Optimal Employment Contracts with Hidden Search

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Abstract

In this paper I explore optimal employment contract design in a random search framework, where workers search on and off the job for employment opportunities similar to that of Lentz (2010) and Bagger and Lentz (2013). The worker determines the frequency by which employment opportunities arrive through a costly choice of search intensity, which is unobserved by the firm and cannot be directly contracted upon. Firms differ in productivity by which they employ workers. Firms compete over workers in terms of utility promises in a fashion otherwise similar to that of Postel-Vinay and Robin (2002). As in Burdett and Coles (2003) and Burdett and Coles (2010), optimal tenure conditional contracts are shown to be back loaded to discourage the worker from generating outside competitive pressure. The analysis establishes existence, uniqueness and provides characterization of the core mechanism. The paper applies the framework to the analysis of firm provided general human capital training. It is shown that more productive firms provide more training and pay higher wages.

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1 Introduction

In a frictional labor market, a firm may be quite certain that it can dictate poor employment terms to an employee and at least in the short run still maintain the employment relationship. However, the firm would naturally be concerned that the worker will be unhappy and go to great lengths to find an outside employer that will offer better terms. Such search is costly. In addition to the significant time and psychological effort it takes for the worker to meet other firms and to market him or herself, it is likely that the effort precludes investments in the current relationship. Also, whatever good will and other intangible assets the worker holds in relation to the current firm are likely to suffer in the process.

In this paper I analyze optimal employment contracts in a random search framework, where workers search on and off the job for employment opportunities. The worker sets the offer arrival frequency through a costly choice of search intensity, which is unobserved by the firm and cannot be directly contracted upon. Firms differ in the marginal productivity by which they employ workers. If an employed worker meets another firm, the two firms engage in Bertrand competition over the worker in as in Postel-Vinay and Robin (2002). It is a maintained limited commitment assumption that the firm cannot commit to not match outside offers. Firms compete over workers in terms of lifetime utility promises. Because a more productive firm has a greater willingness to pay for a worker, a Bertrand competition between two firms over a worker will result in the more productive firm winning the worker at a utility promise equal to the losing firm's willingness to pay.

The worker's incentives to generate outside meetings include additional rent extraction from the current employment relationship, which will cause search to be inefficiently high, as viewed by the contracting parties. Search is hidden and the worker cannot commit to a given search intensity level, which imposes an incentive compatibility constraint on the mechanism design problem.

The employment contract is the mechanism that maximizes firm profits subject to the lifetime utility promise to the worker and subject to the constraint that worker search satisfies the worker's incentive compatibility constraint. The paper's focus on dynamic employment contracts is closely related to Burdett and Coles (2003), Burdett and Coles (2010), Menzio and Shi (2010), and Lamadon (2014), and to make the problem interesting I follow their assumption of risk averse agents and imperfect capital markets that eliminate the possibility of side payments.

I show that the optimal employment contract will be backloaded in order to reduce outside wage pressure on the match. The utility promise in the contract is monotonically increasing in tenure. Wages are shown to be monotonically increasing in the utility promise. Therefore wages increase within a job through two channels in the core model of the paper: One is the contracted upon growth rate in wages through the utility promise path, and second the occasional arrival of an outside job opportunity resulting in a renegotiated and higher utility promise.[1] The worker's lifetime utility increases monotonically over an employment spell, be it within or between jobs, but as in Postel-Vinay and Robin (2002) actual wages may decrease between jobs.

In contrast to Postel-Vinay and Robin (2002) but consistent with Burdett and Coles (2010), the optimal employment contract also implies a decreasing job separation hazard in tenure because the worker's search intensity is declining with duration and limits to the jointly efficient level at the point where the worker is extracting all the surplus from the match.[2] In the Burdett and Coles (2010) framework the declining separation hazard is a result of the worker turning down more outside offers as he or she moves up the wage/tenure profile, which is then associated with an increase in the incidence of inefficient job separation. It is worth noting that in contrast with the Burdett and Coles (2010) framework, all separation in this paper is efficient. Instead search inefficiencies are related to the intensity margin.

The analysis establishes existence, uniqueness, and provides characterization of the core employment contract mechanism. The analysis also considers design constraints with a particular focus on rent division. The Postel-Vinay and Robin (2002) framework has the feature that employers extract all rents associated with a match in excess of the total surplus of a worker's current match. Hence, if the worker is either unemployed or extracting full surplus in a given match, the worker will receive none of the additional rents associated with

[1] In section 5 I show that the framework can easily be extended to allow for human capital growth which then provides a third channel for wage growth.

[2] This prediction can be modified by the inclusion of other events in the employment relationship such as human capital growth. In section 5 I show that if the worker becomes more skilled during an employment relationship, at that instant, the contract implies an upward jump in the worker's search intensity, an unchanged wage, and an upward jump in the growth rate of wages.

meeting a more productive employer. It is an interesting feature of the unrestricted contract design that the worker's inability to commit to a search intensity does not result in any additional rent extraction in the worker's favor. But once the hidden search is coupled with design constraints, additional rents do flow to the worker. The contracting framework in this paper allows ex ante rent extraction due to constraints on the contract or the shape of the utility function. The presence of a minimum wage is one example which is also studied in Flinn and Mabli (2009). A utility function that has the feature that $\lim_{w \to \underline{w}} u'(w) = -\infty$ will also effectively impose a minimum wage $\underline{w}$ on the problem. Finally, the arbitrary constraint of for example a flat wage profile can also by itself result in ex ante rent extraction. In all these cases, it is possible that employers will voluntarily give the worker a strictly higher lifetime utility than what is required by the worker's outside option. Consequently, there will be strictly positive rents to unemployed search and to search where the worker is extracting full surplus from her current match. The paper allows both non-essential worker search and ex ante rent extraction as ways to understand the unemployed worker's exit from unemployment.

The paper applies the framework to the study of firm provided general human capital training. As is well known, outside competition for the worker generates a hold up problem where the worker extracts the rents from the investment paid for by the firm. In a perfectly competitive setting, the typical solution to this problem is for the worker to pay for the training up front through lower initial wages. In the current setting, frictions soften the competition and the firm can reasonably expect some returns to its investment depending on the strength of the outside competition. The firm can manipulate the outside competition pressure through the worker's search intensity incentive compatibility constraint. In addition, the firm has the option to backload wages to at least in part make the worker pay for the training up front. Hence, it is a setup that is very well suited for the study of training. As in Fu (2011), the analysis finds a broadly positive relationship between firm productivity type and the amount of training provided to the worker. However, this conclusion is somewhat modified by the insight that it is possible that a worker in a high productivity firm may search very intensely given a low current utility promise. In this case, the returns to search through rent extraction from the current employment relationship are high, and so the worker searches intensely. In this situation, the firm chooses to provide little training initially. Through the backloading, the workers utility promise will increase, the search intensity will drop, and training will increase. The key insight here is that it is generally true that high productivity firms do tend to provide more training, however, only once the employment relationship has solidified. The paper provides numerical simulation of the steady state equilibrium of the training model.

The paper contributes to (at least) two existing research efforts in the literature. One is the study of optimal wage contracts in models with labor market frictions such as Burdett and Coles (2003) and Burdett and Coles (2010). Obviously the study of wage determination in models with friction is a much larger literature related to the Nobel prize winning research by Peter Diamond, Dale T. Mortensen, and Chris Pissarides. However, this work is typically strong on predictions on the division of rents from match creation rather than the particular path of payments during the match, which is often pinned down with the ultimately arbitrary assumption of flat wage profiles.

The paper is also related to the empirical study of wage dynamics with contributions such as Altonji and Shakotko (1987); Topel (1991); Yamaguchi (2010); Bagger et al. (2013); Kambourov and Manovskii (2009). Yamaguchi (2010) and Bagger et al. (2013) study settings where wages respond to two key accumulation processes: human capital and search capital. The example in Section 5 in the paper include the same two accumulation processes, but in contrast to these two papers, the current framework does not impose a flat piece rate wage contract, but allows the contract to be set optimally by the firm and includes the moral hazard problem in search intensity that implies a separate source of backloading. In addition, in the training example in Section 5, the firm is manipulating both the accumulation processes by the design of the contract.

Of course, the issue of backloading and/or frontloading of contracts also has a long history in the principal-agent moral hazard literature as in for example Rogerson (1985) and Fudenberg et al. (1990). Just as Burdett and Mortensen (1998) and Burdett and Coles (2003) can be viewed as an efficiency wage model, the current analysis has parallels to this literature as well in that the design of the compensation scheme directly impacts the joint value of the match. In this case, the impact is through labor turnover related costs and gains as opposed to the productivity realization of the worker.

The paper proceeds as follows: In Section 2 is analyze the core employment contract design problem with hidden search. Here I establish existence and uniqueness of the contract and provide characterization. The main part of the analysis is set in discrete time and the continuous time formulation is obtained as the limit when period length is taken to zero. In Section 3 I consider constraints on the design program, in particular minimum wages. In Section 4 I briefly touch on socially efficient search, and in Section 5 I provide an application of the framework to the question general human capital training provided by firms.

2 The Basic Mechanism

In this section I present the core wage mechanism along with characterization, existence and uniqueness proofs. For expositional simplicity, the arguments are presented in discrete time. In the following sections I use these results to expand the analysis to a steady state equilibrium and also consider worker heterogeneity, human capital growth, savings, and design constraints. This part of the analysis is done taking the continuous limit of the discrete time environment which allows for simpler expressions in the now more complicated setting.

2.1 Environment

Time is discrete. Each period has length Δ. Workers and firms are infinitely lived and all discount time according to discount factor $\beta(\Delta) = e^{-r\Delta}$. All workers are identical. Firms differ in their match output levels, which are normalized to lie in the unit interval, $p \in [0, 1]$. Workers can be either employed or unemployed. Workers consumer their income, $w\Delta$, in a given period, the utility of which is given by increasing and concave function, $u(w)\Delta$. An unemployed worker has zero income.

Matches are created through a frictional search process. Specifically, through a costly search choice λ the worker sets the probability, $s(\Delta) = 1-e^{-\lambda\Delta}$, that she will meet a vacancy during the period in question. Regardless of employment state, the cost of the choice of λ is given by increasing and convex cost function $c(\lambda)\Delta$. A vacancy is characterized by its output level which is drawn from the cumulative offer distribution, $\Phi(p)$, which has density $\phi(p)$. A match ends either through an exogenous destruction probability, $d(\Delta) = 1 - e^{-\delta\Delta}$, or if the worker quits to another firm as a result of meeting a more attractive vacancy. To save on notation in the discrete formulation of the problem, I will set up the problem where the worker is directly choosing the offer arrival probability, $s \in [0,1]$ at per unit of time cost $\hat{c}(s) = \Delta c\left(\frac{-\ln(1-s)}{\Delta}\right)$.

A firm's match output level is assumed to be time independent. The same is true for the search cost function. Generally, it will be assumed that there are no aggregate shocks and that the economy is in steady state. I make the following assumptions on the model fundamentals.

Assumption 1. $u(\cdot) : \mathbb{R} \to \mathbb{R}$ *is strictly increasing, strictly concave and at least twice differentiable.*

Assumption 2. *The first derivatives of the utility function are bounded, such that for any* $w \in \mathbb{R}$, $u'(w) \in [\underline{u}', \overline{u}']$.

Assumption 3. *The offer distribution has no mass points and everywhere positive mass,* $\forall p \in [0,1]$, $0 < \phi(p) < \infty$.

Assumption 4. $c(\lambda) : \mathbb{R}_+ \to \mathbb{R}$ *is strictly increasing, strictly convex and at least twice differentiable. Furthermore,* $c(0) = c'(0) = 0$.

Firms are constant returns to scale and make decisions at the match level to maximize the expected profits at that match. Each firm posts vacancies and competes over workers as in Postel-Vinay and Robin (2002). If a firm's vacancy meets a currently employed worker, the two firms will engage in Bertrand competition over the worker. The worker will choose employment with the firm that offers the worker the higher expected lifetime utility going forward. Hence, the competition between the firms takes place in terms of lifetime utility promises. Define $\bar{V}(p)$ as the most lifetime utility a productivity p firm is willing to offer a worker through an optimally designed employment contract subject to the constraints of the contracting environment. A utility promise beyond $\bar{V}(p)$ will result in negative expected profits to the firm. It will be shown below that $\bar{V}(p)$ is monotonically increasing p. Consequently, a competition between two firms with productivities p and p', respectively, such that $p < p'$ will result in the worker accepting employment with the productivity p' firm with an employment contract that promises the worker lifetime utility no less than $\bar{V}(p)$. If a vacancy meets an unemployed worker, the firm will hire the worker with an employment contract that offers at a minimum the utility value of unemployment, U.

2.2 Employment Contract Design Constraints

The contractual environment is one of limited commitment: The worker can at any point costlessly quit to unemployment, meaning that the contract must at any point promise the worker no less than the value of unemployment at that point. The firm cannot commit to not match outside offers.[3] Firms can also at any point costlessly lay off a worker, which implies that the contract cannot at any point have negative expected profit value going forward. As will be shown below, in the simple version of the model, the firm's participation constraint turns out to not be binding. All in all this means that the utility promise of an employment contract with a productivity p firm must at any point be in the interval $\left[U, \bar{V}(p)\right]$.

The worker's search intensity choice is not observable by the firm and the worker cannot commit to any particular search choice. As a result, the specified search intensity in the contract must satisfy incentive compatibility. In line with the no borrowing or saving assumption in the simple model, the setup assumes no side payments.

Part of search being unobservable also means that a worker's meetings with outside firms are only observable to her employer if she decides to reveal a meeting. A worker will reveal a meeting in order to induce contract renegotiation, which only happens if both parties agree to do so. The revelation of a meeting with an outside firm that can offer more than the value of the worker's current contract induces renegotiation. As part of the no side payments assumption and in line with the assumption that the firm cannot commit to not match outside offers, the contract cannot make payments conditional on renegotiation or the worker separating from the firm. The renegotiation itself is governed by the nature of the Bertrand competition and is not affected by the history of the contract.

