

Domestic Sovereign Default as Optimal Redistributive Policy

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(PRELIMINARY AND INCOMPLETE)

Abstract

Infrequent but dramatic episodes of outright default on domestic sovereign debt are an important historical fact that remains unexplained. We propose an incomplete-markets, heterogeneous-agents model in which domestic default can be optimal for a utilitarian government that responds to distributional incentives. The government finances the gap between stochastic expenditures and lump-sum taxes by issuing non-state-contingent debt, but it retains the option to default. The distribution of public debt across private agents is endogenous and interacts with the government's optimal default, debt issuance and tax decisions. Repaying is beneficial because it allows the government to access the debt market and provides a mechanism for households to self insure and smooth consumption, but it also increases the need for future tax revenues. Default is optimal when repaying hurts relatively poor agents more than defaulting hurts relatively rich agents, and this occurs along an equilibrium path when public debt is high enough and its ownership is sufficiently concentrated. Unlike standard models of external sovereign default, the model supports realistic debt-output ratios on average (40%) and before default (60%) at a nontrivial default frequency (8%).

Keywords: Public debt, sovereign default, debt crisis

JEL Classifications: ??

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1 Introduction

A central finding of the seminal historical review by Reinhart and Rogoff [22] is that, in data going back to 1750, there are at least 68 episodes of *de jure* sovereign default on domestic debt (i.e. outright defaults by means other than *de facto* defaults via inflation).¹ Furthermore, they document that, while these domestic defaults are less frequent than the more familiar external defaults, they are at least as important in terms of magnitude and the associated macro-economic instability, and many of them triggered external defaults.

De jure domestic defaults took place via mechanisms ranging from forcible conversions to lower coupon rates to unilateral reduction of principal and suspension of payments. For example, Argentina defaulted three times on its domestic debt between 1980 and 2001. Two of these defaults coincided with external defaults (1982 and 2001) and a large scale domestic default in 1989 did not involve external debt. Other examples of recent domestic defaults include countries in Africa, Europe and the Middle East.

The importance of understanding the dynamics of domestic sovereign debt, including outright default, is highlighted by two other key historical facts also identified by Reinhart and Rogoff [22]: Domestic sovereign debt accounts for a large fraction of total public debt in most countries (almost two-thirds on average), and taking this into account helps rationalize why external defaults can occur even when the external debt ratio is not too high.² These patterns have become even more accentuated in recent years. In 2011, the global bond market of local-currency public debt was valued at about U.S.\$30 trillion, roughly 50 percent of the world's GDP and 6 times larger than the market for investment-grade sovereign debt denominated in foreign currencies. Moreover, gross general government debt as a share of GDP reached about 100 percent for advanced economies, compared with 40 percent for developing economies.³ Kumhof and Tanner [14] also document large domestic public debt ratios across industrialized and developing countries using data going back to the early 1990s.⁴

¹We say "at least" because the data do not go back as far as 1750 for many of the countries in their panel data set. Note also that Reinhart and Rogoff [22] do find that *de facto* defaults via reductions in the real value of public debt denominated in local currency through inflation are important, but this still leaves them with the 68 events of *de jure* defaults.

²Note, however, that in our theoretical framework we model domestic public debt as debt that is held domestically, while Reinhart and Rogoff define domestic (external) debt as debt issued under home (foreign) legal jurisdiction. There is a strong link between domestically held public debt and debt issued under domestic legal jurisdiction but the relation is not one to one, as we discuss later in the paper.

³Global bond market and debt ratios are from *The Economist*, Feb. 11, 2012, based on data from Bank of America Merrill Lynch and IMF.

⁴The definition of domestic debt in [14] is in line with our model definition of domestic debt. In particular, their source is BIS data that defines domestic debt as securities issued by residents, targeted at resident investors in domestic currency. This definition is very conservative but covers the borrower side of securities issues and not actual ownership.

In addition to the historical cross-country evidence, the importance of studying domestic public debt and default is emphasized by the recent debt crisis in the European Union. The sharp rise in European public debt ratios was in part a by-product of the 2008 global financial crisis. OECD data show that between 2007 and 2011 the increases in general government debt in Greece, Ireland, Portugal and Spain ranged from 30 to 90 percent of GDP (see Table 1 for information on debt levels and other relevant fiscal variables in 2010). But notwithstanding the effects of the crisis, European countries face deep structural fiscal solvency problems driven by secular high ratios of government expenditures to GDP, and large entitlement programs with adverse demographics and unfunded liabilities. At the end of 2011, debt ratios in *all* large Western European countries, except for Spain, ranged between 85 and 160 percent of GDP.

Table 1: Euro Area Fiscal and Debt Situation in 2010

Moment in (%)	Gov. Debt	Gov. Debt Held by Residents	Gov. Exp.	Gov. Rev.	Primary Balance	Sov. Spreads
France	98.62	31.87	30.97	49.54	-7.08	0.65
Germany	86.88	n.a.	22.35	43.59	-4.28	0.00
Greece	165.10	n.a.	29.47	39.48	-10.76	14.48
Ireland	112.57	14.42	48.82	35.48	-10.06	6.62
Italy	127.74	48.24	28.19	45.76	-4.53	2.60
Portugal	111.94	n.a.	20.61	41.62	-9.79	7.30
Spain	74.13	55.21	27.28	36.28	-9.24	2.70

Note: Author's calculations based on OECD Statistics. All variables except "Gov. Debt Held by Residents" and "Sov. Spreads" reported as a ratio of GDP. "Gov. Debt" corresponds to Total General Government Debt. "Gov. Debt Held by Residents" corresponds to marketable central government debt held by domestic non-financial corporations, financial institutions, other government sectors, households and non-profit institutions. "Gov. Exp." denotes total general government expenditures net of social benefits and social transfers in kind. "Gov. Rev." corresponds to general government revenues. "Primary Balance" corresponds to general government net borrowing (−) or net lending (+) excluding interest payments on consolidated government liabilities. "Sov Spreads" correspond to the difference between interest rates on 10 year government bonds of the given country and Germany. For a given country i , they are computed as $\frac{(1+r^i)}{(1+r^{Ger})} - 1$.

Two features of the European Union make it appealing to approach the European debt crisis from a *de jure* domestic default perspective: First, a large fraction of Euro-zone governments' bonds are held within Europe, so we can think of systemic default across some European countries as a (partial) domestic default from the point of view of the E.U.⁵ Second, members of the

⁵We conduct most of our analysis assuming that when default occurs all debt is repudiated, but we also study an environment with partial default. Note, however, that in our model one single fiscal authority claims all tax revenues, whereas in Europe this only applies to seigniorage collected by the European Central Bank, with other

Euro area share a common currency, which prevents them from unilaterally reducing the real value of debt through inflation (i.e. implementing country-specific *de facto* defaults).⁶ Interestingly, the United States is not in much better fiscal shape than European countries, but with a flexible exchange rate and the global use of its currency, the scope for a *de facto* default through inflation and devaluation is much larger.

Unfortunately, as Reinhart and Rogoff [22] also highlighted, the record of *de jure* domestic sovereign defaults and its associated empirical regularities form part of a “forgotten history” that most of the macroeconomics literature has not studied. In this paper, we aim to make some progress towards explaining these facts. In particular, we propose a model with heterogeneous agents and incomplete financial markets in which infrequent outright domestic sovereign defaults occur as a result of endogenous distributional incentives faced by the government. Intuitively, domestic default emerges as an optimal decision reached when the welfare cost that default imposes on those that are the government’s creditors is lower than the welfare cost that debt repayment would impose on society in general, and particularly on those that do not hold government bonds. In contrast with external default models, however, the government’s social welfare function depends on the utility of all domestic agents, including government bond holders.

The problem is complex, because under incomplete markets the dynamics of the wealth distribution (i.e. the distribution of public bond holdings across private agents), the total issuance of public debt, default risk premia and the social welfare function are all endogenous and interact with each other. Moreover, even with purely idiosyncratic individual income shocks, sovereign default represents a form of aggregate risk, which implies that the distribution of wealth across government bonds and income shocks varies randomly over time, making the distribution itself a state variable.

The model we propose makes important modifications to the workhorse heterogeneous agents model of the wealth distribution and wealth dynamics under incomplete markets to incorporate sovereign default. In the model, as in Aiyagari and McGrattan [3], government debt enhances the liquidity of households by providing a means for self-insurance and consumption smoothing. For simplicity, we assume that public debt is the only asset available to households and that households face idiosyncratic income shocks and are subject to a borrowing limit. The government faces an exogenous stochastic stream of expenditures and levies lump-sum taxes, financing primary deficits with one-period, non-state-contingent bonds but retaining the option

tax revenues controlled individually by each nation. There are agreements that coordinate tax policies at various levels and the recent accord to tighten fiscal ties brings Europe closer to being in line with our assumptions, but we acknowledge that this is still far short from our model’s assumption of a single fiscal authority.

⁶This also applies to other important cases as economies with currency boards (e.g. Argentina prior to the 2001 default) or economies that use a hard currency as medium of exchange (e.g. Ecuador or Panama).

to default. We assume lump-sum taxes to highlight the role of the government's distributional incentives for repayment without blurring the picture with the effects of tax distortions.

Each period, the government decides whether to default or not, how much debt to issue and the level of taxes. The government uses a utilitarian social welfare function to evaluate the payoffs of repayment versus default. We focus mainly on a standard utilitarian welfare function that weights each agent's utility using the economy's endogenous wealth distribution.

Since defaults are not selective by design (i.e. the government cannot default on a particular borrower or group of borrowers), but holdings of public debt are unequally distributed across agents endogenously because of the incompleteness of asset markets, domestic sovereign default has important distributional consequences. If the government defaults, the short-run benefit is the reduction of the level of taxes paid by all agents. The short-run cost is that default reduces the wealth of those with positive holdings of debt. Moreover, since following a default the government is not allowed to issue new debt for a finite set of periods, the costs of a default include more volatile future taxes and the fact that agents lose access to the only asset used to self-insure against idiosyncratic and aggregate shocks. On the other hand, if the government decides not to default, the government chooses the optimal mix between lump-sum taxes and new debt needed to finance government expenditures and to service the previously contracted debt.

The costs and benefits of debt repayment are not equally spread among the population. Taxes represent a larger fraction of total income for those with low debt holdings, so (conditional on a debt repayment) these agents are better off if the government finances debt repayments by borrowing more. Moreover, by increasing the level of debt the government smooths taxes faced by agents and this provides another channel of consumption smoothing. The cost of increasing the level of debt is that expected future taxes are higher and a default more probable.

We study the quantitative predictions of the model using a set of parameter values calibrated to match some statistics for Spain, the largest E.U. member currently facing problems in sovereign debt markets. In the equilibrium produced by this baseline calibration, a large fraction of households are credit constrained, and thus repaying the debt is highly beneficial for the reasons described above. Still, higher public debt increases the need for tax revenues in the future and over time wealth inequality increases. Default thus arises along the equilibrium path when the debt is high enough and the distribution of public bond holdings is sufficiently concentrated. Government debt is about 40 percent of GDP on average, but once the debt reaches about 60 percent of GDP, and conditional on the debt being concentrated in a small fraction of agents, the government finds it optimal to default. This occurs with a frequency of about 8 percent. These results contrast with typical findings of the quantitative literature on external sovereign default, where it has not been possible to account for debt ratios as high as observed

in the data except when the models' predicted default frequency is negligible (see Mendoza and Yue [18]).

The model is also in line with the data in producing rising spreads between the interest rate on public debt and the model-implied risk-free rate as debt builds up leading to a default episode. The real interest rate on public debt itself, however, is generally negative, which means that public debt pricing acts as a proportional tax on debt holdings. This can be the result of two mechanisms: First, at this early stage with a preliminary calibration, in which we match the dispersion and autocorrelation of individual earnings in Spanish data but are still working on calibrating several parameters such as the stochastic process for government expenditures, it might be that the model embodies a level of risk faced by private agents that is higher than what it should be. If risk is too high, a standard finding from the incomplete-markets literature predicts that agents over-accumulate assets, compared with an economy with less risk, and hence the strong demand for public bonds rises its price and lowers its return so that the real interest rate can be negative. The second mechanism is that the government may be strategically managing the level of debt to use non-positive real interest rates as a state- and wealth-contingent tax on debt holdings, thus equipping itself with a "better" tax instrument than taxes that are the same lump sum across all agents.

Agent heterogeneity and wealth inequality driven by non-insurable shocks and market-incompleteness are the focus of a large research program in Macroeconomics. To date, however, this literature has not examined jointly domestic public debt subject to default risk driven by distributional incentives and the evolution of the wealth distribution. Our model is similar to the model of Aiyagari and McGrattan [3] in that domestic government debt plays a role as an insurance device against idiosyncratic income fluctuations, and the distribution of households over debt holdings is endogenously determined. A related literature initiated by Aiyagari, Marcet, Sargent and Seppala [2] studies optimal taxation and public debt dynamics with aggregate uncertainty, but assuming a representative agent. Pouzo [21] considers a similar framework but incorporating default and renegotiation in domestic public debt, while still maintaining the representative agent assumption. Corbae, D'Erasmus and Kuruscu [9] examine optimal taxation also but in a setting with heterogeneous agents, where the endogenous distribution of households over assets determines the set of optimal policies. Moreover, policies affect the evolution of this distribution, so there is a non-trivial feedback effect.⁷

Our work is also related to the growing quantitative literature on external sovereign default based on the classic model of Eaton and Gersovitz [11]. To date this literature has focused

⁷The algorithm we use to solve the model is an extension of the one proposed by Corbae et al. [9], which in turn is an extension of the algorithm proposed by Krusell and Smith [16]. Bachmann and Bai [5] use a similar algorithm in a model with aggregate uncertainty, heterogeneous agents and endogenously determined government policies.

only on a representative agent framework (see, for example Aguiar and Gopinath [1], Arellano [4]).⁸ Some studies in this literature have examined the role of tax and expenditure policies and settings with foreign and domestic lenders. Cuadra, Sanchez and Sapriza [10] extended the model of Arellano [4] to analyze the cyclicity of fiscal policy. A recent strand of literature focuses on the consequences of external sovereign default on domestic agents (see for example Guembel and Sussman [13], Broner, Martin and Ventura [8] and Gennaioli, Martin and Rossi [12]). Like these papers, we assume that default is non-discriminatory. Our work differs in that we abstract from foreign lenders in a infinite horizon setting with heterogeneous agents and an endogenous distribution of debt among domestic households.

Finally, this paper is also related to classic studies on optimal taxation, domestic debt and fiscal solvency which generally assume that debt is always honored (e.g. Barro [6], Lucas and Stokey [17], Bohn [7]). As with the models of wealth dynamics under incomplete markets and the models of external sovereign default, these strands of the Macro literature on public debt abstract from examining the key endogenous links between wealth inequality, public debt dynamics and sovereign default that we uncover here.

