Search Frictions and the Labor Wedge*

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Abstract

This paper assesses whether labor market frictions, in the form of searching and matching, can help explain movements in the labor wedge—the gap between the marginal rate of substitution (MRS) and the marginal productivity of labor in a perfectly competitive business cycle model. Results suggest that those frictions per se are *not* able to explain fluctuations in the labor wedge. However, the introduction of extensive and intensive margin shows that measuring the MRS in terms of total hours artificially introduces procyclicality in the MRS. When the MRS is correctly measured in terms of hours per worker the labor wedge obtained is less variable than the one of the perfectly competitive model. A Frisch elasticity of 2.8, as in most macro models, implies a 20 percent decline in the variability of the labor wedge. A Frisch elasticity closer to micro estimates implies an even higher reduction. Finally, we show that it is possible to measure a strongly procyclical labor wedge as in CKM (2007) even if the actual data generating process does not have any labor wedge but has search frictions that allow for movements in both labor margins.

Key words: Labor Market Search; Business Cycle Accounting; Labor Wedge

JEL classification: E24; E32; J64

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1 Introduction

Real business cycle (RBC) models (i.e. Kydland and Prescott, 1982) impose a strong discipline on the choice of consumption and hours, both intertemporally and intratemporally. In these models, the optimal choice of hours is determined in equilibrium such that the marginal rate of substitution between consumption and leisure (mrs) is equal to the marginal productivity of labor (mpl). However, the data tell that there is a substantial *wedge* between these two quantities that strongly co-varies with the economic cycle. In their seminal work Chari Kehoe and McGrattan (2007) (henceforth CKM) conclude that, along with the efficiency wedge, the labor wedge accounts for most of the fluctuations in output, putting it at the center of their business cycle accounting research program.[1]

We interpret this finding as an indication of a significant misspecification of the prototype RBC model as it relates to the labor market. Search and matching frictions (Mortensen and Pissarides 1994, Pissarides 2000) introduce a wedge between the wage and both the mpl and the mrs, providing a natural framework to address misspecification related to the labor market imperfections. It is indeed tempting to think that these type of frictions will induce endogenous movements in the optimal choice of hours that could manifest themselves as labor wedge. In this paper, we first present a model with labor market frictions—in the form of search and matching—that nests a prototype RBC model *a la* CKM; then, we ask whether the labor wedge, as usually measured, could be an artifact of these frictions and if so to what extent.

We find that a model with Nash bargaining between the firm and the marginal worker over hours and wage alter the firm's perceived benefit of an additional hour at the intensive margin (i.e. hours per employed worker), which is valued in terms of additional marginal output per worker. However, given that search frictions are internalized at the wage bargaining stage the equation that determines the labor wedge is not affected by search frictions directly or explicitly. Nevertheless, since fluctuations in total hours can now be attributed to both the intensive and extensive margin, the marginal rate of substitution, and thus the labor wedge,

[1] The business cycle accounting research program has the goal of identifying promising modeling avenues for dynamic general equilibrium models by measuring the "discrepancy" between the data and a prototype real business cycle model. CKM identify four wedges: the efficiency, labor, investment and government consumption wedge. The labor and efficiency wedge are considered the most important, suggesting that macro models that would like to explain real macro fluctuations should pay more attention to understanding what type of frictions could manifest themselves as these wedges.

differ from the one implied by the prototype RBC model. It turns out that the modification is in the right direction, that is the labor wedge we obtain is less variable and procyclical than the prototype labor wedge. This result is sensitive to the exact parameterization of the labor supply elasticity. We find that, for instance, when the Frisch elasticity is relatively high, such as 2.8, as in most macro models, we can get up to 20 percent decline in the variability of the labor wedge—similarly, we find a reduction in the procyclicality. This result is even stronger, a 40 percent reduction, for Frisch elasticities that are more consistent with the micro estimates.

We complement our results with a numerical exercise where we treat the search model as the data generating process and describe the behavior of the labor wedge that an econometrician would recover in doing business cycle accounting. We show that, even though theoretically there is no labor wedge in the simulated data, one can falsely measure a significantly procyclical and variable labor wedge if the underlying search frictions and the explicit distinction between the intensive and the extensive margin is ignored in the measurement. About 15 percent of the relative variation in the labor wedge and all its comovement with output and total hours could be explained by this misspecification.

The next section discusses the related literature, especially that on the business cycle accounting and the labor wedge. Section 3 presents the idea behind the business cycle accounting exercise in CKM using US data, and puts our following exercise in a context. Section 4 presents an extension of the prototype RBC model with search frictions and discusses how search frictions imply a different labor wedge. Section 5 shows, quantitatively, how search frictions alter the measured wedge, first by using the US data and focusing on only one equilibrium condition, then by using the model generated data and analyzing the behavior of labor wedge in these simulations. Section 6 concludes.

2 Related Literature

This paper is part of the vast literature that studies labor market imperfections in connection with the business cycle, such as in Mortensen and Pissarides (1994), Cole and Rogerson (1999), and Shimer (2005), among others. However, the extension of the standard growth model we use is most closely related to Andolfatto (1996) and Merz (1995, 1999), which embed search frictions into an otherwise standard RBC model.

The focus on the labor wedge makes the paper naturally related to the recent literature on business cycle accounting, which, in different forms, dates back before CKM.[2] For example, Hall (1997) identifies variations in the marginal rate of substitution as an important element to explain aggregate fluctuations. Several other studies in the literature focused on the same equilibrium condition in the labor market and provided different interpretations of it, ranging from changes in labor market institutions, competitive structure of the economy, price-wage markup, changes in regulation, and tax policy (see in particular, Cole and Ohanian 2002, Gali, Gertler and Lopez-Salido 2007, Mulligan 2002, Rotemberg and Woodford 1991 and 1999). Chang and Kim (2007), on the other hand, argues that the apparent distortion in the labor market clearing condition as measured in the aggregate data might be partly due to aggregation bias. They show that a heterogenous-agent economy with incomplete capital markets and indivisible labor can generate this observed wedge. Arseneau and Chugh (2010) also anlayze a general equilibrium matching model with distortions that map into a measured labor wedge. Even though the focus is on optimal tax policy, they show that, as in our paper, labor wedge takes two different forms with matching frictions, one intratemporal and one intertemporal.

More recently, Blanchard and Gali (2010), Cheremukhin and Restrepo-Echavarria (2010) and Shimer (2009, 2010) focus on the variation in the labor wedge. Shimer (2009) reviews the literature and makes a case for focusing the attention on the labor wedge, as it is relatively immune to how the model environment is specified and the expectations are formed. Cheremukhin and Restrepo-Echavarria (2010) lays out an RBC model with search frictions and argues that most of the variation in the labor wedge is attributable to the residual shock to matching efficiency, rather than variations in job destruction or impediments to the bargaining process. Both Blanchard and Gali (2010) and Shimer (2010), differ in their formulation of the search frictions from us, and derive a neutrality result. This result basically implies that fluctuations in the unemployment rate are independent of the aggregate productivity. In Blanchard and Gali (2010), this is due to fact that recruitment of firms is not affected by aggregate productivity. As Shimer (2010) argues, this follows when one assumes that recruitment is only

[2] One can trace the basic idea behind this exercise to early work in the RBC literature as in Prescott (1986) or Ingram, Kocherlakota and Savin (1994).

labor-intensive, not good intensive. We favor a more traditional approach, as in Andolfatto (1996) and Merz (1995) and model recruitment as a good-intensive technology and hence firms respond to productivity shocks by increasing recruitment during booms. We find it reasonable to assume that, at least partially, firms cannot just bear the cost of recruitment by costlessly switching workers from production to recruitment in response to productivity shocks.

3 The prototype real business cycle model

In this section we present a prototype business cycle model with perfectly competitive factor markets. As we will see, when the model is taken to the data, the theoretical prediction that the marginal rate of substitution between consumption and leisure has to be equal to the marginal productivity of labor does not hold; following the literature, we name this discrepancy *labor wedge*.

The economy is populated by a continuum of mass 1 of identical households solving a dynamic optimization problem over the choices of consumption (c_t), savings (x_t) and hours worked (h_t). Markets are complete and household utility function is separable in consumption and leisure. Households maximize their expected utility[3]

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + \psi G(h_t)], \qquad 0 < \beta < 1, \tag{1}$$

where β is the subjective discount factor, c_t is consumption and h_t is per capita hours worked in period t. Households earn wages w_t and receive rental income, r_t, from capital that they rent out to firms. They can invest x_t units each period which adds to new capital next period, k_{t+1}, net of depreciation, δ. Wedges operate as a tax, following CKM we introduce a tax on investment and labor earnings, $[1+\varepsilon_{x,t}]$ and $[1-\varepsilon_{l,t}]$, respectively. Finally, in each period, households also receive a lump sum transfer, T_t, from government. Let $W^h(\cdot)$ and $W^f(\cdot)$ denote the value functions for households and firms respectively. Then, we can summarize the household optimization problem as follows

[3] Usual restricions on the functional form apply, that is $U_c(c) > 0$, $U_{cc}(c) \leq 0$, $G_h(h) < 0$, and $G_{hh}(h) \leq 0$ as well as usual Inada conditions. In addition, we normalize $G(0) = 0$ and impose $G(h) < 0, \forall h > 0$.

$$W^h(\omega_t) = \max_{c_t, k_{t+1}, x_t, l_t} \{U(c_t) + \psi G(h_t) + \beta \mathrm{E}_t W^h(\omega_{t+1}|\omega_t)\} \tag{2}$$

$$s.t.\ c_t + x_t [1 + \varepsilon_{x,t}] = w_t [1 - \varepsilon_{l,t}] h_t + r_t k_t + T_t \tag{3}$$

$$k_{t+1} = (1 - \delta) k_t + x_t \tag{4}$$

where $\omega_t = \{k_t, \Omega_t\}$ is the set of individual and aggregate state variables, respectively. Optimality requires the net wage to be equal to the marginal rate of substitution between consumption and leisure, mrs; given that all households are identical, this condition becomes the relevant labor supply schedule

$$[1 - \varepsilon_{l,t}] w_t = -\psi G_h(h_t) / U_c(c_t) \equiv mrs_t.$$

Firms hire workers and capital to produce according to a neoclassical production function $\varepsilon_{z,t} f(k_t, l_t)$, where $\varepsilon_{z,t}$ is the exogenous productivity shock (or efficiency wedge) and l represents total hours demanded by the firm, the product of hours per worker h and employment n. For future comparison we also write down the static firm optimization in a consistent way.

$$W^f(\omega_t) = \max_{k_t, h_t, n_t} \{\varepsilon_{z,t} f(k_t, h_t n_t) - w_t h_t n_t - r k_t\} \tag{5}$$

Optimality requires the wage to be equal to the marginal productivity of labor,given that all firms are identical this condition becomes the relevant labor demand schedule[4]

$$w_t = \varepsilon_{z,t} f_l(k_t, l_t) \equiv mpl_t.$$

Finally, we assume that the government runs a balanced budget in each period such that tax revenues are equal to government spending $T_t + x_t \varepsilon_{x,t} + w_t l_t \varepsilon_{l,t} = \varepsilon_{g,t}$.

