The Propagation of Technology Shocks: Do Good, Labor and Credit Market Imperfections Matter and How Much?*

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Abstract

Investigating mechanisms of propagation has been central to the real business cycle literature (any dynamic GE model in fact) since its inception. We build a model with three imperfect markets - credit, labor and good market - and attempt to answer the question of which is the main source of persistence and amplification of productivity shocks. We confirm that financial frictions amplify the volatility of economies, however by a smaller margin than in earlier work. In addition, we find that the introduction of good market frictions drastically change the qualitative and quantitative dynamics of the model, leading to significant improvements of second moments. Two factors seem to generate persistence : first, the way past income affects consumer's search for goods; and second, the way past income affects the dynamics of prices. The latter depends on the complementarity or substitutability in consumer's utility between the good that is subject to frictions (manufactured good) and the frictionless numeraire. In particular, under complementarity, prices include both lagged terms (income), contemporaneous terms (productivity and production costs) and lead values (expected surplus growth). Out of the steady-state, the dynamics are therefore slowed down considerably by search frictions in the good market. The model finds positive autocorrelation of labor market variables due to good market frictions, but fails to reach that in the data by a factor 2, while in the absence of goods market frictions there is none.

*Preliminary notes

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1 Introduction

Since its inception the Real Business Cycle literature has faced the same challenge, emphasized in King and Rebelo (2001) and Cogley and Nason (1995): that of the propagation of technological shocks. In the standard RBC model, it is necessary to assume large technological shocks in order to obtain realistic business cycle fluctuations. However, the model fails to generate the amount of autocorrelation in the growth rate of output that we see in the data. This twin failing in the lack of both the amplification and the persistence has generated separate literatures that either argue different values for key parameters or incorporate various frictions to specific markets. In this paper, we build a model with three imperfect markets - credit, labor and goods markets - and attempt to answer the question about which market is the main source of persistence and amplification of these shocks.

An early literature asked whether and how amplification comes from the **labor market**, by either increasing the elasticity of labor supply, e.g. models of indivisible labor as in Hansen (1985) and Rogerson (1988), or the introducing of a market friction in the form of wage rigidity.[1] The importance of the latter for amplifying the reponse of the demand for labor to changes in productivity has received renewed attention in search models of equilibrium unemployment (Pissarides, 1985 and Mortensen and Pissarides, 1994). Indeed, wage rigidity can address the lack of volatility in of job vacancies and unemployment, a deficiency in the canonical model shown in Cole and Rogerson (1999) and Shimer (2005).[2]

Early papers such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997) emphasized the role of **credit markets** in amplifying exogenous shocks to economies and the existence of a *financial accelerator*.[3] Although this literature has focused on the role of credit market imperfections for the elasticity of investment to innovations and detailing a credit channel of monetary policy, the financial accelarator has been shown to be a significant source of labor market dynamics in Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2010).

In the **goods market**, and at least since Keynes' (1936) general theory of employment, interest and money, it has been recognized that disequilibrium in the good market can generate additional unemployment. Several waves of research have attempted to put this intuition into models: Barro and Grossman (1971) and later Benassy (1982, 1986, 1993) and Malinvaud (1977), the neo-Keynesian literature and finally its synthesis with the RBC literature. Note that this previous literature has mostly been centered around the idea of price rigidities leading to excess supply (or demand) of goods and in turn generating disequilibrium in the labor market. In our paper, goods market imperfections arise without price rigidity. The source of persistence and volatility does not rely solely on price rigidi-

[1]Reference long literature from staggered wage contracts to monopolistic competition in the supply of labor services.

[2]See Hall (2005). Alternative is to set wages close enough to the value of leisure (the small surplus assumption in Hagedorn and Manovskii 2008) which itself amounts to raising the degree of wage rigidity as wages are more disconnected from labor productivity. Other mechanisms were suggested in the literature, such as the existence of on-the-job search (Mortensen and Nagypàl 2007) who also discuss the need to introduce additional sources of shocks on the demand side.

[3]See also , Bernanke and Gertler (1995), Bernanke Gertler and Gilchrist (1996).

ties, although the size of the mark-ups of prices over marginal costs will matter for the dynamics.

Our model is built as follows. Following Pissarides (1985), we introduce matching frictions in the labor market, where the number of contacts between unemployed workers and vacant jobs is a constant return to scale matching function. The relevant measure of disequilibrium is therefore the ratio of the two quantities (vacancies over unemployment), the so-called labor market tightness. Following Wasmer and Weil (2004), we further introduce matching frictions in the credit market of firms, where the number of contacts between banks and entrepreneurs is a constant return to scale matching function. The relevant measure of disequilibrium is therefore the ratio of the two quantities (moneyless entrepreneurs and banks endowed with excess liquidity), the so-called credit market tightness. Finally, in this paper, we introduce matching frictions in the good market, where the number of contacts between consumers and sellers is a constant return to scale matching function. The relevant measure of disequilibrium is therefore the ratio of the two quantities (firms willing to sell and consumers willing to consume), the so-called goods market tightness.[4]

Interestingly, there is a growing literature attempting at measuring gross and net creation and destruction flows in the three markets (labor, credit and goods). Following the seminal contributions in the labor markets (Davis and Haltiwanger, 1990, 1992), Del'Arricia and Garibaldi (2005) have measured creation and destruction of loans in US banks, while a recent contribution by Broda and Weinstein (2010) has carefully documented the magnitude of flows of entry and exits of goods, as well as procyclical features of net product creation flows. There has also been a revival of the interest in the impact of demand shocks in the good markets and their implications for the dynamic implications of the RBC literature (see Bai, Rios-Rull and Storesletten, 2010).

