Information Costs and International Currencies

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Abstract

This paper develops a unified framework for examining international payment patterns. Using an open-economy search model with competing monetary assets, I show how *information costs* and *government transaction policies* interact to produce a structure of world payments reminiscent with what is observed in practice. Agents' decision to acquire information shapes exchange patterns by affecting a currency's usefulness in trade— or, its *liquidity.* Due to network effects and complementarities in optimal choices, there is persistence and hysteresis in monetary leadership, even if the payment pattern is socially inefficient. While high inflation can lead to dollarization, a sufficiently large local government can de-dollarize the economy by inducing a switch to national currency use. By using simple transaction policies that promote the use of local money, governments can reverse exchange patterns, despite inertia. A calibration of the generalized model shows how changes in information costs or inflation can make hegemony shift from a dollar-dominated monetary system to a multi-polar regime. The welfare benefits of having an international currency are also discussed and compared with previous estimates.

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1 Introduction

A central issue in international monetary economics concerns the coexistence of competing national monies, possibly with different rates of return, and the emergence of international currencies.[1] While the status of national currencies is typically enforced by legal restrictions, the use of currencies as international media of exchange is largely determined by the "invisible hand." What factors determine which object circulates as an international medium of exchange? How do different payment patterns affect trade, exchange rates, and welfare? While standard (reduced-form) monetary models have little to say on these questions, search-theoretic models have proven particularly insightful as they explicitly formalize the essential role of money, rather than assuming it exogenously. A case in point is Matsuyama, Kiyotaki, and Matsui (1993) and Trejos and Wright (1996), which examine the coexistence of two fiat monies in a two-country world. However, primitive restrictions on asset divisibility and portfolio holdings prevent discussion of inflation and yield insights that lack robustness.[2] Recent works (e.g. Shi (1997) and Lagos and Wright (2005)) have relaxed these restrictions; though without additional frictions, the theory predicts rate-of-return equality across currencies and can neither pin down payments nor exchange rates.

The objective of this paper is to provide a unified framework for exploring both positive and normative aspects of different international monetary systems. To do so, I develop a general equilibrium search model with competing currencies to analyze the factors that make it more or less likely that certain currency regimes emerge, and to study implications for prices, allocations, and welfare. Unlike most international macroeconomic models, this paper does not assume which monies circulate where. In fact, much of this literature restricts agents to only using their domestic currency since this prevents the exchange rate from being indeterminate, as in Kareken and Wallace (1981). However, such an approach is particularly unsatisfactory if currencies have different inflation rates and hence different returns.

To characterize global exchange patterns, the model features three key ingredients. First, the open-economy setting allows for trade between countries— a key determinant of international currency use. Second, payment patterns are pinned down through a costly information acquisition problem, following Lester, Postlewaite, and Wright (2011), who introduce asymmetric information about assets' fundamental values.[3] Here, *information costs* reflect the costly nature of accepting

[1]International currencies are monies used for transactions in regions outside the country of issue. Many historical examples abound; see Krugman (1980) for a review.

[2]In Trejos and Wright (1996) for example, indivisibility implies that increases in currency stock are beneficial. Since absent indivisibility, such policies would be neutral, all policy implications are not robust to this restriction.

[3]Costly information acquisition in random-matching models was pioneered by Kim (1996). In more recent work, Rocheteau (2011) studies how information frictions generate liquidity differentials across assets, while Li, Rocheteau, and Weill (2011) obtain endogenous liquidity constraints from the threat of fraudulent assets.

currencies. For example, sellers must pay an ex-ante transaction fee to administer multi-payment options, install new technologies, or learn how to use new media of exchange such as debit cards or smart cards.[4] Since the decision to acquire information about a currency determines its acceptability in trade, this breaks the indeterminacy that results when multiple assets can be used as payment. Third, *government transaction policies* are introduced to account for the fact that payment outcomes reflect choices made by official bodies, not just private citizens.[5] In practice, governments often seek to affect exchange patterns by adopting policies that favor a certain money. While dollarization may be an endogenous response to high inflation, policies favoring the local money may generate a bias towards national currency use.[6] These policies may be successful— the dollar is driven out of circulation and the country de-dollarizes— while other times they are not— a dual-currency regime prevails and the dollar's international role is strengthened.

In the baseline two-country, two-currency model, agents trade locally and internationally in alternating markets, as in Lagos and Wright (2005). Trade entails exchanging local goods for a portfolio of currencies, with no restrictions on which monies can be used between private citizens. Since a foreign country's conditions and future exchange rate policies cannot be ascertained as well as one's own, this subjects uncertainty on the future value of foreign currency. Differences in asset recognizability— whether trading partners share the same information about its future value— then makes sellers reluctant to accept payment, unless costly information is acquired.[7] This decision to acquire information generates rate-of-return differentials where zero, one, or two international currencies can circulate. Since the use of a particular money itself reinforces that currency's usefulness, complementarities in the trading environment lead to multiple circulation patterns and coordination problems. Moreover, by introducing government, I examine how their transaction policies affect choices made by private agents and hence the set of equilibria.

Overall, the theory implies that the emergence of international currencies is predicated on information costs associated with adopting multiple means of payment. The model illustrates how the network externalities they induce play a crucial role in generating persistence and hysteresis

[4]There are also costs to trade with foreign assets, such as import tariffs or taxes on foreign dividends, as in Geromichalos and Simonovska (2010).

[5]Government transaction policies in search models with indivisible assets was introduced by Aiyagari and Wallace (1997) and furthered by Li and Wright (1998).

[6]Calvo and Rodriguez (1977) define dollarization as the use of foreign assets as a means of payment and unit of account. Much of Latin America and the former Soviet Union is unofficially dollarized, where the dollar is widely used in private transactions, but is not classified as legal tender.

[7]Dating back to Jevons (1875), recognizability, the ability of an object to be easily distinguished from something else, has long been argued to be a desirable property of an asset. A large literature exploring the link between recognizability, information, and liquidity include Brunner and Meltzer (1971), Alchian (1977), Williamson and Wright (1994), Banerjee and Maskin (1996), Berensten and Rocheteau (2004), Lester, Postlewaite, and Wright (2011), and Li, Rocheteau, and Weill (2011).

in monetary leadership. The theory of international currency in this paper therefore emphasizes an important kind of influence on the choice of money as an international medium of exchange.[8] Due to inertia, it is difficult to dislodge an incumbent currency from international use, whose use is associated with low information costs. At the same time, a temporary disruption— such as a change in inflation, information cost, or government size— can permanently shift payment patterns. International currency use therefore reflects both history and hysteresis, consistent with what we observe in practice.

In particular, currency substitution, or dollarization, arises endogenously in response to inflation: as domestic inflation increases, transacting in local currency becomes too costly and locals substitute with foreign money. What's more, a sufficiently large local government can de-dollarize an economy by inducing a switch to national currency use. A relatively weak local government— in the sense that its transactions only comprise a small fraction of the economy's total transactions— can only encourage and promote its currency, but cannot guarantee that foreign money stops circulating. More generally, the analysis reveals that government size matters and can influence the extent to which different monetary regimes prevail. This suggests that as the size and structure of the global economy changes, international currency use may change as well, which will affect welfare across economies.

To illustrate these theoretical findings, the model is generalized to N countries and N currencies and calibrated to match international trade data. The regions of interest in the baseline analysis consist of three trading blocs: the United States, the Eurozone, and China. With the advent of the euro and the rise of China, there is considerable interest in determining whether the dollar is at risk of losing its international role. The analysis reveals that the data favors a regime with national currencies and also one where the dollar is internationally dominant. However, changes in information costs or inflation can generate a shift to a bipolar world where the dollar shares international supremacy with the euro. Furthermore, which currency emerges as international matters for welfare. All else equal, the world benefits from having one international liquidity provider, rather than many, in order to save on information costs. However, due to coordination failures, persistence may induce the world economy to deviate from its optimal outcome. The welfare gains of an international currency under different inflationary scenarios are also discussed and compared with previous estimates.

[8] Similar in spirit to Matsuyama, Kiyotaki, and Matsui (1993), this paper provides microfoundations for insights first suggested by Menger (1892), Kindleberger (1967), and Krugman (1980). There is a large literature on the determinants of international currency use. Most studies focus on the role of currencies in foreign exchange reserves or the invoicing of international trade. This paper is closer in spirit with the latter invoicing role where users care about transaction costs and the economies of scale obtained from tapping into a large network. See also Chinn and Frankel (1997), Rey (2002), Engineer (2005), Mileva and Sigfried (2007), and Devereaux and Shi (2011).

In contrast with earlier dual-currency search models, this paper features divisible assets by extending Lester, Postlewaite, and Wright (2011) to an open-economy setting.[9] Although the authors discuss dollarization and exchange rates, the closed-economy setting and absence of government prevents considerations of international currencies and government policies. Another closely related work is Geromichalos and Simonovska (2010), which introduces liquidation costs to explain asset home bias. However, fiat money is not modeled explicitly and the analysis abstracts from policy considerations. The paper presented here is the first to examine both international currencies and government transaction policies in a third-generation monetary search model.

This paper proceeds as follows. Section 2 describes the environment, and section 3 characterizes equilibrium. Currency regimes and the model's main insights are in Section 4, including discussion of how monetary policy and government transaction policies affect prices, allocations, and welfare. The basic framework is then generalized to an N-country, N-currency model. Section 5 calibrates the model to determine the pattern of world payments and illustrate theoretical results. Section 6 examines the welfare benefits of an international currency and compares to previous estimates. Finally Section 7 concludes and discusses avenues for future research.

2 The Environment

Time is discrete with an infinite horizon. The economy consists of two countries, Home and Foreign, with a continuum of 2 and $2n$ agents, respectively, where $n \in (0, 1)$ denotes relative country size.[10] Each period consists of two sub-periods: in the first, agents meet pairwise in decentralized markets (DM) of each country and trade according to a bilateral bargaining protocol; in the second sub-period, all activity occurs in a competitive market (CM) with market-clearing prices.

There are two perishable consumption goods: a general good identical in both countries and a set of specialized goods. In the CM of both countries, all agents consume the general good x by supplying labor v one-for-one. In the DM of country $s = \{h, f\}$, buyers consume output q_s which only sellers from s can produce. Sellers are immobile and cannot produce the other country's good. Instantaneous utility functions for buyers and sellers are

$$U^b(q_s, x, v) = u(q_s) + x - v,$$

$$U^s(q_s, x, v) = -c(q_s) + x - v.$$

[9]Earlier two-country, two-currency search models include Matsuyama, Kiyotaki, and Matsui (1993), Trejos and Wright (1996), Zhou (1997), Head and Shi (2003), Camera and Winkler (2003), Curtis and Waller (2003), Camera, Craig, and Waller (2004), Liu and Shi (2006), and Li and Matsui (2009).

[10]In Appendix A, I extend the framework to N countries and N currencies.

Functional forms for utilities and cost functions, $u(q)$ and $c(q)$, are the same in both countries; they are assumed to be C^3 with $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $U'(0) = u'(0) = \infty$. Also, let $x^* \in (0, \infty)$ solve $U'(x^*) = 1$ and $q^* \equiv \{q : u'(q^*) = c'(q^*)\}$. All agents discount the future between periods, but not sub-periods with a discount factor $\beta \in (0, 1)$.

There are two fiat currencies, $c = \{h, f\}$, both perfectly divisible and storable. Currency $m_c \in \mathbb{R}_+$ is valued at ϕ_c, the price of money in terms of the general good.[11] Money supplies, M_c, can grow or shrink each period at a constant rate $(\gamma_c - 1)$, where $\gamma_c \equiv \frac{M_c'}{M_c}$. Lump-sum monetary transfers or taxes occur in the CM of each country. A lack of double-coincidence-of-wants rules out barter, and anonymity precludes the use of credit. These frictions make a medium of exchange essential for quid-pro-quo trade, as in Kocherlakota (1998) and Wallace (2001).

Besides their nationalities and exchange roles in the DM, agents are further split between *private citizens* and *government agents*. Each country has a national government, which can dictate a fraction of its citizens to only accept a certain currency. In particular, a subset of sellers in each country produce and consume like private sellers but have exogenous policies regarding what it accepts as payment. All buyers are private citizens, while sellers can either be private sellers or government sellers.[12] Table 1 summarizes the sizes and composition of the world economy. Agents are equally split between buyers and sellers, each of size 1 and n at Home and Foreign, respectively. The fraction of government sellers in each country, $g_h \in [0, 1]$ and $g_f \in [0, 1]$, can be interpreted as government size or the degree of centralized control.

Table 1: Measures of Agents in Each Country

	Home	Foreign
Buyer	1	n
Private Seller	$1 - g_h$	$n(1 - g_f)$
Government	g_h	ng_f
Total	2	$2n$

Agents meet pairwise and at random in the DM. Buyers are mobile while sellers are immobile. With probability $\alpha \in [0, 1]$, a buyer stays in his country of origin and with probability $1 - \alpha$, visits the foreign country. Agents are randomly matched according to a matching function, $M_j =$

[11] Since general goods are homogeneous across countries and market clearing in the CM implies that the law of one price holds, the exchange rate must satisfy $e = \frac{p_i}{p_j} = \frac{\phi_j}{\phi_i}$, where p_i is the price of the CM good in currency i units. Since agents can trade currencies at the market clearing rate, the CM has a natural interpretation as a foreign exchange market.

[12] Government behavior is modeled this way since the purpose is to make precise how the size and influence of government affects realms of circulation and the set of equilibria. Although government can in principle be modeled as both buyers and sellers, only the latter chooses the payments to accept, which is the main focus of this study.

$M(B_j, S_j) = \frac{B_j S_j}{B_j + S_j}$, where B_j and S_j denotes the number of buyers and sellers in country j. At Home, $B_h = \alpha + n(1-\alpha)$, $S_h = 1$, and a buyer meets a seller with probability $e_h = \frac{1}{1+\alpha+n(1-\alpha)}$ while at Foreign, $B_f = \alpha n + 1 - \alpha$, $S_f = n$, and a buyer meets a seller with probability $e_f = \frac{1}{1+\alpha+\frac{1-\alpha}{n}}$. Table 2 presents buyers' meeting probabilities across all meeting types.

