A Monetary Approach to Asset Liquidity*

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First version: September 2007
This version: December 2008

Abstract

This paper offers a monetary theory of asset liquidity—one that emphasizes the role of assets in payment arrangements—and it explores the implications of the theory for the relationship between assets' intrinsic characteristics and liquidity, and the effects of monetary policy on asset prices and welfare. The environment is a random-matching economy where fiat money coexists with a real asset, and no restrictions are imposed on payment arrangements. The liquidity of the real asset is endogenized by introducing an informational asymmetry in regard to its fundamental value. The model delivers the following insights. A monetary equilibrium exists irrespective of the per capita supply of the real asset, provided that inflation is not too high. The illiquidity premium paid to the real asset tends to increase as the asset becomes riskier and more abundant. Monetary policy affects the real asset's return when its quantity is not too large and inflation is in some intermediate range. The model predicts a negative relationship between inflation and the real asset's expected return.

Keywords: Fiat money, payments, private information, liquidity, asset prices
J.E.L. Classification: D82, D83, E40, E50

*I owe a special thank to Ricardo Lagos for his various insights during our repeated discussions. I also thank for their comments and suggestions Murali Agastya, Aditya Goenka, Yiting Li, Ed Nosal, Peter Rupert, Neil Wallace, Pierre-Olivier Weill, Asher Wolinsky, Tao Zhu and seminar participants at the Federal Reserve Bank of Cleveland, Hong Kong University of Science and Technology, National Taiwan University, National University of Singapore, Rice University, Singapore Management University, the Southern Workshop in Macroeconomics (Auckland), the University of California at Irvine, the University of Tokyo, and the 2008 meeting of the Society of Economic Dynamics in Cambridge. I also thank Patrick Higgins for his research assistance. E-Mail: grochete@uci.edu.

1 Introduction

It is a well-accepted notion that assets are valued not only for their future expected dividend streams but also for the liquidity services they provide.[1] A case in point is fiat money, which derives all its value from its role as a medium of exchange. Other assets combine an income stream with some degree of moneyness, e.g., checkable stock and bond mutual funds can easily be transformed into means of payment, government securities or land can serve as collateral for credit transactions. The recognition that some assets have a special role in facilitating trades has important implications for macroeconomics. As shown by Marshall (1992), Bansal and Coleman (1996), Kiyotaki and Moore (2005) and Lagos (2006), among others, it helps understand asset pricing anomalies, and the transmission of monetary policy to assets' returns. A common approach, however, is to take as exogenous the ease with which assets are traded. These shortcuts are undesirable. They occult the crucial link between the liquidity of an asset and its intrinsic characteristics. Moreover, treating the liquidity of assets as exogenous subjects the model to the Lucas critique—the degree of moneyness of an asset is likely to depend on the stance of monetary policy—and they make it ill-equipped for policy analysis.

The aim of this paper is to provide a monetary theory of asset liquidity—one that emphasizes the role of assets in payment arrangements—and to explore the implications of the theory for the relationship between assets' intrinsic characteristics and liquidity, and the effects of monetary policy on asset prices and welfare. Following the literature pioneered by Kiyotaki and Wright (1989), this paper considers economies where some trades occur within bilateral meetings, and a double-coincidence-of-wants problem makes the use of a medium of exchange necessary. Such environments offer a parsimonious way of generating an endogenous demand for liquid assets. I consider an economy where two assets coexist, fiat money and a real asset, and no restrictions are imposed on payment arrangements. Without additional frictions, all forms of wealth are equally good as means of payment, and the theory is unable to pin down uniquely payment arrangements. In order to overcome this difficulty, I assume that the liquidity differential between fiat money and the real asset stems from an informational asymmetry in regard to the fundamental value of the real asset. Specifically, agents paying with the real asset are better informed about its future performance than agents who receive it, which makes it costly to trade.

[1] The theoretical and empirical literature on asset pricing and liquidity is surveyed in Amihud, Mendelson and Pedersen (2005). For a review of the literature which is more directly relevant to this paper, see Section 1.1.

A key insight of the theory is that the asset which is not subject to a private information problem, fiat money in our context, is a strictly preferred means of payment in the following sense. In order to finance their consumption opportunities, individuals spend their cash first, and they use their real assets only if their currency holdings are depleted. Moreover, individuals retain a fraction of their real asset holdings even when their consumption is inefficiently low. As a consequence of the illiquidity of the real asset, a monetary equilibrium exists—one where fiat money is valued—irrespective of the quantity of the real asset, provided that inflation is not too high.

A major insight of Kiyotaki and Wright (1989) was to show that the acceptability of a good depends on its storage cost as well as other fundamentals (e.g., the pattern of specialization) and beliefs. In the same vein, this paper relates the liquidity of the real asset, as apprehended by its transaction velocity, to its dividend process. The asset becomes more illiquid as the dispersion of the dividends across states increases. This relationship between the intrinsic characteristics of an asset and its liquidity is illustrated in a rather dramatic way when the real asset has no value in some state. In this case the real asset becomes fully illiquid and, in the absence of fiat money, all trades shut down. The model also shows that an increase in the supply of the real asset reduces its liquidity value, and raises its rate of return.

A long-lasting challenge of monetary theory—the central issue of the pure theory of money, according to Hicks (1935)—is to explain why fiat money is held when there are capital goods with a higher rate of return. As noticed by Mehra and Prescott (1985), this rate-of-return dominance puzzle echoes the equity premium puzzle—the excessively large difference between the rate of return of equity and risk-free government liabilities. In the model developed in this paper, individuals exhibit a strict preference for currency, which manifests itself by a rate-of-return differential between fiat money (which can readily be reinterpreted as a risk-free bond) and the real asset. The illiquidity premium paid to the real asset is positive even though agents are risk-neutral with respect to their consumption of the dividend good. Moreover, it tends to increase as the asset becomes riskier and more abundant.

This paper borrows some innovations from recent monetary theory (e.g., Lagos and Wright, 2005) to allow for money growth and inflation while keeping the model tractable. As a consequence, the model delivers insights for the linkages between monetary policy and asset prices. Monetary policy affects the real asset's return when the supply of assets is not too large, and inflation is in some intermediate range. An increase in inflation reduces the rate of return of the most liquid asset, and it induces a reallocation of individuals' portfolios towards the real asset. Consequently, the model predicts a negative relationship between inflation and assets' expected returns. The optimal monetary policy is such that the asset price is driven down to its fundamental value given by its dividend stream, and the real asset is illiquid, i.e., its transaction velocity (in some states) and liquidity premium are zero.

The paper is organized as follows. Section 1.1 provides a review of the relevant literature. The environment is described in Section 2 and the social optimum is characterized in Section 3. Section 4 analyzes the bargaining game under incomplete information. Section 5 embeds the bargaining game into a general equilibrium structure and studies the effects of policy and fundamentals on asset liquidity.

1.1 Related literature

The idea of explaining asset liquidity by a private information problem is omnipresent in both the finance and the monetary literature. Asymmetries of informations are used to endogeneize transaction costs in financial markets (e.g., Kyle, 1985; Glosten and Milgrom, 1985), security design (e.g., DeMarzo and Duffie, 1999), and capital structure choices (e.g., Myers and Majluf, 1984). The monetary literature has resorted to private information problems to explain the role of money when goods are of unknown quality (e.g., Williamson and Wright, 1994; Banarjee and Maskin, 1996), when individuals have some private information about their ability to repay their debt (e.g., Jafarey and Rupert, 2001), or to account for Gresham's law (Velde, Weber and Wright, 1999).[2]

The distinctive feature of our environment is the presence of multiple assets traded in bilateral meetings under private information. In the same vein Hopenhayn and Werner (1996) study a three-period nonmonetary game with indivisible assets. The tradeability of an asset depends on the endogenous decision of uninformed agents to accept it, and more tradeable assets exhibit a lower rate of return. Golosov, Lorenzoni, and Tsyvinski (2008) adopt the same ingredients but focus on the transmission of information through decentralized trading, and its implications for long-run efficiency and the value of information.

My paper provides foundations for the trading restrictions that have been imposed in some recent models that have fiat money coexisting with other assets. A number of papers, e.g., Aruoba and Wright (2003), Aruoba, Waller, and Wright (2007), Berentsen, Menzio, and Wright (2007) and Telyukova and Wright (2007),

[2] Berentsen and Rocheteau (2004) introduce a moral hazard similar to Williamson and Wright (1994) into a model with divisible money. The "counterfeit" consumption good is perishable, it has no value, and only a pooling mechanism is considered. Li (1995) constructs a search model, in which there is quality uncertainty about commodity monies.

follow Freeman (1985) and assume that any asset, except money, can be costlessly counterfeited. As a result, these other assets will not be used as a medium of exchange.[3] Lagos (2006) studies a similar environment where one-period risk-free bonds coexist with risky capital and he restricts the use of capital goods as means of payment in a fraction of the meetings.[4]

Lester, Postlewaite, and Wright (2007) and Kim and Lee (2008) further this argument in the context of search monetary models where claims on capital can be counterfeited at no cost and can only be authenticated in a fraction of the meetings. In Lester *et al.*, this fraction of meetings is endogeneized by assuming that agents can invest in a technology to recognize claims on capital. Rocheteau (2008) investigates the case where counterfeits are produced at a positive cost, and shows that the lack of recognizability manifests itself by an endogenous upper bound on the transfer of assets in uninformed matches.

Kiyotaki and Moore (2005) also use moral hazard considerations to explain the partial illiquidity of capital. They assume that the transfer of ownership of capital is not instantaneous so that an agent can steal a fraction of his capital before the transfer is effective. Similarly, Holmstrom and Tirole (1998, 2001) develop a corporate finance approach to liquidity, where a moral hazard problem prevents claims on corporate assets from being written. Kiyotaki and Moore (2005) provide an alternative explanation for why capital may not be perfectly liquid: "*there may be different qualities of capital, and buyers may be less informed than sellers so that there is adverse selection in the second-hand market*." This is precisely the avenue I follow in this paper.[5]

In accordance with the Wallace (1996) dictum, no restrictions on the use of assets as means of payment are made. In the same vein, Aiyagari, Wallace, and Wright (1996), Wallace (1996, 2000) and Cone (2005) emphasize asset divisibility, or lack of divisibility, to explain the coexistence of money and interest-bearing assets and the liquidity structure of asset yields.[6] Lagos and Rocheteau (2008) and Geromichalos, Licari,

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[3] Aruoba and Wright (2003) and Aruoba, Waller, and Wright (2007) also refer to the lack of portability of capital goods to justify the assumption that capital cannot be used as means of payment in decentralized markets. Telyukova and Wright (2007, Section 4) lay down an extension of their model with "Lucas trees," in which agents pay a fixed cost if they use their real assets as means of payment.

[4] Lagos (2006) studies first a version of the model where the use of Lucas' trees as means of payment is unrestricted. The trading restriction is introduced to allow the model to match the equity premium in the data. Shi (2004) adopts a similar assumption in a search model with fiat money and nominal bonds.

[5] Similarly, Zhu (2006, Section 4) discusses how one could introduce capital into his overlapping-generations model with search, and he argues that to maintain the transaction role of money, "one could assume some private information about the quality of capital, similar to the private information problem on the quality of goods in Williamson and Wright (1994)."

[6] In contrast to Aiyagari, Wallace, and Wright (1996), Shi (2004) and Zhu and Wallace (2007), this paper is not an attempt to explain the coexistence of fiat money and risk-free government bonds. All the analysis could be done by substituting fiat currency by risk-free government bonds.

and Suarez-Lledo (2007) study a model where divisible money and capital compete as means of payment. In contrast to my model there is complete information about the value of the real asset. In terms of the results, money is useful provided that the capital stock in the economy is small, and if money and capital coexist they have the same rate of return. In contrast, in my model the presence of money is always useful irrespective of the size of the capital stock, and if money and capital coexist, then capital dominates money in its rate of return.

2 Environment

The environment is similar to the one in Rocheteau and Wright (2005). Time is discrete, starts at $t = 0$, and continues forever. Each period has two subperiods: a morning, where trades occur in a decentralized market (DM), followed by an afternoon, where trades take place in a competitive market (CM). There is a continuum of infinitely-lived agents divided into two types, called *buyers* and *sellers*, who differ in terms of when they produce and consume. The labels *buyers* and *sellers* indicate agents' roles in the DM. Let $\mathcal{B}$ denote the set of buyers, $\mathcal{S}$ the set of sellers, and $\mathcal{J} = \mathcal{B} \cup \mathcal{S}$. The measures of buyers and sellers are normalized to 1. There are two consumption goods, one produced in the DM, and the other in the CM. Consumption goods are perishable.

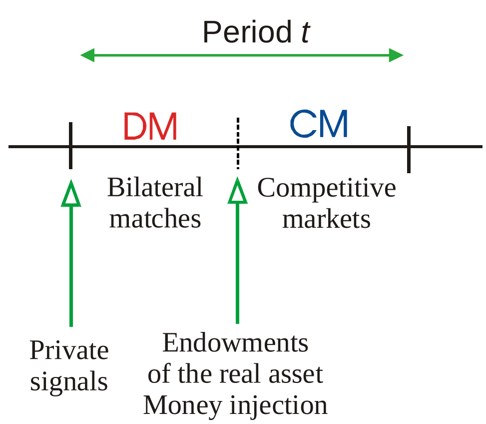

Figure 1: Timing

Buyers and sellers can both produce and consume in the CM. In the DM, however, buyers only consume, while sellers only produce. This heterogeneity will generate a temporal double-coincidence problem. The lifetime expected utility of a buyer from date 0 onward is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[x_t-\ell_t+u(q_t)\right], \tag{1}$$

where x_t is the CM consumption of period t, ℓ_t is the CM disutility of work, q_t is the DM consumption, and $\beta \in (0,1)$ is a discount factor. The utility function $u(q)$ is twice continuously differentiable, $u(0)=0$, $u'(0)=\infty$, $u'(q)>0$, and $u''(q)<0$. The production technology in the CM is linear with labor as the only input, $y_t=\ell_t$.

The lifetime expected utility of a seller from date 0 onward is

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{t}\left[x_t-\ell_t-c(q_t)\right], \tag{2}$$

where q_t is the DM production. The cost function $c(q)$ is twice continuously differentiable, $c(0)=c'(0)=0$, $c'(q)>0$, $c''(q)\geq 0$, and $c(q)=u(q)$ for some $q>0$. Let q^* denote the solution to $u'(q^*)=c'(q^*)$.

At the beginning of the CM, each buyer is endowed with $A>0$ units of a one-period-lived real asset. Because of the absence of wealth effects, who receives the endowment of real assets is irrelevant for the allocations. The asset is perfectly divisible, uncounterfeitable, and perfectly durable over its lifetime. Each unit of the period-t asset yields κ_{t+1} units of CM-output delivered in the CM of $t+1$, and it fully depreciates subsequently. The real dividend can take two values, $\kappa_{t+1}\in\{\kappa_\ell,\kappa_h\}$, where $0<\kappa_\ell<\kappa_h$. The dividend shocks κ_t are independent across time with $\pi_h=\Pr[\kappa_t=\kappa_h]\in(0,1)$ and $\pi_\ell=1-\pi_h$. Denote $\bar{\kappa}=\pi_h\kappa_h+\pi_\ell\kappa_\ell$.[7]

Fiat money is durable, perfectly divisible, and it can be held in any nonnegative amount. The quantity of money per buyer in the DM of period t is denoted M_t. It grows at a constant gross rate, $\gamma\equiv M_{t+1}/M_t$, where $\gamma>\beta$. New money is injected, or withdrawn if $\gamma<1$, by lump-sum transfers, $(\gamma-1)M_t$, or taxes if $\gamma<1$, to the buyers in the CM.[8]

In the CM, there is a competitive market where agents can trade goods, fiat money and the real asset. In the DM, each seller is matched bilaterally with a buyer drawn at random from $\mathcal{B}$. The buyer makes an

[7] In Rocheteau (2007, Appendix D) I show that the model can be generalized to allow for more than two private signals for the buyers. Also, the case where the asset is long-lived complicates significantly the proof for the uniqueness of the equilibrium without generating additional insights.

[8] If $\gamma<1$, the government can force all buyers to pay taxes in the CM. However, it has no enforcement power in the DM. In a related model, Andolfatto (2007) considers the case where the government has limited coercion power–it cannot confiscate output and cannot force agents to work–and agents can avoid paying taxes by not accumulating money balances. He shows that if agents are sufficiently impatient, then the Friedman rule is not incentive-feasible, i.e., there is an induced lower bound on deflation. Also, I restrict my attention to constant money growth rates. One could consider an environment with stochastic inflation, and assume that agents are asymmetrically informed about the future value of money.

offer that the seller accepts or rejects. If the offer is accepted then the trade is implemented. All trades in the DM are *quid pro quo*, and matched agents can transfer any nonnegative quantity of DM-output and any quantity of their asset holdings. Agents can only trade the physical asset, and not claims on future output. In order to guarantee that there is an essential role for a medium of exchange, there is no public record of individuals' trading histories, and agents cannot commit.[9]

An informational asymmetry about the value of the real asset is introduced as follows. Buyers who enter the DM in period t receive a perfectly informative signal about the dividend of the real asset, κ_t. Sellers, in contrast, only learn the realization of the dividend in the CM of the same period.[10] An advantage of having the real asset traded in bilateral meetings in the DM, besides being a realistic feature of many asset markets, is that it prevents the price from revealing the buyers' information at no cost.

3 Social optimum

Consider a social planner who chooses an allocation in order to maximize the sum of the lifetime expected utilities of all agents in the economy. The planner has full command over the resources of the economy, but it has no private information about the future value of the asset, i.e., it observes the realization of the dividend shock, κ_t, at the beginning of the CM in period t.

Let $\mathcal{M}_t$ denote the set of bilateral matches (j, j') composed of one buyer $j \in \mathcal{B}$ and one seller $j' \in \mathcal{S}$ in the DM of period t. The expression for social welfare is then

$$\mathcal{W} = \sum_{t \geq 0} \beta^t \int_{j \in \mathcal{J}} [x_t(j) - \ell_t(j)] \, dj + \sum_{t \geq 0} \beta^t \int_{(j,j') \in \mathcal{M}_t} \{u[q_t(j)] - c[q_t(j')]\} \, d(j, j'). \tag{3}$$

The first integral on the right side of (3) corresponds to the consumption net of the disutility of work for of all agents from $t = 0$ onwards. The second term is buyers' consumption net of sellers' disutility of production

[9] If trading histories were publicly observable, then some good allocations could be implemented through the threat of trigger strategies. See Kocherlakota (1998) for a detailed presentation of this argument.

[10] There are several ways one can interpret this informational asymmetry. One can think of a seller as consolidating the roles of a dealer of assets and a producer. In accordance with the market micro-structure literature, the dealer is uninformed about the future value of the asset (e.g., Glosten and Milgrom, 1985). Alternatively, one could adopt the assumption of Plantin (2008) that agents acquire some private information about the value of an asset by holding it. This assumption is relevant for assets that are not traded publicly, such as securitized pools of loans. Those assets are sold to institutional investors who receive some information from the issuer that is not publicly available. In my model, sellers have no strict incentives to hold the asset, even if they could learn its future dividend in the DM. Buyers have a liquidity motive to hold the asset. Alternative information structures could be considered. For instance, if only a fraction of buyers were informed, then three types of buyers (uninformed, informed in the high state, informed in the low state) would have to be distinguished, but all the insights of the theory would be preserved. One could also assume that a fraction of sellers are informed. Provided that buyers know whether sellers are informed or not, the model remains tractable.

in bilateral matches formed in the DM. The planner is subject to the following feasibility constraints:

$$\int_{j\in\mathcal{J}} x_t(j)dj \quad\leq\quad \int_{j\in\mathcal{J}} \ell_t(j)dj + A\kappa_t, \quad \forall t\geq 0 \tag{4}$$

$$q_t(j) \quad\leq\quad q_t(j'), \qquad \forall(j,j')\in\mathcal{M}_t, \quad \forall t\geq 0. \tag{5}$$

The feasibility constraint (4) requires agents' CM-consumption in period t to be at most equal to the aggregate production in that period, including the output generated by the stock of real assets, $\kappa_t A$. The feasibility condition (5) indicates that the buyer's consumption in a bilateral match is no greater than the seller's production in that match.

Since there is no state variable that links the subperiods, the planner's problem can be rewritten as a sequence of static problems, i.e.,

$$\max_{x_t,\ell_t}\int_{j\in\mathcal{J}}\left[x_t(j)-\ell_t(j)\right]dj \quad \text{s.t. (4)} \tag{6}$$

$$\max_{q_t}\int_{(j,j')\in\mathcal{M}_t}\left\{u[q_t(j)]-c[q_t(j')]\right\}d(j,j') \quad \text{s.t. (5)} \tag{7}$$

The planner is indifferent on how to allocate the CM-goods between agents. The optimal consumption and production in bilateral matches satisfy $q_t(j)=q_t(j')=q^*$ for all $(j,j')\in\mathcal{M}_t$.

4 Payments under private information

I focus on steady-state equilibria where the gross rate of return of fiat money is γ^{-1}. In order to define the payoffs in the bargaining game, it is useful to derive first some properties of the value functions in the CM.

