Wage Rigidities, Reallocation Shocks, and Jobless Recoveries

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Abstract

This paper explores the joint implications of two assumptions in a business cycle model: the key shock is reallocational, rendering some of the capital stock unproductive while not affecting the remainder; and wages are rigid and do not respond to the shock. In a model with an otherwise-frictionless labor market model, an adverse shock leads to a proportional and permanent decline in capital, employment, output, consumption, and investment. In a search model with rigid wages, the impact of the shock eventually disappears but quantitatively the behavior of the economy is very similar, giving rise to a jobless recovery.

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1 Introduction

This paper develops a theoretical model of the business cycle with two important features. First, the key shock is reallocational, rendering some of the existing capital stock unproductive while not affecting the remainder. Second, wages are rigid and so do not fall in response to this shock. The wage rigidity is consistent with individual rationality because of labor search frictions (Hall, 2005). I show that a model with these features robustly generates a simultaneous, roughly proportional, and long-lasting decline in output, employment, consumption, and investment. This is in broad agreement with the recent behavior of the U.S. labor market. In contrast, a model with flexible wages typically generates an investment boom following an adverse reallocation shock and never generates a large decline in employment.

The empirical motivation for this paper is the anemic behavior of the U.S. economy during the 2000s and the disastrous experience since the onset of the financial crisis in 2008 (Figure 1). From the first quarter of 2008 until the second quarter of 2010—the latest available data at the time this paper was written—the employment-population ratio fell by 7.0 log points and average per capita weekly hours fell by 8.3 log points. Over the same time period, real GDP per capita grew 6.9 log points slower than trend and real nondurable and services consumption per capita grew 6.6 log points slower than trend.[1] This extraordinary contraction came on the heels of a decade of weak growth, with essentially no recovery from the mild recession at the start of the decade. Many forecasters predict similarly weak growth in the years ahead and few see any prospect for an early return to the employment level in 2007 and certainly not to the level in 2000. This is an era of jobless recoveries.

The basic building blocks of my theory are not new. Lilien (1982) argued that reallocation shocks may be important at business cycle frequencies and notions that wage rigidities may generate unemployment date back at least to Keynes. But the reallocational view of recessions fell out of favor in part because of the observation that they have difficulty generating a negative comovement between unemployment and job vacancies (Abraham and Katz, 1986). And models of wage rigidities ran afoul of concerns that they did not explain why unemployed workers could not offer to work at less than the prevailing wage (Barro,

[1] I construct a linear trend from 1951 to 2008. Over this time period, the employment-population ratio grew by 0.07 log points per quarter and average weekly hours grew by 0.04 log points, so detrending these variables is quantitatively unimportant. On the other hand, GDP per capita and consumption per capita grew by 0.45 and 0.52 log points per quarterly, respectively, so here detrending is more important. The theory I develop is consistent with balanced growth. That is, trend productivity growth causes trend increases in output and consumption without affecting employment or hours, offering a theoretical justification for the differential treatment of these time series. But in practice it is not important whether I detrend employment and hours.

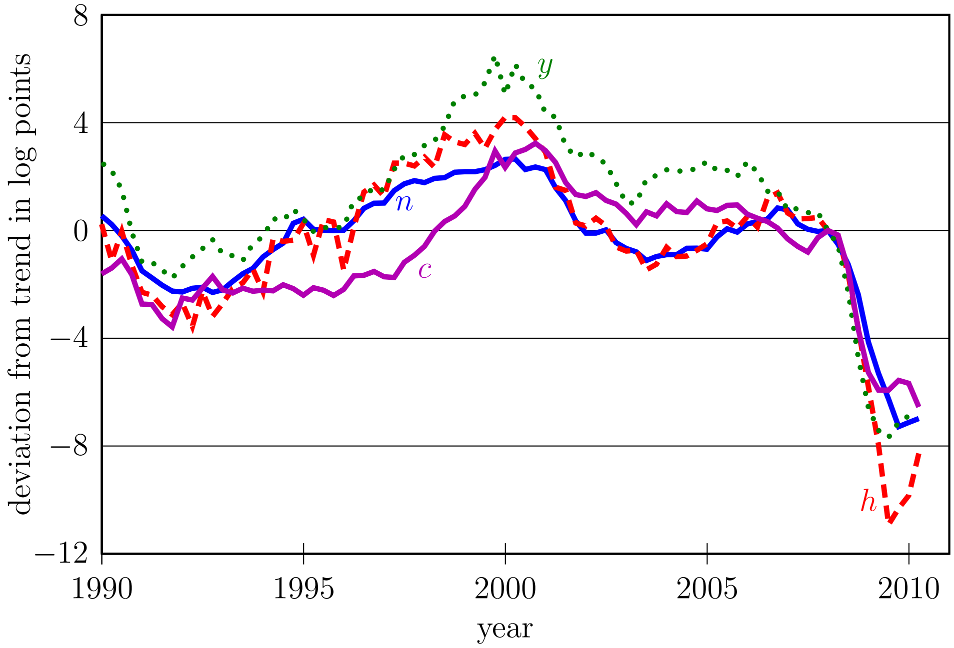

Figure 1: The solid blue line shows employment per capita n; the dashed red line shows hours per capita h; the dotted green line shows GDP per capita y; and the dash-dotted purple line shows consumption per capita c. GDP and consumption are detrended using a linear trend from 1951 to 2008 and expressed as deviations from trend in log points.

1977). Recent research in search theory has alleviated the latter concern by showing that there is an interval of wages at which firms are willing to employ workers and workers are willing to work; moreover, this interval may be quite large even if search frictions are relatively unimportant (Hall, 2005). But most of the subsequent literature has focused on aggregate technology shocks. To my knowledge, no one has explored the economy's behavior following a reallocation shock in a search model with rigid wages.

I start my analysis in a neoclassical growth model with a competitive labor market in which supply and demand are always equal. I model the reallocation shock as a one-time increase in the depreciation rate of capital. Of course, in reality no capital was destroyed during the recession. Instead, some investments that were made prior to the recession, for example in housing and in equipment related to housing construction, turned out to be worth much less than was originally forecast. The recession may have also accelerated the contraction of some declining manufacturing industries. If the capital in these industries cannot easily be adapted to other purposes, the realization that some of the capital stock is ill-suited for current economic circumstances is equivalent to a shock to the size of the capital stock. I show that an adverse reallocation shock always leads to a fall in the wage. Depending on preferences, employment may rise (because individuals are poorer) or fall (because the wage is lower). For example, if the period utility function can be expressed as log consumption minus the disutility of work, employment and investment rise on the impact of an adverse shock, while consumption and output fall. In any case, the model never delivers a quantitatively large decline in employment in response to an adverse reallocation shock.

I then consider an ad hoc wage rigidity in the neoclassical model. I assume that the wage is fixed at the steady state of the neoclassical model and the equilibrium level of employment is determined from firms' labor demand decision, without regard to workers' willingness to supply this labor. This is an old-style model of wage rigidities. I prove that an unanticipated shock that destroys one percent of the capital stock leads to a permanent one percent contraction in output, employment, consumption, and investment. That is, it causes a prolonged recession and jobless recovery. This part of the paper is a warmup to the main exercise, a model with search frictions, but it is useful because I can obtain many results in closed form and the same forces are at work in both models.

In particular, I next show how this logic carries over to a model with search frictions. I augment the neoclassical growth model with a recruiting technology through which a firm uses some of its employees to attract unemployed workers (Shimer, 2010). As Rogerson and Shimer (forthcoming) discuss, the natural analog of the (flexible wage) neoclassical growth model is the social planner's solution to that model. I confirm that the main insights of the frictionless model carry over to the search model. For example, if the period utility function can be expressed as log consumption minus the disutility of work, employment and investment are positively correlated with each other, as are consumption and output, but each of the first pair is negatively correlated with each of the second pair. With strong complementarities between consumption and labor supply, it is possible to generate a positive correlation between these four outcomes; however, employment is still much less volatile than output.

Finally, I introduce rigid wages into the search model. I assume that the wage is fixed at the equilibrium level in the absence of any shocks. Unlike in the frictionless model, I do not impose that workers have to supply whatever labor firms demand at that wage. Instead, I prove that in equilibrium firms are always willing to employ workers at this fixed wage, although they cut back on recruiting if the wage is too high. Workers are also always willing to work at the wage and indeed would happily take a job at a lower wage if one were available. Thus this wage is consistent with the indeterminancy in equilibrium wages identified by Hall (2005). In this case I verify that, even if consumption and leisure are separable, the model can generate the desired correlation between output and consumption, investment and employment, with all four variables having roughly the same volatility. The model is therefore broadly consistent with the empirical patterns that we have observed during the last two years.

The model predicts that employment falls during a recession because firms cut back on hiring, not because the incidence of unemployment rises. I have argued elsewhere that this view accounts for the majority of fluctuations in unemployment (Shimer, 2007), a view that was reaffirmed during the current recession (Elsby, Hobijn, and Şahin, forthcoming). In particular, this approach accounts for the simultaneous increase in unemployment and decline in vacancies that occurs during most recessions. On the other hand, this approach misses out on some features of the current recession, especially the increase in job vacancies during a period of constant unemployment in the second half of 2009 and first half of 2010. I leave this breakdown in the Beveridge curve as a topic for future research.

The paper has three more sections. The next section analyzes the frictionless model. I focus on a deterministic environment for simplicity and cover first the planner's solution, second the decentralized equilibrium, and third the rigid wage model. This develops the intuition for the following section, which analyzes the stochastic model with search frictions. It again contains three subsections which go through the same versions of the model: planner's solution, decentralized equilibrium, and rigid wage model. The final section concludes with a further discussion of the interpretation of reallocation shocks, the implications of these shocks for measured total factor productivity, and the consequences of wage rigidities for countercyclical fiscal policy.