In equilibrium we assume that the labor market is always at full employment, $n_t = 1$; hence, we can replace hours per worker with total hours $h_t = l_t$.[5] The following conditions describe

[4]Notice that once the marginal productivity of labor is equal to the wage the firm is also indifferent between the extensive and intensive margin.

[5]It is worth noting that the household problem could be recast in terms of a big family that equalizes consumption across its members and maximizes the momentary utility function $U(c_t) + \psi n_t G(h_t)$. In the

the competitive equilibrium

$$
\begin{aligned}
U_c(c_t)\,[1+\varepsilon_{x,t}] &= \beta \mathrm{E}_t[U_c(c_{t+1})[\varepsilon_{z,t+1} f_k(k_{t+1}, l_{t+1}) + (1-\delta)\,(1+\varepsilon_{x,t+1})]|\omega_t] && (6)\\
mrs_t(l_t) &= -\psi G_h(l_t)/U_c\,(c_t) = mpl_t\,[1-\varepsilon_{l,t}] && (7)\\
\varepsilon_{z,t} f(k_t, l_t) &= c_t + k_{t+1} - (1-\delta)k_t + \varepsilon_{g,t} && (8)
\end{aligned}
$$

along with a set of unique realizations for the vector of exogenous process $\varepsilon_t = [\varepsilon_{x,t}, \varepsilon_{l,t}, \varepsilon_{z,t}, \varepsilon_{g,t}]$. Notice that the equation that describes the labor market equilibrium is static and is the only one where $\varepsilon_{l,t}$ appears. Under various functional forms—and once a parametrization is chosen—using data on output (y_t), total hours (l_t), and consumption (c_t) is enough to pin down a unique value for the labor tax rate $\varepsilon_{l,t}$. If the actual labor tax rate does not vary substantially at business cycle frequency, $[1-\varepsilon_{l,t}]$ can be interpreted as a discrepancy between the model and the data: a labor wedge (Shimer 2009). While we do not attempt to draw causal implication as in CKM, we analyze the statistical properties of the labor wedge in relation to business cycles. Figure (1) plots $[1-\varepsilon_{l,t}]$ over the period 1959:I to 20010:III. The labor wedge seems to have a low frequency movement, which might be explained by changes in the taxes (see for instance, Ohanian, Rogerson and Raffo, 2008) or changes in the composition of the workforce and the resulting imperfect household aggregation over-time (Cociuba and Ueberfeldt, 2010). However, at the business cycle frequency, there is a lot of variation in the measured labor wedge that is highly unlikely to be explained by high-frequency changes in labor tax. Moreover, the figure confirms the well known result that the labor wedge, $[1-\varepsilon_{l,t}]$, is pro-cyclical (falling during recessions) and is positively correlated with per capita hours worked.

In the next section we explore how a model with frictions in the labor market and with an extensive and intensive labor margin can help explain the observed discrepancy between the marginal rate of substitution and the marginal productivity of labor.

budget constraint labor income would be $(1-\varepsilon_t)w_t n_t h$ with the additional constraint $n_t \leq 1$. In general, the full emplyment constraint would be binding and, thus, we would have the same equilibrium conditions shown in the text. In the next section we will make explicit use of this setting.

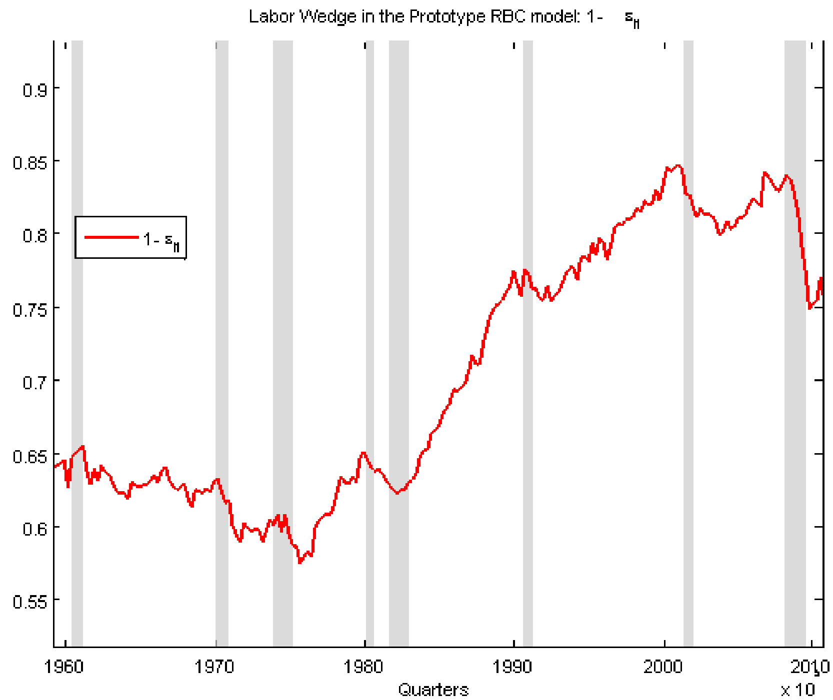

Figure 1: Historical realizations of $[1 - \varepsilon_{lt}]$ according to measurement equation (7) and data on y_t, c_t, and l_t over the period 1959:1 to 2010:III. Shaded areas indicate NBER recession dates.

4 Introducing Search Frictions and Employment Fluctuations

The model economy is almost identical to the one presented in the preceding section except for the presence of search and matching frictions in the labor market and movements in both the intensive and the extensive margin for labor. In other words, even though goods and capital are exchanged in perfectly competitive markets, labor market imperfections allow for unemployment. The model is a decentralized version of Andolfatto (1996) and Merz (1995) and is laid out such that it looks similar to the prototype real business cycle (RBC) model previously described. Search frictions in the model are summarized at an aggregate level by an aggregate matching function and require households and firms to determine employment contracts through Nash bargaining. We first provide the details of the problem for households and firms and then discuss the Nash bargaining outcome.

4.1 Households

The economy is populated by a continuum of infinitely-lived worker-households distributed uniformly along the unit interval implying a constant labor force normalized to one. At any point in time, only a mass $n_t \leq 1$ of households is employed while the remaining $1-n_t$ households are unemployed and searching for a job. Each household has the same utility function as in the previous section. Markets are complete providing perfect insurance against unemployment risk, hence, consumption is equalized across households (Andolfatto 1996 and Merz 1995). It is easy to prove that we can recast the problem in terms of a representative household that maximizes the sum of each individual household expected utility

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t [U(c_t) + \psi n_t G(h_t)], \qquad 0 < \beta < 1. \tag{9}$$

The objective function of the representative household differs from the one of the prototype model because it explicitly takes into account the possibility that employment may vary over time—when $n_t \equiv 1$ we are back to the perfectly competitive model.[6] Each unemployed worker exerts some effort, e_t, to find a job, which costs $c(e_t)$ units of resources.[7] As a result, the budget constraint of the representative households can be written as

$$c_t + x_t [1 + \tau_{x,t}] + c(e_t)(1 - n_t) = w_t [1 - \tau_{l,t}] n_t h_t + r_t k_t + T_t \tag{10}$$

Note that τ_l is the analog of ε_l in the prototype model, i.e. the fraction of labor earnings net of taxes.

Given the complete markets assumption it is easy to write the law of motion for aggregate capital

$$K_{t+1} = (1 - \delta)K_t + X_t \qquad 0 \leq \delta \leq 1. \tag{11}$$

When households solve their problems, they are going to take wage, w_t, and hours worked h_t as given, as they will be determined through Nash Bargaining (see next section). We assume that trade in the labor market is mediated by an aggregate matching function that determines

[6] Notice that we have made use of the normalization $G(0) = 0$. Hence the term $(1 - n_t)\psi G(0)$ does not explicitely appear in the objective function.

[7] Cost of search function, $c(e)$, is assumed to be strictly convex and increasing in e.

the number of jobs formed in each period as a function of the number of job vacancies, V_t, and the aggregate search effort of the household/workers, $(1 - N_t) E_t$

$$M_t = \tau_{\chi,t} V_t^{\gamma} \left[(1 - N_t) E_t\right]^{1-\gamma} \tag{12}$$

where $0 < \gamma < 1$, $\tau_{\chi,t}$ is the period-t realization of a process that governs the efficiency of matching, and the following intuitive restriction applies $M_t \leq \min\{V_t, 1 - N_t\}$. Using the definition of the matching function, we can write the probability of finding a job for an unemployed worker as $p_t = \frac{Mt}{(1-N_t)E_t}$, which is taken as given at household level.