Let $W^b(z,a,\kappa)$ denote the value function of a buyer at the beginning of the CM with a portfolio of z real balances and a units of the real asset when the dividend state is $\kappa\in\{\kappa_\ell,\kappa_h\}$. Similarly, let $V^b(z,a,\kappa)$ denote the value function of the buyer in the DM where κ is the buyer's private information about the future dividend state. The expected lifetime utility of a buyer at the beginning of the CM in period t is given by

$$W^b(z_t,a_t,\kappa_t)=\max_{x_t,\ell_t,z',a'}\left\{x_t-\ell_t+\beta\mathbb{E}V^b\left(z',a',\kappa_{t+1}\right)\right\} \tag{8}$$

$$\text{s.t.}\quad x_t+\gamma z'+\phi_t a'=\ell_t+\kappa_t a_t+z_t+T_t+\phi_t A, \tag{9}$$

where ϕ_t is the price of the real asset in terms of period$-t$ CM output, T_t the lump-sum transfer of real balances by the government, and the expectation is taken with respect to the future dividend state κ_{t+1}. In order to hold z' units of real balances next period the buyer must invest in $\gamma z'$ units in the current period.

According to (8), each buyer chooses his net consumption and his portfolio in order to maximize his expected lifetime utility subject to the budget constraint (9). Substitute $x_t - \ell_t = \kappa_t a_t + z_t - \gamma z' + \phi_t(A - a') + T_t$ from (9) into (8) to obtain

$$W^b(z_t, a_t, \kappa_t) = \kappa_t a_t + z_t + T_t + \phi_t A + \max_{z',a'} \left\{ -\gamma z' - \phi_t a' + \beta \left[\pi_h V^b \left(z', a', \kappa_h \right) + \pi_\ell V^b \left(z', a', \kappa_\ell \right) \right] \right\}. \quad (10)$$

The buyer's value function in the CM is linear in his wealth. Moreover, a buyer's portfolio choice is independent of his initial portfolio when he entered the period.

Following a similar reasoning, the expected lifetime utility of a seller at the beginning of period t is given by

$$W^s(z_t, a_t, \kappa_t) = \kappa_t a_t + z_t + \max_{z',a'} \left\{ -\gamma z' - \phi_t a' + \beta V^s \left(z', a' \right) \right\}, \quad (11)$$

where $V^s(z, a)$ is the value function of the seller upon entering the DM. Recall that the seller has no private information about the future dividend state. The seller's value function in the CM is linear in his wealth.

In the rest of the section, I consider the bargaining game between a buyer holding a portfolio composed of a^b units of the real assets and z^b units of real balances (measured in terms of the CM good), and a seller with a portfolio of a^s units of the real asset and z^s units of real balances. The bargaining game has the structure of a signaling game.[11] A strategy for the buyer specifies an offer $(q, d, \tau) \in \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b]$, where q is the output produced by the seller, d is the transfer of real asset by the buyer, and τ the transfer of real balances, as a function of the buyer's type (i.e., his private information about the future dividend of the real asset). A strategy for the seller is an acceptance rule that specifies a set $\mathcal{A} \subseteq \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b]$ of acceptable offers.

The buyer's payoff in the dividend state κ is

$$\left[u(q) + W^b \left(z^b - \tau, a^b - d, \kappa \right) \right] \mathbb{I}_{\mathcal{A}}(q, d, \tau) + W^b \left(z^b, a^b, \kappa \right) \left[1 - \mathbb{I}_{\mathcal{A}}(q, d, \tau) \right],$$

where $\mathbb{I}_{\mathcal{A}}(q, d, \tau)$ is an indicator function that is equal to one if $(q, d, \tau) \in \mathcal{A}$. If an offer is accepted, then the buyer enjoys his utility of consumption in the DM, $u(q)$, but he forgoes d units of the real asset and τ units of real balances. Using the linearity of the buyer's value function, and omitting the constant terms, the buyer's payoff can be expressed as his surplus $[u(q) - \kappa d - \tau] \mathbb{I}_{\mathcal{A}}(q, d, \tau)$.

Similarly, the seller's (Bernoulli) payoff function is

$$\left[-c(q) + W^s \left(z^s + \tau, a^s + d, \kappa \right) \right] \mathbb{I}_{\mathcal{A}}(q, d, \tau) + W^s \left(z^s, a^s, \kappa \right) \left[1 - \mathbb{I}_{\mathcal{A}}(q, d, \tau) \right],$$

[11] See Appendix B for a more detailed presentation of signaling games.

and his surplus is $[-c(q) + d\kappa + \tau] \mathbb{I}_{\mathcal{A}}(q, d, \tau)$. In order to accept or reject an offer, the seller will have to form expectations about the dividend of the real asset. He will do so by using the offer (q, d, τ) to update his prior belief. Let $\lambda(q, d, \tau) \in [0, 1]$ represent the updated belief of a seller that the buyer holds a high-dividend asset ($\kappa = \kappa_h$). Then, $\mathbb{E}_\lambda [\kappa] = \lambda(q, d, \tau)\kappa_h + [1 - \lambda(q, d, \tau)] \kappa_\ell$.

The equilibrium concept is perfect Bayesian equilibrium.[12] An equilibrium of the bargaining game is a profile of strategies for the buyer and the seller, and a belief system λ. The buyer chooses an offer that maximizes his surplus, taking as given the acceptance rule of the seller. The seller chooses optimally to reject or accept offers given his posterior belief. If (q, d, τ) is an offer made in equilibrium, then $\lambda(q, d, \tau)$ is derived from the seller's prior belief according to Bayes's rule. If (q, d, τ) is an out-of-equilibrium offer, then the seller's belief is arbitrary (to some extent discussed later).

For a given belief system, the set of acceptable offers for a seller is

$$\mathcal{A}(\lambda) = \left\{(q, d, \tau) \in \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b] : -c(q) + \{\lambda(q, d, \tau)\kappa_h + [1 - \lambda(q, d, \tau)] \kappa_\ell\} d + \tau \geq 0\right\}. \quad (12)$$

For an offer to be acceptable, the seller's disutility of production in the DM, $-c(q)$, must be compensated by his expected utility in the next CM, $\mathbb{E}_\lambda [\kappa] d + \tau$. I adopt a tie-breaking rule according to which a seller agrees to any offer that makes him indifferent between accepting or rejecting a trade.[13] The problem of a buyer holding an asset of quality κ is then

$$\max_{q,d,\tau} [u(q) - \kappa d - \tau] \mathbb{I}_{\mathcal{A}}(q, d, \tau) \quad \text{s.t.} \quad (q, d, \tau) \in \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b]. \quad (13)$$

The equilibrium concept is refined by using the Intuitive Criterion of Cho and Kreps (1987).[14] Denote U_h^b the surplus of an h-type buyer and U_ℓ^b the surplus of an ℓ-type buyer in a proposed equilibrium of the bargaining game. This proposed equilibrium fails the Intuitive Criterion if there is an out-of-equilibrium

[12] In the context of a signaling game, the concepts of sequential equilibrium and perfect Bayesian equilibrium are equivalent.

[13] A similar tie-breaking assumption is used in Rubinstein (1985, Assumption B-3). It is made so that the set of acceptable offers is closed, and the buyer's problem has a solution.

[14] The Intuitive Criterion is a refinement supported by much of the signalling literature. An equilibrium that fails the Intuitive Criterion gives an outcome that is not strategically stable in the sense of Kohlberg and Mertens (1986). See Riley (2001) for a survey of the applications of the Intuitive Criterion (and other refinements) in various contexts. It has been used in monetary theory by Nosal and Wallace (2007); in the corporate finance literature by Noe (1989) and DeMarzo and Duffie (1999); in bargaining theory by Rubinstein (1985, Assumption B-1); and, recently, in the literature on global games by Angeletos, Hellwig, and Pavan (2006). In our context, the Intuitive Criterion has the additional advantage of preserving the tractability of the model once the bargaining game is embodied in the general equilibrium structure in Section 5. For sake of completeness, the model is also analyzed under the alternative refinement from Mailath, Okuno-Fujiwara, and Postlewaite (1993) in Appendix C. It is also worth noticing that for some portfolios (e.g., $z^b \geq c(q^*)$) the outcome of any perfect Bayesian equilibrium of the bargaining game is unique.

offer $(\tilde{q}, \tilde{d}, \tilde{\tau}) \in \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b]$ and a buyer's type $\chi \in \{\ell, h\}$ such that the following is true:

$$u(\tilde{q}) - \kappa_\chi \tilde{d} - \tilde{\tau} > U^b_\chi \tag{14}$$

$$u(\tilde{q}) - \kappa_{-\chi} \tilde{d} - \tilde{\tau} < U^b_{-\chi} \tag{15}$$

$$-c(\tilde{q}) + \kappa_\chi \tilde{d} + \tilde{\tau} \geq 0, \tag{16}$$

where $\{-\chi\} = \{\ell, h\} \backslash \{\chi\}$. According to (14), the offer $(\tilde{q}, \tilde{d}, \tilde{\tau})$ would make a χ-type buyer strictly better off if it were accepted. According to (15), the offer $(\tilde{q}, \tilde{d}, \tilde{\tau})$ would make the $-\chi$-type buyer strictly worse off. According to (16), the offer is acceptable provided that the seller believes it comes from a $\chi-$type.

Definition 1 *An equilibrium of the bargaining game is a pair of strategies and a belief system,* $\langle [q(\kappa), d(\kappa), \tau(\kappa)], \mathcal{A}, \lambda \rangle$*, such that: (i)* $[q(\kappa), d(\kappa), \tau(\kappa)]$ *is solution to (13) with* $\kappa \in \{\kappa_\ell, \kappa_h\}$*; (ii)* $\mathcal{A}$ *is given by (12); (iii)* $\lambda : \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b] \rightarrow [0, 1]$ *satisfies Bayes' rule whenever possible, and the Intuitive Criterion.*

The next Lemma narrows the set of possible equilibria by showing that there cannot be a pooling offer in any equilibrium.[15]

Lemma 1 *In equilibrium, there is no pooling offer with* $d \neq 0$*.*

Any equilibrium in which there are transfers of real assets between buyers and sellers is separating. The logic of the argument goes as follows. Suppose there is a pooling offer such that $d > 0$. The buyer in the high-dividend state has the possibility to signal the quality of the real asset by choosing an offer that would raise his payoff relative to the proposed equilibrium, but that would hurt buyers in the low-dividend state. Typically, the trade involves a lower consumption and a smaller transfer of real asset compared to the equilibrium offer, but also a better price for the asset. (See Section 4.1 for a graphical illustration of this argument.) Reciprocally, if $d < 0$, then the buyer in the low-dividend state can reduce his consumption and the quantity of assets he buys from the seller in order to signal the low quality of the asset.

The next Lemma characterizes the equilibrium offers.

[15] It should be clear from the proof of Lemma 1 that the same result would go through if buyers could use mixed strategies.

Lemma 2 *The offer made by a buyer in the low-dividend state is*

$$(q_\ell, d_\ell, \tau_\ell) = \arg\max_{q,\tau,d} \left[u(q) - \kappa_\ell d - \tau\right] \tag{17}$$

$$s.t. \quad -c(q) + \kappa_\ell d + \tau \geq 0 \tag{18}$$

$$-z^s \leq \tau \leq z^b, \quad -a^s \leq d \leq a^b. \tag{19}$$

The offer made by a buyer in the high-dividend state is

$$(q_h, d_h, \tau_h) = \arg\max_{q,\tau,d} \left[u(q) - \kappa_h d - \tau\right] \tag{20}$$

$$s.t. \quad -c(q) + \kappa_h d + \tau \geq 0 \tag{21}$$

$$u(q) - \kappa_\ell d - \tau \leq u(q_\ell) - c(q_\ell) \tag{22}$$

$$-z^s \leq \tau \leq z^b, \quad -a^s \leq d \leq a^b. \tag{23}$$

The only way an ℓ-type buyer can achieve a higher payoff than the one he would get in a game with complete information is by making an offer with $d > 0$ that a seller would attribute to an h-type buyer with positive probability, which has been ruled out by Lemma 1. Hence, buyers in the low-dividend state make their complete information offer (which is always acceptable, irrespective of sellers' beliefs). The solution to (17)-(19) is

$$\begin{aligned} q_\ell &= q^*, \\ \kappa_\ell d_\ell + \tau_\ell &= c(q^*), \end{aligned}$$

if $\kappa_\ell a^b + z^b \geq c(q^*)$, and

$$\begin{aligned} \tau_\ell &= z^b, \\ d_\ell &= a^b, \\ q_\ell &= c^{-1}\left(\kappa_\ell a^b + z^b\right), \end{aligned}$$

otherwise.

According to the Intuitive Criterion, an h-type buyer can always increase his payoff as long as by so doing he does not give incentives to an ℓ-type buyer to imitate him. Hence, from (20)-(23), the buyer maximizes his surplus subject to the participation constraint of the seller, where the seller has the correct belief that he faces an h-type buyer, and subject to the incentive-compatibility condition according to which an ℓ-type buyer does not want to mimic the offer of an h-type buyer.

A belief system consistent with the offers in Lemma 2 is such that $\lambda(q_h, d_h, \tau_h) = 1$ and $\lambda(q_\ell, d_\ell, \tau_\ell) = 0$ if $(q_h, d_h, \tau_h) \neq (q_\ell, d_\ell, \tau_\ell)$, and $\lambda(q_h, d_h, \tau_h) = \pi_h$ and $\lambda(q_\ell, d_\ell, \tau_\ell) = \pi_\ell$ if $(q_h, \tau_h) = (q_\ell, \tau_\ell)$ and $d_h = d_\ell = 0$ (from Bayes' rule). For out-of-equilibrium offers,

$$\begin{aligned} \lambda(q, d, \tau) &= 1 \text{ if (22) holds} \\ &= 0 \text{ otherwise.} \end{aligned}$$

Sellers attribute all offers that would raise the payoff of buyers in the low-dividend state relative to their complete information payoff to ℓ-type buyers, and all other out-of-equilibrium offers to h-type buyers. Offers that violate (22) also violate (18), and since they are attributed to ℓ-type buyers, they are rejected.

Proposition 1 ***(A pecking order theory of payments)***

Consider a match between a buyer holding a portfolio (z^b, a^b) and a seller holding a portfolio (z^s, a^s). There is a solution (q_h, d_h, τ_h) to (20)-(23).

1. *If $z^b \geq c(q^*)$, then*

$$\begin{aligned} q_h &= q^* && (24) \\ \tau_h + \kappa_h d_h &= c(q^*) && (25) \\ d_h &\leq 0. && (26) \end{aligned}$$

2. *If $z^b < c(q^*)$, then $\tau_h = z^b$ and $(q_h, d_h) \in [0, q_\ell] \times [0, a^b]$ is the unique solution to:*

$$\begin{aligned} \kappa_h d_h &= c(q_h) - z^b && (27) \\ u(q_h) - c(q_h) + \left(1 - \frac{\kappa_\ell}{\kappa_h}\right)\left[c(q_h) - z^b\right] &= u(q_\ell) - c(q_\ell), && (28) \end{aligned}$$

where $q_\ell = \min\left[q^*, c^{-1}\left(z^b + \kappa_\ell a^b\right)\right]$. Moreover, if $a^b > 0$ then, $q_h < q_\ell$ and $d_h \in \left(0, a^b\right)$.

Proposition 1 offers a *pecking order* theory of payment choices: agents with a consumption opportunity finance it with cash first, and they use their risky assets as a last resort.[16] If buyers hold enough real balances

[16] The term "pecking order" was coined by Myers (1984, p.581). It describes the predictions of models of capital structure choices under private information. According to the pecking order theory, firms with an investment opportunity prefer internal finance (nondistributed dividends). If external finance is required, then they issue the safest security first, and they use equity as a last resort.

to buy q^* ($z^b \geq c(q^*)$), then they do not transfer any real asset to the sellers. In this sense, fiat money is a preferred means of payment. Even when buyers do not have enough wealth to buy the surplus-maximizing level of output, they choose not to spend all their capital goods. By retaining a fraction of their real asset holdings, buyers signal the high future dividend of the asset, and hence they secure better terms of trade.[17]

The fraction $\theta_h \equiv d_h/a^b$ of his real asset holdings that a buyer spends in the DM is a function of his portfolio and the characteristics of the dividend process. If $u(q) = 2\sqrt{q}$ and $c(q) = q$, then the closed-form solution for θ_h is

$$\begin{aligned}
\theta_h\left(\kappa_h, \kappa_\ell, z^b, a^b\right) &= \frac{\left(\frac{\kappa_h}{\kappa_\ell}\right)^2 \left[1 - \sqrt{1 - \frac{\kappa_\ell}{\kappa_h}\left[2\sqrt{q_\ell} - q_\ell + \left(1 - \frac{\kappa_\ell}{\kappa_h}\right) z^b\right]}\right]^2 - z^b}{\kappa_h a^b} \quad \text{if } \ z^b \leq q^* \\
&= 0 \text{ otherwise,}
\end{aligned}$$

where $q_\ell = \min\left[1, \kappa_\ell a^b + z^b\right]$. This expression points to the differences between the approach in this paper and the approaches of Kiyotaki and Moore (2005) and Lagos (2006). In Kiyotaki and Moore (2005), agents can only sell a fraction $\theta \in (0,1)$ of their illiquid asset (capital) to raise funds; in Lagos (2006), agents can use their illiquid asset ("Lucas' trees") in a fraction of θ of the matches. In both cases, the parameter θ is exogenous. In contrast, in my model buyers spend a fraction θ_h of their capital when the dividend is high, where θ_h is a function of the intrinsic characteristics of the asset (κ_ℓ and κ_h) and the composition of the portfolio held by the buyer (z^b and a^b). Hence, the (il)liquidity of the real asset depends on its intrinsic characteristics, as well as policy, since the buyer's portfolio will be affected by the rate of return of fiat money.

Proposition 2 ***(Asset liquidity and fundamentals)***

Assume $z^b < c(q^*)$ and $a^b > 0$. Then:

1. $\frac{\partial \theta_h}{\partial \kappa_h} < 0$ *and* $\frac{\partial \theta_h}{\partial \kappa_\ell} > 0$.
2. $\lim_{\kappa_\ell \to 0} \theta_h = 0$.

The propensity to spend the real asset in the high-dividend state increases with the size of the low-state dividend, κ_ℓ, and it decreases with κ_h. To understand this result, notice from (22) that ℓ-type buyers enjoy

[17] This result is reminiscent to some of the findings of the liquidity-based model of security design from DeMarzo and Duffie (1999). They consider the problem faced by a firm that needs to raise funds by issuing a security backed by real assets. The issuer has private information regarding the distribution of cash flows of the underlying assets. Using the Intuitive Criterion, they show that a signaling equilibrium exists in which the seller receives a high price for the security by retaining some fraction of the issue.

an informational rent (the difference between the buyer's surplus in the low state and the buyer's surplus in the high state) equal to $(\kappa_h - \kappa_\ell)d_h > 0$. As κ_ℓ gets closer to κ_h, this informational rent shrinks, and the incentive-compatibility constraint is relaxed, which improves the liquidity of the asset in the high-dividend state.[18] Conversely, as $\kappa_h - \kappa_\ell$ increases, the informational asymmetries become more severe, which makes the incentive-compatibiliy condition more binding. In the case where the dividend in the low state approaches 0, the adverse selection problem is so severe that the real asset ceases to be traded. Fiat money becomes the only means of payment.[19]

Proposition 3 ***(Payments and portfolio composition)***

1. If $z^b < c(q^*)$ and $a^b > 0$, then

$$\frac{\partial(\kappa_h d_h)}{\partial z^b} = \frac{u'(q_\ell)/c'(q_\ell) - u'(q_h)/c'(q_h)}{u'(q_h)/c'(q_h) - \frac{\kappa_\ell}{\kappa_h}} < 0. \tag{29}$$

2. If $\kappa_\ell a^b + z^b < c(q^*)$, then

$$\frac{\partial d_h}{\partial a^b} = \frac{u'(q_\ell)/c'(q_\ell) - 1}{\frac{\kappa_h}{\kappa_\ell} u'(q_h)/c'(q_h) - 1} \in (0, 1). \tag{30}$$

As the buyer's real balances increase, the transfer of real assets (expressed in CM output) decreases. The buyer uses his additional real balances to reduce d_h, thereby relaxing the incentive-compatibility constraint (22). This dependence of θ_h on z^b will offer a channel through which monetary policy affects the liquidity of the real asset.

According to (30), the marginal propensity of a buyer to spend his real asset in the high-dividend state is less than one. Provided that $\kappa_\ell a^b + z^b < c(q^*)$, an additional unit of asset increases the surplus of the buyer in the low-dividend state, and hence it relaxes the incentive-compatibility constraint in the high-dividend state, which allows the buyer to spend a fraction of his marginal asset. If $\kappa_\ell a^b + z^b > c(q^*)$, then $q_\ell = q^*$ and $\partial d_h / \partial a^b = 0$. In this case, the liquidity needs in the low-dividend state are satiated and, as a result, an additional unit of the real asset does not affect the incentive-compatibility constraint, and hence the terms of trade, in the high-dividend state.

[18] This result is related to the findings in Banerjee and Maskin (1996), according to which the good that serves as the medium of exchange is the one for which the discrepancy between qualities is smallest.

[19] Strictly speaking, the $\ell-$type buyers can still use the real asset in payments, but because κ_ℓ tends to 0 the amount of output they buy with it approaches 0.

4.1 Nonmonetary equilibrium

In the following, I describe the case where fiat money is not valued, $z^s = z^b = 0$. This special case provides some graphical intuition for the results, and some insights on the role of fiat money.