To close this Introduction, a word of caution about public debt data. As Reinhart and Rogoff [22] noted in motivating their extensive data collection effort, data on domestic public debt is very hard to obtain even though domestic positions in sovereign debt are quantitatively very important. Domestic debt in our model corresponds to debt held by domestic residents, which is in line with the definition of domestic debt that some data sources use (OECD, for example, reports marketable central government debt held by country residents). But the breakdown of public debt in terms of the residence of holders is not always available or reliable. Alternatively, Reinhart and Rogoff [22] defined domestic public debt based on whether the debt is issued under home-country legal jurisdiction or abroad, typically under New York or London law. These two definitions of domestic debt, by residence of holder or by jurisdiction of the issuer, are correlated, but they are not identical, and in some episodes can look very different (as with the famous case of Mexico's Tesobonos in 1994, which were issued under Mexican jurisdiction but with significant holdings outside of Mexico).⁹

The rest of this paper is organized as follows: Section 2 describes the economic environment. Section 3 and 4 construct the concept of recursive competitive equilibrium we solve for. Section 5 provides a simple two-period example that highlights the distributional incentives driving the

⁸See Panizza, Sturzenegger and Zettelmeyer [19] for a recent review of the literature on sovereign debt and default.

⁹Other definitions of domestic debt are based on the residence status of the issuer (i.e. public debt issued by resident entities) and on the currency denomination (i.e. domestic debt is taken to be synonymous with local-currency- denominated debt). None of these definitions are associated with the residence status of the final owner of sovereign debt.

default decision. Sections 6 and 7 present and analyze the results of the model calibrated to Spain. Section 8 presents some extensions. Section 9 concludes.

2 Environment

The economy is inhabited by a continuum of agents with aggregate unit measure. These agents are ex-ante identical but heterogeneous ex-post because of two forms of non-insurable shocks: income fluctuations that are idiosyncratic, and aggregate shocks in the form of fluctuations in government expenditures and the possibility of sovereign default. The government is represented by a social planner with a utilitarian payoff that issues one-period, non-state-contingent debt, levies lump-sum taxes, and has the option to default. Government debt is the only asset available in the economy.

2.1 Preferences and Individual Income

Time is discrete and preferences are given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $\beta \in (0, 1)$ is the discount factor and c_t is individual consumption. The utility function is a standard, twice-continuously differentiable, strictly concave function that satisfies the Inada conditions.

Every period, each agent receives income $y_t \in \mathcal{Y}$. Income is distributed i.i.d. across households and follows an AR(1) process in logs:

$$\log(y_{t+1}) = \rho \log(y_t) + u_t$$

where $0 < \rho < 1$ and u_t is i.i.d. over time and distributed normally with mean zero and standard deviation σ_u . We denote the transition function of y_t by $\pi(y_{t+1}, y_t)$. We let $\pi^*(y)$ denote the stationary distribution associated with $\pi(y_{t+1}, y_t)$.

2.2 Government

Every period, the government can issue one-period, non-state contingent discount bonds $B_{t+1} \in \mathcal{B} \equiv [0, \infty)$ at the price $q_t \geq 0$, collects revenues from lump sum taxes τ_t and pays for exogenous stochastic expenditures equal to $g_t \in \mathcal{G} \equiv \{\underline{g}, \dots, \bar{g}\}$. Government expenditures follow a

Markov process with transition matrix $F(g_{t+1}, g_t)$. These expenditure shocks are persistent but independent of households' income shocks.

The government is not committed to repay the debt. At the beginning of each period, it chooses to default or not on the outstanding debt. After the default decision is made, the level of taxes is determined and, if the government does not default, the new level of debt is chosen. The government is benevolent with a payoff function in the class of utilitarian social welfare functions. We consider different functions in this class. The benchmark case is one with a standard utilitarian social welfare function in which the utility of each agent is weighed according to the economy's wealth distribution, which is also the distribution of individuals of different types (since "types" in this economy differ only by wealth). Thus, this benchmark planner maximizes average welfare. Other utilitarian payoffs can aggregate individual utilities with arbitrary weights, which could be justified with political economy considerations, and can also be extended to incorporate egalitarian concerns (e.g. by introducing penalties defined relative to the utility of the poorest agents). The government, however, cannot discriminate a particular household or set of households when setting taxation, debt and default policies.

2.3 Bond Market and Budget Constraints

Households can hold positions in government bonds defined by $b_{t+1} \in \mathcal{B} \equiv [0, \infty)$, so they are not allowed to borrow. The distribution of households over debt and income is defined by $\Gamma_t(b, y)$. At equilibrium, the discounted price of debt will be such that the following market clearing condition of the government bond market holds:

$$B_{t+1} = \int_{\mathcal{Y} \times \mathcal{B}} b_{t+1} d\Gamma(b, y). \quad (1)$$

If the government does not default at the beginning of $t+1$, each household receives back the corresponding b_{t+1} units of government debt. Taxes are set to satisfy the following government budget constraint:

$$\tau_t = B_t + g_t - q_t B_{t+1}, \quad (2)$$

and the household budget constraint is:

$$c_t + q_t b_{t+1} = y_t + b_t - \tau_t. \quad (3)$$

If the government defaults, the market for government bonds closes and may re-open the following period with probability $\lambda \in [0, 1]$. Thus, default entails temporary exclusion for the

government inasmuch as there is stochastic re-entry to credit markets. Taxes during periods of exclusion are set to satisfy the following government budget constraint:

$$\tau_t = g_t, \tag{4}$$

and the household budget constraint is:

$$c_t = y_t - \tau_t. \tag{5}$$

2.4 Timing:

Let the binary variable $h_t \in \{0, 1\}$ denote if the domestic debt market is open ($h_t = 0$) or closed ($h_t = 1$). The timing of decisions and market participation in the model is as follows:

1. At the start of date t , the distribution of households is given by $\Gamma_t(b, y)$ and the status of the debt market is known to be h_t .
2. The realization of g_t is observed.
3. If $h_t = 0$, the government chooses to default or not, with the default choice denoted by the binary variable $d_t \in \{0, 1\}$:
 - If the government chooses not to default $d_t = 0$, the government debt is repaid, the debt market stays open, new debt is issued, taxes are determined, households decide how much to save b_{t+1} at given price q_t and $h_{t+1} = 0$.
 - If the government defaults $d_t = 1$, the debt market closes and taxes are determined. Then, $h_{t+1} = 0$ with probability λ and $h_{t+1} = 1$ with probability $(1 - \lambda)$.
4. If $h_t = 1$, the government and households have no access to the bond market, and taxes are set to pay government expenditures $\tau_t = g_t$.
5. Agents pay taxes and consume, and date t ends.

3 Equilibrium with Exogenous Government Policy

We start by characterizing a Recursive Markov Competitive Equilibrium for given government policies (default, debt and taxes). We use standard recursive notation where any variable x_t is denoted as x and any variable x_{t+1} is denoted x' . The distribution of wealth $\Gamma(b, y)$ evolves according to an endogenous transition function $\Gamma' = H^{d' \in \{0,1\}}(\Gamma, g, g')$ where $d' \in \{0, 1\}$ denotes

the default decision of the government the following period. If the government decides not to default, Γ' is induced by the bond decision rule of the households and the stochastic income process in the usual way. If the government defaults, all households' bond positions are set to zero and Γ' is determined only by the evolution of the income process. More specifically, if the government defaults, $\Gamma'(b, y) = \pi^*(y)$ for $b = 0$ and zero for any other value of b . Both the wealth distribution and government expenditures are relevant aggregate states for private agents because, together with their associated laws of motion, they are used to predict future prices and government policies. The government default decision is denoted by $d(\Gamma, g)$. The government's decision rule for issuing bonds is denoted $B'(\Gamma, g)$, taxes are $\tau(B', \Gamma, g)$ and the bond pricing function is $q(B', \Gamma, g)$.

3.1 Households' Problem

Define the state variables relevant for an individual agent as (b, y, Γ, g) . Taking as given the recursive functions that characterize the evolution of aggregate states and bond prices $\{H^{d' \in \{0,1\}}(\Gamma, g, g'), B'(\Gamma, g), \tau(B', \Gamma, g), q(B', \Gamma, g), F(g', g)\}$, the optimization problem of an individual agent can be written as follows:

$$V(b, y, \Gamma, g) = (1 - d(\Gamma, g))V^{d=0}(b, y, \Gamma, g) + d(\Gamma, g)V^{d=1}(0, y, \Gamma', g) \quad (6)$$

where $V^{d=0}(b, y, \Gamma, g)$ is the continuation value if the government does not default and $V^{d=1}(0, y, \Gamma', g)$ is the continuation value if default occurs.

The continuation value if the government chooses not to default is:

$$V^{d=0}(b, y, \Gamma, g) = \max_{\{c \geq 0, b' \geq 0\}} \{u(c) + \beta E_{(y', g')|(y, g)}[V(b', y', \Gamma', g')]\} \quad (7)$$

$$\text{s.t.} \begin{cases} c + q(B', \Gamma, g)b' = b + y - \tau(B', \Gamma, g) \\ \tau = B + g - q(B', \Gamma, g)B'(\Gamma, g) \\ \Gamma' = H^{d' \in \{0,1\}}(\Gamma, g, g'). \end{cases}$$

The continuation value if the government chooses to default is:

$$V^{d=1}(0, y, \Gamma, g) = u(y - g) + \beta E_{(y', g')|(y, g)}[\lambda V^{d=0}(0, y', \Gamma', g') + (1 - \lambda)V^{d=1}(0, y', \Gamma', g')]. \quad (8)$$

This continuation value takes into account the fact that the debt market reopens only with probability λ . Note that while in default, all households bond positions are set to zero so the evolution of the distribution is only determined by the stochastic income process. This implies that, while in default, $\Gamma'(b, y) = \pi^*(y)$ for $b = 0$ and zero otherwise.

The solution to the above dynamic programming problem provides the individual decision rule $b' = h(b, y, \Gamma, g)$ as well as value functions $V(b, y, \Gamma, g)$, $V^{d=0}(b, y, \Gamma, g)$ and $V^{d=1}(b, y, \Gamma, g)$. By combining the household bond decision rule, the transition matrix that determines the evolution of idiosyncratic income shocks and the government default decision, we can construct the transition function $H^{d' \in \{0,1\}}(\Gamma, g, g')$.

An important feature of the individual agent's problem is that agents take into account the possibility of default in formulating their optimal choice of bond holdings. This effect is easy to identify by analyzing the first-order condition of this problem with respect to b' for the case of no default (assuming the value function is differentiable):

$$-u'(c)q(B', \Gamma, g) + \beta E_{(y', g')|(y, g)} [V_1(b', y', \Gamma', g')] \leq 0, = 0 \text{ if } b' > 0 \quad (9)$$

where $V_1(\cdot)$ denotes the derivative of the value function with respect to its first argument. Using the envelope condition, the condition can be expressed as:

$$-u'(c)q(B', \Gamma, g) + \beta E_{(y', g')|(y, g)} [(1 - d(\Gamma', g'))u'(c')] \leq 0, = 0 \text{ if } b' > 0 \quad (10)$$

The second term in the left-hand side of the above expression shows that agents evaluate the marginal benefit of one more unit of savings taking into account that consumption, and hence marginal utility, in the next period vary with the default decision. Since in the default state $d(\Gamma', g') = 1$, it is clear that agents assign zero marginal benefit to additional savings in those states where the government defaults.

3.2 Recursive Markov Competitive Equilibrium with Exogenous Policies

Given government policies $d(\Gamma, g)$, $B'(\Gamma, g)$ and $\tau(B', \Gamma, g)$, a **Recursive Markov Competitive Equilibrium** (RMCE) is a value function, household decision rules, and a transition function $H(\Gamma, g, g')$ such that:

1. Given prices and policies, $V(b, y, \Gamma, g)$ and $b' = h(b, y, \Gamma, g)$ solve the households' problem.
2. The price function $q(B', \Gamma, g)$ satisfies the market-clearing condition of the bond market:

$$B' = \int_{\mathcal{Y} \times \mathcal{B}} b'(b, y, \Gamma, g) d\Gamma(b, y). \quad (11)$$

3. If $d' = 0$, the distribution of wealth evolves according to:

$$\Gamma'(b, y; \mathcal{B}_0, \mathcal{Y}_0) = \int_{y' \in \mathcal{Y}_0, b' \in \mathcal{B}_0} \left\{ \int_{\mathcal{Y}, \mathcal{B}} I_{\{b' = h(b, y, \Gamma, g) \in \mathcal{B}_0\}} \pi(y', y) d\Gamma(b, y) \right\} db' dy'.$$

4. The government budget constraint is satisfied every period.

4 Equilibrium with Endogenous Policies

We now characterize a Recursive Markov Equilibrium with Endogenous Policies, which is a subset of the RMCE in which government policies are the solution to the government maximization. As before and with some abuse of notation, we let the default decision be $d(\Gamma, g)$, the government bond function $B'(\Gamma, g)$, taxes $\tau(B', \Gamma, g)$ and the bond pricing function $q(B', \Gamma, g)$. In this recursive equilibrium there is no commitment and the government optimizes policies every period. We start by finding the default decision and then we derive the optimal level of government debt issuance.

4.1 Government's Default Decision

If, at the beginning of the period, there is a positive level of government debt outstanding, the government chooses whether to default or not. By defaulting, the government affects the level of taxes, effectively redistributing resources from those with a positive level of government debt to those with no government debt. The optimal default decision is the solution to the following problem:

$$\max_{d \in \{0,1\}} \{W^{d=0}(\Gamma, g), W^{d=1}(\Gamma, g)\} \quad (12)$$

where

$$W^{d=0}(\Gamma, g) = \int_{\mathcal{Y} \times \mathcal{B}} V^{d=0}(b, y, \Gamma, g) d\hat{\Gamma}(b, y),$$

and

$$W^{d=1}(\Gamma, g) = \int_{\mathcal{Y} \times \mathcal{B}} V^{d=1}(0, y, \Gamma, g) d\hat{\Gamma}(b, y).$$

The government's payoff functions $W^{d=0}(\Gamma, g), W^{d=1}(\Gamma, g)$ are assumed to be in the class of utilitarian social welfare functions, in which each agent's utility is weighed using the weighing function $\hat{\Gamma}(b, y)$. In the benchmark scenario, we assume that $\hat{\Gamma}(b, y) = \Gamma(b, y)$ for all (b, y) (i.e. the government maximizes average welfare). But in principle utilitarian welfare functions do not require this to be the case. Hence, we will also explore in the quantitative analysis cases in which

$\hat{\Gamma}$ is an arbitrary weighing function that can be linked to political economy considerations.¹⁰

4.2 Government's Optimal Debt Level

If the government chooses not to default, it also chooses the new level of government debt taking bond prices as given. To characterize how this choice is made, the government first considers an intermediate step in which it evaluates how any alternative government debt level (denoted by \tilde{B}'), possibly different from the one predicted by the equilibrium law of motion, affects each household. Using these values and the corresponding welfare weights, the government decides the optimal level of debt.

The value for each household of any alternative government debt is the solution to the following one period “deviation” problem:

$$\tilde{V}(b, y, \Gamma, g, \tilde{B}') = \max_{\{c \geq 0, b' \geq 0\}} u(c) + \beta E_{(y', g')|(y, g)}[V(b', y', \Gamma', g')] \quad (13)$$

$$\text{s.t.} \begin{cases} c + q(\tilde{B}', \Gamma, g)b' = b + y - \tau \\ \tau = B + g - q(\tilde{B}', \Gamma, g)\tilde{B}' \\ \Gamma' = \tilde{H}^{d' \in \{0,1\}}(\tilde{B}', \Gamma, g, g'). \end{cases}$$

Note that the continuation value in this deviation problem is given by the solution to the household problem (6).