Our paper goes in this direction and confirms the interest of modelling imperfect goods markets. First, we find, consistent with the earlier literature, that financial frictions amplify the volatility of economies by a factor that can be quantified using a calibration of the share of the financial sector. The multiplier (how much extra volatility we obtain due to financial frictions) is however small compared to what we found in Petrosky-Nadeau and Wasmer (2010) .

Second, the introduction of good market frictions drastically change the qualitative dynamics of the model. In particular, the response on labor market tightnesss to productivity shocks is hump-shaped, or highly persistent, due to the persistence of income dynamics. This persistence arises from the fact that firm profits affect the income of consumers and then the demand side, generating interesting intertemporal linkages. These mechanisms are absent from the standard labor search model and the labor search model augmented with financial frictions which produce virtually no persistence in labor market variables. The exact dynamics depends two factors: first, the way past income affects consumer's search for goods ; and second, the way past income affects the dynamics of prices. The latter depends on the complementarity or substitutability in consumer's utility between the good that is subject to frictions (manufactured good) and the frictionless numeraire. In particu-

[4]Here, we analyze the steady-state properties of the model and develop its dynamics, while Wasmer (2009) focus on its steady-state properties and the ability of the model to match European economies.

lar, under complementarity, prices includes both lagged terms (income), contemporaneous terms (productivity and production costs) and lead values (expected surplus growth).

Third, good market imperfections lead to significant improvement of second moments of the model, bringing the model close to the data to the United Stated in terms of volatility. Out of the steady-state, the dynamics of the labor market are therefore slow and complex only when prices are endogenously determined through bargaining due to consumer search frictions. Moreover, we find that productivity shocks can be either lead to either price declines or increases depending on the relative bargaining power of consumers.In addition, the standard deviation of labor market tighness relative to GDP is augmented by a factor of 3, compared to the standard labor search model. However, at this stage, the model still fails to find the very right positive autocorrelation of labor market tightness, even though it does better than the variants of the models with perfect good markets where the autocorrelation is close to zero.

This paper is organized as follows. In Section 2, we introduce the main dynamic equations of the model. Section 3 discuss some properties of the dynamic equilibrium conditions and the deterministic steady state. Section 4 provides quantitative results for the dynamics.

2 An economy with credit, labor and goods market frictions

Time is discrete. There are three categories of agents. Banks, which have the exclusive ability to transfer value from one period to another. Firms, who have the ability to hire workers, produce output and sell it. Workers, who are required in production. Workers are also the only consumers. At the beginning of each period, nature selects a new value of productivity. All entry decisions are then taken. Financial contracts are negotiated after the productivity has been observed. Wages and prices also reflect productivity changes, in a way specified later on. Next periods are anticipated given expectations on future values of the productivity shocks.

2.1 Entrepreneurs and banks

The lifetime of a firm is divided into 4 different sequences of random length.

Initially, the firm is moneyless and cannot start operating without having first a financial partner (hereafter called a "banker"). For that, it prospects on a credit market and pays a per-period effort cost e assumed to be inelastic. With probability p_t it finds a banker, with complementary probability it remains in this stage (denoted by c like *credit*). We denote by E_c the asset value of the entrepreneur in this stage. At the end of the current period, that is, the time of the meeting between the bank and the firm, both sides agree on the terms of a financial contract whereby the firm is financed by the bank when its cash-flow is negative (in stages 2 and 3) and reimburses when the cash-flow is positive (in stage 4). Importantly, we assume that the agreement between the bank and the firm precludes the financing of any cost beyond the labor markets : only wages and hiring costs will be financed by the bank. This implies in particlar that operating cost of producing will not be covered by the bank. Hence, the firm will start producing only when it has met a demand for its good.

With a banker, the firm accesses the **second stage**, where it prospects on the labor market in order to engage in production. It must pay a per-period cost γ to maintain an active job vacancy. With probability q_t the firm is successful in hiring the worker, with complementary probability it remains in this stage (denoted by l like *labor*). We denote by E_l the asset value of an entrepreneur in this stage. The firm and the worker sign a contract in which the firm pays a wage w_t as long as the firm is active.

In the next, **third stage**, endowed with a banker and a worker, the firm could start producing y_t units of output and attempts to sell it on the goods market and pay operating costs Ω_t. However, remember that the financial contract prevents the firm to produce without generating revenues. Hence, in stage 3, the firm only pays for wages (financed by the bank) and simultaneously searches a consumer at no extra cost. The meeting with a consumer comes with probability λ_t, while with complement probability $1-\lambda_t$ the production remains unsold and is therefore not produced. We denote this stage by g as *good* market and by E_g the asset value of the entrepreneur in this stage. Note that the bank is still financing the firm by transferring the amount of cash necessary to pay the worker. Finally, at the end of the period, and with probability s, the firm is destroyed.

Finally comes the **fourth stage**. Endowed with a banker, a worker and a consumer, the firm can now sell its output and obtain a price P_t for it. With revenue $P_t y_t$, the firm pays the worker w_t, the operating cost Ω_t, pays the bank an amount (pre-determined in the early stage) ρ_t and enjoys the difference. We denote this stage by π standing for profit and by E_π the asset value of being in this stage. At the end of the period, the firm can be destroyed with probability s. The consumer may stop consuming the particular good produced by the firm with probability τ.

