Liquidity, Payment and Endogenous Financial Fragility

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Abstract

We study the fragility of the banking system in relation to its role in liquidity creation. In our framework, fragility stems from the interconnections banks establish to protect themselves from liquidity shocks. In the absence of contractual constraints, banks choose the optimal degree of mutual insurance, because the flexibility with which financial assets can be designed and priced causes all market participants correctly to take into account the economic effects of their own interdependence.

When banks are in the business of providing liquidity, through, for example, payment services, some contractual flexibility is no longer available. In this case, we show that banks have an incentive to become too risky in aggregate, since some of the beneficiaries of the liquidity provision are unable to compensate the banks for maintaining independence. We briefly examine possible regulatory consequences.
1 Introduction

One of the most basic processes in the financial system is the creation of liquidity. Financial instruments are more readily tradable than are the assets that underlie those claims, whether those assets be factories and machinery, or loans to small businesses. Intermediaries create value in part by creating liquidity, and, to a lesser extent, so do corporations issuing tradable equity or debt.

But do they create the “optimal” amount of liquidity? Apparently, financial systems can suffer from “shortages of liquidity.” To be sure, those economists who equate liquidity with central bank reserves, would likely call the very notion of liquidity shortage a chimera, given the central banks’ willingness to supply reserves in effectively unlimited amounts at set interest rates. But this chimera has apparently been spotted several times in the last couple of years, in Europe and in North America, and been blamed for financial and macroeconomic crises, as initial recipients of reserves seem unwilling to pass liquidity on to other institutions “at any price,” despite their apparent ability to receive it in elastic supplies from the source.

Thus the importance of the question: Do financial institutions have the incentive to provide liquidity in efficient amounts? This paper argues that they do not: there is an inherent externality in liquidity provision, because it benefits, not only the immediate recipient but also the recipient’s future counterparties. And since the initial recipient values liquidity precisely because the market is incomplete, it becomes impossible to internalize its value to the next agent in line.

We examine this possibility in the context of interbank linkages. In a modern economy, financial institutions interact in a bewildering variety of ways. The liabilities and assets of financial institutions include claims to and from other financial institutions, creating a high degree of interdependency. Some degree of interdependence is inevitable given the informational role of intermediaries. Intermediaries use superior information to channel capital from owner to user; in a world with a high degree of complexity and specialization some intermediaries will have the job of channeling the capital of other intermediaries. Moreover, in a world where the responsibility for effecting payments is largely taken over by private financial intermediaries, the regular course of commercial activity will impose interdependencies on the institutions that take this role.

A recent literature has argued for an inherent fragility in the financial system, and a variety of mechanisms by which fluctuations in the financial system will propagate through
the economy. To what extent is this propagation endogenous, and what are its determinants? “Financial fragility” is the rubric that has been attached to the posited over-connectedness of banks and financial institutions; the claim being that they arrange their affairs in such a way that they thrive or fail too much in concert. There are numerous reasons given for this asserted connectedness (“herd mentality” or regulatory forbearance for example). Still, even if we accept such claims, we do not know that the financial system delivers the “wrong” degree of fluctuation to the economy. Perhaps the financial system is not “fragile” but “responsive.”

We argue that for the most part, financial institutions have the incentive to get it “right.” The flexibility with which financial assets can be designed and priced causes the market participants correctly to take into account the economic effects of their own interdependence. The externalities inherent in liquidity provision cause this aspect of intermediation to be a crucial exception.

In this paper we argue that the two phenomena–financial fragility and liquidity creation–are linked: the role of banks as creators of liquidity leads to their choice of excessive aggregate risk, and often excessive interdependence. We do this in the following manner. We develop the model of an economy in which there is a desire for liquidity. When the liquidity is provided exogenously, then banks choose the optimal level of interdependence and the optimal level of aggregate risk. But when there is a shortage of exogenously supplied liquidity, which can be supplemented by bank liquidity creation, the banks generally fail to find the correct degree of interdependence.

Of course, “liquidity” itself is a slippery concept, and so attempts to build satisfactory models of it tend to be problematic. Demand for and supply of liquidity only make sense in the context of incomplete markets. Roughly speaking, agents demand liquidity to deal with unforeseen or uninsurable contingencies; ones for which there is no contingent commodity market. Supplies of liquidity enable agents to cope in a less-than-perfect way with those contingencies. A formal attempt to discuss optimal liquidity must thus take into explicit account the presence or absence of other, possibly cheaper means to the same end: for example, the possibility of borrowing on one’s own recognizance. In our model, liquidity demand arises because of uncertainty about and limited meetings with future trading partners, necessitating exchanges for current value rather than promises of future repayment.

Banks, in particular can meet this demand, through the powers of diversification, in effect repackaging illiquid loans into liquid deposits and equity (although similar processes
occur to a lesser degree when non financial institutions issue tradable equity or debt.) As long as banks are solvent, these assets can serve as liquidity, being acceptable in exchange. However, once banks become insolvent, their financial instruments become illiquid. In the current economic crisis, this phenomenon hinges on the lack of transparency of the instruments themselves; in our model we represent this by the transformation of the assets into a form which is not regarded as equally desirable by all the relevant participants in the market.

In our model then, a liquid asset is one that is acceptable in trade. The trades we consider are such that the agent transfers the asset directly to the supplier of the goods he desires; thus liquid asset and medium of payment are synonymous in our framework. More generally the liquidity of the asset can arise because it is acceptable by a third party, so that the ultimate provider of goods receives another, still more liquid asset in exchange. However, we would argue that a medium of payment is simply the most extreme version of a liquid asset; the main difference between the two concepts in practice being a time-scale: media of payment are convertible into desired goods within the day; liquid assets are convertible to desired goods within a short amount of time; depending on context, from a day to a month.

Markets, too, are in the business of creating liquidity, by reducing the costs of willing buyers and sellers to meet. And apparently economic agents in general are able to create their own liquidity, by maintaining the flexibility of their situations, purchasing financial options, or manufacturing real ones. In all these cases the externality is present to a greater or lesser degree, but it is mainly in the context of financial institutions with their complex interlinkages and possibilities for mutual insurance that the externality manifests as aggregate instability.