Job matches that are formed in a period are assumed to become productive in the following period. That is, they only increase employment with a one-period lag. As jobs are created, they are also being destroyed. Letting σ denote the period-t fraction of existing jobs destroyed, the law of motion for employment is given by the expression

$$N_{t+1} = (1 - \sigma) N_t + M_t, \tag{13}$$

where $\sigma \in [0, 1]$. At the household level, employment n_t evolves endogenously according to the following, slightly modified equation of motion

$$n_{t+1} = (1 - \sigma) n_t + p_t(1 - n_t)e_t. \tag{14}$$

Let's define the aggregate productivity shock, $\tau_{z,t}$ as the analog of $\varepsilon_{z,t}$. We can finally write down the representative household problem recursively

$$W^h(\omega_t^h) = \max_{x_t, e_t}\{U(c_t) + \psi n_t G(h_t) + \beta \mathrm{E}_t W^h(\omega_{t+1}^h | \omega_t^h)\} \tag{15}$$

$$\begin{gathered} s.t.\ c_t + x_t\left[1 + \tau_{x,t}\right] + c(e_t)(1 - n_t) = w_t\left[1 - \tau_{l,t}\right] n_t h_t + r_t k_t + T_t \qquad (16) \\ n_{t+1} = (1 - \sigma) n_t + p_t(1 - n_t)e_t \\ k_{t+1} = (1 - \delta)k_t + x_t \end{gathered}$$

taking $w_t = w(\Omega_t)$, $r_t = r(\Omega_t)$, $p_t = p(\Omega_t)$, and equations of motion for aggregate state variables, K_t and N_t as given. For notational simplicity, let $\omega_t^h = \{k_t, n_t, \Omega_t\}$ and $\Omega_t = \{\tau_t, K_t, N_t\}$ denote individual and aggregate state variables for the household respectively, with $\tau_t = [\tau_{l,t}, \tau_{x,t}, \tau_{z,t}, \tau_{g,t}, \tau_{\varkappa,t}]$ being the vector of exogenous processes analogous to ε_t. This optimization problem leads to two conditions that determine the optimal savings and search effort recursively for the large household (details are in the appendix):

$$
\begin{aligned}
U_c(c_t)\left[1+\tau_{x,t}\right] &= \beta \mathrm{E}_t\left[U_c(c_{t+1})(r_{t+1}+(1-\delta)\left[1+\tau_{x,t+1}\right])|\omega_t^h\right]. && (17)\\
U_c(c_t)c_e(e_t)/p_t &= \beta \mathrm{E}_t\left[\begin{array}{c} U_c(c_{t+1})[w_{t+1}\left[1-\tau_{l,t+1}\right]h_{t+1}+c(e_{t+1})]+ \\ \psi G(h_{t+1})+\frac{U_c(c_{t+1})c_e(e_{t+1})}{p_{t+1}}\left(1-\sigma-p_{t+1}e_{t+1}\right)|\omega_t^h \end{array}\right] && (18)
\end{aligned}
$$

The first Euler equation is the standard consumption Euler equation. The second Euler equation determines the households' optimal search behavior. The left hand side in (18) is the expected marginal cost of search for the households in current consumption units, which should be equal to the expected marginal gains from search on the right hand side. If search is successful, the household expects to get utility from the net wage payments, $w_{t+1}\left[1-\tau_{l,t+1}\right]h_{t+1}$, and from economizing on future search costs, $c(e_{t+1})$, and to get disutility from working $G(h_{t+1})$. The final term in brackets represents the net future benefit arising from the expected persistence of a job match.[8] Note that, the household decision for optimal saving is exactly identical to the one in the competitive model (equation 6). Hence, all things equal, labor market frictions do not affect the wedge between marginal utility of consumption and the expected real rate of return on capital in the model. Instead, labor market frictions affect the search behavior, hence the labor supply decision. Next we turn to the demand side and describe firms' problem.

4.2 Firms

As in the previous section firms operate a constant returns to scale production function, $\tau_{z,t}f(k_t, n_t h_t)$ However, while capital is rented in a perfectly competitive market, firms must

[8] Given that any single current-period match survives with probability $1-\sigma$, households' expected utility will increase simply by reducing expected future recruiting costs by the quantity $\frac{U_c(c_{t+1})c_e(e_{t+1})}{p_{t+1}}(1-\sigma)$. The second term in this sum, $-p_{t+1}e_{t+1}\frac{U_c(c_{t+1})c_e(e_{t+1})}{p_{t+1}}$, represents the reduction in the future job-finding rate, due to the current depletion of the unemployment stock.

undergo a costly search process before jobs are created and output is produced. For each job vacancy created in period-t, firms pay κ units of output resulting in period-t "vacancy-posting" costs of κv_t. Jobs must be posted as vacancies before they can be filled. We assume that vacancies adjust in equilibrium such that the value of an additional vacancy is driven to zero. We are going to approach the firms' problem in two steps. In the first step, firms decide how much capital to rent and how many jobs to create taking the rental rate r_t, aggregate state variables, the labor contract $\{w_t, h_t\}$, and the probability of filling a vacancy, q_t, as given[9]

$$W^f(\omega_t^f) = \max_{k_t, v_t, n_{t+1}} \{\tau_{z,t} f(k_t, n_t h_t) - w_t n_t h_t - r_t k_t - \kappa v_t + \tilde{\beta}_t \mathrm{E}_t W^f(\omega_{t+1}^f | \omega_t^f)\} \tag{19}$$

$$\text{such that } f(k_t, n_t h_t) = k_t^{\alpha} (n_t h_t)^{(1-\alpha)} \tag{20}$$

$$n_{t+1} = (1-\sigma)\, n_t + q_t v_t$$

where $\omega_t^f = \{k_t, n_t, \Omega_t\}$, $r_t = r(\Omega_t)$, $w_t = w(\Omega_t)$ with $\Omega_t = \{\tau_t, K_t, N_t\}$ and $\tilde{\beta}_t = \beta \mathrm{E}_t \left[\frac{U_c(c_{t+1})}{U_c(c_t)}\right]$. Firms' problem in (19) impliesa set of first order conditions that determine the optimal level of capital stock rented and the number of vacancies posted by firms.

$$\tau_{z,t} f_{k_t}(k_t, n_t h_t) - r_t = 0 \tag{21}$$

$$\frac{\kappa}{q_t} = \tilde{\beta}_t \mathrm{E}_t \left[\tau_{z,t+1} f_{l_{t+1}}(k_{t+1}, n_{t+1} h_{t+1}) h_{t+1} - w_{t+1} h_{t+1} + (1-\sigma)\, \frac{\kappa}{q_{t+1}} | \omega_t^f\right] \tag{22}$$

The first condition is the familiar relation between the rental rate and the marginal product of capital. The second condition determines the optimal number of vacancies posted by firms. It implies that the marginal cost of filling a vacancy should be equal to the marginal benefit, in expectation. Since each vacancy costs κ and the duration of a vacancy is expected to be $1/q_t$, $\frac{\kappa}{q_t}$ gives the expected marginal cost of filling one vacancy. On the right hand side, we see how each additional filled job produces its marginal product next period net of wage payments, while the third term represents the expected saving in terms of future recruitment costs.

[9] From the aggregate matching function in (12), we know that the probability of filling a vacancy must be $q_t = M_t / V_t$.

4.3 Employment Contract and the Nash Bargaining

Since negotiating a wage has an implicit opportunity cost due to search frictions, we need a mechanism to determine the surplus for each contracting party. It is standard in the search literature to use generalized Nash bargaining for this purpose (see for instance, Pissarides 2000). We follow the same approach and assume that each worker's employment contract, $\{w_t, h_t\}$, is determined through Nash bargaining between the firm and the household. In a setting like this, where there are multiple workers within a firm, it is not entirely clear how to formulate the bargaining problem. Fortunately, Stole and Zwiebel (1996) show that bargaining should happen over the marginal surplus for both parties.[10] [11] Assuming that households (firms) have a bargaining power of $1-\lambda$ (λ) the generalized Nash bargaining problem takes the following form[12]

$$\max_{w_t,h_t} W^h_{n_t}(\omega_t)^{1-\lambda} W^f_{n_t}(\omega_t)^{\lambda} \tag{23}$$

where $W^h_{n_t}(\omega_t)$ is the household's net marginal surplus from having one more worker employed given household's optimal behavior, and $W^f_{n_t}(\omega_t)$ is the firm's net marginal surplus from having one more employee given firm's optimal behavior. As long as each party can extract some surplus, i.e. $W^h_{n_t}(\omega_t) > 0$ and $W^f_{n_t}(\omega_t) > 0$, this Nash bargaining problem yields two conditions that determine equilibrium level of hours, h_t, and wage per hour, w_t (details of the solution are presented in the Appendix)

$$\lambda W^h_n(\omega_t) - (1-\lambda)(1-\tau_{l,t})U_c(c_t)W^f_n(\omega_t) = 0 \tag{24}$$

$$U_c(c_t)\left[1-\tau_{l,t}\right]\left[\tau_{z,t} f_l(k_t, n_t h_t) + \tau_{z,t} f_{ll}(k_t, n_t h_t) n_t h_t\right] + \psi G_h(h_t) = 0 \tag{25}$$

Equation (24) is the starting point to determine the hourly wage and basically divides the total surplus of the marginal match among the household and the firm. Notice that $\tau_{l,t}$ exogenously reduces the household's share of surplus and, at the limit $\tau_{l,t} = 1$, the household's surplus is zero $W^h_{n_t} = 0$. Household risk aversion adds a time-varying dimension to the sharing rule: in

[10] For examples see Merz (1995), Beauchemin and Tasci (2008), Fujita and Nakajima (2009), and Elsby and Micheals (2010), among others.