Finally, we assume as in Pissarides (2000) that any profit opportunities are exhausted by firms, so that at the entry stage, the asset value of a firm must be zero. As in Wasmer and Weil (2004), this implies that $E_{c,t} \equiv 0$ at any time, and that firms facing a destruction shock s have a zero value.

Given these assumptions, the Bellman equations of the entrepreneur, which faces a discount rate r, can then recursively be written as follows, in which we assume that transitions from the credit to the labor market stage occur within a single period:

$$
\begin{aligned}
E_{c,t} &= 0 = -e + p_t E_{l,t} && (1)\\
E_{l,t} &= -\gamma + \gamma + \frac{1}{1+r}\mathbb{E}_t\left[q_t E_{g,t+1} + (1-q_t)E_{l,t+1}\right] && (2)\\
E_{g,t} &= -w_t + w_t + (1-s)\frac{1}{1+r}\mathbb{E}_t\left[\lambda_t E_{\pi,t+1} + (1-\lambda_t)E_{g,t+1}\right] && (3)\\
E_{\pi,t} &= \mathscr{P}_t y_t - w_t - \rho_t - \Omega_t + (1-s)\frac{1}{1+r}\mathbb{E}_t\left[(1-\tau)E_{\pi,t+1} + \tau E_{g,t+1}\right] && (4)
\end{aligned}
$$

The bank's lifetime closely follow that of the entrepreneur. The bank's asset values are denoted by B_j, $j = c, l, g$ or π for each of the stages. In stage c, it prospects on the credit market to find a viable project to finance. We also assume free entry of the banking relationship such that at all times $B_{c,t} = 0$. We denote by κ the screening cost per unit of time of banks in the first stage, and by $\widehat{p_t}$ the Poisson rate at which a bank finds a firm to be financed. In stage l, the bank pays the cost of a vacancy γ and waits for the hiring to be realized. In stage g, the bank now pays the wage cost w_t and waits for the firm to be matched with a consumer. In stage π, the bank cashes in the repayment ρ_t.

The corresponding asset values for the banker are

$$B_{c,t} = 0 = -\kappa + \widehat{p_t} B_{l,t} \tag{5}$$

$$B_{l,t} = -\gamma + \frac{1}{1+r}\mathbb{E}_t\left[q_t B_{g,t+1} + (1-q_t)B_{l,t+1}\right] \tag{6}$$

$$B_{g,t} = -w_t + (1-s)\frac{1}{1+r}\mathbb{E}_t\left[\lambda_t B_{\pi,t+1} + (1-\lambda_t)B_{g,t+1}\right] \tag{7}$$

$$B_{\pi,t} = \rho_t + (1-s)\frac{1}{1+r}\mathbb{E}_t\left[(1-\tau)B_{\pi,t+1} + \tau B_{g,t+1}\right] \tag{8}$$

2.2 Matching and bargaining in the credit market

The matching rates p_t and $\widehat{p_t}$ are made mutually consistent by the existence of a matching function $M_C(\mathscr{B}_{c,t}, \mathscr{N}_{c,t})$, where $\mathscr{B}_{c,t}$ and $\mathscr{N}_{\mathrm{c,t}}$ are respectively the number of bankers and of entrepreneurs in stage c. This function is assumed to have constant returns to scale. Hence, denoting by $\phi_{,t}$ the ratio $\mathscr{N}_{\mathrm{c,t}}/\mathscr{B}_{c,t}$, which is a reflection of the tension in the credit market and that we shall call credit market tightness from the point of view of entrepreneurs, we have

$$p_t = \frac{M_C(\mathscr{B}_{c,t}, \mathscr{N}_{\mathrm{c,t}})}{\mathscr{N}_{\mathrm{c}}} = p(\phi_t) \text{ with } p'(\phi_t) < 0. \tag{9}$$

$$\widehat{p_t} = \phi_t p(\phi_t) \text{ with } \widehat{p}'(\phi_t) > 0. \tag{10}$$

The bank and the entrepreneur engage in bargaining about ρ upon meeting and the Nash-bargaining condition is therefore such that

$$(1-\beta)B_{l,t} = \beta E_{l,t} \tag{11}$$

where $\beta \in (0,1)$ is the bargaining power of the bank relative to the entrepreneur. With $\beta = 0$ the bank leaves all the surplus to the entrepreneur. Note that the value of ρ is determined at the time of the meeting but paid a few periods ahead from the negotiation, when the firms becomes profitable. We assume that there is no commitment problem (as in Wasmer and Weil 2004) so that any new realization of aggregate productivity will not undo the financial contract and there is no renegotiation.[5]

[5] Any possible default is assumed away by the assumption that there is a transfer across firms in case of a default, thanks to a community chest. With risk neutral firms, this assumption does not affect the ex-ante values of firms.