The history of a contract is in the simple model just the path of past utility promises, $\sigma(t) = \{V_\tau\}_{\tau=0}^{t}$. Whatever contracts came before the current contract is irrelevant. A contract specifies a history dependent path of utility promises, search intensity, and wages which given the simplicity of the history can be written as, $C\left(V_0, \bar{V}\right) = \left\{V_t, s_t, w_t\right\}_{t=0}^{\infty}$.

2.3 Worker Lifetime Utility

For now, set $\Delta = 1$ and in the notation suppress the dependence of the arrival rates on the length of a period. Denote by V_t the worker's valuation of her expected discounted stream

[3]Postel-Vinay and Robin (2004) explore the firm's option to commit to not match outside offers.

of future utilities given a current employment contract with history $s(t)$. In the valuation of possible outside employment opportunity offers, it useful to characterize a vacancy by its willingness to pay, $\bar{V}(p)$, rather than the productivity. Define by $F(V)$ the distribution of willingness to pay across vacancies, $F(V) = \Phi(p(V))$, where $p(V) = \bar{V}^{-1}(V)$. Furthermore, define $\hat{F}(V) = 1 - F(V)$. With this and suppressing explicit dependence on the contract history in the notation, the worker's valuation of her current contract with an employer with willingness to pay $\bar{V}$ can be written as,

$$
\begin{aligned}
V_t = {} & u(w_t) - \hat{c}(s_t) + \beta dU \\
& + \beta(1-d)\left[s_t \int_{V_{t+1}}^{\bar{V}} V dF(V) + s_t \bar{V}\hat{F}\left(\bar{V}\right) + \left[s_t F(V_{t+1}) + (1-s_t)\right] V_{t+1}\right] \\
= {} & u(w_t) - \hat{c}(s_t) + \beta\left\{dU + (1-d)\left[s_t \int_{V_{t+1}}^{\bar{V}} \hat{F}(V)\, dV + V_{t+1}\right]\right\}. \qquad (1)
\end{aligned}
$$

In period t, the worker receives utility $u(w_t)$ from wage income and takes disutility $\hat{c}(s_t)$ from the search effort. If the worker is laid off, she receives continuation utility U, which is simply the value of being unemployed. By the assumption of steady state, the value of unemployment is time independent. If her job survives, and she does not receive an outside offer, her continuation value in the employment contract is V_{t+1}. In the case where she does receive an offer, denote the outside employer's willingness to pay by $\bar{V}'$. In the case where $\bar{V}' \leq V_{t+1}$, nothing comes of the meeting. If $V_{t+1} < \bar{V}' \leq \bar{V}$ the worker will force a renegotiation of her contract by engaging the two firms in Bertrand competition over her. Her current employer's willingness to pay, $\bar{V}$, exceeds that of the outside suitor, so she will stay, but will be given a new employment contract with an initial lifetime utility promise of $\bar{V}'$. Finally, if $\bar{V}' > \bar{V}$ the worker will leave her current employer and receive an employment contract with the outside firm with an initial utility promise of her old employer's willingness to pay, $\bar{V}$. The second line follows by integration by parts.

The implicit assumption in the above expression is that an employer will not voluntarily offer more lifetime utility than what is dictated by the Bertrand competition between employers. In section 3, I discuss contract design constraints that can result in a violation of this assumption, in which case equation (1) is modified to reflect that job-to-job transitions can be associated with a new employment contract that has value in excess of the willingness to pay of the old employer.

An unemployed worker receives income flow normalized at zero. Given the case where employers give no more lifetime utility than strictly required by the competition between employers, there are no rents to unemployed search. Any meeting will result in an employer giving the unemployed worker a take it or leave it offer matching the value of unemployment and nothing more.[4] Consequently, the value of unemployment in this case is very simply equivalent to the infinitely discounted stream of utility of unemployed income,

$$U = \frac{u\left(0\right)}{1-\beta}. \tag{2}$$

Once we move to design constraints in section 3 that result in ex ante rent extraction, equation (2) will be appropriately modified to include the now positive option value to unemployed search.

2.4 Incentive Compatibility

The worker's search intensity choice is hidden, so the contract must at any point dictate a s_t that maximizes the worker's lifetime utility going forward,

$$s_t \in \arg\max_{s\geq 0}\left[u\left(w_t\right) - \hat{c}\left(s\right) + \beta\left\{dU + \left(1-d\right)\left[s\int_{V_{t+1}}^{\bar{V}}\hat{F}\left(V\right)dV + V_{t+1}\right]\right\}\right].$$

Given strict convexity of the search cost function, this is a concave optimization problem and the unique maximizer satisfies the first order condition,

$$\hat{c}'\left(s_t\right) = \beta\left(1-d\right)\int_{V_{t+1}}^{\bar{V}}\hat{F}\left(V\right)dV. \tag{3}$$

It is straightforward to show that s_t is strictly declining in V_{t+1} as long as $f\left(V\right) = \partial F\left(V\right)/\partial V > 0$ for all $V \in \left[U, \bar{V}\left(1\right)\right]$.

2.5 Optimal Contract Design

Competition between employers takes place in terms of lifetime utility promises. For a given utility promise, the firm will design an employment contract that maximizes the future

[4] The discrete setting naturally allows modifications of this argument, say, as in Burdett and Judd (1983) where the worker meets multiple employers in a given period and they have to compete with each other Bertrand style over the worker. It is a rather elegant way of introducing ex ante rent extraction into the setup, and the probability of meeting multiple employers in a single period is handled naturally as multiple arrivals of the Poisson arrival process. However, since the paper will be emphasizing the continuous limit of the setting and this competitive mechanism does not survive in the limit, I have chosen to ignore the possibility of multiple meetings in a single period.

discounted profit stream of the match subject to the design constraints and the lifetime utility promise. Stated in terms of the history of the contract, this is a large state space problem. However, following Spear and Srivastava (1987); Atkeson and Lucas (1992); Menzio and Shi (2010) the firm's employment contract design problem can be represented by a recursive formulation of the firm's valuation of a match as a function of the lifetime utility promise to the worker at that point.

For a given optimal employment contract, C, and given the firm's willingness to pay, $\bar{V}(p)$, the firm's valuation of the associated future discounted profit stream can at time t be written recursively as,

$$\Pi(V_t) = p - w_t + \beta(1-d)\left\{s_t \int_{V_{t+1}}^{\bar{V}} \Pi(V)\, dF(V) + \left[1 - s_t \hat{F}(V_{t+1})\right] \Pi(V_{t+1})\right\} \tag{4}$$

$$= p - w_t + \beta(1-d)\left\{s_t \int_{V_{t+1}}^{\bar{V}} \Pi'(V)\, \hat{F}(V)\, dV + \Pi(V_{t+1})\right\}. \tag{5}$$

where steady state has already been invoked to eliminate time dependency of $\Pi(V)$. The per period profit is $p - w_t$ and if the match survives, the firm's continuation value associated with promising the worker a lifetime utility in the next period of V_{t+1} is given by $\Pi(V_{t+1})$. The firm will deliver V_{t+1} to the worker in the case where the worker does not obtain an outside offer or if the worker meets an outside firm with willingness to pay less than V_{t+1}. If the worker meets an outside firm with willingness to pay, $\bar{V}' \in [V_{t+1}, \bar{V}]$, the match will survive, but the employment contract will be renegotiated to deliver $\bar{V}'$ to the worker, which has value $\Pi\left(\bar{V}'\right)$ to the firm. If the outside employer has willingness to pay $\bar{V}' > \bar{V}$, the match ends. The firm's outside value of a match is zero. The second line follows from integration by parts and that by definition, $\Pi\left(\bar{V}\right) = 0$.

I impose incentive compatibility on the employment contract design problem through the first order approach and require that any choice of s_t must satisfy equation (3). Given a lifetime utility promise of $V \in \left[U, \bar{V}\right]$ and suppressing the functional dependence on $\bar{V}$ to save on notation, the firm's valuation of an optimal employment contract is given by,

$$\Pi(V) = \max_{(w,Y,s)\in\Gamma(V)} \left[p - w + \beta(1-d)\left\{s \int_{Y}^{\bar{V}} \Pi'(V)\, \hat{F}(V)\, dV + \Pi(Y)\right\}\right], \tag{6}$$

where for any $V \in \left[U, \bar{V}\right]$ the set of feasible choices of the triple (w, Y, λ) is given by,

$$\Gamma(V) = \Bigg\{ (w, Y, s) \in \mathbb{R}^2 \times [0,1] \,|$$

$$u(w) - \hat{c}(s) + \beta \left\{ dU + (1-d)\left[s \int_Y^{\bar{V}} \hat{F}(V')\,dV' + Y \right] \right\} \geq V \tag{7}$$

$$\hat{c}'(s) = \beta(1-d) \int_Y^{\bar{V}} \hat{F}(V')\,dV' \tag{8}$$

$$U \leq Y \leq \bar{V} \Bigg\}, \tag{9}$$

where the functional dependence on $\bar{V}$ is again suppressed. Equation (7) is the promise keeping constraint stating the contract must deliver no less than the promise of V. Part of establishing consistency of the setup requires showing that this constraint is always binding. Equation (8) is the incentive compatibility constraint. Equation (9) represents the worker and firm participation constraints.

To begin the characterization of the optimal employment contract, Lemma 1 establishes existence and uniqueness of a solution to the problem in (6).

Lemma 1. *For a given continuous and differentiable willingness to pay offer distribution $F(\cdot)$ with support $\left[U, \hat{V}\right]$, there exist for any $\bar{V} \in \left[U, \hat{V}\right]$ a unique solution $\Pi(V)$ to equation (6), for any $V \in \left[U, \bar{V}\right]$. Furthermore, if $\Pi(V)$ is concave in V then it is also differentiable.*

Proof. All the arguments in the proof are conditional on the firm's willingness to pay $\bar{V} \in \left[U, \hat{V}\right]$. For notational convenience, the functional dependence on $\bar{V}$ is suppressed. Equation (6) can also be stated in the form of the functional mapping $T(\Pi)(V)$ by,

$$T(\Pi)(V) = \max_{(w,Y,s)\in\Gamma(V)} \Bigg[p - w +$$

$$\beta(1-d)\left\{ s \int_Y^{\bar{V}} \Pi(V)\,dF(V) + \left[1 - s\hat{F}(Y)\right]\Pi(Y) \right\} \Bigg]. \tag{10}$$

By Blackwell's sufficient conditions, this is a contraction. The profit function is bounded above by the level,

$$\bar{\Pi} = \frac{p}{1 - \beta(1-d)}.$$

$s = 0$ maximizes joint surplus between the worker and the firm in this simple version of the problem without ex ante rent extraction. In addition, a worker hired out of unemployment is the smallest utility promise the firm can ever face, $V = U$, and simply just giving the worker a flat wage $w = 0$ until the match ends is the cost minimizing way to meet the utility promise of $V = U$. This will give the firm an expected profit of $\bar{\Pi}$. The constraints in $\Gamma\left(V|\bar{V}\right)$ for any $V \geq U$ are more restrictive and result in lower profits than $\bar{\Pi}$. Profits are bounded below by,

$$\underline{\Pi} = \frac{p - u^{-1}\left(\bar{V}\left[1-\beta\left(1-d\right)\right] - \beta dU\right)}{1-\beta\left(1-d\right)},$$

which is the profit level for a perpetually flat wage contract with no search at the utility promise $V = \bar{V}$. This is the most restrictive utility promise and a feasible contract choice which may or may not be dominated by the optimal design choice.

To establish monotonicity, consider two functions $\Pi^1(V) \geq \Pi^0(V)$ for any $V \in \left[U, \bar{V}\right]$. Let (w^0, Y^0, s^0) be a maximand of (10) given $\Pi^0(\cdot)$. In that case we get,

$$\begin{aligned}
T\left(\Pi^0\right)(V) &= p - w^0 + \beta(1-d)\left\{s^0\int_{Y^0}^{\bar{V}}\Pi^0(V)\,dF(V) + \left[1 - s^0\hat{F}\left(Y^0\right)\right]\Pi^0(Y)\right\} \\
&\leq p - w^0 + \beta(1-d)\left\{s^0\int_{Y^0}^{\bar{V}}\Pi^1(V)\,dF(V) + \left[1 - s^0\hat{F}\left(Y^0\right)\right]\Pi^1(Y)\right\} \\
&\leq \max_{(w,Y,s)\in\Gamma(V)}\Bigg[p - w + \\
&\quad \beta(1-d)\left\{s\int_{Y}^{\bar{V}}\Pi^1(V)\,dF(V) + \left[1 - s\hat{F}(Y)\right]\Pi^1(Y)\right\}\Bigg] \\
&= T\left(\Pi^1\right)(V).
\end{aligned}$$

The first inequality comes from the assumption $\Pi^1(V) \geq \Pi^0(V)$. The second inequality follows from optimality and the fact that (w^0, Y^0, s^0) must also be a part of the set of feasible choices given Π^1. Hence, monotonicity is satisfied. Turning to discounting, consider an upward shift of a given function by $a > 0$,

$$\begin{aligned}
T(\Pi + a)(V) = & \max_{(w,Y,s)\in\Gamma(V)} \Bigg[p - w + \beta(1-d)\Bigg\{s\int_Y^{\bar{V}} \Pi(V)\,dF(V) \\
& + \left[1 - s\hat{F}(Y)\right]\Pi(Y) + a\left[1 - s\hat{F}\left(\bar{V}\right)\right]\Bigg\}\Bigg] \\
\leq & \max_{(w,Y,s)\in\Gamma(V)} \Bigg[p - w + \beta(1-d)\Bigg\{s\int_Y^{\bar{V}} \Pi(V)\,dF(V) \\
& + \left[1 - s\hat{F}(Y)\right]\Pi(Y)\Bigg] + \beta(1-d)\,a \\
= & \, T(\Pi)(V) + \beta(1-d)\,a \\
< & \, T(\Pi)(V) + \beta' a,
\end{aligned}$$

for some $\beta' \in (\beta(1-d), 1)$. Since $\beta(1-d) < 1$, such a β' exists and hence, discounting is satisfied. Thus, Blackwell's sufficient conditions for a contraction are satisfied. Therefore by the contraction mapping theorem, there exists a unique fixed point to the mapping in equation (10), which is then the unique solution to the problem in equation (6).