2 Frictionless Model

I consider an economy with a representative household and a representative firm. The household contains a unit measure of individuals with identical preferences. Household members are infinitely-lived and discount the future with factor $\beta \in (0,1)$. Labor is indivisible and in period t a fraction n_t of household members are employed. The household maximizes the equal-weighted sum of its members' utility, but the marginal utility of consumption depends on employment status and so in general employed and unemployed household members will not consume the same amount.[2] In particular, the period utility of an employed household member who consumes c_e is

$$\frac{c_e^{1-\sigma}\big(1+(\sigma-1)\gamma\big)^{\sigma}}{1-\sigma}$$

[2] In the frictionless model with a competitive labor market, it would be more correct to refer to household members as nonemployed rather than unemployed, since the activity is in no sense involuntary. I use this notation for consistency with the remainder of the paper.

if $\sigma \neq 1$ and $\log c_e - \gamma$ if $\sigma = 1$; the corresponding value for an unemployed household member who consumes c_u is

$$\frac{c_u^{1-\sigma}}{1-\sigma}$$

if $\sigma \neq 1$ and $\log c_u$ if $\sigma = 1$. This specification of preferences is consistent with balanced growth: a proportional change in labor and non-labor income leaves the houeshold's choice of employment unchanged and simply leads to a proportional change in consumption. The parameter $\gamma > 0$ determines the disutility of work. The parameter $\sigma > 0$ plays several roles. It determines risk-aversion and the intertemporal elasticity of substitution. It is also important for the complementarity between consumption and work and in particular determines the relative consumption of employed and unemployed household members. As I show below, if $\sigma > 1$, employed workers optimally consume more than unemployed workers.

Firms use capital k and labor n to produce output with a standard Cobb-Douglas production technology. The output $Ak^\alpha n^{1-\alpha}$ plus undepreciated capital $(1-\delta_t)k$ is then used both for investment and consumption. For algebraic simplicity, I assume that total factor productivity A is constant. The parameter $\alpha \in (0,1)$ is the capital share of income. I allow for time-variation in the depreciation rate δ but for nw assume it is deterministic.

In what follows, I first solve a benevolent planner's problem, then I discuss the (standard) decentralization as a competitive economy. This is essentially the Hansen (1985) model. Finally I develop the rigid wage model. Throughout I assume parameter values are such that some household members are employed and some are unemployed.

2.1 Planner's Problem

A planner maximizes the total utility of the representative household

$$\sum_{t=0}^{\infty} \beta^t \left(n_t \frac{c_{e,t}^{1-\sigma}\big(1+(\sigma-1)\gamma\big)^\sigma}{1-\sigma} + (1-n_t)\frac{c_{u,t}^{1-\sigma}}{1-\sigma} \right)$$

subject to a law of motion for the capital stock

$$k_{t+1} = Ak_t^\alpha n_t^{1-\alpha} + (1-\delta_t)k_t - n_t c_{e,t} - (1-n_t)c_{u,t},$$

where δ_t is the time-varying depreciation rate of capital.

To solve the model, place a multiplier $\beta^t \lambda_t$ on the law of motion for capital. The first order condition for consumption of employed and unemployed household members is

$$\left(\frac{c_{e,t}}{1+(\sigma-1)\gamma}\right)^{-\sigma} = c_{u,t}^{-\sigma} = \lambda_t.$$

Define total consumption as $c_t = n_t c_{e,t} + (1-n_t)c_{u,t}$. Then the preceding equation implies

$$c_{e,t} = \frac{c_t(1+(\sigma-1)\gamma)}{1+(\sigma-1)\gamma n_t} \text{ and } c_{u,t} = \frac{c_t}{1+(\sigma-1)\gamma n_t}.$$

In particular, the ratio of consumption by the employed to consumption by the unemployed is $c_{e,t}/c_{u,t} = 1+(\sigma-1)\gamma$. If $\sigma > 1$, the employed consume more than the unemployed because working raises the marginal utility of consumption. This is why I refer to σ as the complementarity between consumption and employment.

The preceding expressions imply that the period utility function can be expressed in terms of total consumption,

$$n_t \frac{c_{e,t}^{1-\sigma}\big(1+(\sigma-1)\gamma\big)^{\sigma}}{1-\sigma} + (1-n_t)\frac{c_{u,t}^{1-\sigma}}{1-\sigma} = \frac{c_t^{1-\sigma}\big(1+(\sigma-1)\gamma n_t\big)^{\sigma}}{1-\sigma}.$$

That is, the household acts as if it has a representative member who works a fraction n_t of his time and has an infinite Frisch labor supply elasticity. This is a standard aggregation result. I can then reduce the first order conditions for consumption to

$$\lambda_t = \left(\frac{c_t}{1+(\sigma-1)\gamma n_t}\right)^{-\sigma},$$

so λ_t is the marginal utility of consumption for the hypothetical representative household member.

I assume throughout that some household members work and some are unemployed; this will be the case if the disutility of labor is sufficiently large. At an interior solution for employment, the first order condition for labor is

$$\gamma\sigma\left(\frac{c_t}{1+(\sigma-1)\gamma n_t}\right)^{1-\sigma} = \lambda_t(1-\alpha)A(k_t/n_t)^{\alpha}.$$

Eliminate the marginal utility of consumption λ_t between the first order conditions for c_t and n_t:

$$\frac{\gamma\sigma c_t}{1+(\sigma-1)\gamma n_t} = (1-\alpha)A(k_t/n_t)^{\alpha}.$$

Substitute that back into the first order condition for c_t and solve for n_t:

$$n_t = k_t \lambda_t^{\frac{1}{\alpha\sigma}} \left(\frac{(1-\alpha)A}{\gamma\sigma}\right)^{\frac{1}{\alpha}} \tag{1}$$

Also solve for c_t in terms of the state variable and the marginal utility of consumption using the first order condition for c and equation (1):

$$c_t = \lambda_t^{-\frac{1}{\sigma}} + (\sigma - 1)\gamma k_t \lambda_t^{\frac{1-\alpha}{\alpha\sigma}} \left(\frac{(1-\alpha)A}{\gamma\sigma}\right)^{\frac{1}{\alpha}}. \tag{2}$$

These two equations characterize employment and consumption in terms of the capital stock and the marginal utility of consumption.

Next take the first order condition with respect to k_{t+1} to obtain the Euler equation

$$\lambda_t = \beta\lambda_{t+1}\big(\alpha A(k_{t+1}/n_{t+1})^{\alpha-1} + 1 - \delta_{t+1}\big). \tag{3}$$

Substitute for n_{t+1}/k_{t+1} using equation (1):

$$\lambda_t = \beta\lambda_{t+1}\left(\alpha A^{\frac{1}{\alpha}} \lambda_{t+1}^{\frac{1-\alpha}{\alpha\sigma}} \left(\frac{1-\alpha}{\gamma\sigma}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta_{t+1}\right).$$

This is a first order difference equation in λ alone. In steady state with constant δ, it pins down the marginal utility of consumption λ^*:

$$\lambda^* = \left(\frac{1-\beta(1-\delta)}{\alpha\beta}\right)^{\frac{\alpha\sigma}{1-\alpha}} A^{-\frac{\sigma}{1-\alpha}} \left(\frac{\gamma\sigma}{1-\alpha}\right)^{\sigma}.$$

This depends only on model parameters alone.

Next rewrite the law of motion for the capital stock, eliminating n and c using equations (1) and (2):

$$\begin{aligned} k_{t+1} &= \big(A(n_t/k_t)^{1-\alpha} + 1 - \delta_t\big)k_t - c_t \\ &= \left(A^{\frac{1}{\alpha}} \lambda_t^{\frac{1-\alpha}{\alpha\sigma}} \left(\frac{1-\alpha}{\gamma\sigma}\right)^{\frac{1-\alpha}{\alpha}} \frac{1-\alpha+\alpha\sigma}{\sigma} + 1 - \delta_t\right)k_t - \lambda_t^{-\frac{1}{\sigma}} \end{aligned}$$

In steady state with a constant depreciation rate, the capital stock is

$$k^* = \frac{\lambda^{*-\frac{1}{\sigma}}}{A^{\frac{1}{\alpha}} \lambda^{*\frac{1-\alpha}{\alpha\sigma}} \left(\frac{1-\alpha}{\gamma\sigma}\right)^{\frac{1-\alpha}{\alpha}} \frac{1-\alpha+\alpha\sigma}{\sigma} - \delta} = \frac{A^{\frac{1}{1-\alpha}} \left(\frac{\alpha\beta}{1-\beta(1-\delta)}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{1-\alpha}{\gamma\sigma}\right)}{\left(\frac{1-\beta(1-\delta)}{\alpha\beta}\right)\left(\frac{1-\alpha+\alpha\sigma}{\sigma}\right) - \delta},$$

where the second equation eliminates λ^* to express the steady state capital stock in terms of model primitives alone.