Combining (1), (5) and (11), as well as the definition of $\widehat{p}$ in (10), we can obtain the equilibrium value of ϕ_t denoted by ϕ^* with

$$\phi^* = \frac{\kappa}{e}\frac{1-\beta}{\beta}\forall t \tag{12}$$

The double free-entry condition of both banks and entrepreneurs on credit markets implies a credit market tightness that is constant over time, even out of the steady-state. Also, it is useful to characterizes the total transaction costs paid by both firms and banks in stage c as

$$K(\phi) \equiv \frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} \tag{13}$$

2.3 Matching in the labor market

We assume that matching in the labor market is denoted by $M_L(\mathcal{N}_{l,t}, u_t)$ where u_t is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1. $\mathcal{N}_{l,t}$ is the number of firms in stage l, or the number of "vacancies." The function is also assumed to be constant return to scale, hence the rate at which firms fill vacancies is a function of the ratio $\mathcal{N}_{l,t}/u_t$, that is tightness of the labor market, denoted by θ_t. We have

$$q(\theta_t) = \frac{M_L(\mathcal{N}_{l,t}, u_t)}{\mathcal{N}_{l,t}} \text{ with } q'(\theta_t) < 0.$$

Conversely, the rate at which the unemployed find a job is

$$\frac{M_L(\mathcal{N}_{l,t}, u_t)}{u_t} = \theta_t q(\theta_t) = f(\theta_t) \text{ with } f'(\theta_t) < 0.$$

Once employment, workers earn a wage w_t which we assume, for simplicity, takes the functional form

$$w_t = \chi_w(\mathcal{P}_t y_t)^{\eta_w} \tag{14}$$

where η_w can be interpreted as the elasticity of wages to the marginal product of labor $\mathcal{P}_t y_t$. Gertler and Triggary (2009) argue that the elasticity η_w is close to 0.5, a value we will retain in the calibration section.

Note that, in the spirit of search models, one may want to have a different wage determination schedule, where the wage would be the outcome of Nash-bargaining between the firm and the worker. We decided to avoid to complications implied by Nash-bargaining in this context.[6]

[6] Complications are as follows. First, given that firms pays the worker in two different stages (when it does not produce and when it does), this would imply not one but two wage schedules, with analytical complications but for a small quantitative difference since the surplus value of the firm in each stage are very close and exactly equal when the discount rate is small compared to the rate at which it finds a consumer, a plausible assumption. Hence, a smimilar wage rule in the two stages is a quantitatively good assumption. Second, the full Nash-bargaining game has additional complexities uncovered in Wasmer and Weil (2004): given the wages will depend - negatively - on the flow value of reimbursement to the firm, banks and firms will recognize this ex-ante and therefore have incentives to inflate the debt and the subsequent reimbursement in the entry stage, to expropriate workers. This almost drives down wages to reservation wages, which we want to ignore here by choosing a rather simple wage determination rule.

2.4 Income, demand and matching in the goods market

2.4.1 Disposable income and the demand for goods

The total net flows of profits Π_t in the economy is the sum of banks profits and firms profits and is given by

$$\begin{aligned}\Pi_t &= (y_t\mathscr{P}_t - w_t - \Omega_t - \rho_t)\mathscr{N}_{\pi,t} + \rho_t\mathscr{B}_{\pi,t} - \gamma\mathscr{N}_{l,t} - \kappa\mathscr{B}_{c,t} \\ &= (y_t\mathscr{P}_t - \Omega_t)\mathscr{N}_{\pi,t} - w_t(\mathscr{N}_{g,t} + \mathscr{N}_{\pi,t}) - \gamma\mathscr{N}_{l,t} - \kappa\mathscr{B}_{c,t}\end{aligned}$$

where $\mathscr{B}_c$,$\mathscr{B}_\pi$, $\mathscr{N}_l$, $\mathscr{N}_g$ and $\mathscr{N}_\pi$ respectively the number of banks in stage c and π and firms in stage l, g and π.[7] Note that by construction, the second and the fifth of these stocks are equal. In the second equation, the first term is the revenue of bank/firm in stage 4 net of operating costs, the second term represent wages in the economy, while the remaining terms represent the negative cash-flows of the bank during the first stages due to prospection costs in labor and credit market. These profits net of search costs are pooled and distributed lump sum to workers. The mass 1 of workers (the unemployed and the employed) therefore receive per person and per period Π as a cash transfer.

Further, resources are pooled across categories of workers, as in Merz (1995) and Andofaltto (1996). Therefore, the average income of a representative consumer is simply

$$\begin{aligned}Y_t &= \Pi_t + (1-u_t)w_t \\ &= \Pi_t + (\mathscr{N}_{g,t} + \mathscr{N}_{\pi,t})w_t \\ &= \mathscr{P}_t y_t \mathscr{N}_{\pi,t} - \gamma\mathscr{N}_{l,t} - \kappa\mathscr{B}_{c,t} - \Omega_t\end{aligned}$$

which is indeed the income measure of GDP in this search economy. To simplify this expression, note that the flows into the recruiting stage in every period are $\phi^* p(\phi^*)\mathscr{B}_{c,t}$, which are equal the flows out of the recruiting stage $q(\theta_t)\mathscr{N}_{l,t}$. Therefore $\mathscr{B}_{c,t} = \frac{q(\theta_t)\mathscr{N}_{l,t}}{\phi^* p(\phi^*)}$ such that average income Y_t is $\mathscr{P}_t y_t \mathscr{N}_{\pi,t} - \left(\gamma + \frac{\kappa q(\theta_t)}{\phi p(\phi)}\right)\mathscr{N}_{l,t} - \Omega_t$. This measure corresponds to the protential demand for consumption goods, not all of which will be satisfied due to frictions in the goods market which we detail next.

2.4.2 Goods market matching

Each firm produces a unique variety of a manufactured good. Potential consumers need to search before being able to consume. When they cannot find a good to consume during the period, they may spend their income into a numeraire good not subject to search frictions. Therefore, GDP Y_t is split into manufactured good and the numeraire good. We do not model the supply of the numeraire good which is supplied with a CRS technology and under a zero-profit condition. When a consumer is matched with a manufacturing firm, it buys as much as he can, that is y_t at a price $\mathscr{P}_t$.