Next, for a given social welfare function $\hat{\Gamma}(b, y)$, the optimal government policy is the solution to:

$$\max_{\tilde{B}'} \int_{\mathcal{Y} \times \mathcal{B}} \tilde{V}(b, y, \Gamma, g, \tilde{B}') d\hat{\Gamma}(b, y). \quad (14)$$

The difficulty in finding an equilibrium with endogenous government policies is that the government debt, taxes and default choices induce a different sequence of wealth distributions. In particular, a given choice of \tilde{B}' induces a new set of prices and taxes that generate a new distribution $\Gamma' = \tilde{H}^{d' \in \{0,1\}}(\tilde{B}', \Gamma, g, g')$. This new distribution in turn induces a different set of government policies (default, debt and taxes) that result in a different future distribution and so on. More specifically, a given choice of government debt (\tilde{B}') induces the following sequence

¹⁰Interestingly, a scenario in which some agents are assigned zero weigh in the social welfare function would make the model similar to the standard external default model, in with the sovereign debtor does not care about the utility of its creditors.

of distributions:

$$\begin{aligned}
\Gamma' &= \tilde{H}^{d' \in \{0,1\}}(\tilde{B}', \Gamma, g, g'), \\
\Gamma'' &= H^{d'' \in \{0,1\}}(\tilde{H}^{d' \in \{0,1\}}(\tilde{B}', \Gamma, g, g'), g''), \\
\Gamma''' &= H^{d''' \in \{0,1\}}(H^{d'' \in \{0,1\}}(\tilde{H}^{d' \in \{0,1\}}(\tilde{B}', \Gamma, g, g'), g''), g'''), \\
&\vdots
\end{aligned}$$

4.3 Recursive Markov Equilibrium with Endogenous Policies

A **Recursive Markov Equilibrium with Endogenous Policies** is a value function, households decision rules, government policies and a transition function $H(\Gamma, g, g')$ such that:

1. Given prices and policies, the value function and saving decision rule solve the household problem.
2. The price function $q(B', \Gamma, g)$ is consistent with market clearing, that is

$$B' = \int_{\mathcal{Y} \times \mathcal{B}} b'(b, y, \Gamma, g) d\Gamma(b, y). \quad (15)$$

3. The distribution evolves according to:

$$\Gamma'(b, y; \mathcal{B}_0, \mathcal{Y}_0) = \int_{y' \in \mathcal{Y}_0, b' \in \mathcal{B}_0} \left\{ \int_{\mathcal{Y}, \mathcal{B}} I_{\{b' = h(b, y, \Gamma, g) \in \mathcal{B}_0\}} \pi(y', y) d\Gamma(b, y) \right\} db' dy'. \quad (16)$$

4. The government default decision $d(\Gamma, g)$ solves problem (12).
5. The government debt policy $B'(\Gamma, g)$ is consistent with the solution to (14).
6. The government budget constraint is satisfied every period.

5 A two period example

We illustrate the distributional mechanism driving the model using a simplified two-period example with household heterogeneity but without idiosyncratic uncertainty. The cost of these assumptions is that we need to impose an exogenous initial distribution of wealth characterizing households by their debt and income positions, instead of the endogenous distribution and its dynamics that are part of the equilibrium of the full model. The advantage, however, is that this two-period setup highlights how the equilibrium functions (bond prices, future debt, default and

taxes) react to changes in the initial wealth distribution. This example is only for illustrative purposes and not an attempt to bring the model to the data.

Preferences are given by $u(c) = c^{1-\sigma}/(1-\sigma)$, with $\sigma = 2$ and $\beta = 0.93$. Income is constant and equal to $y = 1$ for every household. Government expenditures are stochastic only in the second period. In particular, we set $g_0 = 0.10$, $g_1 \in \{\underline{g}, \bar{g}\}$ with $\underline{g} = 0.10$, $\bar{g} = 0.25$ and $\Pr(g_1 = \underline{g}) = 0.98$. We also assume that the government repays in period $t = 0$ and has the option to default only at $t = 1$. Initial government debt is set at $B_0 = 0.10$ and debt in period $t = 1$ can take values on $B_1 \in \{0, \dots, 0.40\}$.¹¹

We set the initial wealth distribution by assuming that at $t = 0$ a fraction γ of households are “low-wealth” (L) individuals who hold $b_0^L = 0$, and a fraction $(1 - \gamma)$ are “high-wealth” (H) agents who hold $b_0^H = \frac{B_0 - \gamma b_0^L}{1 - \gamma}$, i.e. the level of debt that is consistent with market clearing. Thus, the parameter γ summarizes all the necessary information about the initial (and future) wealth distribution.¹²

The goal of the exercise is to solve for the equilibrium default and debt decisions and bond prices to provide intuition of the workings of the full model. Taxes in the initial period are given by $\tau_0 = g_0 + B_0 - q_0 B_1$ and taxes in the second period (conditional on no government default) $\tau_1^{d_1=0} = g_1 + B_1 - q_1 B_2$, and in the case of default at $t = 1$ taxes are given by $\tau_1^{d_1=1} = g_1$. Note that, because of the two-period horizon, even if the government repays, the final period’s debt and debt price should satisfy $B_2 = q_1 = 0$. However, in order to make disposable income at $t = 1$ comparable with that of an infinite horizon economy, we set arbitrary terminal values $q_1 = \hat{q}$ and $B_2 = \hat{B}$ (parametrized to $\hat{q} = 0.65$ and $\hat{B} = 0.01$ respectively).

Since period $t = 0$ level of debt and government expenditures are set before agents make any decision and we can derive the equilibrium level of taxes using the government budget constraint, it is sufficient to express the value of the household and bond prices as a function of (B_1, γ) and the default decision in period $t = 1$ as a function of (B_1, g_1, γ) . Thus, for a given value of B_1 and γ a household with initial debt holdings b_0^i for $i = L, H$ chooses b_1^i by solving this maximization problem:

$$v^i(B_1, \gamma) = \max_{b_1^i} \left\{ u(y + b_0^i - q_0(B_1, \gamma)b_1^i - \tau_0) + \beta E_{g_1} \left[(1 - d_1(B_1, g_1, \gamma))u(y + b_1^i - \tau_1^{d_1=0}) + d_1(B_1, g_1, \gamma)u(y - \tau_1^{d_1=1}) \right] \right\}.$$

The continuation value in case of default does not depend on the level of household debt (b_1^i) since when it defaults, the government does not discriminate across households. Taxes in period

¹¹Results correspond to a case where there are 25 equally spaced values of B_1 and the upper bound is not a binding restriction.

¹²We present results for 21 equally spaced values of γ in the set $\{0.10, \dots, 0.90\}$.

1 are affected by the government default decision.

The first-order condition of this problem, evaluated at the equilibrium level of taxes, is:

$$-u'(c_0^i)q_0(B_1, \gamma) + \beta E_{g_1} \left[u'(y - g_1 + b_1^i - B_1 + \hat{q}\hat{B})(1 - d_1(B_1, g_1, \gamma)) \right] \leq 0, = 0 \text{ if } b_1^i > 0$$

At a given γ and B_1 , when the value of g_1 is such that the government chooses to default ($d_1(B_1, g_1, \gamma) = 1$), the marginal value of an extra unit of debt is zero.¹³ Thus, conditional on B_1 , a larger default set (i.e. a larger set of values of g_1 such that the government defaults), implies that the marginal benefit of an extra unit of savings decreases. This implies that, everything else equal, a higher default probability results in a lower demand for government bonds, a lower equilibrium bond price, and higher taxes. This has important redistributive implications, because by affecting the bond supply the government affects the expected probability of default and changes the equilibrium bond price. From the households perspective, the individual bond decision has no marginal effect on $d_1(B_1, g_1, \gamma)$.

The equilibrium price is the value of $q_0(B_1, \gamma)$ that clears the debt market. That is, at $q_0(B_1, \gamma)$, the following condition needs to hold

$$B_1 = \gamma b_1^L(B_1, \gamma) + (1 - \gamma)b_1^H(B_1, \gamma),$$

where B_1 in the left-hand-side of this expression represents the public bonds supply, and the right-hand-side is the government bond demand.

At $t = 1$, when contemplating to default or not, the government solves:

$$\max_{d \in \{0,1\}} \{W_1^{d=0}(B_1, g_1, \gamma), W_1^{d=1}(g_1, \gamma)\}, \quad (17)$$

where $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(B_1, g_1, \gamma)$ denote social welfare at the beginning of period 1 in the case of no default and default respectively. Using the government budget constraint to substitute for the corresponding values of $\tau_1^{d=0}$ and $\tau_1^{d=1}$ and an utilitarian social welfare function, the government payoffs can be expressed as:

$$W_1^{d=0}(B_1, g_1, \gamma) = \gamma u(y - g_1 + b_1^L - B_1 + \hat{q}\hat{B}) + (1 - \gamma)u(y - g_1 + b_1^H - B_1 + \hat{q}\hat{B}) \quad (18)$$

and

$$W_1^{d=1}(g_1, \gamma) = u(y - g_1). \quad (19)$$

To solve for the price and default function, at a given B_1 and γ , we iterate on $q_0(B_1, \gamma)$, $b_1^i(B_1, \gamma)$ and the default decision rule $d_1(B_1, g_1, \gamma)$ until the bond market clears and the bond

¹³The utility in the case of default equals $u(y - g_1)$, independent of b_1^i .

and default decisions are consistent with government's optimization.

The remaining step to complete the solution of the equilibrium is to determine the optimal debt choice of the government. This step is similar to a Ramsey problem in the sense that the government chooses its debt policy internalizing all the price and quantity effects we just described. More specifically, the government chooses B_1 to maximize the "indirect" social welfare function:

$$W_0(\gamma) = \max_{B_1} \{ \gamma v^L(B_1, \gamma) + (1 - \gamma) v^H(B_1, \gamma) \}. \quad (20)$$

It is more intuitive to present the results of this example by starting in period 1 and going backwards. First, we analyze the government's default decision at $t = 1$ for a given value of B_1 . We then present the implications of the default decision on households' decision rules, prices and taxes. Finally, we describe the optimal government debt choice at period 0.

Combining the payoff functions under default and no default, it follows that the government defaults if:

$$\gamma \left[u(y - g_1 + \overbrace{(b_1^L - B_1)}^{\leq 0} + \hat{q}\hat{B}) - u(y - g_1) \right] + \\ (1 - \gamma) \left[u(y - g_1 + \overbrace{(b_1^H - B_1)}^{\geq 0} + \hat{q}\hat{B}) - u(y - g_1) \right] \leq 0$$

Notice that all households lose g_1 of their income to government absorption regardless of the default choice, and that utility under default is the same for all agents ($u(y - g_1)$).

The distributional effects of a default are implicit in the above default condition. The assumption that low-wealth households do not hold bonds and high-wealth households own bonds as needed to clear the bond market implies that $(b_1^L - B_1) \leq 0$ and $(b_1^H - B_1) \geq 0$. Thus, for a given B_1 , the payoff under repayment allocates (weakly) lower welfare for L agents and higher for H agents, and the gap between the latter and the former is larger the larger B_1 . Moreover, since the default payoffs are the same for both types of agents, this is also true of the *difference* in welfare under repayment v. default: It is higher for H agents than for L agents and it gets larger as B_1 rises. To induce default, however, is necessary not only that L agents have a smaller difference in the payments of repayment v. default, but that the difference is negative (i.e. they must attain lower welfare under repayment than under default), which requires $B_1 - \hat{q}\hat{B} > 0$. In turn, the government budget constraint implies that this holds if taxes are higher in the repayment state ($\tau_1^{d=0} = g_1 + B_1 - \hat{q}\hat{B}$ under repayment v. $\tau_1^{d=1} = g_1$ under default).¹⁴

¹⁴This condition can also be stated in terms of the primary fiscal balance. For repayment to be welfare-reducing for the poor, there must be a primary surplus in the repayment state (i.e. $\tau_1 - g_1 > 0 \rightarrow B_1 - \hat{q}\hat{B} > 0$).

In summary, if repayment requires higher taxes than default, default is always preferable than repayment for L agents and viceversa for H agents.

Since we can also write the consumption allocations under repayment as $c_1^L = y - \tau_1^{d=0} + b_1^L$ and $c_1^H = y - \tau_1^{d=0} + b_1^H$, the distributional effects of default can also be interpreted in terms of how the changes in taxes and wealth caused by a default affect each agent's consumption (and hence utility). First, since $b_1^H > b_1^L = 0$, default wipes out the net worth of H agents but has no effect on the net worth of L agents, thus reducing the welfare of the former without affecting the latter (or more generally, hurting L agents less if we simply assume $b_1^H > b_1^L$, without assuming $b_1^L = 0$). Second, with regard to taxes, we established that for default incentives to exist τ_1 must be higher under repayment than under default, but both types of agents face the same tax rate. This still has distributional implications, however, because marginal utility is higher for L agents, and thus they suffer more if taxes rise under repayment.

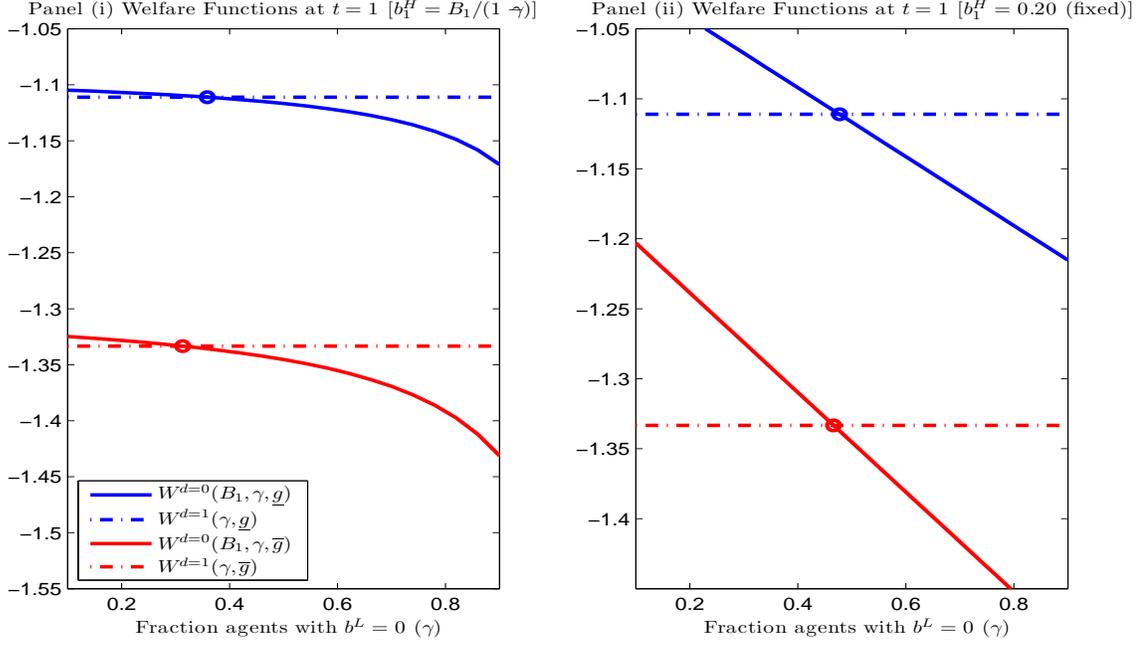
The distribution of wealth determines the weight the utilitarian planner assigns to the gains and losses that default imposes on the different agents. As γ increases, the fraction of L agents is larger, and thus the value of repayment for the government decreases because it weights more the welfare loss that L agents endure under repayment. Hence, with sufficient inequality default becomes optimal because the utilitarian planner attains higher social welfare under default. Note that differences in γ also affect the date-0 bond decision rules b_1^L and b_1^H and hence the market price of bonds q_0 , even for a given supply of bonds B_1 .