Table 2: Buyer's Meeting Probabilities

	Home Private Seller	Foreign Private Seller	Home Govt	Foreign Govt
Home Buyer	$\alpha e_h(1-g_h)$	$(1-\alpha)e_f(1-g_f)$	$\alpha e_h g_h$	$(1-\alpha)e_f g_f$
Foreign Buyer	$(1-\alpha)e_h(1-g_h)$	$\alpha e_f(1-g_f)$	$(1-\alpha)e_h g_h$	$\alpha e_f g_f$

The parameters n, g_h, g_f, and α control national and international trade frictions. Since $n \in (0,1)$, international meeting probabilities differ across countries. α parameterizes the degree of economic integration between countries: as $\alpha \to \frac{1}{2}$, countries become more integrated and meeting a foreigner is more likely, while $\alpha \to 1$ corresponds to a closed economy where only locals trade.

Along with search frictions, the DMs also feature an information problem about the different currencies, following Lester, Postlewaite, and Wright (2011). Before being matched, a private seller from $s = \{h, f\}$ can be informed or uninformed about the two currencies: they can learn about both, only the domestic currency, or only the foreign currency. Let $k = \{hf, h, f\}$ index these events. However, acquiring information k comes at disutility cost $\psi_s^k \geq 0$. It is common knowledge in a match whether the seller is informed, and sellers do not accept currencies they cannot recognize.[13] Hence payment k is accepted if and only if ψ_s^k is incurred.[14]

Unlike private sellers' endogenous information choice, government sellers adopt an exogenous trading rule, following Aiyagari and Wallace (1997) and Li and Wright (1998). Government transaction policies are specified by $\Gamma \equiv \{(\tau_h^h, \tau_h^f), (\tau_f^h, \tau_f^f)\}$, where $\tau_s^c = 1$ if government sellers from s accepts currency c, and $\tau_s^c = 0$ otherwise. Since the most typical case in practice is when governments only accept the domestic currency, $\tau = \{(1,0),(0,1)\}$ is considered the baseline policy. Policies $\tau^H = \{(1,0),(1,1)\}$, $\tau^F = \{(1,1),(0,1)\}$, and $\tau^U = \{(1,1),(1,1)\}$ are also considered, where foreign currency is accepted in addition to the national money. Given Γ, terms of trade are

[13]For example, sellers will reject payment if it is costless to produce worthless counterfeits: if sellers accepted unrecognizable currencies, buyers would just hand over counterfeits in each exchange. This assumption simplifies the pricing mechanism, as discussed by Rocheteau (2008).

[14]There are several interpretations of the cost ψ_s^k, which broadly reflects the cost of accepting currency or dealing with multiple assets. For example, the transactions necessary for dealing with multiple currencies are costly to administer and require obtaining information about different monies. There are also costs to install new technologies, such as credit cards, debit cards, and "smart cards." One can also imagine that there is a cost of verifying asset value if some agents can costlessly produce fraudulent assets, as in Lester, Postlewaite, and Wright (2011). This paper's approach of taking a broad interpretation of this cost follows the tradition of Freeman and Kydland (2000), where the fixed cost of accepting credit is similarly interpreted broadly.

determined through the same bargaining protocol as with interactions with private sellers.[15]

3 Equilibrium

This section describes the equilibrium of the two-country, two-currency model. I focus on a stationary equilibrium where output q_s in both countries is constant over time.

3.1 Centralized Markets (CM) Value Functions

In the centralized markets, a representative buyer of each country chooses consumption of the general good x, labor v, and real balances to bring forward next period. Since the price of currency can change over time due to monetary injections each period, portfolios are expressed in real terms: let $z \equiv (z_h, z_f) = (\phi_h m_h,\ \phi_f m_f) \in \mathbb{R}^2_+$ represent a Home buyer's real balances, while $\hat{z} \equiv (\hat{z}_h, \hat{z}_f) = (\phi_h \hat{m}_h,\ \phi_f \hat{m}_f) \in \mathbb{R}^2_+$ corresponds to a Foreign buyer. In what follows, hatted variables will refer to Foreign buyers. Variables with a prime denote next period's choices. Finally, let $\mathbf{W(z)} \equiv \{W(z), \hat{W}(\hat{z})\}$ and $\mathbf{V(z)} \equiv \{V(z), \hat{V}(\hat{z})\}$ denote buyers' value functions in the CM and DM, respectively, which represents the lifetime expected value from holding $\mathbf{z} \equiv \{z, \hat{z}\}$.

In the beginning of the CM, a representative Home buyer faces the following maximization problem:

$$W(z) = \max_{x,v,z'} \{x - v + \beta V(z')\} \tag{1}$$

$$x + (\phi_h m'_h + \phi_f m'_f) = v + (z_h + z_f) + T_h \tag{2}$$

$$z'_h = \phi'_h m'_h \geq 0, z'_f = \phi'_f m'_f \geq 0 \tag{3}$$

The portfolio taken into the next DM is $z' = (z'_h, z'_f)$, while $T_h \equiv (\gamma_h - 1)\phi_h M_h$ is the lump-sum transfer of Home currency by the government. Substituting $m'_c = \frac{z'_c}{\phi'_c}$ into the budget constraint, recalling that $\frac{\phi_c}{\phi'_c} = \pi_c$, and then substituting $x - v$ from the constraint into the objective yields

$$W(z) = z_h + z_f + T_h + \max_{z_h, z_f} \{-\pi_h z'_h - \pi_f z'_f + \beta V(z')\}. \tag{4}$$

An analogous expression can be derived for a Foreign buyer, with hats on all relevant variables

[15] Another approach would be for governments to specify the price and quantity in a match, as in Li and Wright (1998) and Li (2002). Alternatively, government can be modeled as an institution that imperfectly monitors transactions between private agents. For example, Lotz and Rocheteau (2002) consider how legal restrictions affect the adoption of a new currency.

and value functions:

$$\hat{W}(\hat{z}) = \hat{z}_h + \hat{z}_f + T_f + \max_{\hat{z}_h, \hat{z}_f}\{-\pi_h \hat{z}_h' - \pi_f \hat{z}_f' + \beta \widehat{V}(\hat{z}')\}. \tag{5}$$

A buyer's lifetime utility at the beginning of the CM is thus the sum of his real balances, the country-specific transfer, and the continuation value at the beginning of the next DM minus the investment in real balances. This follows from the fact that $\pi_c z_c'$ must be acquired in the current period to hold z_c' next period.

A few results from the CM value function are worth highlighting. First, $W(z)$ is linear in a buyer's total wealth $\overline{z} = z_h + z_f$: $W'(\overline{z}) = 1$. Second, there are no wealth effects since z' is independent of z. This follows from the linearity of the utility function, which helps reduce the dimensionality of the state space as will soon be evident. Taking first order conditions, optimal money holdings must satisfy for each currency $c = \{h, f\}$:

$$-\pi_c + \beta \frac{\partial V(z')}{\partial z_c} \leq 0; -\pi_c + \beta \frac{\partial \hat{V}(\hat{z}')}{\partial \hat{z}_c} \leq 0.$$

Provided that DM value functions $\mathbf{V(z)} = \{V(z), \hat{V}(\hat{z})\}$ are strictly concave, there will generally be a unique portfolio that satisfies market clearing where all buyers in a country demand the same real balances. A caveat is when the two currencies are perfect substitutes; in this case, buyers can hold different portfolios but they will have the same total value. I next specify agents' decision problems in the DM, which will imply the strict concavity of $\mathbf{V(z')}$.

3.2 Information Acquisition Problem

Before matches are formed, consider the seller's information choice problem. In the DM, private sellers from $s = \{h, f\}$ can be informed or uninformed about currency $c = \{h, f\}$ by choosing a strategy $(\chi_s^h, \chi_s^f) \in \{0, 1\}^2$, where $\chi_s^c = 1$ if the seller acquires information about currency c. I restrict attention to equilibria where all sellers from a country pursue the same strategies, though the strategies of a representative seller from different countries can differ. Let λ_s denote the probability of being a private seller from s: $\lambda_h = (1 - g_h)/2(1 + n)$, $\lambda_f = n(1 - g_f)/2(1 + n)$. For any type-$k$ meeting, where $k = \{hf, h, f\}$ indexes meetings where both currencies, only the Home currency, or only the Foreign currency is accepted, a private seller's expected surplus is given by

$$\Pi(q_s^k) \equiv -\psi_s^k + \lambda_s(1 - \theta)[u(q_s^k) - c(q_s^k)]. \tag{6}$$

Sellers become informed by weighing the cost ψ_s^k with the gains from trade: by accepting k, the seller can extract a fraction $(1-\theta)$ of the match surplus when output q_s^k is traded. Thus, sellers accept k as payment if the expected surplus from doing so outweighs the expected surplus of accepting a different $k^- = \{h, f, hf\}\backslash k$:

$$(\chi_s^h, \chi_s^f) = \begin{cases} (1,0): & \Pi(q_s^h) > max\{\Pi(q_s^{hf}), \Pi(q_s^f)\} \\ (0,1): & \Pi(q_s^f) > max\{\Pi(q_s^{hf}), \Pi(q_s^h)\} \\ (1,1): & \Pi(q_s^{hf}) > max\{\Pi(q_s^h), \Pi(q_s^f)\}. \end{cases}$$

Since sellers will reject currencies they do not recognize and it is common knowledge whether the seller is informed, trade occurs under full information.

3.3 Terms of Trade

Terms of trade are determined according to a proportional bargaining rule, following Kalai (1977).[16] Under proportional bargaining, a buyer acquires q in exchange for payment p and receives a constant share $\frac{\theta}{1-\theta}$ of the seller's surplus, where $\theta \in [0,1]$ measures the buyer's bargaining power. Bargaining occurs in DM_s, where s denotes the seller's country of origin.

Given the model specification, terms of trade will depend on buyers' portfolios, private sellers' information choice, and governments' transaction policy. Since payment depends on which currencies are accepted, let l_s^k denote the maximum liquid wealth that a Home buyer can transfer to a seller from s in a type-k meeting, where $l_s^k = \chi_s^h z_h + \chi_s^f z_f$ in private encounters. Similarly, maximum transferable wealth to a government seller that accepts k under Γ is $l_s^k = \tau_s^h z_h + \tau_s^f z_f$. Liquid wealth for Foreign buyers is $\hat{l}_s^k = \chi_s^h \hat{z}_h + \chi_s^f \hat{z}_f$ and $\hat{l}_s^k = \tau_s^h \hat{z}_h + \tau_s^f \hat{z}_f$ for private and government meetings, respectively. In what follows, Foreign buyers are not discussed if the implications are understood.

Under proportional bargaining, output $q_s^k \equiv q^k(z, \chi_s, \Gamma_s)$ and payment $p_s^k \equiv p^k(z, \chi_s, \Gamma_s)$ solves the following problem:

$$max_{q,p}[u(q_s^k) - p_s^k] \tag{7}$$

$$u(q_s^k) - p_s^k = \frac{\theta}{1-\theta}[p_s^k - c(q_s^k)] \tag{8}$$

$$p_s^k \leq l_s^k. \tag{9}$$

The bargaining problem maximizes trade surplus, subject to each party receiving a constant

[16] Other pricing mechanisms can be used, such as the generalized Nash bargaining solution. Note however that if the buyer makes a take-it-or-leave-it offer, sellers will have no incentive to incur the fixed cost to accept currencies since they do not receive any surplus from trade.

share, and a feasibility constraint that places no restrictions on the currencies that can be used by private citizens. Quantity traded q_s^k must then satisfy

$$q_s^k \in argmax\theta[u(q_s^k) - c(q_s^k)]$$

$$(1-\theta)u(q_s^k) + \theta c(q_s^k) \leq l_s^k.$$

The transfer of wealth from buyers to sellers is thus $\omega(q_s^k) \equiv \theta c(q_s^k) + (1-\theta)u(q_s^k)$. In type-$k$ meetings between a Home buyer and a seller from s, $q_s^k \equiv q^k(z, \chi_s, \Gamma_s)$ solves

$$\omega(q_s^k) = \min\{\omega(q_s^*), l_s^k\} \tag{10}$$

where

$$\omega(q_s^k) \equiv \theta c(q_s^k) + (1-\theta)u(q_s^k). \tag{11}$$

Terms of trade in meetings with a Foreign buyer solve an analogous expression, with hats on relevant variables.

The bargaining solution simply says that when liquid wealth is scarce, $l_s^k < \omega(q_s^*)$, the buyer pays his maximum liquid wealth l_s^k to the seller and gets $q_s^k < q_s^*$ where $q_s^k = \omega^{-1}(l_s^k)$. Notice that quantity traded depends only on buyers' portfolio, private sellers' information strategy χ, and governments' transaction policy Γ. The first-best $q_s^* \equiv \{q_s : u'(q_s^k) = c'(q_s^k)\}$ is exchanged if the buyer holds enough liquid wealth: $l_s^k \geq \theta c(q_s^*) + (1-\theta)u(q_s^*)$. In this case, payment to the seller will be exactly $\omega(q_s^*) = \theta c(q_s^*) + (1-\theta)u(q_s^*)$. Let $\mathbf{l}_s^k \equiv \{l_s^k, \hat{l}_s^k\}$ and $\mathbf{q}_s^k \equiv \{q_s^k, \hat{q}_s^k\}$ denote liquid wealth and output, respectively, for all buyers. DM quantities are then $\mathbf{q}_s^k = \min\{\mathbf{q}_s^*, \omega^{-1}(\mathbf{l}_s^k)\}$, which varies across matches according to agents' nationalities and the currencies accepted.

3.4 Decentralized Markets (DM) Value Function

Given the bargaining solution, DM value functions simplify greatly. Since the seller's surplus does not depend on z and $\hat{z}$, the DM value function can be written solely in terms of the buyer's problem. In what follows, let a Home buyer's meeting probabilities at Home and Foreign be denoted by μ_h and μ_f for private encounters and ν_h and ν_f for government encounters, where $\mu_h = \alpha e_h(1-g_h)$, $\mu_f = (1-\alpha)e_f(1-g_f)$, $\nu_h = \alpha e_h g_h$, and $\nu_f = (1-\alpha)e_f g_f$. Similarly for a Foreign buyer, let $\hat{\mu}_h = (1-\alpha)e_h(1-g_h)$, $\hat{\mu}_f = \alpha e_f(1-g_f)$, $\hat{\nu}_h = (1-\alpha)e_h g_h$, and $\hat{\nu}_f = \alpha e_f g_f$. Also, let ρ_h^k and ρ_f^k denote the probability of a type-k meeting with a Home and Foreign seller, respectively.