Given concavity of the solution to (6) it is also differentiable. The proof is a straightforward application of Benveniste and Scheinkman reproduced as Theorem 4.10 in Stokey and Lucas (1989). Take the solution to (6) at some point $V_0 \in \left[U, \bar{V}\right]$, $\Pi(V_0)$. Let (w_0, s_0, Y_0) be the maximand of the problem and let $\tilde{V}_0 = w_0 - \hat{c}(s_0) + \beta\Big\{dU + (1-d)\Big[s_0 \int_{Y_0}^{\bar{V}} \hat{F}(V')\,dV' + Y_0\Big]\Big\}$. Define D as a neighborhood around V_0. Then, define the function for $V \in D$,

$$\widetilde{\Pi}(V) = p - \tilde{w}(V) + \beta(1-d)\left\{s_0\int_{Y_0}^{\bar{V}} \Pi(V)\,dF(V) + \left[1 - s_0\hat{F}(Y_0)\right]\Pi(Y_0)\right\},$$

where

$$\tilde{w}(V) = u^{-1}\left(\tilde{V}_0 + (V - V_0) + \hat{c}(s_0) - \beta\left\{dU + (1-d)\left[s_0\int_{Y_0}^{\bar{V}} \hat{F}(V')\,dV' + Y_0\right]\right\}\right).$$

Sufficient conditions for existence of $\tilde{w}(V)$ can be something like $\lim_{w\to-\infty} u(w) = -\infty$ and $\lim_{w\to\infty} u(w) = \infty$, but is much stronger than what is needed anywhere else in the proofs. By construction, the wage $\tilde{w}(V)$ in combination with search intensity s_0 and continuation promise Y_0 will give the worker exactly current lifetime utility promise V. Given concavity of $u(\cdot)$ this implies that $\widetilde{\Pi}(V)$ is a concave function in V and by Assumption 1, it is differentiable in V. Furthermore, since $(\tilde{w}(V), Y_0, s_0) \in \Gamma(V)$, by optimality $\widetilde{\Pi}(V) \leq \Pi(V)$ and $\widetilde{\Pi}(V_0) = \Pi(V_0)$. Hence, by Benveniste and Scheinkman it follows that $\Pi(V_0)$ is differentiable at V_0. □

Lemma 2 provides the key first conditions that the optimal contract must satisfy.

Lemma 2. *Assume the profit function $\Pi(V)$ is differentiable. Under assumptions 1-4, for a given willingness to pay $\bar{V} \in \left[U, \bar{V}(1)\right]$ such that $\Pi\left(\bar{V}\right) = 0$, an optimal contract given by wages $w(V)$, search intensity $s(V)$, and next period's lifetime utility promise $Y(V) \in \left[U, \bar{V}\right]$ must for any $V \in \left[U, \bar{V}\right]$ satisfy the conditions,*

$$\nu(V) = \frac{1}{u'(w(V))} \tag{11}$$

$$s(Y(V)) = \hat{c}'^{-1}\left(\beta(1-d)\int_{Y(V)}^{\bar{V}} \hat{F}(V')\,dV'\right) \tag{12}$$

$$\mu(Y(V)) = \frac{-\beta(1-d)\int_{Y(V)}^{\bar{V}} \Pi'(V')\hat{F}(V')\,dV'}{\hat{c}''(\lambda(Y(V)))} \tag{13}$$

$$\nu(V) + \Pi'(Y(V)) = -\Psi(Y(V)) \tag{14}$$

$$\Psi(Y(V)) = \frac{\mu(Y(V))\hat{F}(Y(V))}{1 - s(Y(V))\hat{F}(Y(V))} \tag{15}$$

where $\nu(V)$ and $\mu(V)$ are the Lagrange multipliers on the (7) and (8) constraints, respectively. By Assumption 2, the promise keeping constraint is always binding, that is, $\nu(V) > 0$ for all $V \in \left[U, \bar{V}\right]$. In addition, the incentive compatibility constraint is binding for any $Y(V) < \bar{V}$ but is not binding for $Y(V) = \bar{V}$, that is $\mu(Y(V)) \geq 0$ with strict inequality for $Y(V) < \bar{V}$ and equality for $Y(V) = \bar{V}$.

Proof. Equations (11)-(14) are the first order conditions of the Lagrangian associated with the problem in (6) with respect to w, s, and Y, respectively. By assumption 2, the promise keeping constraint is always strictly binding, $\nu(V) > 0$, since the first derivatives of the utility function are bounded. The incentive compatibility constraint (12) implies that $s(Y(V)) > 0$ for all $Y(V) \in [U, \bar{V})$ and by equation (13) and Assumption 4 it follows that $\mu(Y(V)) > 0$ for all $Y(V) \in [U, \bar{V})$. For $Y(V) = \bar{V}$, and since by construction $\Pi\left(\bar{V}\right) = 0$ it follows that s does not enter into the profit function. At zero profits, the firm is indifferent whether the match ends soon or later due to worker search. Hence, the firm will want to use the worker's search intensity only to create slack in the promise keeping constraint, which is perfectly aligned with the worker's incentive compatibility constraint of maximizing lifetime utility. Hence, the incentive compatibility constraint (8) is not a binding constraint in this case, $\mu\left(\bar{V}\right) = 0$. ◻

Proposition 1 establishes that $\Pi(V)$ must be strictly decreasing and strictly concave in V over the support, $V \in \left[U, \bar{V}\right]$. In order to establish concavity, the proof makes the sufficient condition that the second derivative of the cost function be decreasing in the search intensity choice, as summarized in assumption 5:

Assumption 5. *For any $s_1 > s_0 \geq 0$, the search cost function is such that $\hat{c}''(s_1) \leq \hat{c}''(s_0)$.*

The assumption is sufficient (but not necessary) to ensure that $\Psi(\cdot)$ is decreasing which is by itself stronger than necessary to establish concavity. Assumption 5 is somewhat heavy handed, but it has the advantage of being simple and it provides a sufficient condition for the nice characteristic that $\mu(V)$ will be everywhere decreasing in V, that is, the incentive compatibility constraint will be less binding the higher the utility promise.[5] With concavity established, Proposition 1 provides the key result that there exists a unique optimal employment contract, that it backloads wages and that lifetime utility is increasing in tenure as it goes toward the firm's willingness to pay.

Proposition 1. *Given assumptions 1-5, there exists for any productivity $p \in [0,1]$ firm a unique profit function $\Pi\left(V|\bar{V}(p)\right)$ as defined in equation (6) where $\bar{V}(p)$ is the firm's willingness to pay. The profit function is differentiable, strictly decreasing and strictly concave over the support $V \in \left[U, \bar{V}(p)\right]$, with $\Pi\left(\bar{V}(p)|\bar{V}(p)\right) = 0$. The willingness to pay is given by, $\bar{V}(p) = \left[u(p) + \beta d U\right] / \left[1 - \beta(1-d)\right]$. Wages are backloaded, $w(V) \leq w(Y(V))$, with strict inequality for $V < \bar{V}(p)$.*

Proof. The proof uses the corollary to the contraction mapping theorem that if a contraction mapping $T : X \to X$ maps functions in the subset $S \subseteq X$ into a subset $S' \subset S$, then the

[5] A third order derivative condition also figures prominently in Rogerson (1985) and also in relation to establishing concavity/convexity. However, the contexts and mechanisms are quite different and seem to shed little light on each other. In addition, simulations suggest that Assumption 5 is far from necessary in establishing concavity. Examples are available upon request.

unique fixed point $v = Tv$ must belong to S', that is $v \in S'$. In this case S will be the set of weakly decreasing and weakly concave functions and S' will the be set of strictly decreasing and strictly concave functions and X is the set of continuous and bounded functions.

For a given willingness to pay $\bar{V}$, the profit function $\Pi(V)$ is the unique fixed point to the contraction mapping in equation (10). Take some decreasing and concave function $\Pi(V)$ over the support $V \in \left[U, \bar{V}\right]$. Furthermore, assume $\Pi(V)$ is differentiable. In this case, the solution to the maximization problem on the right hand side of equation (10) is characterized by equations (11)-(15) in Lemma 2. It is straightforward to show that the second order sufficient conditions for a concave problem are also satisfied in this case, so the first order equations define a unique maximizer. In addition, by the envelope theorem, it immediately follows that,

$$\frac{\partial T(\Pi)(V)}{\partial V} = -\nu(V),$$

where $-\nu(V)$ is the Lagrange multiplier on the promise keeping constraint as defined in Lemma 2. Thus, it follows that,

$$\frac{\partial T(\Pi)(V)}{\partial V} = \frac{-1}{u'(w(V))} < 0, \tag{16}$$

where the strict inequality follows from Assumption 2. Hence, $T(\Pi)(V)$ is strictly decreasing in V. In addition it must be that $T(\Pi)(V)$ is strictly concave, that is $\partial T(\Pi)(V)/\partial V$ is strictly decreasing in V. Assume to the contrary that there exist some $U \leq V_0 < V_1 \leq \bar{V}$ such that $\partial T(\Pi)(V_0)/\partial V_0 \leq \partial T(\Pi)(V_1)/\partial V_1$. By equation (16) this implies $w(V_0) \geq w(V_1)$. Since the promise keeping constraint is always binding the continuation utility promises must satisfy,

$$V = u(w(V)) + S(Y(V)) + \beta(1-d)Y(V) + \beta dU, \tag{17}$$

where $S(V) = \max_s \left[\beta(1-d)s\int_V^{\bar{V}} \hat{F}(V')\,dV' - \hat{c}(s)\right]$ is strictly decreasing in V by Assumptions 3 and 4. By the promise keeping constraints it follows that it must be that $Y(V_0) < Y(V_1)$. By equations (14) and (16) the continuation utility promise must satisfy,

$$\frac{\partial T(\Pi)(V)}{\partial V} - \Pi'(Y(V)) = \Psi(Y(V)).$$

From this one can make the following series of arguments,

$$\begin{aligned}\frac{\partial T(\Pi)(V_0)}{\partial V_0} &= \Psi(Y(V_0)) + \Pi'(Y(V_0)) \\ &\geq \Psi(Y(V_0)) + \Pi'(Y(V_1)) \\ &> \Psi(Y(V_1)) + \Pi'(Y(V_1)) \\ &= \frac{\partial T(\Pi)(V_1)}{\partial V_1},\end{aligned}$$

where the first inequality follows from concavity of $\Pi(V)$. The second inequality comes from Assumptions 3 and 5. Assumption 3 ensures that there is positive mass over all of the offer distribution support and so as the continuation utility promise strictly increases, the probability that the worker receives a better offer strictly declines. Consequently, the worker will search strictly less. Assumption 5 is sufficient to guarantee that a strict increase in the continuation utility promise is associated with a strictly less binding incentive compatibility constraint in the sense that $\mu(Y(V))$ is strictly declining in $Y(V)$. In total this implies that $\Psi(Y(V))$ is strictly declining in $Y(V)$. Hence, one obtains that $\partial T(\Pi)(V_0)/\partial V_0 > \partial T(\Pi)(V_1)/\partial V_1$ which is a contradiction. Hence, it must be $T(\Pi)(V)$ is strictly concave over the support $V \in [U, \bar{V}]$.

By the Benveniste and Scheinkman argument in the proof of Lemma 1, as long as $T(\Pi)(V)$ is concave it is also differentiable. By the corollary to the contraction mapping theorem, since $T(\Pi)$ maps decreasing and concave functions into strictly decreasing and strictly concave functions, the fixed point must be strictly decreasing and strictly concave because the set of weakly increasing and weakly concave functions is a closed subset of the set of bounded and continuous functions. By the fact that concavity is mapped into concavity always, it follows that differentiability is also maintained in the iteration of the mapping as it converges to the fixed point. Therefore it must be that the profit function $\Pi(V)$ defined in (6) must be strictly decreasing, strictly concave, and differentiable over the support $V \in [U, \bar{V}]$.

By construction $\Pi(\bar{V}) = 0$. Furthermore, by equation (14) the continuation utility promise at the willingness to pay satisfies that $\Pi'(\bar{V}) = \Pi'(Y(\bar{V})) \Rightarrow Y(\bar{V}) = \bar{V}$. By the incentive compatibility constraint, $\lambda\left(\bar{V}\right)=0$, which gives,

$$0=\Pi\left(\bar{V}\right)=p-w\left(\bar{V}\right)+\Pi\left(\bar{V}\right)$$
$$\Downarrow$$
$$w\left(\bar{V}\right)=p.$$

Hence, from the always binding promise keeping constraint, it follows that,

$$\bar{V}\left(p\right)=u\left(p\right)+\beta\left(1-d\right)\bar{V}+\beta dU$$
$$\Updownarrow$$
$$\bar{V}\left(p\right)=\frac{u\left(p\right)+\beta dU}{1-\beta\left(1-d\right)}.$$

Therefore, the willingness to pay is strictly increasing in p. Hence, define $\hat{V}=\bar{V}\left(1\right)$, and in combination with Assumption 3 the conditions in Lemma 1 for existence and uniqueness are satisfied.

For the fixed point $\Pi\left(V\right)=T\left(\Pi\right)\left(V\right)$ it must be that,

$$\Pi'\left(V\right)-\Pi'\left(Y\left(V\right)\right)=\Psi\left(Y\left(V\right)\right)\geq 0,$$

which by Π strictly decreasing and strictly concave implies that $\bar{V}\geq Y\left(V\right)\geq V$ for all $V\in\left[U,\bar{V}\right]$ with strict inequalities for $V<\bar{V}$ and equality for $V=\bar{V}$. Furthermore, again by concavity, wages are strictly increasing in the lifetime utility promise, which delivers that $w\left(V\right)<w\left(Y\left(V\right)\right)$ for any $V<\bar{V}$ and $w\left(\bar{V}\right)=w\left(Y\left(\bar{V}\right)\right)=p$. That is, wages are strictly increasing for utility promises below the willingness to pay. The optimal contract promises increasing lifetime utility to the point of delivering exactly the firm's willingness to pay after which it is flat. The contract asymptotes to the willingness to pay in that $\bar{V}>Y\left(V\right)>V$ for any $V<\bar{V}$ by the strict concavity of the profit function and by the fact that $\Psi\left(\bar{V}\right)=0$. □

By Proposition 1 it follows that $w\left(V\right)$ is increasing in the utility promise. For lifetime utility promises strictly less than the firm's willingness to pay At the willingness to pay the worker is extracting full surplus which is delivered in a flat wage profile, $w\left(\bar{V}\left(p\right)\right)=p$ with $Y\left(\bar{V}\left(p\right)\right)=\bar{V}\left(p\right)$ and $s\left(\bar{V}\left(p\right)\right)=0$.