To illustrate the mechanics of the distributional incentives of default at work, Panel (i) of Figure 1 plots the welfare functions $W^{d=0}(B_1, g_1, \gamma)$ and $W^{d=1}(g_1, \gamma)$ as functions of γ for values of $g_1 \in \{\underline{g}, \bar{g}\}$ and a given $B_1 = 0.10$. In this chart, $W^{d=0}(B_1, g_1, \gamma)$ does not consider households' decision rules about b_1^i . Instead, in Panel (i) we assume that b_1^H adjusts with γ to clear the government bond market ($b_1^H = \frac{B_1}{1-\gamma}$) and in Panel (ii) we simply fixed $b_1^H = 0.20$. In both cases, we set $b_1^L = 0$

Panel (i) shows how $W^{d=0}$ changes in response to both the change in weights of the utilitarian planner and the changes in households' bond holdings with the assumed decision rules. Social welfare in the default state is lower with higher g_1 and, as explained, it is independent of γ . In contrast, social welfare under repayment decreases with γ because taxes needed for repayment represent a larger portion of income to L agents and as γ increases the weight assigned to L agents also increases. The circles in the plot mark the threshold $\hat{\gamma}$ such that the government chooses to default at $t = 1$ for any $\gamma > \hat{\gamma}$.

To separate the effects of the changes in weights and changes in households debt holdings, Panel (ii) shows again the welfare functions but now evaluated at $b_1^H = 0.20$ which is generally not the market clearing value ($b_1^H = 0.20$ only clears the debt market when $\gamma = 0.50$). Clearly, if the debt holdings of agents are kept unchanged, the welfare function loses its curvature and

Figure 1: Welfare Default vs No-Default $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(g_1, \gamma)$

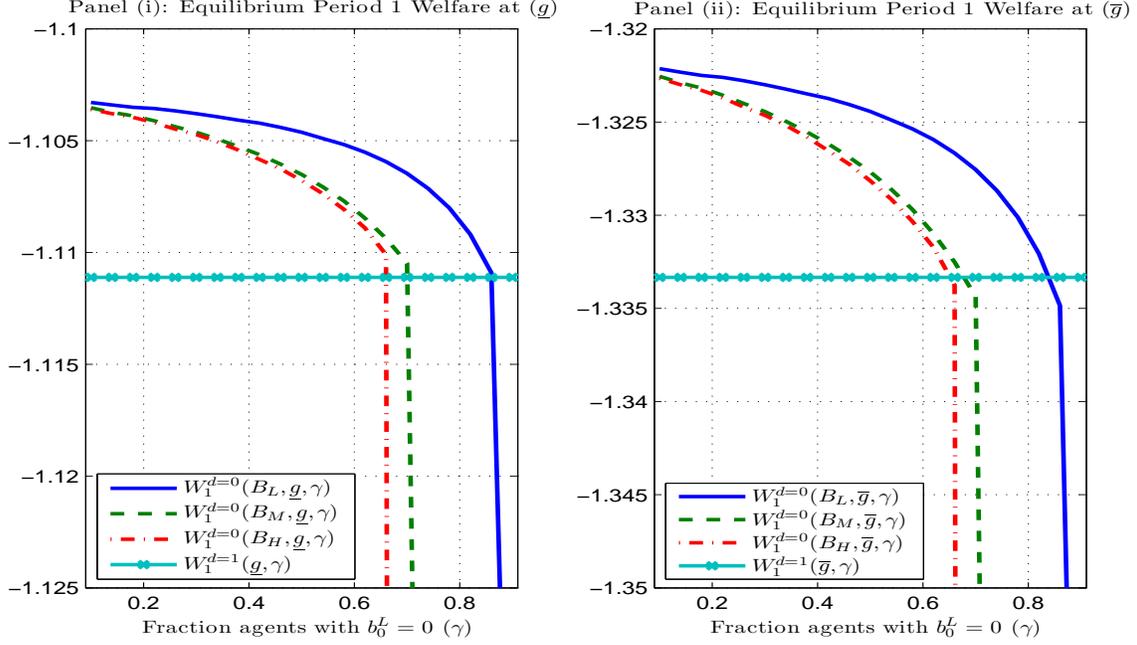


is simply the linear combination of two constants. Comparing Panels (i) and (ii), it is evident that $\hat{\gamma}$ is higher when we ignore the response of b_1^H to the change in γ . This occurs because the welfare loss of each individual H agent in the event of a default is larger when $b_1^H = 0.20$ than when $b_1^H = \frac{B_1}{1-\gamma}$ and $\gamma < 0.50$, and hence it takes a higher value of γ to make the welfare gain of the L agents offset the larger loss of H agents in the case of default.

Now we study the results for the “recursive” equilibrium of this two-period model, for which we have solved optimal decision rules, default and debt choices of the government, as well as bond prices and taxes. Panel (i) of Figure 2 shows date-1 equilibrium welfare functions for given values of government debt $B_1 = \{B_L, B_M, B_H\} = \{0.033, 0.05, 0.20\}$ at $g_1 = \underline{g}$. Panel (ii) present the same functions at $g_1 = \bar{g}$. In this chart, the distributions of household debt in period 1 are “recursive” equilibrium distributions, i.e. for each B_1 we solved for the price and private decision rules that clear the market contingent on the default decision.

The relation between wealth inequality or debt dispersion (measured by γ) and default in Figure 2 is similar to that observed in Figure 1. These figures show that there is a negative relation between the level of debt and default. At higher levels of debt, lower values of γ trigger a default. For example, when the debt is high ($B_1 = 0.20$) and $g_1 = \bar{g}$, if $\gamma > 0.62$ households are better off (in terms of the utilitarian average) if the government defaults since debt repayment would result in higher taxes. What it is also visible from this figure is that the default threshold value of γ is lower when government expenditures are $g_1 = \bar{g}$ than when $g_1 = \underline{g}$. This is also

Figure 2: Equilibrium Welfare $W_1^{d=0}(B_1, g_1, \gamma)$ and $W_1^{d=1}(g_1, \gamma)$



reflected in the government default decision, presented in Figure 3.

Consistent with the previous plots, Figure 3 shows again that the government has higher incentives to default when government debt is higher or when the distribution of debt holdings is more unequal (higher γ).

Figure 4 presents the value of γ such that the government is indifferent between defaulting and repaying in period 1. We denote this threshold by $\hat{\gamma}(B_1, g_1)$.

This figure shows that the threshold is decreasing in B_1 (i.e. less dispersion is needed to induce the government to default when debt is higher) and that is decreasing in g_1 (when government expenditures are higher the government has stronger incentives to default). This last feature of $\hat{\gamma}$ is very important to generate default in equilibrium. If, for a given value of B_1 , the threshold $\hat{\gamma}$ is the same for both values of g_1 , the probability of default equals one, so the equilibrium price has to equal zero and the government has not access to that debt level (i.e. there is a zero probability of default in equilibrium). On the other hand, if, for a given B_1 , the threshold is different across values of g_1 , the probability of default will be positive but strictly less than one for $\gamma \in [\hat{\gamma}(B_1, \underline{g}), \hat{\gamma}(B_1, \bar{g})]$, so the price can be positive and the government has access to this debt level (i.e. there is default with positive probability).

Figure 5 shows the equilibrium price function $q_0(B_1, \gamma)$ as well as the bond price if the government were not allowed to default on the debt $q_0^{RF}(B_1, \gamma)$ (i.e. the “risk free” price).

The equilibrium bond price is positive only for combinations of γ and B_1 such that the

Figure 3: Equilibrium Government Default Policy

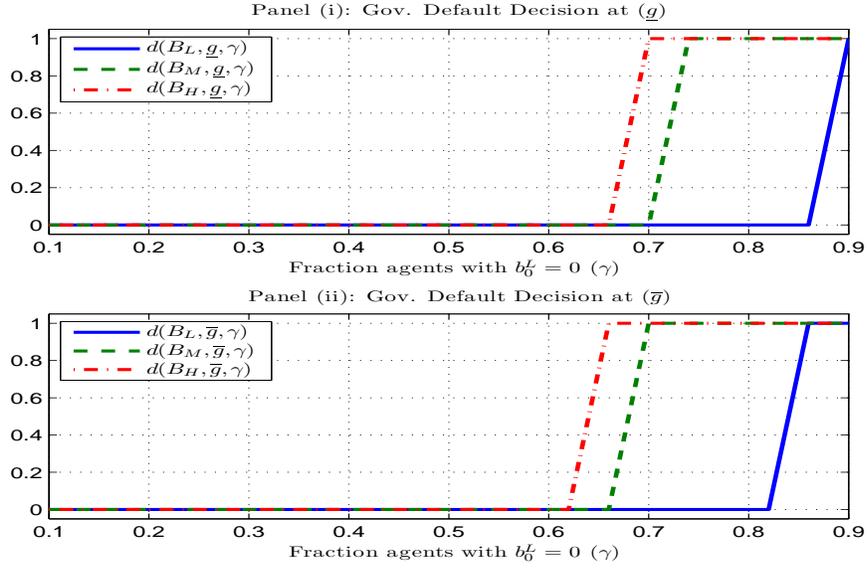


Figure 4: Default Threshold $\hat{\gamma}(B_1, g_1)$

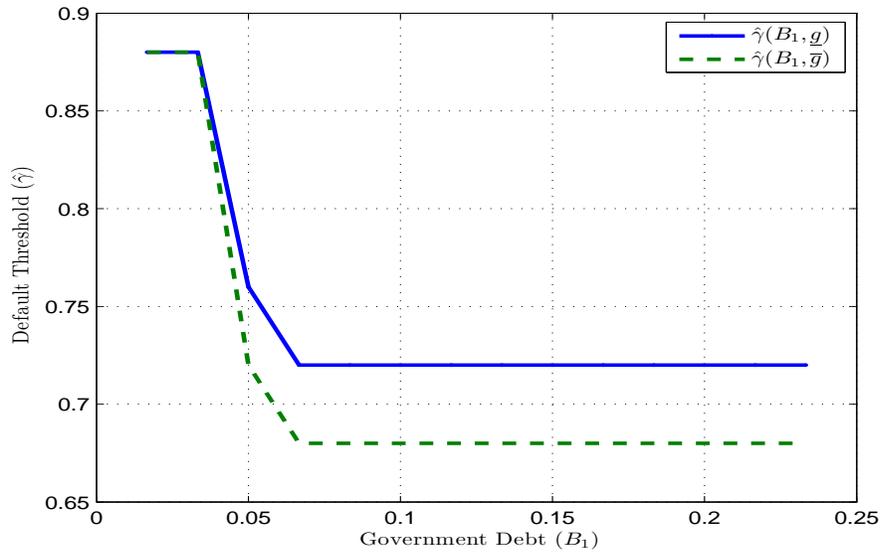
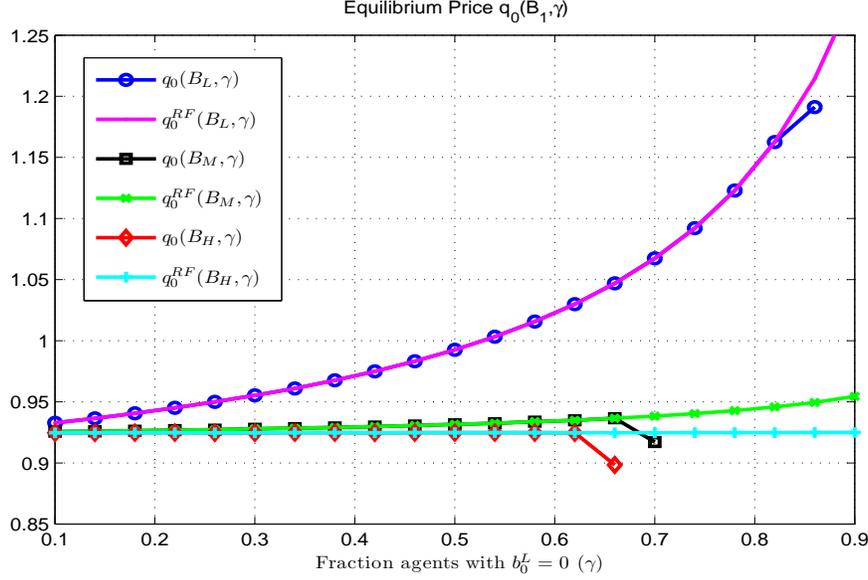


Figure 5: Equilibrium Bond Price

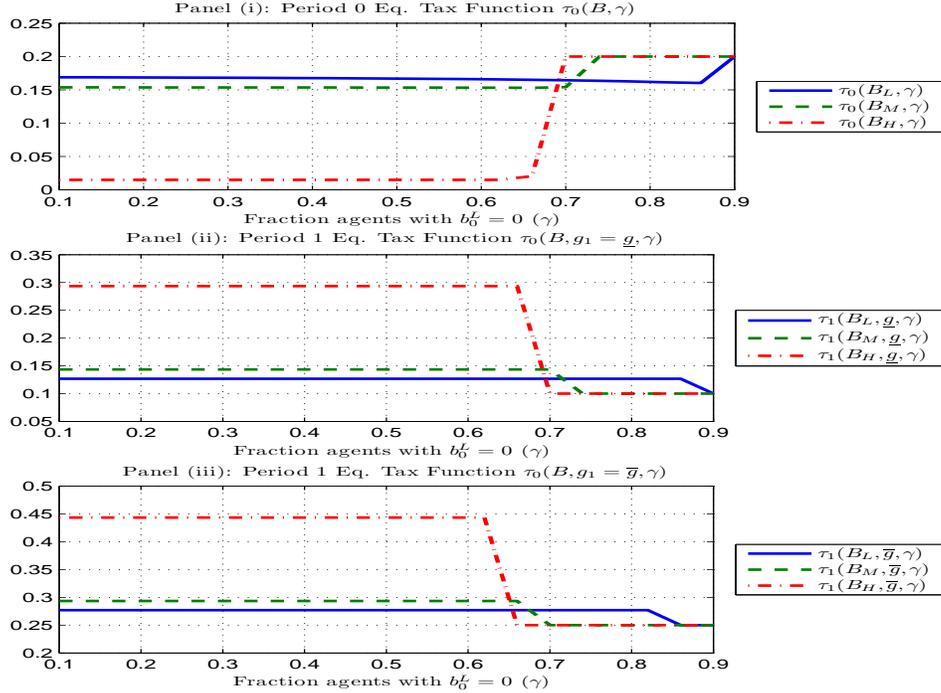


probability of default is less than one. For example, when $B_1 = 0.05$, the default probability equals 1 for values of $\gamma > 0.7$, so the bond price goes to zero. Three important characteristics are worth mentioning about this figure: **(i)** the equilibrium price is decreasing in B_1 (shifts across lines). For any given γ , as the government borrows more, the interest rate needed to create the incentives for households to demand a larger amount of debt increases. For values where the risk free rate coincides with the equilibrium interest rate, this change is driven exclusively by supply and demand factors. More specifically, as we show below, in many cases $b_1^L = 0$, so the marginal investor (i.e. the one determining the bond price) corresponds to an H household. Then, the equilibrium price is the one that is consistent with their first-order condition. For any value of b_0^H , as B_1 increases, lower values of q_0 are needed to generate the corresponding increase in b_1^H that clears the bond market. **(ii)** default risk reduces the price even further (i.e. increases interest rates). This can be seen by comparing the equilibrium bond price with the risk free price. For example, when $\gamma = 0.7$ and $B_1 = 0.05$ the probability of default equals 2% since the government will choose to default in case $g_1 = \bar{g}$. The equilibrium price equals zero when the default probability equals 1 and the distance between the risk free rate and the equilibrium price is maximized. That is, for cases where the government would prefer to default for any value of future government expenditures the equilibrium interest rate becomes infinity. **(iii)** for regions with no default risk, the bond price is increasing in γ . The rationale for the rise in debt prices as γ rises is more subtle than the effect of debt level on prices, and results from changes in the wealth distribution. A larger value of γ implies a more dispersed wealth distribution (i.e. higher b_0^H). Hence, a smaller fraction of the population demands a larger amount of debt and

this pushes prices up.