The theoretical underpinnings of our model are the “usual suspects” in the information and contracting literature: moral hazard, imperfect information, and externalities. Indeed the efficiency result is simply an instance of the general principal of constrained optimality in a decentralized ex ante contracting environment with common information.\(^1\) The inefficiency arises when liquidity arrangements are introduced because ex ante contracting is no longer possible among all interested parties. Nonetheless, we think that this distinction is particularly relevant when financial intermediaries run payments systems. In general financial institutions are able to devise complex and flexible arrangements to cover numerous contingencies; thus in general, it is natural to model financial contracts in a complete contracting environment. However

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payments arrangements are intended to be universal. When such systems are decentralized, there is no way for all the potential recipients of payments to agree ahead of time to complex terms for the arrangement. The natural starting point is to assume no ex ante contracting over payments arrangements between all pairs of potential transactors, even if the intermediary institutions themselves retain contracting flexibility.

The rest of the paper is organized as follows. The next section reviews the related literature and compares it with our contribution. Section 3 presents our model and begins by examining a bank’s independent capital decisions. Section 4 extends the model to examine incentives for mutual insurance or cross-guarantees among banks. Section 5 expands this setting to include a liquidity provision role for banks. Section 6 discusses the implications of our model for regulation, and section 7 provides a brief conclusion.

2 Related literature

Our paper is related to two important strands of the literature on financial intermediation: the literature that explains banks based on their liquidity services, and the literature on the stability of the banking system.

In our setting banks are valuable because they create liquidity. Bryant (1980) and Diamond and Dybvig (1983) were among the first to show the importance of banks’ liquidity services: banks are valuable because they offer securities that insure consumers against idiosyncratic shocks to their consumption needs over time. Subsequently, Gorton and Pennacchi (1990) provided a liquidity theory of banks based on their ability to create securities that are not sensitive to private information. In their model, banks provide liquidity not by designing securities that insure investors against a shock to their preferences for the timing of consumption, but instead by designing securities that protect uninformed investors from the costs they would incur when trading with investors that possess superior information. Cavalcanti, Erosa and Temzelides (1999) provided yet another liquidity-based theory of banks. They show that banks can play a valuable role when they create inside money, that is, when they create an instrument which can be used at no cost to trade with third parties. Finally, Holmstrom and Tirole (1998) and Kashyap, Rajan and Stein (2002) show that banks can play a valuable role by offering firms access to liquidity through loan commitments.

Our paper is closest to the literature on inside money in that in our setting banks are valuable because they create a financial instrument — deposits — which agents with different
needs can use at no cost to trade among themselves. Our key result, however, does not require restriction of payments to deposits, nor does it hinge on the assumption that bank debt is the sole medium of exchange.

Our paper is also related to the strand of the financial intermediation literature that investigates the stability of the banking system. While Bryant (1980) and Diamond and Dybvig (1983) were critical to our understanding that a bank is an inherently unstable financial institution, these papers provide only a limited contribution to our understanding of the fragility of a banking system because they consider a system-wide run akin to a generalized panic, and in their settings runs arise as self-fulfilling prophecies (“sunspots”). In subsequent work, researchers expanded this literature to investigate the implications of having multiple banks and to study the determinants of the stability of the banking system. De Bandt (1995), Aghion, Bolton and Dewatripont (1999), Chen (1999), and Rochet and Tirole (1996) show that a full-scale financial collapse can occur in a multibank setting when the failure of a bank provides depositors of other banks with information about aggregate conditions. Acharya (2000) and Acharya and Yorulmazer (2005), in turn, show that banks' incentives to choose correlated portfolios of assets driven by strategic advantages of failing together creates the conditions for a joint failure. Lastly, Freixas, Parigi and Rochet (2000), and Allen and Gale (2000) show that the collapse of the banking system may arise as a result of the financial interlinkages that banks establish among them to manage their liquidity needs.

Our paper is closer to the latter studies and in particular to Allen and Gale (2000). They consider a setting in which banks are subject to idiosyncratic as well as aggregate asset risk. The financial linkages between banks arise from the mutual insurance arrangements that banks establish with one another. Banks' insurance arrangements make them mutually dependent, implying that loss of value in one bank – driven by a liquidity shock for example

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2Gorton (1985), Postlewaite and Vives (1987), Jacklin and Bhattacharya (1988), Chari and Jagannathan (1988) and Allen and Gale (1998) show that the release of information on the banks’ financial condition can trigger, but they too continued to focus on models with a single representative bank.

3Leitner (2005), Babus (2006), and Castiglionesi and Navarro (2007) rely on the theory of networks to show how contagion can arise when banks belong to a financial network, and Kiyotaki and Moore (1997) and Lagunoff and Schreft (2001) are examples of papers in which shocks can spread from one agent to another, leading to financial contagion, because agents are interlinked through some financial connection.

4In Freixas, Parigi and Rochet (2000), a financial crisis can arise as a result of a coordination failure among depositors: if depositors believe that there will be enough resources in their future location, they will liquidate their investment in their original location, making it optimal for depositors in other regions to do the same.
– can cause sufficient loss of value in a second bank to precipitate a run there, and so on, culminating in a system failure. A financial crisis can arise because banks’ insurance agreement protects them against idiosyncratic risk without providing protection against aggregate risk.

We too focus on the implications of the financial interlinkages between banks. However, we depart from Allen and Gale (2000) in an important way. Key to the propagation of failure in their model is Allen and Gale’s assumption that banks act as if the aggregate shock is a zero-probability event. This makes banks indifferent about the degree of mutual insurance. But when the aggregate shock does occur, the degree of mutual insurance does have different implications. In their setting, if insurance is carried out in a “daisy chain” each bank in the sequence bears the full brunt of the shock, propagating it without dissipating it. If the insurance arrangement were instead universal then shocks would dissipate as losses spread out among the other participants. In our setting we drop the assumption that the aggregate shock is a zero probability event and consider the effect on banks’ choice of insurance arrangement.5

Our paper is also related to Acharya (2000) because we too investigate the determinants of the stability of the banking system.6 In his model financial fragility arises because banks choose to correlate their portfolios of assets. In our model in contrast, financial fragility arises because of the mutual insurance arrangement that banks establish to protect themselves from liquidity shocks. We agree that financial institutions do have wide discretion to invest in projects and therefore a temptation to alter the correlation of their projects; nonetheless, fragility through the asset side will likely involve a much slower process than the fragility resulting from interbank liabilities.