As shown in Lemma 1, there is no equilibrium of the bargaining game with a pooling offer. The proof is illustrated in the left panel of Figure 2. Consider an equilibrium with a pooling offer $(\bar{q}, \bar{d})$ with $\bar{d} > 0$. (An offer with $\bar{d} < 0$ would not be acceptable since the seller would receive nothing in exchange for some output and some asset.) The surpluses of the two types of buyers at the proposed equilibrium are denoted $U_\ell^b \equiv u(\bar{q}) - \kappa_\ell \bar{d}$ and $U_h^b \equiv u(\bar{q}) - \kappa_h \bar{d}$. The indifference curves U_ℓ^b and U_h^b in Figure 2 represent the set of offers (q, d) that generate the equilibrium surpluses. They exhibit a single-crossing property, which is key to obtain a separating equilibrium. The participation constraint of a seller who believes he is facing an h-type buyer is represented by the frontier $U_h^s \equiv \{(q, d) : -c(q) + \kappa_h d = 0\}$. The offer $(\bar{q}, \bar{d})$ is located above U_h^s since it is accepted when $\lambda < 1$. The shaded area indicates the set of offers that raise the utility of an h-type buyer (offers to the right of U_h^b), but reduce the utility of an ℓ-type buyer (offers to the left of U_ℓ^b), and are acceptable by sellers provided that $\lambda = 1$ (offers above U_h^s). These offers satisfy (14)-(16) with $\chi = h$ so that the proposed equilibrium with a pooling offer $(\bar{q}, \bar{d})$ violates the Intuitive Criterion. In order to separate himself, an h-type buyer reduces his DM consumption as well as his transfer of asset to the seller. Provided that the reduction in q is sufficiently large relative to the reduction in d, an ℓ-type buyer would not choose such an offer because his asset is less valuable than the one of an h-type buyer.

The equilibrium offers are as described in Lemma 2. The buyer in the low-dividend state makes his complete information offer while the buyer in the high-divided state makes the least-cost separating offer. Buyers' offers are illustrated in the right panel of Figure 2 in the case where the constraint $d_\ell \leq a^b$ does not bind. The offer of the ℓ-type buyer is at the tangency point between the iso-surplus curve of the seller, $U_\ell^s \equiv -c(q) + \kappa_\ell d = 0$, and the iso-surplus curve of the buyer, U_ℓ^b. In order to satisfy the seller's participation constraint and the incentive-compatibility condition, type-h buyers make offers to the left of U_ℓ^b, and above U_h^s. The utility-maximizing offer is at the intersection of the two curves. As shown in the Figure, and proved in Proposition 1, $q_h < q_\ell \leq q^*$. Buyers in the high-dividend state always consume less than in the low-dividend state. Despite this inefficiently low consumption, buyers retain a fraction of their real asset holdings (Proposition 1).

Turn to the normative properties of the equilibrium. If $\kappa_\ell a^b \geq c(q^*)$, then the value of the low-dividend

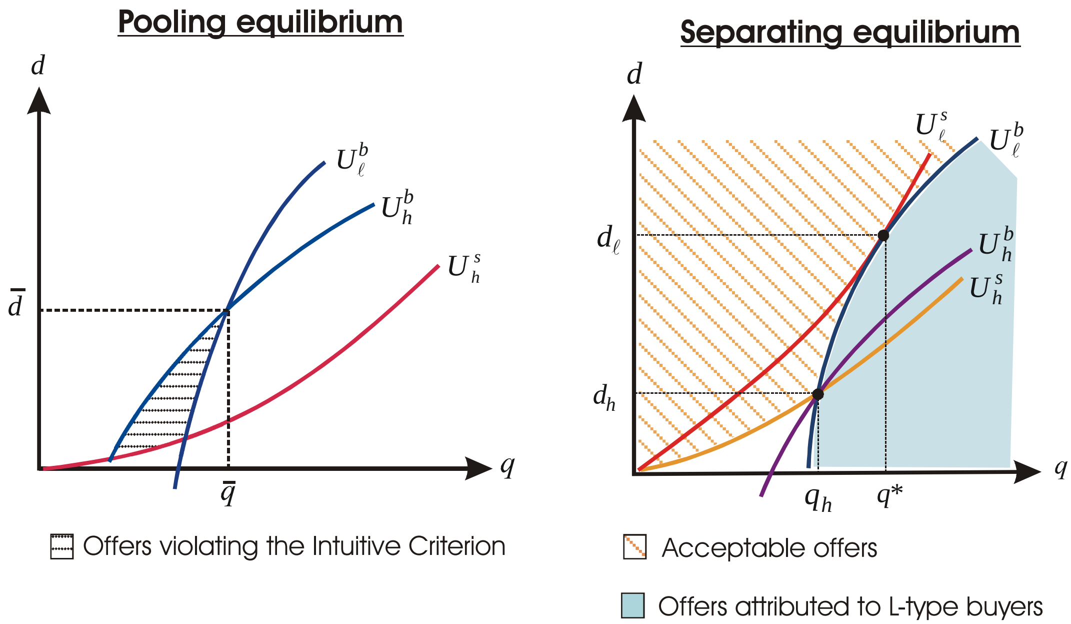

Figure 2: Pooling vs separating equilibria

asset is large enough to trade the first-best quantity, q^*. Under complete information, the economy would achieve its first best. In contrast, if the quality of the asset is private information, then the equilibrium allocation is inefficient. The ℓ-type buyers consume q^*, but h-type buyers consume $q_h < q^*$. If $\kappa_\ell a^b < c(q^*)$, then the quantities traded in the DM are inefficiently low in all matches, i.e., $q_h < q_\ell < q^*$.[20] The inefficiency induced by the private information problem can be shown in a rather dramatic way by looking at the case where κ_ℓ approaches 0, i.e., the asset is valueless in one state. Then, from Proposition 2, θ_h goes to 0 so that buyers do not spend any of their real asset holdings, and the market shuts down ($q_h, q_\ell \to 0$).

As revealed by Proposition 1, by holding real balances the buyer can overcome the inefficiency associated with the private information problem. In particular, if $z^b \geq c(q^*)$, then the first-best allocation is obtained, and buyers do not use the real asset as means of payment in the high-dividend state.

[20] One could also ask whether there exists an incentive-feasible trading mechanism that implements the first-best allocation in the absence of fiat money. Consider a direct mechanism that maps the buyer's type κ into an offer (q, d). Suppose $q_h = q_\ell = q^*$. Then, incentive-compatibility requires $d_h = d_\ell = d$. So the outcome is pooling. The trade (q^*, d) satisfies the seller's individual rationality constraint if $-c(q^*) + \bar{\kappa} d \geq 0$. Similarly, buyers are willing to participate if $u(q^*) - \kappa_h d \geq 0$. Thus, the first-best is incentive-feasible provided that $a^b \geq c(q^*)/\bar{\kappa}$ and $\kappa_h/\bar{\kappa} \leq u(q^*)/c(q^*)$, i.e., there is no shortage of the asset, and the discrepancy between the dividends in the different states is not too large.

5 Asset prices, liquidity, and monetary policy

This section incorporates the bargaining game studied in Section 4 into the general equilibrium structure laid down in Section 2, and it investigates the implications of the model for the relationship between monetary policy, asset liquidity, and asset returns. In order to prevent the asset price in the CM from revealing buyers' private information, it is assumed that buyers learn the future dividend of the real asset when they enter the DM, after they chose their portfolios. The analysis of the bargaining game is simplified by assuming that the buyer's and seller's portfolios are common knowledge in a match in the DM.[21]

The sequence of events is as follows. (See Figure 1.) Agents make a portfolio choice in the CM. At the beginning of the subsequent period, buyers receive a private and fully informative signal about the future dividend of the real asset, while sellers are uninformed. Then, buyers get matched with sellers. An implication of this timing is that the buyer's portfolio does not convey any information about κ. The outcome of the bargaining game between a buyer and a seller in the DM is as described in Section 4. From Proposition 1, the seller's portfolio is irrelevant for the determination of the agents' surpluses in the DM. Hence, the terms of trade $[q(z^b, a^b, \kappa), \tau(z^b, a^b, \kappa), d(z^b, a^b, \kappa)]$ are functions of the buyer's portfolio and his private signal. They solve (17)-(19) if $\kappa = \kappa_\ell$, and (20)-(23) if $\kappa = \kappa_h$.[22]

The missing block of the model is the determination of agents' portfolio choices. The expected lifetime utility of a buyer entering the DM with z units of real balances, a units of the real asset, and a private signal κ, is

$$V^b(z, a, \kappa) = u\left[q(z, a, \kappa)\right] + W^b\left[z - \tau(z, a, \kappa), a - d(z, a, \kappa), \kappa\right]. \tag{31}$$

Using the linearity of W^b, (31) can be reexpressed as

$$V^b(z, a, \kappa_\chi) = S^\chi\left(z, a\right) + z + \kappa_\chi a + W^b\left(0, 0, \kappa_\chi\right), \quad \chi \in \{\ell, h\}, \tag{32}$$

where $S^\chi\left(z, a\right)$ is the buyer's surplus in the DM when the dividend state is χ, i.e.,

$$S^\chi\left(z, a\right) \equiv u\left[q(z, a, \kappa_\chi)\right] - \kappa_\chi d(z, a, \kappa_\chi) - \tau(z, a, \kappa_\chi) \text{ for } \chi \in \{\ell, h\}.$$

[21] This assumption is made in order to avoid having to specify the agents' beliefs regarding the portfolio held by their partner in the match. It will be shown in the following that the surplus functions in the DM are weakly monotone increasing in the agent's asset holdings. Hence, if agents had the possibility to show their portfolios in a pre-stage of the bargaining game, I conjecture that there would be an equilibrium where they would do so truthfully.

[22] The solutions to (17)-(19) and (20)-(23) might not be unique, e.g., if $z^b > c(q^*)$. With no loss, one can select a solution such that $d \geq 0$ since it is feasible irrespective of the seller's portfolio.

Substituting V^b by its expression given by (32) into (10), the buyer's portfolio problem in the CM reduces to

$$[z(j), a(j)] \in \arg \max_{(z,a)\in\mathbb{R}_{2+}} \left\{ -iz - \left(\frac{\phi - \beta\bar{\kappa}}{\beta} \right) a + \pi_h S^h(z,a) + \pi_\ell S^\ell(z,a) \right\}, \quad \forall j \in \mathcal{B}, \tag{33}$$

where $i = (\gamma - \beta)/\beta$ is the cost of holding real balances, and $(\phi - \beta\bar{\kappa})/\beta$ is the cost of investing in the real asset. According to (33), buyers choose their portfolios in order to maximize their expected surplus in the DM, net of the cost of holding real balances and capital.

Since sellers obtain no surplus from the DM trades, their expected lifetime utility upon entering the DM is

$$V^s(z,a) = \mathbb{E}W^s(z,a,\kappa) = z + \bar{\kappa}a + \mathbb{E}W^s(0,0,\kappa). \tag{34}$$

Substitute V^s by its expression given by (34) into (11) to reduce the seller's portfolio problem to

$$[z(j), a(j)] \in \arg \max_{(z,a)\in\mathbb{R}_{2+}} \left\{ -iz - \left(\frac{\phi - \beta\bar{\kappa}}{\beta} \right) a \right\}, \quad \forall j \in \mathcal{S}. \tag{35}$$

Finally, the clearing of the asset market implies

$$\int_{j\in\mathcal{J}} a(j)dj = A. \tag{36}$$

In the following, $Z \equiv \int_{j\in\mathcal{J}} z(j)dj$ represents aggregate real balances, and M_{t+1}/Z is the price level in the CM of period t.

Definition 2 *An equilibrium is a list of portfolios, terms of trade in the DM, the price of capital, and aggregate real balances,* $\langle [z(j), a(j)]_{j\in\mathcal{J}}, [q(\cdot), d(\cdot), \tau(\cdot)], \phi, Z \rangle$ *such that:*

(i) $[z(j), a(j)]$ *is solution to (33) for all* $j \in \mathcal{B}$ *and (35) for all* $j \in \mathcal{S}$*;*

(ii) For all $(z,a) \in \mathbb{R}_{2+}$*,* $[q(z,a,\kappa), d(z,a,\kappa), \tau(z,a,\kappa)]$ *is solution to (17)-(19) if* $\kappa = \kappa_\ell$ *and (20)-(23) if* $\kappa = \kappa_h$*;*

(iii) ϕ *solves (36);*

(iv) $Z = \int_{j\in\mathcal{J}} z(j)dj$*.*

The next two lemmas characterize the buyers' and sellers' portfolio choices.

Lemma 3 ***(Sellers' portfolio choices)***

Consider the seller's portfolio problem in (35).

1. *For all* $i > 0$, $z = 0$.

2. *If* $\phi > \beta\bar{\kappa}$, *then* $a = 0$. *If* $\phi = \beta\bar{\kappa}$, *then* $a \in [0, \infty)$. *If* $\phi < \beta\bar{\kappa}$, *then (35) has no solution.*

The proof, which is immediate from (35), is omitted. Holding fiat money is costly, due to inflation and discounting, and sellers get no surplus from their trades in the DM. Hence, they hold no real balances. Similarly, sellers hold the real asset only if its price is equal to its fundamental value, as defined by the expected discounted dividend.

Let S_z^χ and S_a^χ denote the partial derivatives of the surplus function for $\chi \in \{\ell, h\}$. These quantities represent the liquidity values of fiat money and the real asset, at the margin, in the DM in the dividend state χ.

Lemma 4 ***(Buyers' portfolio choices)***

Consider the buyer's portfolio problem in (33). If $\phi > \beta\bar{\kappa}$, *then there is a unique solution to (33), and it satisfies*

$$-i + \pi_h S_z^h(z, a) + \pi_\ell S_z^\ell(z, a) \quad \leq \quad 0 \quad \text{“} = \text{”} \quad \text{if } z > 0, \tag{37}$$

$$-\frac{\phi - \beta\bar{\kappa}}{\beta} + \pi_h S_a^h(z, a) + \pi_\ell S_a^\ell(z, a) \quad \leq \quad 0 \quad \text{“} = \text{”} \quad \text{if } a > 0, \tag{38}$$

where

$$S_z^\ell \quad = \quad \frac{S_a^\ell}{\kappa_\ell} = \frac{u'(q_\ell)}{c'(q_\ell)} - 1 \tag{39}$$

$$S_z^h \quad = \quad \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] \left[\frac{u'(q_\ell)/c'(q_\ell) - \kappa_\ell/\kappa_h}{u'(q_h)/c'(q_h) - \kappa_\ell/\kappa_h}\right] \tag{40}$$

$$S_a^h \quad = \quad \kappa_h \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] \frac{\kappa_\ell}{\kappa_h} \left[\frac{u'(q_\ell)/c'(q_\ell) - 1}{u'(q_h)/c'(q_h) - \frac{\kappa_\ell}{\kappa_h}}\right]. \tag{41}$$

If $\phi = \beta\bar{\kappa}$, *then* z *is uniquely determined, and* $a \in [\frac{c(q^*)-z}{\kappa_\ell}, \infty)$. *If* $\phi < \beta\bar{\kappa}$, *then there is no solution to (33).*

If the price of the real asset is greater than its fundamental value, i.e., $\phi - \beta\bar{\kappa} > 0$, then the composition of the buyer's optimal portfolio is unique. This result is a consequence of Lemma 1 according to which fiat money and the real asset are imperfect substitutes, i.e., fiat money is a preferred means of payment.

From (37) and (38), for an asset to be held, its cost must be equal to the marginal benefit that the asset confers in the DM. According to (39), a marginal unit of asset (expressed in terms of CM output) allows the buyer to purchase $1/c'(q_\ell)$ units of DM output when the dividend state is low; this additional output is valued according to the marginal surplus of the match, $u'(q_\ell) - c'(q_\ell)$. The first term in brackets on the right side of (40) is the liquidity value of real balances in the high-dividend state, in the complete information economy. It is the analog of (39). This term is multiplied by $1 + \frac{\partial(\kappa_h d_h)}{\partial z^b} < 1$ (see (29)) because, in the private information economy, the buyer with an additional unit of real balances reduces his transfer of real asset in order to mitigate the informational asymmetry in the match. Similarly, the first two terms on the right side of (41) correspond to the liquidity value of the real asset in the high-dividend state in the complete information economy. This liquidity component is multiplied by the marginal propensity to spend the real asset, $\frac{\partial d_h}{\partial a^b} \in [0, 1)$, which is less than one in the private information economy (see (30)).

If the real asset is priced according to its fundamental value, $\phi = \beta\bar{\kappa}$, then the buyer's choice of the real asset is indeterminate: buyers accumulate enough wealth to buy the first-best quantity of output when $\kappa = \kappa_\ell$. The choice of real balances, however, is always unique.

The next proposition proves the existence of an equilibrium.

Proposition 4 ***(Equilibrium allocations and prices)***

1. *An equilibrium exists, and it is such that ϕ, Z, and $(q_\ell, q_h, d_h, \tau_h)$ are uniquely determined. Moreover, $\phi \in [\beta\bar{\kappa}, \beta\bar{\kappa} + i\beta\kappa_\ell]$.*

2. *For all $A > 0$, there is a $i_0(A) > 0$ such that the equilibrium is monetary ($Z > 0$) if and only if $i < i_0(A)$.*

An equilibrium exists, and it is essentially unique. Interestingly, the equilibrium has a simple recursive structure. Aggregate real balances, Z, are determined from (37) where $a = A$. Given Z, the asset price, ϕ, comes from (38). Given ϕ, one can use Lemmas 3 and 4 to determine the agents' portfolios. Given (z^b, a^b) and (z^s, a^s), Lemma 2 generates the allocation in the DM.

In Figure 3, $A^d(\phi)$ represents the aggregate demand for the real asset. From the proof of Proposition 4, $A^d(\phi)$ is upper-hemi continuous, and any selection from $A^d(\phi)$ is decreasing. Hence, the asset price is unique, at the intersection of A and $A^d(\phi)$.

The equilibrium is monetary for all A provided that the cost of holding real balances, i, is sufficiently

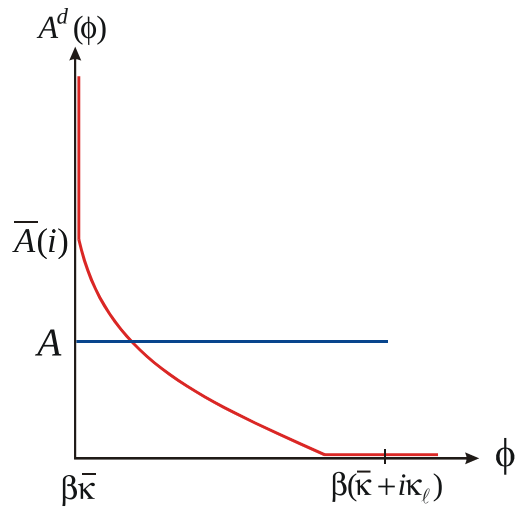

Figure 3: Aggregate demand of the real asset, $A^d(\phi)$.

low. From (37) and (39)-(40), the liquidity return of fiat money is

$$i = \pi_\ell \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right] + \pi_h \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] \left[\frac{\kappa_h u'(q_\ell)/c'(q_\ell) - \kappa_\ell}{\kappa_h u'(q_h)/c'(q_h) - \kappa_\ell}\right]. \tag{42}$$

If A is large, then the return of fiat money is $\pi_h (\kappa_h - \kappa_\ell) \left[\frac{u'(q_h)/c'(q_h)-1}{\kappa_h u'(q_h)/c'(q_h)-\kappa_\ell}\right]$, which is bounded away from 0 since $q_h < q^*$. Money is useful, even for large values of A, because it overcomes the illiquidity of the real asset in the high-dividend state, i.e., it relaxes the incentive-compatibility constraint faced by buyers. In contrast, in the complete information economy, fiat money is valued only if the stock of the real asset is not large enough to allow buyers to trade q^* when $\kappa = \kappa_\ell$.[23] (See Figure 4).

From (38) and (41)-(39), the equilibrium price of the real asset is

$$\phi = \beta\bar{\kappa} + \pi_\ell \beta \kappa_\ell \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right] + \pi_h \beta \kappa_h \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] \left[\frac{\kappa_\ell u'(q_\ell)/c'(q_\ell) - \kappa_\ell}{\kappa_h u'(q_h)/c'(q_h) - \kappa_\ell}\right]. \tag{43}$$

The first term on the right side of (43) is the fundamental value of the asset, and the last two terms are the liquidity values in the DM in the different states. In order to isolate the contribution of the private information friction to the liquidity of the asset, one can rewrite the asset price as $\phi = \beta\bar{\kappa}\left(1 + \mathcal{L}^1 + \mathcal{L}^2\right)$

[23] The return of fiat money in the complete information economy is $\pi_\ell \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right] + \pi_h \left[\frac{u'(q_h)}{c'(q_h)} - 1\right]$. The difference with the right side of (42) stems from the last term in brackets, $1 + \kappa_h \frac{\partial d_h}{\partial z^b}$, which captures the fact that when the buyer accumulates additional real balances, he reduces the quantity of real asset he transfers to the seller $\left(\frac{\partial d_h}{\partial z^b} < 0\right)$.

with

$$\mathcal{L}^1 = \pi_\ell \frac{\kappa_\ell}{\bar{\kappa}} \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1 \right] + \pi_h \frac{\kappa_h}{\bar{\kappa}} \left[\frac{u'(q_h)}{c'(q_h)} - 1 \right] \geq 0 \tag{44}$$

$$\mathcal{L}^2 = \pi_h \frac{\kappa_h}{\bar{\kappa}} \left[\frac{u'(q_h)}{c'(q_h)} - 1 \right] \left[\frac{\kappa_\ell u'(q_\ell)/c'(q_\ell) - \kappa_h u'(q_h)/c'(q_h)}{\kappa_h u'(q_h)/c'(q_h) - \kappa_\ell} \right] \leq 0. \tag{45}$$

In the complete information economy, $\phi = \beta\bar{\kappa}\left(1 + \mathcal{L}^1\right)$.[24] The private information friction adds the component $\mathcal{L}^2$, which is negative. This negative contribution results from the last term on the right side of (45), $\frac{\partial d_h}{\partial a_b} - 1 < 0$, according to which the buyer's marginal propensity to spend the real asset is less than one.