Figure 6 presents the resulting tax policy functions in period 0 and period 1. They are a function of B_1 , g_1 , the equilibrium price and the corresponding default decision.

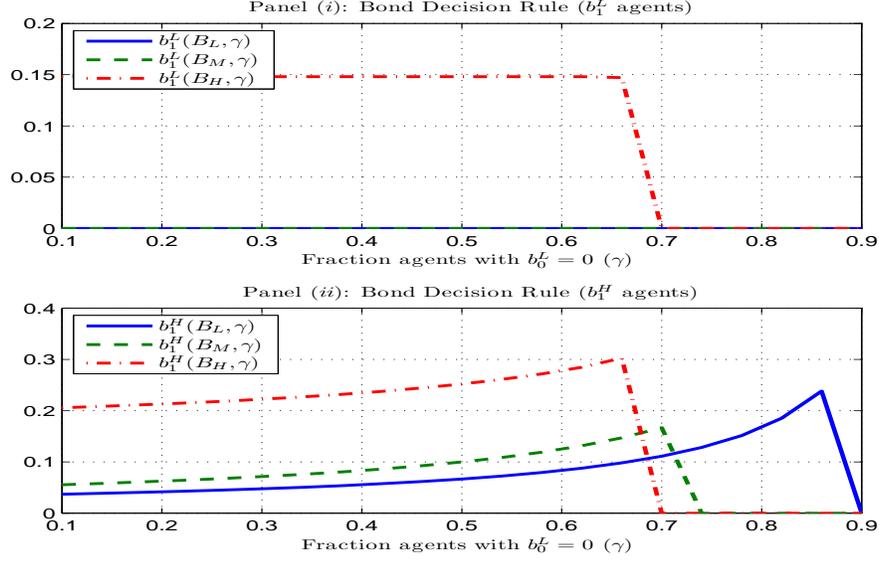
Figure 6: Equilibrium Tax Policy



Panel (i) presents τ_0 as a function of B_1 (new debt issuance). In regions where the government finds optimal not to default ($\gamma < 0.70$), higher debt results in lower τ_0 . On the other hand, in regions where the probability of a government default is positive, τ_0 increases since the government has to pay a higher interest rate or it cannot issue new debt at all (values of γ such that τ_0 does not vary across debt levels). Panels (ii) and (iii) present τ_1 as a function of B_1 , for g_1 equal to \underline{g} and \bar{g} respectively. When the government does not default, taxes are increasing in the level of debt (i.e. higher debt service results in higher taxes). If the government defaults, taxes are equal to g_1 for any value of debt. Thus, in Figure 6 taxes drop once γ is high enough for default to be optimal.

At equilibrium, the households' optimal bond decision rules capture the effects of the equilibrium set of taxes and prices. These decision rules are presented in Figure 7. This figure shows that in many cases $b_1^L = 0$ and $b_1^H = \frac{B_1}{1-\gamma}$, so the H -type households are effectively the "marginal" investor. In particular, poor agents continue to choose zero debt except in the case that $B_1 = B_H$, in which they choose a positive amount of debt holdings for states where the

Figure 7: Equilibrium HH bond decision rules b_1^i



government does not default. The shape of b_1^H is derived from our assumption on the initial distribution of debt holdings (recall that $b_0^H = 0$ and $b_0^H = \frac{B_0}{1-\gamma}$).

Now, having examined the date 1 features of the model as well as the price, taxes and households' decision rules in period 0, we can complete the presentation of the solution of the model and show the optimal debt choice of the government in period 0 (equation (20)). To provide some intuition about the solution, it is illustrative to rearrange the government first-order condition as follows:

$$\begin{aligned}
 & q_0 [\gamma u'(c_0^L) + (1-\gamma)u'(c_0^H)] - \beta E [(1-d_1)[\gamma u'(c_1^L) + (1-\gamma)u'(c_1^H)]] \\
 & \quad + \underbrace{\frac{\partial q_0}{\partial B_1}}_{\leq 0} \left[\underbrace{\gamma u'(c_0^L)}_{\geq 0} (B_1 - b_1^L) + (1-\gamma)u'(c_0^H) \underbrace{(B_1 - b_1^H)}_{\leq 0} \right] \\
 & \quad + \beta E_{g_1} \left\{ \Delta d_1 \left[\underbrace{\gamma [u(y - g_1) - u(c_1^L)]}_{> \text{ or } < 0} + (1-\gamma) \underbrace{[u(y - g_1) - u(c_1^H)]}_{< 0} \right] \right\} \leq 0.
 \end{aligned}$$

This expression can be broken into four terms. The first two are identical to the negative of the first order conditions of the household. Since they are evaluated from the perspective of the social planner, they represent the social marginal benefit and cost of one more unit of debt at a given debt price and "default propensity" of the government. The planner takes into account that, as the level of debt increases, all agents pay less taxes today but pay more taxes in the following period, and H agents postpone more consumption today to buy debt. The third term

corresponds to the effect of issuing more debt on the equilibrium price of government bonds. Since L agents are net borrowers ($(b_1^L - B_1) \leq 0$) and H agents are net savers ($(b_1^H - B_1) \geq 0$), a lower value of q_0 has a differential effect. Net savers receive a higher return on the bond as the total stock of government debt increases. The final term corresponds to the change in the default propensity of the government in the following period. A default is a cost for H agents who hold higher debt than the average and possibly a benefit for L agents with low or zero debt (recall we showed earlier that repayment causes a welfare loss for L agents that do not hold debt if $B_1 - \hat{q}\hat{B} > 0$, or equivalently if at $t = 1$ taxes in the repayment state are higher than in the default state).

If all households are unconstrained in their borrowing, so that their Euler equations hold with equality, and $\Delta d_1 = 0$, then the above optimality condition simplifies to:

$$\frac{u'(c_0^L)}{u'(c_0^H)} = \frac{-(B_1 - b_1^H)(1 - \gamma)}{(B_1 - b_1^L)\gamma},$$

Hence, under those assumptions we obtain the intuitive result that the social planner would want to set the ratio of date-0 marginal utilities of the two agents equal to their weighted relative savings ratio. Moreover, if L agents are constrained (i.e. $b_1^L = 0$) and $\Delta d_1 = 0$, then the optimality condition yields this result:

$$\underbrace{\gamma [q_0 u_0^L] - \beta E [(1 - d_1) u'(c_1^L)]}_{>0} + \frac{\partial q_0}{\partial B_1} [\gamma u'(c_0^L) B_1 + (1 - \gamma) u'(c_0^H) (B_1 - b_1^H)] = 0.$$

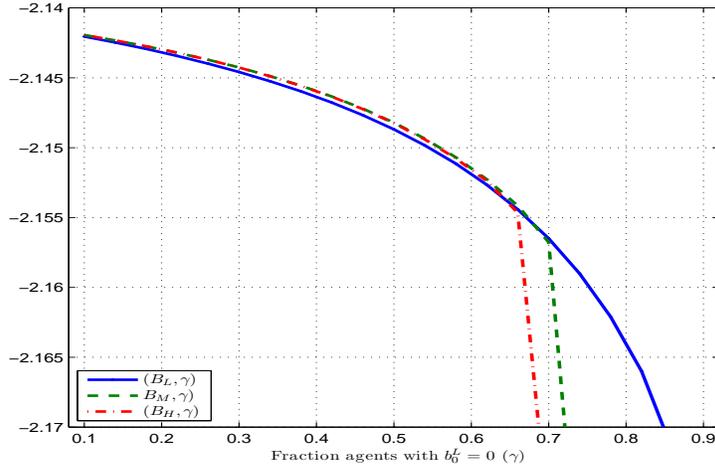
Thus, when the borrowing constraint is binding for some agents, the optimal level of debt issued by the government increases, because at the debt level consistent with the unconstrained optimality condition the planner would have $u'(c_0^L)/u'(c_0^H) > -(1 - \gamma)(B_1 - b_1^H)/(\gamma B_1)$, and hence the marginal benefit of borrowing more to reduce τ_0 and allocate more consumption to L agents exceeds the cost of making H agents save more to buy the debt.

Figure 8 shows the indirect social welfare function evaluated as to $t = 0$ as a function of γ and for the three values of B_1 considered earlier:

This figure shows that the planner attains a higher equilibrium payoff with a lower level of debt choice as γ rises. The reason is that, a higher level of debt induces a reduction in τ_0 and increases τ_1 (see Figure 6). For low γ , this tax reallocation makes little difference for the welfare of both agents (in fact, for $\gamma < 0.25$ the welfare of L agents is actually slightly higher with more debt). On the other hand, as γ rises above 0.4, the welfare of L agents is significantly lower as the expected value of taxes increases.

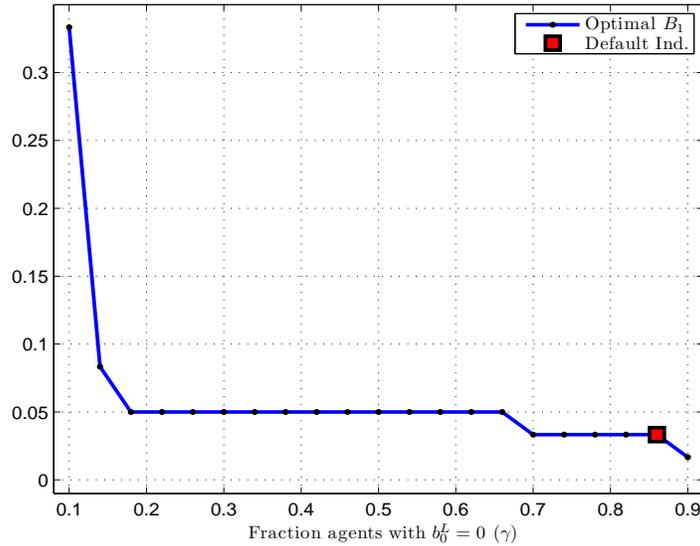
Finally, Figure 9 shows the optimal debt level chosen by the government as a function of γ . The debt level identified by a square is an optimal debt choice with a positive probability of

Figure 8: Equilibrium Initial Welfare as a function of B_1



default.

Figure 9: Equilibrium Optimal Government Debt Policy



In line with the results and the intuition provided above, the optimal debt level falls as γ increases. Thus, this simple framework yields the interesting prediction that a higher concentration of wealth (i.e. a larger fraction of low-wealth individuals) supports a lower level of public debt, taking into account the possibility of domestic default driven by distributional incentives. Figure 9 shows that default is a zero probability event unless wealth concentration is sufficiently high. More specifically, we found an equilibrium with default at $\gamma = 0.86$ where at $t = 0$ the government chooses $B_1 = 0.033$ and defaults in $t = 1$ when g_1 realizes to be equal to \bar{g} . At

higher values of γ , the government prefers an even lower level of debt and there is no default.

6 Quantitative Predictions of the Dynamic Model

In this Section, we study the quantitative predictions of the model based on particular functional forms and a set of calibrated parameter values. The goals are to show that the model’s distributional incentives support an equilibrium with domestic sovereign default, and to study the quantitative characteristics of this equilibrium.

6.1 Functional Forms and Parameter Values

The utility function retains the same time-separable, CRRA form of the two-period example, hence we need to set values for the subjective discount factor β and the coefficient of relative risk aversion σ . Individual income y is now stochastic and set to follow an AR(1) process with variance σ_u^2 and first-order autocorrelation ρ , and represented as a Markov process using Tauchen’s (1986) quadrature method with 11 nodes. The process for government expenditures is represented by a realization vector $g \in \{\underline{g}, \dots, \bar{g}\}$ with 5 evenly-spaced nodes and a transition matrix $F(g', g)$, with diagonal values set to $0 < \bar{F} < 1$ and non-diagonal values equal to $(1 - \bar{F})/4$.¹⁵ In addition, we need to set a value for the government’s exogenous probability of re-entry to the debt market after a default, λ . Hence, the full set of parameters that need to be defined in order to solve the model are:

$$\Theta = \{\beta, \sigma, \rho, \sigma_u, \underline{g}, \bar{g}, \bar{F}, \lambda\}.$$

At this stage, we set parameter values using some standard values from the literature and a rough calibration to actual data. We set $\sigma = 2$, which is a standard value. Most of the other parameters are calibrated using data from Spain, because Spain is one of the key EU countries currently facing distress in public debt markets and for which the are available estimates of the idiosyncratic income process. In particular, we set $\rho = 0.90$ and $\sigma_u = 0.20$ to be consistent with Spain’s cross-sectional dispersion of earnings reported by Pijoan-Mas and Sanchez Marcos [20].¹⁶ The values of $\underline{g}, \bar{g}, \bar{F}$ and β are arbitrary but we intend to set their values to match the following moments: mean government expenditures as a share of GDP (25.83%), standard deviation of government expenditures in logs (0.0254), first-order autocorrelation of government

¹⁵This is only a preliminary parametrization. We are currently working in a parametrization where the government expenditures to GDP are model as an AR(1) process and this process is approximated using Tauchen (1986).

¹⁶They estimate the average residual dispersion of labor earnings at about 0.45 (see their Figure 8).

expenditures (0.91), average debt to GDP ratio (54.63%).¹⁷

Table 2 shows all the parameter values.