2.6 The Continuous Time Limit

The continuous time limit of the model and its solution are obtained by taking the limit $\Delta \to 0$ for the results above. In this case, the design problem can be written as,

$$[r+\delta]\,\Pi(V) = \max_{\left(w,\dot{V},\lambda\right)\in\Gamma(V)} \left[p - w + \lambda \int_V^{\bar{V}(p)} \Pi'(V')\,\hat{F}(V')\,dV' + \Pi'(V)\,\dot{V}\right],$$

where the set of feasible choices $\Gamma(V)$ is given by,

$$\Gamma(V) = \left\{ \left(w,\dot{V},\lambda\right) \in \mathbb{R}^3 \right|$$

$$u(w) - c(\lambda) + \delta U + \lambda \int_V^{\bar{V}(p)} \hat{F}(V')\,dV' + \dot{V} \geq (r+\delta)\,V \tag{18}$$

$$c'(\lambda) = \int_V^{\bar{V}(p)} \hat{F}(V')\,dV' \Bigg\}. \tag{19}$$

Lemma 3 provides the continuous time expressions that the solution must satisfy. The lemma is left without proof since it is a simple restatement of results in Lemma 2 and Proposition 1 taking $\Delta \to 0$. In the statement of the results, the lemma makes the assumption that the second derivative of the profit function exists. If not, the law of motion $\dot{V}(V)$ defined in equation (22) is stated in terms of $\dot{\Pi}'(V)$.

Lemma 3. *Let the unique solution to the design problem in equation (6) in the continuous time limit, $\Delta \to 0$, be represented by the wage and search intensity functions, $w(V)$ and $\lambda(V)$ respectively, as well as a law of motion of the utility promise, $\dot{V}(V) = \lim_{\Delta\to 0}[Y(V) - V]/\Delta$. The contract is for a given utility promise $V \in \left[U, \bar{V}(p)\right]$ characterized by,*

$$\Pi'(V) = \frac{-1}{u'(w(V))} \tag{20}$$

$$c'(\lambda(V)) = \int_V^{\bar{V}} \hat{F}(V')\,dV' \tag{21}$$

$$\dot{V}(V) = \frac{\hat{F}(V) \int_V^{\bar{V}} \Pi'(V')\,\hat{F}(V')\,dV'^{\prime}}{c''(\lambda(V))\,\Pi''(V)} \tag{22}$$

$$(r+\delta)\,V = u(w(V)) - c(\lambda(V)) + \delta U + \lambda(V) \int_V^{\bar{V}} \hat{F}(V')\,dV' + \dot{V}(V) \tag{23}$$

$$(r+\delta)\,\Pi(V) = p - w(V) + \lambda(V) \int_V^{\bar{V}} \Pi'(V)\,\hat{F}(V)\,dV + \Pi'(V)\,\dot{V}(V) \tag{24}$$

$$U = \frac{u(0)}{r} \tag{25}$$

$$(r+\delta)\,\bar{V}(p) = u(p) + \delta U, \tag{26}$$

where the willingness to pay distribution is defined by $F(V) = \tilde{F}\left(\bar{V}^{-1}(V)\right)$.

2.7 Savings

The framework assumes no savings and borrowing, which simplifies the analysis considerably. Since an obvious solution to the moral hazard problem in the paper is for the worker to buy the match up front (which is ruled out by the assumption of no side payments), it is not surprising that the firm would adopt a more aggressive backloading strategy if the worker can smooth consumption by use of already accumulated savings. In the extreme, if the worker has enough savings, the optimal backloading strategy would resemble an up front payment for the job and then quickly settle into a flat wage scheme equalling the output of the match. This would resolve the inefficiency that results from the worker searching to extract rents from the current relationship. However, this presumes a situation where the worker has made past consumption sacrifices to accumulate savings. And if indeed the worker were to receive rents from eliminating or reducing the inefficiencies in future relationships, the worker would be motivated to make consumption sacrifices to accumulate savings. However, in the baseline version of the model the rents go to the firm, and as such, the worker has no incentive to accumulate savings for the purpose of eliminating inefficiencies due to hidden search. Therefore the worker has no incentives to accumulate savings for the purpose of eliminating search inefficiencies. Of course, one can imagine introducing other incentives to save, such as life cycle savings, and one can possibly obtain a quantitative impact on the baseline analysis this way. But this is beyond the scope of this paper.

In the next section, the analysis will consider wage mechanism deign constraints such as minimum wages that will result in ex ante rent extraction by the worker. A bargaining setup as in Cahuc et al. (2006) also has the feature that the worker can extract rents from a future match over and above the total match surplus in the current match. In those setups, the worker will receive some fraction of the increased rents due to accumulated savings. However, only in the case where the worker receives all the rents from a future match will savings not be under-accumulated.

3 Design Constraints

It is noteworthy that in the core problem, the worker's lack of ability to commit ex ante to a search intensity level does not by itself result in any rent extraction by the worker. Any rents that flow to the worker are purely a result of the offer matching process. For example, an unemployed worker has in the core problem no gains from search as a future employer will offer a contract that exactly matches the value of unemployment, and no more. The same is true for any worker who is currently extracting full rents from a given employment relationship. This changes with the imposition of constraints on the choice of wage path in the contract. In the following, I consider two constraints: (1) A minimum wage, that is the wage must at any point in time t be greater than some level, $w_t \geq \underline{w}$, and (2) a constraint on the change in wages, specifically impose that the wage profile must be flat, $\dot{V}_t = 0$.

The design constraints that are considered in this section all have the implication that the promise keeping constraint is no longer necessarily binding for all utility promises, that is, it is possible that $\nu(V) = 0$ for some $V \in \left[U, \bar{V}\right]$. This modifies the worker's valuation of search since the worker may receive more lifetime utility from an employer than what is dictated by the Bertrand competition between firms. For this purpose, define the object $\underline{V}(p)$ as the least lifetime utility an employer will want to give a worker irrespective of the worker's outside option. Since the worker always has unemployment as the outside option, if that option is binding, set $\underline{V}(p)$ equal to the value of unemployment, $\underline{V}(p) = \max(\underline{V}(p), U)$. For expositional purposes, also define the minimum utility promise in terms of a firm's willingness to pay rather than its productivity, $\tilde{\underline{V}}\left(\bar{V}\right) = \underline{V}\left(\bar{V}^{-1}\left(\bar{V}\right)\right)$. With this object, the worker's expected lifetime utility in equation (1) is modified as follows,

$$
\begin{aligned}
V_t = u(w_t) - \hat{c}(s_t) + \beta dU + \beta(1-d)\Bigg[s_t \int_{V_{t+1}}^{\bar{V}} V dF(V) + \\
s_t \int_{\bar{V}}^{V(1)} \max\left[\bar{V}, \tilde{\underline{V}}(V)\right] dF(V) + \left[s_t F(V_{t+1}) + (1-s_t)\right] V_{t+1} \Bigg] \\
= u(w_t) - \hat{c}(s_t) + \beta dU + \beta(1-d)\Bigg[s_t \Bigg[\int_{V_{t+1}}^{\bar{V}} \hat{F}(V) dV + \\
\int_{\bar{V}}^{V(1)} \max\left[0, \tilde{\underline{V}}(V) - \bar{V}\right] dF(V) \Bigg] + V_{t+1} \Bigg].
\end{aligned} \tag{27}
$$

Consequently, the incentive compatibility constraint becomes,

$$\hat{c}'(s_t) = \beta(1-d)\left[\int_{V_{t+1}}^{\bar{V}} \hat{F}(V)\,dV + \int_{\bar{V}}^{V(1)} \max\left[0, \underline{\tilde{V}}(V) - \bar{V}\right] dF(V)\right]. \tag{28}$$

The modified incentive compatibility constraint highlights that the worker's access to hidden search may lead to ex ante rent extraction given particular constraints on the wage contract. Thus, even though firms make take-it-or-leave-it offers to unemployed workers, there may still be strictly positive returns to unemployed search. And by the same token, the jointly efficient search within a given match may not be zero, since the worker may be able to extract rents from an outside firm in excess of the maximal rents of the current relationship. This is in contrast to the standard Postel-Vinay and Robin (2002) setup, where all rents to match creation fall to the firms.

Flinn and Mabli (2009) also argue the issue of ex ante rent extraction associated with minimum wages in a Cahuc et al. (2006) setup with assumed flat wage profiles and exogenous offer arrival rates. Common to their analysis and to the one in this paper is the firm expectation that its workers will on occasion meet outside employers and the subsequent contract renegotiation will result in either wage increases or separation. In the Cahuc et al. (2006) paper, the flat wage contract is then set so as to match whatever match surplus division is dictated by the Bertrand competition and wage bargaining at the outset of the match. InFlinn and Mabli (2009) this wage may be below the minimum wage in which case wages are set as a corner solution at the minimum wage and the worker ends up with a greater match surplus share than that dictated by the competitive pressures in combination with the standard bargaining.

This has similarities to what happens in the analysis at hand. However, the incentive compatibility constraint interacts with lower wage bound in a different way. The worker's implicit threat to search incents the firm to promise a greater utility level from the next period. It is an interesting aspect of the analysis that the inability to commit to a search intensity level does not give the worker any match rents in the baseline case in the model. But this is because the firm can simply drop the current wage freely with no adverse effects on behavior. The minimum wage limits this mechanism and sets the stage for the result that the worker's inability to commit to a particular search intensity effectively delivers rents to the worker.

3.1 Minimum wages

A minimum wage can be imposed on the problem in the form of $w_t \geq \underline{w}$ or it can be a consequence of a utility function specification that violates assumption A2. A utility function that has the characteristic that $\lim_{w \to \underline{w}^+} u'(w) = -\infty$ will have a similar impact on the optimal contracting problem as the simple minimum wage constraint as above. So, to keep things simple, I will maintain assumption A2 and let minimum wage constraints be expressed in the form of $w_t \geq \underline{w}$. In this case, the firm's optimal contract design problem is still given as in equation (6), but the constraint set is modified by,

$$\begin{aligned}
\Gamma\left(V|\bar{V}\right) = \Bigg\{ & (w, Y, s) \in \mathbb{R}^2 \times [0,1] \,| \\
& u(w) - \hat{c}(s) + \Bigg\{ dU + (1-d)\left[s\left[\int_Y^{\bar{V}} \hat{F}(V)\, dV + \right.\right. \\
& \left.\left. \beta \int_{\bar{V}}^{V(1)} \max\left[0, \underline{\tilde{V}}(V) - \bar{V}\right] dF(V) \right] + Y \right] \Bigg\} \geq V && (29) \\
& \frac{\hat{c}'(s)}{\beta(1-d)} = \int_Y^{\bar{V}} \hat{F}(V')\, dV' + \int_{\bar{V}}^{V(1)} \max\left[0, \underline{\tilde{V}}(V') - \bar{V}\right] dF(V') && (30) \\
& w \geq \underline{w} && (31) \\
& U \leq Y \leq \bar{V} \Bigg\}. && (32)
\end{aligned}$$

In this case one obtains the first order conditions,

$$\begin{aligned}
-1 + \nu(V)\, u'(w(V)) + \eta(V) = 0 & \\
\Pi'(V) = & -\nu(V) \\
\mu(Y(V)) = & \frac{-\beta(1-d)\int_{Y(V)}^{\bar{V}} \Pi'(V')\, \hat{F}(V')\, dV'}{\hat{c}''(s(Y(V)))} \\
\nu(V) + \Pi'(Y(V)) = & -\Psi(Y(V)) \\
\Psi(Y(V)) = & \frac{\mu(Y(V))\, \hat{F}(Y(V))}{1 - s(Y(V))\, \hat{F}(Y(V))}.
\end{aligned}$$

If the promise keeping constraint is not binding, one obtains that $\nu(V) = 0 \Rightarrow \Pi'(V) = 0$. The continuation utility promise is the solution to the equation,

$$\Pi'(Y(V)) = -\Psi(Y(V)) < 0,$$

which means that the continuation utility will put the problem into a strictly decreasing part of the profit function. It also follows that $\eta(V) = 1$ and then, necessarily, $w(V) = \underline{w}$. The search intensity is directly determined by the continuation utility choice,

$$s(Y(V)) = \hat{c}'^{-1}\left(\beta(1-d)\left[\int_{Y(V)}^{\bar{V}} \hat{F}(V')\,dV' + \int_{\bar{V}}^{\hat{V}} \max\left[0, \underline{\tilde{V}}(V') - \bar{V}\right] dF(V')\right]\right).$$

The resulting actual lifetime utility that is provided in this case is the minimum lifetime utility the firm will want to provide to the worker,

$$\underline{\tilde{V}}\left(\bar{V}\right) = u(\underline{w}) + S(Y(V)) + \beta\left[dU + (1-d)Y(V)\right], \tag{33}$$

where,

$$S(Y) = \max_{s}\left[-\hat{c}(s) + \beta(1-d)s\left[\int_{Y}^{\bar{V}} \hat{F}(V)\,dV + \int_{\bar{V}}^{\hat{V}} \max\left[0, \underline{\tilde{V}}(V) - \bar{V}\right] dF(V)\right]\right].$$

Minimum wages are part of the institutional makeup of many labor markets and if nothing else one would typically want to impose non-negativity of wages on the analysis. The framework will have the implication that across a range of different types of firms wage contracts can turn out to start at the minimum wage. While the worker will spend only one period at the minimum wage the dynamics away from it may be slow enough to generate a resemblance of a mass point at the minimum wage, consistent with data.

