phi_H(k)dk + (1-\pi)\int_{\underline{k}}^{k_A^*} \phi_H(k)dk]$. The supply and demand for managers in other cohorts can be obtained in a similar fashion. Adding up the demand for managers and the supply of managers in each cohort, we get

$$\text{Demand for managers } = \frac{1}{1-\theta(1-\delta)} \int\limits_{\underline{k}}^{k_A^*} \frac{\phi_H(k)}{n(k;\beta)} dk$$

$$\text{Supply of managers} = \frac{1}{\delta(1-\theta(1-\delta))}[[1-\theta(1-\delta)] \int\limits_{k_A^*}^{\overline{k}_H} \phi_H(k)dk + (1-\theta)(1-\delta) \int\limits_{\underline{k}}^{k_A^*} \phi_H(k)dk]$$

In equilibrium, supply must equal demand. Equating the above two expressions and after a bit of algebra, we obtain the following :

$$\int\limits_{\underline{k}}^{k_A^*} \frac{\phi_H(k)}{n(k;\beta)} dk = \int\limits_{k_A^*}^{\overline{k}_H} \phi_H(k)dk + (1-\theta)(\frac{1-\delta}{\delta})$$

In order to derive the properties of k_A^*, we use the Implicit Function Theorem. Differentiating the above equation w.r.t. π,

$$\frac{\partial k_A^*}{\partial \pi} \frac{\phi_H(k_A^*)}{n(k_A^*;\beta)} = -\frac{\partial k_A^*}{\partial \pi} \phi_H(k_A^*) - (\frac{1-\delta}{\delta})$$

Therefore,

$$\frac{\partial k_A^*}{\partial \pi} [\frac{\phi_H(k_A^*)}{n(k_A^*;\beta)} + \phi_H(k_A^*)] = -(\frac{1-\delta}{\delta})$$

Since the LHS is positive while the RHS is negative, $\frac{\partial k_A^*}{\partial \pi} < 0$. In a similar fashion it can be shown that $\frac{\partial k_A^*}{\partial \delta} < 0$.

Differentiating the labour market clearing condition w.r.t. β,

$$\frac{\partial k_A^*}{\partial \beta} \frac{\phi_H(k_A^*)}{n(k_A^*;\beta)} + \int\limits_{\underline{k}}^{k_A^*} \frac{-\phi_H(k)}{n(k;\beta)^2} \frac{\partial n(k;\beta)}{\partial \beta} dk = -\frac{\partial k_A^*}{\partial \beta} \phi_H(k_A^*)$$

Re-arranging terms, we have

$$\frac{\partial k_A^*}{\partial \beta} [\frac{\phi_H(k_A^*)}{n(k_A^*;\beta)} + \phi_H(k_A^*)] = \int\limits_{\underline{k}}^{k_A^*} \frac{\phi_H(k)}{n(k;\beta)^2} \frac{\partial n(k;\beta)}{\partial \beta} dk$$

Since $\frac{\partial n(k;\beta)}{\partial \beta} > 0$, both the LHS and the RHS are positive. Therefore, $\frac{\partial k_A^*}{\partial \beta} > 0$

Proof of Lemma 6: Let G f.o.s.d. H. Let g and h be the corresponding densities. Also, let $\xi(k)$ be the demand for manager per worker, where the worker has knowledge k. Since the span of control is only a function of the worker's knowledge, a worker with knowledge k works in a firm of size $n(k)$. Hence $\xi(k)$ is simply the reciprocal of $n(k)$. Therefore $\xi'(k) < 0$ (this follows from $n'(k) > 0$). Also, let k^* be the threshold under H.

We shall prove the lemma by contradiction. Let k^* also be the threshold for G. We can have two possibilities - (i) $g(k) < h(k)$ for all $k < k^*$. In this case, the demand for managers under $G = \int_{\underline{k}}^{k^*} \xi(k)g(k)dk < \int_{\underline{k}}^{k^*} \xi(k)h(k)dk =$ demand for managers under H. But the supply of managers under $G = 1 - G(k^*) > 1 - H(k^*) =$supply of managers under H. Hence at k^*, there is an excess supply of managers under G. This means that the threshold for G must be greater than k^*. (ii) There are n intervals $A_i \subset [\underline{k}, k^*]$, $i = 1, ...n$ such that

$$\begin{aligned} g(k) &> h(k) \forall k \in A_i, \forall i \\ g(k) &< h(k) \text{ otherwise} \end{aligned}$$