Table 2: Model Parameters

Parameter		Value	Target
Risk Aversion	σ	2	Standard Value RBC Lit.
Autocorrelation	ρ	0.90	Spain earnings data [†]
Std Dev Error	σ_u	0.20	Spain earnings data [†]
Gov. Exp. Grid	\underline{g}	0.08	Std Dev $\log(G/Y)$ Spain (0.0254)
Gov. Exp. Grid	\bar{g}	0.10	Avg. (G/Y) Spain (25.83%)
Gov. Exp. Transition	\bar{F}	0.95	Corr($G/Y, G'/Y'$) Spain (0.91)
Discount Factor	β	0.93	Avg. Government Debt Level (54.63%)
Re-entry Probability	λ	0.25	

Note: [†] Spain's earnings data from Pijoan-Mas and Sanchez Marcos [20].

Data on G/Y and government debt from OECD Statistics based on author's calculations. Government expenditures corresponds to Total Government Expenditures net of Social Benefits and Transfers in kind Data on government expenditures is available from 1995 to 2010. Government debt corresponds to Total General Government Debt. Data on government debt is available from 1980 to 2010.

6.2 Numerical Solution Method

Solving the model requires keeping track of the fact that the evolution of bond prices as well as government decisions depend on the wealth distribution of households, and in turn households need to forecast future prices in order to choose their level of savings and consumption. As is standard in models of heterogeneous agents with aggregate risk, the wealth distribution is an infinitely dimensional object, and hence it is not possible to include it as a state variable. To get around this issue, we extend the algorithm proposed by Krusell and Smith [16] to accommodate an environment with government default and conjecture that prices can be predicted by a finite set of moments.¹⁸ In particular, we assume that current prices q , the bond decision rule B' and the default decision d can be written as functions of a finite set of moments $M = \{m_1, \dots, m_N\}$ of the wealth distribution and the current value of the aggregate shock represented by the realization of government expenditures g . Finally, we let M' be a function of M and g .

¹⁷We use data from the OECD. Government expenditures corresponds to Total Government Expenditures net of Social Benefits and Transfers in kind. Data on government expenditures is available from 1995 to 2010. Government debt corresponds to Total General Government Debt. Data on government debt is available from 1980 to 2010. The value of this ratio in 2010 is 74.13.

¹⁸The algorithm is also related to that of Krusell and Smith [15].

In the results presented below, we set $N = 1$ with $m_1 = B = \int bd\Gamma(y, b)$, i.e. the average debt holdings. We are currently working on extending the set of moments.¹⁹

6.3 Policy, Payoff and Pricing Functions in the Baseline Model

We start with a set of baseline results generated under the assumption that the government’s utilitarian social welfare function uses the economy’s wealth distribution to weight individual utility ($\hat{\Gamma}(b, y) = \Gamma(b, y)$). As we show later, this baseline scenario supports an equilibrium path with domestic default driven by distributional incentives.

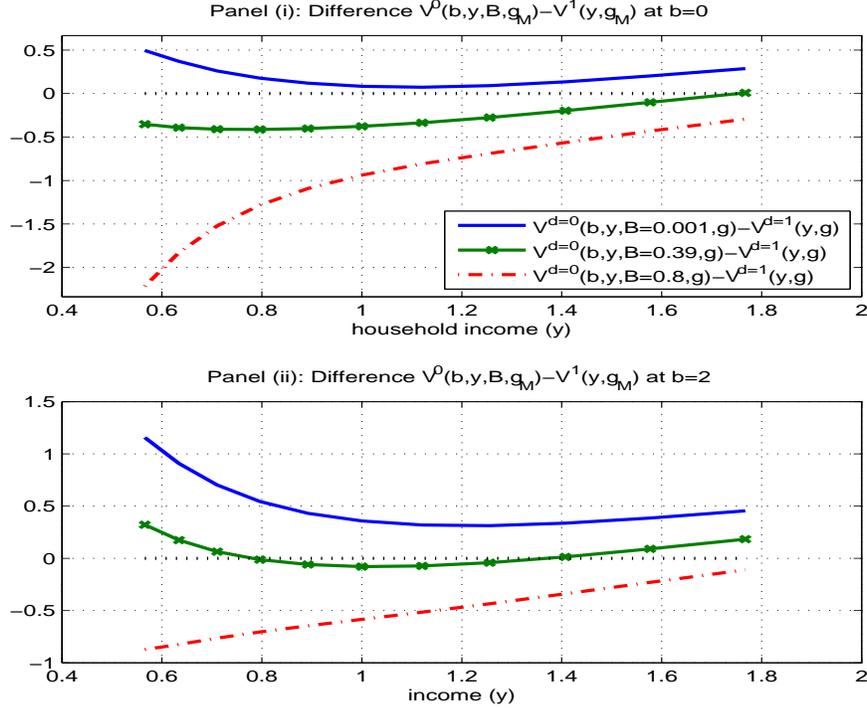
Consider first the incentives for default in the baseline case. Figure 10 plots the net gain of continuation over default (i.e. $V^{d=0} - V^{d=1}$) as a function of individual income y for three different values of outstanding public debt B and two levels of individual debt holdings: Panel (i) for agents without individual debt holdings ($b = 0$), and Panel (ii) for agents with high individual debt holdings ($b = 2$). For high levels of debt, the net gain is increasing and concave in y for both low and high wealth agents. As B falls, however, the net gain becomes a convex function of y that is decreasing in y at low income levels and increasing in y at high income levels. This happens because, when B is low (high), agents are trading off the benefit (cost) of paying lower (higher) taxes for the cost (benefit) of reduced (increased) access to an asset that provides a vehicle for self-insurance and consumption smoothing.

The same tradeoff also determines the critical income level at which the net gain of continuation changes sign (i.e. the value of y that identifies an agent indifferent between a government default and repayment). According to Panel (i), when $B = 0.001$, the net gain of continuation is always positive for agents with $b = 0$ at all income levels, so these agents always prefer repayment. As B and the implied taxes to service the debt rise, however, the net gain of continuation falls for all income levels. For $B = 0.39$, the net gain of continuation is negative (i.e. default is preferred) except for high-income individuals with $y \geq 1.7$. This occurs because, keeping b constant, high-income individuals would like to save a larger portion of their income than low-income individuals for self-insurance reasons, so for high-income agents public debt repayment despite the cost of the higher taxes needed to service the debt. In contrast, for low-income agents, the marginal cost of higher taxes dominates, and thus default is preferable. Finally, the net gain of repayment is negative at all income levels for $B = 0.80$, because at this high debt level the effect of higher taxes dominates for all values of y .

Similar mechanisms are at work in Panel (ii), which uses $b = 2$, instead of 0. As in Panel (i), repayment is preferred to default for all income levels when $B = 0.001$. In contrast with Panel (i), however, we find that for $B = 0.39$ the net gain of continuation changes sign twice as income

¹⁹Appendix 1 describes the algorithm in detail, including a description of the aggregate law of motion of the wealth distribution and measures of fitness of the algorithm.

Figure 10: Household $V^{d=0}(b, y, B, g)$ and $V^{d=0}(y, g)$ (for different B)

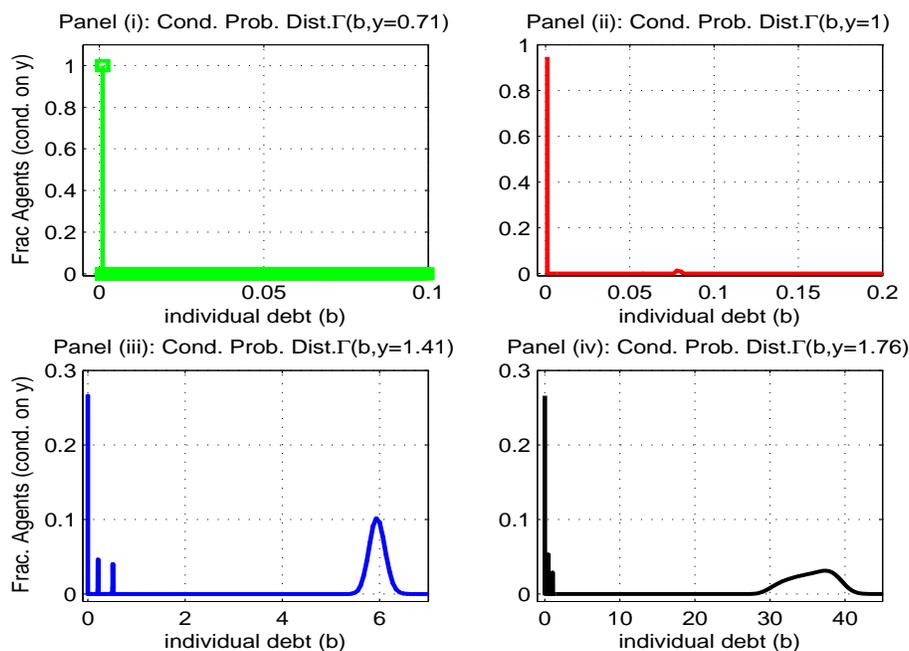


risers. This is an important point because it shows that whether default is welfare-enhancing or welfare-reducing varies non-linearly with individual income as b rises. When $b = 2$, relatively low- and high-income agents prefer repayment, while agents with y between $[0.8, 1.4]$ prefer default. The intuition for high-income agents is the same as above. For low-income individuals (but with high financial wealth, since $b = 2$), a government default implies that their main source of income, which is interest on public debt holdings, is wiped out. Thus, even though they also face higher taxes under repayment than under default, they still prefer repayment. Finally, for $B = 0.80$, again agents at all income levels prefer default because taxes are too high.

Next we examine the model's distribution of wealth. Figure 11 presents "average" distributions of households over debt conditional on income. These distributions are averages over a simulated sample of 5000 periods with the first 2000 truncated and shown in four panels each conditional on an income level for $y = [0.71, 1, 1.41, 1.76]$.²⁰ Panels (i) and (ii) show that approximately all agents with incomes at or below average income ($y = 1$) are up against the $b = 0$ bound as a binding borrowing limit. Debt holdings are higher for agents with $y > 1$, although still roughly 1/4 of high income agents are at the $b = 0$ bound (see panels (iii) and

²⁰To be precise, this distribution is constructed as the average of $\Gamma_t(b, y)$ over T simulated time periods. The average distribution is given by $\sum_{t=t_0}^T \frac{\Gamma_t(b, y)}{(T-t_0)}$. We simulate the economy for $T = 5000$ periods and drop the first $t_0 = 2000$ periods.

Figure 11: “Average” Conditional Cumulative Distribution $\Gamma(b, y)$



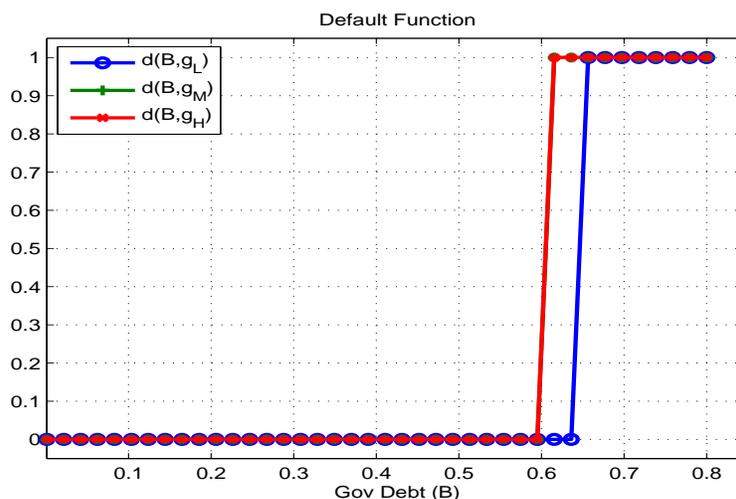
(iv)). Moreover, roughly 2/3rds of agents with $y = 1.41$ hold debt levels around $b = 6$ and about half of the agents with $y = 1.76$ hold debt levels in the 30 – 40 range.

Consider next the default policy of the government. Figure 12 shows the government default policy as a function of B for different values of government expenditures $g \in \{g_L, g_M, g_H\}$ with $g_L < g_M < g_H$.

For low levels of government expenditures ($g = g_L$) default is chosen only when $B \geq 0.65$, but with medium or high levels of government expenditures default is chosen when $B \geq 0.60$. Thus, government default is chosen more often at higher levels of debt and government expenditures. That is, the size of the government (as measured by the level of debt service and government expenditures) is an important factor influencing the default decision. Implicit in this relationship is, however, the connection between the default decision, the wealth distribution, the distribution of net repayment gains, and the level of public debt reviewed above.

Figure 13 shows the equilibrium price function for government bonds as a function of new debt issuance (B'). We observe that the model features the conventional result that the bond price decreases as net borrowing $B' - B$ increases. This can be seen by movements along the lines (i.e increases in B' , keeping B fixed) or by movements across lines (i.e increases in B , keeping B' fixed). The intuition is similar to that presented for the two period example. Even at levels of debt where there is no risk of default, the government needs to pay a higher interest rate to induce households to absorb the new level debt. Supply and demand factors are the ones

Figure 12: Equilibrium Default Function (for different g)



determining the bond price when the default probability is zero. Default risk induces an even larger reduction in bond prices as B' increases. More specifically, if B' is higher than 0.60, we reach states where default happens with probability 1 (see $d(B, g_M)$ in Figure 12) and the price drops to zero.