[7] As a note, total employment consists of all firms on the goods market, selling or making profits, that is, $\mathscr{N}_g + \mathscr{N}_\pi = \mathscr{N} = 1 - u$.

At any point in time, there are unmatched consumers and matched consumers in this economy. Denote by $\mathscr{C}_{0,t}$and $\mathscr{C}_{1,t}$ their numbers, which are also shares as the total number of consumers is of mass 1. We assume the imperfection on goods market of associating unmatched consumers $\mathscr{C}_{0,t}$ with disposable income Y_t with unmatched goods $\mathscr{N}_{g,t}$ can be summarized by a constant returns to scale function $M_G(\bar{e}_t\mathscr{C}_{0,t}, \mathscr{N}_{g,t})$, which is also the flow number of contacts between $\mathscr{C}_0$ unmatched consumers producing on average $\bar{e}_t$ units of effort (that will be determined later on), and $\mathscr{N}_g$ firms producing and attempting to sell their product. [8]

Thus the meeting rates between consumers and firms per unit of effort are given by:

$$\frac{M_G(\bar{e}_t\mathscr{C}_{0,t}, \mathscr{N}_{g,t})}{\mathscr{N}_{g,t}} = \lambda(\xi_t) \quad \text{with} \quad \lambda'(\xi_t) > 0$$
$$\frac{M_G(\bar{e}_t\mathscr{C}_{0,t}, \mathscr{N}_{g,t})}{\bar{e}_t\mathscr{C}_{0,t}} = \tilde{\lambda}(\xi_t) \quad \text{with} \quad \tilde{\lambda}'(\xi_t) < 0$$

where $\xi_t = \frac{\bar{e}_t\mathscr{C}_{0,t}}{\mathscr{N}_{g,t}}$ is the natural concept for tightness in the good market (from the point of view of consumers). Note that $\tilde{\lambda}_t$ is the probability that an unmatched consumer finds a suitable firm from which to buy goods and $\lambda(\xi_t) = \xi_t\tilde{\lambda}(\xi_t)$. The higher ξ_t, the higher the demand from consumers relative to the production awaiting to be consumed and an increase in the effective demand for new goods $e_{t-1}\mathscr{C}_{0,t}$ relative to the availability of unmatched good $\mathscr{N}_{g,t}$ reduces the duration of search for producers. This creates a feedback from the goods market to the labor market in that it improves the returns to hiring a worker as sales are more likely to materialize.

3 Dynamic equilibrium conditions

We first describe the evolution of stocks in this economy, from which a law of motion for tightness on the goods market is derived. Whereas goods market tightness is a backward looking variable, tightness on the labor market is forward looking and determined through a job creaton condition that reflects frictions on each market - credit, labor and goods.

3.1 Stocks of consumers, employment and unemployment

Potential consumers $\mathscr{C}_0$ become consumers the period after meeting a producer, and a fraction $0 < \tau < 1$ of current consumers separate from their product only to return to the pool

[8]This specification is different of that in Wasmer (2009)and will lead to search effort responding endogeneously to changes in disposable income and play an important role in generate persistence to productivity shocks.

of potential consumer the following period:[9]

$$\begin{aligned}
\mathscr{C}_{0,t+1} &= (1-\tilde{\lambda}_t)\mathscr{C}_{0,t} + \tau\mathscr{C}_{1,t} && (15)\\
\mathscr{C}_{1,t+1} &= (1-\tau)\mathscr{C}_{1,t} + \tilde{\lambda}_t\mathscr{C}_{0,t} && (16)\\
\mathscr{N}_{g,t+1} &= (1-s)(1-\lambda_t)\mathscr{N}_{g,t} + q(\theta_t)\mathscr{N}_{l,t} + (1-s)\tau N_{\pi,t} && (17)\\
\mathscr{N}_{\pi,t+1} &= (1-s)(1-\tau)\mathscr{N}_{\pi,t} + (1-s)\lambda_t\mathscr{N}_{g,t} && (18)
\end{aligned}$$

The evolution of the stock of firms in stages g and π, given by equation (17) and (18), assume that newly recruited workers begin employment the following period and that successful meetings with potential costumers generate sales the next period as well.

Finally, the dynamics of aggregate unemployment and employment are then given by the following equations:

$$\begin{aligned}
u_{t+1} &= s(1-u_t) + (1-f(\theta_t))u_t && (19)\\
1-u_t &= \mathscr{N}_{g,t} + \mathscr{N}_{\pi,t} && (20)
\end{aligned}$$

where s is an exogenous firm destruction shock.

3.2 The dynamics of goods market tightness

Noting that $q(\theta_t)\mathscr{N}_{l,t} = f(\theta_t)u_t$, the evolution of goods market tightness ξ_t, from equations (15) and (17) can be expressed as:

$$\xi_{t+1} = \bar{e}_{t+1}(Y_{t+1})\frac{(1-\tilde{\lambda}_t)\mathscr{C}_{0,t} + \tau\mathscr{C}_{1,t}}{(1-s)(1-\lambda_t)\mathscr{N}_{g,t} + (1-s)\tau\mathscr{N}_{\pi,t} + f(\theta_t)u_t} \qquad (21)$$

Clearly, after an intial positive shock to disposable income, tightness on the goods market will drop at first as firms hire new workers in response to a change in productivity. This increase in the number of firms prospecting in the goods market relative to the number of unmatched consumers and finally causes a decline in the hazard rate $\lambda(\xi)$ for firms, as it will be more difficult for a given producer to bring his good to market. Therefore the strongest response of labor market tightness to a productivity shock will not occur on impact due to goods market frictions. It will follow the evolution of conditions on the goods market, because productivity shocks affect income dynamics only after one period. The dynamic of firm's decision, in turn, will react to income variations after one period. This is likely to generate complex and interesting dynamics.