Rank the A_is such that $A_i < A_j \Rightarrow maxA_i < minA_j$. We proceed as follows - We know that $\underline{k} < minA_1 = a_1(say)$ (otherwise H would f.o.s.d. G). Let $B = [\underline{k}, a_1]$. Then it must be the case that $g(k) < h(k)$ for all $k \in B$. G f.o.s.d. H implies that

$$\int_B g(k)dk + \int_{A_1} g(k)dk < \int_{A_1} h(k)dk + \int_B h(k)dk$$

Re-arranging the above equation,

$$\int_{A_1} [g(k) - h(k)]dk < \int_B [h(k) - g(k)]dk$$

Multiplying both sides by $\xi(a_1)$,

$$\int_{A_1} \xi(a_1)[g(k) - h(k)]dk < \int_B \xi(a_1)[h(k) - g(k)]dk$$

Now, $\xi'(k) < 0$ implies that $\xi(a_1) < \xi(k) \forall k \in B$ and $\xi(a_1) > \xi(k) \forall k \in A_1$. Replacing $\xi(a_1)$ in the above equation,

$$\int_{A_1} \xi(k)[g(k) - h(k)]dk < \int_B \xi(k)[h(k) - g(k)]dk$$

Here we are using the fact that $h(k) - g(k) > 0 \forall k \in B$ and $g(k) - h(k) > 0 \forall k \in A_1$. We re-arrange again to obtain

$$\int_B \xi(k)g(k)dk + \int_{A_1} \xi(k)g(k)dk < \int_{A_1} \xi(k)h(k)dk + \int_B \xi(k)h(k)dk$$

The LHS and the RHS are the demand for managers by workers in $B \cup A_1$ under G_1 and G_2 respectively. Define $maxA_1 = a_1'$ and $minA_2 = a_2$. Let $C = [a_1', a_2]$. G f.o.s.d. H implies that

$$\int_B g(k)dk + \int_{A_1} g(k)dk + \int_C g(k)dk + \int_{A_2} g(k)dk < \int_B h(k)dk + \int_{A_1} h(k)dk + \int_C h(k)dk + \int_{A_2} h(k)dk$$

Re-arranging, we have

$$\int_{A_2} [g(k) - h(k)]dk < (\int_B [h(k) - g(k)]dk - \int_{A_1} [g(k) - h(k)]dk) + \int_C [h(k) - g(k)]dk$$

Multiplying both sides by $\xi(a_2)$,

$$\int_{A_2} \xi(a_2)[g(k)-h(k)]dk < \xi(a_2)(\int_B [h(k)-g(k)]dk - \int_{A_1} [g(k)-h(k)]dk) + \int_C \xi(a_2)[h(k)-g(k)]dk$$

Since $\xi'(k) < 0$, we have

$$\begin{aligned}\int_{A_2} \xi(k)[g(k)-h(k)]dk &< \xi(a_2)(\int_B [h(k)-g(k)]dk - \int_{A_1} [g(k)-h(k)]dk) \\ &+ \int_C \xi(k)[h(k)-g(k)]dk\end{aligned}$$

Again, using $\xi'(k) < 0$ in the above inequality

$$\begin{aligned}LHS &< \xi(a_1)(\int_B [h(k)-g(k)]dk - \int_{A_1} [g(k)-h(k)]dk) \\ &+ \int_C \xi(k)[h(k)-g(k)]dk \\ &\int_B \xi(k)[h(k)-g(k)]dk - \int_{A_1} \xi(k)[g(k)-h(k)]dk \\ &+ \int_C \xi(k)[h(k)-g(k)]dk\end{aligned}$$

Re-arranging gives us that the demand for managers by workers in $B \cup A_1 \cup C \cup A_2$ under G is less than that under H. We can repeat this argument by expanding the set till we reach k^*. But then we have shown that the demand for managers under G is less than that under H. However the supply of managers under G is greater than that under H. Therefore, at k^*, there is an excess supply of managers under G. Hence the threshold under G has to be greater than k^*.