Consider a representative Home buyer. Using the linearity of $W(z)$ and the fact that proportional bargaining implies $u(q) - \omega(q) = \theta[u(q) - c(q)]$ yields the following DM value function:

$$V(z) = \mu_h \sum_k \rho_h^k \theta[u(q_h^k(z,\chi)) - c(q_h^k(z,\chi))] + \mu_f \sum_k \rho_f^k \theta[u(q_f^k(z,\chi)) - c(q_f^k(z,\chi))] \tag{12}$$

$$+\nu_h \theta[u(q_h^k(z,\Gamma)) - c(q_h^k(z,\Gamma))] + \nu_f \theta[u(q_f^k(z,\Gamma)) - c(q_f^k(z,\Gamma))] + z_h + z_f + W(0,0).$$

Similarly for a Foreign buyer:

$$\hat{V}(\hat{z}) = \hat{\mu}_f \sum_k \rho_f^k \theta[u(\hat{q}_f^k(\hat{z},\chi)) - c(\hat{q}_f^k(\hat{z},\chi))] + \hat{\mu}_h \sum_k \rho_h^k \theta[u(\hat{q}_h^k(\hat{z},\chi)) - c(\hat{q}_h^k(\hat{z},\chi))] \tag{13}$$

$$+\hat{\nu}_f \theta[u(\hat{q}_f^k(\hat{z},\Gamma)) - c(\hat{q}_f^k(\hat{z},\Gamma))] + \hat{\nu}_h \theta[u(\hat{q}_h^k(\hat{z},\Gamma)) - c(\hat{q}_h^k(\hat{z},\Gamma))] + \hat{z}_h + \hat{z}_f + \hat{W}(0,0).$$

In both expressions, the first term on the right-hand-side is the buyer's expected trade surplus in domestic encounters with a private seller. The second term is the expected payoff in private international meetings, and the payoff when encountering the national government and the foreign government is given by the third and fourth terms, respectively. Note that each of these terms depend on the currencies accepted. The last three terms result from the linearity of $W(z)$ and $\hat{W}(\hat{z})$ and is the value of proceeding to the CM with one's portfolio intact. The value function with no government results when $g_h = g_f = 0$.

Next, lead the DM value function forward by one period and substitute into the CM value function to yield the buyer's objective function. Letting the interest rate on an illiquid nominal bond denominated in currency c be $1 + i_c = \frac{\phi_c}{\phi_c' \beta}$, optimal real balances for Home buyers thus solve

$$\max_{z_h, z_f} \{-i_h z_h - i_f z_f + \mu_h \sum_k \rho_h^k \theta[u(q_h^k(z,\chi)) - c(q_h^k(z,\chi))] + \mu_f \sum_k \rho_f^k \theta[u(q_f^k(z,\chi)) - c(q_f^k(z,\chi))] \tag{14}$$

$$+\nu_h \theta[u(q_h^k(z,\Gamma)) - c(q_h^k(z,\Gamma))] + \nu_f \theta[u(q_f^k(z,\Gamma)) - c(q_f^k(z,\Gamma))]\}.$$

Here $i_c = \frac{\phi_c}{\phi_c' \beta} - 1$ represents the cost of holding currency c. The objective function says that a buyer chooses a portfolio to maximize expected surplus, net of the cost of holding money. A similar expression can be derived for Foreign buyers. In the appendix, I show that the objective function is strictly jointly concave. The first-order conditions with respect to z_c, $c = \{h, f\}$, are:

$$-i_c + \mu_h \sum_k \rho_h^k \theta[u'(q_h^k) - c'(q_h^k)] \frac{dq_h^k}{dz_c} + \mu_f \sum_k \rho_f^k \theta[u'(q_f^k) - c'(q_f^k)] \frac{dq_f^k}{dz_c} \tag{15}$$

$$+\nu_h\theta[u'(q_h^k) - c'(q_h^k)]\frac{dq_h^k}{dz_c} + \nu_f\theta[u'(q_f^k) - c'(q_f^k)]\frac{dq_f^k}{dz_c} \leq 0$$

where

$$\frac{dq^k}{dz_c} = \begin{cases} \frac{1}{\omega'(q^k)} = \frac{1}{\theta c'(q^k)+(1-\theta)u'(q^k)} & : l^k < \omega(q^k) \\ 0 & : l^k \geq \omega(q^k). \end{cases}$$

We are now in a position to define the equilibrium of the economy.

3.5 Equilibrium

Definition. *Given government transaction policies Γ, a stationary monetary equilibrium is a list of quantities traded $\boldsymbol{q}_s^k \equiv \{q_s^k, \hat{q}_s^k\}$, real balances $(\boldsymbol{z}_h, \boldsymbol{z}_f) \equiv \{(z_h, z_f), (\hat{z}_h, \hat{z}_f)\}$, and information strategies $\Omega \equiv \{(\chi_h^h, \chi_h^f), (\chi_f^h, \chi_f^f)\}$ $\forall$ $s = \{h, f\}, k = \{hf, h, f\}$ that satisfy (i) terms of trade under the proportional bargaining rule, (ii) buyers' portfolio choice problem, (iii) sellers' information choice problem, and (iv) market clearing in money markets.*

Consider first a Home buyer. In a monetary equilibrium where $(z_h, z_f) > 0$, DM output across all meeting types must satisfy

$$\begin{aligned} i_h = {} & \mu_h[\rho_h^{hf} L(q_h^{hf}) + \rho_h^h L(q_h^h)] + \mu_f[\rho_f^{hf} L(q_f^{hf}) + \rho_f^h L(q_f^h)] \\ & +\nu_h[\rho_h^{hf} L(q_h^{hf}) + \rho_h^h L(q_h^h)] + \nu_f[\rho_f^{hf} L(q_f^{hf})] \end{aligned} \tag{16}$$

$$\begin{aligned} i_f = {} & \mu_h[\rho_h^{hf} L(q_h^{hf}) + \rho_h^f L(q_h^f)] + \mu_f[\rho_f^{hf} L(q_f^{hf}) + \rho_f^f L(q_f^f)] \\ & +\nu_h[\rho_h^{hf} L(q_h^{hf})] + \nu_f[\rho_f^{hf} L(q_f^{hf}) + \rho_f^f L(q_f^f)] \end{aligned} \tag{17}$$

where

$$L(q^k(z)) \equiv \frac{\theta[u'(q^k) - c'(q^k)]}{\theta c'(q^k) + (1-\theta)u'(q^k)}.$$

Buyers wish to bring currencies into the DM since these objects facilitate trade across different meeting types, but doing so is costly as captured by the terms i_h and i_f on the right sides of (16) and (17). The liquidity premium, $L(q^k(z))$, is the marginal payoff an agent gets from his liquid wealth that can be used to acquire more output in the DM instead of carrying it over to the subsequent CM.

Similarly, a Foreign buyer with $(\hat{z}_h, \hat{z}_f) > 0$ trades off the cost and benefit of holding each currency:

$$i_h = \hat{\mu}_h[\rho_h^{hf} L(\hat{q}_h^{hf}) + \rho_h^h L(\hat{q}_h^h)] + \hat{\mu}_f[\rho_f^{hf} L(\hat{q}_f^{hf}) + \rho_f^h L(\hat{q}_f^h)] \tag{18}$$
$$+\hat{\nu}_h[\rho_h^{hf} L(\hat{q}_h^{hf}) + \rho_h^h L(\hat{q}_h^h)] + \nu_f[\rho_f^{hf} L(\hat{q}_f^{hf})]$$

$$i_f = \hat{\mu}_h[\rho_h^{hf} L(\hat{q}_h^{hf}) + \rho_h^f L(\hat{q}_h^f)] + \hat{\mu}_f[\rho_f^{hf} L(\hat{q}_f^{hf}) + \rho_f^f L(\hat{q}_f^f)] \tag{19}$$
$$+\hat{\nu}_h[\rho_h^{hf} L(\hat{q}_h^{hf})] + \hat{\nu}_f[\rho_f^{hf} L(\hat{q}_f^{hf}) + \rho_f^f L(\hat{q}_f^f)].$$

Intuitively, the equilibrium conditions equate the marginal benefit of liquidity to its cost, given by the nominal interest rates of each currency, i_h and i_f. As a result, a currency demands a liquidity premium only if it is accepted in trade, as determined by χ and Γ. In this case, output falls short of the first-best: when $\chi^c = 1$ or $\tau^c = 1$, $i_c > 0$, $L(q^k(z)) > 0$, and $q^k < q^*$. When no sellers accept a currency, it will not be valued. Notice that $L(q^k(z))$ is strictly decreasing over the relevant range: $L'(q^k(z)) < 0$ for $q^k \in [0, q^*]$.[17] As in standard monetary search models, the Friedman Rule, which requires $i_c \to 0$, also arises here: as $\gamma_c \to \beta$ ($i_c \to 0$), $L(q^*(z)) = 0$ and $q^k = q^* \ \forall\ \theta \in [0, 1]$. By reducing the cost of holding money to zero, the Friedman Rule maximizes the demand for real balances and hence output traded. In what follows, I focus on equilibria where $\gamma_c \geq \beta$.[18]

The following lemma summarizes some basic properties of optimal portfolio holdings.

Lemma 1. *Consider any equilibrium where $\gamma_c > \beta$ and $i_h \neq i_f$ (currencies are not perfect substitutes).*

1. *All buyers of the same nationality will hold the same portfolios.*
2. *Buyers of different nationalities will generally hold different portfolios.*
3. *When there are no asymmetries in meeting arrangements— i.e., when countries and governments are of equal sizes ($n = 1$, $g_h = g_f$) and perfectly integrated ($\alpha = \frac{1}{2}$)— then all buyers, irrespective of country origin, will hold the same portfolios.*

Proof. Part *(1)* of Lemma 1 can be verified by examining the buyer's maximization problem in (14). In Appendix B, I show that the objective function is strictly jointly concave. Due to the

[17] Differentiating $L(q^k(z)) = \frac{\theta[u'(q^k) - c'(q^k)]}{\theta c'(q^k) + (1-\theta)u'(q^k)}$ yields $L'(q^k(z)) = \frac{\theta[u''(q^k)c'(q^k) - u'(q^k)c''(q^k)]}{[\theta c'(q^k) + (1-\theta)u'(q^k)]^3} < 0$, given the assumptions on the concavity of $u(q)$ and convexity of $c(q)$. Since the same conditions that make $L'(q^k(z)) < 0$ make $V(z)$ strictly concave, this ensures a unique stationary monetary equilibrium that solves the first order conditions.

[18] This just means the net gain of carrying money across periods is either negative or zero. If $\frac{\phi_c}{\phi_c'} > \beta$ for $c = \{h, f\}$, then the optimal portfolio is uniquely characterized by the buyer's portfolio choice problem. If $\frac{\phi_c}{\phi_c'} = \beta$, then any $z \geq z^*$ is optimal. No equilibrium exists if $\frac{\phi_c}{\phi_c'} < \beta$. As in Lagos and Wright (2005), so long as the cost of carrying money across periods is positive, the distribution of money holdings is degenerate each period.

strict concavity of the objective, each buyer from a particular country has a degenerate demand for both currencies. This follows from the linearity of the CM value function. Parts *(2)* and *(3)* follow directly from inspection of equations (16) − (19) and the bargaining solution.

The intuition of Lemma 1 is that buyers from different countries hold different portfolios due to the asymmetry in the matching process for the two countries: since the probability of meeting a foreigner can differ depending on one's nationality, buyers will allocate portfolio weights accordingly. Without any asymmetry in meeting arrangements, then buyers' nationalities cease to matter and will all hold symmetric portfolios. Given the model specification, this requires that $n = 1$, $g_h = g_f$, and $\alpha = \frac{1}{2}$, which implies that it is equally likely to meet compatriots as foreigners.

Finally, market clearing in the CM implies that for each currency, aggregate supply must equal aggregate demand. By Lemma 1, each buyer from a particular country has a degenerate demand for both currencies. As a result, all buyers from the same country hold the same portfolio when currencies are not perfect substitutes. Total demand for each currency is $2z_h + 2n\hat{z}_h$ and $2z_f + 2n\hat{z}_f$. To close the model, market clearing implies $2z_h + 2n\hat{z}_h = \phi_h M_h \equiv Z_h$ and $2z_f + 2n\hat{z}_f = \phi_f M_f \equiv Z_f$. Given Z_h and Z_f such that money markets clear, sellers' strategies χ, and government policies Γ, output traded in each country $q_s^k(z) = \omega^{-1}(l_s^k)$ can be obtained from the bargaining solution.

4 Currency Regimes

Having defined monetary equilibrium and described some of its main features, I now examine the types of currency regimes that arise in the dual-currency economy. Given government transaction policies under Γ, a currency regime is defined as a pair of strategies for each private seller, $\Omega \equiv \{(\chi_h^h, \chi_h^f), (\chi_f^h, \chi_f^f)\}$, that satisfies sellers' information choice problem. A laissez-faire regime corresponds to $g_h = g_f = 0$ equilibria with no government intervention.

In each regime, the share of transactions requiring different currencies is endogenous and pinned down by the model's information structure. Since equilibria are characterized in terms of sellers' strategies, a particular regime will sustain an equilibrium if the expected surplus of choosing $k = \{hf, h, f\}$ outweighs the surplus of any other choice $k^- = \{hf, h, f\}\backslash k$. Instead of studying all possible permutations, the focus is on the most representative monetary regimes: local circulation of currencies and international circulation of one or both currencies. Table 3 summarizes the currency regimes discussed in the text. In the following, the implications and existence of these types of equilibria are discussed.

Table 3: **Equilibrium Currency Regimes for Two-Country, Two-Currency Model**

Ω		Circulation Pattern
A	{(1,0)(0,1)}	Two National Currencies
H	{(1,0)(1,1)}	One International (Home) and National (Foreign) Currency
U	{(1,1)(1,1)}	Two International Currencies
M	{(1,0)(1,0)}	One International (Home) Currency

Regime A: Two National Currencies $A = \{(1,0),(0,1)\}$

Consider first the government's baseline policy $\tau = \{(1,0),(0,1)\}$, where each government only accepts its domestic currency. Under τ, this equilibrium gives rise to the emergence of two national currencies where only the local money is accepted in domestic transactions.[19] This coincides with a common assumption in many international macroeconomic models, though the outcome is endogenous in this model.