Among other recent papers examining financial fragility are Wagner (2007), which argues that banks have an incentive to excessive mutual insurance, inducing a likelihood of joint failure. Unlike Acharya, Wagner’s result stems simply from a desire for diversification.7 Unlike our paper, Wagner’s assumed externality is internal to the banking system and, as he notes, this means that the result depends on non-cooperative oligopolistic behavior by the

5More recently, Brusco and Castiglionesi (2007) also attempted to model contagion in the banking system without relying on unexpected shocks but they focus on banks’ gambling behavior which occurs when banks are not sufficiently capitalized.

6In different context Allen and Gale (2004) also challenge the conventional wisdom that financial crises are bad from a welfare point of view.

7Rampini (1999) also shows that systemic risk, as defined by a substantial correlation of default, may arise as a result of optimal risk sharing.
banks. Our assumed externality is from the banking system to the rest of the economy; thus our result holds even when banks can contract cooperatively with one another.

Underpinning our structure is the assumption that banks need to issue debt contracts. One way to justify this assumption is illustrated in the appendix, which presents a simplified version of a framework developed by Diamond and Rajan (2001). Their model, in turn, builds on the intuition of Calomiris and Kahn (1991) that bank failure, while expensive ex post, can nonetheless be constructive for ex ante incentives. In our version of the environment it is not even necessary to introduce risk aversion as a motive for interbank liabilities: these liabilities can aid in increasing the bank’s available capital or reducing the cost of raising capital.

3 The model

There are three periods, 0, 1, and 2. There are three types of agent, “consumers,” “artisans,” and “bankers.” There are $N$ types of consumer. A consumer derives utility from three different goods, one of which (“gold”) is storable, and the other two of which (“vanilla ice cream” and “special ice cream”) are not. “Special ice cream” comes in $N$ different flavors; an individual of type $i$ derives utility only from type $i$ of special ice cream. He derives disutility from input (labor). An individual of type $i$ receives utility

$$U_i = g + c_0 + c_i - wl$$

where the first three terms are, respectively, the number of units consumed of gold, vanilla ice cream and special ice cream of type $i$, and $w$ is the marginal disutility of labor. Without loss of generality, gold is only consumed in period 2. Ice cream can be consumed in period 1 or 2 (but it must be consumed when produced). Some consumers receive an endowment of one unit of labor in period 0; others receive a unit of gold. The aggregate gold endowment in the economy is $G$.

There are $N$ types of artisan; each specializes in production of a particular kind of special ice cream. Each artisan is endowed with one unit of labor in period 0 and a technology which can transform labor into up to 1 unit of period 1 special ice cream of a particular type, through a linear production function. The marginal product of labor is $f > 1$, so that the artisan never uses more than his own labor endowment. An artisan receives no utility from period 1 consumption; otherwise his utility function is just like that of a consumer. We assume $w < f$ so that specialty ice cream production is socially valuable. There are $K$ artisans of
There are $M$ bankers. A banker has input endowment $C$; any additional input he must purchase from others. He has a production function which generates vanilla ice cream. The production function is stochastic: by using $L$ units of labor input in period 0, banker $j$ can produce $Y(\theta_j, L)$ units of vanilla ice cream in period 2, where each bank’s productivity draw is independent and identically distributed. To keep calculations simple we will work with the following production function:

$$Y(\theta_j, L) = \begin{cases} 
0 & \text{if } L < 1 \\
\theta_j & \text{if } L \geq 1
\end{cases}$$

This guarantees that the bank chooses to employ either zero or one unit of labor. More general production functions will give similar results. Let $F$ represent the distribution function, with support $[a, b]$; when needed we will let $F^*(\theta)$ represent the joint distribution $F(\theta_1)\ldots F(\theta_M)$. We assume

$$w < \int_a^b \theta_j \, dF(\theta_j).$$

Since labor is employed before the productivity draw is realized, this means it is always socially valuable to produce vanilla ice cream. We will assume that the size of the labor force is greater than $M + NK$ so that there is never a shortage of period 0 labor. A banker’s utility function is just like a consumer’s.

The banker’s production can be shut down prematurely in period 1 (liquidated). If so the output is $\delta Y$ units of vanilla ice cream, where $0 \leq \delta < 1$.10

At time 0, bankers do not know their production draw, and neither bankers nor consumers know their consumption type. Both are revealed in period 1. For simplicity, we will assume that in that realization there is no aggregate uncertainty about the underlying demand

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8We assume competitive behavior on the part of the bankers; thus, more precisely there is a unit mass of each of $M$ different types of bank. We maintain the discrete description for ease of exposition; however in the final sections of the paper it will necessitate considering actions taken by “fractions” of a bank.

9We can regard the banker as an agent with a special technology and labor endowment. Alternatively, we can regard the value $C$ as representing the bank’s initial capital; the difference between capital and value of total input will be financed by issuing debt.

10In an earlier version, we consider a more general distribution of $\theta_j$ and $\delta$ as functions of $L$; the results are unaffected.
for specialized ice cream: $1/N$ of all bankers and consumers will prefer each different type of specialized ice cream.

The following table presents a time line of the model.

<table>
<thead>
<tr>
<th>Period</th>
<th>Actions</th>
</tr>
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</table>
| 0      | Contracts written  
         | Production decisions made |
| 1      | Bank investment results revealed  
         | Liquidation decision made  
         | Consumers learn tastes for specialty goods  
         | Consumers and suppliers trade; period 1 production consumed |
| 2      | Investments mature; agents receive and consume payoffs |

### 3.1 Spot markets only

Suppose that there are competitive spot markets within each period and the only way to carry wealth across periods is through gold holdings. In period 0 there is a market for labor. In periods 1 and 2 there are spot markets for ice cream.