[11] Implicitly, we assume that the contracted hours apply to every worker. However, given that $G(\cdot)$ is convex, it is easy to prove that homogenous hours minimize the overall worker disutility.

[12] We drop the distinction between the state variables for the households and the firms, since in equilibrium, $\omega^h_t = \omega^f_t = \omega_t$.

times of high marginal utility households claim a relatively higher surplus. Equation (25) is the first order condition with respect to hours, as we will see this equation will play a crucial role in the analysis of the labor wedge. Those last two conditions complete the necessary set of equations that fully characterize the equilibrium. Next, we combine these set of conditions to describe the recursive competitive equilibrium.

4.4 Equilibrium

Since all households and firms are identical, in equilibrium, individually efficient allocations coincide with the aggregate, i.e. $k_t = K_t$, $c_t = C_t$, $l_t = L_t$, $n_t = N_t$, therefore the relevant state is Ω_t. Then the competitive equilibrium of this economy is characterized by a list, $k_t(\Omega_t)$, $l_t(\Omega_t)$, $c_t(\Omega_t)$, $n_t(\Omega_t)$, $w_t(\Omega_t)$, $r_t(\Omega_t)$ that satisfy equilibrium conditions implied by household and firm optimization (17-18) and (21-22), Nash bargaining (24) and (25), as well as equations of motion for aggregate states, (11) and (13).[13] It is also possible to pin down the aggregate resource constraint by exploiting the free entry condition for firms which imposes that all the flow profits net of recruitment expenses are exhausted in equilibrium

$$y_t = c_t + x_t + \varepsilon_{g,t} + \kappa v_t + c(e_t)(1 - n_t)$$

Note that, search frictions do not affect the consumption Euler equation and the resource constraint in a significant way.[14] However, fluctuations in the extensive and intensive margin, along with the search decision give rise to additional equilibrium conditions for employment, search effort, e_t, and job vacancies, v_t, that are absent in the prototype business cycle model. For the purpose of this paper, we can focus our analysis on the static equilibrium conditions that describe the labor wedge (25) and (24).

[13] There is a possibility of multiple steady state equilibria in this model due to the complementarities between firms' recruitment effort and workers' search effort. Intuitively, if firms expect that workers will not search as hard, the returns to firms' recruitment will diminish, hence the number of vacancies posted. This will in turn provide workers with the incentive to search less, thereby fulfilling firms' expectations in the first place. However, one can show that given constant returns to scale of the matching function and assuming enough convexity in $c(e)$, we will have a unique steady state equilibrium in this model. We calibrate the model such that $c(e)$ has enough convexity in the numerical exercises.

[14] At least quantitatively for the resource constraint, since most calibrations does not imply big search costs relative to output.

4.5 The Wage Determination

It is possible to derive an explicit wage equation using the equilibrium conditions from household and firm optimization (see appendix for details)

$$w_t h_t = (1-\lambda)\left[\tau_{z,t} f_n(k_t, l_t) + \kappa\frac{(1-\sigma)}{q_t}\right] + \frac{\lambda}{1-\tau_{l,t}}\left[-\frac{\psi G(h_t)}{U_c(c_t)} - c(e_t) - \frac{(1-\sigma-p_t e_t)}{p_t} c_{e_t}(e_t)\right]. \tag{26}$$

The wage bill for an additional worker $w_t h_t$ is a function of the marginal productivity of employment, $\tau_{z,t} f_n(k_t, l_t)$, search frictions and the *extensive* marginal rate of substitution,$-\psi G(h_t)/U_c(c_t)$, which reflects, at the social level, the marginal disutility of an additional worker, $-\psi G(h_t)$, for any given h. It is useful to introduce the *mrs* and *mpl* by noting that $-\psi G(h_t)/U_c(c_t) = mrs_t h_t \eta(h_t)$, with $\eta_t = G(h_t)/[h_t G_h(h_t)] \in [0,1]$, and the marginal productivity of employment $\tau_{z,t} f_n(k_t, l_t) = mpl_t h_t$. We can thus rewrite the wage equation in a compact form

$$w_t = (1-\lambda) mpl_t + \lambda\frac{mrs_t}{1-\tau_{l,t}}\eta_t + \Phi_t/h_t. \tag{27}$$

where $\Phi_t \lesseqgtr 0$ is the only term that involves search frictions explicitly.[15] If we combine the wage equation (27) with the condition on hours, equation (25), we can relate both the mrs_t and the mpl_t to the wage

$$w_t = mpl_t[1-\lambda(1-\eta_t s_t^L)] + \Phi_t/h_t \tag{28}$$

$$w_t = \frac{mrs_t}{1-\tau_{l,t}}\frac{[1-\lambda(1-\eta_t s_t^L)]}{s_t^L} + \Phi_t/h_t \tag{29}$$

where we have introduced the output elasticity to total hours $s_t^L = 1 + \frac{f_{ll}(k_t,l_t)l_t}{f_l(k_t,l_t)} \in [0,1]$, which is approximately the labor share.[16] Those two equations are very instructive and resemble the labor supply and labor demand equations of the perfectly competitive model. However, search frictions coupled with Nash bargaining introduce two time varying wedges between the wage and both the marginal productivity of labor and the marginal rate of substitution. In

[15] In the text we have implicitely defined $\Phi_t \equiv (1-\lambda)\kappa\frac{(1-\sigma)}{q_t} - \frac{\lambda}{1-\tau_{l_t}}[c(e_t) + \frac{(1-\sigma-p_t e_t)}{p_t}c_{e_t}(e_t)]$. Notice that its sign is ambiguos in principle, in fact, we have $\Phi > 0$ when $\lambda = 0$ and $\Phi < 0$ when $\lambda = 1$.

[16] In the search model, due to the search frictions s^L is not exactly the labor share—while it is in the perfectly competitive model.

fact, even if $\tau_{l,t}$ were constant, the wage would not perfectly co-vary with the *mpl* and the *mrs* over the cycle, being affected by movements in the search frictions Φ_t and in the way the match surplus is split. In general, the wage will be between the *mrs* and the *mpl*, however, when λ is small, it might be possible to observe $w_t > mpl_t$. In part this is due to the presence of s_t^L which reduces the firm's perceived benefit of labor. In fact, firms value the benefit of increasing hours worked h across workers in terms of its effect on the marginal productivity of an extra worker (not in terms of the average worker productivity). However, while the cost is still linear in h, the marginal productivity of employment is not: as long as there is curvature in the production function, i.e., $s^L < 1$, the marginal productivity of employment increases with h less than one-to-one.

Finally, it is also interesting to note that the firm bargaining power parameter, λ, is inversely related to the wage and enters symmetrically in both equations (28) and (29). As we will see, this implies that the bargaining power per se does not affect the labor wedge, as long as there is an interior solution of the Nash Bargaining problem, i.e., $\lambda \in (0, 1)$.

In the following section we describe in more detail our labor wedge and compare it with the one of the competitive model.

4.6 The labor wedge

Eliminating the wage from (28) and (29) we have an expression that is analogous to the one of the perfectly competitive model (equation 7).

$$mrs_t = mpl_t \left[1 - \tau_{l,t}\right] s_t^L \tag{30}$$

It is striking that while the bargaining process has created a wedge between *mrs* and *mpl*, search frictions per se do not appear directly in this equation. Mechanically, this is simply because they enter symmetrically and additively in the labor demand and supply equations, hence, when those are equated search frictions perfectly offset and cancel out. The same is true also for λ and η which affect *mrs* and *mpl* exactly in the same proportion. Intuitively, one reason for this result is that the bargaining process internalizes search frictions through the wage rather than hours. In part this may be due to the fact that search frictions are inherently intertemporal—

i.e. it takes time and resources to match unemployed workers with vacant positions—while the labor wedge is inherently intratemporal. The effects of intertemporal frictions are absorbed by movements in the wage and the extensive margin, and not in the labor wedge.

In order to understand the effects of search frictions on the extensive margin, it is crucial to observe that the variation in this margin is governed by mainly two decisions; workers' search effort and firms' vacancy posting decision. Optimality conditions for search effort and vacancies (see equations 18 and 22) along with the law of motion for employment will govern the movements at the extensive margin. Using equations (18) and (22), noting that the terms in expectations are each party's marginal surplus from a match, one can write down a simple equation that relates market tightness and search effort to movements in the value of expected surpluses:

$$\frac{\kappa\theta_t}{e_t c_e(e_t)} = \frac{\mathrm{E}_t\left[U_c(c_{t+1})W^f_{n_{t+1}}\right]}{\mathrm{E}_t\left[W^h_{n_{t+1}}\right]}. \tag{31}$$

Variations in the relative values of expected surpluses will determine fluctuations in market tightness, θ_t, and e_t, implying fluctuations in employment. Note also that the Nash bargaining outcome requires a specific sharing rule determining the relation between $W^f_{n_t}$ and $W^h_{n_t}$ for all t (see equation 24). Once this is taken into account, it is easy to see how fundamental parameters related to search frictions explicitly affect the movements in the extensive margin through search effort and market tightness. Up to a covariance term equation (31) can be written as[17]

$$\frac{\kappa\theta_t}{e_t c_e(e_t)} \simeq \frac{\lambda}{1-\lambda} E_t \frac{1}{1-\tau_{l,t+1}} \tag{32}$$

Equation (32) shows, how the labor wedge (that will be pinned down by equation 34 below) affects the cost of searching and, thus, employment fluctuations. In turn, since the employment law of motion has to hold, movements in the labor wedge will probably induce fluctuations in the matching function process, τ_χ, interpretable as the extensive labor wedge. We leave the study of the extensie wedge to future work and focus on the static labor wedge instead.