3.3 The dynamics of labor market tightness

Let $S_{j,t} = E_{j,t} + B_{j,t}$ with $j = c, l, g, \pi$ be the total surplus of a bank-entrepreneur relationship in the various stages, which we define as a firm. The dynamic equations for the value

[9] Some verifications: $\mathscr{C}_{0,t+1} + \mathscr{C}_{1,t+1} = \mathscr{C}_{0,t} + \mathscr{C}_{1,t} \equiv 1$ and $\mathscr{N}_{g,t+1} + \mathscr{N}_{\pi,t+1} = (1-s)[\mathscr{N}_{g,t} + \mathscr{N}_{\pi,t}] + q(\theta_t)\mathscr{N}_{l,t} = 1 - u_{t+1}$.

of a firm in each state can be obtained in summing the corresponding equations for entrepreneurs and banks, that is (1) to (4) and (5)to (8). We have, after rearrangement, the corresponding asset values:

$$\begin{aligned}
S_{c,t} &= 0 \Leftrightarrow K(\phi^*) = S_{l,t} && (22)\\
S_{l,t} &= -\gamma + \frac{1}{1+r}\mathbb{E}_t\left[q(\theta_t)S_{g,t+1} + (1-q(\theta_t))S_{l,t+1}\right] && (23)\\
S_{g,t} &= -w_t + \frac{1-s}{1+r}\mathbb{E}_t\left[\lambda_t S_{\pi,t+1} + (1-\lambda_t)S_{g,t+1}\right] && (24)\\
S_{\pi,t} &= \mathscr{P}_t y_t - w_t - \Omega + \frac{1-s}{1+r}\mathbb{E}_t\left[(1-\tau)S_{\pi,t+1} + \tau S_{g,t+1}\right] && (25)
\end{aligned}$$

Equation (22) states that the value of a firm in the hiring stage is equal to the sum of capitalized search costs paid by each side in the previous stage and, since $\phi^*$is time-invariant, so is the expected value of the the surplus in stage l.

Equation (23) contains the information on how labor market tightness responds to changes in the expected value of a worker to the firm. This is more clearly seen by using (22) to express (23) as $\left(\frac{r+q(\theta_t)}{1+r}\right)K(\phi^*) = -\gamma + \frac{1}{1+r}\mathbb{E}_t\left[q(\theta_t)S_{g,t+1}\right]$, or, calling $o_t(r) \equiv r\left(\frac{1/q(\theta_t)-1}{1+r}\right)$ a term vanishing as the discount rate goes to zero,

$$K(\phi^*)(1+o_t(r)) + \frac{\gamma}{q(\theta_t)} = \frac{1}{1+r}\mathbb{E}_t S_{g,t+1} \quad (26)$$

which equates the average cost of creating a job (the left-hand side, equal to the financial costs properly discounted $K(\phi)$ and the expected costs of search on the labor market $\gamma/q(\theta_t)$) to the discounted expected value of a worker to the firm in the goods market stage (the right-hand side). A few words of comparison with the canonical search model are warranted here. First, the costs of financial intermediation enter the left hand side of the equation and place a lower bound on the value of a "vacancy" to firm. Absent credit market frictions the average cost of creation depends on the flow cost of a vacancy γ and congestion on the labor market. Second, the expected value on the right hand side corresponds to the ability to produce and sell a good once a consumer has been located. Under frictionless goods markets the right hand side is simply the value of the fourth stage. Thus the current model nest the canonical search model when $K(\phi^*)$ tends to zero and the goods market friction is removed.

A log-linear approximation around the deterministic steady state of the job creation condition (26),

$$0 \;=\; -\eta_L\widehat{\theta}_t + \frac{S_g}{S_g - K(\phi)}\mathbb{E}_t\widehat{S}_{g,t+1} \quad (27)$$

where η_L is the elasticity of the job filling rate with respect to labor market tightness clear shows how labor market tightness is a forward looking variable and, as in the canonical framework, responds to expected changes in the value of labor to the firm proportionally to the inverse of the elasticity of the labor matching function. Frictions in credit markets raise this elasticity by a factor $\frac{S_g}{S_g - K(\phi)}$. The difference $S_g - K(\phi)$ is the surplus to the firm of hiring a worker, and the amplification from credit market, of the financial accelerator, is decreasing in this surplus. As Petrosky-Nadeau and Wasmer (2010) show, large volatitilites of labor market tightness over the business cycle can be achieve by assuming a *small firm surplus*, indenpently of the value of the labor surplus.

The goods market will thus affect the dynamics of the labor market through its impact on the dynamics of the value of a worker to the firm, S_g. The latter depends on the evolution of the hazard rate λ_t: an improved goods mark for firms reduces period for which the cost of labor are not offset by a revenue for selling the manufactured goods.

Summarizing, the dynamic solution is therefore a 12-uple (θ_t, ξ_t, ϕ^*, u_t, $\mathscr{C}_{0,t}$, $\mathscr{C}_{1,t}$, $\mathscr{N}_{l,t}$, $\mathscr{N}_{g,t}$, $\mathscr{N}_{\pi,t}$, $S_{l,t}$, $S_{g,t}$, $S_{\pi,t}$) solving (23), (21), (12), (19), the definition of labor market tightness, (15), (16), (17), (18), (22), (24) and (25).