3.2 Flat wage contracts

Given the focus in the existing literature on flat wage contracts, one can consider the impact of constraining wages to be flat until renegotiated. In this case, the design problem is as in Section 3.1 except that the continuation utility is eliminated from the set of control variables and the constraint set is written as,

$$\Gamma\left(V|\bar{V}\right)=\Big\{(w,Y,s)\in\mathbb{R}^2\times[0,1]\,|$$

$$Y\geq V \tag{34}$$

$$\frac{\hat{c}'(s)}{\beta(1-d)}=\int_Y^{\bar{V}}\hat{F}(V')\,dV'+\int_{\bar{V}}^{\hat{V}}\max\left[0,\underline{\tilde{V}}(V')-\bar{V}\right]dF(V') \tag{35}$$

$$u(w)-\hat{c}(s)+\beta\Big\{dU+(1-d)\Big[s\Big[\int_Y^{\bar{V}}\hat{F}(V)\,dV+\int_{\bar{V}}^{\hat{V}}\max\left[0,\underline{\tilde{V}}(V)-\bar{V}\right]dF(V)\Big]+Y\Big]\Big\}=Y \tag{36}$$

$$U\leq Y\leq\bar{V}\Big\}. \tag{37}$$

In this case, the promise keeping constraint (34) may not be binding, as the firm trades off giving the worker more utility in return for a lower search intensity and greater overall joint match surplus. Again, the design constraint allows the worker to benefit from the inability to commit to a search intensity.

4 Socially Efficient Search

The Mortensen rule, Mortensen (1982), of efficient search dictates that the initiator or "match maker" of a match also receive all the surplus from it. In the partial setting that I have worked with so far, workers are fully responsible for the creation of matches, yet in the no ex ante rent extraction version of the model all the rents associated with unemployed search go to the firms.[6] Clearly, workers in this state must search too little from a social point of view. It is actually quite bad: If the model does not provide the unemployed worker with a costless strictly positive base level of offer arrivals, unemployment becomes a trap and the steady state economy collapses to zero employment. On the other hand, consider a worker currently employed with the most productive firm. Any search by this worker is socially inefficient since it cannot result in the creation of a more productive match. Yet, as long as the worker is receiving less than all of the rents from the match, he or she will engage in a strictly positive level of search, which is socially too much. Thus, it should be clear that the economy will not be socially efficient. This section will document the extent and the sources

[6] This is also true for the search by workers who are extracting all the rents from their current relationships.

of the inefficiency.

The direct application of the Mortensen rule is to give the worker the entire production flow from a match. Otherwise, constrain the social planner to not be able to smooth consumption across individuals and states. For simplicity, consider the continuous time version of the model. Let the social welfare criterion for a given state of the economy, $G(p)$, be given by,

$$\mathcal{W} = \max \int_0^1 V(p)\, dG(p),$$

where

$$V(p) = \max_{s(t)} E\left[\int_0^\infty e^{-rt}\left[u(p) - c(\lambda(t))\right]\right]$$

is the discounted stream of future utilities and $\lambda(t)$ is the choice of state conditional search intensities. The functional equation for this expression is,

$$\begin{aligned}
(r+\delta) V(p) &= \max_{\lambda}\left[u(p) - c(\lambda) + \delta V(0) + \lambda \int_p^1 \left[V(p') - V(p)\right] d\Phi(p')\right] \\
&= \max_{\lambda}\left[u(p) - c(\lambda) + \delta V(0) + \lambda \int_p^1 V'(p')\, \hat{\Phi}(p')\, dp'\right] \\
&= \max_{\lambda}\left[u(p) - c(\lambda) + \delta V(0) + \lambda \int_p^1 \frac{u'(p)\, \hat{\Phi}(p')\, dp'}{r+\delta+\hat{\Phi}(p')\, \lambda(p')}\right],
\end{aligned}$$

where the second line follows from integration by parts and the third by the envelope condition. Following the notation scheme, $\hat{\Phi}(p) = 1 - \Phi(p)$. Therefore, the social planner will for each worker dictate a state dependent search intensity characterized by the first order conditions,

$$c'(\lambda^*(p)) = \int_p^1 \frac{u'(p)\, \hat{\Phi}(p')\, dp'}{r+\delta+\hat{\Phi}(p')\, \lambda^*(p')},\ p \in [0,1].$$

$\lambda^*(p)$ is the constrained socially optimal level of search. Obviously, it is not tenure dependent. It is monotonically decreasing in the type of the firm.

If on the other hand, firms spend resources as part of the match creation process, then the Mortensen rule dictates that for search to be efficient, firms must receive some of the ex post rents from matches. As in the case of the Hosios rule, the setting calls for a specific balance in rent extraction. One can imagine labor market policies and institutions, such as for example the minimum wage as playing a possibly important role in this case.

5 Extension

So far the analysis has focused on the core mechanism and has left out what is possibly empirically important dynamics and heterogeneity. In what follows I present an application of the framework to human capital dynamics, specifically that of firm provided general human capital training. The application presents an understanding of how frictions allow for training to be provided by softening the classic hold up problem, where the worker can appropriate the rents from the training through competition from other firms for his services. It does so in a framework that allows the firm to make the worker "pay" for training up front through lower initial wages, but the instrument is limited by the worker's risk aversion. In addition, the firm can design wages so as to control the extent of the competitive pressure on the match by manipulating the worker's incentives to search for outside offers.

The model predicts that more productive firms are in a better position to provide training since the competitive pressure on them is smaller. This generates a positive correlation between accumulated search capital and the growth rate of human capital. This is an important point for the analysis of the impact of frictions and heterogeneity on wage dispersion, since human capital heterogeneity now becomes tied to firm productivity heterogeneity and frictions in terms of heterogeneity in the accumulation of search capital. The backloading of the contract due to the hidden search also introduces a dynamic that training is increasing in the worker's tenure with the firm, which is a natural because the relationship is becoming cemented and the outside competitive pressure is diminishing.

5.1 Training

Frictions soften competition over workers and may allow firms to reap the rewards from training its workers with general skills. Otherwise, the classic answer to the question of how general training is provided is for workers to pay for it up front through reduced wages. The framework in this paper is particularly well suited to study the provision of general human capital: Exposure to outside competition is in part dictated by the firm's position in the productive hierarchy, but it is also a result of search behavior which can be be affected through the design of the wage mechanism. Furthermore, the contract design allows for wage profile flexibility in tenure and considers risk averse workers.

The provision of training generally depends on the firm's position in the productivity hierarchy and the promised utility to the worker which both impact the competitive pressure on the match. In addition, the impact of the outside competition depends on the complementarity between worker skill and firm productivity in the match output function as well as the degree of concavity in the utility function.

If the match output function is supermodular in worker skill and firm productivity, then some of the rents generated from training will fall into the hands of a more productive outside firm, should the worker meet such a firm. That is not true if the production function is modular: In this case, training increases the workers productivity by exactly the same amount in all firms, and an outside firm will in the Bertrand competition with the incumbent be forced to deliver all the rents from training to the worker. Hence, in this case, all the rents from training stay within the contractual relationship between the worker and the firm providing the training, regardless of the firm's position in the hierarchy.

In the supermodular case, the intensity of the hold-up problem is greater the further down the productivity hierarchy the firm is. Thus, in isolation, one would in this case expect more productive firms to provide more training. This is the key result in Fu (2011) where the question is studied in a Burdett and Mortensen (1998) wage posting model and also assuming a supermodular production function.[7] The result is also related to shin Hwang et al. (1998) where firms produce not only an output good but also an amenity that can only be consumed by the employed worker. The correlation between wages and intensity of amenity provision depends on the assumed correlation between output efficiency and amenity production efficiency. In this particular case, the ability to take returns from provision of training is negatively correlated with the degree of outside competitive pressure, and so more training is associated with more productive firms.

Besides the possibility of rent capture by third parties, training incentives are also affected by non-smooth wages in the optimal contract: Consider the training provided by the most productive firm. Regardless of the production function characteristics, there are in such a match no concerns that rents from training will be captured by outside parties. However,

[7]Including productive heterogeneity on the worker side in the Burdett and Mortensen (1998) setup is generally difficult, but if one restricts attention to piecewise contracts in human capital efficiency units and otherwise design the model so that it is generally multiplicative in the worker's efficiency units of human capital, then the firms wage posting problem becomes one of establishing a human capital efficiency unit compensation rate. This is the approach in Fu (2011).

outside competitive pressure will force the worker's wages up over time and the provision of training accentuates this fact. The concavity of the worker's utility function limits the firm's ability to extract the productive gains from training because it is associated with a non-smooth delivery of wages. Thus, less training is provided the lower the utility promise. The risk aversion channel does however favor training provision in the more productive firms conditional on full worker rent extraction. In this case, the impact of non-smooth wages associated with training is the lowest for the most productive firms.

The insight that more productive firms will provide more training does carry through in the analysis in this section, however with modifications. How much training is provided depends on the worker's search intensity. A highly productive firm that gives little utility to its worker must expect that she is searching intensely for outside offers and therefore face greater competitive pressure. The analysis in this paper will find that training is increasing in the duration of the employment relationship. Wages and utility promises are backloaded in the contract and so a worker's search intensity is decreasing in the duration. As the relationship solidifies, the training intensity increases. More productive firms do tend to provide more training, however this conclusion is in part a result of longer employment spell durations.

5.1.1 The model

Turning to the specifics of the model, I take the continuous time framework setup above and add human capital dynamics. For expositional purposes I assume that all workers are initially identical, but it is straightforward to allow for ex ante productive worker heterogeneity as well. All workers are born inexperienced into unemployment and become experienced at some Poisson rate γ if employed, which I will also refer to as the training intensity. The training intensity γ is provided by the firm at increasing and convex training cost $\chi(\gamma)$.[8] An inexperienced worker has productivity $p_0(p)$ whereas an experienced worker's productivity is $p_1(p)$, where $p_0(p) < p_1(p)$, for all $p \in [0,1]$. At any instant there is a flow of newborns into the unemployment pool. Any worker dies at Poisson rate m. Normalize the total labor force at unity. In this case, the flow of newborns into the unemployment pool equals m.

[8] For simplicity I assume that all firms have access to the same training technology, but this could be relaxed.

Define the effective discount rate, $\rho = r + m$.

The wage contract design problem of an experienced worker is no different from that analyzed in section 2.6 since there is no more potential for human capital growth in this case. In the following, I present the contract design problem of a productivity p firm. To keep notation in the following simple, I suppress the dependence of all the values on the type of the firm since it is constant in all the expressions. Thus, the profit value function $\Pi_i(V)$ is the profit of an optimally designed contract to the productivity p firm given a utility promise of V to an experience level $i \in \{0, 1\}$ worker.

Firm p's contract design problem for an inexperienced worker is given by,

$$(\rho + \delta)\,\Pi_0(V) = \max_{\left(w,\dot{V},X,\lambda,\gamma\right)\in\Gamma_0(V)} \Bigg[p_0 - w - \chi(\gamma) + \lambda \int_V^{\bar{V}_0} \Pi_0'(V')\,\hat{F}_0(V')\,dV' \\ + \gamma\left[\Pi_1(X) - \Pi_0(V)\right] + \Pi_0'(V)\,\dot{V} \Bigg],$$

where the set of feasible choices $\Gamma_0(V)$ is given by,

$$\Gamma_0(V) = \Bigg\{ \left(w, \dot{V}, X, \lambda, \gamma\right) \in \mathbb{R}^3 \times \mathbb{R}_+^2 \Bigg| u(w) - c(\lambda) + \delta U + \\ \lambda \int_V^{\bar{V}_0(p)} \hat{F}_0(V')\,dV' + \gamma X + \dot{V} \geq (\rho + \delta + \gamma)\,V \tag{38}$$

$$c'(\lambda) = \int_V^{\bar{V}_0(p)} \hat{F}_0(V')\,dV' \tag{39}$$

$$U_1 \leq X \leq \bar{V}_1(p) \Bigg\}. \tag{40}$$

X is the continuation utility promise in case the worker becomes experienced.

The Appendix details the numerical solution algorithm to the model. The following provides some comments on the characteristics of the optimal contract. The optimal contract must satisfy the conditions,

$$\Pi_0'(V) = \frac{-1}{u'(w_0(V))} \tag{41}$$

$$\Pi_1'(X(V)) = \Pi_0'(V) - \frac{\eta_0(V)}{\gamma(V)} \tag{42}$$

$$\chi'(\gamma(V)) = \Pi_1(X(V)) - \Pi_0(V) - \Pi_0'(V)(X(V) - V) \tag{43}$$

$$\mu_0(V) c''(\lambda_0(V)) = -\int_V^{\bar{V}_0(p)} \Pi_0'(V') \hat{F}_0(V') dV' \tag{44}$$

$$c'(\lambda_0(V)) = \int_V^{\bar{V}_0(p)} \hat{F}_0(V') dV' \tag{45}$$

$$\dot{V}_0(V) = \frac{-\mu_0(V) \hat{F}_0(V)}{\Pi_0''(V)}, \tag{46}$$

where $\eta_0(V)$ is the Lagrange multiplier on the worker's participation constraint. Specifically, $\eta_0(V) > 0 \Rightarrow X(V) = U_1$. Equations (20 (41), and (42) imply that as long as the worker's participation constraint is not binding, then wages will be smooth across a jump in the worker's human capital, that is $w_1(X(V)) = w_0(V)$. As long as the worker is just the tiniest bit risk averse, the contract will have this feature. Of course, the growth rate in utility promise and wages may well jump across the skill level increase, but wages will be smooth. It is worth noting this feature because it is distinctly different from the oft used assumption of piecewise contracts in human capital.

Equation (43) states that training is chosen so that the marginal costs of training equals the gains. The gains consist of the direct increase in profit value to the firm as the worker becomes experienced, $\Pi_1(X(V)) - \Pi_0(V)$ as well as the profit value to the firm from loosening the utility promise constraint, $-\Pi_0'(V)(X(V) - V)$.