The period 2 market is easy to analyze: the gold price of vanilla ice cream is 1 since that is the marginal rate of substitution for all agents. There is no period 2 market for special ice cream: it is only produced in period 1 and there is no way to transfer it to period 2.

The period 1 market is stochastic, depending on realized demand and supply. As of period 1, supply of type $i$ ice cream is inelastic; call it $S_i$. If the gold price of ice cream is greater than 1, the demand for it is zero. Then at a price of 1, the demand for special type $i$ ice cream is completely elastic up to the amount $G/N$, the aggregate gold holdings of all agents with a taste for type $i$ ice cream. At a price $p_i$ less than 1, the market demand is $G/(Np_i)$. Thus the equilibrium price of type $i$ ice cream is $\min\{1, G/(NS_i)\}$. Furthermore, the equilibrium consumer surplus in the market is $\max\{0, S_i - G/N\}$; the equilibrium profit

11This is easiest to see when the supply of vanilla ice cream in period 1 is 0. If the bank supplies positive amounts of vanilla ice cream in period 1, then equilibrium is only a little more complicated: Vanilla ice cream will have a price identical to that of the highest priced special ice cream, for which it will be a perfect substitute. But demand for ice cream by the bank stakeholders increases one-for-one with their period 1 wealth, which increases 1 for 1 with their supply of vanilla ice cream, so that the common price $p$ will remain $\min\{1, G/(NS_i)\}$. 

11
of type $i$ producers in the market is $\min\{S_i, G/N\} - S_i w f^{-1}$; and so the social surplus from
the market is $S_i (1 - w f^{-1})$, independent of $G$.

Note that, since the price for vanilla ice cream in period 1 is never greater than the
price in period 2, banks prefer not to stop production prematurely.

In period 0, labor is elastically supplied at a price of 1. Since their expected profits
from production are positive, banks will desire to employ 1 unit of labor each. The supply of
specialty ice cream depends on the anticipated period 1 price of special ice cream. If $p_i = w f^{-1}$
special ice cream $i$ is supplied elastically up to a total of $K$ units. Thus

$$S_i = \min\{K, \frac{f G}{w N}\}$$

Note therefore that the demand for and supply of ice cream is limited by the availability of
gold in the economy.

### 3.2 Financing generalized production

We will assume $C < 1$, so that bankers will desire to purchase additional inputs. They finance
their demands for input by issuing debt (deposits). The debt works as follows: The bank
promises to pay depositors an amount $D$ (denominated in ice cream). If the bank’s value
is less than this amount, the bank is bankrupt. Bankruptcy entails costs; specifically it
requires premature termination of production, and the division of the liquidated firm among
the depositors.

Many authors have provided rationales for using deposits for financing banks; we simply
assume that debt finance is used. Note, however, that debt financing entails an inefficiency:
ex post, it would be in stakeholders’ interest to continue the bank, although the contractual
arrangement requires that it be liquidated.\(^{12}\)

First, consider the case where $\delta = 0$, so that there is no salvage value to a bankrupt
bank. Now an individual bank will choose $D$ so as to acquire $1 - C$ units of labor. In expectation
the amount received by employees must be $w$, so that the bank must raise from outside the
amount $w(1 - C)$. If $a \geq w(1 - C)$, then it is possible to issue riskless debt, which will have a
face value $w(1 - C)$. Otherwise, provided there exists a value $D$ such that

$$(1 - F(D)) D = w(1 - C)$$

\(^{12}\)See the appendix for a model justifying these assumptions.
then the bank can be financed by issuing risky debt with face value \( D > a \). In fact, the bank will choose the minimum value of \( D \) for which the equality holds.

More generally the bank will choose the minimum value of \( D \) for which

\[
(1 - F(D))D + \delta \int_a^D \theta_j \, dF(\theta_j) = w(1 - C).
\]

In other words, the debt holders also include the liquidation value of the bank in estimating the expected payoff from holding debt.\(^{13}\)

4 Cross holdings

Because of the cost associated with bankruptcy, it may be worthwhile for bankers to engage in mutual insurance. This insurance could take many forms: cross-holdings of shares, offsetting deposits and the like. The simplest form that it could take is consolidation of the companies. If all bankers are consolidated into a single company, the company’s production function is, in effect,

\[
\max\{L, M\} \sum_{j=1}^{\max\{L, M\}} \theta_j
\]

where \( L \) is the total number of units of labor employed by the consolidated bank. Let \( F' \) be the distribution function for \( \theta' = \sum_{j=1}^M \theta_j \).

It is feasible to use debt finance with a face value \( D' \), provided that there exists a value \( D' \) under which

\[
(1 - F'(D'))D' + \delta \int_{Ma}^{D'} \theta' \, dF'(\theta') = Mw(1 - C)
\]

Again, the consolidated bank will choose the lowest value of \( D' \) such that this condition holds.

4.1 Efficiency of Cross Holding Choices

Since the bankers are ex ante identical, there is no difficulty in deciding whether or not they will consolidate: they simply choose the configuration which maximizes their joint expected profits, subject to the requirement that the depositors receive their required return on deposits.\(^{13}\)

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\(^{13}\)Since the price of ice cream in terms of gold in period 1 is never more than 1, depositors will at least weakly prefer to consume their liquidated vanilla ice cream, rather than trade it for gold.
Thus the expected payoff for all bankers, given that they produce and remain independent, is

\[ M \int_{D}^{b} (\theta_j - D) \, dF(\theta_j) = M(E[\theta_j] - w(1 - C)) - M(1 - \delta) \int_{\theta_j < D} \theta_j \, dF(\theta_j) \]

and their expected payoff if all bankers consolidate is

\[ \int_{D'}^{M_{th}} (\theta'/D') \, dF'(\theta') = M(E[\theta_j] - w(1 - C)) - (1 - \delta) \int_{\theta'/D'} \theta' \, dF'(\theta') \]

In each case, the right side is simply the social benefit from the production less the cost of the labor. This social surplus is net of the expected deadweight loss from bankruptcy. Since the choice is in effect maximizing social welfare, an immediate consequence is

**Proposition 1** Under debt financing, banks choose the constrained efficient level of output and interdependence.

In the case where a bankrupt bank has no value \((\delta = 0)\), the results are more specific

**Proposition 2** If \(\delta = 0\), then banks always choose complete consolidation. Complete consolidation lowers the face value of debt and reduces the probability that any individual bank is bankrupt. It can increase or decrease the probability that all banks go bankrupt.