3.4 Optimal consumer decisions

3.4.1 Belmann equations of consumers

We now introduce the demand side of the economy. Recall that consumers want to consume manufactured goods but may not necessarily buy them before they are matched with a firm. Let us denote by $D_{1,t}$ and $D_{0,t}$ the asset values of being matched or unmatched. The generic indirect utility of consuming both goods is denoted by $v(c_1, c_0)$ where c_1 and c_0 are the optimal consumption patterns. Finally, let σ(e) be the cost of consumer search effort with $\sigma'(e) > 0$ and $\sigma''(e) \geq 0$. We have:

$$D_{1,t} = v(c_{1,t}, c_{0,t}) + \frac{1-s}{1+r}\mathbb{E}_t\left[\tau D_{0,t+1} + (1-\tau)D_{1,t+1}\right] + \frac{s}{1+r}\mathbb{E}_t D_{0,t+1} \quad (28)$$

$$D_{0,t} = v(0, c_{0,t}) - \sigma(e_t) + \frac{1}{1+r}\mathbb{E}_t\left[e_t\tilde{\lambda}_t D_{1,t+1} + (1-\tilde{\lambda}_t e_t)D_{0,t+1}\right] \quad (29)$$

We now assume that consumers, whenever possible, put the maximum of their income into the manufactured good y_t because it yields a higher marginal utility than the numeraire, and then consume what is left $Y_{t-1} - \mathscr{P}_t y_t$ into the numeraire. The equations therefore become

$$D_{1,t} = v(y_t, Y_{t-1} - \mathscr{P}_t y_t) + \frac{1-s}{1+r}\mathbb{E}_t\left[\tau D_{0,t+1} + (1-\tau)D_{1,t+1}\right] + \frac{s}{1+r}\mathbb{E}_t D_{0,t+1} \quad (30)$$

$$D_{0,t} = v(0, Y_{t-1}) - \sigma(e_t) + \frac{1}{1+r}\mathbb{E}_t\left[e_t\tilde{\lambda}_t D_{1,t+1} + (1-\tilde{\lambda}_t e_t)D_{0,t+1}\right] \quad (31)$$

3.4.2 Optimal search effort

The optimal individual search effort is simply given by

$$\sigma'(e_t) = \frac{\tilde{\lambda}_t}{1+r}\mathbb{E}_t\left[(D_{1,t+1} - D_{0,t+1})\right] \quad (32)$$

and it follows that all consumers exert the same effort :

$$e_t^* = \bar{e}_t$$

Optimality condition (32) implies that consumer search effort is increasing in the expected capital gain from consuming the manufactured good increases. Both disposible income and the dynamics of the price $\mathscr{P}$, which we discuss next, play a determining role in this respect.

3.4.3 The dynamics of prices and surplus: general case

So far we remained silent on price determination. Consistent with the search literature, we postulate that the price $\mathscr{P}_t$ is bargained between a consumer and a firm. Call $\delta \in (0,1)$ the share of the goods surplus going to consumers. The total surplus is here $G_t = (S_{\pi,t} - S_{g,t}) + (D_{1,t} - D_{0,t})$. We therefore have for all t's, assuming price renegotiation each period:

$$S_{\pi,t} - S_{g,t} = (1-\delta)G_t \tag{33}$$

$$D_{1,t} - D_{0,t} = \delta G_t \tag{34}$$

Here, we will ignore any strategic interaction of the type described earlier (footnote #) between banks and firms in the early stage : bargaining over the loan repayment ρ is assumed not to take into account the future effect on prices.

The dynamic of the total consumer-seller surplus satisfies a dynamic equation:

$$\begin{aligned} G_t = {} & v(y_t, Y_{t-1} - \mathscr{P}_t y_t) - v(0, Y_{t-1}) - \sigma(e_t) + \mathscr{P}_t y_t - \Omega \\ & + \frac{1-s}{1+r}\left[(1-\tau) - (1-\delta)\lambda_t\right]\mathbb{E}_t G_{t+1} - \delta \frac{\bar{e}_t \tilde{\lambda}_t}{1+r}\mathbb{E}_t G_{t+1} \end{aligned} \tag{35}$$

Given the definitions of the asset values (24) and (25) for the firm and (30) and (31) for the consumer, we can find an implicit equation for the price $\mathscr{P}_t$ that satisfies the sharing rule. This equation is, after simplification using equations (33) and (34) at time t and t+1:

$$\delta \mathscr{P}_t y_t = (1-\delta)\left[v(y_t, Y_{t-1} - \mathscr{P}_t y_t) - v(0, Y_{t-1}) - \sigma(e_t)\right] + \delta\left(\Omega + \frac{(1-\delta)}{1+r}\left[(1-s)\lambda_t - e_t \tilde{\lambda}_t\right]\mathbb{E}_t G_{t+1}\right) \tag{36}$$

Interestingly, the price depends on lagged value of income, contemporaneous firm' repayment terms and production costs, and also expected capital gains (or losses) of the surplus of each side. If agents anticipate a downturn next period, surplus values will fall and price start to go down, leading to immediate transmission of bad news into current pricing decisions. This equation summarizes the intertemporal transmission mechanisms best: current pricing is affected by past shocks through income, current shocks and production costs, and finally by expectations about future surplus values.