Under $\tau = \{(1,0),(0,1)\}$, equilibrium conditions imply that $q_h^h(z)$, $q_f^f(z)$ for a Home buyer solves

$$i_h = (\mu_h + \nu_h)L(q_h^h)$$

$$i_f = (\mu_f + \nu_f)L(q_f^f).$$

For a Foreign buyer, $\hat{q}_h^h(\hat{z})$, $\hat{q}_f^f(\hat{z})$ must satisfy similar equations. The equations above relate the demand by agents from different countries for the two currencies to the cost of holding it. DM quantities in each meeting type can be obtained independently from these equations. Note that output q_s^c and $\hat{q}_s^c$ also represents the DM value of currency c in country s and thus its purchasing power for Home and Foreign buyers, respectively. Relative demands for Home and Foreign currency, respectively, are related through the following expressions:

$$\alpha L(q_h^h) = (1-\alpha)nL(\hat{q}_h^h)$$

$$(1-\alpha)L(q_f^f) = \alpha nL(\hat{q}_f^f).$$

[19] National-currency equilibria are difficult to obtain in earlier dual-currency search models with indivisible money and divisible goods, as pointed out by Yiting Li (in correspondence). To obtain such an equilibrium, Trejos and Wright (2001) must assume that sellers cannot recognize the nationality of the buyer, but only the nationality of the currency the buyer carries. The seller thus makes the same decision whenever he meets any buyer with a given money. Without this assumption, there are no frictions for sellers to reject foreign money and thus national currencies cannot exist. A related literature on legal restrictions— for example, Li and Wright (1998) and Curtis and Waller (2000)— can help overcome this problem. Similarly in this paper, government transaction can help guarantee national currency use. What's more crucial is the model's information structure: without information costs, government policies alone cannot generate this outcome.

Notice that Home and Foreign buyers will have symmetric demands for their domestic and foreign currency if countries are of equal sizes. More generally however, relative demands will depend on country sizes and international trade frictions.

Real balances can then be obtained from the bargaining solution: for Home currency, $z_h = \omega(q_h^h) > 0$, $\hat{z}_h = \omega(\hat{q}_h^h) > 0$, and for Foreign currency, $z_f = \omega(q_f^f) > 0$, $\hat{z}_f = \omega(\hat{q}_f^f) > 0$. Buyers hold positive balances of both monies, where they exchange their domestic currency in local meetings and the other currency in international meetings. Since sellers only accept their domestic money, only national currencies change hands.[20] Finally, market-clearing prices can be obtained by summing over all demand and equating to a currency's supply:[21]

$$2z_h + 2n\hat{z}_h = \phi_h M_h$$

$$2z_f + 2n\hat{z}_f = \phi_f M_f.$$

Both currencies are valued since each have exchange roles in the issuing country: $i_h > 0$ and $i_f > 0$. In this case, $\pi_c = \gamma_c$. Neither currency is fundamentally priced since each is essential for some meetings, even if one is being issued at a higher rate and thus has a higher inflation rate. Hence low-return currencies can circulate in equilibrium despite the existence of a competing, higher-return currency. What's more, due to the presence of government agents that always accept its national money, there no longer exists an equilibrium where that money is not valued, contrary to the laissez-faire case.

The value of each currency depends on the economy's fundamentals, such as country size and search frictions. All else equal, a decrease in national search frictions increases output and thus the value of its currency: $\frac{\partial q_h^h}{\partial \alpha} > 0$ for Home currency and $\frac{\partial \hat{q}_f^f}{\partial \alpha} > 0$ for Foreign currency. Since less trade frictions imply more encounters with sellers that only accept the local money, each currency will be essential in a larger fraction of meetings and hence demand a higher liquidity premium. Further, $\frac{\partial q_h^h}{\partial i_h} < 0$ and $\frac{\partial q_f^f}{\partial i_f} < 0$, so that inflation reduces output. Since both currencies are valued

[20] Note however that it is a misnomer to label this regime an "autarky" equilibrium since there will be trade between agents of different nationalities. For example, a Home buyer holds both monies since they may meet a foreigner that only accepts Foreign currency. Since trade occurs between agents from different countries— though only the seller's domestic currency changes hands— this differs from the "autarky" regime discussed in Matsuyama, Kiyotaki, and Matsui (1993).

[21] In a symmetric environment where all buyers, irrespective of their nationality, hold the same portfolio, parameters must be such that $n = 1$, $g_h = g_f$, and $\alpha = \frac{1}{2}$. This means that the two economies are of equal sizes and perfectly integrated. In this case, $z_c = \hat{z}_c$, which reduces the market-clearing conditions to $4z_h = \phi_h M_h$ and $4z_f = \phi_h M_f$. This is analogous to what would arise in standard one-country monetary search models. The model presented here therefore nests a one-country, two-currency model for an appropriate choice of parameters. More generally however, demand for real balances are in proportion to country size.

at potentially different rates of return, the nominal exchange rate $e = \frac{\phi_f}{\phi_h}$ will equal

$$e = \frac{\omega(q_f^f) + n\omega(\hat{q}_f^f)}{\omega(q_h^h) + n\omega(\hat{q}_h^h)} \frac{M_h}{M_f}.$$

As expected, the exchange rate depends on fundamentals and monetary factors in the two countries. For example, if the Home country increases its money supply then its currency depreciates relative to the Foreign currency. The exchange rate is also affected by search frictions through dependence on q.

Turning to existence, this regime will constitute an equilibrium so long as private sellers always find it optimal to accept only the domestic currency. This requires that *(i)* all sellers want to accept the local currency; *(ii)* no seller wants to accept the non-local currency instead of the local currency; and *(iii)* no sellers want to accept both currencies. Let $S_j^k \equiv [u(q_j^k) - c(q_j^k)]$ denote match surplus when a seller from j accepts k as payment. With government, these conditions all hold when

$$g_h > max\{1 - \frac{2(1+n)(\psi_h^{hf} - \psi_h^h)}{(1-\theta)(S_h^{hf} - S_h^h)}, 1 - \frac{2(1+n)(\psi_h^f - \psi_h^h)}{(1-\theta)(S_h^f - S_h^h)}\} \equiv max\{g_{h1}, g_{h2}\}$$

$$g_f > max\{1 - \frac{2(1+n)(\psi_f^{hf} - \psi_f^f)}{n(1-\theta)(S_f^{hf} - S_f^f)}, 1 - \frac{2(1+n)(\psi_f^h - \psi_f^f)}{n(1-\theta)(S_f^h - S_f^f)}\} \equiv max\{g_{f1}, g_{f2}\}$$

$$q_h^h \geq q_h^f; q_f^f \geq q_f^h.$$

Here, national currency equilibria exist due to information costs. Since sellers do not incur the cost to accept foreign money, buyers matched with domestic sellers pay with their local currency. As shown above, government transaction policies can help guarantee this outcome. When $g_h > max\{g_{h1}, g_{h2}\}$ and $g_f > max\{g_{f1}, g_{f2}\}$, a large government sector ensures that this equilibrium will exist. Thus the key to national currency circulation is private sellers' information choice and the presence of large national governments that adopt policies only accepting the local money. Under such a policy, there always exist an equilibrium where local money circulates while the foreign one does not.

Regime H and F: One International Currency and One National Currency $H = \{(1,0),(1,1)\}, F = \{(1,1),(0,1)\}$

This class of equilibria corresponds to the emergence of an international currency that circulates both locally and abroad. In regime $H = \{(1,0),(1,1)\}$, Home sellers accept the Home currency while Foreign sellers accept both: as a result, the Home currency becomes an international currency while the Foreign currency only circulates locally. Government acceptance strategies consistent with this pattern include the baseline policy $\tau = \{(1,0),(0,1)\}$ and a policy where the Foreign government also accepts the Home currency, $\tau^H = \{(1,0),(1,1)\}$.

Under τ, DM quantities $q_h^h(z)$, $q_f^{hf}(z)$, $q_f^f(z)$ for Home buyers must satisfy

$$i_h = (\mu_h + \nu_h)L(q_h^h) + \mu_f L(q_f^{hf})$$

$$i_f = \mu_f L(q_f^{hf}) + \nu_f L(q_f^f).$$

As before, $\hat{q}_h^h(\hat{z})$, $\hat{q}_f^{hf}(\hat{z})$, $\hat{q}_f^f(\hat{z})$ for Foreign buyers satisfy similar equations.

In any monetary equilibrium where both currencies are valued, buyers hold positive balances of the two monies. From the bargaining solution, $z_h = \omega(q_h^h) > 0$ and $\hat{z}_h = \omega(\hat{q}_h^h) > 0$ for Home currency and $z_f = \omega(q_f^{hf}) - \omega(q_h^h) > 0$, $\hat{z}_f = \omega(\hat{q}_f^{hf}) - \omega(\hat{q}_h^h) > 0$ for Foreign currency. This holds so long as $q_s^k < q^*$, in which case $i_h > 0$ and $i_f > 0$. Real balances can then be inserted in the market-clearing conditions $2z_h + 2n\hat{z}_h = \phi_h M_h$ and $2z_f + 2n\hat{z}_f = \phi_f M_f$ to obtain CM prices. Both currencies are valued only if $i_h > i_f$ since $i_h - i_f = (\mu_h + \nu_h)L(q_h^h) - \nu_f L(q_f^f)$. Due to liquidity differentials, currencies bear different returns.

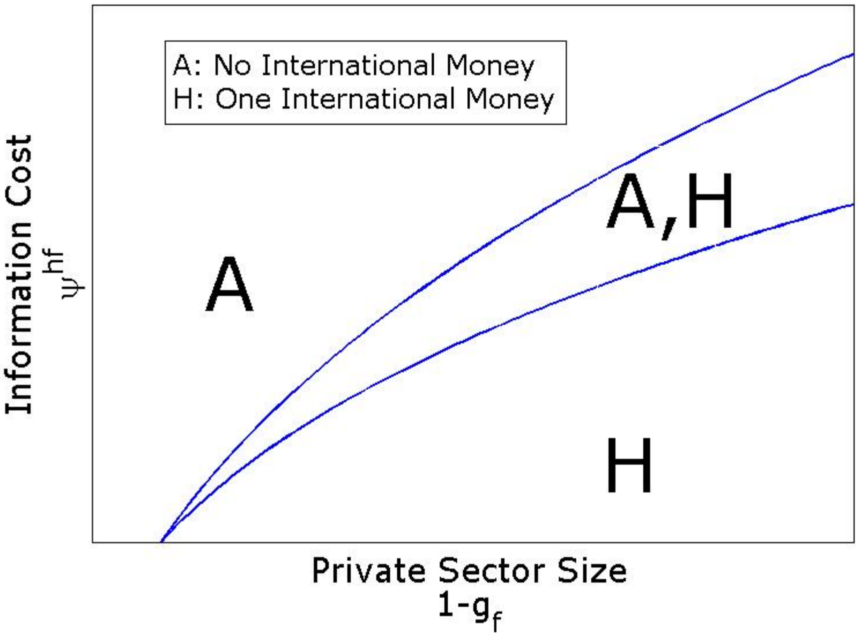
In this model, the emergence of international currencies is predicated on information costs associated with adopting multiple means of payment. An international medium of exchange is valued when a subset of sellers get informed about both currencies. In particular, Foreign sellers get informed about both currencies rather than just their local money so long as the expected benefit of doing so outweighs the cost:

$$\Sigma^{hf} \equiv \lambda_f(1-\theta)\{[u(q_f^{hf}) - c(q_f^{hf})] - [u(q_f^f) - c(q_f^f)]\} > \psi_f^{hf} - \psi_f^f.$$

Figure 1 compares the existence of equilibria for regimes A and H as a function of private sector size $1 - g_f$ and the cost of accepting both currencies ψ^{hf}. For a given country size, a sufficiently large ψ^{hf} would rule out dual-currency circulation. As ψ^{hf} falls however, an international currency may circulate. It is also possible to characterize how the policy variable g_f affects multiplicity, which occurs when ψ^{hf} is not too large. As government size falls, it becomes more likely that both currencies are used. As government gets large however, there are now more sellers that only accept the local money: the expected benefit Σ^{hf} falls, thereby making it more likely that national currencies are used. Figure 1 also illustrates the possibility that a local currency may survive and coexist with an international medium of exchange even without government restrictions ($g_f = 0$), as in Matsuyama, Kiyotaki, and Matsui (1993).

By formalizing the role of currency in payments, the model provides a channel through which

Figure 1: **Information Costs and Government Policy**



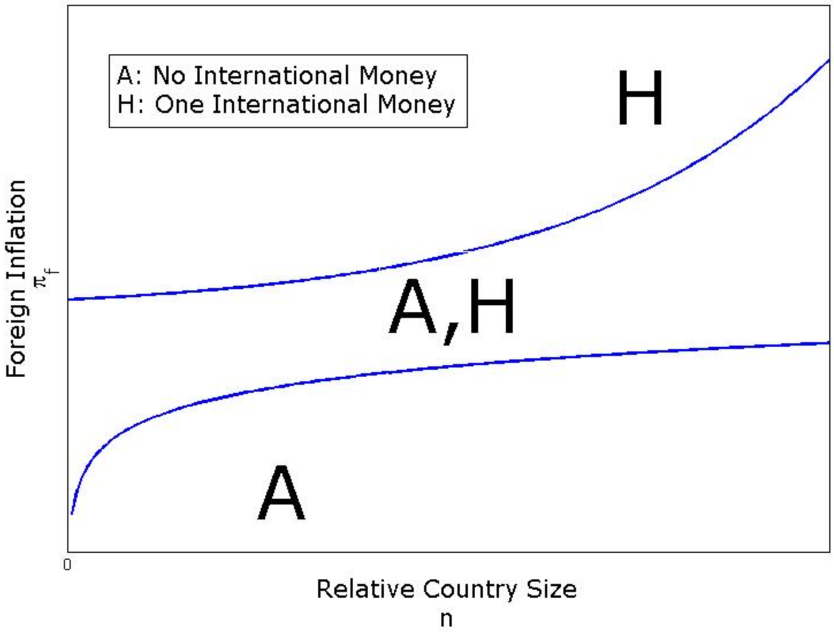
monetary policy affects prices and exchange rates. The following table summarizes the effects of inflation and monetary policy in the two countries.