Going back to the static condition (30), one can see that the only distortion present, sum-

[17] This is similar to the 'intertemporal labor wedge' derived by Arseneau and Chugh (2010).

marized by s_t^L, is induced by the bargaining problem. However, once we assume that the production function takes a Cobb-Douglas form, such that $f_{ll}(k,l)l = -\alpha f_l(k,l)$, the marginal productivity of workers change one-to-one with h, we have that $s_t^L = 1-\alpha$. Hence, the wedge introduced by s_t^L is irrelevant at business cycle frequency. The constant $(1-\alpha)$ acts like a tax on employment, which, in this context, is observationally equivalent to a labor income tax; in the baseline calibration α takes a value of about 0.35 reducing substantially the steady state value required for τ_l.[18]

We can finally compare the two fundamental equations that are used to back out the labor wedge from the data in the competitive and non-competitive labor market model

$$[1-\varepsilon_{l,t}] = \frac{mrs_t(l_t)}{mpl_t} = -\frac{\psi}{mpl_t U_c(c_t)} G_h(n_t h_t) \tag{33}$$

$$[1-\tau_{l,t}] = \frac{mrs_t(h_t)}{mpl_t}\frac{1}{1-\alpha} = -\frac{\psi}{mpl_t U_c(c_t)}\frac{G_h(h_t)}{1-\alpha} \tag{34}$$

The major difference between the prototype labor wedge $[1-\varepsilon_{l,t}]$ and ours is that, in the perfectly competitive RBC model the lack of distinction between the extensive and intensive margin has induced the use of total hours worked in the evaluation of the mrs instead of hours per worker. Intuitively, the presence of n_t in the prototype labor wedge equation may introduce a strongly procyclical element—given that $G_h > 0$. Employment is instead not present in the second equation when we back out our discrepancy, $[1-\tau_{l,t}]$. Since movements in both margins are not equally significant in deriving the business cycle frequency fluctuations in total hours, this distinction about the intensive-extensive margin is clearly important. Figure (2) shows the decomposition of total hours into its components. Total hours in the U.S. is clearly procyclical. When total hours is assumed to follow a path where one of the margins is fixed at its historical average and the other margin is assumed to follow the actual path in the data, one recognizes the well-known fact that most of the fluctuations in total hours comes from the extensive margin, not the intensive margin (Shimer 2009). This stylized fact is particularly relevant when interpreting the results of our exercise.

To see why this might affect the measured labor wedge, consider a simple constant Frisch

[18] While a different functional form for the production function would imply a time-varying s^L, we do not believe that this will much help in explaining the cyclical properties of the labor wedge.

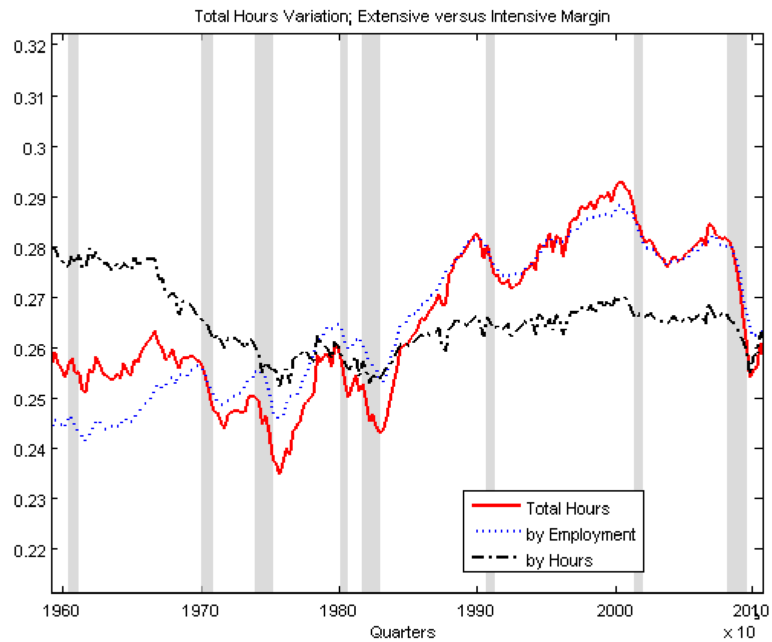

Figure 2: Total Hours decomposition into hous per worker versus employment. Plots for only employment and hours are generated keeping the respective margin moving where the other margin is assumed to be fixed at the historical average. Shaded areas indicate NBER recession dates.

elasticity utility function of the form $G(x) = -x^{(1+\phi)}/(1+\phi)$. Then we can write a simple relation describing the mapping between $[1-\varepsilon_{l,t}]$ and $[1-\tau_{l,t}]$

$$[1-\varepsilon_{l,t}] = (1-\tau_{l,t})(1-\alpha)n_t^{\phi} \tag{35}$$

This shows how some of the observed procyclicality of $[1-\varepsilon_{l,t}]$ induced by employment does not have to be present in our labor wedge. In other words, even if we could interpret all the movements in τ_l as due to actual changes in labor tax, equation (35) suggests that we would still find a strongly procyclical labor wedge $[1-\varepsilon_{l,t}]$ in the prototype model merely because of procycilcal employment fluctuations in the data—the higher ϕ the stronger the procyclicality.

5 Results

In this section we present the results of our quantitative experiments. The labor wedge is a reduced form expression of a model's inability to explain the data. Ideally, we would like to have a truly exogenous and small wedge, that, excluding tax movements, behaves as a measurement error. In the first part of the quantitative exercise, we compare the behavior and statistical properties of the labor wedge in the prototype model with the one in the search model using U.S. data. We show that we can account for somewhere between 15 to more than 40 percent of the fluctuations in the prototype labor wedge. Next, we generate artificial data from our model and compute the labor wedge as in CKM. In this experiment we show that even though the data generating process does not have an exogenous labor wedge—we shut down τ_l— the econometrician would still measure a strongly procyclical labor wedge as in CKM.

5.1 Mapping the Models to the U.S. data

As we mentioned in previous sections, employment fluctuations make the measurement of the prototype mrs differ from the one in our search model. This difference is crucial for the labor wedge, because most of its variation is associated with mrs and not mpl fluctuations. Figure (3) plots the log-detrended mrs and mpl jointly with the log detrended prototype labor wedge. In line with the model, the log mrs, mpl, and $[1 - \varepsilon_{l,t}]$ have been detrended by subtracting the HP-trend of the log of consumption, output, and consumption-output ratio, respectively.[19] While some low-frequency movements in the labor wedge may be driven by the mpl, particularly between 1982-90, most of the business cycle movements are largely due to fluctuations in the mrs. Hence, seen from the lens of the perfectly competitive model, labor supply is less than implied by the model during expansions and it does not fall as much as required by the model during recessions. Given that the search labor wedge defined in (30) basically modifies the mrs reducing its procyclicality, as we have explained in the example in (35), the search model points in the right direction.

Quantitative Analysis We recover the labor wedge in the prototype model (the prototype labor wedge) and in the model with labor market search frictions (search labor wedge), from equations (33) and (34), respectively. Conducting this partial accounting exercise requires us to calibrate the disutility of labor, $G(\cdot)$, α, and ψ. We use quite a general functional form for, $G(\cdot)$, that has non-constant Frisch elasticity of substitution: $G(x) = [(1-x)^{(1-\phi)} - 1]/(1-\phi)$.

[19] The calibration follows strictly the one of CKM. Series are demeaned for ease of comparison.

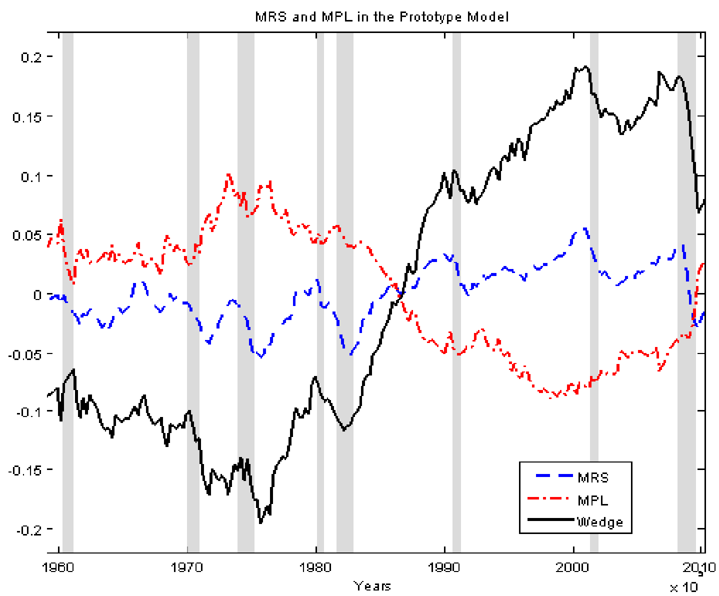

Figure 3: De-meaned log-detrended mrs, mpl and the prototype labor wedge, $[1-\varepsilon_{l,t}]$. They are detrended by subtracting the HP-filter trend of $\log(c_t)$, $\log(y_t)$ and $\log(c_t/y_t)$ respectively. Shaded areas indicate NBER recession dates.