The last part of this proposition yields an ambiguous result because, while consolidation with a given face value increases the joint probability of bankruptcy, the reduction in face value decreases the probability. When we allow for more general types of salvage value, then the banks may prefer to remain independent; even so, under debt financing, banks choose the constrained efficient level of integration—that is, the level that minimizes the expected deadweight cost from bankruptcy.

### 4.2 Extensions

We have only discussed the possibility of the two extremes of bank interdependence: complete independence and full mutual insurance. We could imagine a variety of intermediate cases, and with them a variety of limited or contingent forms of recourse between the holders of one institution’s liabilities and the assets of the other institution. Kahn and Winton (2004) show in particular that, when the choice of investments is not observable by the debt holders, the introduction of more complex subsidiary structures, with limited recourse among them, can improve a bank’s choice of investments. Nonetheless, it will still be the case that a financial
institution with these instruments available to it will still pick the constrained socially optimal
degree of interdependence between the various affiliated institutions.

The presentation above implicitly incorporates a variety of competitive assumptions: in particular, no bank possesses monopoly power over the capital market. A consolidated set of banks would very likely possess some such market power, and that would be a deterrent to allowing them to join together. In the presence of antitrust regulation on interest rate pricing, it is conceivable that mutual insurance arrangements could provide a back-door to such market power.

In Allen and Gale (2000) a second issue arises: aggregate shocks have the potential to change the shadow value of capital.\textsuperscript{14} That paper’s “aggregate shock” is in fact concentrated at a single bank. Thus taking the model literally, a solution would be to cut that single bank out of the insurance arrangement in the case of an “aggregate shock.” Arrangements in which the degree of insurance provided by one bank to another varied depending on aggregate conditions are extremely plausible—indeed historically documented. Roberds (1995) cites a variety of examples where early clearing houses provided a limited form of mutual insurance which only arose in times of panic.

Nonetheless, it is also plausible that interbank arrangements are imperfect and that some of them would on occasion mistakenly treat aggregate (non-insurable) shocks as if they were insurable, (as appears to have been the case in the past few years). However a systematic restriction of this sort requires quite stringent restrictions on clauses permitted in contracts: It means treating aggregate shocks as nonobservable, at least within the relevant time frame. It means treating market prices generated—including pricing of intermediary liabilities themselves—as either unobservable, or as somehow not including aggregate information. And it means prohibiting the use of one firm’s bankruptcy as a triggering clause in the contracts of other intermediaries.

5 Liquid financial assets

Now suppose that financial assets of the bank at period one can also be used to pay for specialized ice cream. Let $A/N$ be the value of financial assets (denominated in gold) held by stakeholders who have a taste for $i$ type ice cream. This value is stochastic; thus price in the

\textsuperscript{14}See also Diamond and Rajan (2000) and (2003).
spot market for special ice cream is also stochastic. Arguing as in the previous section, we can
determine that demand for specialized ice cream is 0 if \( p_i > 1 \) and \((G + A)/(Np_i)\) if \( p_i < 1 \). Thus

\[
p_i = \min\{1, \frac{G + A}{NS_i}\}
\]

in other words, in equilibrium the spot market behaves as before, with \( G + A \) replacing \( G \).

Without loss of generality, we can assume that the residual value of the bank is held
by the banker in the form of equity, which can also be used for purchases (this is the natural
assumption, since the value of equity is certain as of period 1). Thus the value of the assets
that can be used to purchase specialized ice cream is now \( G + A \), where \( A \) is the total value
of all banks. In other words, the total value of all liquid assets is the value of the gold plus
the value of the assets in banks which ex post remain solvent; other banks are worthless for
trading purposes.

### 5.1 Capital Structure

With the prices for period 1 determined, and the supply of special ice cream given, we can
now investigate the bank’s choice of capital structure, and the face value of debt required by
depositors. We start with independent banks.

If the bank fails, the depositor receives \( \delta \theta_j \) of vanilla ice cream. If the bank remains
open the depositor receives \( D \) of liquidity (denominated in gold). Recall that the gold price
of special ice cream in the spot market is never greater than 1. Thus the depositor at least
weakly prefers to spend all his wealth on special ice cream and his utility is \( D/p_i \). In short,
the depositor’s payoff if investing in an independent bank is

\[
D/p_i \text{ if } \theta_j > D, \\
\delta \theta_j \text{ otherwise}
\]

Note that since liquidated ice cream can not be used to pay for special ice cream (it
will melt before the artisans want it), the price of special ice cream does not appear in the
second part of the formula.\(^\text{15}\)

\(^{15}\)In other words, melting ice cream is illiquid!
Meanwhile, the independent banker’s payoff is
\[ (\theta_j - D)/p \text{ if } \theta_j > D \]
\[ 0 \text{ otherwise} \]

Thus the total value of the bank is:
\[ \theta_j/p_i \text{ if } \theta_j > D \]
\[ \delta \theta_j \text{ otherwise} \]

The expected value to the depositors of the bank’s deposits is thus
\[ \int_{\theta_j > D} D \max[1, \frac{S_i}{N(G+A)}] \, dF^*(\theta) + \delta \int_{\theta_j < D} \theta_j \, dF^*(\theta). \tag{2} \]

Note that \( A \) is also dependent on \( \theta \). For the depositor willingly to invest, (2) must be at least equal to the amount of capital the banker wishes to raise, which for the independent bank is \( w(1-C) \). The banker maximizes firm value (and his own payoff) by choosing the minimum value of \( D \) for which this is the case.