$\frac{\partial q^{hf}}{\partial i_h} = 0$	$\frac{\partial q^{h}}{\partial i_h} < 0$	$\frac{\partial \phi_h}{\partial i_h} < 0$	$\frac{\partial \phi_f}{\partial i_h} > 0$	$\frac{\partial e}{\partial i_h} > 0$
$\frac{\partial q^{hf}}{\partial i_f} < 0$	$\frac{\partial q^{h}}{\partial i_f} > 0$	$\frac{\partial \phi_h}{\partial i_f} > 0$	$\frac{\partial \phi_f}{\partial i_f} < 0$	$\frac{\partial e}{\partial i_f} < 0$

For sake of illustration, suppose that the Home currency is the U.S. dollar and the Foreign currency is the peso. For example, if γ_f is increased, the peso inflates, which decreases ϕ_f. This also raises ϕ_h. The exchange rate $e = \frac{\phi_f}{\phi_h}$ falls and dollars appreciate. As the dollar is more valuable, sellers also have more incentive to accept it. As in Lester, Postlewaite, and Wright (2011), the model's general equilibrium effects makes currency substitution an endogenous response to local inflation. This situation arises precisely in dollarized economies where high inflation makes transacting in the local currency more costly so that citizens instead adopt the U.S. dollar.[22]

[22]In practice, currency substitution typically arises under high inflation, where the use of foreign currency in transactions may not be rapidly abandoned after it has been accepted. Remarkably, dollarization in some Latin American and Asian countries has continued and accelerated in recent years even following inflation stabilization, consistent with the model's predictions. For example, Oomes (2001) documents that dollarization in Russia quickly increased from almost zero to over 70% in the 1990s, and failed to decrease despite stabilization. Similarly, Guidotti and Rodriguez (1992) report that dollarization in Bolivia went from close to zero in 1985 to nearly 50% in 1987.

Figure 2: **Monetary Policy and Inflation**



Figures 2 illustrates the effect of policy parameters on uniqueness or multiplicity. Consider a Foreign seller's decision to accept both currencies rather than just the local money. When Foreign inflation is low, the benefit of adopting an additional medium of exchange is also low. As monetary policy approaches the Friedman rule $i_f \to 0$ ($\pi_f \to \beta$), output q^f and q^{hf} goes to q^*: the expected benefit of acquiring information Σ^{hf} gets small and there is an equilibrium where sellers do not get informed and the economy ends up in regime A. Intuitively, there is no reason to make a costly investment when a single currency z_f delivers q^* in all meetings. As inflation increases however, it becomes more costly to use the national money, which decreases ϕ_f and increases the value of the alternative asset ϕ_h. This raises the incentive to acquire information and as before, can generate multiple circulation patterns. The model therefore implies that currency substitution may be a purely expectational phenomenon: an international money may emerge even if the fundamentals of the economy are consistent with an equilibrium with national currency use.

Similarly, γ_h increases ϕ_f and decreases the value of domestic money ϕ_h.[23] Costly information

[23] To see this, differentiate to get $\frac{\partial \phi_f}{\partial i_h} = \frac{\omega(q_f^f)-\omega(q_h^h)}{M_f \Delta} L'(q_f^{hf})\omega'(q_h^h)$, where $L'(q_f^{hf}) < 0$, $\omega'(q_h^h) > 0$, and $\Delta < 0$. Thus, $\frac{\partial \phi_f}{\partial i_h} > 0$.

acquisition therefore puts a discipline on monetary policy if its currency circulates abroad. To maximize seigniorage revenue, monetary authorities would like a high money growth rate γ_h or inflation rate. But if they engineer a rate of money growth that is too high, foreign sellers may no longer find it optimal to accept its currency, as ϕ_h declines when γ_h goes up.

Further, the model illustrates how inertia, or persistence in currency use, is generated by network externalities and complementarities in payments. As shown above, currency substitution occurs in response to increases in local inflation. However, dollarization does not subside even when inflation stabilizes. To illustrate, consider a country with low inflation, π_f and agents only accept the local money. If π_f increases to a sufficiently higher level, private sellers will incur the information cost to accept the alternative asset— the economy dollarizes. Suppose however that high inflation is temporary and π_f reverts back to its initial low level. Since the dollarization outcome can still be an equilibrium, agents can continue to coordinate on this regime even after inflation stabilizes. This captures the phenomenon of hysteresis: even though the inflationary episode was only temporary, the economy's payment system changes permanently. Due to multiplicity, history, not just fundamentals and policies, determines current outcomes. Episodes of dollarization and euroization in response to high inflation support this idea.[24]

A new feature in this model is that government transaction policies can further influence the extent of currency substitution and dollarization through its effect on a currency's liquidity properties. By introducing government, we see how their transaction policies affect currency value. Since the values of the two monies are interdependent, changes in one country's acceptance policy will affect the values of both currencies. Under the baseline policy τ, equilibrium conditions imply $i_h - i_f = (\mu_h + \nu_h)L(q_h^h) - \nu_f L(q_f^f)$. All else equal, an increase in g_f reduces the value of dollars and increases the value of pesos: $\frac{\partial \phi_h}{\partial g_f} < 0$ and $\frac{\partial \phi_f}{\partial g_f} > 0$. Since the exchange rate $e = \frac{\phi_f}{\phi_h}$ increases, dollars will depreciate, and as before, this will affect sellers' information choice problem. Consider a Foreign seller's decision to accept dollars. Recall that their expected surplus from accepting both dollars and the local money is

$$\Pi^{hf} \equiv \psi_f^{hf} + \lambda_f(1-\theta)[u(q_f^{hf}) - c(q_f^{hf})]\}.$$

Since q_f^{hf} and z_h fall from the increase in g_f, Π^{hf} will also fall. This then makes private sellers less inclined to accept dollars, which further decreases its value. Thus strong transaction policies

[24] In the dollarized nations Argentina, Bolivia, Uruguay and Vietnam, inflation reached over 300% in the late 1980s. However, the fact that over the subsequent decade inflation has decreased dramatically has not lowered dollarization, as reported by Reinhart, Rogoff, and Savastano (2003). Similarly, periods of turbulence from the 1980s to the early 2000s led to significant euroization in Central and Eastern Europe. While the economic and political environment has stabilized in the subsequent period for many of these countries, de facto euroization continues to be present.

in the Foreign country— in the sense that its transactions comprise a large part of the economy's total transactions— can make its currency appreciate, which reduces the dollar's international role. More generally, changes in government size will have direct effects on currency values, thereby influencing the monetary equilibrium attained.

Now consider a change in the policy of one of the governments: suppose the new policy follows $\tau^H = \{(1,0),(1,1)\}$ where the Foreign government also accepts dollars in addition to its local money. Equilibrium conditions imply $i_h^H - i_f^H = (\mu_h + \nu_h)L(q_h^h) > 0$. Now that the share of the world population accepting dollars has increased, this makes acquiring information about it more desirable. Due to general equilibrium effects in the decentralized markets, private sellers have a stronger incentive to accept dollars. Consider again a Foreign seller. They will accept both currencies rather than just pesos by comparing the expected benefit with the cost:

$$\Sigma^{hf} \equiv \lambda_f(1-\theta)\{[u(q_f^{hf}) - c(q_f^{hf})] - [u(q_f^f) - c(q_f^f)]\} > \psi_f^{hf} - \psi_f^f.$$

As there are now more meetings where sellers accept dollars, the expected benefit Σ^{hf} increases. Again, the mechanism behind this increase operates through the model's information structure and the way liquidity is endogenized. As a result, the model implies that governments can strengthen a currency's international role through its acceptance policies.

In order for this regime to exist, private Home sellers must only accept the domestic currency while private Foreign sellers must accept both. This requires that

$$g_h > max\{1 - \frac{2(1+n)(\psi_h^{hf} - \psi_h^h)}{(1-\theta)(S_h^{hf} - S_h^h)}, 1 - \frac{2(1+n)(\psi_h^f - \psi_h^h)}{(1-\theta)(S_h^f - S_h^h)}\} \equiv max\{g_{h3}, g_{h4}\}$$

$$g_f < min\{1 - \frac{2(1+n)(\psi_f^{hf} - \psi_f^f)}{n(1-\theta)(S_f^{hf} - S_f^f)}, 1 - \frac{2(1+n)(\psi_f^{hf} - \psi_f^h)}{n(1-\theta)(S_f^{hf} - S_f^h)}\} \equiv min\{g_{f3}, g_{f4}\}$$

$$q_h^h \geq q_h^f.$$

So long as the Home government is large enough and the Foreign government is small enough— that is, $g_h > max\{g_{h3}, g_{h4}\}$ and $g_f < min\{g_{f3}, g_{f4}\}$— an equilibrium where dollars emerge as the single international currency will exist. Another equilibrium, $F = \{(1,1),(0,1)\}$, has symmetric properties as regime $H = \{(1,0),(1,1)\}$, so discussion is omitted.

Since dollars circulates abroad, the Foreign government may want to consider using its transaction policy to drive dollars out of circulation in its country. However, so long as private Foreign sellers want to accept both currencies rather than just the local money, the government cannot completely de-dollarize the economy for fixed levels of g_h, g_f. Hence legal tender laws may not be sufficient to rule of international currency use. For dollars to cease circulating abroad, it must be that private sellers no longer find it optimal to accept both currencies. This requires that $g_f > max\{g_{f3}, g_{f4}\}$. As a result, Foreign sellers only accept their local money. Thus a sufficiently large local government can de-dollarize the economy by inducing a switch to national currency equilibria. If instead g_f is less than this threshold then the government can only encourage and promote its currency, but cannot guarantee that the other stops circulating.

Regime U: Two International Currencies $U = \{(1,1),(1,1)\}$

Now consider an equilibrium where all sellers accept both currencies, leading to the emergence of two international currencies that are unified and become perfect substitutes. To facilitate presentation, assume that countries are of equal sizes and perfectly integrated so that $n = 1$ and $\alpha = \frac{1}{2}$. In this equilibrium, this assumption makes the presentation easier to follow without sacrificing generality, and eliminates subscripts on variables.

With no government, equilibrium conditions simplify to

$$i_h = (\mu_h + \mu_f)L(q^{hf})$$

$$i_f = (\mu_h + \mu_f)L(q^{hf}).$$

When both monies are valued, it must be that

$$i_h = i_f.$$

Hence the two currencies are equally liquid and are valued only if they have the same rate of return. As in Kareken and Wallace (1981), citizens are indifferent between currencies with equal returns, so that the two are perfect substitutes. Letting $i = i_h = i_f$, output traded in the DM, q^{hf}, must satisfy $i = (\mu_h + \mu_f)L(q^{hf})$. However, this need not imply that the growth rates of the two currencies are equal since the necessary condition $i_h = i_f$ does not prevent the two currencies from being issued at different rates $\gamma_h \neq \gamma_f$.

When monies circulate at par, agents may hold different portfolios, but they will have the same total value. Market clearing implies $Z_h + Z_f = \omega(q^{hf}) > 0$. Notice that an increase in the growth rate of one country's money reduces the real value of both monies: since they are perfect substitutes, citizens treat different currencies as simply different denominations of a single international money. The first-best level of output $q^{hf} = q^*$ is achieved under the Friedman Rule $i = 0$, in which case $\omega(q^{hf}) = \omega(q^*)$. Even so however, this is not socially efficient since all sellers must pay a real cost $\psi^{hf} > 0$.

Further, the exchange rate is indeterminate since q^{hf} is uniquely determined while Z_h and Z_f must satisfy a single condition. Suppose the two monies grow at different constant rates $\gamma_h \neq \gamma_f$. Let γ denote the growth rate of the world money supply $M = M_h + eM_f$. For any e, $M' = \gamma_h M_h + e\gamma_f M_f$. γ must then satisfy $\gamma \equiv \frac{M'}{M} = \nu\gamma_h + (1-\nu)\gamma_f$, where $\nu = \frac{M_h}{M} = \frac{M_h}{M_h + eM_f}$. Hence the growth rate of the world money supply is just a weighted average of the growth rates for the two countries. An equilibrium will consist of constant values for q^{hf} and e, plus paths for ϕ_h, ϕ_f satisfying $i = (\mu_h + \mu_f)L(q^{hf})$ and $\omega(q^{hf}) = \phi_h[M_h + eM_f]$. While q^{hf} is uniquely determined, e and ϕ_h must satisfy a single equality, which makes the relative value of the two monies indeterminate.[25]

In general, exchange rate indeterminacy arises naturally in multi-asset economies where citizens are free to use the money of any country. Intuitively, this is unsurprising as $e = \frac{\phi_f}{\phi_h}$ is just the relative price of two intrinsically worthless objects. The exchange rate can therefore be whatever people believe it to be. As there is nothing to pin it down, it will fluctuate as beliefs fluctuate. For example, the dollar may fall against the euro not because of changes in real economic conditions, but because all citizens believe it will fall. This is consistent with the exchange rate volatility present in the data since the United States' 1971 abandonment of foreign exchange controls.[26] The model suggests this volatility has to do with the presence of international currency traders that can hold multiple assets.

One way to eliminate indeterminacy in this model is by having governments adopt transaction policies so that currencies no longer circulate at par. For example, the policy $\tau^H = \{(1,0),(1,1)\}$ where the Foreign government accepts both currencies will generally imply $i_h \neq i_f$. Consider now the baseline policy $\tau = \{(1,0),(0,1)\}$, which implies

$$i_h = (\mu_h + \mu_f)L(q^{hf}) + g_h L(q^h)$$

$$i_f = (\mu_h + \mu_f)L(q^{hf}) + g_f L(q^f).$$

There will be liquidity differentials so long as $g_h \neq g_f$. If instead governments are of equal size, τ ensures that monies remain perfect substitutes.

Furthermore, the model's information costs also constrains indeterminacy. If the cost of recognizing both currencies ψ^{hf} is sufficiently high so that no sellers accept both, there is no longer

[25]To see this result when $\gamma_h = \gamma_f = \gamma$, let the world money supply in ϕ_h units be $M = M_h + eM_f$, growing at constant rate γ, where $e = \frac{\phi_f}{\phi_h} > 0$. A representative citizen's currency portfolio is constant: $\omega(q^{hf}) = Z_h + Z_f = \phi_h M_h + \phi_f M_f$. Since $\omega(q^{hf}) = \phi_h M$ is constant, ϕ_h and ϕ_f must be decreasing at rate γ. As both e and ϕ_h must satisfy a single condition $\omega(q^{hf}) = \phi_h[M_h + eM_f]$, there will be indeterminacy: there can exist equilibria where only one currency is valued and equilibria where both currencies are valued.