The next Proposition determines the condition under which the liquidity premium, defined as $\mathcal{L} = \mathcal{L}^1 + \mathcal{L}^2$, is positive, and the expected return of the real asset, denoted $R_a = \bar{\kappa}/\phi$, is less than the discount rate.

Proposition 5 ***(Liquidity premium and asset returns)***

For all $i > 0$, there is $\bar{A}(i) \in [0, c(q^*)/\kappa_\ell]$ such that:

1. *For all $A \geq \bar{A}(i)$, $\mathcal{L} = 0$ and $R_a = \beta^{-1}$.*
2. *For all $A < \bar{A}(i)$, $\mathcal{L} > 0$ and $R_a < \beta^{-1}$. Moreover, $\frac{\partial \mathcal{L}}{\partial A} < 0$ and $\frac{\partial R_a}{\partial A} > 0$.*

The liquidity premium emerges if the real asset is relatively scarce, i.e., $A < c(q^*)/\kappa_\ell$, and inflation is sufficiently large, $i > \bar{A}^{-1}(A)$. If these conditions hold, buyers' wealth in the low dividend state is not large enough to allow buyers to ask for the first-best quantity, i.e., $q_\ell < q^*$. An increase in the supply of the real asset reduces the liquidity premium, and raises the rate of return of the real asset.[25] On the contrary, if the real asset is sufficiently abundant to allow buyers to consume q^* in the low-dividend state, then its price is equal to its fundamental value–which is independent of monetary policy–and its expected rate of return is equal to the gross discount rate.[26]

The next proposition describes the effects of monetary policy on the liquidity and expected return of the real asset. The liquidity of the real asset is measured by its transaction velocity (or turnover) in the high-dividend state, $\mathcal{V}_h \equiv d_h/A$.[27]

[24] See Appendix D for a derivation of the asset price in the complete information economy.

[25] The result according to which the size of the liquidity premium depends on the supply of the asset is consistent with the findings of Krishnamurthy and Vissing-Jorgensen (2008) for Treasury debt. One could replace fiat money in my model by default-free, real bonds. An increase in the supply of bonds would then reduce the liquidity premia of all assets.

[26] The expression for $\bar{A}(i)$ is provided in the proof of Proposition 5. If $i < i_0(A)$, i.e., fiat money is valued, it can be shown that $\bar{A}(i)$ is strictly increasing.

[27] This definition of asset liquidity is also the one used in Wallace (2000). Kiyotaki and Wright (1989) have argued that

Proposition 6 *(Monetary policy and asset prices)*

1. *If* $i < i_0(A)$*, then* $d\mathcal{V}_h/di > 0$*.*

2. *If* $i < i_0(A)$ *and* $A < \bar{A}(i)$*, then* $d\mathcal{L}/di > 0$ *and* $dR_a/di < 0$*.*

3. *As* $i \to 0$*,* $\mathcal{V}_h \to 0$*,* $\mathcal{L} \to 0$ *and* $R_a \to \beta^{-1}$*.*

A lower rate of return of fiat money induces buyers to hold fewer real balances, and to turn to the real asset as a means of payment. As a consequence, the turnover of the real asset ($\mathcal{V}_h$) increases. Because the real asset is in fixed supply, an increase in the aggregate demand for the real asset translates into a higher price. As a corollary, the model predicts a negative relationship between inflation and expected asset returns.[28]

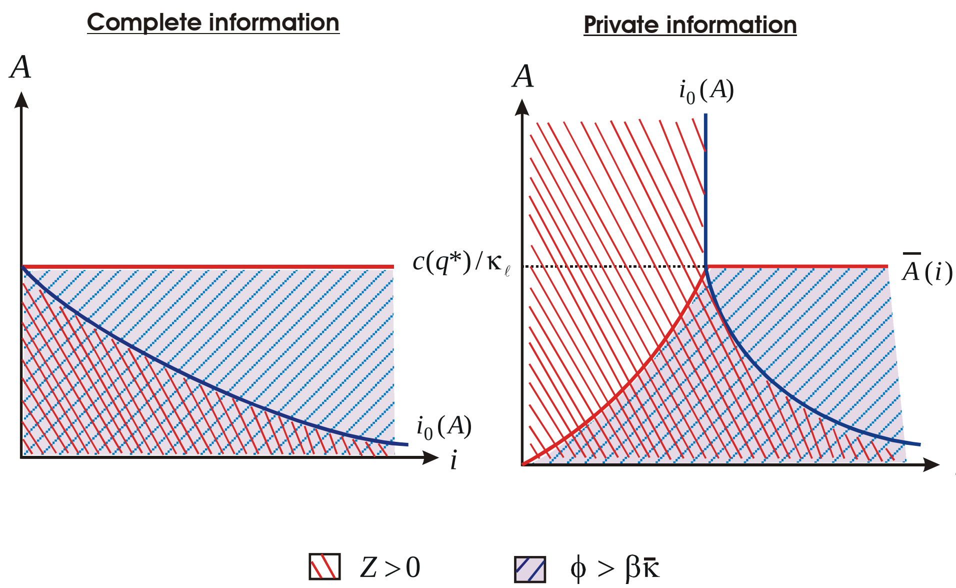

Figure 4: Liquidity premium and the value of money

velocity is not always a good measure of moneyness. They showed that objects can have a high transactions velocity even though they are accepted in bilateral trade with low probability, because the endogenous stock of the object is low. To avoid this concern, the stock of the asset in my model is kept fixed. Since the payment arrangement in the low-dividend state is indeterminate when $\phi = \beta\bar{\kappa}$, I focus on the velocity in the high-dividend state. The velocity of the asset in the low-dividend state is equal to 1 if $A < \bar{A}(i)$. In the case where $A > \bar{A}(i)$, one could adopt the convention that buyers use their money first, i.e., $\mathcal{V}_\ell = [c(q^*) - z]/\kappa_\ell A$ and $d\mathcal{V}_\ell/di > 0$ provided that $i < i_0(A)$.

[28] The negative relationship between equity returns and inflation has been extensively documented. See Marshall (1992) for references. Theoretical models of this relationship are provided by Danthine and Donaldson (1986) and Marshall (1992). Both models assume the liquidity services of fiat money through a money-in-the-utility-function assumption or a shopping time technology.

As i tends to zero, then q_ℓ and q_h approach q^*. Hence, the optimal policy drives the cost of holding money to 0. In the high-dividend state, buyers trade with money only ($d_h \to 0$), while in the low-dividend state buyers are indifferent between using money or the real asset as means of payment.[29] The price of the real asset converges to its fundamental value ($\phi \to \beta\bar{\kappa}$).

The next Proposition looks at the implications of the model for the structure of asset returns. The rate of return of fiat money is $R_z = \gamma^{-1}$, and the rate of return of the real asset is $R_a = \bar{\kappa}/\phi = \beta^{-1}\left(1 + \mathcal{L}^1 + \mathcal{L}^2\right)^{-1}$.

Proposition 7 ***(Distribution of asset returns)***

In any monetary equilibrium, $R_a > R_z$.

The expected rate of return of the real asset is always greater than the rate of return of fiat money (provided that it is valued). Equivalently, $i > \mathcal{L}^1 + \mathcal{L}^2$. The liquidity return of fiat money in the DM is greater than the liquidity return of the real asset. The rate-of-return differential between fiat money and the real asset is not generated by restrictions on payment arrangements, as in cash-in-advance models. Moreover, because of linear preferences with respect to the CM consumption, the riskiness of the real asset does not affect its rate of return through the standard risk-aversion component of the pricing kernel.[30] Risk matters here for two reasons.

First, the riskiness of the real asset generates a covariance between its dividend and the marginal value of wealth in the DM (see (44)). This covariance term is negative in the complete information economy because a high dividend is associated with a high wealth. As a consequence, the liquidity return of the real asset is lower than the liquidity return of fiat money, which explains why a rate-of-return dominance pattern can also emerge from a complete information economy.[31] In contrast, this covariance term is positive in the private information economy because DM output is lower in the high-dividend state relative to the low-dividend state. (See Proposition 1.)

Second, the riskiness of the real asset makes the informational asymmetry between buyers and sellers relevant. It is because the dividend of the real asset can take different values, and because buyers have

[29] Since the equilibrium correspondence is only upper-hemi continuous at $i = 0$, the focus is on the equilibria that are obtained by taking the limit as i approaches 0. There are several ways to get the optimal monetary policy to differ from the Friedman rule. If one assumes limited coercion power by the government, then the Friedman rule might not be incentive-feasible. See footnote 8. Also, the Friedman rule may not longer be optimal if agents have strictly concave preferences and face idiosyncratic trading shocks. See Zhu (2006) and Waller (2007).

[30] Krishnamurthy and Vissing-Jorgensen (2008) also distinguish the security motive for holding an asset and risk aversion. They define surety as "a value investors place on a sure cash-flow above and beyond what would be implied by the pricing kernel."

[31] See the Appendix D.

some private information about the future value of the asset, that fiat money becomes a preferred means of payment. The private information problem reduces the liquidity premium that accrues to the real asset, and it increases the rate of return differential between fiat money and risky capital. To see this, notice from Proposition 4 that the liquidity premium of the real asset, $(\phi - \beta\bar{\kappa})/\beta\bar{\kappa}$, is bounded above by $\kappa_\ell i/\bar{\kappa}$, which tends to zero as the dividend in the low state becomes small. Moreover, provided that the stock of the real asset is sufficiently large ($A > \bar{A}(i)$), the rate of return of the real asset is maximum and equal to the gross discount rate, $R_a = \beta^{-1}$. In this case, the rate of return of the asset is the one that would prevail in a cash-in-advance economy where the real asset cannot serve as means of payment in the DM.[32] In contrast, under complete information the rate of return of the real asset in any monetary equilibrium is bounded away from β^{-1}.

The next Proposition investigates how the discrepancy between the dividend in different states affects the structure of asset returns.

Proposition 8 ***(Asset prices and fundamentals)***

1. *If $\kappa_\ell/\kappa_h = 1$, then a monetary equilibrium exists if and only if $A < c(q^*)/\bar{\kappa}$ and $i < \frac{u'[c^{-1}(\bar{\kappa}A)]}{c'[c^{-1}(\bar{\kappa}A)]} - 1$. Under these conditions, $R_a = R_z = \gamma^{-1}$.*

2. *If $\kappa_\ell \to 0$, then a monetary equilibrium always exists. Moreover, $\mathcal{V}_h \to 0$, $R_a \to \beta^{-1} > R_z$.*

If the real asset is safe, then a monetary equilibrium exists only if there is a shortage of the real asset to be used as means of payment. The rate of return of the real asset is then equal to the rate of return of fiat money. Graphically, the vertical portion of the curve $i_0(A)$ in Figure 4 coincides with the vertical axis. In contrast, if the dividend in the low state becomes very small, then the turnover of the asset in the high-dividend state goes to 0, and its rate of return approaches its maximum given by the discount rate.

The Proposition above is complemented by a simple numerical example based on the following specifications: $u(q) = 2\sqrt{q}$, $c(q) = q$, $\beta = 0.95$, $\kappa_\ell = 1 - \sigma$, $\kappa_h = 1 + \sigma$, $\pi_h = \pi_\ell = 0.5$ and $A = 1$. The mean of the dividend is equal to 1 while its variance is σ^2. I consider the effects of a change in σ on the turnover of the asset and its liquidity premium.

[32] These results can have interesting empirical implications for asset pricing puzzles (provided that one reinterprets currency as risk-free bonds). Lagos (2006) showed that a standard search model of exchange can generate an equity premium as large as in the data (for plausible degrees of risk aversion) provided that equity is partially illiquid. While the illiquidity arises from legal or institutional restrictions in Lagos (2006), it is directly related to the dividend process here.

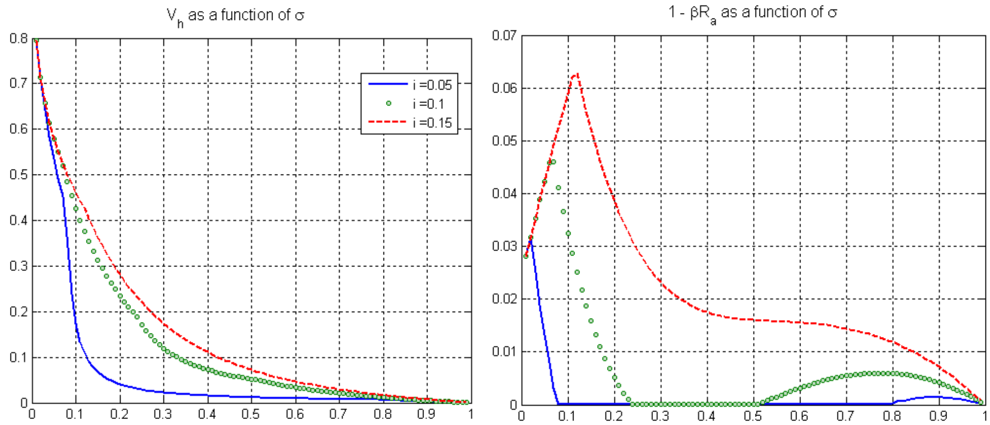

Figure 5: Asset liquidity

The left panel of Figure 5 represents the turnover of the asset in the high state, $\mathcal{V}_h = d_h/A$. As σ increases, the fraction of the asset that is used as means of payment in the high-dividend state decreases. As σ approaches one, the real asset becomes fully illiquid, i.e., fiat money is the only means of payment in the DM.

The right panel of Figure 5 plots the difference between the price of the asset and its fundamental value as a fraction of the asset price, $\frac{\phi - \beta\bar{\kappa}}{\phi} = 1 - \beta R_a$. Recall that the rate of return of fiat money is constant and equal to γ^{-1}. If σ is close to 0 (i.e., the asset is safe), then the equilibrium is non-monetary. An increase in the riskiness of the asset raises its liquidity value because the buyer's wealth constraint tightens in the low-dividend state. Above a threshold for σ fiat money becomes valued. An increase in the riskiness of the real asset makes fiat money more valuable, and it reduces the liquidity value of the asset. The liquidity premium tends to 0 as σ approaches one.[33]

[33] The negative relationship between liquidity and risk is consistent with Longstaff, Mithal, and Neis (2005) and Krishnamurthy and Vissing-Jorgensen (2008) who argue that assets that are less risky (e.g., Tresuriy securities or the highest grade corporate bonds) have a higher liquidity (convenience) value. Also, Amihud, Mendelson and Pedersen (2005, p.47) argue that is difficult to distinguish empirically the effects of illiquidity and risk on stock returns because of a strong relationship between the two.

6 Conclusion

The objective of this paper is to provide a monetary theory of asset liquidity that emphasizes the role of assets in payment arrangements. The main ingredients of the model are the presence of multiple assets, fiat money and risky equity, that are traded in both centralized and decentralized markets, and an informational asymmetry between agents paying with an asset, and agents receiving the asset. I have explored the implications of the theory for the relationship between assets' intrinsic characteristics and liquidity, and the effects of monetary policy on asset prices and welfare.

A recurrent theme of this paper is that the liquidity of the real asset, measured either by its turnover or by its liquidity premium, depends on the properties of its dividend process. An asset which is riskier tends to be less liquid. In support of this finding, Krishnamurthy and Vissing-Jorgensen (2008) argue that half of the convenience yield of Treasury security relative to corporate bonds can be explained by a *surety* motive, where *surety* is the "value investors place on a sure cash-flow above and beyond what would be implied by the pricing kernel". Similarly, Longstaff, Mithal, and Neis (2005) find that the highest-rated corporate bonds exhibit a lower yield spread relative to Treasury securities once the default component is removed. Spiegel and Wang (2005) find a strong negative correlation between liquidity and idiosyncratic risk in stock returns.

If one reinterprets fiat money as risk-free government bonds, then the model has macroeconomic implications for the risk-free rate and equity premium puzzles. It predicts that the rate of return of government bonds is less than the rate of time preference (the risk-free rate puzzle) and risky equity commands a higher return than risk-free bonds (the equity premium puzzle), despite agents' being risk-neutral. By introducing a more standard pricing kernel, Lagos (2006) shows that a monetary model with exogenous liquidity constraints can generate the observed equity premium for plausible measures of risk aversion. My model can relate the illiquidity of the equity to its dividend process, thereby providing a greater check on the theory.

My model also predicts a negative relationship between the liquidity component of asset returns and the supply of assets. This is consistent with Krishnamurthy and Vissing-Jorgensen (2008) who find a negative relationship between the yield spread between corporate bonds and Treasury securities and the U.S. government debt-to-GDP ratio, based on annual observations from 1925 to 2005. Interestingly, this aggregate demand for Treasury debt is reminiscent to the aggregate money demand used by Lucas (2000) to assess the welfare cost of inflation.[34] Microfounded models–such as the search-theoretic model I have been building

[34] Lucas found that the welfare cost of 10 per cent inflation is about 1 per cent of GDP per year. Krishnamurthy and

upon–that proved useful to assess the welfare cost of inflation can also provide new insights for the social value of liquid assets.

In terms of monetary policy, Lagos and Rocheteau (2008), Geromichalos, Licari, and Suarez-Lledo (2007), and Lagos (2006) find that, in the absence of liquidity constraints, a monetary equilibrium ceases to exist if the supply of assets is sufficiently large. In contrast, my model has the more realistic prediction that a monetary equilibrium exists irrespective of the supply of assets, provided that the inflation rate is not too large. The rate of return of the asset is negatively correlated with inflation, in accordance with Danthine and Donaldson (1986) and Marshall (1992). In contrast to earlier works, my model can be used to relate the inflation-elasticity of the asset price to the characteristics of the asset.

The natural next step is to construct a calibrated version of the model incorporating more realistic features, such as risk aversion, infinitely-lived assets, and a richer information structure. A more standard pricing kernel could be obtained by adopting the three-sector model of Lagos (2006), or the overlapping-generations model of Zhu (2008). Our approach could also be useful to revisit the relationship between inflation and capital, as in Aruoba, Waller, and Wright (2007). I leave these quantitative investigations of the model to future research.

Vissing-Jorgensen found that "the value of the liquidity provided by the current level of Treasuries is around 0.95 per cent of GDP per year".

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A. Proofs of lemmas and propositions

Proof of Lemma 1. The proof shows that any equilibrium with a pooling offer $(\bar{q}, \bar{d}, \bar{\tau})$ can be dismissed by the Intuitive Criterion. It distinguishes the case where $\bar{d} > 0$ from the case where $\bar{d} < 0$.

(i) Suppose first that $\bar{d} > 0$. By definition, the buyers' payoffs at the proposed equilibrium are $U_h^b = u(\bar{q}) - \kappa_h \bar{d} - \bar{\tau} \geq 0$ and $U_\ell^b = u(\bar{q}) - \kappa_\ell \bar{d} - \bar{\tau} \geq 0$. A necessary condition for $(\bar{q}, \bar{d}, \bar{\tau})$ to be acceptable when $\lambda(\bar{q}, \bar{d}, \bar{\tau}) < 1$ is $-c(\bar{q}) + \kappa_h \bar{d} + \bar{\tau} > 0$. Define $F \equiv \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b]$ the subspace of $\mathbb{R}^3$ of feasible offers. Let

$$O_1 \equiv \{(q, d, \tau) \in F : -c(q) + \kappa_h d + \tau > 0\}.$$

Then, O_1 is open in F, and it contains $(\bar{q}, \bar{d}, \bar{\tau})$. Let

$$O_2 \equiv \{(q, d, \tau) \in F : \kappa_\ell \left(\bar{d} - d\right) < [u(\bar{q}) - \bar{\tau}] - [u(q) - \tau] < \kappa_h \left(\bar{d} - d\right)\}.$$

Then, O_2 is open in F, it is not empty, and its closure is $\overline{O}_2 \ni (\bar{q}, \bar{d}, \bar{\tau})$.[35] Consequently, any open ball centered at $(\bar{q}, \bar{d}, \bar{\tau})$ has a non-empty intersection with O_2. Moreover, by definition of an open set, there exists a radius $\varepsilon > 0$ such that the open ball $\mathcal{B}((\bar{q}, \bar{d}, \bar{\tau}), \varepsilon) \subset O_1$, and hence $\mathcal{B}((\bar{q}, \bar{d}, \bar{\tau}), \varepsilon) \cap O_2 \neq \varnothing$. Consequently, there is $(\tilde{q}, \tilde{d}, \tilde{\tau}) \in O_1 \cap O_2$ that satisfies (14)-(16) with $\chi = h$, and the proposed pooling equilibrium fails the Intuitive Criterion.