For the consolidated bank the corresponding formulas replace \( \theta_j \) by \( \theta' \), \( D \) by \( D' \), and \( w(1-C) \) by \( Mw(1-C) \).

For completeness, note that the payoff to the consumers who hold gold (either because they did not deposit it initially or because they were paid in gold for their period zero labor) is
\[ \max\{G, GNS_i\} \]
so that holders of gold actually do best as their proportion of the economy’s financial wealth increases—that is, as the value of the banks falls.

5.2 Artisan profits and the role of liquidity

Assuming that artisans cannot write ex ante contracts for their output, their expected profit per unit produced is
\[ E_0 p_i - wf^{-1} \tag{3} \]
where \( E_0 p_i \) is the period 0 expectation of the right side of (1). If (3) is negative, supply is 0; if it is positive supply of special ice cream \( i \) is equal to \( K \), the industry capacity. Thus \( S_i \) is determined by the following pair of conditions,
\[ E[\min\{1, (G + A)/(NS_i)\}] \geq w f^{-1}; \quad S_i \leq K \tag{4} \]
which hold with complementary slackness. Note therefore that the introduction of financial
assets increases the supply of special ice cream.

In this world, financial assets play two roles: they represent value of the output pro-
duced by the bank, and they represent purchasing power for their holders. To the extent
that the holders do not capture the entire surplus from subsequent transactions, the value
they place on the shares is less than the social value that should be attached to them. If a
government were to issue additional promissory notes, (redeemable in vanilla ice cream) and
if it had the taxing power to back those notes, then welfare in the economy would increase:
it is as if the gold constraint disappeared, making the price of ice cream always equal to 1
In this case $S_i = K$. The limitations imposed by the shortage of liquid assets is simply an
example of “cash-in-the-market” pricing (see, among others, Allen and Gale (2007), Acharya
and Yorulmazer 2007)).

5.3 Equilibrium and the aggregate value of liquid assets

In the independent case, the liquid value of a bank is $y_j$, where

$$y_j = \theta_j \text{ if } \theta_j \geq D;$$

$$0, \text{ otherwise.}$$

In the consolidated case, the liquid value of a bank is $y'$, where

$$y' = \sum_{j=1}^{M} \theta_j, \text{ if } \sum_{j=1}^{M} \theta_j \geq D';$$

$$0, \text{ otherwise.}$$

Thus $A$ is defined as follows

$$A(\theta; D, D') \equiv \pi \sum_{j=1}^{M} y_j + (1 - \pi)y'$$

where $\pi$ is the proportion of each type of bank in the economy that remains independent.

In this model, an equilibrium is, in principle, quite complex. An equilibrium specifies
the period 0 supply decisions of all producers and the period 0 capital structures and inter-
dependence choices of all banks. It specifies the spot prices and trade for specialty goods in
period 1 as a function of the realized values \( \theta_i \) for all banks and the realized preferences of consumers.

We want to look at competitive equilibria. Similarly, depositors must be indifferent between depositing in a consolidated bank, depositing in an independent bank, and not depositing at all. For convenience we will assume a certain degree of symmetry: We will assume that consumers are allocated among the banks in such a way that all types have identical aggregate wealth in any realization.

Given supplies \( \{S_i\} \), a short run equilibrium is a triple \((D, D', \pi)\) satisfying the following conditions:

\[
D = \min D^* \text{ st. } \int_{\theta_1 > D^*} D^* \max[1, \frac{S_i}{N(G + A(\theta; D, D'))}] \, dF^*(\theta) + \delta \int_{\theta_1 < D^*} \theta_1 \, dF^*(\theta) \geq w(1 - C).
\]

\[
D' = \min D^* \text{ st. } \int_{\sum \theta_i > D^*} D^* \max[1, \frac{S_i}{N(G + A(\theta; D, D'))}] \, dF^*(\theta) + \delta \int_{\sum \theta_i > D^*} \sum \theta_i \, dF^*(\theta) \geq Mw(1 - C).
\]

\[
\pi \in \arg \max \pi \int_{\theta_1 > D^*} \theta_1 \max[1, \frac{S_i}{N(G + A(\theta; D, D'))}] \, dF^*(\theta) + \delta \int_{\theta_1 < D^*} \theta_1 \, dF^*(\theta) + (1 - \pi) M^{-1} \{ \int_{\sum \theta_i > D^*} \sum \theta_i \max[1, \frac{S_i}{N(G + A(\theta; D, D'))}] \, dF^*(\theta) + \delta \int_{\sum \theta_i > D^*} \sum \theta_i \, dF^*(\theta) \}.
\]

In other words, all banks make value maximizing choices, picking the lowest feasible levels of face value of debt, and making the value maximizing consolidation decision. Note that \( D' \) and \( D \) appear in the denominators, rather than \( D^* \); this is appropriate given we are assuming competitive behavior, since that implies that banks are price takers.

Standard fixed-point arguments demonstrate

**Proposition 3** Given \( \{S_i\} \), for \( w \) sufficiently small a short-run equilibrium exists in which banks take positive amounts of deposits.

A long run equilibrium is a quadruple \((S, D, D', \pi)\) where \((D, D', \pi)\) is a short run equilibrium given \( S_i = S \) for all \( i \), and \( S \) satisfies conditions (4) with complementary slackness.

In other words, in the long run equilibrium, artisans are also picking supplies to maximize expected profits; without loss of generality we assume they behave identically. Note that we do not assume that all banks behave identically; in fact we can demonstrate the opposite.
Proposition 4 For $G$ and $w$ sufficiently small, in long run equilibrium $\pi \notin \{0, 1\}$.

The intuition behind this result is that when $G$ is small, it is extremely valuable to be the only bank surviving. If all other banks are consolidated, there will be states in which an independent bank will survive when all others fail, and vice versa. As in Allen and Gale (2007), the equilibria in this framework will have asymmetric behavior by the banks—indeed this is enough to demonstrate the inefficiency of the arrangement. Briefly, banks will find it advantageous to survive when other banks have failed. If all banks were consolidated, it becomes advantageous to be an independent bank, and vice versa. Thus the equilibrium must have a mixture of consolidated and independent banks, all of identical value.