The often used CRRA form $G(x) = -x^{1+\phi}/(1+\phi)$ delivers fairly similar results. Given the chosen form for $G(\cdot)$, and restricting our analysis to models that allow for a balanced growth path $U(c_t) = \log(c_t)$, we can write down two equations that implicitly define the prototype and search labor wedge[20]

$$\frac{\overline{\psi} c_t}{(1-l_t)^{\overline{\phi}}} = \frac{y_t}{l_t}(1-\alpha)\left[1-\varepsilon_{l,t}\right] \tag{36}$$

$$\frac{\widetilde{\psi} c_t}{(1-h_t)^{\widetilde{\phi}}} = \frac{y_t}{l_t}(1-\alpha)^2(1-\tau_{l,t}) \tag{37}$$

Data on y_t, l_t, h_t and c_t pin down a unique labor wedge for each period in both equations once we calibrate α, $\overline{\phi}$ ($\widetilde{\phi}$), and $\overline{\psi}$ ($\widetilde{\psi}$). We choose $\alpha = 0.35$ to match the average labor share in the prototype model. Turning to preferences, we set the parameter $\overline{\psi}$ ($\widetilde{\psi}$) such that in steady state ε_l (τ_l) is 0.4, consistent with the tax wedge measured by Prescott (2004). Finally, we adjust $\overline{\phi}$, to get the same steady state labor elasticity in both models. In the baseline calibration we

[20]The same parameter, e.g., ψ, will take different values when calibrated in one of the two models. With a loose notation, we denote with a $\overline{\psi}$ ($\widetilde{\psi}$) the parameter value calibrated for the perfectly competitive model (labor search model).

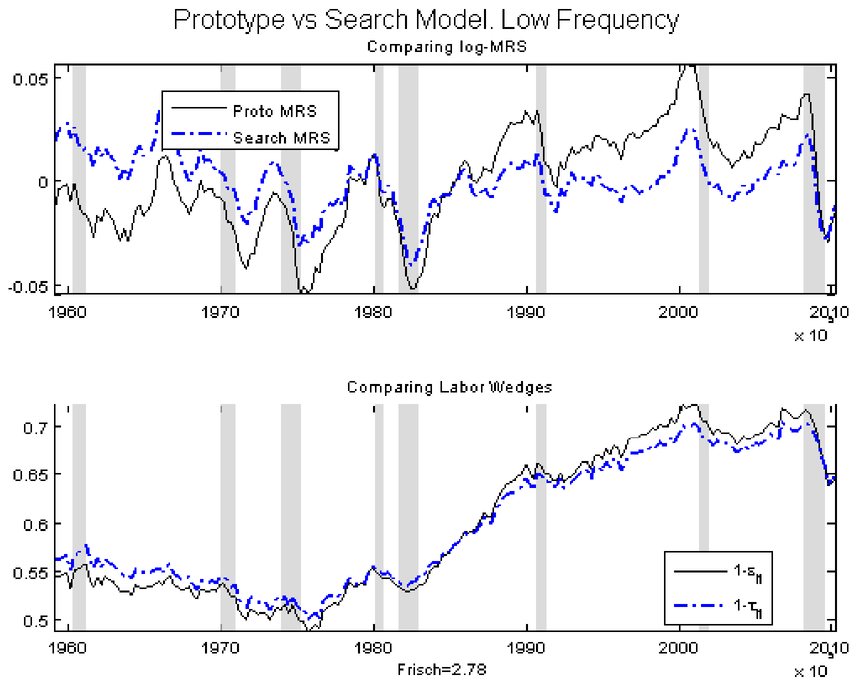

Figure 4: Prototype and search mrs and respective labor wedges, $[1-\varepsilon_{l,t}]$ and $[1-\tau_{l,t}]$, where Frisch elasticity is around 2.8. Detrending uses HP-filter trend of $\log(c_t)$ for mrs and $\log(c_t/y_t)$ for labor wedges. Shaded areas indicate NBER recession dates.

choose the limiting case, $\overline{\phi}=1$, which is also used by CKM and implies a steady state Frisch elasticity of about 2.8, then, in order to get the same elasticity in the search model, we set $\widetilde{\phi}=0.6$. (Next we will explore a lower elasticity case).

Figure (4) shows the results for the 'low-frequency case' when variables are detrended using HP-trends in output and consumption—low-frequency movements are not necessarily filtered out. The upper panel presents the mrs for both models while the lower panel shows the related wedges. There is a clear low-frequency cycle in the wedges, mainly due to the mpl, that has not been captured by the trend in the consumption-labor ratio. The rise in the labor wedge in the second half of the sample is due to a declining mpl, whether we assume competitive labor markets or not. As suggested by Figure (4), the search labor wedge is less volatile than the prototype labor wedge, by about 23 percent (see Table 1). This is entirely due to our measurement of the mrs, which is about 40 percent less volatile than the mrs measured through (36). This obviously follows from the fact that the intensive margin is the less variable margin in total hours, which makes the measurement of the mrs in the search model less variable than the one of the prototype model.

Table 1: Moments for High Frisch Elasticity (2.78)					
	Y_t	$[1-\varepsilon_{l,t}]$	$[1-\tau_{l,t}]$	mrs_t^{proto}	mrs_t^{search}
Std. Dev.	0.015	0.015	0.012	0.013	0.010
	(0.015)	(0.074)	(0.057)	(0.024)	(0.014)
Corr(x_t, x_{t-1})	0.862	0.749	0.666	0.893	0.865
	(0.862)	(0.988)	(0.983)	(0.966)	(0.922)
Cross Correlations					
	Y_t	L_t	C/Y		
$[1-\varepsilon_{l,t}]$	0.514	0.866	-0.162		
	(0.183)	(0.992)	(-0.086)		
$[1-\tau_{l,t}]$	0.410	0.796	-0.043		
	(0.173)	(0.978)	(-0.066)		

All variables are HP-filtered, except L_t.

Moments in () are for the 'low-frequency' case.

As shown in Table 1 the correlations of the labor wedge with cyclical variables are substantially different from zero (excluding the consumption-output ratio). Results are particularly evident at business cycle frequency—in this case each variable in Table 1, but L_t, has been HP-filtered (shown in Figure 5). Given a relatively high Frisch elasticity (about 2.78), at business cycle frequency the search labor wedge is 20 percent less volatile than the prototype wedge (see Table 1). As in the low-frequency case, the nature of the decline stems from the measurement of the mrs. Moreover, the correlation with total hours is reduced by about 8 percent while the one with output by 20 percent.

While the calibration of α and ψ has no implication for our analysis, results are clearly affected by the choice of the parameter that governs the Frisch elasticity ϕ. So far, the Frisch elasticity we have used in the numerical exercises was 2.78, in line with most macro model and chosen to be comparable with CKM. However, this elasticity is at high end of the estimates

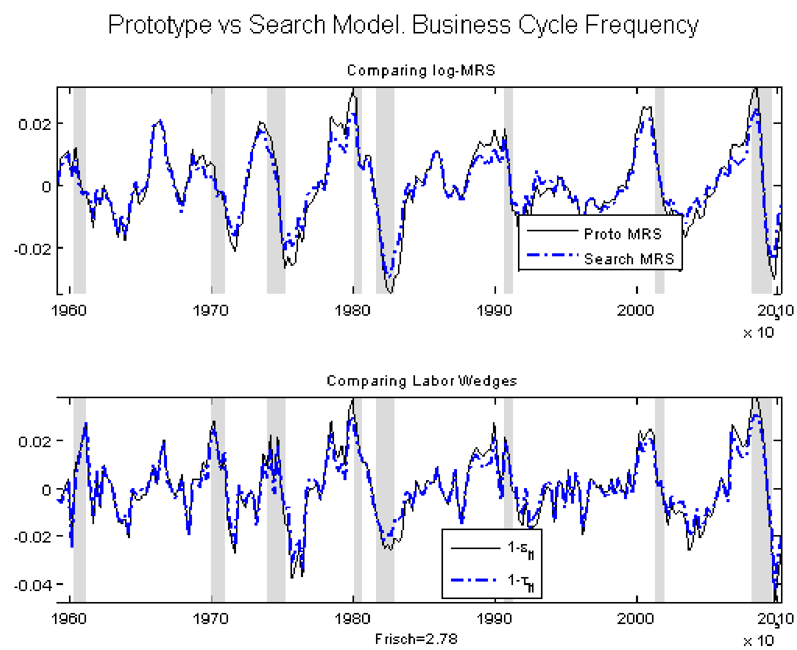

Figure 5: Prototype and search mrs and respective labor wedges, $[1 - \varepsilon_{l,t}]$ and $[1 - \tau_{l,t}]$ where Frisch elasticity is around 2.8. All expressed as deviations from their respective HP-filter trend. Shaded areas indicate NBER recession dates.

found in the micro literature (Blundell and MaCurdy 1999). In what follows we will show that our results are indeed amplified when the Frisch elasticity is chosen consistent with most of the micro estimates.

Low Frisch Elasticity The major problem behind the failure of the prototype model, as well as our extension of it, is that hours do not vary as much in the data as implied by our models. This has been well-recognized in the context of macro models. Higher aggregate micro elasticities tend to produce smaller wedges at the expense of a large micro literature that argues for lower individual labor elasticities. Here, we replicate our exercise by targeting a Frisch elasticity of 0.5 on average, which is more in line with these studies. Figures (6), (7) and Table 2 present our results.[21]

While the overall volatility is higher across variables, there is a dramatic reduction in the volatility of the labor wedge with search frictions of more than 40 percent, both at low frequency (Figure 6) and at business cycle frequency (Figure 7). Once again, this reduction is entirely

[21] This requires $\phi = 5.54$ in (36) and $\phi = 3.33$ in (37) for the given sample averages of L and h.