(ii) Suppose next that $\bar{d} < 0$. The participation constraint of the seller implies $\bar{\tau} > 0$. Since $\bar{d} < 0$ and $\lambda(\bar{q}, \bar{d}, \bar{\tau}) \in (0, 1)$ then

$$0 \leq -c(\bar{q}) + \lambda(\bar{q}, \bar{d}, \bar{\tau})\kappa_h \bar{d} + \left[1 - \lambda(\bar{q}, \bar{d}, \bar{\tau})\right] \kappa_\ell \bar{d} + \bar{\tau} < -c(\bar{q}) + \kappa_\ell \bar{d} + \bar{\tau}.$$

Define

$$\begin{aligned} O_1 &\equiv \{(q, d, \tau) \in F : -c(q) + \kappa_\ell d + \tau > 0\} \\ O_2 &\equiv \{(q, d, \tau) \in F : \kappa_h \left(\bar{d} - d\right) < [u(\bar{q}) - \bar{\tau}] - [u(q) - \tau] < \kappa_\ell \left(\bar{d} - d\right)\}. \end{aligned}$$

In order to show that $O_2 \neq \varnothing$, one can construct an offer $(q, d, \tau) \in F$ such that $q = \bar{q}$, $\bar{d} - d < 0$, $\tau - \bar{\tau} < 0$, and $\kappa_\ell < \frac{\tau - \bar{\tau}}{\bar{d} - d} < \kappa_h$, where $\tau - \bar{\tau}$ and $\bar{d} - d$ can be made arbitrarily close to 0. By the same reasoning as in

[35] To show that O_2 is not empty, one can construct an element $(q, d, \tau) \in F$ such that $\tau = \bar{\tau}$ and $\kappa_\ell < \frac{u(\bar{q}) - u(q)}{\bar{d} - d} < \kappa_h$ where $u(\bar{q}) - u(q) > 0$ and $\bar{d} - d > 0$ can be made arbitrarily small. To show that $(\bar{q}, \bar{d}, \bar{\tau})$ is in the closure of O_2, consider a sequence $\{(q_n, d_n, \tau_n)\}_{n=1}^{\infty}$ such that $\tau_n = \bar{\tau}$, $u(\bar{q}) - u(q_n) > 0$, $\bar{d} - d_n > 0$ and $\kappa_\ell < \frac{u(\bar{q}) - u(q_n)}{\bar{d} - d_n} < \kappa_h$ for all $n \in \mathbb{N}$ and $(q_n, d_n) \to (\bar{q}, \bar{d})$. All the terms of the sequence are in O_2 and it converges to $(\bar{q}, \bar{d}, \bar{\tau})$.

(i), there is $(\tilde{q}, \tilde{d}, \tilde{\tau}) \in O_1 \cap O_2$ that satisfies (14)-(16) with $\chi = \ell$, and the proposed pooling equilibrium fails the Intuitive Criterion. ■

Proof of Lemma 2. **(i) Offer by the ℓ-type buyer.** Provided that $d \geq 0$, any offer that satisfies (18) is acceptable since

$$-c(q) + \{\lambda(q,d,\tau)\kappa_h + [1-\lambda(q,d,\tau)]\,\kappa_\ell\}\, d + \tau \geq -c(q) + \kappa_\ell d + \tau \geq 0,$$

for all $\lambda(q,d,\tau) \in [0,1]$. Moreover, the complete information offer is such that $\kappa_\ell d + \tau \geq 0$, and hence the requirement $d \geq 0$ is not binding. Consequently, the complete information payoff for the ℓ-type buyer can always be achieved. A payoff strictly greater than the complete information payoff is obtained only if (18) is violated, i.e.,

$$-c(q) + \kappa_\ell d + \tau < 0. \tag{46}$$

But an offer is acceptable if

$$-c(q) + \{\lambda(q,d,\tau)\kappa_h + [1-\lambda(q,d,\tau)]\,\kappa_\ell\}\, d + \tau \geq 0. \tag{47}$$

From (46) and (47), $\kappa_\ell d < \{\lambda(q,d,\tau)\kappa_h + [1-\lambda(q,d,\tau)]\,\kappa_\ell\}\, d$, and hence $d > 0$ and $\lambda(q,d,\tau) > 0$. This has been ruled out by Lemma 1.

(ii) Offer by the h-type buyer. Suppose there is an equilibrium where the ℓ-type buyer achieves his complete information payoff, $U_\ell^b = u(q_\ell) - c(q_\ell)$, and the expected payoff of the h-type is $U_h^b \in [0, \bar{U})$, where $\bar{U}$ is the payoff associated with the solution to (20)-(23). For $\varepsilon > 0$, define U^ε as

$$U^\varepsilon = \max_{q,\tau,d} \left[u(q) - \kappa_h d - \tau\right] \tag{48}$$

$$\text{s.t.} \quad -c(q) + \kappa_h d + \tau \geq 0 \tag{49}$$

$$u(q) - \kappa_\ell d - \tau \leq U_\ell^b - \varepsilon \tag{50}$$

$$-z^s \leq \tau \leq z^b, \quad -a^s \leq d \leq a^b. \tag{51}$$

The set of acceptable and feasible offers is compact, and it is nonempty provided that $\varepsilon < U_\ell^b$. From the Theorem of the Maximum, U^ε is continuous in ε, and $\lim_{\varepsilon \to 0} U^\varepsilon = \bar{U}$. Hence, there is an $\varepsilon > 0$ such that $U^\varepsilon > U_h^b$. The associated offer satisfies (14)-(16), i.e., the proposed equilibrium violates the Intuitive Criterion. Finally, suppose that $U_h^b > \bar{U}$. Then, either the seller's participation constraint, (21), or the incentive-compatibility constraint, (22), are violated. Suppose $-c(q) + \kappa_h d + \tau < 0$. The offer is acceptable if (47) holds, which implies $\kappa_h d < \{\lambda(q, d, \tau)\kappa_h + [1 - \lambda(q, d, \tau)]\,\kappa_\ell\}\, d$, and hence $d < 0$ and $\lambda(q, d, \tau) < 1$. This has been ruled out by Lemma 1. If $u(q) - \kappa_\ell d - \tau > U_\ell^b$ then ℓ-type buyers can achieve a payoff strictly greater than their complete information payoff, which contradicts (i). ■

Proof of Proposition 1. The buyer's objective function in (20) is continuous, and it is maximized over a non-empty, compact set. Hence, by the Theorem of the Maximum, there is a solution to (20)-(23). If $a^b = 0$, then the maximization to (20) subject to (21) gives $q_h = \min\left[q^*, c^{-1}(z^b)\right] = q_\ell$, $\tau_h + \kappa_h d_h = c(q_h)$, and $d_h \leq 0$. The incentive-compatibility condition (22) implies

$$u(q_h) - c(q_h) + (\kappa_h - \kappa_\ell)d_h \leq u(q_\ell) - c(q_\ell),$$

which is satisfied. This solution is consistent with Parts 1 and 2 of the Proposition. In the following, I focus on the case where $a^b > 0$.

*Part 1 of the Proposition***.** I investigate in turn the conditions under which the constraints (21) and (22) are slack. First, suppose that the incentive-compatibility condition (22) is slack. Then, $q_h = \min\left[q^*, c^{-1}(\kappa_h a^b + z^b)\right] \geq q_\ell$. Since from (21) $c(q_h) = \kappa_h d_h + \tau_h$, then (22) becomes

$$u(q_h) - c(q_h) + d_h(\kappa_h - \kappa_\ell) \leq u(q_\ell) - c(q_\ell).$$

If $z^b < c(q^*)$, then $d_h > 0$ and (22) is violated, which is a contradiction. If $z^b \geq c(q^*)$, then $q_h = q_\ell = q^*$ and the inequality above implies $d_h \leq 0$, as in Part 1 of the Proposition.

Second, suppose that the seller's participation constraint (21) is slack. Substitute $u(q_h)$ by its expression given by (22) at equality into the objective function (20) to get

$$U_h^b = \max_{d \in [-a^s, a^b]} \left[(\kappa_\ell - \kappa_h)\, d + U_\ell^b\right] = U_\ell^b + (\kappa_h - \kappa_\ell)a^s,$$

and $d_h = -a^s$. If $a^s > 0$, then $u(q_h) - c(q_h) \geq U_h^b > U_\ell^b = u(q_\ell) - c(q_\ell)$, which requires $q_\ell < q^*$ and $q_h > q_\ell$. From (18) and (21), $\kappa_h d_h + \tau_h \geq c(q_h) > c(q_\ell) = z^b + \kappa_\ell a^b$, and hence $\tau_h > z^b + \kappa_\ell a^b + \kappa_h a^s$. This inequality violates feasibility. If $a^s = 0$, then $U_h^b = U_\ell^b = u(q_\ell) - c(q_\ell)$. Since $d_h = 0$, $U_h^b \leq \max[u(q) - \tau]$ subject to $-c(q) + \tau = 0$. Hence, $U_h^b = U_\ell^b$ if and only if $z^b \geq c(q^*)$. In that case, $q_h = q^*$ and $\tau_h = c(q^*)$, which is consistent with Part 1 of the Proposition.

Part 2 of the Proposition. I show first that the constraint $\tau_h \leq z^b$ is binding when $z^b < c(q^*)$. If $\tau_h \leq z^b$ is slack, then $q_h = q^*$, $d_h \leq 0$ and $\tau_h + \kappa_h d_h = c(q^*)$. This solution maximizes (20) subject to (21), and the constraint $d_h \leq 0$ guarantees that (22) holds. However, $\tau_h \geq c(q^*)$ is in contradiction with $z^b < c(q^*)$.

Since (21) is binding and $\tau_h = z^b$, d_h is given by (27). Substitute d_h by its expression into (22) at equality to get (28). For all $q_h \in [0, q_\ell]$ the left side of (28) is strictly increasing. It is nonpositive at $q_h = 0$, and greater than $u(q_\ell) - c(q_\ell)$ at $q_h = q_\ell$ if $c(q_\ell) > z^b$. This last inequality holds from (17)-(19). Indeed, if $z^b < c(q^*)$, then $c(q_\ell) = \min\left[c(q^*), z^b + \kappa_\ell a^b\right] > z^b$ since I focus on the case $a^b > 0$. Hence, there is a unique $q_h \in (0, q_\ell)$ solution to (28). The objective in (20) $u(q_h) - c(q_h) = u(q_\ell) - c(q_\ell) - \left(1 - \frac{\kappa_\ell}{\kappa_h}\right)\left[c(q_h) - z^b\right]$ is decreasing in q_h for any solution to (28). Hence, the unique solution in $(0, q_\ell)$ delivers a maximum to the problem (20)-(23). Given a unique q_h, d_h is determined by (27). Finally, $c(q_h) = z^b + \kappa_h d_h < c(q_\ell) = \tau_\ell + \kappa_\ell d_\ell \leq z^b + \kappa_\ell a^b$ implies $d_h < a^b$. From (28), $q_h < q_\ell$ implies $c(q_h) - z^b > 0$ and, from (27), $d_h > 0$. ■

Proof of Proposition 2.

(i) From Proposition 1, if $z^b < c(q^*)$, then q_h is the unique solution in $[0, q_\ell]$ to (28). Differentiate (28) to obtain

$$\begin{aligned}\frac{\partial q_h}{\partial \kappa_h} &= -\frac{\frac{\kappa_\ell}{\kappa_h} d_h}{u'(q_h) - \frac{\kappa_\ell}{\kappa_h} c'(q_h)} < 0,\\ \frac{\partial q_h}{\partial \kappa_\ell} &= \frac{\left[u'(q_\ell)/c'(q_\ell) - 1\right] a^b + d_h}{u'(q_h) - \frac{\kappa_\ell}{\kappa_h} c'(q_h)} > 0,\end{aligned}$$

where $q_\ell = \min\left[q^*, c^{-1}(\kappa_\ell a^b + z^b)\right]$ and $d_h > 0$ (from Proposition 1 and the assumption $a^b > 0$). From (27), $\frac{\partial d_h}{\partial \kappa_\ell} = \frac{c'(q_h)}{\kappa_h}\frac{\partial q_h}{\partial \kappa_\ell} > 0$ and

$$\frac{\partial d_h}{\partial \kappa_h} = -\frac{u'(q_h) d_h}{\kappa_h u'(q_h) - \kappa_\ell c'(q_h)} < 0.$$

(ii) From (28), as κ_ℓ approaches to 0 q_h tends to the solution to

$$u(q_h) - z^b = u(q_\ell) - c(q_\ell) = u(q_\ell) - z^b,$$

where I have used that $q_\ell = c^{-1}(z^b)$ when $z^b < c(q^*)$. Consequently, $q_h \to q_\ell$ and, from (27),

$$d_h = \frac{c(q_h) - z^b}{\kappa_h} \to 0.$$

■

Proof of Proposition 3. From Proposition 1, if $z^b < c(q^*)$, then q_h is the unique solution in $[0, q_\ell]$ to (28). Differentiating (28),

$$\frac{\partial q_h}{\partial z^b} = \frac{u'(q_\ell)/c'(q_\ell) - \frac{\kappa_\ell}{\kappa_h}}{u'(q_h) - \frac{\kappa_\ell}{\kappa_h}c'(q_h)} > 0.$$

From (27), $\frac{\partial(\kappa_h d_h)}{\partial z^b} = c'(q_h)\frac{\partial q_h}{\partial z^b} - 1$, and hence (29). The assumption $a^b > 0$ implies $q_\ell > q_h$ (Proposition 1) and $\frac{\partial(\kappa_h d_h)}{\partial z^b} < 0$. The expression for $\frac{\partial d_h}{\partial a^b}$ in (30) is obtained by a similar reasoning. ■

Proof of Lemma 4. The following cases are distinguished: $\phi > \beta\bar{\kappa}$, $\phi = \beta\bar{\kappa}$ and $\phi < \beta\bar{\kappa}$.

(i) $\phi > \beta\bar{\kappa}$**.**

Equations (37) and (38) are the first-order conditions with respect to z and a of the problem (33). First, compute the first and second partial derivatives and the cross-partial derivatives of the surplus functions $S^\ell(z,a)$ and $S^h(z,a)$. From Lemma 2, $S^\ell(z,a) = \hat{\mathcal{S}}_\ell(z + \kappa_\ell a)$ with

$$\begin{aligned} \hat{\mathcal{S}}_\ell(z+\kappa_\ell a) &= u \circ c^{-1}(z + \kappa_\ell a) - z - \kappa_\ell a \ \text{ if } z + \kappa_\ell a < c(q^*) \\ &= u(q^*) - c(q^*) \ \text{ otherwise.} \end{aligned}$$

Therefore, $S^\ell_a = \kappa_\ell \hat{\mathcal{S}}'_\ell$, $S^\ell_z = \hat{\mathcal{S}}'_\ell$, $S^\ell_{zz} = \hat{\mathcal{S}}''_\ell$, $S^\ell_{az} = \kappa_\ell \hat{\mathcal{S}}''_\ell$, and $S^\ell_{aa} = (\kappa_\ell)^2 \hat{\mathcal{S}}''_\ell$. From Proposition 1, if $z < c(q^*)$, then q_h solves (28), i.e.,

$$u(q_h) - c(q_h) + \left(1 - \frac{\kappa_\ell}{\kappa_h}\right)[c(q_h) - z] = \hat{\mathcal{S}}_\ell(z + \kappa_\ell a).$$

Totally differentiating the equation above,

$$\begin{aligned} \left[u'(q_h) - \frac{\kappa_\ell}{\kappa_h}c'(q_h)\right]\frac{\mathtt{d}q_h}{\mathtt{d}z} &= 1 - \frac{\kappa_\ell}{\kappa_h} + \hat{\mathcal{S}}'_\ell \\ \left[u'(q_h) - \frac{\kappa_\ell}{\kappa_h}c'(q_h)\right]\frac{\mathtt{d}q_h}{\mathtt{d}a} &= \kappa_\ell \hat{\mathcal{S}}'_\ell. \end{aligned}$$

Notice that $\frac{\mathtt{d}q_h}{\mathtt{d}z} > 0$ for all $z < c(q^*)$, and $\frac{\mathtt{d}q_h}{\mathtt{d}a} > 0$ for all (z,a) such that $z + \kappa_\ell a < c(q^*)$. From Proposition 1, the seller's participation constraint (21) holds at equality so that $S^h(z,a) = u(q_h) - c(q_h)$. Hence,

$$\begin{aligned} S^h_z(z,a) &= [u'(q_h) - c'(q_h)]\frac{\mathtt{d}q_h}{\mathtt{d}z} = \Delta(q_h)\left(1 - \frac{\kappa_\ell}{\kappa_h} + \hat{\mathcal{S}}'_\ell\right) \\ S^h_a(z,a) &= [u'(q_h) - c'(q_h)]\frac{\mathtt{d}q_h}{\mathtt{d}a} = \Delta(q_h)\kappa_\ell \hat{\mathcal{S}}'_\ell \end{aligned}$$

where

$$\Delta(q) \equiv \frac{u'(q) - c'(q)}{u'(q) - \frac{\kappa_\ell}{\kappa_h}c'(q)} = 1 - \frac{1 - \frac{\kappa_\ell}{\kappa_h}}{u'(q)/c'(q) - \frac{\kappa_\ell}{\kappa_h}}.$$

For all $q \in [0, q^*]$, $\Delta(q) \in [0,1]$ and, since $u'(q)/c'(q)$ is decreasing in q, $\Delta'(q) < 0$. Furthermore,

$$\begin{aligned}
S^h_{zz} &= \Delta'(q_h)\frac{\mathtt{d}q_h}{\mathtt{d}z}\left(1 - \frac{\kappa_\ell}{\kappa_h} + \hat{\mathcal{S}}'_\ell\right) + \Delta(q_h)\hat{\mathcal{S}}''_\ell \\
S^h_{za} &= \Delta'(q_h)\frac{\mathtt{d}q_h}{\mathtt{d}z}\kappa_\ell\hat{\mathcal{S}}'_\ell + \Delta(q_h)\kappa_\ell\hat{\mathcal{S}}''_\ell \\
&= \Delta'(q_h)\frac{\mathtt{d}q_h}{\mathtt{d}a}\left(1 - \frac{\kappa_\ell}{\kappa_h} + \hat{\mathcal{S}}'_\ell\right) + \Delta(q_h)\kappa_\ell\hat{\mathcal{S}}''_\ell \\
S^h_{aa} &= \Delta'(q_h)\frac{\mathtt{d}q_h}{\mathtt{d}a}\kappa_\ell\hat{\mathcal{S}}'_\ell + \Delta(q_h)\left(\kappa_\ell\right)^2\hat{\mathcal{S}}''_\ell.
\end{aligned}$$

For all $z < c(q^*)$, $S^h_{zz} < 0$. Consequently, the first leading principal minor of the Hessian matrix associated with (33), $\pi_h S^h_{zz} + \pi_\ell S^\ell_{zz}$, is negative for all $z < c(q^*)$.

The determinant of the Hessian matrix associated with (33) is

$$|\mathbb{H}| = \left(\pi_h S^h_{zz} + \pi_\ell S^\ell_{zz}\right)\left(\pi_h S^h_{aa} + \pi_\ell S^\ell_{aa}\right) - \left(\pi_h S^h_{za} + \pi_\ell S^\ell_{za}\right)^2.$$

It can be decomposed as $|\mathbb{H}| = \Gamma_1 + \Gamma_2 + \Gamma_3$ where

$$\begin{aligned}
\Gamma_1 &= (\pi_\ell)^2\left[S^\ell_{zz}S^\ell_{aa} - \left(S^\ell_{za}\right)^2\right] \\
\Gamma_2 &= (\pi_h)^2\left[S^h_{zz}S^h_{aa} - \left(S^h_{za}\right)^2\right] \\
\Gamma_3 &= \pi_h\pi_\ell\left[S^h_{zz}S^\ell_{aa} + S^\ell_{zz}S^h_{aa} - 2S^h_{za}S^\ell_{za}\right].
\end{aligned}$$

Since $S^\ell(z,a) = \hat{\mathcal{S}}_\ell(z + \kappa_\ell a)$, $\Gamma_1 = 0$. After some calculation,

$$\begin{aligned}
\Gamma_2 &= (\pi_h)^2\left(1 - \frac{\kappa_\ell}{\kappa_h}\right)\Delta\Delta'\hat{\mathcal{S}}''_\ell\kappa_\ell\left(\frac{\mathtt{d}q_h}{\mathtt{d}z}\kappa_\ell - \frac{\mathtt{d}q_h}{\mathtt{d}a}\right) \\
\Gamma_3 &= \pi_h\pi_\ell\left(1 - \frac{\kappa_\ell}{\kappa_h}\right)\Delta'\hat{\mathcal{S}}''_\ell\kappa_\ell\left(\frac{\mathtt{d}q_h}{\mathtt{d}z}\kappa_\ell - \frac{\mathtt{d}q_h}{\mathtt{d}a}\right),
\end{aligned}$$

where Δ and Δ' are evaluated at $q = q_h$. Therefore,

$$|\mathbb{H}| = \left(\frac{\mathtt{d}q_h}{\mathtt{d}z}\kappa_\ell - \frac{\mathtt{d}q_h}{\mathtt{d}a}\right)\left(1 - \frac{\kappa_\ell}{\kappa_h}\right)\Delta'\hat{\mathcal{S}}''_\ell\kappa_\ell\pi_h\left(\pi_h\Delta + \pi_\ell\right),$$

with

$$\kappa_\ell\frac{\mathtt{d}q_h}{\mathtt{d}z} - \frac{\mathtt{d}q_h}{\mathtt{d}a} = \left[u'(q_h) - \frac{\kappa_\ell}{\kappa_h}c'(q_h)\right]^{-1}\kappa_\ell\left(1 - \frac{\kappa_\ell}{\kappa_h}\right) > 0, \quad \forall q_h \leq q^*.$$

Hence, $|\mathbb{H}| > 0$ for all $z + \kappa_\ell a < c(q^*)$.

I now show that there is a unique solution to (33). First, the solution to (33) is such that $z + \kappa_\ell a \leq c(q^*)$. Suppose $z + \kappa_\ell a > c(q^*)$. Then, $\hat{\mathcal{S}}'_\ell = 0$ and $S^h_a(z,a) = S^\ell_a(z,a) = 0$. The first-order condition for a, (38), implies $a=0$. If $z>c(q^*)$, then $q_h=q_\ell=q^*$ and hence $S^h_z(z,a)=S^\ell_z(z,a)=0$. The first-order condition for z, (37), implies $z=0$. A contradiction.