I am primarily interested in how the economy behaves away from steady state. To start, assume δ is constant but the capital stock starts away from its steady state value. I focus on the behavior of the dynamic system

$$\lambda_t = \beta\lambda_{t+1}\Big(\alpha A^{\frac{1}{\alpha}}\lambda_{t+1}^{\frac{1-\alpha}{\alpha\sigma}}\left(\tfrac{1-\alpha}{\gamma\sigma}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta\Big),$$
$$k_{t+1} = \left(A_t^{\frac{1}{\alpha}}\lambda_t^{\frac{1-\alpha}{\alpha\sigma}}\left(\tfrac{1-\alpha}{\gamma\sigma}\right)^{\frac{1-\alpha}{\alpha}}\tfrac{1-\alpha+\alpha\sigma}{\sigma} + 1 - \delta_t\right)k_t - \lambda_t^{-\frac{1}{\sigma}}.$$

In particular, I log-linearize it in a neighborhood of the steady state. Using $\hat{\lambda}_t$ to denote the log-deviation of λ from steady state, and similarly for $\hat{k}_t$, I obtain

$$\hat{\lambda}_{t+1} = \frac{\alpha\sigma}{\alpha\sigma + (1-\alpha)(1-\beta(1-\delta))}\hat{\lambda}_t,$$

$$\hat{k}_{t+1} = \frac{(1-\beta(1-\delta))(1+\alpha(\sigma-1)) - \beta\delta\alpha^2\sigma}{\beta\sigma^2\alpha^2}\hat{\lambda}_t + \frac{1-\alpha+\alpha\sigma-\beta(1-\alpha)(1-\delta)}{\beta\alpha\sigma}\hat{k}_t.$$

The eigenvalues of this system are

$$0 < \frac{\alpha\sigma}{(1-\alpha)(1-\beta(1-\delta))+\alpha\sigma} < 1 < \frac{(1-\alpha)(1-\beta(1-\delta))+\alpha\sigma}{\beta\alpha\sigma}.$$

Thus it is saddle path stable. The eigenvector associated with the stable arm is $(\hat{k}, \hat{\lambda}) = (1, e_\lambda)$, where

$$e_\lambda \equiv -\sigma\frac{\frac{(1-\alpha)(1-\beta(1-\delta))+\alpha\sigma}{\beta\alpha\sigma} - \frac{\alpha\sigma}{(1-\alpha)(1-\beta(1-\delta))+\alpha\sigma}}{\frac{(1-\beta(1-\delta))(1-\alpha+\alpha\sigma)}{\beta\alpha^2\sigma} - \delta}$$

is the elasticity of the marginal utility of consumption λ with respect to the capital stock along the saddle path. One can confirm that $e_\lambda < 0$ and so the saddle path is downward sloping. That is, when the capital stock is below steady state, the marginal utility of consumption is above steady state.

Next, suppose that the economy is at a steady state and there is an unanticipated, one-time, temporary, one percentage point increase in δ at $t = 0$. The steady state is unchanged, but one percent of the capital stock is destroyed. Following this event, the economy stays on the saddle path, and so on impact λ must increase by $-e_\lambda$ percent. Eventually λ falls back to its steady state value as the capital stock is rebuilt. I am interested in how other model outcomes behave along this transition path. The discussion that follows focuses on the log-linear dynamics in a neighborhood of the steady state.

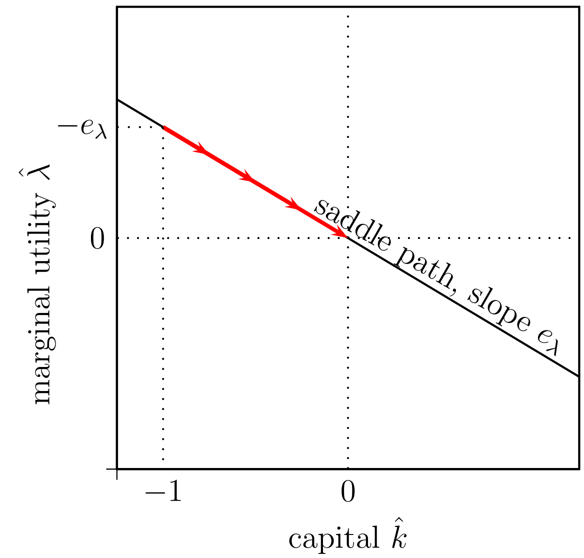

Figure 2: Transitional dynamics of the marginal utility of consumption

To start, equation (1) implies labor productivity $(k/n)^\alpha$ moves inversely with λ,

$$(k_t/n_t)^\alpha \propto \lambda_t^{-\frac{1}{\sigma}}.$$

Since a one percentage point increase in the depreciation rate reduces the capital stock by one percent, it raises λ by $-e_\lambda$ percent; therefore, labor productivity must fall upon the impact of the shock, by $-e_\lambda/\sigma$ percent. It then gradually increases back to its steady state value.

Next consider the impact on employment. Equation (1) implies

$$\hat{n}_0 = \hat{k}_0 + \frac{1}{\alpha\sigma}\hat{\lambda}_0 = -\frac{\alpha\sigma + e_\lambda}{\alpha\sigma}$$
$$= \frac{\frac{\alpha(1-\alpha)\beta\sigma}{1-\beta(1-\delta)}\left(1-\delta(1-\alpha)-\frac{\alpha\delta\sigma}{1-\beta(1-\delta)}\right)}{\left(1-\alpha+\frac{\alpha\sigma}{1-\beta(1-\delta)}\right)\left(1-\alpha+\alpha\sigma-\frac{\alpha^2\beta\delta\sigma}{1-\beta(1-\delta)}\right)}$$

If $\sigma \leq 1$, one can verify that this is negative, so employment rises on the impact of the adverse shock and then gradually falls back to its steady state value. This seems like the most natural case. Households are poor and since leisure is a normal good, they cut back on it and so supply more labor. This accelerates the economy's recovery to steady state. But this need not be the case. If

$$\sigma > \frac{(1-\beta(1-\delta))(1-\delta(1-\alpha))}{\alpha\delta} \equiv \bar{\sigma},$$

a number larger than 1, then employment initially falls after the adverse shock before rising back to its steady state value. In this case, substitution effects dominate income effects. Following the destruction of capital, the marginal product of labor is low, which discourages households from supplying much labor despite their relative poverty. Still, for any σ the elasticity of employment with respect to the capital stock is bounded above by

$$\frac{(1-\alpha)\beta\delta}{1-\beta(1-\delta(1-\alpha))} < 1.$$

For example, if $\alpha = 1/3$, $\beta = 0.996$, and $\delta = 0.0046$, numbers that I later argue are reasonable on a monthly basis, a one percent decline in the capital stock cannot reduce employment by more than 0.43 percent. Since employment falls by less than capital, it also falls by less than output, which is, of course, consistent with the finding in the previous paragraph that labor productivity is low when the capital stock is below steady state.

One can also compute the behavior of consumption, although the expressions are messier. If $\sigma \leq 1$, it is possible to prove analytically that when the capital stock is below steady state, consumption is also below steady state. This is also true when $\sigma \geq \bar{\sigma}$, the case where a reduction in the capital stock lowers employment. At intermediate values of σ, perhaps surprisingly, consumption may rise when the capital stock falls. The explanation is instructive. When $\sigma < \bar{\sigma}$, employment rises upon the impact of the shock. This raises the marginal utility of consumption when $\sigma > 1$, and so encourages workers to consume. The income effect from the destruction of capital can be dominated by the incentive to smooth the marginal utility of consumption.

Finally, I turn to investment, $i = Ak^{\alpha}n^{1-\alpha} - c$. In general, the elasticity of investment with respect to the capital stock is also a messy expression, but I find numerically that when $\sigma \leq 1$, a reduction in the capital stock raises investment as the economy returns to steady state. This can be reversed, however, if σ is sufficiently large. In particular, when $\sigma = \bar{\sigma}$, a one percent reduction in the capital stock reduces investment by α percent.

In summary, at low values of σ, an adverse shock to the capital depreciation rate raises employment and may raise consumption. But if $\sigma \geq \bar{\sigma}$, the shock causes an event that looks broadly like a recession, with consumption and employment both falling. Because employment falls by significantly less than does the capital stock, this still pushes down labor productivity. Similarly, investment declines, but only if consumption-labor complementarities are strong.

2.2 Decentralized Equilibrium

The welfare theorems apply, and so a competitive equilibrium decentralizes the social planner's problem. This feature of the neoclassical growth model is well understood, but I write down the decentralized economy formally as a prelude to the discussion of a model in which wages do not clear the labor market.

Consider a typical household which starts period t with assets a. Representing its problem recursively, I obtain

$$V_t(a) = \max_{c_e, c_u, n, a'} n \frac{c_e^{1-\sigma}\big(1+(\sigma-1)\gamma\big)^{\sigma}}{1-\sigma} + (1-n)\frac{c_u^{1-\sigma}}{1-\sigma} + \beta V_{t+1}(a'),$$

where next period's assets are $a' = (a + w_t n - n c_e + (1-n)c_u)/q_t$ and q_t is the price of a one-period bond. As in the social planner's problem, the household equates the marginal utility of consumption for employed and unemployed individuals, giving

$$c_{e,t} = \frac{c_t(1+(\sigma-1)\gamma)}{1+(\sigma-1)\gamma n_t} \text{ and } c_{u,t} = \frac{c_t}{1+(\sigma-1)\gamma n_t},$$

where $c_t = n_t c_{e,t} + (1-n_t)c_{u,t}$ is total consumption. The first order condition, expressed in terms of total consumption, is

$$\lambda_t \equiv \left(\frac{c_t}{1+(\sigma-1)\gamma n_t}\right)^{-\sigma} = \beta V'_{t+1}(a_{t+1})/q_t,$$

where λ_t is again the marginal utility of consumption for a hypothetical representative agent with period utility function $c_t^{1-\sigma}\big(1+(\sigma-1)\gamma n_t\big)^{\sigma}/(1-\sigma)$. The first order condition for labor (still assuming an interior solution) is

$$\gamma\sigma\left(\frac{c_t}{1+(\sigma-1)\gamma n_t}\right)^{1-\sigma} = \beta V'_{t+1}(a_{t+1})/q_t.$$