To illustrate the hedonic aspect to wages in this example, consider the wage at the maximum utility promise,

$$\begin{aligned} w_1\left(\bar{V}_1(p), p\right) &= p_1(p) \\ w_0\left(\bar{V}_0(p), p\right) &= p_0(p) - \chi\left(\gamma\left(\bar{V}_0(p), p\right)\right) + \gamma\left(\bar{V}_0(p), p\right) \Pi_1\left(Y\left(\bar{V}_0(p), p\right)\right). \end{aligned}$$

This is the particularly simple case where all the rents from the relationship are delivered to the worker. For the experienced worker, there is no training and so there is no compensating differential to wages. However, for the inexperienced worker, wages do reflect the cost of the training, $\chi\left(\gamma\left(\bar{V}_0(p), p\right)\right)$, that is being provided. However, the cost subtraction is modified by the firm's expected profit gains when the worker becomes experienced, $\gamma\left(\bar{V}_0(p), p\right)\Pi_1\left(Y\left(\bar{V}_0(p), p\right)\right)$. In principle, it is possible that the expected profits to the firm exceed the cost, in which case the worker is paid more than productivity, $p_0(p)$.

5.1.2 Steady state equilibrium

For experience level $i = 0, 1$, denote by u_i the mass of experience level i workers that are unemployed. Similarly, let e_i denote the mass of employed experience level i workers. In addition, denote by $g_i(V, p)$ the mass of experience level $i = 0, 1$ workers that are employed in productivity p firms with utility promise V. Denote by $\lambda_i = \lambda_i(U_i, 0)$ the search intensity of an unemployed experience level $i = 0, 1$ worker. Denote by $G_i(V, p)$ the associated cumulative distribution function. In steady state, worker inflows and outflows must equal each other, which implies,

$$\begin{aligned}
u_0(\lambda_0 + m) &= e_0\delta + m \\
u_1(\lambda_1 + m) &= e_1\delta \\
e_0(\bar{\gamma} + m + \delta) &= u_0\lambda_0 \\
e_1(m + \delta) &= u_1\lambda_1 + e_0\bar{\gamma}
\end{aligned}$$

where the first equation utilizes that the flow of newborns into the unemployment pool equals m by the normalization that $u_0+u_1+e_0+e_1 = 1$. Furthermore, $\bar{\gamma} = \int_0^1 \int_U^{\bar{V}_0(p')} \gamma(V', p')\, dG_0(V', p')$. The steady state match distribution is characterized by the same type of equality between inflows and outflows which implies,

$$
\begin{aligned}
u_0\lambda_0\Phi(p) = e_0\Bigg\{ & \hat{F}_0(V)\int_0^p\int_U^{\min\left[\bar{V}_0(p'),V\right]}\lambda_0(V',p')\,dG_0(V',p') \\
& + (\delta+m)\,G_0(V,p) + \int_{\bar{V}_0^{-1}(V)}^p \dot{V}_0(V,p')\,g_0(V,p')\,dp' \\
& + \int_0^p\int_U^{\min\left[\bar{V}_0(p'),V\right]}\gamma(V',p')\,dG_0(V',p')\Bigg\} \qquad (47)\\
u_1\lambda_1\Phi(p) = e_1 & \hat{F}_1(V)\int_b^p\int_U^{\min\left[\bar{V}_1(p'),V\right]}\lambda_1(V',p')\,dG_1(V',p') \\
& + e_1(\delta+m)\,G_1(V,p) + e_1\int_{\bar{V}_1^{-1}(V)}^p \dot{V}_1(V,p')\,g_1(V,p')\,dp' \\
& - e_0\int_0^p\int_U^{\bar{V}_0(p')}\gamma(V',p')\,I\left[Y_1(V',p')\le V\right]dG_0(V',p').
\end{aligned}
$$

It is worth noting that keeping track of the distribution of utility promises in the match distribution is quite straightforward in the continuation time formulation. The above expressions equalize the flows in and out of the pools of experience i workers in productivity p or less firms that are in receipt of utility V or less, $e_iG_i(p,V)$. The change in utility promise over time in the contracts results in an outflow equal to, $\int_{\bar{V}_0^{-1}(V)}^p \dot{V}_0(V,p')\,g_0(V,p')\,dp'$. Given that utility promises change smoothly over time in the optimal contracts, the only workers that can leave the pool at a given point in time, are the ones in the pool that receive exactly V, which can happen in a range of different productivity $p' \in \left[\bar{V}_0^{-1}(V),p\right]$ firms. These workers leave the pool at rate, $\dot{V}_0(V,p')$.

The Appendix details the numerical details of solving for $G_i(V,p)$. It turns out to be an almost lower triangular system and it solves rather simply by forward recursion.

5.1.3 Numerical example

To illustrate the details of the optimal contract the following provides a numerical solution of the model steady state equilibrium. Make the following specifications.

$$
\begin{aligned}
u(w) &= \frac{1-\exp(-\alpha w)}{\alpha}\\
c(\lambda) &= \frac{\left(c_0\left(\lambda-\underline{\lambda}\right)\right)^{1+c_1}}{1+c_1}\\
\chi(\gamma) &= \frac{\left(\chi_0\gamma\right)^{1+\chi_1}}{1+\chi_1},
\end{aligned}
$$

where $\underline{\lambda}$ is a "free" level of search that the worker is given reflecting an assumption that worker search is possibly not an essential input into the creation of matches. In this formulation, the search intensity choice is constrained such that $\lambda \in [\underline{\lambda}, \infty)$.[9] Assume $\Phi(\cdot)$ is a beta distribution with parameters (β_0, β_1). Let the production function for $i = 0, 1$ be given by $p_i(p) = 0.2(1+i) + p$. Table 1 provides the parameterization of the model.

Table 1: Parameterization

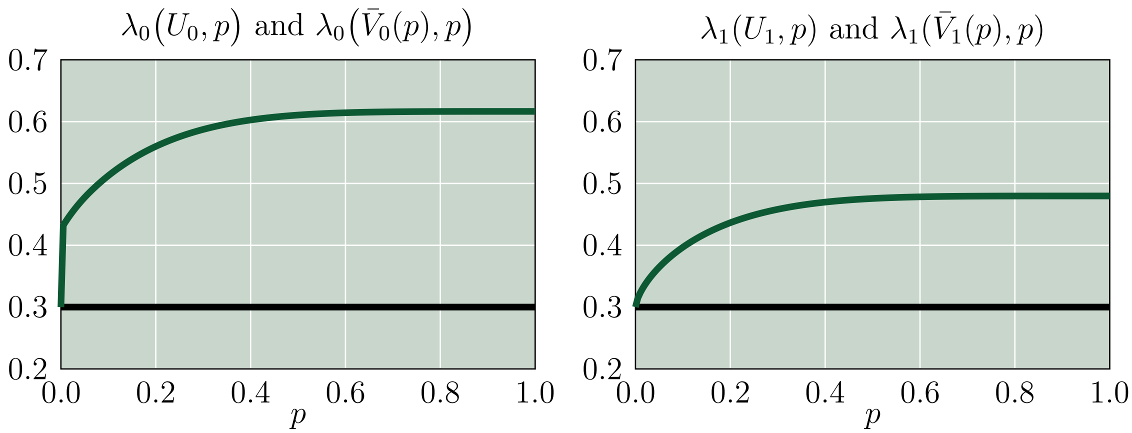
α	c_0	c_1	β_0	β_1	r	δ	m	$\underline{\lambda}$	χ_0	χ_1
3	2	1	2	3	0.05	0.2	0.02	0.3	10	3

The parameters have been chosen with the purpose of providing a useful illustration of the key mechanisms in the model more so than fitting to reasonable empirical outcomes. I have specifically chosen a parameterization which illustrates that the model can well produce negative wages, just like the basic Postel-Vinay and Robin (2002) model. Most empirical applications would impose non-negativity in which case, the parameterization in this example would involve ex ante rent extraction. That is not the focus of the current parameterization, but it illustrates that design constraints such as lower wage bounds may be important features to include in an empirical study. This of course is also the point in Flinn and Mabli (2009). On the other hand, it is also quite possible that with an empirically reasonable firm productivity support, lower wage bounds just happen to not bind.

In figure 1, I show how the optimal employment contracts dictate search intensities at the value bounds for the firm type in question. Given that the example does not have a minimum wage or other design constraints that allow for ex ante rent extraction, the worker has no incentive to search when given the firm's willingness to pay, and therefore search is just at the zero cost level, $\underline{\lambda}$. When offered less than that, the worker searches more and as can be seen, the more potential there is for rent extraction, the more the worker searches. The hardest searching worker in this example is the inexperienced worker that has just been hired directly out of unemployment into the best firm. The inexperienced worker hired directly out of unemployment searches more intensely than an experienced worker in the same situation for two reasons: The higher level of income for the experienced worker is associated with an income effect that reduces search intensity supply. In addition, a given

[9] This is a minor violation of Assumption A4, but poses no problem for the analysis.

Figure 1: Offer arrival rate at value promise support bounds.



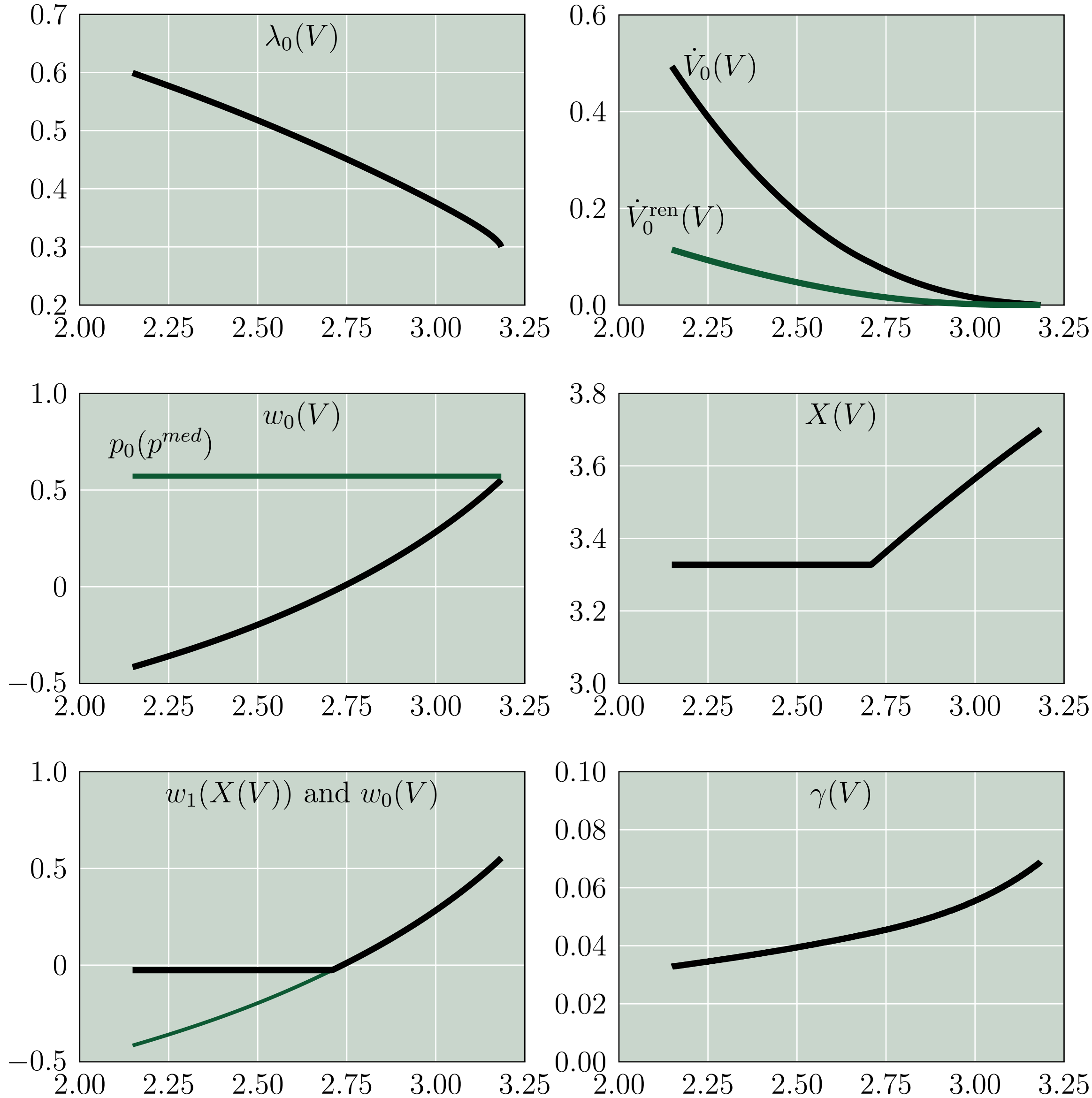
Note: Offer arrival rate at lower value support bound in green and at upper support bound in black.

firm type can also provide training to an inexperienced worker, which increases the match surplus (net of unemployment value) relative to that of an experienced worker. Thus, the inexperienced worker's search has greater rent extraction potential.

Figure 2 shows the median firm type employment contract for an inexperienced worker. The worker will start employment with the firm at some initial utility promise depending on the worker's labor market history. The left top panel shows the worker's search intensity as a function of the current utility promise. The worker's incentive to search for outside options are proportional to the possible rents the worker has yet to extract from the current match. Once at full rent extraction, the worker's search is reduced to $\underline{\lambda}$.