[26]Obstfeld and Rogoff (1996) document that the magnitude of these fluctuations cannot be traced to changes in economic fundamentals of similar magnitude.

an equilibrium where the two currencies circulate at par.[27] When instead all sellers accept both currencies, $U = \{(1,1),(1,1)\}$ requires that *(i)* all sellers want to accept both currencies; *(ii)* no sellers want to accept only the domestic currency; and *(iii)* no sellers want to accept only the foreign currency. An equilibrium where all private sellers accept both currencies exists when

$$g_h < min\{1 - \frac{2(1+n)(\psi_h^{hf} - \psi_h^h)}{(1-\theta)(S_h^{hf} - S_h^h)}, 1 - \frac{2(1+n)(\psi_h^f - \psi_h^h)}{(1-\theta)(S_h^f - S_h^h)}\} \equiv min\{g_{h5}, g_{h6}\}$$

$$g_f < min\{1 - \frac{2(1+n)(\psi_f^{hf} - \psi_f^f)}{n(1-\theta)(S_f^{hf} - S_f^f)}, 1 - \frac{2(1+n)(\psi_f^h - \psi_f^f)}{n(1-\theta)(S_f^h - S_f^f)}\} \equiv min\{g_{f5}, g_{f6}\}.$$

Regime M: One International Currency $M = \{(1,0),(1,0)\}$

With no government, the emergence of a single international money occurs when all private sellers only accept the Home currency. Since the Foreign currency is never accepted, the Home currency emerges as the sole medium of exchange. By introducing governments however, the Foreign currency may be valued if it's accepted by the local government. Contrary to the laissez-faire case, introducing governments of strictly positive measure ensures that there is no longer a equilibrium such that one currency circulates but the other does not.

Equilibrium conditions imply that DM quantities must satisfy

$$i_h = (\mu_h + \mu_f + \nu_h)L(q_h^h)$$

$$i_f = \nu_f L(q_f^f).$$

With no government intervention, Foreign currency is not valued since it has no role in exchange: when $g_f = 0$, $\nu_f = 0$ and $i_f = 0$. With government however, it's clear that Foreign currency will now have value: with $g_f > 0$, $i_f > 0$. Thus, having a government policy that favors its national money ensures that it stays in circulation and maintains value, at least locally.

4.1 Multiple Equilibria

As shown in the previous subsections, the model implies the emergence of distinct currency regimes characterized by different payment patterns and realms of circulation. There can be multiple

[27] In an overlapping generations monetary model, Martin (2006) derives an analogous result and rules out indeterminacy of the exchange rate by assuming that there is a fixed cost for sellers to accept two currencies. If the fixed cost of accepting two currencies is sufficiently high so that no household decides to do so, then the exchange rate is no longer indeterminate since the equilibria with perfect substitute currencies will cease to exist.

equilibria, where the share of transactions requiring different currencies is not uniquely determined by fundamentals. Proposition 1 summarizes how the government size parameter affects multiplicity.

Proposition 1. *Suppose that governments of strictly positive measure adopt a given transaction policy Γ.*

1. *If $max\{g_{f1}, g_{f2}\} < min\{g_{f3}, g_{f4}, g_{f5}, g_{f6}\}$, there will be an equilibrium where Foreign sellers get informed about both currencies and one where they only get informed about their domestic currency for any $g_f \in [max\{g_{f1}, g_{f2}\}, min\{g_{f3}, g_{f4}, g_{f5}, g_{f6}\}]$ since existence conditions for regimes A, H, and U can be simultaneously satisfied.*

2. *If $max\{g_{h1}, g_{h2}, g_{h3}, g_{h4}\} < min\{g_{h5}, g_{h6}\}$ there will be an equilibrium where Home sellers get informed about both currencies and one where they do not for any $g_h \in [max\{g_{h1}, g_{h2}, g_{h3}, g_{h4}\}, min\{g_{h5}, g_{h6}\}]$.*

The intuition for multiplicity operates through the general equilibrium interaction between buyers and sellers: what sellers accept depend on what buyers carry, and what buyers carry depend on what sellers accept. When more sellers accept a currency, it becomes more liquid and thus more valuable in exchange. Since buyers now want to hold more of this currency in their portfolio, its price increases. Now that it's more valuable, sellers have more incentive to accept it. Due to this complementarity, multiple equilibria can arise. Multiplicity and coexistence of different currency regimes is a pervasive feature of the model. Thus, the regime that the economy ends up in will depend a large degree on expectations, since the circulation of fiat monies is grounded upon self-fulfilling beliefs and confidence in others.

As an example, suppose the two regions are the United States and the European Union. The model suggests that if the two regions are similar sizes and share comparable fundamentals, competition with the euro can dampen the use of cash dollars.[28] Since the theory implies that countries with the same underlying fundamentals will have currencies that are perfect substitutes, this suggests that the euro could be a serious competitor for the dollar. However, this need not imply that the dollar will lose its dominant international role. For these same fundamentals, there can also exists an equilibrium where only one international currency emerges in which the dollar maintains its dominant role. Since a small change in fundamentals can induce a permanent regime shift, the current status of dollar as premier global currency cannot be taken for granted.

[28]For the period 1990 to 2007, Hellerstein and Ryan (2009) report that the Eurozone is nearly the size of the United States, with comparable growth and inflation rates. Given these fundamentals, Section 5 shows how the dollar may lose its premier international status.

4.2 Government Transaction Policies

Given the model's multiplicity, governments can help coordinate the economy on certain equilibria. Introducing government allows consideration of how their transaction policies affect circulation patterns, as shown in the previous subsections. The following proposition summarizes some insights from this analysis.

Proposition 2. *Suppose that governments of strictly positive measure adopt a given transaction policy* Γ. *Assume* $q_h^h \geq q_h^f; q_f^f \geq q_f^h$. *Then the following cases may arise depending on parameter values:*

(a) A sufficiently large government sector can guarantee the existence of Regime A, an equilibrium with two local currencies. That is, $A = \{(1,0),(0,1)\}$ *exists when* $g_h > max\{g_{h1}, g_{h2}\}$ *and* $g_f > max\{g_{f1}, g_{f2}\}$.

(b) Home governments must be sufficiently large and Foreign governments must be sufficiently small for existence of Regime H, an equilibrium where the Home currency is international while the Foreign currency is national. That is, $H = \{(1,0),(1,1)\}$ *exists when* $g_h > max\{g_{h3}, g_{h4}\}$ *and* $g_f < min\{g_{f3}, g_{f4}\}$.

(c) A sufficiently large government sector can rule out the existence of Regime U, an equilibrium with a unified currency where the two currencies are perfect substitutes. That is, $U = \{(1,1),(1,1)\}$ *ceases to exist when* $g_h > max\{g_{h5}, g_{h5}\}$ *and* $g_f > max\{g_{f5}, g_{f6}\}$.

The threshold values g_i *are provided in the paper's main text.*

Proposition 2 illustrates that if governments are sufficiently large, it can guarantee the existence of an equilibrium where only national monies circulate and rule out an equilibrium where a unified currency circulates internationally. Governments that only accept the domestic currency can thus guarantee the local circulation of their currency. Contrary to the laissez-faire case, introducing governments of strictly positive measure ensures that there is no longer a equilibrium such that one currency circulates but the other does not.

In addition, Proposition 2 also implies that countries with larger governments are more likely to have its currency circulate internationally. This is consistent with the fact that historically, countries with stable political systems and the capacity to enforce the rule of law had currencies that were widely traded and accepted internationally. Similarly, the model also implies that countries with weak transaction policies, or a smaller government sector, are more likely to dollarize and transact in foreign money.

In addition, strong transaction policies— in which a government's transactions make up a large part of the economy's total transactions— may extend to the point of driving foreign money out of domestic circulation. While high domestic inflation can lead to dollarization, a sufficiently large local government can induce a switch to national currency equilibria and hence de-dollarize the economy. To de-dollarize successfully, the model suggests that economic authorities must set up the proper constraints for residents to want to transact and hold local currency by increasing its usability. In Peru for example, the government switched its public lending program to local currency in 1988, and in Angola in 2001, public payments for wages, goods, and services lead to declines in dollarization. Consequently, both countries had successfully market-driven de-dollarization during the last decade.[29] As in the model, both market forces and government intervention reinforce each other.

As a historical example, the Ukrainian government in 1992 was confronted with the problem of making citizens use its new fiat currency, the hryvnia, in a dual-currency economy. In the transition period, both the hryvnia and the old currency, the karbovanets, were used in circulation. To help drive out the old currency, the government ended up restricting its use so that only the hrynvnia could be accepted in transactions. For example, merchants were required to give change only in hryvnia. As in the present model, the launching of a new currency succeeded when authorities imposed constraints that enhanced its acceptability by locals: since a subset of citizens could only accept that currency, it became more attractive for internal transactions. Similarly in Estonia, governments triggered a switch to the kroon in 1992 by resisting to accept the alternative asset, the Russian ruble.[30]

4.3 Welfare

Having examined prices and some policy implications, I now discuss optimality and the model's normative implications. Given the model's multiplicity, society may prefer certain equilibria over others. Social welfare in each country $j = \{h, f\}$ is measured as the sum of private agents' match surpluses net of information costs. Welfare for country j is thus

$$W_j = \rho_j^{hf}[u(q_j^{hf}) - c(q_j^{hf}) - \psi_j^{hf}] + \rho_j^{h}[u(q_j^{h}) - c(q_j^{h}) - \psi_j^{h}] + \rho_j^{f}[u(q_j^{f}) - c(q_j^{f}) - \psi_j^{f}]$$

The following two propositions summarize the model's main welfare implications.

Proposition 3. *Equilibria where currency $c = \{h, f\}$ circulates internationally dominate national-currency equilibria in terms of welfare if the expected benefit of accepting both currencies exceeds the expected cost:*

$$S_j^{hf} - S_j^{c} > \psi_j^{hf} - \psi_j^{c}$$

[29] See for example, Kokenyne, Ley, and Veyrune (2010).

[30] These examples and others can be found in Melliss and Cornelius (1994). A related discussion applied to launching a new currency is in Lotz and Rocheteau (2002).

where $S_j^k \equiv [u(q_j^k) - c(q_j^k)]$.

Proposition 3 simply says that there are potential welfare gains from having one currency— or two that serve as perfect substitutes. However, a unified currency regime where the two monies circulate internationally and on par is not necessarily preferable to having multiple monies circulate— e.g. a regime with two local currencies. Since welfare depends on the tradeoff between the cost and benefit of acquiring information, having a single currency will only be socially preferred so long as the cost of accepting both currencies is sufficiently low. Though welfare comparisons across different equilibria may be ambiguous, the relevant insight is not to ignore information costs once trade patterns are endogenous.

Further, since dual-currency regimes only emerge if information costs are minimal, the model suggests that countries seeking to promote its currency for international use may find it worthwhile to adopt campaigns to improve the flow of information internationally. Even without any government intervention regarding its transaction policy, this makes private citizens more inclined to use that currency and increases social welfare.[31]

Proposition 4. *If countries adopt the Friedman rule, a single-currency equilibrium achieves the first-best allocation. If it is costless to accept one of the currencies, such an equilibrium is also socially efficient.*

Thus, if it is costless to accept one of the currencies but costly to accept the other, the equilibrium that maximizes welfare is a regime with a single international money where all private citizens use the currency with no information cost. In this case, countries satiate the demand for liquidity under the Friedman rule. Proposition 4 suggests that a monetary union where member countries all share a single currency and adopt a common monetary policy characterized by the Friedman rule is both optimal— since it achieves the first-best allocation— and socially efficient since no resources are spent on information costs.

In addition, the optimal outcome can also be achieved with currency unification where the two currencies become perfect substitutes. To see this, suppose that the monetary authorities of both countries set the money growth rate that maximizes the welfare of its citizens. Since the Friedman rule maximizes welfare for both countries, the non-cooperative outcome would be $\gamma = \gamma_h = \gamma_f = \beta$.

[31] A leading example concerns the threat of counterfeit money. In a related model with costly counterfeiting, Li, Rocheteau, and Weill (2011) show that the possibility to counterfeit money can affect its value and hence welfare, even if no counterfeiting occurs in equilibrium. Consistent with this result, the U.S. government responds actively to the threat of counterfeiting with periodic note redesigns and public educational programs. Recently, educational campaigns for both domestic and international users of U.S. dollars were introduced, namely the New Color of Money program in 2003.

This same outcome would also occur if countries instead could cooperate by jointly maximizing aggregate welfare of the world.[32]

5 Quantitative Analysis

The preceding sections presented a simple two-country, two-currency search model that is amenable to policy analysis. To generate additional insight, the framework is generalized to an arbitrary number of countries and currencies and calibrated to match international trade data. Since much of the set-up and analysis carries over from the two-country, two-currency model, the basic environment and equilibrium of the N-country, N-currency model is in Appendix A.

The generalized framework allows for the study of many different international payment patterns from first principles. Recall that the theory has three distinct features: (*i*) the emergence of international currencies is predicated on information costs and network effects; (*ii*) strategic complementarities in the trading environment leads to coordination problems and multiple equilibria; and (*iii*) governments can ensure the existence of certain equilibria while ruling out other ones.

In this section, a calibration of the general framework is used to characterize the structure of world payments in a multi-country setting and illustrate the model's main theoretical predictions. The regions of interest in the baseline analysis consist of three trading blocs: the United States, the Eurozone, and China. The declining dollar, the advent of the euro and the recent rise of China has renewed considerable interest in determining whether the dollar is at risk of losing its international role.[33] This paper provides a new theoretical framework to evaluate this issue. The strategy taken here is to let the data and calibration procedure narrow down the set of equilibria to ones that are empirically plausible. Turning to normative considerations, the general framework can also be used to estimate the welfare benefits of alternative monetary regimes.

5.1 Calibration

To calibrate the model, the global economy is split into three trading blocs, or regions: the United States (region A), the Eurozone (region B), and China (region C).[34] After discussing parameters

[32] This argument clearly abstracts from other relevant considerations, such as the government's fiscal policy. In a follow-up paper, I examine optimal monetary and fiscal policy in a currency union using a related model.

[33] Eichengreen (2010) describes a future in which the dollar and the euro would be the dominant global currencies, with a potential international role for the Chinese renminbi as well.