Eliminate the marginal value of assets next period between these equations to get that the marginal rate of substitution between consumption and leisure is equal to the wage,

$$\frac{\gamma\sigma c_t}{1+(\sigma-1)\gamma n_t} = w_t.$$

Also, the envelope condition is

$$V'_t(a_t) = \beta V'_{t+1}(a_{t+1})/q_t.$$

Combining with the consumption first order condition gives

$$V_t'(a_t) = \lambda_t,$$

which in turn implies the consumption Euler equation:

$$\lambda_t = \frac{\beta}{q_t}\lambda_{t+1}.$$

Next, write the problem of a firm that owns k units of capital as

$$J_t(k) = \max_{n,i,k'} Ak^\alpha n^{1-\alpha} - w_t n - i + q_t J_{t+1}(k'),$$

where $k' = (1-\delta_t)k + i$. The first order condition for labor is

$$(1-\alpha)A(k_t/n_t)^\alpha = w_t,$$

equating the marginal product of labor to the wage. The first order condition for investment is

$$1 = q_t J_{t+1}'(k_{t+1}).$$

The envelope condition is

$$J_t'(k_t) = \alpha A(k_t/n_t)^{\alpha-1} + (1-\delta_t)q_t J_{t+1}'(k_{t+1}).$$

Eliminate next period's marginal value of capital between the last two equations to get

$$J_t'(k_t) = \alpha A(k_t/n_t)^{\alpha-1} + 1 - \delta_t.$$

Plug this into the envelope condition and solve for q_t:

$$\frac{1}{q_t} = \left(\alpha A(k_{t+1}/n_{t+1})^{\alpha-1} + 1 - \delta_{t+1}\right).$$

Now use this to eliminate q_t from the household Euler equation.

$$\lambda_t = \beta\lambda_{t+1}\left(\alpha A(k_{t+1}/n_{t+1})^{\alpha-1} + 1 - \delta_{t+1}\right).$$

This is identical to the Euler equation (3) in the planner's problem. Since the relationship between consumption and employment on the one hand, and capital and the marginal utility of consumption on the other hand, is unchanged, the competitive equilibrium solves the planner's problem.

One important prediction that comes out of the decentralization concerns the behavior of wages. Recall that an adverse shock to the depreciation rate of capital lowers labor productivity. With a Cobb-Douglas production function, the wage is proportional to labor productivity and so the wage falls as well. I now turn to a model in which the wage cannot adjust.

2.3 Rigid Wage

In this section, I assume that the wage is fixed exogenously at some level $\bar{w}$ and households must supply whatever labor firms demand. The model is otherwise unchanged. In particular, a household with assets a solves

$$V_t(a) = \max_{c_e, c_u, a'} n \frac{c_e^{1-\sigma} \big(1 + (\sigma - 1)\gamma\big)^{\sigma}}{1 - \sigma} + (1 - n) \frac{c_u^{1-\sigma}}{1 - \sigma} + \beta V_{t+1}(a'),$$

where $a' = (a + \bar{w} n_t - n_t c_e - (1 - n_t) c_u)/q_t$. The household Euler equation is unchanged:

$$\lambda_t = \frac{\beta}{q_t} \lambda_{t+1},$$

where λ_t is still the marginal utility of consumption for a hypothetical representative agent. This, combined with the budget constraint $q_t a_{t+1} = a_t + \bar{w} n_t - c_t$, completely describes the household's behavior. In particular, the household does not choose how much to work.

The firm's problem is unchanged. This gives us the first order condition for employment,

$$(1 - \alpha) A (k_t / n_t)^{\alpha} = \bar{w},$$

as well as the envelope condition

$$\frac{1}{q_t} = \left(\alpha A (k_{t+1}/n_{t+1})^{\alpha - 1} + 1 - \delta_{t+1} \right).$$

I can again combine this with the household Euler equation, eliminating employment using the firms' first order condition for labor,

$$\lambda_t = \beta \left(\alpha A^{\frac{1}{\alpha}} \left(\tfrac{1-\alpha}{\bar{w}} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta_{t+1} \right) \lambda_{t+1} = \beta R_t \lambda_{t+1},$$

where

$$R_t \equiv \alpha A^{\frac{1}{\alpha}} \left(\tfrac{1-\alpha}{\bar{w}} \right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta_t$$

is the gross marginal product of capital. In addition, eliminate employment from the resource constraint to get

$$k_{t+1} = \left(A^{\frac{1}{\alpha}} \left(\tfrac{1-\alpha}{\bar{w}}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta_t \right) k_t - c_t.$$

The marginal utility of consumption is a function of consumption and employment, while employment depends on the capital stock and the wage. Thus I can invert this to express consumption as a function of marginal utility, the capital stock and the wage,

$$c_t = \lambda_t^{-\frac{1}{\sigma}} \left(1 + (\sigma - 1)\gamma \left(\tfrac{(1-\alpha)A}{\bar{w}}\right)^{\frac{1}{\alpha}} k_t \right).$$

This is again a pair of difference equations for capital and the marginal utility of consumption.

Now suppose that the economy starts from a steady state where $\beta R = 1$. This might, for example, be the steady state decentralized equilibrium of an economy with a competitive labor market. There is a one time, transitory, unanticipated increase in δ, destroying one percent of the capital stock. Thereafter, the depreciation rate returns to normal, but the wage is not allowed to adjust. In particular, $\beta R = 1$ implies that the wage satisfies

$$\frac{1}{\beta} = \alpha A^{\frac{1}{\alpha}} \left(\tfrac{1-\alpha}{\bar{w}}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta.$$

I claim that there is an equilibrium of the economy in which consumption, employment, investment, and output all fall by one percent and remain permanently at this depressed level. That is, for any $k_t \leq k^*$,

$$c_t = \left(\frac{1 - \beta(1-\delta)}{\alpha\beta} - \delta \right) k_t \text{ and } n_t = \left(\frac{1 - \beta(1-\delta)}{\alpha\beta A} \right)^{\frac{1}{1-\alpha}} k_t.$$

To prove this, we simply verify that at this level of consumption, the capital stock is constant. Eliminate consumption from the resource constraint using our conjecture to obtain

$$k_{t+1} = \left(A^{\frac{1}{\alpha}} \left(\tfrac{1-\alpha}{\bar{w}}\right)^{\frac{1-\alpha}{\alpha}} + 1 - \delta - \left(\frac{1 - \beta(1-\delta)}{\alpha\beta} - \delta \right) \right) k_t.$$

When $\beta R = 1$, the equation reduces to $k_{t+1} = k_t$ and so consumption, employment, and output all permanently remain one percent below the initial steady state. Since consumption and employment are constant, marginal utility is constant as well, consistent with $\beta R = 1$. Finally, for the capital stock to remain one percent below steady state, investment must also fall by one percent.

If $\sigma = 1$, so consumption and the disutility of work are separable, it is straightforward to prove that this is the unique equilibrium. The assumption that $\beta R = 1$ implies that marginal utility is constant. With separable utility, this implies that consumption is constant as well. Any higher level of consumption than the one defined above is unaffordable, while any lower level is inconsistent with the transversality condition. I conjecture that the uniqueness result holds more generally but do not have a proof.

To summarize, the frictionless model shows that if wages are flexible, consumption, employment, and investment may decline, but only if consumption-labor complementarities are strong. Even then, employment always declines by less than output. But with rigid wages, the model naturally produces a permanent and equal decline in consumption, employment, investment, and output in response to a one time shock that reduces the size of the capital stock. The question that cannot be answered in the frictionless model is why wages would not adjust following the shock. There are unemployed workers who would like to work at less than the prevailing wage and firms that would be willing to hire them. What prevents these workers from offering to work at that wage? Search frictions, which I introduce next, provide a natural answer.

3 Search Model

I now extend the neoclassical growth model by assuming that firms have access to two technologies. One uses capital and labor to produce output. The other uses labor to recruit more workers. Each period, every firm divides its workforce between these two tasks, production and recruiting. This allocation determines current output and the evolution of the aggregate employment rate, which is now a state variable. In addition, I assume that the depreciation rate is stochastic. I again start by looking at the planner's solution and then discuss its decentralization before turning to the economy with a rigid wage.

3.1 Planner's Problem

At the start of each period, a representative household containing a measure 1 of individuals owns k units of capital and has an employment rate n. A planner chooses how much each employed and unemployed individual consumes, c_e and c_u, and the amount of labor to devote to production and to recruiting. Next period's employment rate is then determined as

$$n' = (1 - x)n + f(\theta)(1 - n), \tag{4}$$

where $f : \mathbb{R} \mapsto [0, 1]$ is an increasing function of the recruiter-unemployment ratio θ. A fraction x of the employed workers exogenously lose their job, while a fraction $f(\theta)$ of the unemployed workers find a job. Next period's capital stock is

$$k' = Ak^{\alpha}(n - \theta(1-n))^{1-\alpha} + (1-\delta)k - nc_e - (1-n)c_u. \tag{5}$$

The first term is a standard Cobb-Douglas production function with capital share α. The amount of labor devoted to production is $n - \theta(1-n)$ since n is total employment, θ is the recruiter-unemployment ratio, and $1-n$ is unemployment. The second term is the undepreciated portion of the capital stock. I assume that the depreciation rate between periods t and $t+1$, δ in the notation above, follows a first order Markov process with transition probabilities $\pi(\delta'|\delta)$ and that δ is known at the time period t decisions are made. The last two terms are the consumption of employed and unemployed workers.

Let $V(k, n, \delta)$ denote the expected sum of utilities of the household members when the aggregate capital stock is k, aggregate employment is n, and the depreciation rate is δ. The planner maximizes this object subject to the technological constraints in equations (4) and (5). Expressing the planner's problem recursively gives

$$V(k,n,\delta) = \max_{c_e, c_u, \theta, k', n'} n\frac{c_e^{1-\sigma}\big(1+(\sigma-1)\gamma\big)^{\sigma}}{1-\sigma} + (1-n)\frac{c_u^{1-\sigma}}{1-\sigma} + \beta\sum_{\delta'}\pi(\delta'|\delta)V(k', n', \delta').$$

This is a straightforward extension of the frictionless model to an environment in which employment is a state variable and hiring is subject to search frictions.