So one can restrict (z,a) to the compact set $\{(z,a)\in\mathbb{R}_{2+}: z+\kappa_\ell a\leq c(q^*)\}$ and, from the Theorem of the Maximum, a solution to (33) exists, and it satisfies the first-order conditions (37)-(38). Since $\mathbb{H}$ is negative definite for all (z,a) such that $z+\kappa_\ell a<c(q^*)$, i.e., the leading principal minors of $\mathbb{H}$ alternate in sign with the first one being negative, the solution to (33) is unique.

(ii) $\phi=\beta\bar{\kappa}$**.**

From the first-order condition for a, (38), $S^h_a(z,a)=S^\ell_a(z,a)=0$, which requires $z+\kappa_\ell a\geq c(q^*)$. The first-order condition for z, (37), implies

$$-i+\pi_h\Delta\left[q_h(z)\right]\left(1-\frac{\kappa_\ell}{\kappa_h}\right)\leq 0,\quad \text{``}=\text{''}\quad \text{if } z>0, \tag{52}$$

where I have used that $\hat{\mathcal{S}}'_\ell=0$. From (28), $q_h(z)$ is implicitly defined by

$$u(q_h)-c(q_h)+\left(1-\frac{\kappa_\ell}{\kappa_h}\right)[c(q_h)-z]=u(q^*)-c(q^*)\ \text{ if }\ z<c(q^*), \tag{53}$$

and $q_h(z)=q^*$ if $z\geq c(q^*)$. For all $z\geq c(q^*)$, $\Delta(q_h)=\Delta(q^*)=0$, and (52) implies $z=0$. A contradiction. Since $\Delta'<0$ and $q'_h(z)>0$ for all $z\in(0,c(q^*))$, and since the function on the left side of (52) is continuous in z, there is a unique $z\in[0,c(q^*)]$ solution to (52). Consequently, $a\in[\frac{c(q^*)-z}{\kappa_\ell},\infty)$.

(iii) $\phi<\beta\bar{\kappa}$**.**

Since $S^h_a(z,a)\geq 0$ and $S^\ell_a(z,a)\geq 0$ there is no solution to the first-order condition for a, (38). ■

Proof of Proposition 4. The proof proceeds in three steps. First, it establishes the existence and uniqueness of the market-clearing price ϕ. Second, it derives the condition for a monetary equilibrium. Third, it characterizes the allocations in the DM.

(i) Existence and uniqueness of ϕ.

Define $A^d(\phi)\equiv\left\{\int_{j\in\mathcal{J}}a(j)dj : a(j) \text{ solution to (33) if } j\in\mathcal{B} \text{ and to (35) if } j\in\mathcal{S}\right\}$. If $\phi>\beta\bar{\kappa}$ then, from Lemma 3, the solution to (35) is such that $a^s=0$ and, from Lemma 4, there is a unique solution (z^b,a^b) to the problem (33). Hence, $A^d(\phi)=\{a^b\}$. Moreover, since (z^b,a^b) can be restricted to the compact set $\{(z,a)\in\mathbb{R}_{2+}: z+\kappa_\ell a\leq c(q^*)\}$, and since the objective function in (33) is continuous, the Theorem of the Maximum implies that $A^d(\phi)$ is continuous. Assuming an interior solution, and totally differentiating

(37)-(38),

$$\mathbb{H}\cdot\left(\begin{array}{c} dz^b \\ da^b \end{array}\right)=\left(\begin{array}{c} di \\ \frac{d\phi}{\beta} \end{array}\right)$$

where $\mathbb{H}=[H_{ij}]_{(i,j)\in\{1,2\}^2}$ is the Hessian matrix associated with (33). Since $|\mathbb{H}|>0$ (see proof of Lemma 4), $\mathbb{H}$ is invertible and

$$\left(\begin{array}{c} dz^b \\ da^b \end{array}\right)=\frac{1}{|\mathbb{H}|}\left(\begin{array}{cc} H_{22} & -H_{12} \\ -H_{21} & H_{11} \end{array}\right)\left(\begin{array}{c} di \\ \frac{d\phi}{\beta} \end{array}\right).$$

Consequently, for all $\phi>\beta\bar{\kappa}$, $da^b/d\phi=\frac{H_{11}}{\beta|\mathbb{H}|}<0$ where $H_{11}=\pi_h S^h_{zz}+\pi_\ell S^\ell_{zz}$. If the solution to (33) is such that $z^b=0$ then, from (38),

$$\frac{da^b}{d\phi}=\beta^{-1}\left[\pi_h S^h_{aa}(0,a)+\pi_\ell S^\ell_{aa}(0,a)\right]^{-1}<0.$$

So, $A^d(\phi)$ is decreasing provided that $a>0$. Last, from (37)-(38), if $A_d(\phi)=\{0\}$, then $A_d(\phi')=\{0\}$ for all $\phi'>\phi$. Moreover, $A^d(\phi)=\{0\}$ if $\phi>\beta\bar{\kappa}+i\beta\kappa_\ell$. To see this, rewrite (38) as

$$\frac{-\phi+\beta\bar{\kappa}}{\beta\kappa_\ell}+\pi_h\Delta(q_h)\hat{\mathcal{S}}'_\ell+\pi_\ell\hat{\mathcal{S}}'_\ell\leq 0.$$

From the comparison with (37), i.e.,

$$-i+\pi_h\Delta(q_h)\hat{\mathcal{S}}'_\ell+\pi_\ell\hat{\mathcal{S}}'_\ell+\pi_h\Delta(q_h)\left(1-\frac{\kappa_\ell}{\kappa_h}\right)\leq 0,$$

if $\frac{\phi-\beta\bar{\kappa}}{\beta\kappa_\ell}>i$, then (38) holds with a strict inequality and $a^b=0$.

If $\phi=\beta\bar{\kappa}$, then $a^s\in[0,\infty)$ (from Lemma 3) and $a^b\in\left[\frac{c(q^*)-\hat{z}(i)}{\kappa_\ell},\infty\right)$ (from Lemma 4), where $\hat{z}(i)$ is the unique solution to (52)-(53). Hence, $A^d(\beta\bar{\kappa})=\left[\frac{c(q^*)-\hat{z}(i)}{\kappa_\ell},\infty\right)$. In order to show that $A^d(\phi)$ is upper-hemi continuous at $\phi=\beta\bar{\kappa}$, consider a sequence $\{\phi_n\}_{n=0}^\infty$ such that $\phi_n>\beta\bar{\kappa}$ for all $n\in\mathbb{N}$ and $\phi_n\to\beta\bar{\kappa}$. Then, for all $n\in\mathbb{N}$ the buyer's portfolio choice, (a^b_n,z^b_n), is unique and such that $\kappa_\ell a^b_n+z^b_n<c(q^*)$. Moreover, (37)-(38) being continuous, (a^b_n,z^b_n) converges to (z^b_∞,a^b_∞) such that $\kappa_\ell a^b_\infty+z^b_\infty\leq c(q^*)$ and

$$\begin{array}{rclll} -i+\pi_h S^h_z(z^b_\infty,a^b_\infty)+\pi_\ell S^\ell_z(z^b_\infty,a^b_\infty) & \leq & 0 & \text{``}=\text{''} & \text{if } z^b_\infty>0 \\ \pi_h S^h_a(z^b_\infty,a^b_\infty)+\pi_\ell S^\ell_a(z^b_\infty,a^b_\infty) & \leq & 0 & \text{``}=\text{''} & \text{if } a^b_\infty>0. \end{array}$$

Hence, $z^b_\infty=\hat{z}(i)$ and $\kappa_\ell a^b_\infty+z^b_\infty=c(q^*)$, i.e., $a^b_\infty=\frac{c(q^*)-\hat{z}(i)}{\kappa_\ell}\in A^d(\beta\bar{\kappa})$.

To summarize: $A^d(\phi)$ is upper-hemi continuous, any selection from $A^d(\phi)$ is decreasing whenever $a>0$, $A^d(\phi)=\{0\}$ for all $\phi>\beta\bar{\kappa}+i\beta\kappa_\ell$ and $\infty\in A^d(\beta\bar{\kappa})$. Hence, there is a unique $\phi\in[\beta\bar{\kappa},\beta\bar{\kappa}+i\beta\kappa_\ell]$ such that $A\in A^d(\phi)$. (See Figure 3.)

(ii) Monetary equilibrium.

From Lemma 4, for given ϕ there is a unique z^b solution to the buyer's problem. Consequently, $Z = \int_{j\in\mathcal{B}} z(j)dj = z^b$. Since $A \in A^d(\phi)$, Z satisfies (37) with $a = A$, i.e.,

$$-i + \pi_h S_z^h(Z, A) + \pi_\ell S_z^\ell(Z, A) \leq 0 \quad \text{“}=\text{”} \quad \text{if } Z > 0.$$

Since $S_{zz}^h < 0$ for all $Z < c(q^*)$, $\frac{dZ}{di} < 0$ whenever $Z > 0$, and there exists $i_0(A) = \pi_h S_z^h(0, A) + \pi_\ell S_z^\ell(0, A)$ such that $Z > 0$ for all $i < i_0$. Since $S_z^h(0, A) < \infty$ and $S_z^\ell(0, A) < \infty$ for all $A > 0$, then $i_0(A) < \infty$. Furthermore, since $S_z^h(0, A) = \Delta(q_h)\left(1 - \frac{\kappa_\ell}{\kappa_h} + \hat{\mathcal{S}}_\ell'\right) > 0$ for all $q_h < q^*$ (from Proposition 1), then $i_0(A) > 0$.

(iii) DM allocations.

From (i) ϕ is unique. From Lemma 4, if $\phi > \beta\bar{\kappa}$, then there is a unique solution to (33). From Proposition 1, if $\kappa = \kappa_h$, then (q_h, d_h, τ_h) is unique; if $\kappa = \kappa_\ell$, then q_ℓ and $\tau_\ell + \kappa_\ell d_\ell$ are uniquely determined. If $\phi = \beta\bar{\kappa}$, then $a(j)$ can vary across buyers and sellers but $z^b = Z$ is unique, and $z^b + \kappa_\ell a(j) \geq c(q^*)$ for all $j \in \mathcal{B}$ (see Lemma 4). Consequently, $q_\ell = q^*$ and, from (28), q_h is independent of $a(j)$ for all $j \in \mathcal{B}$, and it solves (53). ■

Proof of Proposition 5. Define $\bar{A}(i) = [c(q^*) - \hat{z}(i)]/\kappa_\ell$ where $\hat{z}(i)$ is the unique solution to (52)-(53). As shown in the proof of Proposition 4, $A^d(\beta\bar{\kappa}) = [\bar{A}(i), \infty)$ and $A^d(\phi) = \{a^b\}$ for all $\phi > \beta\bar{\kappa}$, with $a^b < \bar{A}(i)$. Thus, from the market-clearing condition $A \in A^d(\phi)$, if $A \geq \bar{A}(i)$, then $\phi = \beta\bar{\kappa}$ and $R_a = \bar{\kappa}/\phi = \beta^{-1}$; If $A < \bar{A}(i)$, then $\phi > \beta\bar{\kappa}$ and $R_a = \bar{\kappa}/\phi < \beta^{-1}$. Finally, any selection from $A^d(\phi)$ is strictly decreasing in ϕ for all $\phi > \beta\bar{\kappa}$ (provided that $0 \notin A^d(\phi)$). Consequently, the solution to $A \in A^d(\phi)$ is such that $d\phi/dA < 0$ for all $A < \bar{A}(i)$. Hence, $\frac{d\mathcal{L}}{dA} < 0$ and $\frac{dR_a}{dA} > 0$ for all $A < \bar{A}(i)$. ■

Proof of Proposition 6.

(i) According to Proposition 4, if $i < i_0(A)$, then $Z > 0$. Differentiating (37) at equality, and using (29),

$$\kappa_h \frac{d(d_h)}{di} = \frac{u'(q_\ell)/c'(q_\ell) - u'(q_h)/c'(q_h)}{u'(q_h)/c'(q_h) - \frac{\kappa_\ell}{\kappa_h}} \left[\pi_h S_{zz}^h(Z, A) + \pi_\ell S_{zz}^\ell(Z, A)\right]^{-1},$$

which is strictly positive if $Z < c(q^*)$. (To see this, recall that $q_\ell > q_h$ and $S_{zz}^h < 0$.) From (37), $i > 0$ implies $Z < c(q^*)$. Consequently, $\frac{d\mathcal{V}_h}{di} = \frac{d(d_h)}{Adi} > 0$.

(ii) From (38),

$$\phi = \beta\bar{\kappa} + \beta\left[\pi_h S_a^h(Z, A) + \pi_\ell S_a^\ell(Z, A)\right], \tag{54}$$

where Z is the function of i implicitly defined by (37). If $i < i_0(A)$, then $Z > 0$. Differentiate (37) and (54) to obtain

$$\frac{d\phi}{di} = \beta \left[\frac{\pi_h S^h_{az}(Z, A) + \pi_\ell S^\ell_{az}(Z, A)}{\pi_h S^h_{zz}(Z, A) + \pi_\ell S^\ell_{zz}(Z, A)} \right].$$

If $A > \bar{A}(i)$, then $\phi > \beta\bar{\kappa}$ and $Z + \kappa_\ell A < c(q^*)$. Consequently, S^h_{az},S^ℓ_{az},S^ℓ_{zz},$S^h_{zz} < 0$ (from the proof of Lemma 4), and $\frac{d\phi}{di} > 0$. This implies $d\mathcal{L}/di > 0$ and $dR_a/di < 0$.

(iii) From (37), as $i \to 0$, $S^h_z, S^\ell_z \to 0$ and hence $q_h, q_\ell \to q^*$ and $Z \to c(q^*)$. From Lemma 1, $d_h \to 0$ and $\mathcal{V}_h \to 0$. From (54), $S^h_a, S^\ell_a \to 0$ implies $\phi \to \beta\bar{\kappa}$. ■

Proof of Proposition 7. From (38),

$$\phi = \beta\bar{\kappa} + \pi_h \beta S^h_a(z, a) + \pi_\ell \beta S^\ell_a(z, a).$$

Substitute S^h_a and S^ℓ_a by their expressions to get

$$\frac{\phi}{\bar{\kappa}} = \beta \left\{ 1 + \frac{\kappa_\ell}{\bar{\kappa}} \left[\pi_h \Delta(q_h) \hat{\mathcal{S}}'_\ell + \pi_\ell \hat{\mathcal{S}}'_\ell \right] \right\}. \tag{55}$$

Similarly, from (37), and after replacing S^h_z and S^ℓ_z by their expressions,

$$i = \pi_h \Delta(q_h) \left(1 - \frac{\kappa_\ell}{\kappa_h} + \hat{\mathcal{S}}'_\ell \right) + \pi_\ell \hat{\mathcal{S}}'_\ell. \tag{56}$$

Using the definition $1 + i = \gamma/\beta$, (56) can be rewritten as

$$\gamma = \beta \left\{ 1 + \pi_h \Delta(q_h) \left(1 - \frac{\kappa_\ell}{\kappa_h} + \hat{\mathcal{S}}'_\ell \right) + \pi_\ell \hat{\mathcal{S}}'_\ell \right\}. \tag{57}$$

From (55) and (57),

$$\begin{aligned} \gamma - \frac{\phi}{\bar{\kappa}} &= \frac{1}{R_z} - \frac{1}{R_a} \\ &= \beta \left\{ \left(1 - \frac{\kappa_\ell}{\kappa_h} \right) \pi_h \Delta(q_h) + \left(1 - \frac{\kappa_\ell}{\bar{\kappa}} \right) \hat{\mathcal{S}}'_\ell \left[\pi_h \Delta(q_h) + \pi_\ell \right] \right\}. \end{aligned}$$

From (37), $\Delta(q_h) > 0$ (since $S^h_z > 0$) for all $i > 0$. Moreover, $\kappa_\ell < \bar{\kappa} < \kappa_h$. Hence, $R_a > R_z$. ■

Proof of Proposition 8. (i) From (28), as $\kappa_\ell/\kappa_h \to 1$, then $q_h \to q_\ell$. From (42), in any monetary equilibrium $i = \frac{u'(q_\ell)}{c'(q_\ell)} - 1$ and, from (43), $\phi = \beta\bar{\kappa}(1 + i)$ and $R_a = \frac{\bar{\kappa}}{\phi} = \gamma^{-1}$. The condition for a monetary equilibrium to exist is $Z = c(q_\ell) - \bar{\kappa}A > 0$, i.e., $q_\ell > c^{-1}(\bar{\kappa}A)$. This requires $i < \frac{u'\left[c^{-1}(\bar{\kappa}A)\right]}{c'\left[c^{-1}(\bar{\kappa}A)\right]} - 1$. The right side of the inequality is strictly positive if and only if $c^{-1}(\bar{\kappa}A) < q^*$, i.e., $\bar{\kappa}A < c(q^*)$.

(ii) From Proposition 2, as $\kappa_\ell \to 0$, then $d_h \to 0$ and $\mathcal{V}_h = \frac{d_h}{A} \to 0$. From (37), the condition for the existence of a monetary equilibrium is

$$-i + \pi_h S_z^h(0, A) + \pi_\ell S_z^\ell(0, A) > 0.$$

Since $S_z^\ell(0, A) = \max\left\{\frac{u'\left[c^{-1}(\kappa_\ell A)\right]}{c'\left[c^{-1}(\kappa_\ell A)\right]} - 1, 0\right\} = \infty$, then a monetary equilibrium always exists. As $\kappa_\ell \to 0$, then $S_a^h(Z, A), S_a^\ell(Z, A) \to 0$ for all $Z > 0$. From (43), $\phi \to \beta\bar{\kappa}$ and $R_a = \frac{\bar{\kappa}}{\phi} \to \beta^{-1}$. ■

Supplementary appendices

B. Refinements of sequential equilibrium

I describe two refinements of sequential equilibrium in the context of the bargaining game studied in Section 4: the Intuitive Criterion and the undefeated equilibrium.

A typical signaling game has the following structure. There are two players: a sender of information and a receiver of information. In the context of the bargaining game in this paper, the sender is the buyer who makes an offer and the receiver is the seller who accepts or rejects the offer. The timing of the game is:

1. Nature draws a type t for the sender according to some (commonly known) probability distribution $\pi(t)$. Here, the set of types is $T = \{\ell, h\}$ and $\pi(\ell) = \pi_\ell$ and $\pi(h) = \pi_h$.

2. The sender (buyer) privately observes the type t, and he sends an offer o to the receiver (seller). Here, an offer is a triple $(q, d, \tau) \in \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b]$ where q is the output, d is the transfer of the real asset, and τ is the transfer of real balances.

3. The receiver observes the offer o and takes an action r. Here, the set of actions is $\{Y, N\}$. If $r = Y$ then the offer is accepted; If $r = N$ then the offer is rejected.

The payoff of the buyer is $U^b(t, o, r) = [u(q) - \kappa_t d - \tau]\, \mathbb{I}_{\{r=Y\}}$. The payoff of the seller is $U^s(t, o, r) = [-c(q) + \kappa_t d + \tau]\, \mathbb{I}_{\{r=Y\}}$. After receiving the offer o, the seller forms a posterior probability assessment over the set of types of the buyer, $\lambda\,(t\,|o)$. The best response of the seller is

$$\text{BR}(\lambda, o) = \arg \max_{r \in \{Y,N\}} \sum_{t \in \{\ell,h\}} \lambda\,(t\,|o)\, U^s(t, o, r).$$

In the context of the bargaining game, the best response of the seller can be reexpressed as

$$\text{BR}(\lambda, o) = \arg \max_{r \in \{Y,N\}} [-c(q) + [\lambda\,(h\,|o)\,\kappa_h + \lambda\,(\ell\,|o)\,\kappa_\ell]\, d + \tau]\, \mathbb{I}_{\{r=Y\}}.$$

I adopt the tie-breaking rule according to which $r = Y$ whenever $\text{BR}(\lambda, o) = \{Y, N\}$.

Let $\text{O}\colon T \to \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b]$ denote a strategy for a buyer. It is a mapping from the set of types to the set of feasible offers. Let $\text{R}\colon \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b] \to \{Y, N\}$ denote a strategy for a seller. It is a mapping from the set of feasible offers to the set $\{Y, N\}$. A (pure strategy) sequential equilibrium is a profile of strategies (O^*,R^*) and a seller's belief system, λ^*, such that the following is true.