The top right hand panel shows the rate of change in the utility promise over tenure with the firm. If the worker meets an outside firm, it is possible that the contract renegotiation will result in a jump in the utility value with the firm to some higher level V' in which case employment will continue from that point under the new contract. The right top panel shows the two sources of utility promise growth within the worker's current job. $\dot{V}_0(V)$ is the contracted upon change in utility over time. $\dot{V}_0^{\text{ren}}(V)$ is the expected change in the inexperienced worker's lifetime utility due to contract renegotiation caused by outside offer arrival. For an experience level $i = 0, 1$ worker, expected growth due to renegotiation is

Figure 2: Median firm employment contract, inexperienced worker



Note: Lower left panel: Solid black line shows $w_1(X(V))$. The solid green line shows $w_0(V)$.

defined by,

$$\begin{aligned}\dot{V}_i^{\text{ren}}(V,p) &= \lambda_i(V,p)\int_V^{\bar{V}_i(p)}[V'-V]\,dF_i(V') \\ &= \lambda_i(V,p)\left[\int_V^{\bar{V}_i(p)}\hat{F}_i(V')\,dV' - \hat{F}\left(\bar{V}_i(p)\right)\left[\bar{V}_i(p)-V\right]\right].\end{aligned}$$

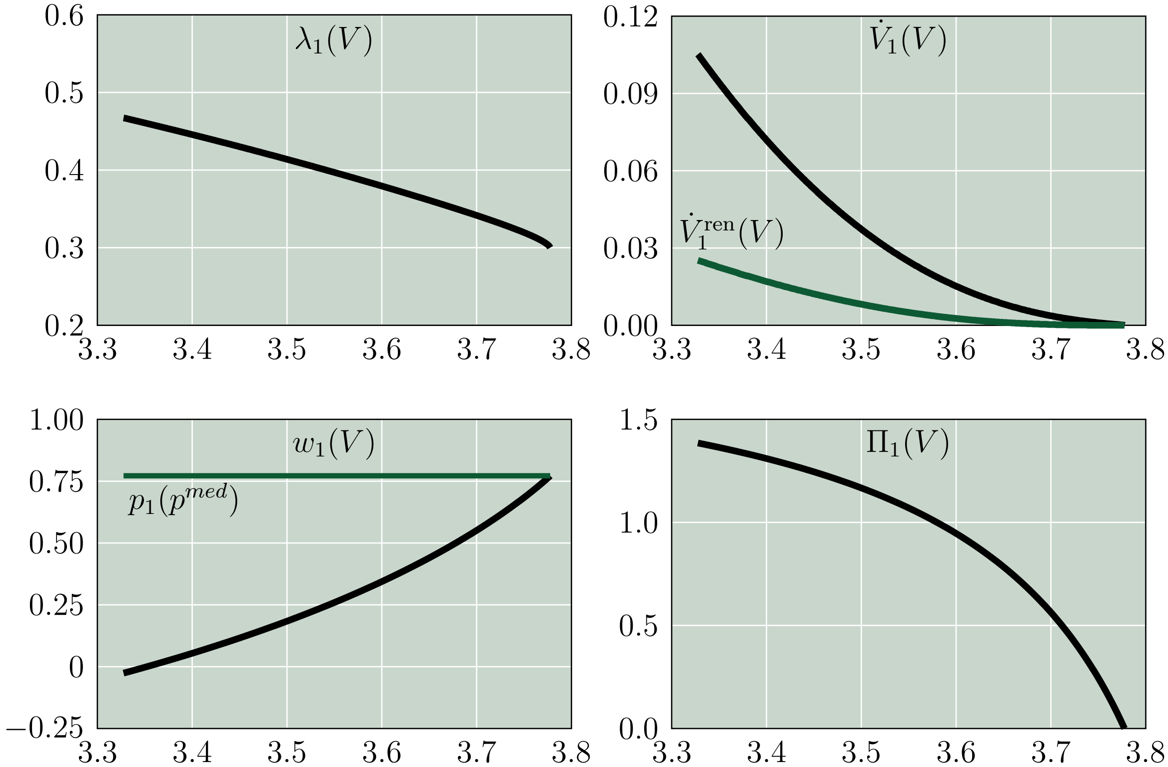
In a wage contract without back loading such as (Postel-Vinay and Robin, 2002) all of within job wage growth comes from the $\dot{V}_i^{\text{ren}}(V)$ channel. In the current example, the contract is delivering substantially more wage growth through contracted upon backloading than through the renegotiation channel. Thus, the model delivers within job wage growth motivated by the threat of contract renegotiation and wasteful search, but in this case, with much less actual outside offer matching.

The middle left panel shows the inexperienced worker wage as a function of the utility promise. It also shows the output of the match $p_0(p^{med})$. In accordance with Proposition 1, wages are increasing in the utility promise. In this case, the wage at the firm's willingness to pay is slightly less than the output of the match which reflects the costs and profit gains to the firm from providing training to the worker. The right middle panel shows the continuation utility promise as the worker moves from low skill to high skill. For low utility promises, the choice of $X(V)$ is at the lower bound, $X(V) = U_1$. The implication for the wage path as the worker moves from low skill to high skill are shown in the lower left panel. The solid green line is the wage for the inexperienced worker and the solid black line is the wage the worker receives immediately following becoming experienced. When the choice of $X(V)$ is not in a corner, it is seen that the wage does not change as the worker becomes high skill, but in the corner the wage jumps up.

The bottom right hand panel shows the training intensity which is increasing in the utility promise. For low utility promises, the worker is searching more intensely which increases the competitive pressure on the match which in turn reduces the firm's returns to training the worker. As the utility promise increases, the worker's search intensity decreases and training intensity increases.

Figure 3 shows the median firm employment contract for the experienced worker. Backloading of wages is also a central feature of the contract in this case, but less so here because the income effect has lowered the elasticity of the worker's search intensity response to the utility promise. In addition, since training is no longer valuable, the net surplus is relatively smaller. As a result, the utility promise growth rate is smaller and while not directly shown, this is also the case for the wage growth rate. As was the case for the inexperienced worker, wage growth is primarily due to the built in growth in the contract as opposed to renegotiation and the associated outside offer matching. The lower right hand panel shows the profit

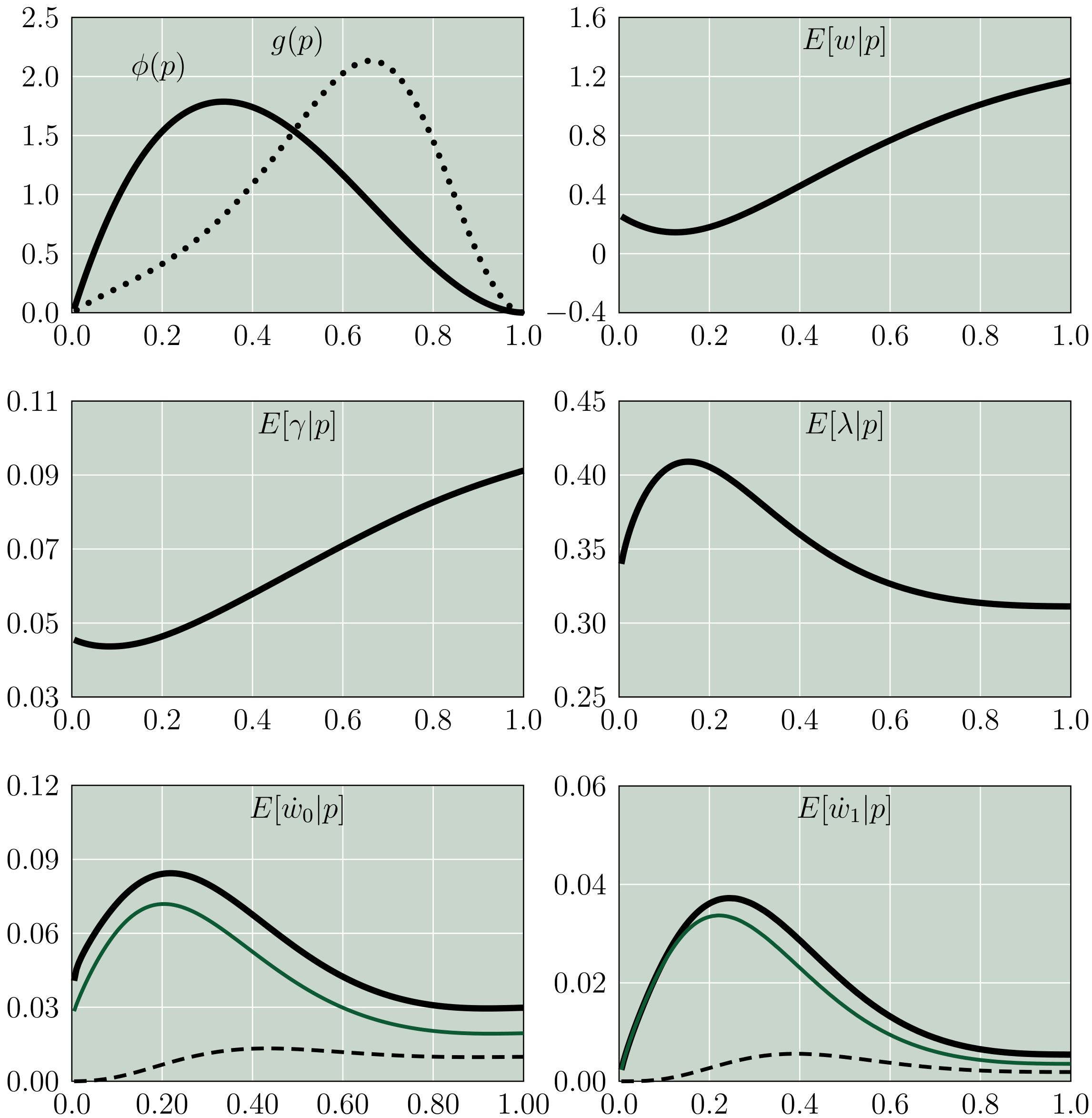
Figure 3: Median firm employment contract, experienced worker



value to the firm as a confirmation of the result in Proposition 1 that it is monotonically decreasing and concave, and takes zero value at the willingness to pay.

The following figures show some of the characteristics of the steady state. The top left panel in Figure 4 shows the firm type offer distribution $\phi(p)$ and the steady state match distribution over firm types $g(p)$, which stochastically dominates $\phi(p)$ as it must in this model. The top right panel and the two middle panels show the firm type conditional average wage, training intensity, and search intensity in steady state. In all three cases, the relationship is non-monotone, and for much the same reason: The non-monotonicity of wages in firm productivity is a general feature of the Postel-Vinay and Robin (2002) type wage setting process and is also emphasized in Bagger and Lentz (2013). If anything the backloading in the paper at hand amplifies the feature. Consider the case of an experienced worker employed by a type $p = 0$ firm. The outside option of unemployment is equal to the maximal utility promise $\bar{V}_1(0) = U_1$ that the firm can offer. Consequently, the wage contract is simply $w_1(U_1, 0) = p_1(0) = 0.4$. Now, consider the wage in matches with a

Figure 4: Steady State



parbox5.75in

Note: Lower graphs: Solid black line shows total within job wage growth rate. Dashed black line shows within job wage growth attributable to renegotiation, only. The solid green line shows within job wage growth that is directly contracted upon, only.

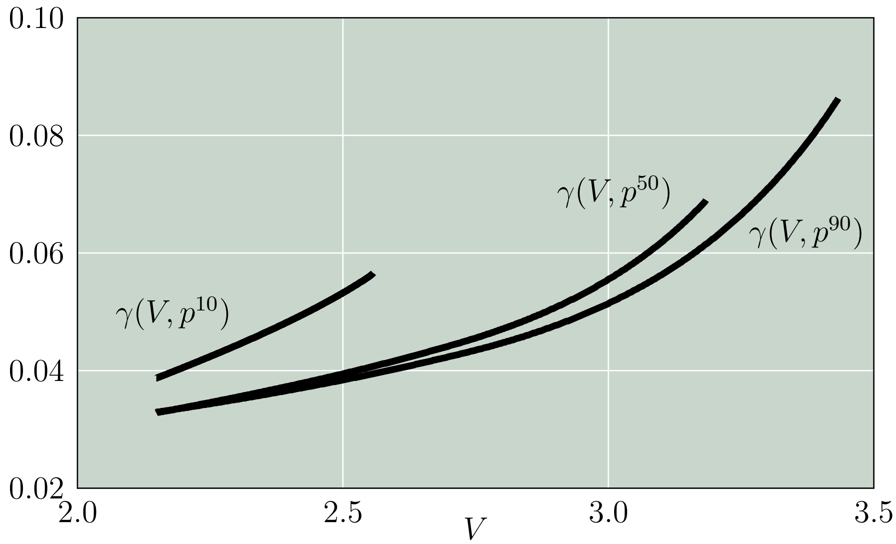
somewhat more productive firm type, $p = 0.1$. If a worker happened to obtain a bargaining position equal to the firm's willingness to pay, $\bar{V}_1(0.1)$, the worker would again be receiving a flat profile exactly equal to match output, $w_1\left(\bar{V}_1(0.1), 0.1\right) = p_1(0.1) = 0.5$, which of course exceeds that of the $p = 0$ type firm. However, this is not the typical kind of worker that is employed in this firm. The match distribution has more mass towards a bargaining position of unemployment because the arrival of outside offers most often result in the worker leaving the firm rather than staying with a renegotiated contract. Given a bargaining position of unemployment, wages are substantially lower in anticipation of higher wages with future firms *and* as part of the backloading of the wage contract with the current firm. This is also why the average worker in the $p = 0.1$ firm searches more intensely than the one in the $p = 0$ type firm. And because of the associated greater competitive pressure, the average training intensity is lower in the $p = 0.1$ firm.

Of course, this non-monotonicity should not distract the analysis from the more important effect that in general, more productive firms pay higher wages and supply more training.

The two lower panels in Figure 4 show the average firm type conditional *within job* wage growth rates for low and high skill workers. This is average observed growth rate of a randomly selected individual from a given firm conditional on the worker staying with the firm. The bold line shows the total wage growth rate. The thin solid line is the wage growth rate built into the contract, and the dashed thin line is the growth rate in wages that is contributed to renegotiation of wages due to outside offer arrivals. For the high skill workers the bold line is the sum of the growth rates of the two thin lines. For low skill workers, wages can jump due to skill increase, but only if their utility promise as a low skill worker is less than the value of unemployment as a high skill worker, in which case the firm's choice of continuation utility promise is at a corner. Otherwise, the wage path will be flat over a skill jump. Consequently, the expected total wage growth rate of a low skill worker exceeds the sum of the contractual wage growth rate and growth due to renegotiation. This is true more so for low type firms than high type firms due to the greater likelihood that wages must jump when the worker's skill jumps for these firms.

The lower panels in Figure 4 illustrate another important point: In lower productivity firms the vast majority of wage growth is built directly into the contract and almost none is due to outside offer matching. In more productive firms the two sources become more evenly split. If one were to ask a manager in a low productivity firm whether wages are set and workers are retained by matching outside offers, the answer could well be no. If a worker receives an outside offer, the outside firm as almost always better and the worker leaves. In

Figure 5: Training intensity in the 10th, 50th, and 90th percentile firms.