The first order conditions for consumption by employed and unemployed workers implies

$$\left(\frac{c_e}{1+(\sigma-1)\gamma}\right)^{-\sigma} = c_u^{-\sigma} = \beta\sum_{\delta}\pi(\delta'|\delta)V_k'.$$

As is now standard, I let $c \equiv nc_e + (1-n)c_u$ denote total household consumption, which implies

$$c_e = \frac{c(1+(\sigma-1)\gamma)}{1+(\sigma-1)\gamma n} \text{ and } c_u = \frac{c}{1+(\sigma-1)\gamma n} \tag{6}$$

and

$$\lambda \equiv \left(\frac{c}{1+(\sigma-1)\gamma n}\right)^{-\sigma} = \beta\sum_{\delta'}\pi(\delta'|\delta)V_k', \tag{7}$$

where λ is the marginal utility of consumption.

The first order condition for recruiting is

$$f'(\theta)\beta\sum_{\delta'}\pi(\delta'|\delta)V_n' = (1-\alpha)A\kappa^{\alpha}\beta\sum_{\delta'}\pi(\delta'|\delta)V_k', \tag{8}$$

where

$$\kappa \equiv \frac{k}{n - \theta(1-n)}$$

is the ratio of capital to producers. Combining equations (7) and (8) gives

$$f'(\theta)\beta \sum_{\delta'} \pi(\delta'|\delta) V_n' = \lambda(1-\alpha)A\kappa^{\alpha}. \tag{9}$$

The envelope conditions are

$$V_k = \beta\big(\alpha A \kappa^{\alpha-1} + 1 - \delta\big) \sum_{\delta'} \pi(\delta'|\delta) V_k'$$

for capital and

$$V_n = \big((1+\theta)(1-\alpha)A\kappa^{\alpha} - c_e + c_u\big)\beta \sum_{\delta'} \pi(\delta'|\delta) V_k' + (1 - x - f(\theta))\beta \sum_{\delta'} \pi(\delta'|\delta) V_n' + \frac{c_e^{1-\sigma}\big(1 + (\sigma-1)\gamma\big)^{\sigma} - c_u^{1-\sigma}}{1-\sigma}$$

for employment. Use equations (6), (7), and (9) to eliminate c_e, c_u, V_n', and V_k' from these equations. This delivers expressions for the marginal value of capital

$$V_k = \lambda\big(\alpha A \kappa^{\alpha-1} + 1 - \delta\big)$$

and the marginal value of employment

$$V_n = \left(1 + \theta + \frac{1 - x - f(\theta)}{f'(\theta)}\right) \lambda(1-\alpha)A\kappa^{\alpha} - \gamma\sigma \left(\frac{c}{1 + (\sigma-1)\gamma n}\right)^{1-\sigma}.$$

Finally, substitute these back into the envelope conditions to get

$$\lambda = \beta \sum_{\delta'} \pi(\delta'|\delta) \lambda' \left(\alpha A \kappa'^{\alpha-1} + 1 - \delta'\right) \tag{10}$$

and

$$(1-\alpha)A\kappa^{\alpha}\lambda = f'(\theta)\beta \sum_{\delta'} \pi(\delta'|\delta) \times \left(\lambda'(1-\alpha)A\kappa'^{\alpha}\left(1 + \theta' + \frac{1 - x - f(\theta')}{f'(\theta')}\right) - \gamma\sigma\left(\frac{c'}{1 + (\sigma-1)\gamma n'}\right)^{1-\sigma}\right), \tag{11}$$

where λ and λ' are the current and future marginal utility of consumption. Equation (10) is a standard expression for the optimal consumption-investment decision, essentially unchanged from equation (3) in the frictionless model. Increasing the capital stock by one unit today requires forgoing one unit of consumption, and so the left hand side is the utility cost of investment. Next period output increases by the marginal product of capital $\alpha\kappa'^{\alpha-1}$ and investment can be reduced by $1-\delta$ while keeping the capital stock unchanged two periods hence. Multiplying this by the discounted marginal utility of consumption gives the benefit from the investment.

Equation (11) is special to the search model, but does not depend directly on the reallocation shock. This determines the optimal level of recruiting versus production. An additional producer raises output today by the marginal product of labor $(1-\alpha)A\kappa^{\alpha}$. This is valued at the marginal utility of consumption, and so the left hand side is the utility cost of recruiting. Each additional recruiter attracts $f'(\theta)$ additional workers to the firm next period. This reduces utility directly by the marginal disutility of working. In addition, it allows the planner to shift resources into production next period, while keeping employment at a fixed level two periods hence. More precisely, given the level of employment next period n', the planner sets the number of producers $l' = n' - \theta'(1-n')$ to achieve a desired employment level in two periods, $n'' = (1-x)n' + f(\theta')(1-n')$. Implicitly differentiating, I obtain

$$\frac{\partial l'}{\partial n'} = 1 + \theta' + \frac{1 - x - f(\theta')}{f'(\theta')}$$

when n'' is held fixed. These are the additional workers available to produce next period for each worker who is hired this period, and the output is valued at next period's marginal utility of consumption.

In summary, the model has two endogenous state variables, n and k, two controls, c and θ, with κ defined for convenience as the capital-producer ratio and λ as the marginal utility of consumption. The four equations (4), (5), (10), and (11) describe the equilibrium. The consumption of employed and unemployed workers can then be recovered from equation (6).

I solve the model numerically. More precisely, I calibrate the model parameters and log-linearize it in a neighborhood of the steady state. I set the model time period to be one month in order to capture the typical short duration of an unemployment spell. I first graph theoretical impulse responses to a one-time shock to the depreciation rate. I then simulate 708 months (59 years) of data from the model using monte carlo. Since much of my real-world data is only available at quarterly frequencies, I generate quarterly averages of the monthly model-generated data before detrending it using an HP filter with smoothing parameter 1600. Finally, I compute a variety of model-generated moments using the detrended data and repeat the procedure to reduce sampling uncertainty.

I first describe the calibration. I think of a time period as a month and set the discount factor to $\beta = 0.996$, just under five percent annually. I fix $\alpha = 0.33$ to match the capital share of income in the National Income and Product Accounts. The value of total factor productivity A is irrelevant for the business cycle statistics that I report here.

I assume that the depreciation rate is independently and identically distributed over time, $\log \delta = \log \bar{\delta} + \varsigma \upsilon$, where υ is an i.i.d. shock with mean 0 and standard deviation 1. I set the mean depreciation rate at $\bar{\delta} = 0.0046$, which implies that the steady state capital-output ratio is 3.2 when expressed on an annual basis. This is the average capital-output ratio in the United States since 1948.[3] The standard deviation ς is immaterial in the log-linearized model. I also consider an environment where the depreciation rate is serially correlated, $\log \delta' - \log \bar{\delta} = \rho(\log \delta - \log \bar{\delta}) + \varsigma \upsilon'$ for some $\rho \in (0, 1)$.

I turn next to the parameters that determine flows between employment and unemployment. Shimer (2005) measures the average exit probability from employment to unemployment in the United States at $x = 0.034$ per month, and I stick with that number here. Although there are many estimates of the matching function f in the literature (see the survey by Petrongolo and Pissarides, 2001), most papers assume that firms create job vacancies in order to attract unemployed workers and so estimate matching functions using data on unemployment and vacancies. The technology in this paper is slightly different, with firms using workers to recruit workers. Unfortunately I am unaware of any time series showing the number of workers (or hours of work) devoted to recruiting, and so the choice of f is somewhat arbitrary. Still, following much of the search and matching literature, I focus on an isoelastic function, $f(\theta) = \bar{\mu}\theta^{\eta}$, and in particular at the symmetric case, $\eta = 0.5$. I discuss below the importance of this parameter. To pin down the efficiency parameter in the matching function $\bar{\mu}$, I build on evidence in Hagedorn and Manovskii (2008) and Silva and Toledo (2009). Those papers argue that recruiting a worker uses approximately 4 percent of one worker's quarterly wage, i.e., a recruiter can attract approximately 25 new workers in a quarter, or 8.33 in a month. I use this fact and data on the average unemployment rate to determine $\bar{\mu}$. I proceed in several steps. First, from (1), the steady state employment rate satisfies

$$n = \frac{f(\theta)}{x + f(\theta)}.$$

Setting $n = 0.95$, the average share of the labor force employed during the post-war period, and $x = 0.034$, this implies $f(\theta) = 0.646$ in steady state. Second, the functional form

[3] More precisely, I use the Bureau of Economic Analysis's Fixed Asset Table 1.1, line 1 to measure the current cost net stock of fixed assets and consumer durable goods. I use National Income and Product Accounts Table 1.1.5, line 1 to measure nominal Gross Domestic Product.

$f(\theta) = \bar{\mu}\theta^{\eta}$ implies

$$\bar{\mu} = \frac{f(\theta)}{\theta^{\eta}} = f(\theta)^{1-\eta}\mu(\theta)^{\eta},$$

where the second equation follows because $\mu(\theta) \equiv f(\theta)/\theta$. From this equation, I set $\bar{\mu} = 2.32$, consistent with $f(\theta) = 0.646$, $\mu(\theta) = 8.33$, and $\eta = 1/2$. Note that this implies that the recruiter-unemployment ratio is $\theta = f(\theta)/\mu(\theta) \approx 0.078$. It follows that the share of recruiters in employment is $\theta(1-n)/n \approx 0.004$, with 99.6 percent of employees devoted to production. Thus in this calibration, the implicit hiring costs are small, at least on average.

Finally, I consider the model both with $\sigma = 1$ and with higher values which allow for consumption-labor complementarity. I set the parameter γ, governing the taste for leisure, to obtain a five percent unemployment rate along the balanced growth path; the exact value depends on the other calibrated parameters

In the deterministic frictionless model, I found that when $\sigma = 1$, an increase in δ raised employment and investment and reduced consumption. Although employment is no longer a control variable, this result carries over to the stochastic model with search frictions. Figure 3 shows how the model economy responds to a one time increase in the depreciation rate at $t = 0$. Investment jumps up on impact before settling down to a new higher value, while employment takes a few months to increase due to the search frictions. On the other hand, consumption, output (of the final good), labor productivity (output of the final good divided by employment of both producers and recruiters) and the wage all fall.[4] It is worth stressing that, despite the large movements in labor productivity, this is unrelated to any technology shock. Total factor productivity is, by assumption, constant.