Proof of Proposition 4: The derivation of the threshold is the same as in the proof of Proposition 3. Equating the supply of managers and the demand for managers, we have

$$\int_{\underline{k}}^{k_I^*} \frac{\phi_H(k) + \phi_F(k)}{2n(k;\beta)} dk = \int_{k_I^*}^{\overline{k}_F} \frac{\phi_H(k) + \phi_F(k)}{2} dk + (1-\theta)(\frac{1-\delta}{\delta})$$

Re-arranging, we have

$$\int_{\underline{k}}^{k_I^*} \frac{\phi_H(k)}{n(k;\beta)} dk - \int_{k_I^*}^{\overline{k}_H} \phi_H(k)dk - (1-\theta)(\frac{1-\delta}{\delta}) = \int_{k_I^*}^{\overline{k}_H} \phi_F(k)dk + (1-\theta)(\frac{1-\delta}{\delta}) - \int_{\underline{k}}^{k_I^*} \frac{\phi_F(k)}{n(k;\beta)} dk$$

Not that the LHS is the excess demand for managers in the Home country if the threshold is k_I^*, while the RHS is the excess supply of managers in the Foreign country if the threshold is k_I^*. If $k_I^* = k_A^*$, the LHS is equal to 0, i.e.

$$\int_{\underline{k}}^{k_A^*} \frac{\phi_H(k)}{n(k;\beta)}dk - \int_{k_A^*}^{\overline{k}_H} \phi_H(k)dk = (1-\theta)(\frac{1-\delta}{\delta})$$

Since $\phi_F(k)$ f.o.s.d. $\phi_H(k)$, from Lemma 6, we know that

$$\int_{\underline{k}}^{k_A^*} \frac{\phi_F(k)}{n(k;\beta)}dk - \int_{k_A^*}^{\overline{k}_H} \phi_F(k)dk < (1-\theta)(\frac{1-\delta}{\delta})$$

Therefore, for $k_I^* = k_A^*$, the RHS is positive. But this means that $k_I^* \neq k_A^*$. In particular, since the LHS is increasing in k_I^* and the RHS is decreasing, it must be the case that $k_I^* > k_A^*$.

Proof of Theorem 2: We know that an allocation A is a Pareto improvement over allocation B if $u(x_i^A) \geq u(x_i^B)$ for all i, and $u(x_j^A) > u(x_j^B)$ for some j. This suggests that in order to show that A *is not* a Pareto improvement over B, it is sufficient to show that $\exists$ individuals 1 and 2 s.t. $u(x_1^A) \geq u(x_1^B) \Rightarrow u(x_2^A) < u(x_2^B)$ and vice versa. From Lemma 6, we have

$$k_{A,NL}^* < k_{I,NL}^*$$

where NL refers to no-learning. If there are incumbent firms in the Home economy, this also means that

$$k_{I,NL}^* < \overline{k}_H$$

The above inequality suggests that under Integration, there are incumbent Home managers who continue to operate ($k \in [k_{I,NL}^*, \overline{k}_H]$). At the same time, under Autarky, $m_{A,NL}^{-1}(k_{A,NL}^*) = \underline{k} \Rightarrow m_{A,NL}^{-1}(k_{I,NL}^*) > \underline{k}$ (follows from PAM). While under Integration, $m_{A,NL}^{-1}(k_{I,NL}^*) = \underline{k}$, i.e., under Integration, the manager with knowledge $k_{I,NL}^*$ has a worse match. The present value of $k_{I,NL}^*$ is just the period profits $\pi_{I,NL}(k_{I,NL}^*)$ divided by δ.

$$\begin{aligned}
\pi_{A,NL}(k_{I,NL}^*) &= (\sigma f(k_{I,NL}^*) - w_{A,NL}(m_{A,NL}^{-1}(k_{I,NL}^*)))n(m_{A,NL}^{-1}(k_{I,NL}^*)) \\
&\geq (\sigma f(k_{I,NL}^*) - w_{A,NL}(k))n(k) \quad \forall k \\
&\geq (\sigma f(k_{I,NL}^*) - w_{A,NL}(\underline{k}))n(\underline{k})
\end{aligned}$$

Note that $\pi_{I,NL}(k_{I,NL}^*) = (\sigma f(k_{I,NL}^*) - w_{I,NL}(\underline{k}))n(\underline{k})$. Therefore, the relation between $\pi_{A,NL}(k_{I,NL}^*)$ and $\pi_{I,NL}(k_{I,NL}^*)$ depends on the relation between $w_{A,NL}(\underline{k})$ and $w_{I,NL}(\underline{k})$. Let us consider the following cases -

(a) $w_{A,NL}(\underline{k}) < w_{I,NL}(\underline{k})$: In this case, $\pi_{A,NL}(k_{I,NL}^*) > \pi_{I,NL}(k_{I,NL}^*) \Rightarrow \underline{k}$ is strictly better-off under Integration but $k_{I,NL}^*$ is strictly worse-off.