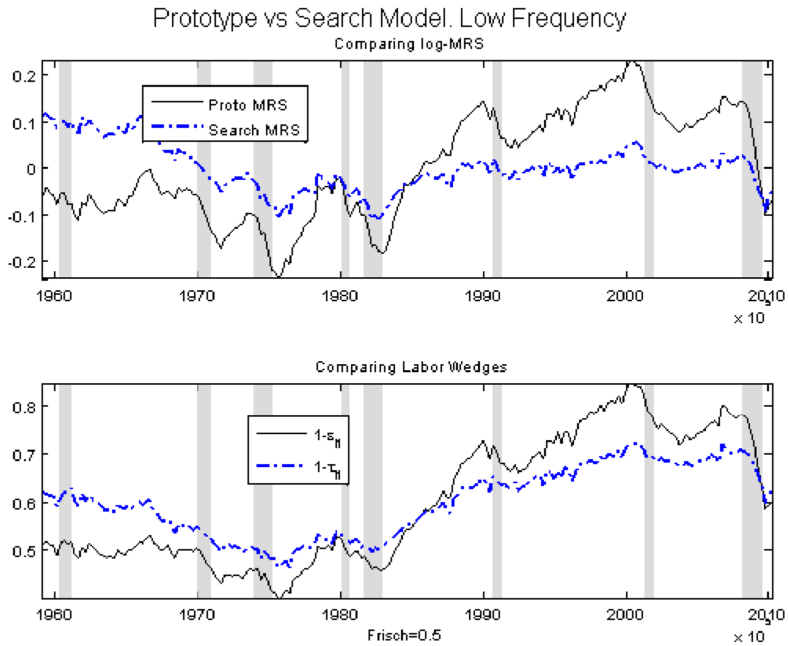

Figure 6: Prototype and search mrs and respective labor wedges, $[1-\varepsilon_{l,t}]$ and $[1-\tau_{l,t}]$ where Frisch elasticity is 0.5. Detrending uses HP-filter trend of $\log(c_t)$ for mrs and and $\log(c_t/y_t)$ for labor wedges. Shaded areas indicate NBER recession dates.

due to the lower volatility in mrs as measured by our model. The reduction in correlation with output is about 22 percent at business cycle frequency. Note however, that a lower Frisch elasticity increases the overall standard deviation in both wedges relative to high elasticity case. Given that wages are not very volatile in the data, most macro models favor a high elasticity. However, while in the prototype model the volatility of the mrs triples, the mrs volatility in the low elasticity case is only 1.46 times the one in the prototype high elasticity case, which makes the labor search model much more promising in reconciling micro and macro estimates.

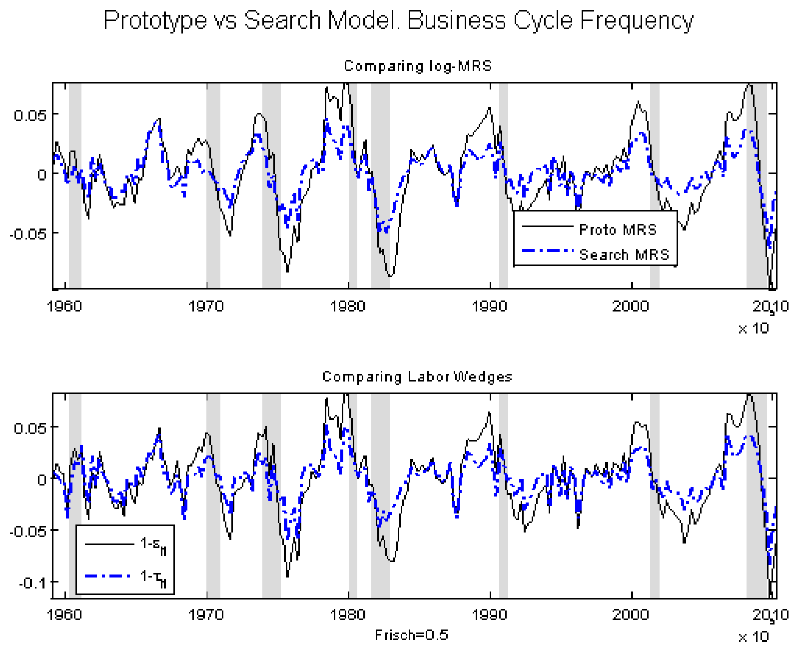

Figure 7: Prototype and search mrs and respective labor wedges, $[1-\varepsilon_{l,t}]$ and $[1-\tau_{l,t}]$ where Frisch elasticity is 0.5. All expressed as deviations from their respective HP-filter trend. Shaded areas indicate NBER recession dates.

Table 2: Moments for Low Frisch Elasticity (0.5)

	Y_t	$[1-\varepsilon_{l,t}]$	$[1-\tau_{l,t}]$	mrs_t^{proto}	mrs_t^{search}
Std. Dev.	0.015	0.037	0.021	0.035	0.019
	(0.015)	(0.165)	(0.082)	(0.114)	(0.052)
$\text{Corr}(x_t, x_{t-1})$	0.862	0.818	0.59	0.875	0.732
	(0.862)	(0.99)	(0.971)	(0.987)	(0.962)
Cross Correlations					
	Y_t	L_t	C/Y		
$[1-\varepsilon_{l,t}]$	0.721	0.975	-0.452		
	(0.238)	(0.998)	(-0.153)		
$(1-\tau_{l,t})$	0.56	0.869	-0.283		
	(0.229)	(0.813)	(-0.132)		

All variables are HP-filtered, except L_t.

Moments in () are for the 'low-frequency' case.

Our comparison between the statistical properties of the two wedges, so far points to a potential misspecification in the prototype model. We can argue that an econometrician measuring the labor wedge based on (36) might misinterpret some endogenous variation in hours as variation in the labor wedge.

5.2 Mapping with the simulated data

The previous section has analyzed the labor wedge based solely on one equilibrium condition. In this section, instead, we use the model as the data generating process and analyze what it implies for the labor wedge of the prototype RBC model. More specifically, we calibrate our model and simulate artificial data by shutting off all the exogenous shocks (and wedges) except the efficiency wedge, Z_t, which means that $[1 - \tau_{l,t}]$ will be constant at 0.6 over time. Then, using the simulated data on consumption, output and labor, we measure a labor wedge as in CKM using (36). Hence, the null hypothesis is no movements in the prototype labor wedge, i.e., $\varepsilon_{l,t} = .4 \; \forall t$.

To calibrate parameters related to search frictions we follow an approach that is similar to Merz (1995) and Andolfatto (1996), and target first moments of the labor market variables, n and h. We follow a standard approach for parameters that are not related to search frictions. More specifically, we set $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.36$. Note that $1 - \alpha$ is not equal to labor's share because of the search frictions, but as long as the total recruiting costs is a relatively small fraction of output, labor's share is not far from $1 - \alpha$. The total recruiting cost share, i.e. $\kappa v/y$, is set to 1.5 percent. We follow Merz (1995) and assume a strictly convex search cost function $c(e) = c_0 e^{\mu}$, where c_0 is normalized to 1 and the output share of search costs born by workers, $c(e)(1 - n)$, is targeted to be 0.5 percent. Unfortunately, there is no guidance in the literature over these parameters and we try to minimize the resource cost of search and recruitment by this calibration. It turns out this is not far from what has been done in the literature before and our results are not sensitive to the exact share of these costs (see Andolfatto 1996). What is more sensitive is the parameter μ, that determines the degree of convexity in cost of worker's search effort. As we have argued in section 4.4, in order to have a unique steady state equilibrium we need to have enough convexity in this function. In particular, as μ converges to 1 we will have multiplicity of equilibria. In our calibration μ is implied by the relative ratios of the search/recruitment costs in output, i.e. $\mu = 3$.[22] The elasticity of matching function and bargaining power of workers (firms) are calibrated such that the Hosios (1990) condition holds, which implies $\gamma = \lambda$. The elasticity of job matches with respect to vacancies, γ, is set to 0.5, which lies in the middle of the range of estimates reported by Petrongolo and Pissarides (2001). The quarterly rate of transition from employment to non-employment, σ, is set at 0.15, following calibration of Andolfatto (1996) and references therein. Five parameters, μ, ϕ, ψ, κ and χ are calibrated to match five moments, $\kappa v/y = 0.015$, $c(e)(1-n)/y = 0.005$, $n = 0.7074$, $h = 0.3752$, and $q = 0.9$.[23]

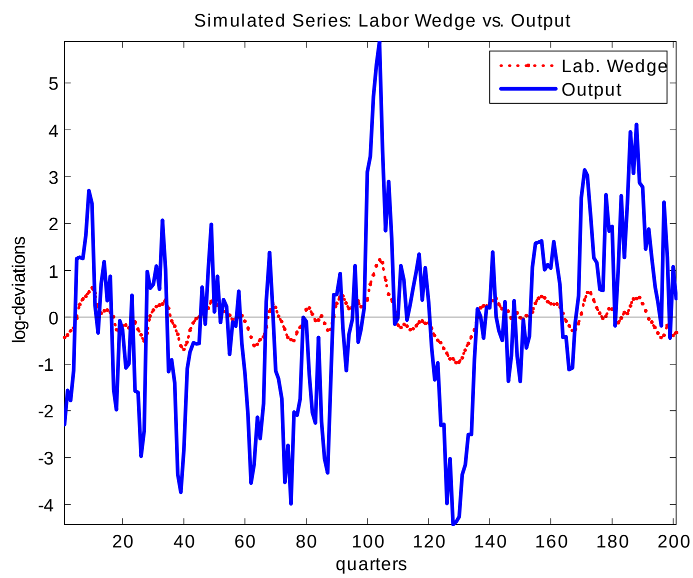

Figure 8: Labor wedge and output from the simulated data.

Figure (8) shows simulated output series and the prototype labor wedge derived under the baseline calibration described above. The standard deviation of ε_t^L relative to output is 0.15 which makes the null hypothesis of no fluctuations clearly rejected. In other words, this implies that doing the business cycle accounting as in CKM would falsely detect the presence of a labor wedge when reality is well described by the labor search model. Moreover, the correlation

[22] Our results are essentially same when $\mu = 1.5$.