1. For all $t \in T$, $\text{O}^*(t) \in \arg\max_{o'} U^b(t, o', \text{R}^*(o'))$

2. For all o, $\text{R}^*(o) \in \text{BR}(\lambda^*(o), o)$

3. λ^* satisfies Bayes' rule whenever possible

In the context of the bargaining game studied in Section 4, the buyer's strategy can be simplified by noticing the following. First, $U^b(t, o, \text{R}^*(o)) = 0$ for all o such that $\text{R}^*(o) = N$. Second, from the tie-breaking rule, $U^b(t, o, \text{R}^*(o)) = 0$ and $\text{R}^*(o) = Y$ if $o = (0, 0, 0)$. Hence, with no loss, the buyer can choose an offer among those that are accepted by sellers, i.e.,

$$\text{O}^*(t) \in \arg\max_{o} U^b(t, o, Y) \quad \text{s.t.} \quad Y \in BR(\lambda^*(o), o).$$

The Intuitive Criterion

The Cho-Kreps (1987) refinement is based on the idea that out-of-equilibrium actions should never be attributed to a type who would not benefit from it under any circumstances. For a subset $K \subseteq T$, let $\text{BR}(K, o)$ denote the set of best responses for the seller to beliefs concentrated on K, i.e.,

$$\text{BR}(K, o) = \bigcup_{\{\lambda:\lambda(K)=1\}} \text{BR}(\lambda, o).$$

Suppose $K = T = \{\ell, h\}$. Then,

$$\begin{aligned} BR(T, o) &= \{Y\} \quad \text{if} \quad -c(q) + \kappa_\ell d + \tau > 0 \\ &= \{N\} \quad \text{if} \quad -c(q) + \kappa_h d + \tau < 0 \\ &= \{Y, N\} \quad \text{otherwise.} \end{aligned}$$

Consider a proposed equilibrium where the payoff of a buyer of type t is denoted U_t^*. According to Cho and Kreps (1987, p.202), this proposed equilibrium fails the Intuitive Criterion if there exists an out-of-equilibrium offer o' and a type $t \in T$ such that:

1. $U_t^* > \max_{r \in BR(\{\ell,h\},o')} U^b(t, o', r)$

2. $U_{\tilde{t}}^* < \min_{r \in BR(T \backslash \{t\},o')} U^b(\tilde{t}, o', r)$ for all $\tilde{t} \in T \backslash \{t\}$

According to the first requirement, the unsent offer o' would reduce the payoff of the buyer with type t compared to his equilibrium payoff irrespective of the inference the seller draws from o'. Consequently, the seller should attribute the offer o' to a buyer with type $\tilde{t}$. If he does so, the second requirement specifies that the buyer with type $\tilde{t}$ should obtain a higher utility with o' compare to his equilibrium payoff.

In the bargaining game, the buyer's equilibrium payoff is bounded below by 0. Hence, the second condition implies $\min_{r \in BR(T \setminus \{t\}, o')} U^b(\tilde{t}, o', r) > 0$, which requires that the offer o' is acccepted when the seller's belief is restricted to $T \setminus \{t\}$, i.e., $Y \in BR(T \setminus \{t\}, o')$. (Recall from the tie-breaking rule that if $\{Y, N\} = BR$ then the seller accepts the offer.) This implies $Y \in BR(\{\ell, h\}, o')$, and hence the first requirement becomes $U_t^* > U^b(t, o', Y)$, i.e., the payoff of the type$-t$ buyer at the proposed equilibrium is greater than what he would obtain if he would make the offer o' and the offer was accepted. This leads to the definition in the text according to which a proposed equilibrium fails the Intuitive Criterion if there is an out-of-equilibrium offer that satisfies (14)-(16).

Undefeated sequential equilibria

Mailath, Okuno-Fujiwara and Postlewaite (1993) proposed an alternative to the Intuitive Criterion. Their refinement is based on the notion of *undefeated equilibrium*.

An equilibrium is composed of a strategy for buyers, O, that specifies an offer for each type, an acceptance rule for sellers, R, and a belief system for sellers, λ. According to Mailath, Okuno-Fujiwara and Postlewaite (1993, p.254, Definition 2) an equilibrium (O', R', λ') defeats (O,R, λ) if there exists an offer o' such that:

1. For all t, $\mathrm{O}(t) \neq o'$ and $K \equiv \{t \in T \,|\mathrm{O}'(t) = o'\} \neq \varnothing$
2. For all $t \in K$, $U^b\left[t, o', \mathrm{R}'(o')\right] \geq U^b\left[t, \mathrm{O}(t), \mathrm{R}(\mathrm{O}(t))\right]$ with a strict inequality for one t in K
3. $\lambda(t\,|o') \neq p(t)\pi(t)/\sum_{t'} p(t')\pi(t')$ for at least one t in K where $p(t) = 1$ if $t \in K$ and $U^b\left[t, o', \mathrm{R}'(o')\right] > U^b\left[t, \mathrm{O}(t), \mathrm{R}(\mathrm{O}(t))\right]$ and $p(t) = 0$ if $t \notin K$.

For a sequential equilibrium to be defeated there must exist an out-of-equilibrium offer that is used in an alternative sequential equilibrium by a subset K of buyers' types (requirement 1). For all buyers with types in K, their payoff at the alternative equilibrium must be greater than the one at the proposed equilibrium with a strict inequality for at least one type (requirement 2). Finally, the belief system in the proposed equilibrium does not update sellers' prior belief conditional on the buyer's type being in K (requirement 3).

C. Undefeated monetary equilibria

In this section I determine a condition on the inflation rate under which the equilibrium outcome obtained by using the Intuitive Criterion is robust to the use of the alternative refinement of Mailath, Okuno-Fujiwara and Postlewaite (1993).[36]

Consider two sequential equilibria of the bargaining game, a proposed equilibrium ($\text{O},\text{R},\lambda$) and an alternative equilibrium ($\text{O'},\text{R'},\lambda'$), where O: $T \to \mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b]$ specifies an offer for each type, R: $\mathbb{R}_+ \times [-a^s, a^b] \times [-z^s, z^b] \to \{Y, N\}$ is an acceptance rule for sellers, and λ a belief system for sellers. (See the Appendix B for a definition of the sequential equilibrium of the game.) Let U_h^b denote the payoff of the h-type buyer at the proposed equilibrium and U_ℓ^b the payoff of the ℓ-type buyer. Consider an out-of-equilibrium offer $\tilde{o} = (\tilde{q}, \tilde{d}, \tilde{\tau})$ ($\text{O}(t) \neq \tilde{o}$ for all $t \in T$) such that $\text{O'}(t) = \tilde{o}$ for all $t \in K \subset T$ with $K \neq \varnothing$, i.e., the out-of-equilibrium offer is made by some types of buyers in the alternative equilibrium. If K is the singleton $\{t\}$, then the proposed equilibrium ($\text{O},\text{R},\lambda$) is defeated by ($\text{O'},\text{R'},\lambda'$) if

$$u(\tilde{q}) - \kappa_t \tilde{d} - \tilde{\tau} > U_t^b \; . \tag{58}$$

The payoff of the buyer of type t is strictly greater at the alternative equilibrium than at the proposed equilibrium. The reason for why the buyer of type t does not offer $\tilde{o}$ at the proposed equilibrium is because the seller does not attribute this offer to a $t-$type buyer with probability one, i.e., $\lambda(t\,|\tilde{o}) < 1$. (All through the analysis I assume the tie-breaking rule according to which sellers accept offers that make them indifferent between accepting and rejecting.) Consequently, the proposed equilibrium is defeated.

Consider next the case where $K = \{\ell, h\}$, the alternative equilibrium is pooling. Then, the proposed equilibrium ($\text{O},\text{R},\lambda$) is defeated by ($\text{O'},\text{R'},\lambda'$) if

$$u(\tilde{q}) - \kappa_t \tilde{d} - \tilde{\tau} > U_t^b \;\text{ for all } t \in \{\ell, h\}. \tag{59}$$

The offer is rejected at the proposed equilibrium because $\lambda(t\,|\tilde{o}) \neq \pi_h$. If the inequality above is weak for one type t, then the proposed equilibrium is defeated if $\lambda(t\,|\tilde{o}) > \pi(t)$, i.e., the offer should not be attributed to the type who is indifferent with a probability greater than the occurrence of this type.

[36] As Mailath, Okuno-Fujiwara and Postlewaite (1993, p.265) put it,

> "[T]here is no reason that different refinements shouldn't be employed in the analysis of a single game. Various implausibilities may be exhibited in different equilibria of a game, and hence, considering different refinements of the equilibrium set for a single game is like looking at the game from different vantage points".

Lemma 5 *Consider a match between a buyer holding z^b real balances and a^b units of the real asset and a seller holding z^s real balances and a^s units of the real asset. The (separating) equilibrium of the bargaining game that satisfies the Intuitive Criterion is the only undefeated equilibrium if*

$$\bar{U}_h^b \equiv u(q^p) - \kappa_h d^p - \tau^p < U_h^b \equiv u(q_h) - \kappa_h d_h - \tau_h, \tag{60}$$

where (q_h, d_h, τ_h) is the solution to (20)-(23) and

$$(q^p, d^p, \tau^p) = \arg\max_{q,d,\tau} \{u(q) - \kappa_h d - \tau\} \tag{61}$$

$$s.t. \quad -c(q) + (\pi_h \kappa_h + \pi_\ell \kappa_\ell)\, d + \tau \geq 0 \tag{62}$$

$$u(q) - \kappa_\ell d - \tau \geq u(q_\ell) - c(q_\ell) \tag{63}$$

$$(d, \tau) \in \left[-a^s, a^b\right] \times \left[-z^s, z^b\right], \tag{64}$$

where $q_\ell = \min\left[q^*, c^{-1}(\kappa_\ell a^b + z^b)\right]$. If $\bar{U}_h^b > U_h^b$ then there is an undefeated equilibrium and it is pooling. If $\bar{U}_h^b = U_h^b$ then there is both a pooling and a separating undefeated equilibrium.

Proof. A solution to (61)-(64) exists. To see this, notice that $(q_\ell, d_\ell, \tau_\ell)$ with $d_\ell \geq 0$ satisfies (62)-(64) so that the set of offers that satisfy (62)-(64) is not empty and it is compact. The objective function in (61) being continuous a solution exists.

First, I establish that among the separating sequential equilibria, the only one that can be undefeated is the one that satisfies the Intuitive Criterion. Consider a sequential equilibrium that is separating and denote $\hat{U}_h^b$ the payoff of the h-type buyer at this equilibrium. Suppose that the offer $(\hat{q}_h, \hat{d}_h, \hat{\tau}_h)$ of the h-type at the proposed equilibrium is different from (q_h, d_h, τ_h) that solves (20)-(23) so that $\hat{U}_h^b < U_h^b = u(q_h) - \kappa_h d_h - \tau_h$. From (58) the proposed equilibrium is defeated.

Second, suppose (60) holds. From (61)-(64), $\bar{U}_h^b$ is the highest payoff an h-type buyer can reach at a pooling equilibrium. Hence, from (58), any pooling equilibrium is defeated by the Pareto-efficient separating equilibrium.

Third, suppose that $\bar{U}_h^b > U_h^b$. The pooling equilibrium (q^p, d^p, τ^p) defeats the Pareto-efficient separating equilibrium. To see this, notice that the h-type buyer strictly prefers the pooling equilibrium while ℓ-type buyers prefer weakly the pooling equilibrium. Hence, $\lambda(h\,|(q^p, d^p, \tau^p))$ should be at least equal to π_h at the separating equilibrium, in which case h-type buyers would have a profitable deviation. Moreover, since (q^p, d^p, τ^p) is the preferred equilibrium of the h-type buyer it cannot be defeated by another pooling equilibrium.

Fourth, if $\bar{U}_h^b = U_h^b$ then both the Pareto-efficient separating equilibrium and the pooling equilibrium (q^p, d^p, τ^p) are undefeated by a similar reasoning as above. ■

Following Mailath, Okuno-Fujiwara and Postlewaite (1993), the *lexicographically maximum sequential equilibrium* (LMSE) corresponds to the pooling offer (q^p, d^p, τ^p) if $\bar{U}_h^b \geq U_h^b$, and to the separating equilibrium given by the Intuitive Criterion otherwise.[37] Lemma 5 shows that the LMSE is undefeated (Mailath *et al.*, 1993, Theorem 1) and if it is completely separating, it is the only undefeated (pure strategy) sequential equilibrium (Mailath *et al.*, 1993, Theorem 2). In the following, I assume that in every match where a buyer and a seller bargain over the terms of trade the equilibrium of the bargaining game corresponds to the LMSE.

The definition of a symmetric equilibrium for the whole economy when the outcome of the bargaining games in all bilateral matches corresponds to the LMSE is as follows.

Definition 3 *An equilibrium is a list of portfolios, buyers' offers in the DM, the price of capital, and aggregate real balances,* $\left\langle \left\{(a^b, z^b), (a^s, z^s)\right\}, \{(q_h^*, d_h^*, \tau_h^*), (q_\ell^*, d_\ell^*, \tau_\ell^*)\}, \phi, Z \right\rangle$ *such that:*

(i) (a^b, z^b) *is solution to (10) and* (a^s, z^s) *is solution to (11);*

(ii) If (60) holds then (q_h^*, d_h^*, τ_h^*) *is solution to (20)-(23) and* $(q_\ell^*, d_\ell^*, \tau_\ell^*)$ *solves (17)-(19); Otherwise,* $(q_h^*, d_h^*, \tau_h^*) = (q_\ell^*, d_\ell^*, \tau_\ell^*) = (q^p, d^p, \tau^p)$ *solution to (61)-(64).*

(iii) ϕ *solves (36);*

(iv) $Z = z^b + z^s$.

Since the seller's participation constraint is binding in all matches, the seller's choice of portfolio solves (35), i.e.,

$$(z^s, a^s) \in \arg \max_{z \geq 0, a \geq 0} \left\{ -iz - \left(\frac{\phi - \beta \bar{\kappa}}{\beta} \right) a \right\}.$$

As in Proposition 4, $\phi \geq \beta \bar{\kappa}$ in any equilibrium. So I focus on equilibria where $(z^s, a^s) = (0, 0)$.

Next I characterize the solution to (61)-(64) omitting the constraint (63). The constraint (63) is needed to guarantee that the pooling outcome can be sustained for some beliefs (e.g., $\lambda(h \,|(q, d, \tau)) = 0$ for all $(q, d, \tau) \neq (q^p, d^p, \tau^p)$). However, if (60) is violated this constraint is automatically satisfied. To see this,

[37] Consider two sequential equilibria with the associated profile of payoffs for the buyers (u_h, u_ℓ) and (u_h', u_ℓ'). The first equilibrium lexicographically dominates the second one if $u_h > u_h'$ or $u_h = u_h'$ and $u_\ell > u_\ell'$.

notice that any solution to (61)-(62)-(64) with $a^s = 0$ (and hence $d \geq 0$) is such that (21) is not binding. Consequently, if (60) does not hold then (22) must be violated.

Let $\hat{q}$ be the solution to $u'(q) = \frac{\kappa_h}{\bar{\kappa}} c'(q)$.

Lemma 6 *The solution to*

$$(\tilde{q}, \tilde{d}, \tilde{\tau}) = \arg\max_{q,d,\tau} \{u(q) - \kappa_h d - \tau\} \tag{65}$$

$$s.t. \quad -c(q) + (\pi_h \kappa_h + \pi_\ell \kappa_\ell) d + \tau \geq 0, \tag{66}$$

$$(d, \tau) \in \left[0, a^b\right] \times \left[0, z^b\right], \tag{67}$$

is such that:

1. *If $z^b \geq c(q^*)$ then $\tilde{q} = q^*$, $\tilde{d} = 0$ and $\tilde{\tau} = c(q^*)$.*

2. *If $z^b \in [c(\hat{q}), c(q^*))$ then $\tilde{q} = c^{-1}(z^b)$, $\tilde{d} = 0$ and $\tilde{\tau} = z^b$.*

3. *If $z^b < c(\hat{q})$ and $\bar{\kappa} a^b + z^b \geq c(\hat{q})$ then $\tilde{q} = \hat{q}$, $\tilde{d} = \left[c(\hat{q}) - z^b\right]/\bar{\kappa}$ and $\tilde{\tau} = z^b$.*

4. *If $\bar{\kappa} a^b + z^b < c(\hat{q})$ then $\tilde{q} = c^{-1}(\bar{\kappa} a^b + z^b)$, $\tilde{d} = a^b$ and $\tilde{\tau} = z^b$.*

Proof. Substitute $q = c^{-1}(\bar{\kappa} d + \tau)$ from (66) at equality into the objective (65) to rewrite the problem as

$$\max_{d,\tau} \left\{u \circ c^{-1}(\bar{\kappa} d + \tau) - \kappa_h d - \tau\right\}$$
$$\text{s.t.} \quad 0 \leq \tau \leq z^b \quad \text{and} \quad 0 \leq d \leq a^b$$

(i) If the constraint $\tau \leq z^b$ is not binding then the first-order condition with respect to τ gives $q = q^*$ and the first-order condition with respect to d gives $d = 0$. From (66) $\tau = c(q^*)$. The condition $\tau \leq z^b$ can then be rewritten as $c(q^*) \leq z^b$.

(ii) If both $\tau \leq z^b$ and $d \geq 0$ are binding then $q = c^{-1}(z^b)$. The first-order condition for τ implies $u'(q)/c'(q) \geq 1$, i.e., $q \leq q^*$, and the first-order condition for d implies $u'(q)/c'(q) \leq \kappa_h/\bar{\kappa}$, i.e., $q \geq \hat{q}$.

(iii) If only $\tau \leq z^b$ is binding then $q = \hat{q}$ and, from (66), $d = \left[c(\hat{q}) - z^b\right]/\bar{\kappa}$. The condition $0 \leq d \leq a^b$ implies $c(\hat{q}) - \bar{\kappa} a^b \leq z^b \leq c(\hat{q})$.

(iv) If both $\tau \leq z^b$ and $d \leq a^b$ are binding then $q = c^{-1}(z^b + \bar{\kappa} a^b)$. From the first-order condition for d, $q \leq \hat{q}$ and hence $z^b + \bar{\kappa} a^b \leq c(\hat{q})$. ■

A key feature of the pooling outcome is that if the buyer's real balances are large enough to allow him to consume $q \in [\hat{q}, q^*)$ (part 2 of Lemma 6) then the buyer chooses not to spend any of his real asset. I will use this result to show that if the buyer's real balances are sufficiently large, then the separating equilibrium given by the Intuitive Criterion is undefeated.

Lemma 7 *For all matches where $a^b > 0$ and $z^b \in [c(\hat{q}), c(q^*))$, where $\hat{q}$ is the solution to $u'(q) = \frac{\kappa_h}{\bar{\kappa}} c'(q)$, the unique undefeated equilibrium of the bargaining game is separating.*

Proof. The proof is by contradiction. Suppose that (60) is violated so that the LMSE corresponds to the pooling outcome (q^p, d^p, τ^p). From Lemma 6, if $z^b \in [c(\hat{q}), c(q^*))$ then $d^p = 0$. The solution (q_h, d_h, τ_h) to (20)-(23) is such that $d_h > 0$ and $q_h = c^{-1}(z^b + \kappa_h d_h)$. Hence, $u(q_h) - c(q_h) > u(q^p) - c(q^p)$ where $q^p = c^{-1}(z^p)$. A contradiction. ■

The following Proposition makes use of Lemma 7 to show that the equilibrium outcome when the LMSE is imposed in all matches is identical to the equilibrium outcome when the Intuitive Criterion is applied to the bargaining games in all matches provided that the inflation rate is not too large.

Proposition 9 *There is $i_0 > 0$ such that for all $i \in (0, i_0)$ the equilibrium outcome under the undefeated criterion is identical to the one obtained under the Intuitive Criterion.*

Proof. From Lemma 7 if $z^b \geq c(\hat{q})$ then the LMSE of the bargaining game is separating. Suppose that the buyer chooses a portfolio (z^b, a^b) such that the outcome of the bargaining game is pooling and equal to (q^p, d^p, τ^p). Then, $z^b < c(\hat{q})$ and, from Lemma 6, $q^p \leq \hat{q}$. The expected surplus of the buyer net of the cost of holding his portfolio is then

$$-iz^b - \left(\frac{\phi - \beta\bar{\kappa}}{\beta}\right) a^b + u(q^p) - c(q^p) \leq u(\hat{q}) - c(\hat{q}).$$

Let $\Psi(i)$ be defined as

$$\Psi(i) \equiv \max_{z,a} \left[-iz - \left(\frac{\phi - \beta\bar{\kappa}}{\beta}\right) a + \pi_h S^h(z, a) + \pi_\ell S^\ell(z, a)\right],$$

where $S^h(z, a)$ and $S^\ell(z, a)$ are the surplus functions obtained under the separating outcome of the bargaining game and ϕ is the equilibrium price at the equilibrium under the Intuitive Criterion. From (38),

$$\frac{\phi - \beta\bar{\kappa}}{\beta} = \pi_h S^h_a(z^b, A) + \pi_\ell S^\ell_a(z^b, A),$$

where, from the proof of Proposition 4, $z^b(i)$ is a continuous function of i and $\lim_{i \to 0} z^b(i) = c(q^*)$. The function Ψ is continuous, $\lim_{i \to 0} \Psi(i) = u(q^*) - c(q^*) > u(\hat{q}) - c(\hat{q})$. Hence, there exists a nonempty, open interval $(0, i_0)$ such that $\Psi(i) > u(\hat{q}) - c(\hat{q})$ and $z^b(i) > c(\hat{q})$. The buyer chooses a portfolio that correspond to a separating equilibrium of the bargaining game, and any portfolio corresponding to a pooling outcome generates an expected surplus net of the cost of the portfolio, $-iz^b - \left(\frac{\phi - \beta\bar{\kappa}}{\beta}\right) a^b + u(q^p) - c(q^p)$, that is smaller than the one obtained under the separating outcome. ■

This last Proposition suggests that the separating outcome predicted by the Intuitive Criterion is consistent with other refinements for signaling games provided that the inflation is not too large.

D. Asset pricing under symmetric information

In order to isolate the role of the private information friction for liquidity and asset prices, I analyze in this Appendix the economy with symmetric information: buyers and sellers in the DM have the same information about the future dividend of the asset.