(b) $w_{A,NL}(\underline{k}) \gg w_{I,NL}(\underline{k})$: Then it is possible to have, $\pi_{A,NL}(k_{I,NL}^*) < \pi_{I,NL}(k_{I,NL}^*) \Rightarrow k_{I,NL}^*$ is strictly better-off under Integration but $\underline{k}$ is strictly worse-off.

(c) $w_{A,NL}(\underline{k}) = w_{I,NL}(\underline{k})$: In this case, $\pi_{A,NL}(k_{I,NL}^*) \geq \pi_{I,NL}(k_{I,NL}^*)$. This is not a negation of Pareto improvement. However let us choose the agent with knowledge $k_{I,NL}^* + \epsilon$ such that $m_{I,NL}^{-1}(k_{I,NL}^* + \epsilon) < m_{A,NL}^{-1}(k_{I,NL}^* + \epsilon)$. Since $m(.)$ is continuous, we can always find such an ϵ.

Moreover, since $m(.)$ is a function, its inverse must be strictly monotonic. Hence $m^{-1}_{I,NL}(k^*_{I,NL} + \epsilon) > m^{-1}_{I,NL}(k^*_{I,NL}) = \underline{k}$. Now

$$w'_{A,NL}(\underline{k}) = \frac{(\sigma f(k'_{A,NL}) - w_{A,NL}(\underline{k}))n'(\underline{k})}{n(\underline{k})} < \frac{(\sigma f(k'_{I,NL}) - w_{I,NL}(\underline{k}))n'(\underline{k})}{n(\underline{k})} = w'_{I,NL}(\underline{k})$$

Combined with $w_{A,NL}(\underline{k}) = w_{I,NL}(\underline{k})$, this means that in the neighborhood of $k = \underline{k}$, $w_{A,NL}(k) < w_{I,NL}(k)$. Hence,

$$\begin{aligned}\pi_{A,NL}(k^*_{I,NL} + \epsilon) &= (\sigma f(k^*_{I,NL} + \epsilon) - w_{A,NL}(m^{-1}_{A,NL}(k^*_{I,NL} + \epsilon)))n(m^{-1}_{A,NL}(k^*_{I,NL} + \epsilon)) \\ &\geq (\sigma f(k^*_{I,NL} + \epsilon) - w_{A,NL}(m^{-1}_{I,NL}(k^*_{I,NL} + \epsilon)))n(m^{-1}_{I,NL}(k^*_{I,NL} + \epsilon))\end{aligned}$$

Using the fact that $w_{A,NL}(k^*_{I,NL} + \epsilon) < w_{I,NL}(k^*_{I,NL} + \epsilon)$, we have

$$\pi_{A,NL}(k^*_{I,NL}+\epsilon) > (\sigma f(k^*_{I,NL}+\epsilon) - w_{I,NL}(m^{-1}_{I,NL}(k^*_{I,NL}+\epsilon)))n(m^{-1}_{I,NL}(k^*_{I,NL}+\epsilon)) = \pi_{I,NL}(k^*_{I,NL}+\epsilon)$$

Therefore $k^*_{I,NL} + \epsilon$ is strictly worse-off.

Hence, for all the 3 cases (a), (b) and (c), we have shown that at least one individual is worse-off. Since these cases are exhaustive, the result follows.

Proof of Theorem 3: We shall proceed as follows - First, we shall find the condition under which $\overline{k}_H$ is better-off under Integration. Since $\overline{k}_H$ is a manager under both Autarky and Integration, we have to show that $V_{M,I}(\overline{k}_H) > V_{M,A}(\overline{k}_H)$. Since $\overline{k}_H$ is matched with k^*_A under Autarky and $\widetilde{k}$ under Integration, this implies that

$$[\sigma f(\overline{k}_H) - w_A(k^*_A)]n(k^*_A) < [\sigma f(\overline{k}_H) - w_I(\widetilde{k})]n(\widetilde{k})$$

Re-arranging, we have

$$\sigma f(\overline{k}_H)[n(k^*_A) - n(\widetilde{k})] < w_A(k^*_A)n(k^*_A) - w_I(\widetilde{k})n(\widetilde{k})$$