Figure 13: Equilibrium Price (for different B at $g = g_M$)

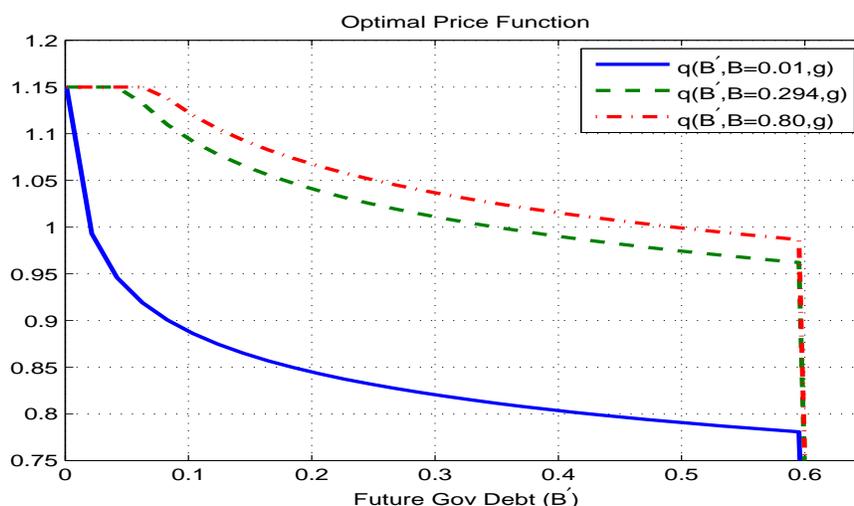
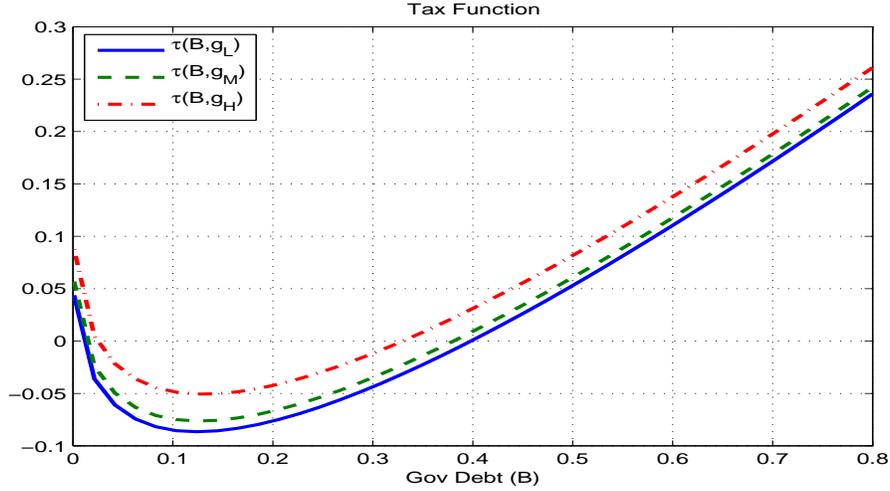


Figure 14 presents the tax decision rule as a function of outstanding debt (B). Figure 14 shows that the tax decision rule is a convex function of outstanding debt. For low levels of debt, there is a negative relation between τ and B . The reason is that at low values of B , the optimal borrowing policy implies $B' > B$ and the price of the bond does not drop enough to compensate for this change. Thus, total resources from borrowing qB' are increasing in B' and larger than B when B is low, so the government is able to substitute taxes for new debt. As the level of

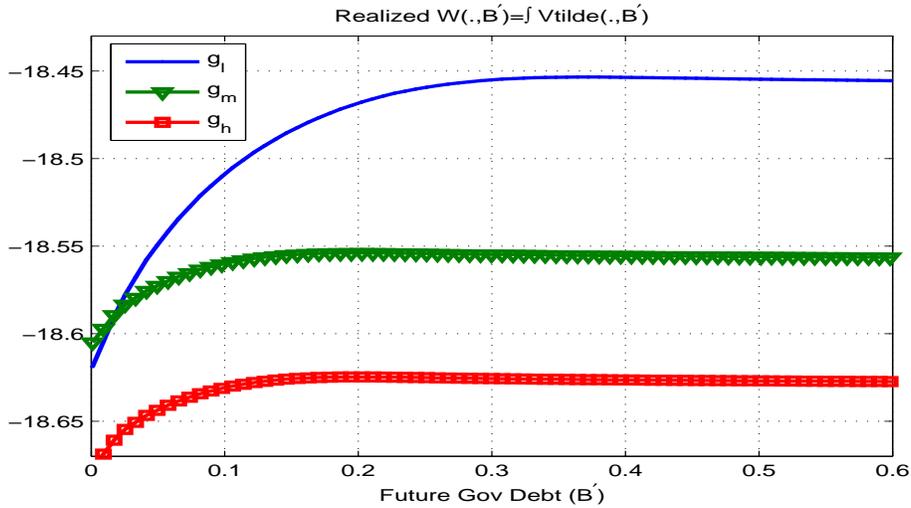
Figure 14: Equilibrium Tax Function



debt goes beyond 0.15, taxes are a strictly increasing function of current debt. Increasing the level of debt implies higher future taxes.

Figure 15 presents the “average” payoff of the government along the equilibrium path for different values of government expenditures.²¹ This average payoff combines the distribution

Figure 15: Government Objective: Debt Problem



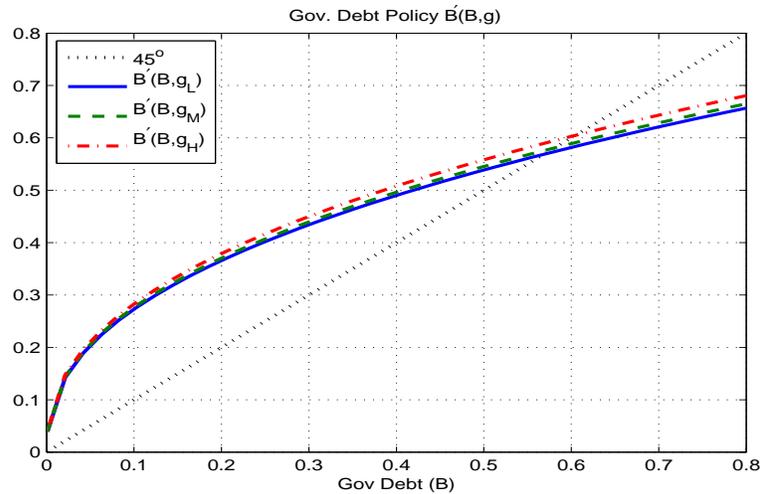
presented in Figure 11 with the household values presented in Figure 10 along the equilibrium path. Since many households are at the borrowing limit, the shape of this payoff function inherits mainly the properties of the payoff of those households with low debt. When B' is low,

²¹This figure is constructed as our “average” distribution presented in Figure 11. More specifically, it corresponds to $\sum_{t=t_0}^T \frac{\int \tilde{V}(b, y, g, B, B') d\Gamma_t(b, y)}{(T-t_0)}$ along the simulated periods.

a marginal unit of government debt reduces the level of current taxes without generating a large increase in expected debt service. However, as B' reaches higher values, there are two negative effects playing a role. First, households anticipate a much higher debt service (and taxes) in future periods. Second, the premium needed to clear the debt market makes an increase in future debt welfare-reducing for those with no debt holdings. On the other hand, for households with high levels of debt, this last effect dominates and the objective is almost always increasing in B' .

Figure 16 presents the debt decision rule of the government (B') as a function of B for the three levels of government expenditures mentioned earlier. Figure 16 shows that B' is increasing

Figure 16: Government Debt Policy



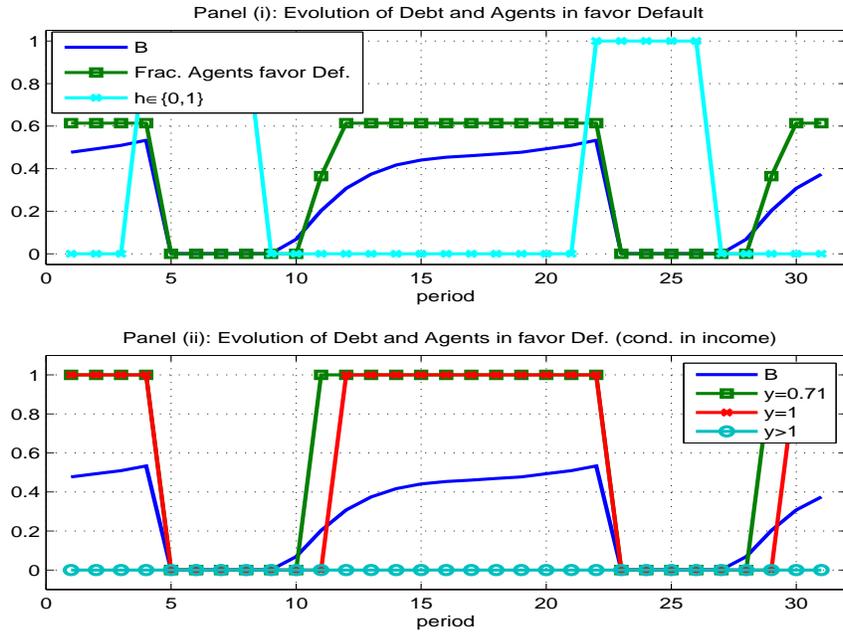
in B and g . This relation is important in determining the evolution of taxes as well as default. Higher level of government expenditures induce the government to borrow more and pushes it towards debt levels where default is optimal. There is however, a point where the government reduces its level of debt for all values of current debt even when government expenditures are high. This is a combination of the price function and the tax function that we presented above.

6.4 Default Dynamics in the Baseline Model

We now examine the paper's key result: That default can arise in equilibrium due to distributional incentives. We study first how the payoff function of the utilitarian government evolves over time. At each date t , the government uses each period's wealth distribution to weight the net gain of repayment of each agent for the same period. The top panel of Figure 17 shows the evolution of the fraction of agents with negative net repayment (i.e. those that favor default) and the equilibrium allocation of B over a simulation of 35 periods, which includes periods not

in default (identified as $h = 0$) and periods in default (labeled $h = 1$).

Figure 17: Household Preferences over Default



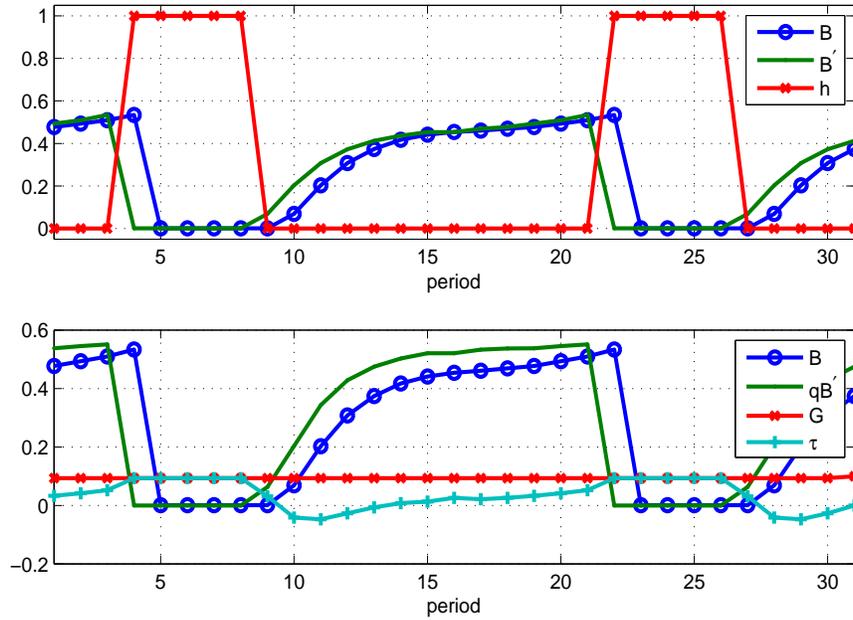
Notice that default does not necessarily occur when more than 50 percent of agents prefer default, because the default decision rule (12) considers the values of continuation and repayment of all agents. In fact, from $t = 0$ to 11 roughly 60 percent of agents have a negative repayment gain, but the government does not default. This is because, as long as B is relatively low (i.e. lower than about 58 percent), the weighted cost of defaulting imposed on the other 40 percent of agents who prefer repayment outweighs the benefit to the 60 percent that prefer default. The government thus chooses optimally to repay and continues issuing debt with the insurance benefits mentioned earlier, and without necessitating taxes that are "too large" to service the debt. This is no longer true when B reaches 58 percent and at that point ($t = 12$) the government chooses to default. After that, by $t = 17$ the government re-enters the debt market and begins to borrow again. The fraction of agents that prefer default increases very quickly and by $t = 20$ is back up at about 60 percent, while the debt builds up gradually in the upswing towards the next default.

The bottom panel of Figure 17 is similar to the top panel but it breaks down the fraction of agents that favor default across three income groups ($y = 0.71, y = 1, y > 1$). This breakdown shows that in fact high-income individuals (with $y > 1$) never prefer default, so for them default is always welfare-reducing. Similarly, low-income individuals never prefer repayment (whenever $B > 0$), while a fraction of agents with average income do prefer repayment when the debt is

low.

The top panel of Figure 18 displays the evolution of debt B , future debt B' . The bottom panel displays the evolution of government debt B , resources collected from debt issuance qB' , government expenditures g and taxes τ . This figure shows that the model is able to generate

Figure 18: Evolution of Debt and Taxes



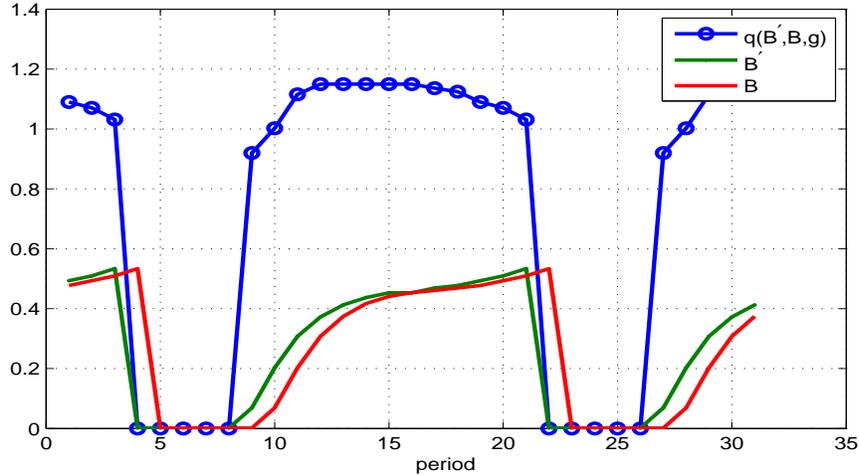
default along the equilibrium path. Government debt goes through phases in which increases gradually, until it reaches a level of near 0.60 and default occurs, followed by re-entry about five periods later. In the early stages after re-entry, the increase in government debt allows a reduction in taxes. As debt increases, however, further debt issuance brings in increases in borrowed resources (qB') at a diminishing rate, shifting taxes into an increasing path. When the level of government debt is high enough and repaying the level of debt would imply a level of taxes higher than the level of taxes in the case of no repayment, the government chooses to default. During the default period, taxes equal the level of government expenditures g . Once the government re-enters the bond market, the cycle starts again with some differences depending on the path of government expenditure.

Notice that interesting similarity with the model of optimal taxation under incomplete markets studied by Aiyagari et al. (2002). Volatile taxes are a feature of their model with exogenous bounds on government debt. In our economy, the upper bound on debt is derived endogenously by allowing the government to default. Note, however, that in our setup the government is running a primary balance deficit since $g > \tau$. What allows the government to sustain this

primary deficit is the fact that real interest rates are negative, i.e. $q > 1$ in most periods.

Figure 19 shows the evolution of debt and bond prices. This figure shows that in most

Figure 19: Evolution of Debt and Prices



periods the real return on government bonds is below zero (i.e. $q > 1$). Only when debt is very low or high enough so the economy is close to a government default the return becomes positive. This is another channel of redistribution. While the only available taxes are lump sum, a negative equilibrium interest rate acts as a proportional tax on bond holdings. There is also the possibility that under our parametrization households face more risk than in the data, and as it is standard in the incomplete markets literature, the return on the asset becomes negative for high levels of risk.

Figure 20 shows the evolution of debt and the difference between the welfare value of repayment and default (i.e. $W^{d=0}$ and $W^{d=1}$).

As the level of debt increases the distance between the continuation (i.e. repayment) and default is reduced, until it becomes negative. This relation is monotonic but we will show below that the distribution of households over debt and measures of dispersion are highly correlated with this difference as well.

Figure 21 shows the relation between the difference ($W^{d=0} - W^{d=1}$) with taxes, government expenditures, current debt, and future debt. This figure makes clear the positive relation between debt and default. As the value of debt increases, the continuation value decreases and default becomes a likely outcome. This difference is almost independent of government expenditures and highly negatively correlated with taxes.