Note: The productivity percentiles are given by, $p^{10} = 0.045$, $p^{50} = 0.372$, and $p^{90} = 0.688$.

more productive firms the answer would be that on occasion the firm retains a worker by matching outside offers which would then result in a raise.

Finally, Figure 5 shows the training intensity in the 10th, 50th, and 90th percentile firms. The figure reveals an interesting feature of the employment contracts: For a given utility promise, training is decreasing in the productivity of the firm, which is a reflection of search intensity being increasing in firm productivity for a given utility promise. Figure 1 also makes this point. For a given utility promise, the rents that the worker can extract from the current relationship are increasing in the productivity of the firm, and so the worker searches harder. The greater exposure to outside competition lowers the firm's returns to training the worker and so it chooses a lower training intensity.

The result is not in contradiction with the generally positive correlation between training and firm productivity that is illustrated in Figure 4 because workers tend to receive greater utility with more productive firms. Figure 5 emphasizes the point that to obtain high training intensity, it is not enough to simply meet a good firm, the worker must accumulate bargaining position either through the contracted upon growth rate in the utility promise or through meetings with outside firms.

6 Concluding remarks

The paper presents a tractable framework for the study of wage dynamics with rich productive heterogeneity on both worker and firm sides and optimally chosen wage-tenure contracts. The model emphasizes the role of hidden search which gives the firm incentive to backload wage contracts so as to preempt workers from engaging in search that is primarily motivated by rent extraction from the current match.

The paper presents proof of existence and uniqueness of the wage mechanism as well as characterization of its key properties. It is shown that the design problem can be formulated as a contraction mapping which then provides the core of the proof strategies. With this, existence and uniqueness is established and the firm's profit value of the match is shown to be monotonically decreasing, concave, and differentiable in the utility promise to the worker. The contract will be backloaded meaning that the utility promise to the worker is monotonically increasing in tenure. Wages are shown to be increasing in the utility promise and the worker's search intensity is decreasing in the promise. Therefore, even in the absence of outside offer arrivals, the worker's wages will be increasing in tenure and the separation hazard will be declining.

The example in the paper demonstrates the ease with which the framework can accommodate empirically relevant accumulation processes in for example human capital. This is encouraging evidence that the framework can provide a useful foundation for the empirical study of wage dynamics. The example also has considerable independent interest in that it demonstrates a plausible mechanism for firm provided general human capital training and it links search frictions with human capital accumulation. It is shown that more productive firms will provide more training. For a given firm, training intensity is increasing in the utility promise because the higher utility promise cements the worker's position with the firm by reducing his search for outside options.

A Solution Algorithm for Continuous Time Training Model.

A.1 The optimal contract

In the continuous case, there is no contraction mapping to work with for a value function iteration scheme. Of course, one can iterate on equation (10) for very small Δ, however one would expect very slow convergence. Instead, one can adopt the following algorithm: Given the assumption that human capital can only grow, the model lends itself to a solution that starts at the experienced level where there is no training decision, and with a solution for wage contracts for experienced workers in hand, one can then proceed to solve for wage contracts for inexperienced workers.

First, determine $\bar{V}_1(p)$ and U_1. With this, the optimal contract for a given type can be solved for a given firm type independent of the other firm types (which also suggests an obvious parallelization strategy if multiple processors are available). For any given firm type, the contracting problem is initialized at the willingness to pay, $\bar{V}_1(p)$. Here it is known that,

$$\begin{aligned}
\dot{V}_1\left(\bar{V}_1(p), p\right) &= 0 \\
w_1\left(\bar{V}_1(p), p\right) &= p_1(p) \\
\lambda_1\left(\bar{V}_1(p), p\right) &= 0 \\
\Pi_1\left(\bar{V}_1(p), p\right) &= 0 \\
\Pi_1'\left(\bar{V}_1(p), p\right) &= \frac{-1}{u'\left(p_1(p)\right)}.
\end{aligned}$$

For numerical purposes, discretize the state space $\{V_{1i}\}_{i=1}^N$ such that $V_{11} = U$ and $V_{1N} = \bar{V}_1(p)$. There will be more curvature in the problem as V goes to $\bar{V}_1(p)$ and so it may make sense to adopt a grid that is denser on the right hand side, although this is not essential. Equation (21) immediately provides the search choice for any utility promise grid point $\{\lambda_{1i}\}_{i=1}^N$. It then remains to determine $\{w_{1i}, \dot{V}_{1i}, \Pi_{1i}, \Pi_{1i}'\}_{i=1}^N$. Define a distance function $D_{1i}(w) = \left(V_{1i} - \tilde{V}_{1i}(w)\right)^2$, where $\tilde{V}_{1i}(w)$ is obtained as follows,

1. Set $w_{1i} = w$.

2. By equation (20), this provides $\Pi_{1i}' = -1/u'(w_{1i})$.

3. Equation (20) also implies that $\Pi''(V) = u''(w(V))\, w'(V)/u'(w(V))^2$. Approximate $w_{1i}' \approx (w_{1i+1} - w_{1i})/(V_{1i+1} - V_{1i})$, which then provides $\Pi_{1i}'' = u''(w_{1i})\, w_{1i}'/u'(w_{1i})^2$.

4. Equation (22) now provides $\dot{V}_{1i} = \hat{F}(V_{1i}) \int_{V_{1i}}^{\bar{V}_1(p)} \Pi_1'(V') \hat{F}_1(V') dV' / [c''(\lambda_{1i}) \Pi_{1i}'']$. The integral evaluation is done by interpolation over the points $\{\Pi_{1\iota}'\}_{\iota=i}^{N}$.

5. By equation (24), this finally delivers the implied utility promise for the given wage guess, $(\rho+\delta)\tilde{V}_{1i} = u(w_{1i}) - c(\lambda_{1i}) + \delta U_1 + \lambda_{1i} \int_{V_{1i}}^{\bar{V}_1(p)} \hat{F}_1(V') dV' + \dot{V}_{1i}$.

The optimal wage contract is found such that $D_i(w_i) = 0$ for all $i = 1, \ldots, N$. Start the algorithm from $i = N-1$ and loop down to $i = 1$.

With the optimal contracts for experienced workers in hand, proceed to solve for the optimal inexperienced worker contract for a given firm type, p. First step is to determine firm p's willingness to pay, $\bar{V}_0(p)$. When the firm is delivering utility $V = \bar{V}_0(p)$ to the inexperienced worker, there is for the worker no additional surplus to extract from the match and the incentive compatibility constraint is non-binding implying that the Lagrange multiplier is zero, $\mu_0(p) = 0$. As a consequence the utility promise growth rate is also zero at the willingness to pay, $\dot{V}_0\left(\bar{V}_0(p), p\right) = 0$. Therefore, one obtains that $\bar{V}_0(p)$ satisfies,

$$(\rho+\delta+\gamma)\bar{V}_0(p) = u\left(w_0\left(\bar{V}_0(p), p\right)\right) + \delta U_0 + \gamma\left(\bar{V}_0(p), p\right) X\left(\bar{V}_0(p), p\right),$$

where

$$w_0\left(\bar{V}_0(p), p\right) = p_0(p) - \chi\left(\gamma\left(\bar{V}_0(p), p\right)\right) + \gamma\left(\bar{V}_0(p), p\right) \Pi_1\left(X\left(\bar{V}_0(p), p\right)\right),$$

which follows from the definition of the willingness to pay, $\Pi_0\left(\bar{V}_0(p), p\right) = 0$. The training rate and the utility promise at the transition from inexperienced to experienced satisfy the first order conditions,

$$\chi'\left(\gamma\left(\bar{V}_0(p), p\right)\right) = \Pi_1\left(X\left(\bar{V}_0(p), p\right)\right) + \frac{1}{u'\left(w_0\left(\bar{V}_0(p), p\right)\right)}\left[X\left(\bar{V}_0(p), p\right) - \bar{V}_0(p)\right]$$

and

$$X\left(\bar{V}_0(p), p\right) = \begin{cases} U_1 & \text{if } \Pi_1'(U_1, p) < \Pi_0'\left(\bar{V}_0(p), p\right) \\ \Pi_1'^{-1}\left(\Pi_0'\left(\bar{V}_0(p), p\right), p\right) & \text{if } \Pi_1'(U_1, p) \geq \Pi_0'\left(\bar{V}_0(p), p\right). \end{cases}$$

This is 3 equations in 3 unknowns, and can be solved for a given firm type independent of knowing the optimal contracts for any other firm type.

Given the value of $\bar{V}_0(p)$ proceed as above to define a grid $\{V_{0i}\}_{i=1}^{N}$ over the interval $\left[U_0, \bar{V}_0(p)\right]$. Again, define the distance $D_{0i}(w) = \left(V_{0i} - \tilde{V}_{0i}(w)\right)^2$ as follows,

1. set $w_i = w$.

2. By equation (**??**) this then yields $\Pi'_{0i} = u'(w_i)$.

3. Also by equation (**??**) for an interior solution one obtains, $\Pi'_1(X_i) = \Pi'_{0i}$. If at bound, then $X_i = U_1$.

4. By $\Pi_0(V) = -\int_V^{\bar{V}_0} \Pi'_0(V')\,dV'$, the value in step 2 can be used to infer Π_{0i}.

5. The training choice is then found by $\gamma_i = \chi'^{-1}\left[\Pi_1(Y_i) - \Pi_{0i} - \Pi'_{0i}(Y_i - V_{0i})\right]$.

6. Equation (**??**) also implies that $\Pi''_0(V) = u''(w_0(V))\,w'_0(V)/u'(w_0(V))^2$. Approximate $w'_i \approx (w_{i+1} - w_i)/(V_{i+1} - V_i)$, which then provides $\Pi''_{0i} = u''(w_i)\,w'_i/u'(w_i)^2$.

7. And, again, from here one can obtain the utility promise growth rate,
$$\dot{V}_{0i} = \hat{F}_0(V_i)\int_{V_i}^{\bar{V}_0} \Pi'(V')\,\hat{F}_0(V')\,dV'/\left[c''(\lambda_{0i})\,\Pi''_{0i}\right].$$

8. With all of this in hand, the implied utility promise is given by, $(\rho + \delta + \gamma)\,\tilde{V}_{0i}(w) = u(w) - c(\lambda_{0i}) + \delta U_0 + \lambda_{0i}\int_{V_{0i}}^{\bar{V}_0(p)} \hat{F}_0(V')\,dV' + \gamma_i X_i + \dot{V}_{0i}$.

The optimal contract is found by solving $D_{0i}(w_i) = 0$ for $i = 1, \ldots, N$. Again, start from $i = N - 1$ and loop down to $i = 1$. It's worth noting that there are some checks that can be done for the sake of accuracy. The algorithm above integrates over Π'_{0i} to find Π_{0i}. This needs to be consistent with the actual profit expression, $(\rho + \delta + \gamma_i)\,\Pi_{0i} = p_0 - w_{0i} - \chi(\gamma_i) + \lambda_{0i}\int_V^{\bar{V}_0(p)} \Pi'_0(V')\,\hat{F}_0(V')\,dV' + \gamma_i \Pi_1(X_i) + \Pi'_{0i}\dot{V}_{0i}$.

A.2 Steady state

For a given average training rate, $\bar{\gamma}$, the unemployment and employment stocks are given by,

$$\begin{aligned}
u_0 &= \frac{m\left[\bar{\gamma} + m + \delta\right]}{(\lambda_0 + m)(\bar{\gamma} + m) + \delta m} \\
e_0 &= \frac{u_0 \lambda_0}{\bar{\gamma} + m + \delta} \\
u_1 &= \frac{\delta \bar{\gamma} e_0}{m(\lambda_1 + \delta + m)} \\
e_1 &= \frac{u_1(\lambda_1 + m)}{\delta}.
\end{aligned}$$

The match distribution steady state conditions can be written as,

$$
\begin{aligned}
(\bar{\gamma}+m+\delta)\,\Phi(p) = & \int_0^p \int_U^{\min\left[\bar{V}_0(p'),V\right]} \left\{\delta+m+\gamma(V',p')+\hat{F}_0(V)\,\lambda_0(V',p')\right\} dG_0(V',p') \\
\Updownarrow & + \int_{\bar{V}_0^{-1}(V)}^p \dot{V}_0(V,p')\,g_0(V,p')\,dp' \qquad (48) \\
\frac{\lambda_1\delta}{\lambda_1+m}\Phi(p) = & \int_0^p \int_U^{\min\left[\bar{V}_1(p'),V\right]} \left\{\delta+m+\hat{F}_1(V)\,\lambda_1(V',p')\right\} dG_1(V',p') + \\
& \int_{\bar{V}_1^{-1}(V)}^p \dot{V}_1(V,p')\,g_1(V,p')\,dp' + \\
& -\frac{e_0}{e_1}\int_0^p \int_U^{\bar{V}_0(p')} \gamma(V',p')\,I\left[X(V',p')\leq V\right] dG_0(V',p')\,. \qquad (49)
\end{aligned}
$$

The solution algorithm starts by solving for $G_0(V,p)$ by use of equation (48). For a given $\bar{\gamma}$, the discretized system of equations can be written as a lower triangular linear equation system, which solves quickly by trivial forward recursion. Except for the true value of $\bar{\gamma}$, the solution will involve $G_0\left(1,\bar{V}_0(1)\right)\neq 1$, which is of course a violation of the definition of $G_0(V,p)$. However, this is only a multiplicative level problem on the probability density. Hence, rescale the density so that the solution is characterized by, $G_0\left(1,\bar{V}_0(1)\right)=1$. With this, the correct average training intensity can be determined, $\bar{\gamma}=\int_0^1\int_U^{\bar{V}_0(p')}\gamma(V',p')\,dG_0(V',p')\,.$

With a solution for $G_0(V,p)$ in hand, proceed to solve for $G_1(V,p)$ by use of equation (49). Again, for a given discretization of the state space, the system of equations can be written up as a lower triangular set of linear equations and solved quickly by forward recursion.

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