Now,

$$\begin{aligned} w_A(k^*_A)n(k^*_A) - w_I(\widetilde{k})n(\widetilde{k}) &= \frac{\delta + (1-\theta)(1-\delta)}{\delta}[\sigma f(k^*_A)n(\underline{k})\frac{n(k^*_A)}{1+n(k^*_A)} - \sigma f(k^*_I)n(\underline{k})\frac{n(k^*_I)}{1+n(k^*_I)}] \\ &+ \frac{(1-\theta)(1-\delta)}{\delta}[f(\overline{k}_F)n(k^*_I)\frac{n(k^*_I)}{1+n(k^*_I)} - f(\overline{k}_H)n(k^*_A)\frac{n(k^*_A)}{1+n(k^*_A)}] \\ &+ \frac{\delta + (1-\theta)(1-\delta)}{\delta}[\frac{n(k^*_A)}{1+n(k^*_A)}\int_{\underline{k}}^{k^*_A} f(m_A(k))n'(k)dk \end{aligned}$$

$$+\frac{n(k^*_I)}{1+n(k^*_I)}\int_{\widetilde{k}}^{k^*_I} f(m_I(k))n'(k)dk] - \int_{\underline{k}}^{\widetilde{k}} f(m_I(k))n'(k)dk$$

Let us consider each term on the RHS.

$$\sigma f(k_A^*)n(\underline{k})\frac{n(k_A^*)}{1+n(k_A^*)} - \sigma f(k_I^*)n(\underline{k})\frac{n(k_I^*)}{1+n(k_I^*)} > \sigma f(\underline{k})n(\underline{k})\frac{n(\underline{k})}{1+n(\underline{k})} - \sigma f(\overline{k}_H)n(\underline{k})\frac{n(\overline{k}_H)}{1+n(\overline{k}_H)}$$

$$f(\overline{k}_F)n(k_I^*)\frac{n(k_I^*)}{1+n(k_I^*)} - f(\overline{k}_H)n(k_A^*)\frac{n(k_A^*)}{1+n(k_A^*)} > f(\overline{k}_F)n(\underline{k})\frac{n(\underline{k})}{1+n(\underline{k})} - f(\overline{k}_H)n(\overline{k}_H)\frac{n(\overline{k}_H)}{1+n(\overline{k}_H)}$$

$$-\int_{\underline{k}}^{\widetilde{k}} f(m_I(k))n'(k)dk > -f(\overline{k}_H)[n(\overline{k}_H) - n(\underline{k})]$$

$$\frac{n(k_A^*)}{1+n(k_A^*)}\int_{\underline{k}}^{k_A^*} f(m_A(k))n'(k)dk > 0, \; \frac{n(k_I^*)}{1+n(k_I^*)}\int_{\widetilde{k}}^{k_I^*} f(m_I(k))n'(k)dk > 0$$

Replacing them in the above equation,

$$\begin{aligned} w_A(k_A^*)n(k_A^*) - w_I(\widetilde{k})n(\widetilde{k}) \;\; &> \;\; \frac{\delta + (1-\theta)(1-\delta)}{\delta}[\sigma f(\underline{k})n(\underline{k})\frac{n(\underline{k})}{1+n(\underline{k})} - \sigma f(\overline{k}_H)n(\underline{k})\frac{n(\overline{k}_H)}{1+n(\overline{k}_H)}] \\ &\quad + \frac{(1-\theta)(1-\delta)}{\delta}[f(\overline{k}_F)n(\underline{k})\frac{n(\underline{k})}{1+n(\underline{k})} - f(\overline{k}_H)n(\overline{k}_H)\frac{n(\overline{k}_H)}{1+n(\overline{k}_H)}] \\ &\quad - f(\overline{k}_H)[n(\overline{k}_H) - n(\underline{k})] \\ &= \;\; A(say) \end{aligned}$$

Furthermore,

$$\begin{aligned} f(\overline{k}_H)[n(k_A^*) - n(\widetilde{k})] \;\; &< \;\; f(\overline{k}_H)[n(\overline{k}_H) - n(\underline{k})] \\ &= \;\; B(say) \end{aligned}$$