After time-averaging and detrending model-generated data, I confirm the main results. Output and consumption are positively correlated, as are employment and investment; however, each of output and consumption is negatively correlated with each of employment and investment. The basic problem is that a one-time increase in the depreciation rate pushes up employment and investment as the planner moves the economy back towards the old steady state. But the increase in employment is not enough to make up for the shortage of capital and so output and consumption are pushed down.

Figure 3 also shows that in the baseline model, consumption is about twice as volatile as output, while employment is far less volatile. The same result shows up in the time-averaged and detrended data. Putting these findings together, the frictionless model has trouble generating either a recession or a jobless recovery in response to a shock to the depreciation rate.

[4] I discuss the decentralization of the social planner's problem, and so wage determination, in the next section.

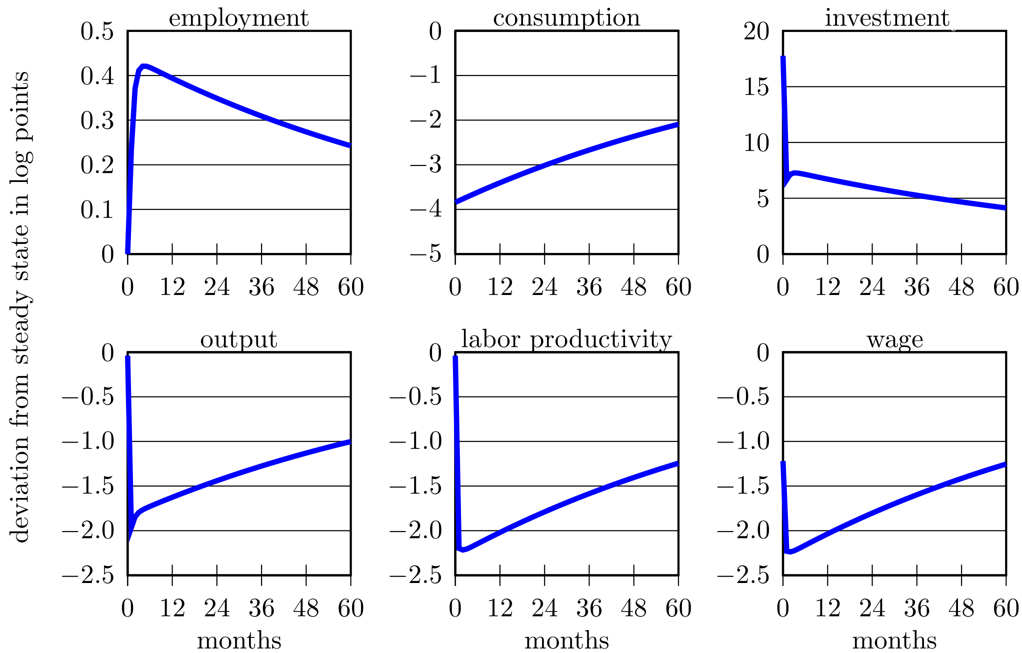

Figure 3: Impulse response to a transitory increase in the depreciation rate δ at $t = 0$, social planner's solution. Parameterization is the baseline: $\sigma = 1$, $\rho = 0$, and $\eta = 0.5$.

When I raise σ, the results improve, but only slowly. For example, set $\sigma = 2$. The calibration target for unemployment implies $\gamma = 0.63$, and so employed workers optimally consume $1+(\sigma-1)\gamma = 1.63$ times as much as unemployed workers. This is far larger than the reduction in consumption at retirement would suggest is plausible (Aguiar and Hurst, 2005). Still, the signs of the pairwise correlations between output, consumption, employment, and investment are unchanged.

With $\sigma = 10$, $\gamma = 0.24$ and employed workers optimally consume 3.15 times as much as unemployed workers. This strong complementarity between consumption and employment is enough to generate a positive pairwise correlation between the four variables. In particular, a one-time increase in the depreciation rate lowers each within a period of the shock (Figure 4). The low level of employment and investment imply that the recovery is extremely slow. Still, the resulting recession does not look like the recent U.S. experience because employment declines only four percent as much as output. This is a robust prediction of the model. Labor productivity always falls following an increase in the depreciation rate, which means that the decline in the capital stock must exceed the decline in employment, with the decline in output lying somewhere in between.

Finally, I consider the effect of other parameters on model outcomes. I focus on the case of $\sigma = 10$, since only in this case can the model generate a positive correlation between output,

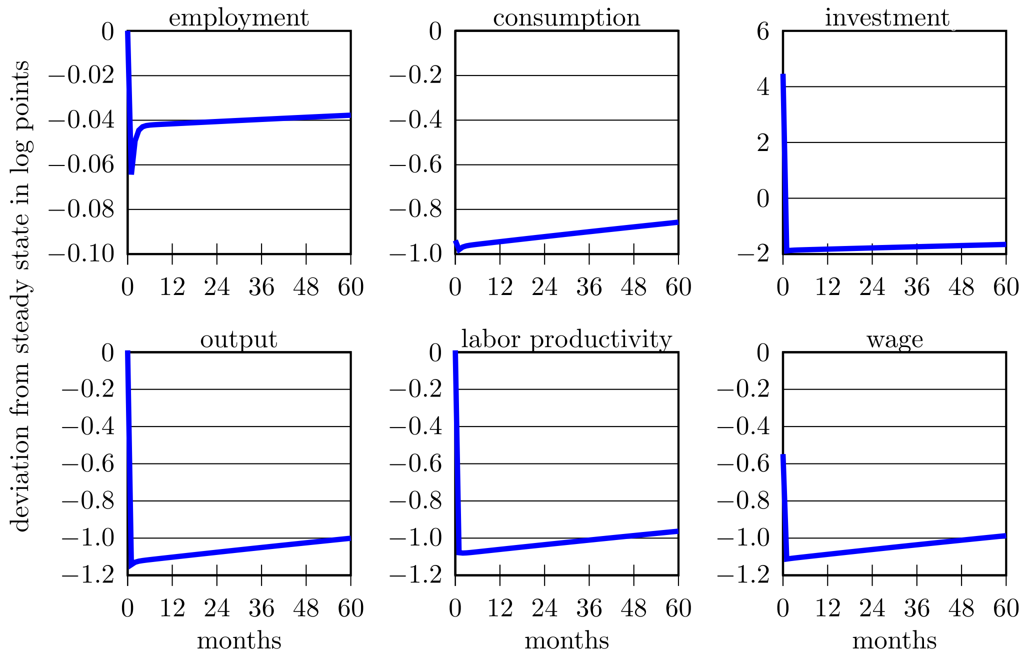

Figure 4: Impulse response to a transitory increase in the depreciation rate δ at $t = 0$, social planner's solution. Parameterization is $\sigma = 10$, $\rho = 0$, and $\eta = 0.5$.

employment, consumption, and investment. I find that the elasticity of the matching function is important for the volatility of employment. At one extreme, if $f(\theta) = \bar{\mu}$, employment is fixed at $n = x/(x + \bar{\mu})$. At the other, if $f(\theta) = \bar{\mu}\theta$, employment is more volatile than in the baseline calibration. Intuitively, diminishing returns to recruiting limits the willingness of the planner to increase recruiting in response to a shock that would otherwise encourage him to raise the level of employment. This issue disappears in the limit with $\eta = 1$. Still, with $\sigma = 10$ and $\eta = 1$, employment is only 0.36 times as volatile as output and so the model continues to have difficulty generating the large decline in employment that we have witnessed in recent years. The other comovements are not much affected by the specification of the matching function.

I also introduce autocorrelated shocks, setting $\rho = 0.98$. In this case, I lose the positive correlation between consumption on the one hand and employment and investment on the other hand. This is because the latter two outcomes rise following an increase in the depreciation rate before eventually falling below trend; on the other hand, the income effect drives down consumption immediately.

Table 1 summarizes the main comovements for the social planner's solution both in the baseline calibration and in the alternatives that I discussed above.

parameters			std. dev. (x)/ std. dev. (y)				
σ	ρ	η	n	c	i	θ	δ
1	0	0.5	0.23	2.10	4.55	9.51	202.59
10	0	0.5	0.04	0.86	2.08	1.65	172.03
10	0	1	0.36	0.90	1.75	7.53	133.08
10	0.98	0.5	0.56	2.04	10.85	22.94	69.24

			correlation (x, y)				
σ	ρ	η	n	c	i	θ	δ
1	0	0.5	−0.99	0.97	−0.85	−0.98	0.07
10	0	0.5	0.91	0.97	0.87	0.84	0.13
10	0	1	0.92	0.98	0.87	0.86	0.11
10	0.98	0.5	0.38	0.05	0.48	0.41	0.34

Table 1: Summary of results for social planner's solution. The top panel shows the relative standard deviation of five variables (employment n, consumption c, investment i, recruiter-unemployment ratio θ, and the shock δ compared to the standard deviation of output. Bottom panel shows the pairwise correlations of these variables with output.