More interestingly, Figure 22 shows the relation between the continuation value and two measures of debt dispersion: skewness and kurtosis. Each observation is computed from the distribution that arise during our simulations.

Figure 20: Evolution of Debt and Welfare

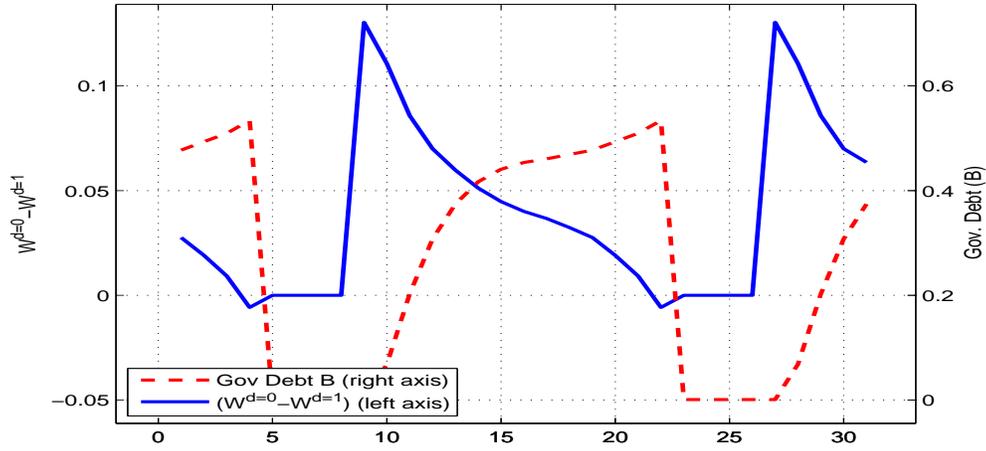


Figure 21: Welfare, Debt and Taxes

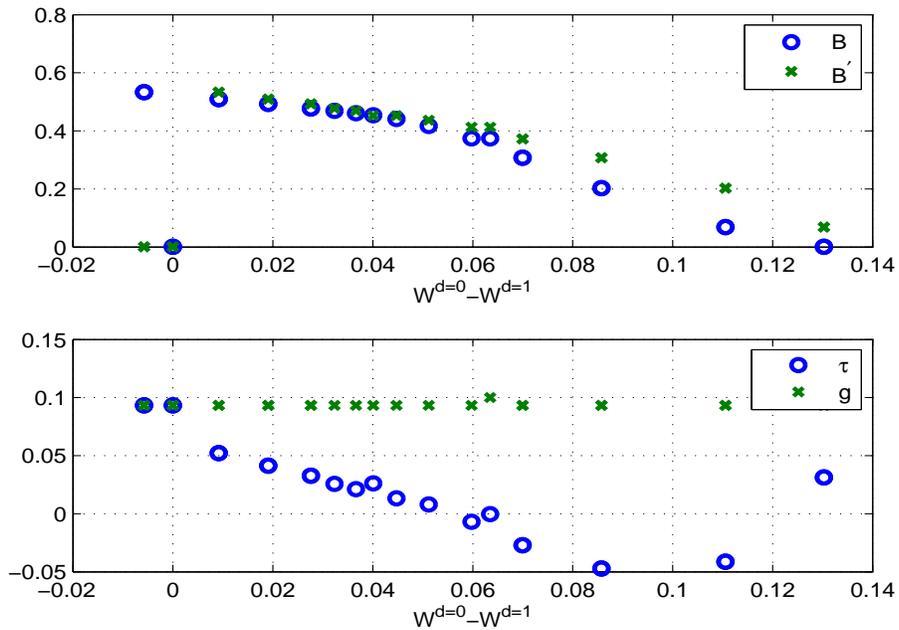
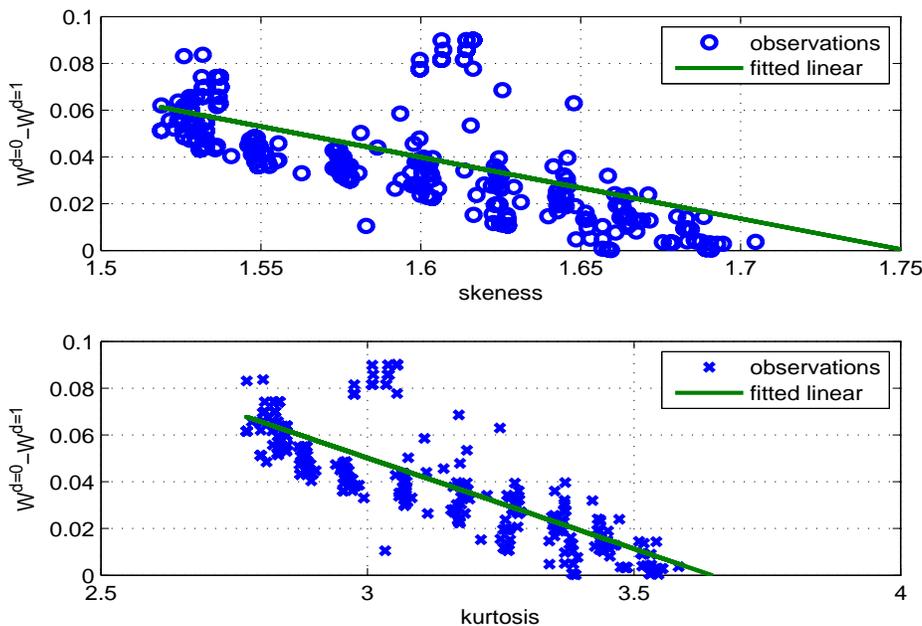


Figure 22: Evolution of Welfare and Dispersion



Both measures present the same pattern. As dispersion increases the continuation value decreases. This shows that household heterogeneity is an important determinant of the debt and default decision.

7 Conclusion

We showed that domestic sovereign defaults can be rationalized as the optimal action of a utilitarian social planner that responds to distributional incentives in a model with heterogeneous agents and incomplete markets. In this model, the distribution of bond holdings across private agents is endogenous and interacts with the government's optimal default, debt issuance and tax decisions. Default is optimal when the weighted net welfare loss of servicing the debt for sufficiently poor agents exceeds the corresponding weighed net welfare gain of sufficiently rich agents.

Defaults are not selective, and hence the choice of default versus repayment has important redistributive tradeoffs. The short-run benefit of default is the reduction in the level of lump-sum taxes common to all agents. The costs are, in the short-run, the reduction of wealth for agents with positive holdings of government debt and, in the long-run, more volatile taxes and the loss of a vehicle that agents use to self-insure against idiosyncratic and aggregate shocks. On the other hand, if the government repays the debt, it selects an optimal mix of lump-sum taxes

and new debt to finance government expenditures and service outstanding debt. Taxes represent a larger fraction of disposable income for those with low debt holdings, so (conditional on debt repayment) these agents are better off if the government finances its outlays by borrowing more. The cost of increasing the level of debt is that expected future taxes are higher and a default more probable.

The model's quantitative predictions based on a rough calibration to data for Spain show that the model produces a high degree of concentration in the distribution of public debt holdings, with a large fraction of agents holding zero debt. Moreover, the model supports realistically high debt-output ratios, equivalent to 40 percent on average and up to 60 percent in the debt run-up leading to a default, and optimal default events occur with a frequency of about 8 percent. These are important findings in light of well-known limitations of standard quantitative models of external sovereign default that can produce debt ratios as high as in the data only at negligible default frequencies.

This research is still at a preliminary stage. In further work we plan to strengthen the calibration of the model to the Spanish data, and thus improve our assessment of the model's ability to match the key empirical regularities of debt dynamics. A limitation of our analysis under our current rough calibration is that real interest rates are, on average, much lower than in the data (in fact, they are negative). Furthermore, we plan to extend our research in three important directions. One to allow for partial default, which provides a natural framework to study a situation akin to *de facto* defaults via inflation. The second to incorporate external lenders, so that we can investigate the interaction between domestic and foreign defaults highlighted in the Reinhart-Rogoff facts. And third, we are also considering to examine a model with secondary markets for sovereign debt after default, along the lines of Broner et. al (2010).

8 Appendix

8.1 Solution Method and Computational Algorithm

As explained in the main body of the paper, we extend the algorithm proposed by Krusell and Smith [16] to accommodate an environment with government default. We assume that current prices q , the government bond decision rule B' and the default decision d can be written as a function of a finite set of moments from the distribution $M = \{m_1, \dots, m_N\}$ and the current value for government expenditures g . Finally, we let M' be a function of M and g .²² In this version, we set $N = 1$ and assume that $m_1 = B = \int b d\Gamma(y, b)$, i.e. the average debt holdings.

The equilibrium functions are assumed to take the following form:

$$\begin{aligned} q &= H^q(B', m_1, g) \equiv \delta_0 + \delta_1 B' + \delta_2 B + \delta_3 g, \\ B' &= H^B(B, g) \equiv \alpha_0 + \alpha_1 B + \alpha_2 g, \\ d &= d(B, g) \equiv \gamma_0 + \gamma_1 B + \gamma_2 g \end{aligned}$$

In what follows we redefine the household and government problems.

8.1.1 Household Problem

Given government policies, the problem of the household can be written as follows:

$$V(b, y, B, g) = (1 - d(B, g))V^{d=0}(b, y, B, g) + d(B, g)V^{d=1}(y, g)$$

where $V^{d=0}(b, y, B, g)$ denote the continuation value if the government chooses not to default and $V^{d=1}(b, y, B, g)$ is the continuation value if the government chooses to default. The continuation value in the case of no default is given by:

$$V^{d=0}(b, y, B, g) = \max_{b' \geq 0} u(b + y - \tau - qb') + \beta E_{y', g'} [V(b', y', B', g') | y, g]$$

$$\text{s.t.} \begin{cases} q = H^q(B', B, g) \\ B' = H^B(B, g) \\ \tau = B + g - qB' \end{cases}$$

²²Note that, since government debt is constant, average debt is (trivially) included as a state variable.

The value of default is defined as follows (assuming permanent autarky after default):

$$V^{d=1}(y, g) = u(y - g) + \beta E_{y', g'}[\lambda V^{d=0}(0, y, 0, g) + (1 - \lambda)V^{d=1}(y', g')|y, g].$$

Note that, in equilibrium, the government budget constraint needs to be satisfied, so we can define τ as a ‘residual’ endogenous variable: $\tau = B + g - qB'$ (provided B' and q are defined).

8.1.2 Government’s Default Decision

At any given distribution, the government evaluates

$$W(B, g) = \max_{d \in \{0, 1\}} \left\{ \int V^{d=0}(b, y, B, g) d\hat{\Gamma}, \int V^{d=1}(y, g) d\hat{\Gamma} \right\}$$

That is, after the government expending shock is realized, the government chooses to repay the debt and borrow again or default.

The optimal debt problem and the auxiliary problem to obtain market clearing are similarly to those previously defined. They are solved after the government has decided not to default.

8.1.3 Optimal Government Debt (Deviation Problem)

For given H^q , the problem of the household at an arbitrary level of future debt B' can be written as follows:

$$\tilde{V}(b, y, B, g, \tilde{B}') = \max_{b' \geq 0} u(b + y - \tau - qb') + \beta E_{y', g'}[V(b', y', \tilde{B}', g')|y, g]$$

$$\text{s.t.} \begin{cases} q = H^q(B', B, g) \\ \tau = B + g - q\tilde{B}' \end{cases}$$

The solution to this problem provides the value function $\tilde{V}(b, y, B, g, \tilde{B}')$. That is, each household has its own valuation of the future government policy. Note the effect of B' on τ .

At a given distribution, the government chooses the optimal level of debt by solving the following problem:

$$\max_{B'} \int_{b, y} \tilde{V}(b, y, B, g, B') d\hat{\Gamma}(b, y)$$

In equilibrium, the function H^B is consistent with the solution of this problem. Different aggregation mechanisms can be easily analyzed here.

8.1.4 Auxiliary Problem: Price Function

As in Krusell, P. and A. Smith (1997), it is convenient to solve for the equilibrium market clearing price along the simulation and for that reason we need to define the solution to household problem for an arbitrary set of prices. More specifically, for given H^B , the problem of the household at an arbitrary price \hat{q} can be written as follows:

$$\max_{b' \geq 0} u(b + y - \tau - \hat{q}b') + \beta E_{y', g'} [V(b', y', B', g') | y, g]$$

$$\text{s.t.} \begin{cases} B' = H^B(B, g) \\ \hat{\tau} = B + g - \hat{q}B' \end{cases}$$

The solution to this problem provides the decision rule $b' = \hat{h}(b, y, B, g, \hat{q})$. In this problem, agents choose the level of debt based on an arbitrary value of q and perceive future prices and policies to evolve according to equilibrium functions. For any given distribution is then possible to achieve market clearing:

$$B' = \int_{b, y} \hat{h}(b, y, B, g, \hat{q}) d\Gamma(b, y).$$

where $\Gamma(b, y)$ denotes the distribution of households over debt and income.

8.1.5 Computational Algorithm

1. Guess Aggregate Functions: d , H^B , H^q and τ .
2. Solve Household Problem.
3. Solve Government Deviation Problem (solution to B').
4. Solve Auxiliary problem (to obtain decision rules as function of q).
5. Simulation:
 - (a) Simulate a sequence of government expenditures $\{g_t\}_{t=0}^T$.
 - (b) Guess Γ_0 . From the initial distribution Γ_0 , compute the initial aggregate level of debt B_0 .
 - (c) Using $\tilde{V}(b, y, B, g, B')$ and Γ_t , solve the deviation problem to obtain B_{t+1} and τ_t .
 - (d) Using Γ_t , B_{t+1} and $b' = \hat{h}(b, y, B, g, \hat{q})$ obtain the equilibrium price q_t :

$$B_{t+1} = \int_{b, y} \hat{h}(b, y, B_t, g_t, q_t).$$

- (e) Using Γ_t , $b' = \hat{h}(b, y, B, g, \hat{q})$ and the equilibrium price q_t obtain the new distribution Γ_{t+1} .
 - (f) Solve default decision. If default, set Γ_t such that all agents hold zero debt.
 - (g) If $t < T$, return to (c). If $t = T$ finish simulation.
6. Using the sequences of $\{g_t, B_t, q_t, d_t\}_{t=\hat{t}}^T$ update the aggregate functions, where \hat{t} is a period $\hat{t} > 0$ such that the model is simulated long enough to make all results independent of the initial distribution.
 7. Using the new functions for H^B and H^q update τ .
 8. If new aggregate functions are close to starting aggregate function you are done. If not, return.

8.1.6 Computed Aggregate Functions

Table 3: Equilibrium Price and Gov Policies

	$\ln(q)$	$\ln(B')$	d
constant	-0.0564	-0.1243	0.08865
s.e.	0.0310	0.07512	6.9491
$\ln(g)$	-0.0055	0.07986	0.04585
s.e.	0.0128	0.0312	0.0086
$\ln(B')$	-0.0724	-	-
s.e.	0.0126	-	-
$\ln(B)$	0.0350	0.4224	-1.1494
s.e.	0.0041	0.00168	0.084
$\ln(B)^2$	-	-	2.15786
s.e.	-	-	0.2598
R^2	0.92	0.98	0.86

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