Hence the sufficient condition for $\overline{k}_H$ to be strictly better-off under Integration is that $A > B$. After a bit of algebra, this condition reduces to

$$(1-\theta)(\frac{1-\delta}{\delta}) > \frac{2f(\overline{k}_H)[n(\overline{k}_H) - n(\underline{k})] + n(\underline{k})[\mu(\overline{k}_H)f(\overline{k}_H) - \mu(\underline{k})f(\underline{k})]}{\mu(\underline{k})[f(\overline{k}_F) + f(\underline{k})]n(\underline{k}) - \mu(\overline{k}_H)[n(\underline{k}) + n(\overline{k}_H)]f(\overline{k}_H)}$$

where $\mu(\underline{k}) = \frac{n(\underline{k})}{1+n(\underline{k})}$ and $\mu(\overline{k}_H) = \frac{n(\overline{k}_H)}{1+n(\overline{k}_H)}$. Of course, this only ensures that $\overline{k}_H$ is strictly better off. We need to show that every Home agent can be made better off.

Notice that for $k \in [\underline{k}, k_A^*]$, agents are workers under both regimes. For $k \in [k_A^*, k_I^*]$, agents are workers under Integration but managers under Autarky. Finally for $k \in [k_I^*, \overline{k}_H]$, agents are managers under both regimes. In the steady-state, $V_{M,i}(k) = \frac{1}{\delta}\pi_i(k)$, $i \in \{A, I\} \Rightarrow V'_{M,i}(k) = \frac{1}{\delta}\pi'_i(k) =$

$\frac{1}{\delta}f'(k)n(m_i^{-1}(k))$. For $k \in [k_I^*, \overline{k}_H]$,

$$\begin{aligned} m_I^{-1}(k) &< m_A^{-1}(k) \Rightarrow \frac{1}{\delta}f'(k)n(m_I^{-1}(k)) < \frac{1}{\delta}f'(k)n(m_A^{-1}(k)) \\ &\Rightarrow V'_{M,I}(k) < V'_{M,A}(k) \end{aligned}$$

Suppose $V_{M,I}(\overline{k}_H) > V_{M,A}(\overline{k}_H)$. Since $V_{M,A}(.)$ is decreasing at a faster rate than $V_{M,I}(.)$ in the neighborhood $[k_I^*, \overline{k}_H]$, this implies that $V_{M,I}(k) > V_{M,A}(k)$ for $k \in [k_I^*, \overline{k}_H]$. In particular, $V_{M,I}(k_I^*) > V_{M,A}(k_I^*)$. For $k \in [k_A^*, k_I^*]$,

$$V_{W,I}(k) = \frac{1}{\delta + (1-\theta)(1-\delta)}w_I(k) + \frac{(1-\theta)(1-\delta)}{\delta(\delta + (1-\theta)(1-\delta))}\sigma f(m_I(k))n(k)$$

$$\Rightarrow V'_{W,I}(k) = \frac{1}{\delta + (1-\theta)(1-\delta)}\sigma f(m_I(k))n'(k) + \frac{(1-\theta)(1-\delta)}{\delta(\delta + (1-\theta)(1-\delta))}\sigma f'(m_I(k))n(k)m'_I(k)$$

Also, $V'_{M,A}(k) = \frac{1}{\delta}f'(k)n(m_A^{-1}(k))$. When $\delta = 1$, $V'_{W,I}(k) = f(m_I(k))n'(k)$ and $V'_{M,A}(k) = f'(k)n(m_A^{-1}(k))$. Now,

$$\frac{f'(k)}{f(m_I(k))} \geq \frac{f'(\underline{k})}{f(m_I(k))} > \frac{f'(\underline{k})}{f(\overline{k}_H)} > \frac{n'}{(}\underline{k})n(\underline{k}) \geq \frac{n'}{(}m_A^{-1}(k))n(m_A^{-1}(k)) > \frac{n'(k)}{n(m_A^{-1}(k))}$$

Hence, $V'_{M,A}(k) > V'_{W,I}(k)$. Therefore, $\exists\ \delta_1$ s.t. $\forall \delta > \delta_1$, $V'_{M,A}(k) > V'_{W,I}(k)$ and hence $V_{W,I}(k) > V_{M,A}(k)$ for $k \in [k_A^*, k_I^*]$. In particular, $V_{W,I}(k_A^*) > V_{M,A}(k_A^*)$. For $k \in [\underline{k}, k_A^*]$,

$$V'_{W,A}(k) = \frac{1}{\delta + (1-\theta)(1-\delta)}f(m_A(k))n'(k) + \frac{(1-\theta)(1-\delta)}{\delta(\delta + (1-\theta)(1-\delta))}f'(m_A(k))n(k)m'_A(k)$$

and

$$V'_{W,I}(k) = \frac{1}{\delta + (1-\theta)(1-\delta)}f(m_I(k))n'(k) + \frac{(1-\theta)(1-\delta)}{\delta(\delta + (1-\theta)(1-\delta))}f'(m_I(k))n(k)m'_I(k)$$