3.2 Decentralization

As a prelude to introducing the rigid wage model, I discuss how the social planner's solution can be implemented through Nash bargaining over wages in a decentralized economy. Let $s \equiv (k, n)$ denote the endogenous aggregate state variables. I assume markets are complete. Rather than specify trade in securities that insure against aggregate risk, I assume that insurance takes place within a large household (Merz, 1995). Write the problem of a household with assets a as

$$V(a, \delta, s) = \max_{c_e, c_u, \{a'(\delta')\}} n \frac{c_e^{1-\sigma}\big(1 + (\sigma - 1)\gamma\big)^{\sigma}}{1 - \sigma} + (1 - n)\frac{c_u^{1-\sigma}}{1 - \sigma} + \beta \sum_{\delta'} \pi(\delta'|\delta) V(a'(\delta'), \delta', s')$$

subject to the budget constraint

$$a + w(\delta, s)n = nc_e + (1 - n)c_u + \sum_{\delta'} q(\delta'; \delta, s)a'(\delta'),$$

where $w(\delta, s)$ is the wage in state (δ, s), $q(\delta'; \delta, s)$ is the price of a security in state (δ, s) that pays off one unit of consumption if the next exogenous state is δ', and $a'(\delta')$ is the household's purchase of those securities; note that the next endogenous state s' is known at time t and so I do not explicitly include it in the asset price. As in the rigid wage model without search frictions, I assume that the household cannot control its employment rate n.

Place a Lagrange multiplier λ on the budget constraint. The first order condition for the consumption of employed and unemployed workers is

$$\left(\frac{c_e}{1+(\sigma-1)\gamma}\right)^{-\sigma} = c_u^{-\sigma} = \lambda.$$

As usual, let $c = nc_e + (1-n)c_u$ denote total consumption. Then

$$c_e = \frac{c(1+(\sigma-1)\gamma)}{1+(\sigma-1)\gamma n} \text{ and } c_u = \frac{c}{1+(\sigma-1)\gamma n}$$

and λ is the marginal utility of consumption for a hypothetical representative consumer with period utility

$$n\frac{c_e^{1-\sigma}\big(1+(\sigma-1)\gamma\big)^{\sigma}}{1-\sigma} + (1-n)\frac{c_u^{1-\sigma}}{1-\sigma} = \frac{c^{1-\sigma}\big(1+(\sigma-1)\gamma n\big)^{\sigma}}{1-\sigma}.$$

Next, the first order condition for the purchase of assets that pay off in state δ' is

$$\lambda q(\delta';\delta,s) = \beta\pi(\delta'|\delta)V_a(a'(\delta'),\delta',s').$$

The envelope condition is

$$V_a(a,\delta,s) = \lambda.$$

Combining these gives the consumption Euler equation:

$$\lambda = \frac{\beta\pi(\delta'|\delta)\lambda'}{q(\delta';\delta,s)}, \tag{12}$$

where λ and λ' are current and future marginal utility of consumption. Note that the household problem does not deliver a first order condition for employment, since the household cannot directly affect the level of employment.

Next, write the problem of a firm that owns $\tilde{k}$ units of capital and employs $\tilde{n}$ workers as

$$J(\tilde{k},\tilde{n},\delta,s) = \max_{i,\nu,\tilde{k}',\tilde{n}} A\tilde{k}^{\alpha}(\tilde{n}(1-\nu))^{1-\alpha} - w(\delta,s)\tilde{n} - i + \sum_{\delta'} q(\delta';\delta,s)J(\tilde{k}',\tilde{n}',\delta',s')$$

subject to the constraints

$$\tilde{k}' = (1-\delta)\tilde{k} + i,$$
$$\tilde{n}' = (1-x+\nu\mu(\theta))\tilde{n}.$$

Here ν represents the fraction of employees that the firm puts into recruiting. In addition, $\mu(\theta) \equiv f(\theta)/\theta$ is the number of hires per recruiter. Note that recruiting is constant returns to scale at the firm level, although it generally has diminishing returns at the aggregate level. In equilibrium, the firm's level of capital and employment is equal to the economy-wide level, $\tilde{k} = k$ and $\tilde{n} = n$.

The first order conditions for investment and recruiting are:

$$1 = \sum_{\delta'} q(\delta'; \delta, s) J_k(k', n', \delta', s'),$$
$$(1-\alpha)A\kappa^{\alpha} = \mu(\theta) \sum_{\delta'} q(\delta'; \delta, s) J_n(k', n', \delta', s'),$$

where κ is again the capital-producer ratio. The envelope conditions are

$$J_k(k, n, \delta, s) = \alpha A \kappa^{\alpha-1} + (1-\delta) \sum_{\delta'} q(\delta'; \delta, s) J_k(k', n', \delta', s'),$$
$$J_n(k, n, \delta, s) = (1-\nu)(1-\alpha)A\kappa^{\alpha} - w + (1 - x + \nu\mu(\theta)) \sum_{\delta'} q(\delta'; \delta, s) J_n(k', n', \delta', s'),$$

where I denote the current wage as w for simplicity. Eliminate next period's variables using the first order conditions:

$$J_k(k, n, \delta, s) = \alpha A \kappa^{\alpha-1} + 1 - \delta,$$
$$J_n(k, n, \delta, s) = (1-\alpha)A\kappa^{\alpha} \left(\frac{1 - x + \mu(\theta)}{\mu(\theta)} \right) - w.$$

Next combine these with the envelope conditions to get

$$1 = \sum_{\delta'} q(\delta'; \delta, s) \left(\alpha A \kappa'^{\alpha-1} + 1 - \delta' \right),$$
$$(1-\alpha)A\kappa^{\alpha} = \mu(\theta) \sum_{\delta'} q(\delta'; \delta, s) \left((1-\alpha)A\kappa'^{\alpha} \left(\frac{1 - x + \mu(\theta')}{\mu(\theta')} \right) - w' \right),$$

where w' is the wage in the next period if the exogenous state is δ'. Finally, eliminate q using the consumption Euler equation (12):

$$\lambda = \beta \sum_{\delta'} \pi(\delta'|\delta) \lambda' \left(\alpha A \kappa'^{\alpha-1} + 1 - \delta' \right), \tag{13}$$

$$(1-\alpha)A\kappa^{\alpha}\lambda = \beta\mu(\theta) \sum_{\delta'} \pi(\delta'|\delta) \lambda' \left((1-\alpha)A\kappa'^{\alpha} \left(\frac{1 - x + \mu(\theta')}{\mu(\theta')} \right) - w' \right). \tag{14}$$

Equation (13) is identical to the planner's first order condition for investment, equation (10). Equation (14) differs from equation (11), however because of the presence of the wage.

Suppose that workers bargain with employers to set wages. The outcome is described by the Nash bargaining solution. A worker's threat point while bargaining is that he becomes unemployed, while a firm's threat point is that it loses the worker. Moreover, let $\phi \in (0,1)$ denote the worker's bargaining power. Shimer (2010) proves that the wage satisfies

$$w = \phi(1-\alpha)A\kappa^{\alpha}(1+\theta) + (1-\phi)\frac{\gamma\sigma c}{1+(\sigma-1)\gamma n}. \tag{15}$$

This is a weighted average of two terms. The first term is the marginal product of labor. This accounts both for the output the worker produces $(1-\alpha)A\kappa^{\alpha}$, and for the fact that, if bargaining fails, the size of the firm shrinks and the firm must place additional workers into recruiting in order to maintain the same size the next period. The second term is the marginal rate of substitution between consumption and leisure. In the frictionless model, these two terms are equal and both are equal to the wage. With search frictions, the marginal product of labor generally exceeds the marginal rate of substitution and the wage lies between these two levels. At this wage, firms are happy to hire workers and households are happy to supply labor.

In general, the equilibrium does not decentralize the planner's problem. But suppose the matching function $f(\theta)$ has a constant elasticity, $f(\theta) = \bar{\mu}\theta^{1-\eta}$, so $\mu(\theta) = \bar{\mu}\theta^{-\eta}$ for some $\eta \in [0,1]$. If $\eta = \phi$, one can verify that equations (14) and (15) reduce to the planner's condition in equation (11), and so the decentralized equilibrium is efficient. This is a generalization of the Mortensen (1982)–Hosios (1990) efficiency condition. Conversely, any other wage path that has a different expected present value leads to an inefficient equilibrium.[5]

3.3 Rigid Wage Model

The rigid wage model is identical to the decentralization above, except that the wage does not satisfy the Nash bargaining solution, but is instead constant at some level regardless of the sequence of shocks. I verify that under appropriate conditions, firms do not want to fire their workers at that wage and workers do not want to quit their jobs. Thus, in contrast to the neoclassical growth model, in the search model I do not need to specify differently how the level of employment is determined in the flexible and rigid wage economies.

[5] Other wage paths may have the same expected present value but specify spread risks differently across workers and firms. For example, an employed worker may earn a constant wage while employed, although the wage depends on the labor market conditions when he is hired. This changes measured wages but does not alter the efficiency of equilibrium (Shimer, 2004).

Before proceeding to the business cycle analysis, some comparative statics are useful for understanding how the search model differs from the frictionless one. In the model without search frictions and a rigid wage, the steady state was degenerate unless a borderline condition, $\beta R = 1$, held. This is no longer true with search frictions. Instead, the steady state version of the first order condition for investment, equation (13), pins down the capital-producer ratio κ:

$$1 = \beta\left(\alpha A \kappa^{\alpha-1} + 1 - \delta\right).$$

Then the steady state version of the first order condition for recruiting, equation (14) pins down the recruiter-unemployment ratio θ as a function of the wage:

$$w = (1-\alpha)\kappa^{\alpha} A\left(1 - \frac{x + \frac{1}{\beta} - 1}{\mu(\theta)}\right).$$

Note that this condition ensures that the wage is always smaller than the full marginal product of labor, $(1-\alpha)A\kappa^{\alpha}(1+\theta)$. Thus firms are always willing to employ workers at the equilibrium wage. Next, the steady state equation for employment pins down n:

$$n = \frac{f(\theta)}{x + f(\theta)}.$$

Using $\kappa = k/(n - \theta(1-n))$, one then obtains the level of capital k. Finally, the steady state equation for capital pins down consumption:

$$c = \left(A\kappa^{\alpha-1} - \delta\right)k.$$

The wage does not affect κ, but an increase in the wage raises $\mu(\theta)$ and so reduces θ. This in turn lowers employment. In general, the effect on capital and consumption is ambiguous, although it may be natural to focus on parameters for which an increase in recruiters raises the steady state number of producers $n - \theta(1-n)$; this is always the case at the social optimum. But the important point to note is that a range of values for the wage is consistent with a well-behaved steady state equilibrium.