5.4 Welfare Analysis

Proposition 5 When $G$ and $w$ are sufficiently small, in a long run equilibrium, a decrease in the riskiness of the financial sector would be welfare improving.

The proof is a specific application of the techniques of Greenwald and Stiglitz (1986).

Proof (Outline): In equilibrium a bank is indifferent between consolidating and remaining independent, because to first order its value is unaffected by the choice. However, to first order, the choice is not a matter of indifference to the artisans: Their utility is a concave function of the financial wealth in the economy. Given all agents’ risk neutrality, this utility improvement is offset by the utility loss of the remaining agents in the economy (holders of gold). However the artisans make a first order response to the increase in expected utility by increasing their choices of output levels to produce, and since output is socially valuable, this causes an unambiguous increase in the economic surplus of the economy. Q.E.D.

In other words, at least locally, the banks choice of mutual insurance leads to an excessively risky economy. Of course, if the producers could also be brought into the collective agreement ex ante, this inefficient choice could perhaps be bargained away. But if producers do not meet consumers or intermediaries until after the contracting period then this bias persists. This inability to meet ex ante is simply our way of modeling the need for liquidity: the need to hold some assets for exchange later because it is not possible to handle all contingencies through ex ante contracts.

The inefficiency we find is due to the shortage of liquidity, and to the externality that banks impose when they supply liquidity. For $G$ sufficiently large, of course, this externality
ceases to be an issue, because there is never a liquidity shortage in the economy:

**Proposition 6** For $G$ sufficiently large, banks choose the efficient level of consolidation.

In general we cannot state whether it is a decrease or increase in consolidation which would reduce overall riskiness; nonetheless, we can show the results for some simple examples, to begin to understand the considerations that lead to one conclusion or the other.

## 6 Implications for Regulation

The plausibility of the contract limitation whereby sellers are unable to contract with banks requires a large number of banks in the system. The payments system, for example, has a natural scale economy: it is most effective when it is universal, encouraging production by extending the market for product. To the extent that payments systems are decentralized, with producers potentially receiving payment from any of a large number of purchasers delivered through any of a large number of intermediaries, it is unreasonable to assume that a potential recipient of payment can pre contract with all potential customers, or with their representative intermediaries. As a result, there is no incentive for those intermediaries to maintain the effectiveness and stability of that system.

Natural remedies include public maintenance or subsidy for the payments system, or for liquidity provision in general. A related regulatory remedy is for the government to intervene to protect major liquidity-providing institutions in the case of widespread failure—a policy of “too strategically important to fail.” In general this subsidy can take the form of ready provision of liquidity in response to aggregate shocks.\(^\text{16}\)

The policy implications of this model have some similarities with those of Acharya (2000): we want to provide banks with incentives not to fail when others are failing—that is, to be more forbearing of idiosyncratic failures than of collective failures. Like Acharya’s analysis, ours thus leads to an immediate policy dilemma: the conclusion runs into the face of the standard recommendation that idiosyncratic failure is a signal of malfeasance, and thus needs to be punished more severely.

The different source of bank interdependences that the two papers consider has some policy implications, though. Recall that Acharya (2000) consider bank interdependences arising from their assets while we investigate interdependences arising from banks’ liabilities. Accounts

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\(^{16}\)Ready provision of liquidity by the government is also the remedy in Allen and Gale (2000).
relying on interdependence of bank liabilities differ, for example, from accounts relying on interdependence of bank assets in terms of the operative time scale. The combination of large interbank liabilities, high leverage, and ease of adjusting the financial portions of the balance sheet make the liability side of the balance sheet important for short-term economic stability. Given the relative illiquidity of bank assets the problems of excessive interdependence are therefore more likely to be of relevance for stabilization policy on the liability side.

Furthermore, our framework emphasizes that it is the decentralization of liquidity provision that causes the potential instability of the financial sector. If the payment system in particular, or liquidity creation in general, is concentrated in the hands of a small number of institutions—and in particular if the system is arranged like the credit card industry, where payments recipients also establish extensive ex ante contractual relations with the payments mechanism (through, e.g., credit card merchant accounts)—then there is scope for the internalization of costs imposed by overdependence among financial institutions. Loosely speaking, then, the arguments we have provided might better justify intervention by financial authorities in America than in Europe.

7 Final remarks

Fragility stems from the interconnections banks establish to protect themselves from liquidity shocks. Mutual recourse for bank liabilities is often advantageous. The question we have addressed is whether banks have an incentive to increase fragility beyond the socially desirable level. When aggregate shocks are no longer a zero probability event banks find cross guarantees by other banks valuable, and are no longer indifferent about the form this insurance takes. We have built a framework examining the consequences for interbank insurance when individual bank fragility stems from a moral hazard problem.

As long as ex ante contracting between equally-uninformed parties is feasible, then banks and debt holders can reach constrained optimal arrangements for themselves. If cross guarantees impose a risk, then the price of debt adjusts accordingly. Nonetheless, cross guarantees may be suboptimal, since they place too high a correlation on individual bank failures. If banks also provide liquidity services, then the social costs may be higher from having all fail than from having some fail, thus creating a justification for a regulatory intervention, when these systems are decentralized.

Of course, in our structure, one easy way to eliminate the problem is for the central
bank permanently to provide assets to substitute for gold as a means of payment, not just in times of crisis. There are clearly two limitations on this policy: the credibility of the central bank itself, and the problems of credit worthiness of the initial recipients of this liquidity and their counterparties. Extending the model of this paper to take these additional considerations into account is a task that is timely, as well as important.
Appendix

In this appendix we show how a debt contract within our framework can be rationalized by a simplified version of the model of Diamond and Rajan.

Each bank raises capital by issuing two sorts of liabilities: “deposits” with a total face value of $D$ and “junior bonds” with a total face value of $R$. We assume that if $Y < D$ the bank is liquidated, depositors receive $X$ and the bank and the bond holders receive zero. If $Y > D$ depositors receive $D$, the bond holders receive $\min\{X - D, R\}$ and the banker receives $(Y - z)_+$ where

$$z = \begin{cases} D & \text{if } D > X \\ X & \text{if } D + R > X > D \\ D + R & \text{if } X > D + R \end{cases}$$

Figure 1 illustrates this division of payoffs as a function of $Y$ for the case where $X$ is a constant proportion of $Y$). We justify these payoffs in the following subsection.