[34] The Eurozone, or the Euro Area, consists of the 17 European Union member states that have adopted the euro as their common currency and sole legal tender. These countries include Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Luxembourg, Malta, Netherlands, Portugal, Slovakia, and Spain. Data from each of the 17 member countries are averaged into an aggregate measure for the Eurozone, with country weights determined by GDP.

that can be easily estimated or fixed independently to their empirical counterparts, I describe the calibration procedure for the remaining parameters. This procedure uses the model's equations and the parameters calibrated independently in order to find parameter values that minimize the sum of squared residuals between moments in the data and corresponding outcomes generated by the theory.

Following Lagos and Wright (2005), functional forms for utility and cost functions are $U(x) = Blnx$, $u(q) = ln(q+b) - ln(b)$, and $c(q) = q$, where $b \in (0,1)$.[35] The parameter b is set to $b = 0.0001$ which ensures a solution to the bargaining problem. Since $x^* = B$, the preference parameter B proxies production in the centralized markets, or the extent of the tax base. In this model, the centralized market can be interpreted as a home production sector where only domestic goods are consumed. B is therefore calibrated to match the target of domestically-produced consumption goods share in total consumption, which is about 0.5 for the United States.[36] The discount factor is set to $\beta = 0.966$, consistent with an annual real interest rate of 3.5%.

Country sizes are calibrated to match volume of trade as a fraction of world trade (where "world" here is defined as the the U.S., Eurozone, and China) which is 0.39 for the United States, 0.32 for the Eurozone, and 0.29 for China. Since the model implies that gross money growth rates are also gross inflation rates in a steady state equilibrium, γ_A, γ_B, γ_C are set to average annual inflation rates for the period 2000 to 2010, which is about 3% for the U.S., 2% for the Eurozone, and 5% for China. Data on trade volume and inflation are obtained from Source OECD and the World Bank. I also consider different inflation scenarios in the quantitative exercise. For the baseline calibration, the bargaining power parameter is set to $\theta = 0.5$, consistent with an egalitarian bargaining rule.

Since in the model, government sellers always accept a particular currency, government size parameters, g_A, g_B, and g_C, proxy a country's degree of centralized control or legal restrictions. These values are calibrated with an index of government size I construct using data on property rights, monetary freedom, and government expenditures as a fraction of GDP. The property rights index is from the Heritage Foundation and measures rule of law and contract enforcement in a particular country. The index of monetary freedom is also from the Heritage Foundation. Data on government expenditures and GDP is from Source OECD. The resulting government size index is then

$$GS_i = \frac{PR_i + (100 - MF_i) + 100 * \frac{G_i}{GDP_i}}{3},$$

[35]This specification departs from the baseline specification, where $U(x) = x$, in order to pin down output in the centralized market. With a linear specification, CM output and hours worked is indeterminate.

[36]The calibration procedure for B follows a similar approach as Gomis-Porqueras, Kam, and Lee (2011). Notice that this implies that citizens from all countries produce the same number of goods in the centralized market. This assumption is mainly used to facilitate computation.

Table 4: **Bilateral Trade: Model vs. Data (2000-2010)**

Parameter	Target Moment	Model	Data
α_A	World trade share (U.S.)	41.1%	41.1%
α_B	World trade share (Eurozone)	37.9%	37.9%
α_C	World trade share (China)	41.8%	41.8%
α_{AB}	Share of E.U. trade with U.S.	14.0%	14.0%
α_{AC}	Share of China trade with U.S.	13.6%	13.6%
α_{BA}	Share of U.S. trade with E.U.	11.7%	17.8%
α_{BC}	Share of China trade with E.U.	17.0%	17.0%
α_{CA}	Share of U.S. trade with China	10.3%	14.9%
α_{CB}	Share of E.U. trade with China	15.4%	13.8%

where $PR_i \in [0, 100]$ is the property rights index, $MF_i \in [0, 100]$ is the monetary freedom index, and $\frac{G_i}{GDP_i} \in [0, 1]$ is government spending as a fraction of GDP for each region.

The next set of parameters are domestic trade frictions, α_i, and international trade frictions for each country pair, α_{is}.[37] The parameter α_i governs how often a citizen meets compatriots, and is calibrated to match world output shares from the WTO. The six international meeting parameters α_{is} are calibrated to match bilateral trade data for the period 2000 to 2010 obtained from the European Commission Bilateral Affairs, as summarized in Table 4. Due to an accounting constraint that the total measure of meetings between agents from country i with agents from country s have to be the same as the total measure of meetings between agents from country s with agents from country i, three of the meeting probabilities will not precisely match its targeted value. These values are then backed out using calibrated values for n_i, subject to the accounting constraint.

The final set of parameters are the information costs for accepting different currencies, which is the model's main mechanism for generating liquidity differentials. Since there are now three currencies, private sellers can potentially accept any combination in this set. For each seller, the cost of accepting two or more currencies is assumed to take an additive structure $\psi_s^{sij} = \psi_s^s + \psi_s^i + \psi_s^j$. The cost of accepting one's domestic currency is set to zero, $\psi_s^s = 0\ \forall s \in \{a, b, c\}$, while the cost of accepting a foreign currency, $\psi_s^k \geq 0$, will be pinned down by the calibration procedure, resulting

[37] In the generalized model, countries are of size $2n_A, 2n_B$, and $2n_c$, where $\sum_i n_i = 1$. Buyers are mobile while sellers are immobile. With probability α_i, a buyer meets a sellers in his country of origin i and with probability α_{ij}, meets a sellers from the foreign country $j \neq i$, where $\alpha_{ij} + \alpha_{ik} < 1 - \alpha_i$. Agents are randomly matched according to a matching function, $M_s = M(\mathcal{B}_s, \mathcal{S}_s) = \frac{\mathcal{B}_s \mathcal{S}_s}{\mathcal{B}_s + \mathcal{S}_s}$, where $\mathcal{B}_s$ and $\mathcal{S}_s$ denotes the number of buyers and sellers in country s. As in Kannan (2009), α_{is} is not independent of α_{si} due to the accounting constraint $n_i \alpha_{is} = n_s \alpha_{si}$.

Table 5: **Information Costs and Currency Portfolios**

Parameter	Target Moment	Source
ψ_b^a	E.U trades invoiced in dollars = 32%	Goldberg and Tille (2005)
ψ_c^a	China trades invoiced in dollars = 52%	Goldberg and Tille (2005)

in a total of six parameters. These parameters can be summarized by the following matrix:

$$\Xi = \begin{bmatrix} 0 & \psi_a^b & \psi_a^c \\ \psi_b^a & 0 & \psi_b^c \\ \psi_c^a & \psi_c^b & 0 \end{bmatrix}.$$

To determine values for information costs, which are unobservable quantities, I use data on the extensive margin of foreign currency holdings— whether or not a country holds a particular foreign currency— and, when available, the intensive margin— how much of a particular currency is held in a particular country. Information on the extensive margin corresponds to private sellers' acceptance decision $\chi_s = (\chi_s^a, \chi_s^b, \chi_s^c) \in \{0,1\}^3$, while the intensive margin will partially determine the composition of currency portfolios.[38]

In the U.S., only dollars circulate, so that $\chi_A = (1,0,0)$. In the Eurozone and China, dollars are used in international trade invoicing, as reported in Goldberg and Tille (2007). In addition, the Bank for International Settlements reports that U.S. dollars represent most of China's settlement of international trade while the euro is only used in transactions in regions on the periphery of the Eurozone. This results in $\chi_B = (1,1,0)$ and $\chi_C = (1,0,1)$. Next I use data on the intensive margin of foreign currency holdings to construct ratios of foreign currency holdings that will pin down the information costs for accepting dollars in Europe and China, as summarized in Table 5. Goldberg and Tille (2005) report that the share of dollar-denominated trade in Europe ranges from 20% in Italy to 71% in Greece. I use the reported European average of 32.4% of dollar-invoiced trade to target the fraction of dollars in Eurozone portfolio holdings z_a^b/z^b, where $z^b = z_a^b + z_b^b + z_c^b$. Similarly in Asia, estimates of dollar-denominated range trade from 52% to 84%. I use the lower bound of 52% to determine the fraction of dollars in China's portfolio holdings, z_a^c/z^c, where $z^c = z_a^c + z_b^c + z_c^c$. Information costs are then calculated so that the strategy set $\Omega = (1,0,0),(1,1,0),(1,0,1)$ is an equilibrium.

[38]Freeman and Kydland (2000) undertake a similar calibration procedure to determine values for transaction costs by using data on currency-deposit ratios.

5.2 Parameter Estimates

Table 6 summarizes the calibration results for the three region model.

Table 6: **Calibrated Parameters for 3-Region Model (2000-2010)**

	Interpretation	Value	Target
b	–	0.0001	Ensures solution to bargaining problem
B	CM production	0.769	Domestic consumption share = 0.5
β	Discount factor	0.966	Annual real interest rate = 3.5%
γ_A	Gross inflation (U.S.)	1.03	Average inflation rate (U.S.) = 3%
γ_B	Gross inflation (Eurozone)	1.02	Average inflation rate (Eurozone) = 2%
γ_C	Gross inflation (China)	1.05	Average inflation rate (China) = 5%
θ	Bargaining power	0.5	Egalitarian bargaining solution
n_A	Country size (U.S.)	0.39	Relative trade volume (U.S.) = 0.387
n_B	Country size (Eurozone)	0.32	Relative trade volume (Eurozone) = 0.322
n_C	Country size (China)	0.29	Relative trade volume (China) = 0.291
g_A	Government size (U.S.)	0.52	Government size index (U.S.) = 52.3
g_B	Government size (Eurozone)	0.41	Government size index (Eurozone) = 41.2
g_C	Government size (China)	0.38	Government size index (China) = 37.6

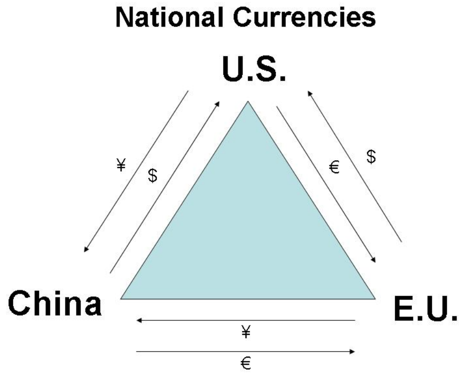
With this calibration, the model can endogenously generate currency portfolios and quantities traded for a given circulation pattern. Under each candidate equilibrium, the buyer's portfolio problem will determine quantities traded. Real balances can then be calculated using the bargaining solution. The next sub-section discusses the payment arrangements that emerge and uses the calibrated model to illustrate the main theoretical predictions.

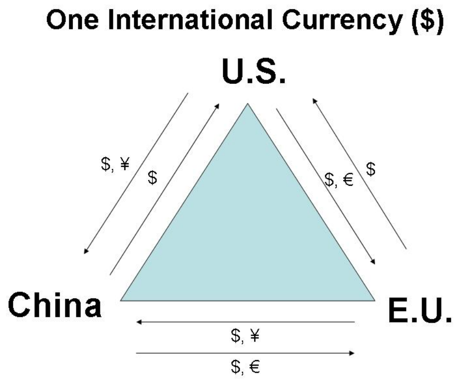
5.3 System of World Payments

In the baseline analysis, it is assumed that government sellers from each country only accept its domestic currency. As a result, all currencies will be valued in equilibrium. Note that even with legal restrictions on payment patterns in government transactions, all private citizens are free to use any asset in their portfolio, which does not rule out an international role for any currency.

The calibrated model yields two types of payment patterns: *(i)* national currency circulation, and *(ii)* one international currency (the dollar). Since the cost of accepting a domestic currency is zero, it is not surprising to see that an equilibrium with national currencies exists. Hence the same forces which lead to convergence on many national currencies leads the world to converge on a single international money, the dollar.

Given that this payment pattern reflects the current state of affairs, what factors may cause





hegemony to shift? The theory implies that a sufficiently large reduction in information costs, such as an improvement in transaction technologies, can induce a switch from a world with one international currency, the dollar, to a world with two international currencies, the dollar and the euro. Figure 3 illustrates this effect and summarizes how circulation patterns depend on the cost of accepting euros ψ^{EUR} and the size of the Eurozone, n_{EUR}. When it is too costly to accept euros, there will be an equilibrium where national currencies circulate and an equilibrium where only the dollar is international. A sufficiently low ψ^{EUR} however leads to a case where two international monies emerge: in this case, both the dollar and euro circulate abroad. As n_{EUR} increases, it becomes more likely to meet a citizen from the Eurozone, a fraction of which only accept euros, which increases the threshold level of ψ^{EUR}. It therefore becomes more likely that the dollar shares the international spotlight with the euro as the economic power of the Eurozone increases.