It only remains to check whether the wage exceeds the marginal rate of substitution between consumption and leisure, so workers are willing to participate in the labor market:

$$w \geq \frac{\gamma\sigma c}{1 + (\sigma-1)\gamma n}.$$

Unfortunately, I cannot proceed analytically. Instead, suppose suppose I fix the wage at the socially optimal level when $\delta = \bar{\delta} = 0.0046$. I then examine how changes in δ affect whether this condition holds. In our baseline calibration, I find that as δ varies between 0.00456 and 0.00468, the steady state job finding probability falls from 1 to 0 and so the employment rate falls from $1/(1+x)$ to 0. Moreover, for any δ in this interval, the wage exceeds the marginal rate of substitution between consumption and leisure and so workers are happy to take a job when they find one. As δ continues to fall, it is natural to fix the job finding probability at $f(\theta) = 1$ and so $\mu(\theta) = 1/\theta$. Eventually, if $\delta < 0.0026$, workers' participation constraint is violated. At this fixed wage, households are no longer willing to supply all the labor that firms demand. But over a large range of parameters, the rigid wage is consistent with workers willingly supplying labor to firms and firms willingly hiring workers.

This example illustrates another point: although a steady state equilibrium exists for a range of δ even when the wage is fixed, the steady state employment rate is extremely sensitive to δ. A 1 percent increase in δ reduces the steady state employment rate from $n = 0.95$ to 0.89. A 2 percent increase in δ shuts down the labor market entirely. This occurs because the increase in δ lowers the steady state capital-producer ratio κ. This in turn lowers the marginal product of labor and so reduces the profitability of recruiting a worker at a fixed wage. With less recruiting, employment falls as well.

To study the model out of steady state, I use the same parameter values as in the social planner's problem, with the wage set at a level such that the equilibrium of the deterministic economy with $\delta = \bar{\delta} = 0.0046$ is socially optimal. I find that in the rigid wage model, there is a strong positive comovement between output, employment, consumption, and investment for a broad range of paramterizations of the model. Moreover, output, consumption, and employment have nearly the same volatility, with investment somewhat more volatile. The higher volatility of investment occurs because an adverse shock causes a one period investment boom before investment collapses by about the same amount as the other three outcomes. On the other hand, labor productivity scarcely moves after the shock and the wage is constant by assumption.

These findings hold regardless of the value of the complementarity parameter σ and regardless of the elasticity of the matching function. This is in contrast to the flexible wage model, where the specification of preferences was critical for the model's predictions and the matching technology was critical for the volatility of employment. If the shocks are strongly autocorrelated, the correlation between investment and other outcomes starts to breakdown, but the results are otherwise unchanged. Table 2 summarizes these comovements in the rigid wage model, while Figure 5 shows the response to a shock at time 0 both with $\sigma = 1$ and with $\sigma = 10$.

Finally, note that in a flexible wage economy, the wage would fall after this adverse shock.

parameters			std. dev. (x)/ std. dev. (y)				
σ	ρ	η	n	c	i	θ	δ
1	0	0.5	0.98	1.05	1.94	39.86	70.98
10	0	0.5	0.98	0.97	1.29	39.97	69.49
10	0	1	1.00	0.99	1.15	20.42	58.16
10	0.98	0.5	1.05	1.42	3.95	42.29	26.21

			correlation (x, y)				
σ	ρ	η	n	c	i	θ	δ
1	0	0.5	0.97	0.94	0.53	0.94	0.20
10	0	0.5	0.97	0.99	0.91	0.94	0.20
10	0	1	0.96	1.00	0.93	0.92	0.16
10	0.98	0.5	0.96	0.80	0.09	0.95	−0.18

Table 2: Summary of results for the rigid wage model. The top panel shows the relative standard deviation of five variables (employment n, consumption c, investment i, recruiter-unemployment ratio θ, and the shock δ) compared to the standard deviation of output. The bottom panel shows the pairwise correlations of these variables with output.

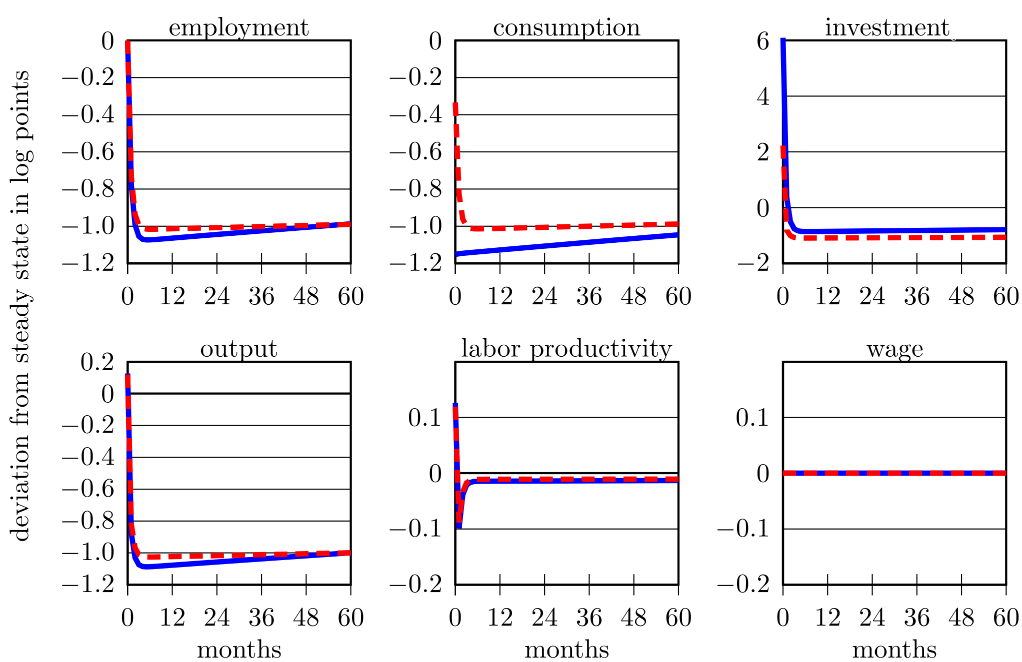

Figure 5: Impulse response to a transitory increase in the depreciation rate δ at $t = 0$, rigid wage model. The solid blue line shows $\sigma = 1$, the dashed red line shows $\sigma = 10$. In both parameterizations, $\rho = 0$ and $\eta = 0.5$.

It follows that the rigid wage is always higher than the marginal rate of substitution between consumption and leisure. Put differently, when the unemployment rate is high because wages are too high, individuals are happy when they have an opportunity to work. In addition, the wage is less than the marginal product of labor as long as the shock is not so large that firms stop recruiting new workers. Thus the model can handle a sharp slowdown in recruiting following an adverse reallocation shock without necessitating any change in the wage.

4 Conclusion

This paper argues that reallocation shocks which destroy the value of some portion of the capital stock can explain a persistent decline in output, employment, consumption, and investment if wages do not fall after the shock. This is consistent with individually rational behavior in a search model: firms are still willing to hire workers at the high wage, but they simply cut back on recruiting; and workers are happy when they are able to get a job at this high wage. The conclusion is robust to the specification of preferences, the stochastic process for shocks, and the nature of the matching technology, arguably the most controversial ingredients in the theory.

I have modeled reallocation shocks as an increase in the depreciation rate of capital. In a richer model, one could imagine firms producing heterogeneous goods. The final good, used both for consumption and investment, is an aggregate of the intermediate goods. When final goods are invested, the capital becomes specific to a particular variety of intermediate good. Finally, idiosyncratic shocks change the composition of intermediate goods used to produce the final good. When a shock reduces the demand for a particular variety, effectively the associated capital becomes worthless. In reduced form, periods when the composition of final goods changes sharply, for example because of technological change or increased availability of imports, look like periods when the depreciation rate of capital is high. However, a model that explicitly considers an environment with heterogeneous goods may be able to account for the rise in layoffs that typically occurs early in a recession.[6]

The theory that I explored in this paper may also explain endogenous total factor productivity fluctuations. The important assumption is that the capital stock that we measure in the data does not account for the loss in value from the depreciation shocks. This means that during a recession, the measured capital stock exceeds the true capital stock and the Solow residual declines. This is consistent with evidence that total factor productivity is

[6]Den Haan, Ramey, and Watson (2000) develop a search and matching model in which labor is specific to a firm and so an adverse idiosyncratic productivity shock results in a surge in layoffs. In an extension, they consider a model where capital is also specific to the firm, but only for one period. After that, capital can be reallocated to any other firm. They do not consider the implications of wage rigidities in their framework.

typically procyclical (Kydland and Prescott, 1982).

I have ignored the government sector in this paper. One interesting question is what impact countercyclical government spending has in this type of model. It is straightforward to introduce a government financed by lump-sum taxes. In particular, with lump-sum taxes, Ricardian equivalence holds and so the timing of taxation is irrelevant. In a flexible wage model, wasteful government spending, or government spending that enters additively into individuals' utility functions, raises labor supply by making households feel poorer. This carries over to the search model as well. An increase in government spending lowers wages and labor productivity and raises output, though by less than the increase in spending. With rigid wages, an increase in government spending crowds out private investment. The decline in the capital stock then acts much like a depreciation shock, reducing employment, consumption, and output. Thus this model, despite its arguably Keynesian rigid wage and noncompetitive labor market, predicts that countercyclical fiscal expansions will have perverse and unintended consequences.

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