Explanation for the payoffs

Suppose the bank has the ability to issue only debt instruments. These instruments vary in two ways: seniority and “representation.” Once $Y$ and $X$ are revealed, in period 1, the bank will attempt to renegotiate the face value of its instruments; the bank is assumed unable to commit not to renegotiate. Renegotiation takes the form of the following simple game: The banker proposes a revised face value of every instrument. If all debt holders unanimously assent to the revision, it occurs, otherwise the bank is liquidated according to the initial terms, with those rejecting the renegotiated arrangement coming first within their class. For an instrument without representation, debtholders assent to or reject the renegotiation proposal individually. An instrument with representation is assumed to have a “spokesperson” representing the interests of the holders collectively (since their preferences are identical, this poses no problem); the “spokesperson” individually provides assent or rejection for the class as a whole. (Thus all members of a class with representation receive identical payment).

In this game the bank issues junior bonds (with representation) and deposits (no representation). Once $Y$ and $X$ are revealed, the banker proposes new face values for junior bonds and deposits $(D', R')$. Once the announcement is made, each depositor simultaneously announces acceptance or rejection of the revision. At the same time, the representative of
the bond holders announces acceptance or rejection. If any party rejects, then the firm is liquidated, and the value \(X\) is divided in period 1 with individuals receiving up to their face value according to the following seniority: Rejecting depositors come first, accepting depositors come second, all bondholders come third, and the banker comes last. If all agents accept, then the value \(Y\) will be divided in period 2 according to the same seniority. In this game, if some depositors are arbitrarily small, the unique subgame perfect equilibrium has payoffs as described above. In this game the banker proposes \((D', R') = (D, \min\{R, (X - D)_+\})\), the representative of the bondholder always accepts, and the depositors reject when \(X < D\). To see this, note that the new \(D'\) always equals the old \(D\) (if it were less, some small depositor would object; if greater, the banker’s profits are reduced). And the bondholders receive in every state exactly the amount they would receive if the bank were liquidated.

This structure for a bank captures the central insight of Diamond and Rajan (2001): demandable debt (deposits) can enable the bank to borrow more than it could borrow under simple debt, by making it more difficult to renegotiate arrangements after the fact, whereas simple debt will have limited ability to generate funds because of ex-post hold-up. When the bank is able to pay the depositors but not the junior bond holders, the bond holders are negotiated down to a lower payment without actually liquidating the bank; the depositors can not be held up and therefore can receive more than the threat value \(X\). (In Figure 1, the graphical depiction of this effect is the triangle of depositors’ payoff which lies above the line \(X\).) On the other hand, the impossibility of renegotiation with the depositors means that the demandable debt arrangement is inflexible, leading in some circumstances to additional expenses of liquidation. (the empty triangle between the \(X\) and \(Y\) lines).

\[
\max_{D, K_D, K_R} \int_{Y > D} Y \ dF(Y, X; K_D + K_R) + \int_{Y < D} X \ dF(Y, X; K_D + K_R) - (K_D + K_R)
\]

again, subject to (5) and (7). Note that this simplified expression is also the social surplus associated with the bank’s activities. Thus an economy of such banks achieves a constrained efficient outcome. Capital structure for independent banks

A banker can raise capital from deposits or junior bonds. The amount of capital raised through deposits \(K_D\) and bonds \(K_R\) depends on the initial terms \((D, R)\) and the implicit return they generate. The return that the depositors will receive is

\[
D \int_{Y > D} dF(Y, X; K_D + K_R) + \int_{Y < D} X \ dF(Y, X; K_D + K_R).
\]
Figure 1: Bank payoffs

Note that this is independent of the choice of $R$. The return that the bond holders receive is

$$
\int_{D < X < D + R} [X - D] \ dF(Y, X; K_D + K_R) + R \int_{X > D + R} dF(Y, X; K_D + K_R).
$$

The banker’s objective is to choose an initial capital structure $(D, R, K_D, K_R)$ so as to maximize

$$
\int_{Y > D} [Y - z] \ dF(Y, X; K_D + K_R)
$$

subject to the following two individual rationality constraints for the two classes of investors:

$$
D \int_{Y > D} dF(Y, X; K_D + K_R) + \int_{Y < D} X \ dF(Y, X; K_D + K_R) \geq K_D \quad (5)
$$

$$
\int_{D < X < D + R} [X - D] \ dF(Y, X; K_D + K_R) + R \int_{X > D + R} dF(Y, X; K_D + K_R) \geq K_R \quad (6)
$$

Issuing deposits imposes an efficiency loss; therefore we wish to raise as much funding as possible through junior bonds. It is clear that the constraints in the above problem are relaxed by setting $R$ arbitrarily high, thus we can drop $R$ from the problem and constraint

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17Note that we are assuming that the choice of investments—that is the choice of the bank’s distribution of asset values—is observable at the time of collecting the period zero investment inputs. If not considerations as in Kahn and Winton (2004) apply.

18Diamond and Rajan (2001) discuss additional considerations which would limit the bond issue. These considerations are not needed for our purposes.
(6) simplifies to
\[
\int_{D<X} [X - D] dF(Y, X; K_D + K_R) \geq K_R. \tag{7}
\]
Even so the amount of capital that can be raised through bonds alone is limited to the expected value of \(X\). If additional capital is needed, deposits must be introduced. Deposits will arise if the bank wishes to raise more capital than it can through junior bonds alone:

\[K^* > EX(K^*)\]

for

\[K^* \in \text{arg max} [EY(K^*) - K^*]\]

—in other words, were we to solve the problem assuming \(D = 0\), constraint (7) would be binding.

In the context of payment in our model, it is most consistent to assume that all financial assets can be used as payment, since there are no informational constraints on any of them.
References