Complete information

Consider a match in the DM between a buyer and a seller. The buyer holds z^b real balances (expressed in terms of the next CM output) and a^b units of the real asset. The seller holds z^s real balances and a^s units of the real asset. The future dividend of each unit of capital is κ and it is common knowledge in the match. The strategy of a buyer in the DM is a triple (q, d, τ) solution to

$$(q, d, \tau) = \arg\max_{q,\tau,d} [u(q) - \kappa d - \tau] \tag{68}$$

$$\text{s.t.} \quad -c(q) + \kappa d + \tau \geq 0, \tag{69}$$

$$\text{s.t.} \quad -z^s \leq \tau \leq z^b, \quad -a^s \leq d \leq a^b. \tag{70}$$

If $\kappa a^b + z^b \geq c(q^*)$ then the solution to (68)-(70) is $q = q^*$ and $\kappa d + \tau = c(q^*)$. Otherwise, $q = c^{-1}(\kappa a^b + z^b)$, $d = a^b$ and $\tau = z^b$. Let denote $\hat{\mathcal{S}}\left(\kappa a^b + z^b\right) = u(q) - c(q)$ where $q = \min\left[q^*, c^{-1}(\kappa a^b + z^b)\right]$.

Following the same reasoning as in the text, the buyer's portfolio choice in the CM is:

$$[z(j), a(j)] \in \arg\max_{z\geq 0, a\geq 0} \left\{-iz - \left(\frac{\phi - \beta\bar{\kappa}}{\beta}\right) a + \pi_h \hat{\mathcal{S}}(z + \kappa_h a) + \pi_\ell \hat{\mathcal{S}}(z + \kappa_\ell a)\right\}, \quad \forall j \in \mathcal{B} \tag{71}$$

where $i = (\gamma - \beta)/\beta$ is the cost of holding real balances, and $(\phi - \beta\bar{\kappa})/\beta$ represents the cost of investing in the real asset. Similarly, the seller's portfolio decision in the CM is

$$[z(j), a(j)] \in \arg\max_{z\geq 0, a\geq 0} \left\{-iz - \left(\frac{\phi - \beta\bar{\kappa}}{\beta}\right) a\right\}, \quad \forall j \in \mathcal{S}. \tag{72}$$

The clearing condition for the market for the real asset requires the fixed stock of the asset to be held by buyers and sellers,

$$\int_{j\in\mathcal{J}} a(j)dj = A. \tag{73}$$

Finally, the aggregate real balances are defined by

$$Z = \int_{j\in\mathcal{J}} z(j)dj. \tag{74}$$

An equilibrium is a list of portfolios, a profile of buyers' offers in the DM, aggregate real balances, and the price of capital that satisfy (68)-(74).

From (72) the seller's portfolio choice is such that $z(j) = 0$ (since $i > 0$), $a(j) = 0$ if $\phi > \beta\bar{\kappa}$ and $a(j) \in [0, \infty)$ if $\phi = \beta\bar{\kappa}$. The following lemma characterizes the buyer's portfolio choice.

Lemma 8 *Assume $\phi > \beta\bar{\kappa}$. If $(\phi - \beta\bar{\kappa})/\beta\kappa_\ell \neq i$ or $\pi_\ell \hat{\mathcal{S}}'[\kappa_\ell c(q^*)/\kappa_h] < i$ then the buyer's problem (71) admits a unique solution. It satisfies*

$$-i + \pi_h \hat{\mathcal{S}}'(z + \kappa_h a) + \pi_\ell \hat{\mathcal{S}}'(z + \kappa_\ell a) \leq 0 \quad \text{“} = \text{”} \quad \textit{if } z > 0 \tag{75}$$

$$-\left(\frac{\phi - \beta\bar{\kappa}}{\beta}\right) + \pi_h \kappa_h \hat{\mathcal{S}}'(z + \kappa_h a) + \pi_\ell \kappa_\ell \hat{\mathcal{S}}'(z + \kappa_\ell a) \leq 0 \quad \text{“} = \text{”} \quad \textit{if } a > 0. \tag{76}$$

If $(\phi - \beta\bar{\kappa})/\beta\kappa_\ell = i$ and $\pi_\ell \hat{\mathcal{S}}'[\kappa_\ell c(q^*)/\kappa_h] \geq i$ then any (z, a) such that $\pi_\ell \hat{\mathcal{S}}'(z + \kappa_\ell a) = i$ and $z + \kappa_h a \geq c(q^*)$ is solution to (71). If $\phi = \beta\bar{\kappa}$ then any $(z, a) \in \{0\} \times [c(q^*)/\kappa_\ell, \infty)$ is solution to (71). Finally, if $\phi < \beta\bar{\kappa}$ then there is no solution to (71).

Proof. Since $\hat{\mathcal{S}}'(z + \kappa a) = u'(q)/c'(q) - 1$, where $q = \min\left[q^*, c^{-1}(z + \kappa a)\right]$, then $\hat{\mathcal{S}}$ is concave and the buyer's problem (71) is concave as well. The first-order conditions (75) and (76) are then necessary and sufficient. Three cases are distinguished.

(i) $\phi > \beta\bar{\kappa}$. The solution to (71) cannot be such that $z + \kappa_\ell a > c(q^*)$. Indeed, if $z + \kappa_\ell a > c(q^*)$ then $\hat{\mathcal{S}}'(z + \kappa_h a) = \hat{\mathcal{S}}'(z + \kappa_\ell a) = u'(q^*)/c'(q^*) - 1 = 0$. But then (75)-(76) imply $z = a = 0$. A contradiction. So, one can restrict (z, a) to the compact set $\{(z, a) \in \mathbb{R}_{2+} : z + \kappa_\ell a \leq c(q^*)\}$ and from the Theorem of the Maximum a solution to (71) exists. Next, I show that the problem (71) is strictly jointly concave for all (z, a) such that $z + \kappa_h a < c(q^*)$. The Hessian matrix associated with (71) is

$$\mathbb{H} = \begin{pmatrix} \pi_h \hat{\mathcal{S}}_h'' + \pi_\ell \hat{\mathcal{S}}_\ell'' & \pi_h \kappa_h \hat{\mathcal{S}}_h'' + \pi_\ell \kappa_\ell \hat{\mathcal{S}}_\ell'' \\ \pi_h \kappa_h \hat{\mathcal{S}}_h'' + \pi_\ell \kappa_\ell \hat{\mathcal{S}}_\ell'' & \pi_h (\kappa_h)^2 \hat{\mathcal{S}}_h'' + \pi_\ell (\kappa_\ell)^2 \hat{\mathcal{S}}_\ell'' \end{pmatrix}$$

where $\hat{\mathcal{S}}_h'' \equiv \hat{\mathcal{S}}''(z + \kappa_h a)$ and $\hat{\mathcal{S}}_\ell'' \equiv \hat{\mathcal{S}}''(z + \kappa_\ell a)$. For all (z, a) such that $\kappa_h a + z < c(q^*)$, $\hat{\mathcal{S}}_\ell'' < 0$ and $\hat{\mathcal{S}}_h'' < 0$ and

$$|\mathbb{H}| = (\kappa_h - \kappa_\ell)^2 \pi_h \pi_\ell \hat{\mathcal{S}}_\ell'' \hat{\mathcal{S}}_h'' > 0.$$

So, $\mathbb{H}$ is negative definite and any solution to (75)-(76) such that $z + \kappa_h a < c(q^*)$ corresponds to a strict local maximum and hence, from the concavity of the objective, it corresponds to the global maximum.

Suppose next that the solution is such that $z+\kappa_h a \geq c(q^*)$. Then, $\hat{\mathcal{S}}'(z+\kappa_h a)=0$ and from (75)-(76),

$$\pi_\ell \hat{\mathcal{S}}'(z+\kappa_\ell a)=\min\left[i,\frac{(\phi-\beta\bar{\kappa})}{\beta\kappa_\ell}\right],$$

with $z=0$ if $i>\frac{(\phi-\beta\bar{\kappa})}{\beta\kappa_\ell}$ and $a=0$ if $i<\frac{(\phi-\beta\bar{\kappa})}{\beta\kappa_\ell}$. If $i=\frac{(\phi-\beta\bar{\kappa})}{\beta\kappa_\ell}$ then any pair (z,a) such that $z+\kappa_h a \geq c(q^*)$ and $\pi_\ell \hat{\mathcal{S}}'(z+\kappa_\ell a)=i$ is solution to (71). For such pairs to exist, $\pi_\ell \hat{\mathcal{S}}'\left[\frac{\kappa_\ell}{\kappa_h}c(q^*)\right] \geq i$.

(ii) $\phi=\beta\bar{\kappa}$. The first-order condition for a requires $\hat{\mathcal{S}}'(z+\kappa_h a)=\hat{\mathcal{S}}'(z+\kappa_\ell a)=0$ and hence any $a \geq [c(q^*)-z]/\kappa_\ell$ is part of a solution. But then (75) implies $z=0$. Thus, $a\in[c(q^*)/\kappa_\ell,\infty)$.

(iii) $\phi<\beta\bar{\kappa}$ then the first-order condition for a admits no solution. ■

The following proposition establishes the existence of an equilibrium and the uniqueness of the price of the real asset and the allocation of the DM output conditional on the realization of κ.[38] Denote q_ℓ and q_h the output levels in the DM when $\kappa=\kappa_\ell$ and $\kappa=\kappa_h$, respectively.

Proposition 10 *There exists an equilibrium, and (ϕ,q_ℓ,q_h) is uniquely determined. If $A \geq c(q^*)/\kappa_\ell$ then the equilibrium is nonmonetary ($Z=0$) and $\phi=\beta\bar{\kappa}$. If $A<c(q^*)/\kappa_\ell$ then $\phi>\beta\bar{\kappa}$ and there is $i_0>0$ such that for all $i<i_0$ an equilibrium is monetary ($Z>0$).*

Proof. The proof proceeds in four parts. It first characterizes the asset demand correspondence

$$A^d(\phi)\equiv\left\{\int_{j\in\mathcal{J}} a(j)dj : a(j) \text{ solution to (71) or (72)}\right\}.$$

Second, it shows that ϕ is uniquely determined. Third, it establishes that the DM output levels, q_ℓ and q_h, are unique. Finally, it determines the conditions for fiat money to be valued.

(i) Existence. Consider first the case $\phi>\beta\bar{\kappa}$. The solution to (72) is such that $a(j)=0$ for all $j\in\mathcal{S}$. As shown in the proof of Lemma 8 (Part (i)), any solution (z,a) to (71) lies in the compact set $[0,c(q^*)]\times[0,c(q^*)/\kappa_\ell]$. Since the objective in (71) is continuous, the Theorem of the Maximum guarantees that $A^d(\phi)$ is nonempty and upper-hemi continuous. Since the objective in (71) is concave, $A^d(\phi)$ is convex-valued. From (75) and (76), it can be checked that $A^d(\phi)=\{0\}$ for all $\phi>\beta\bar{\kappa}+i\beta\kappa_h$ and $A_d(\phi)=\{a\}$ where $a>0$ solves

$$\pi_h\kappa_h\hat{\mathcal{S}}'(\kappa_h a)+\pi_\ell\kappa_\ell\hat{\mathcal{S}}'(\kappa_\ell a)=\frac{\phi-\beta\bar{\kappa}}{\beta}$$

[38] Buyers' portfolios are not always uniquely determined. If $(\phi-\beta\bar{\kappa})/\beta\kappa_\ell=i$, and provided that $\hat{\mathcal{S}}'(z+\kappa_h a)=0$, real balances and capital are perfect substitutes. If $\phi=\beta\bar{\kappa}$ then buyers hold any quantity of capital above the level that satiates their liquidity needs in the DM, $\hat{\mathcal{S}}'=0$, and they hold no real balances.

for all $\phi < \beta\bar{\kappa} + i\beta\kappa_\ell$ (since $z = 0$). Moreover, $a \to c(q^*)/\kappa_\ell$ as $\phi \to \beta\bar{\kappa}$. Consider next the case $\phi = \beta\bar{\kappa}$. For all $j \in \mathcal{S}$, $a(j) \in [0, \infty)$. From Lemma 8, $A^d(\beta\bar{\kappa}) = [c(q^*)/\kappa_\ell, \infty)$. Hence, from this characterization of $A^d(\phi)$, there is a $\phi \in [\beta\bar{\kappa}, \beta\bar{\kappa} + i\beta\kappa_h]$ such that $A \in A^d(\phi)$.

(ii) Uniqueness. In order to prove that ϕ is uniquely determined, I show that any selection from A^d is decreasing in ϕ for all $\phi > \beta\bar{\kappa}$: if $a_1 \in A^d(\phi_1)$ and $a_2 \in A^d(\phi_2)$ for $\phi_2 > \phi_1$ then $a_2 < a_1$ unless $a_2 = a_1 = 0$. Consider $\phi_2 > \phi_1$, and the associated buyers' portfolio choices (z_1, a_1) and (z_2, a_2). By revealed preferences,

$$
\begin{aligned}
-\frac{\phi_1}{\beta} a_1 + \Psi(z_1, a_1) &\geq -\frac{\phi_1}{\beta} a_2 + \Psi(z_2, a_2) \\
-\frac{\phi_2}{\beta} a_2 + \Psi(z_2, a_2) &\geq -\frac{\phi_2}{\beta} a_1 + \Psi(z_1, a_1),
\end{aligned}
$$

where $\Psi(z, a) \equiv -iz + \bar{\kappa}a + \pi_h \hat{\mathcal{S}}(z + \kappa_h a) + \pi_\ell \hat{\mathcal{S}}(z + \kappa_\ell a)$. These last two inequalities yield

$$
\phi_1(a_1 - a_2) \leq \beta \left[\Psi(z_1, a_1) - \Psi(z_2, a_2) \right] \leq \phi_2 \left(a_1 - a_2 \right).
$$

Since $\phi_2 > \phi_1$ then $a_1 \geq a_2$. Suppose $a_1 = a_2 > 0$. From (76), $z_2 < z_1$ (where I have used that $\hat{\mathcal{S}}' < 0$ if $z + \kappa a < c(q^*)$). But $z_1 \neq z_2$ is inconsistent with (75). A contradiction.

(iii) The allocation (q_ℓ, q_h). From (i) and (ii), there exists a unique $\phi \geq \beta\bar{\kappa}$ such that $A \in A^d(\phi)$. If $A \geq c(q^*)/\kappa_\ell$ then $\phi = \beta\bar{\kappa}$. Since $a(j) \geq c(q^*)/\kappa_\ell$ for all $j \in \mathcal{B}$ then $q_h = q_\ell = q^*$. If $A < c(q^*)/\kappa_\ell$ then $\phi > \beta\bar{\kappa}$ and, from Lemma 8, (z^b, a^b) is uniquely determined unless $(\phi - \beta\bar{\kappa})/\beta\kappa_\ell = i$ and $\pi_\ell \hat{\mathcal{S}}' [\kappa_\ell c(q^*)/\kappa_h] \geq i$ in which case $q_\ell = c^{-1}\left[\hat{\mathcal{S}}'^{-1}(i/\pi_\ell)\right]$ and $q_h = q^*$. (See proof of Lemma 8.) For given (z^b, a^b) the problem (68)-(69) determines uniquely q_h and q_ℓ.

(iv) Suppose an equilibrium is nonmonetary. Then, $z(j) = 0$ and $a(j)$ is the unique solution to (76) for all $j \in \mathcal{B}$. Hence, $a(j) = A$. From (75) $Z = 0$ implies

$$
-i + \pi_h \hat{\mathcal{S}}'(\kappa_h A) + \pi_\ell \hat{\mathcal{S}}'(\kappa_\ell A) \leq 0.
$$

Define $i_0 = \pi_h \hat{\mathcal{S}}'(\kappa_h A) + \pi_\ell \hat{\mathcal{S}}'(\kappa_\ell A)$. By the contrapositive, if $i < i_0$ then the equilibrium is monetary. Provided that $\kappa_\ell A < c(q^*)$, $\hat{\mathcal{S}}'(\kappa_\ell A) > 0$ and $i_0 > 0$. Finally, if $\kappa_\ell A \geq c(q^*)$ then $\phi = \beta\bar{\kappa}$ and $Z = 0$. ∎

If the economy-wide stock of real assets is large enough to allow agents to trade q^* in the DM for the lowest realization of κ then fiat money is not valued.[39] If the aggregate stock of real assets is too low relative

[39] This result is in accordance with Lagos and Rocheteau (2008) who show that money is useful in the presence of capital in the Lagos-Wright environment if the first-best level of capital stock provides enough wealth for agents to trade the first best level of output in the DM, i.e., there is no shortage of capital to be used as means of payment.

to agents' liquidity needs (in terms of means of payment) then the price of the real asset increases above its fundamental value and fiat money can be valued provided that i is sufficiently low.

The expression for the price of the real asset in equilibrium is obtained from (71) by taking the first order condition for a, i.e.,

$$\phi = \beta\bar{\kappa} + \pi_h \beta\kappa_h \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] + \pi_\ell \beta\kappa_\ell \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right]. \tag{77}$$

It has two components. The first component is its fundamental value, $\beta\bar{\kappa}$. The second component is the liquidity value of capital in the DM, the last two terms on the right-hand side of (77). This liquidity value arises because capital can help relaxing buyers' budget constraint in a bilateral match.

To determine the liquidity value of fiat money, take the first-order condition of (71) with respect to z, and assume that the solution is interior (so that fiat money is valued),

$$i = \pi_h \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] + \pi_\ell \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right]. \tag{78}$$

The cost of holding fiat money, the left side of (78), is equal to the liquidity value of money in the DM in both states, the right side of (78).

The next proposition compares the (gross) rates of return of money and the real asset, $R_z = \gamma^{-1}$ and $R_a = \bar{\kappa}/\phi$, respectively. Let ρ denote the covariance between the return of the real asset, κ, and the marginal return of wealth in the DM, $[u'(q)/c'(q) - 1]$, i.e.,

$$\rho = \pi_h (\kappa_h - \bar{\kappa}) \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] + \pi_\ell (\kappa_\ell - \bar{\kappa}) \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right]. \tag{79}$$

Proposition 11 *In any monetary equilibrium,*

$$R_a = \gamma^{-1} \left\{\frac{\rho}{\bar{\kappa}(1+i)} + 1\right\}^{-1} > R_z. \tag{80}$$

Proof. The expression for ϕ given by (77) can be rearranged as

$$\phi = \beta \left\{\bar{\kappa}(1+i) + \pi_h (\kappa_h - \bar{\kappa}) \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] + \pi_\ell (\kappa_\ell - \bar{\kappa}) \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right]\right\}. \tag{81}$$

where I have used the fact that, from (75), $i = \left\{\pi_h \left[\frac{u'(q_h)}{c'(q_h)} - 1\right] + \pi_\ell \left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right]\right\}$ in any monetary equilibrium. Substitute ρ by its expression given by (79) into (81) to get

$$\phi = \beta(1+i)\bar{\kappa} \left\{\frac{\rho}{\bar{\kappa}(1+i)} + 1\right\}.$$

Divide by $\bar{\kappa}$ and use the definition $\gamma = \beta(1+i)$ to get (80). In order to show that $R_a > R_z$ it is enough to establish that $\rho < 0$. Notice that $\pi_h(\kappa_h - \bar{\kappa}) + \pi_\ell(\kappa_\ell - \bar{\kappa}) = 0$. From (75), and since $\hat{\mathcal{S}}''(\kappa a + z) < 0$ whenever $\hat{\mathcal{S}}'(\kappa a + z) > 0$, in any monetary equilibrium $0 \leq \hat{\mathcal{S}}'(\kappa_h a + z) < \hat{\mathcal{S}}'(\kappa_\ell a + z)$. Since $\frac{u'(q_h)}{c'(q_h)} < \frac{u'(q_\ell)}{c'(q_\ell)}$ then

$$\pi_h(\kappa_h - \bar{\kappa})\left[\frac{u'(q_h)}{c'(q_h)} - 1\right] < -\pi_\ell(\kappa_\ell - \bar{\kappa})\left[\frac{u'(q_\ell)}{c'(q_\ell)} - 1\right]$$

and $\rho < 0$. ■

The real asset has a higher rate of return than fiat money in any monetary equilibrium. This result holds even though agents are risk-neutral with respect to their CM consumption. This rate-of-return differential arises because of the negative correlation between the marginal utility of wealth in the DM and the dividend of the real asset, i.e., the real asset yields a high dividend in matches when the marginal value of wealth in the DM is low, and a low dividend in matches where the marginal value of wealth is high.[40] In contrast, the value of money is constant and uncorrelated with the marginal utility of wealth in the DM. Finally, as $\kappa_h - \kappa_\ell \to 0$ then $\rho \to 0$ and $R_a = \gamma^{-1}$, i.e., money and capital have the same rate of return.

Incomplete information

I now describe succinctly the case where both buyers and sellers are uninformed about the future value of κ. Buyers choose their portfolios in order to maximize $-iz - \left(\frac{\phi - \beta\bar{\kappa}}{\beta}\right)a + \hat{\mathcal{S}}(z + \bar{\kappa}a)$. If $A < c(q^*)/\bar{\kappa}$ then $\phi > \beta\bar{\kappa}$ and there is $i_0 > 0$ such that for all $i < i_0$ an equilibrium is monetary. Moreover, if a monetary equilibrium exists then $\phi = \beta\bar{\kappa}(1+i)$ and $1 + i = u'(q)/c'(q)$ where q is the quantity produced and consumed in bilateral matches in the DM. In this case, $R_a = R_z$, i.e., fiat money and capital have the same rate of return.

[40] This result is analogous to the one in Lagos (2006) who finds that even in the absence of legal restrictions on the use of assets as means of payment his model can be consistent with an equity-premium puzzle, i.e., a too large return differential between bonds and equity.