When $\delta = 1$, $V'_{W,A}(k) = f(m_A(k))n'(k) > f(m_I(k))n'(k) = V'_{W,I}(k)$. Therefore, $\exists\ \delta_2$ s.t. $\forall \delta > \delta_2$, $V'_{W,A}(k) > V'_{W,I}(k)$ and hence $V_{W,I}(k) > V_{w,A}(k)$ for $k \in [k_A^*, k_I^*]$. Hence, if we choose $\delta^* = \max\{\delta_1, \delta_2\}$, $\forall \delta > \delta^*$, $V_{W,I}(k) > V_{w,A}(k)$ for $k \in [\underline{k}, \overline{k}_H]$.

Appendix B

Derivation of the length of time : In our paper, because of PAM, workers do not work for the same manager for two consecutive periods. But we can think of each period as consisting of several sub-periods. In each of these sub-periods, the worker learns a little bit and accordingly, his wage also increases gradually. This wage increase does not take place in our model. This is because the workers are implicitly tied to their managers for one period. We can think of the rising wages as what the managers would have paid if the workers could potentially leave at any point in time. We make some simplifying assumptions : First, the wage growth takes place at a constant rate every year. Second, although wages are growing, the manager's earnings remain constant. And finally, the marginal increment to knowledge is constant over the sub-periods. The annual rate of growth of wage due to knowledge accumulation turns out to be 2% (Basically we are solving for g, where g satisfies $(1+g)^{10} = 1.25$). In our model, the difference between the earnings of the worker and the manager is only due to a difference in their level of knowledge. In the U.S., on average, the manager's wage is a little more than twice the wage of the worker (Source : U.S. Bureau of Labor Statistics). Worker here refers to an individuals engaged in any non-managerial occupation. Assuming that the wage of the worker continues to rise at the rate of 2% per year, this essentially means that the worker would catch up with the manager in around 34 years (Here we are using the second assumption). With an uniform learning distribution, the expected knowledge accumulated by the worker in one period lies half way between the worker's initial knowledge and the knowledge of his manager. The third assumption then implies that in order to catch up with the manager, the worker's knowledge has to rise by a constant amount every period for 34 years; the worker will reach the half-way stage in 17 years. Hence, the length of time is 17 years.

Appendix C

Income groups : We have a total of 117 countries. The countries are classified according to their per capita GDP in 1970.

High-income : Switzerland, Luxembourg, United States, Denmark, Sweden, Australia, Canada, Netherlands, New Zealand, Norway, France, Germany, Belgium, United Kingdom, Japan, Finland, Austria, Italy, Iceland.

Middle-income (High) : Barbados, Argentina, Spain, Israel, Greece, Puerto Rico, Ireland, Gabon.

Middle-income (Low) : Hong Kong, South Africa, Uruguay, Portugal, Namibia, Hungary, Mexico, Cyprus, Venezuela, Trinidad & Tobago, Chile, Singapore, Peru, Poland, Costa Rica, Nicaragua, El Salvador, Panama, Mauritius, Brazil.

Low-income (High) : Turkey, Iran, Papua New Guinea, Jamaica, Guatemala, Angola, Seychelles, Zambia, Colombia, Cote d'Ivoire, Guyana, Equatorial Guinea, Fiji, Algeria, Taiwan, Guinea, Malaysia, Tunisia, Korea, Republic of, Bolivia, Paraguay, Zimbabwe, Ecuador, Philippines, Comoros, Mozambique, Romania, Dominican Republic, Syria, Egypt, Morocco, Central African Republic, Thailand, Jordan, Mauritania, Honduras.

Low-income (Low) : Ghana, Senegal, Togo, Gambia, The, Cameroon, Madagascar, Sri Lanka, Cape Verde, Sierra Leone, Botswana, Niger, Pakistan, Congo, Republic of, Chad, Benin, Bangladesh, India, Congo, Dem. Rep., Guinea-Bissau, Kenya, Haiti, Burundi, Lesotho, Nigeria, Indonesia, Rwanda, Nepal, China, Mali, Tanzania, Burkina Faso, Malawi, Ethiopia, Uganda.