[23] Note that, in this quantitative exercise, shocks to the matching efficiency is shut down, as well as τ_l and g,. The latter two parameters are set to 2/5 and 0.16, respectively, following the avergaes in the data and the measured labor wedge from the previous section. Our targets for n and h are the means for employment and hours per worker respectively. The calibration target for q follows from the average duration of a posted vacancy based on van Ours and Ridder (1992).

between the labor wedge is strongly procyclical with a correlation with output and total hours of about 0.85 and 0.96, respectively. Hence, a simple extension of the prototype RBC model with search frictions is consistent with a significantly procyclical labor wedge, which could be entirely due to endogenous movements in both the extensive and the intensive margins. Those results support the view that labor search frictions may induce endogenous fluctuations in hours and employment that will manifest themselves as a procyclical and variable labor wedge as in CKM or Shimer (2009).

Table 3: Simulation Exercise

	Simul. Data	Actual Data
$std([1-\varepsilon_{l,t}])/std(y_t)$	0.15	1.00
$corr([1-\varepsilon_{l,t}], y_t)$	0.85	0.51
$corr([1-\varepsilon_{l,t}], l_t)$	0.96	0.87

6 Conclusion

The business cycle accounting literature has identified the relation between the marginal productivity of labor (mpl) and the marginal rate of substitution (mrs) as a weakness common to many perfectly competitive RBC models. More precisely, the observed wedge between the mrs and mpl seems to conceal one of the keys that would allow us to improve our understanding of what drives economic cycles. This paper has explored the role of labor-market search models in addressing that weakness and providing insights on the mechanisms that lead to business cycle fluctuations.

Our results indicate that labor-market search frictions per se do not provide a mechanism that could directly alter movements in the wedge between mpl and mrs as observed in the data. The main reason for this is that the bargaining process between firms and workers internalize search frictions through the wage decision rather than hours decision.

Instead, the advantage of the labor-search model is due to its natural ability to distinguish between fluctuations in total hours and hours per worker leaving no doubt on the fact that the mrs has to be measured in terms of hours per worker. Since fluctuations in employment account for the majority of the movements in total hours, confounding hours per worker with total hours leads to a substantial mismeasurement of the mrs and, consequently, a serious misspecification of the model, which shows up as labor wedge. Our findings show that, at business cycle frequency, about 20 percent of the observed volatility and most of the procyclicality of the labor wedge can be attributed to fluctuations in the extensive margin (employment) through their effect on the mrs.

In addition to that, we also cast additional light on how the divergent finding between macro and micro estimates of the labor supply elasticity is exacerbated by the use of total hours in the measurement of the mrs. The search model is able to take much lower values for the Frisch elasticity without dramatically increasing the volatility of the mrs—which is the main reason for why macro estimates give high values of the labor demand elasticity.

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Appendix

A Data

We use data from NIPA tables to construct our measures for real output (y_t), consumption (c_t), and government expenditures ($\varepsilon_{g,t}$), which also includes net exports[24]. Labor market variables, employment (n_t) and average hours per worker (h_t) are taken from Cociuba, Prescott and Ueberfeldt (2009). All the data are from 1959:Q2 to through 2010:Q3 and seasonally adjusted at an annualized rate when relevant. Output, y_t,and some of its components,c_t , and $\varepsilon_{g,t}$ are all deflated by the GDP deflator. Real output, y_t is defined as the quarterly gross domestic product net of sales taxes. Consumption, c_t, is the sum of non-durable goods purchases and services. Finally, government expenditures, $\varepsilon_{g,t}$, includes government consumption expenditures (including federal, state and local governments) and net exports of goods and services. In effect we lump together government consumption and net exports. For the purpose of our exercises in this paper this distinction is not important. For the labor market data, we follow Cociuba, Prescott and Ueberfeldt (2009) and use their data to incorporate military hours and employment into total hours and total employment figures as estimated by the Current Population Survey of the BLS.

B Households' Decision Problem

First order necessary conditions and costates for the household's problem (15)-(16) are,

$$x_t \quad : \quad -U_c(c_t)\left[1+\tau_{x,t}\right] + \beta \mathrm{E}_t W^h_{k_{t+1}}(\omega^h_{t+1}|\omega^h_t)\frac{\partial k_{t+1}}{\partial x_t} = 0 \tag{38}$$

$$e_t \quad : \quad -U_c(c_t)c_e(e_t)(1-n_t) + \beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t)\frac{\partial n_{t+1}}{\partial e_t} = 0 \tag{39}$$

$$k_t \quad : \quad W^h_{k_t}(\omega^h_t|\omega^h_{t-1}) = U_c(c_t)r_t + \beta \mathrm{E}_t W^h_{k_{t+1}}(\omega^h_{t+1}|\omega^h_t)\frac{\partial k_{t+1}}{\partial k_t} \tag{40}$$

$$n_t \quad : \quad W^h_{n_t}(\omega^h_t|\omega^h_{t-1}) = U_c(c_t)[w_t\left[1-\tau_{l,t}\right]h_t + c(e_t)] + \psi G(h_t) + \beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t)\frac{\partial n_{t+1}}{\partial n_t} \tag{41}$$

Combining (38) and (40) will give the consumption Euler equation in (17). Note that equation (39) implies that $\beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t) = \frac{U_{c_t}(c_t)c_{e_t}(e_t)}{p_t}$. Substituting this expression for $\beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t)$ in (41) and taking expectations using (39) gives the households' choice of effort in (18).

C Firms' Decision Problem

First order necessary conditions and costates for the firms' problem (19)-(20) are,

$$k_t \quad : \quad \tau_{z,t} f_k(k_t, n_t h_t) - r_t = 0 \tag{42}$$

$$v_t \quad : \quad -\kappa + q_t \tilde{\beta} \mathrm{E}_t W^f(\omega^f_{t+1}|\omega^f_t) = 0 \tag{43}$$

$$k_{t+1} \quad : \quad W^f_{n_t}(\omega^f_t|\omega^f_{t-1}) = \tau_{z,t} f_k(k_t, n_t h_t)h_t - w_t h_t + (1-\sigma)\tilde{\beta}_t \mathrm{E}_t W^f_{n_{t+1}}(\omega^f_t|\omega^f_{t-1}) \tag{44}$$

[24] We pulled this data from Haver Analytics database.

Equation (42) gives us the equilibrium rental rate on capital in (21). Note that equation (43) implies that $\tilde{\beta}_t \mathrm{E}_t W^f_{n_{t+1}}(\omega^f_{t+1}|\omega^f_t) = \frac{\kappa}{q_t}$. Substituting this expression for $\tilde{\beta}_t \mathrm{E}_t W^f(\omega^f_{t+1}|\omega^f_t)$ in (44) and taking expectations using (42) gives the firms' choice of vacancies in (22).

D Details of Employment Contract

Employment contract is given by the solution to the problem defined in (23). Note that we have the necessary expressions for each party's marginal surplus from respective decision problems. $W^h_{n_t}(\omega_t)$ is defined in (41) and $W^f_{n_t}(\omega_t)$ is defined in (44).

$$W^h_{n_t}(\omega^h_t) = U_c(c_t)[w_t\,[1-\tau_{l,t}]\,h_t + c(e)] + \psi G(h_t) + (1-\sigma-p_t e)\,\beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t) \quad (45)$$

$$W^f_{n_t}(\omega^f_t) = \tau_{z,t} f_{l_t}(k_t, n_t h_t) h_t - w_t h_t + (1-\sigma)\,\tilde{\beta}_t \mathrm{E}_t W^f_{n_{t+1}}(\omega^f_{t+1}|\omega^f_t) \quad (46)$$

As long as there are gains from trade, first order conditions are given by two conditions that determine optimal hours and wage.

$$w_t \quad : \quad \lambda \frac{W^f_{nw}(\omega^f_t)}{W^f_n(\omega^f_t)} + (1-\lambda)\frac{W^h_{nw}(\omega^h_t)}{W^h_n(\omega^h_t)} = 0$$

$$h_t \quad : \quad \lambda \frac{W^f_{nh}(\omega^f_t)}{W^f_n(\omega^f_t)} + (1-\lambda)\frac{W^h_{nh}(\omega^h_t)}{W^h_n(\omega^h_t)} = 0$$

where the cross-partial derivatives could be evaluated by using (45) and (46) to arrive at (24) and (25) in the text. Then one can use the wage equation, (24), along with (45) and (46) to derive an explicit wage equation in (26). To do so, multiply both sides of (45) with λ and (46) with $(1-\lambda)U_c(c_t)\,[1-\tau_{l,t}]$.

$$\begin{aligned}
\lambda W^h_n(\omega^h_t) &= \lambda U_c(c_t)[w_t\,[1-\tau_{lt}]\,h_t + c(e)] + \\
&\quad \lambda \psi G(h_t) + \lambda\,(1-\sigma-p_t e)\,\beta \mathrm{E}_t W^h_n(\omega^h_{t+1}|\omega^h_t) \\
(1-\lambda)U_c(c_t)\,[1-\tau_{l_t}]\,W^f_n(\omega^f_t) &= (1-\lambda)U_c(c_t)\,[1-\tau_{l,t}]\,[\varepsilon_{z,t} f_l(k_t, n_t h_t) h_t - w_t h_t] \\
&\quad +(1-\lambda)U_c(c_t)\,[1-\tau_{l,t}]\,(1-\sigma)\,\tilde{\beta}_t \mathrm{E}_t W^f_n(\omega^f_{t+1}|\omega^f_t)
\end{aligned}$$

Substituting for $\beta \mathrm{E}_t W^h_{n_{t+1}}(\omega^h_{t+1}|\omega^h_t)$ and $\tilde{\beta} \mathrm{E}_t W^f_{n_{t+1}}(\omega^f_{t+1}|\omega^f_t)$ and subtracting the second line from the first with some additional algebra gives the wage equation expressed in (26). Finally, it is straightforward to see that the optimality in hours from Nash bargaining problem implies the labor wedge equation expressed in (30).