The theory also implies that changes in inflation is a channel through which monetary policy

Figure 3: **Circulation Patterns and Information Costs**

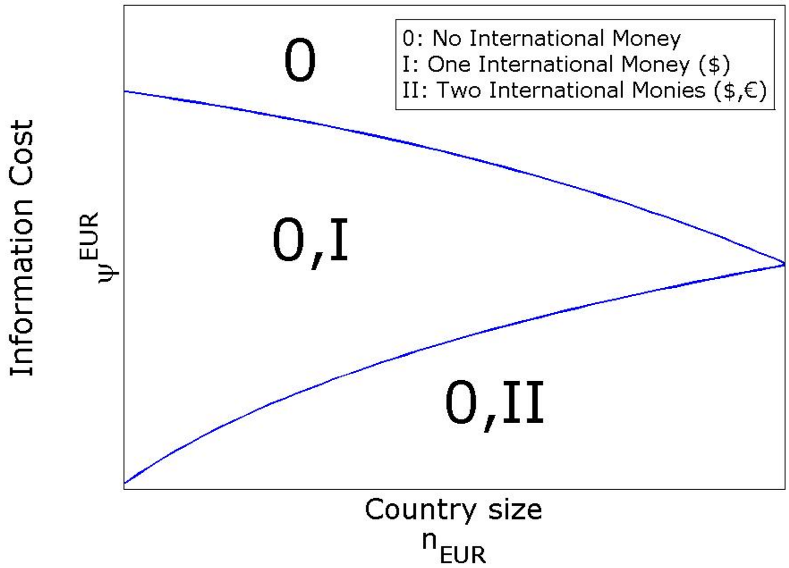


Figure 4: **Costly Information Acquisition and Inflation**

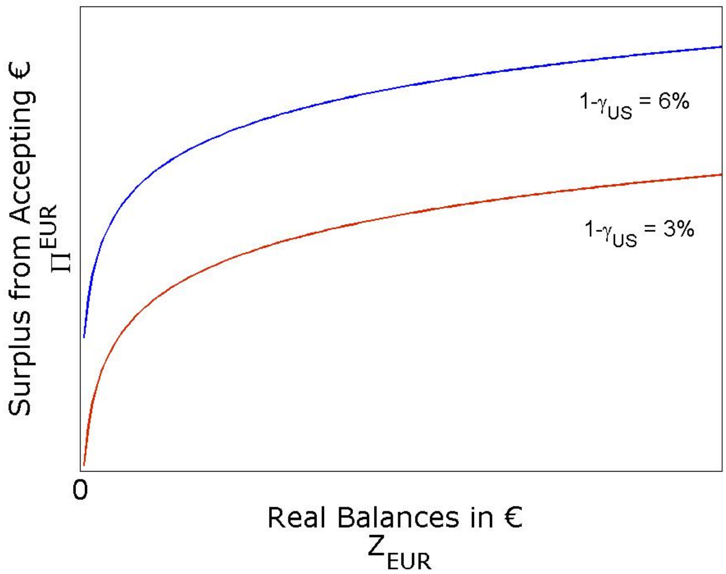
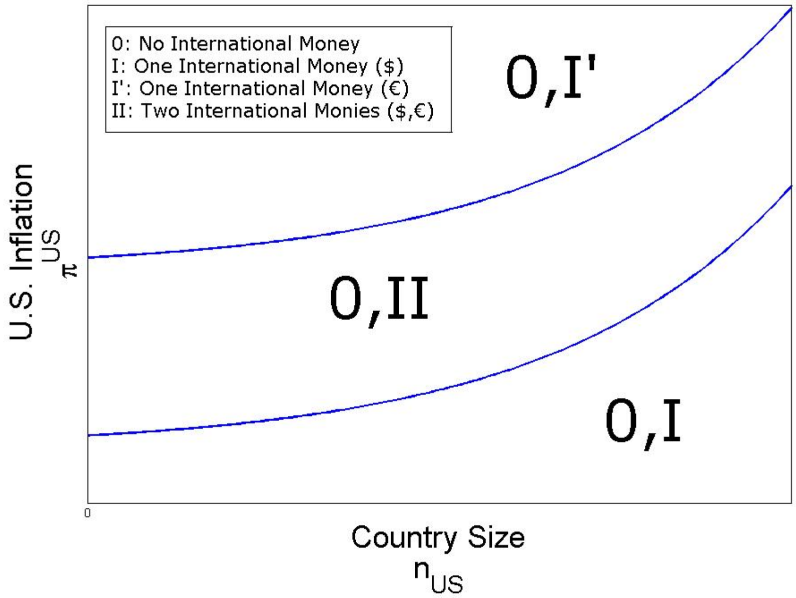


Figure 5: **Circulation Patterns and Inflation**



can influence prices and hence circulation patterns. Figure 4 shows that the expected surplus from accepting euros shifts up when inflation in the U.S. increases from 3% to 6%. Due to complementarities in optimal choices, this can lead to multiple circulation patterns, including a regime where the dollar is international and one where both dollars and euros are international.

Figure 5 illustrates inflation's effect on international currency circulation. When U.S. inflation is low, there will be an equilibrium where only the dollar is internationally used. As inflation is in an intermediate range, it now becomes costlier to hold dollars which increases the expected surplus from accepting euros. Sellers in the U.S. substitute away from dollars into euros, leading the euro to circulate alongside the dollar in international transactions. If U.S. inflation increases further, it may be possible that sellers from Europe no longer hold dollars and switch to solely accepting euros. In this case, euros circulate abroad while the dollar loses its international status and only circulates at home. Also notice that international currency use may be a purely expectational phenomenon: an international money may emerge even if the fundamentals of the economy are consistent with an equilibrium with national currency use.

6 Welfare

(This section is preliminary and incomplete.)

For the scenarios described so far, it is possible to rank each equilibria in terms of welfare. Under a given regime Ω, welfare in each region $i \in \{A, B, C\}$ can be decomposed as the sum of net consumption in the CM, seigniorage transfers, and private agents' match surpluses net of information costs for all combinations for currencies k that can be accepted:

$$V(z^i, \Omega) = U(x^*) - x^* + T_i + \sum_s \mu_{is} \sum_k \rho_s^k [u(q_s^k(\Omega)) - q_s^k(\Omega) - \psi_s^k] + \sum_s \nu_{is} [u(q_s^k(\Omega)) - q_s^k(\Omega)]$$

Welfare therefore depends on three factors: consumption of the general good net of production, seigniorage transfers, and utility from trading in the decentralized markets. Since calibrated values of gross inflation rates all exceed unity, $T_i \equiv (\gamma_i - 1)\phi_i M_i$ is the positive seigniorage revenue from monetary injections, which is increasing with the inflation rate γ_i.[39] The theory therefore implies two distinct sources of the welfare benefits of having an international currency. First is the increase in welfare due to increased seigniorage revenues, namely in the demand for domestic real balances by foreigners. Second is the change in welfare due to increased trade. When a currency becomes international, it is more widely used in denominating transactions which increases trade.

Table 7: **Welfare Rankings for Three-Region Model**

	U.S.	Eurozone	China
National currencies	3	3	3
One international currency (dollar)	1	2	1
Two international currencies (dollar, euro)	2	1	2

Table 7 summarizes rankings of equilibria in terms of welfare, where 1 is the best outcome, followed by 2 and 3. From the perspective of the U.S., the best scenario is one where it issues the sole international medium of exchange. Compared with a world with national currencies, welfare for the U.S. is higher since international currency use increases seigniorage revenues and opens up more transactions with the Eurozone, resulting in more trade. From an economic point of view, the usual logic is that money follows trade. When bilateral trade with a certain country is important, settling payment in its national currency is attractive since it finds ready domestic sellers willing to accept it. Here however, trade also follows money. By accepting the dollar, a seller from Europe opens up trade with the U.S. that would not have occurred if the dollar had not been accepted.

[39] Since seigniorage revenues are lump-sum transfers, they do not affect quantity traded q_s^k but will affect hour worked and welfare.

The next best case for the U.S. is when both the dollar and euro are internationally used. Not only do domestic residents have to incur the cost to accept the euro, but since there are now more people that can use the euro, less people are using the dollar, resulting in less seigniorage revenues and less trade.

For the Eurozone, the best scenario is one where it shares the international spotlight with the dollar. Although it must make seigniorage transfers to the U.S. since there is still private demand for dollars, this is outweighed by the seigniorage revenues it receives from U.S. residents and the increase in trade. The next best case is when there are no international monies, followed by a world where only the dollar is international.

6.1 Welfare Benefits of an International Currency

Pushing further, one may also consider how large are these gains and losses? To study the welfare effects of transitions from one equilibrium to another, I follow the approach in Lucas (1987) and ask how much consumption agents demand, or are willing to give up, as compensation to move from regime Ω to another regime Ω'. This exercise is particularly relevant as there has been great interest in evaluating the consequences of transferring the dollar's premier international status to the euro or yuan.[40]

In steady state, for any equilibrium given by Ω, ex-ante welfare can be written as

$$V(z^i, \Omega) = U(x^*) - x^* + T_i + \sum_s \mu_{is} \sum_k \rho_s^k [u(q_s^k(\Omega)) - q_s^k(\Omega) - \psi_s^k] + \sum_s \nu_{is}[u(q_s^k(\Omega)) - q_s^k(\Omega)]$$

where $q(\Omega)$ is the equilibrium value for q given Ω. Suppose we move from Ω to a different equilibrium Ω', but also adjust consumption of all goods x and q by a common factor Δ. The amount $1 - \Delta$ then measures the percentage gain, or loss if $1 - \Delta < 0$, of consumption faced by the agent each period. Adjusted or compensated welfare then becomes

$$V_\Delta(z^i, \Omega') = U(x^*\Delta) - x^* + T_i + \sum_s \mu_{is} \sum_k \rho_s^k [u(q_s^k(\Omega')\Delta) - q_s^k(\Omega') - \psi_s^k] + \sum_s \nu_{is}[u(q_s^k(\Omega')\Delta) - q_s^k(\Omega')].$$

The compensating variation value $1 - \Delta$ that solves $V_\Delta(z^i, \Omega') = V(z^i, \Omega)$ is then the welfare benefit or cost of moving from regime Ω to Ω'. If $1 - \Delta > 0$, agents are indifferent between being in Ω and being in Ω', with consumption reduced by $1 - \Delta$ percent. Equivalently, agents are willing to give up $1 - \Delta$ percent of consumption to be in regime Ω' rather than regime Ω.

[40] Kannan (2009) provides a similar study of the welfare benefits of an international currency. However cash-in-advance constraints prevent agents from using certain currencies in their portfolio, which will affect welfare estimates.

Table 8 summarizes consumption equivalent welfare changes for transitions in the equilibria of interest. Regime 0 denotes national currency use, regime I is where the dollar is international, and regime II is where the dollar and euro share the international role.

Table 8: **Welfare Changes in Consumption Equivalent Terms** (%)

	$\gamma_{US} = 1.03$ 0 to I	$\gamma_{US} = 1.05$ 0 to I	$\gamma_{US} = 1.06$ 0 to II	$\gamma_{US} = 1.10$ 0 to II
$1 - \Delta_{US}$	2.97	3.20	1.96	1.72
$1 - \Delta_{EUR}$	0.23	0.09	1.82	1.13
$1 - \Delta_{CHN}$	0.36	0.21	0.19	0.08

For the U.S., the welfare benefit of having the dollar as the sole international currency is about 2.97% of consumption. This gain derives from two sources. The first source is from increased seigniorage revenues from foreigners. The second source however comes from international exchange: due to the increased acceptability of the dollar, there is now more surplus from international transactions in the decentralized markets. As more people use the dollar, its value goes up, which increases the amount of goods that can be purchased for a given unit. Both the Eurozone and China benefit from using the dollar, owing to increased trade. The gain is larger for China due to a lower inflation tax (3%) than its domestic level (5%). Although Europe now must pay a higher inflation tax in a fraction of its transactions, this does not outweigh the increased surplus from more trade with the U.S. Column 3 shows that if U.S. inflation rises from 3% to 5%, welfare in the U.S. increases to 3.20% of consumption due to more seigniorage revenue abroad. However welfare in the Eurozone and China falls due to the higher inflation tax .

The analysis also shows that for the world as a whole, the welfare benefits are less when there are two international currencies. For the U.S., the gain from having two international monies relative to having no international money is about 1.96% of consumption, which is slightly more than the gain for the Eurozone. On net, having two international currencies results in more trade surplus, though this increase is lower for the U.S. and China relative to a world with one international currency.

The welfare gains of international currency use in this paper are larger than previous estimates. Portes and Rey (1998) report that the gains from increased seigniorage revenue for the U.S. is about 0.2% of GDP. Kannan (2009) estimates that the welfare benefit of a currency's increased international role is 1.17% of consumption, compared to a gain of close to 3% for the U.S. and 1.82% for the Eurozone in this paper. The larger estimates in this paper arise from the complementarities between money and trade owing to the decisions of private citizens to accept foreign currency. These complementarities and network effects are larger than in Kannan (2009) which restricts all local trade to be conducted in local currency. Since all previous welfare estimates were based on models with cash-in-advance constraints and restrictions on asset portfolios, the way in which liquidity is endogenized matters and can be crucial for welfare calculations.

Overall, which currency emerges as international is also important for welfare. All else equal, the world benefits from having one international liquidity provider, rather than many, in order to save on information costs. These costs coordinate agents on a single international currency, and in the words of Kindleberger (1967), "world efficiency is enhanced when all countries learn the same language." However, due to coordination failures, persistence in currency use may induce the world economy to deviate from its optimal outcome.

7 Conclusion

This paper constructed a two-country, two-currency search model to investigate the role of information costs and government transaction policies on international payment systems. The analysis revisited classical issues in international monetary economics, such as the emergence of international currencies, the effect of inflation on dollarization, and the determination of exchange rates. Instead of assuming the payments used in each country, citizens' acceptance decision is made endogenous through an information acquisition problem, in a similar spirit as Lester, Postlewaite, and Wright (2011). Further, government transaction policies are introduced to examine how certain policies— namely ones which favor the use of a country's national money— affect private agents' acceptance decisions and hence the set of equilibria. Fairly innocuous policies of the kind considered ended up implying the connections observed in practice between currencies and countries.[41]

Recent events have shown that the future international status of the dollar— and the international monetary system more generally— is uncertain. Any shift from the current regime will affect exchange rates and the composition of portfolios, things that many reduced-form models take as given. In sharp contrast, this paper offers a new analytical framework to analyze the effects of switching from a hegemonic monetary system, dominated by the dollar, to a more multi-polar regime. The theory has shown that as the composition of the global economy change, payment patterns may change as well, which affects welfare across countries. The quantitative analysis has shown that the data favors a regime with national currencies and also one where the dollar is internationally dominant. However, a reduction in information costs or increase in U.S. inflation can generate a shift to a bipolar world where the dollar shares international supremacy with the euro.

For several decades, the dollar has served as the dominant medium of exchange and unit of

[41] Wallace (1996) comments that to have more realistic implications, it is desirable for models of international currencies to incorporate the type of government policies examined in this paper.

account for international trade. Due to inertia, the theory implies that it is very difficult to dislodge the incumbent currency, whose use is associated with low information costs. Even so, the theory is also consistent with a switch to a parallel invoicing system that includes the euro if information costs fall and if agents expect others to start using it as well. This brings to mind the shift in the interwar period where the dollar and the pound sterling shared the international role, from a scenario where the pound reigned supreme prior to World War I. Just as the U.S. now has to share the world stage with other economies, the dollar may have to make room for other international currencies.

Finally, the framework presented in this paper need not have a literal interpretation as multiple countries and currencies. It is also applicable to many historical settings, such as ante-bellum United States during the National Banking Era. Recognizability and counterfeiting concerns were pervasive issues, with widespread circulation of multiple bank notes, of which quality was uncertain. The different regions of the model could be interpreted as different state banks, each issuing its own note. I consider this application in a follow-up paper and delegate similar historical applications to future work.

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