3.4.4 Specific cases:

There are three polar cases to consider. First, assume that consumption is additively linear in manufactured good and the numeraire, with a marginal utility Φ>1 for the manufactured good. Given this perfect substitutability, the price equation will be such that

$$\delta \mathscr{P}_t y_t = (1-\delta)\left[\Phi y_t - \sigma(e_t)\right] + \delta\left(\Omega + \frac{(1-\delta)}{1+r}\left[(1-s)\lambda_t - e_t\tilde{\lambda}_t\right]\mathbb{E}_t G_{t+1}\right) \quad (37)$$

and therefore, lagged income will have disappeared.

The second case is the one with complementarity between goods, such that $v(0,Y_{t-1}) = 0$. In this case, the term Y_{t-1} will enter in the price dynamic equation (36) with a positive coefficient.

The last case is the one in which the consumer has to choose between manufactured good and the numeraire, a sort of extreme complementarity. Hence, past income only affect the unmatched consumer and in this case, it will affect equation (36) with a negative coefficient instead.

4 Quantitative Dynamic results

The dynamic system is solved for a log-linear approximation around the deterministic steady state, with details provided in the appendix. We are interested, in particular, in separating the propagation of shocks to labor productivity into amplification and persistence, measured as short-run autocorrelations in growth rates. In addition, we solve for three comparative models in which we remove and combine various frictions in order to asses their relative importance. This section begins by discussing the calibration strategy for the full model, which is replicated for the comparison models, before presenting results.

4.1 Calibration strategy

The model is calibrated to quarterly data, and a summary of parameter values for the baseline calibration can be found in Table 1. We follow the strategy developped in Petrosky-Nadeau and Wasmer (2010), fixing the value of parameters for which there are direct empirical coutnerparts or good estimated values, and performing an optimized search to the remaining parameter values subject to a set of constraints and target.

We set the risk free rate to $r = 0.01$, consistent with the average return on three month Treasure bills for the post-war period. Labor productivity is assumed to follow an AR(1) of the form

$$\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y,$$

with $\varepsilon_t^y \sim (0, \sigma_y^2)$ and $0 < \rho_y < 1$. Staying within the real business cycles literature, the relevant parameters are chosen as $\rho_y = 0.975$ and $\sigma_y = 0.0072$ (e.g., King and Rebelo, 1999).

Table 1: Baseline parameter values

Labor market			**Credit market**		
vacancy cost	γ	0.25	bank's barg. weight	β	0.27^*
matching elasticity	η_L	0.5	matching elasticity	ε	0.5
matching level param.	χ_L	0.55^*	matching level param.	χ_C	1.9^*
job seperation rate	s	0.09	search costs	$\kappa = e$	0.05^*
wage elasticity	η_w	0.5			
Goods market			**Technology**		
matching elasticity	η_G	0.5	labor productivity	y	1
matching level param.	χ_G	0.8^*	persistence param.	ρ_y	0.975
goods exit rate	τ	0.1	risk free rate	r	0.01
consumer barg. weight	δ	0.5			

The calibration of the labor marke closely follows the labor literature. The labor matching function is assumed to be Cobb-Douglas $M_L(\mathcal{N}_l, u) = \chi_L \mathcal{N}_l^{1-\eta_L} u^{\eta_L}$, with an elasticity set to $\eta_L = 0.5$, in the mid-range of value reported in the survey by Petrongolo and Pissarides (2001). We set the quarterly job separation rate to $s = 0.0.6$ as suggest by the work of Davis et al. (2007). We set the elasticity of the wage to labor productivity to $\eta_w = 0.5$, consistent with the value reported in Gertler and Trigari (2009). The level parameter χ_w in the wage equation is chosen such that the wage is equal to two thirds of the maginal product of labor, steering clear of the assumption of a small labor surplus. The cost of a vacancy γ is set to 0.25 such that the average cost of filling a vacancy is about 3% of the workweek of labor, consistent with the estimates in Baron et al. (1997). The level parameter in the labor matching function χ_L is adjusted such that the steady state equilibrium rate of unemployment is approximately 8%.

The calibration of the credit market requires choosing parameters of the credit matching function, assumed to be of the form $M_c(\mathcal{B}, N_c) = \chi_C \mathcal{E}^{1-\eta_C} \mathcal{B}^{\eta_C}$, the costs of prospecting on credit markets and the bargaining weight β. We assume symmetry in prospecting costs $\kappa = e$. The remaining parameters, χ_C and β, adjust to accommodate our targets, which include the share of the financial sector in GDP[10]

$$\Sigma = \frac{\mathcal{B}_\pi \rho - \mathcal{B}_g w - \mathcal{B}_l \gamma - \mathcal{B}_c \kappa}{Y} \tag{38}$$

to approximately 2%.

Finally, on the goods market, we also assume a Cobb-Douglas goods matching function $M_G(\mathcal{C}_0, N_g) = \chi_G C_0^{1-\eta_G} \mathcal{N}_g^{\eta_G}$, setting the elasticity of the goods matching function η_G to 0.5 and perform a sensibility analysis to this choice below. Let σ be the ratio sales / production observed in the data. The corresponding value in our model is simply the ratio

[10]The derivation of the steady state repayment ρ is detailed in the appendix.

Table 2: Steady state values

Labor market:			**Goods market:**		
filling rate	$q(\theta)$	0.31	firm hazard	$\lambda(\xi)$	0.79
finding rate	$f(\theta)$	0.9	consumer hazard	$\tilde{\lambda}(\xi)$	0.81
unemployment	u	8%	non-producing firms	$\frac{\mathcal{N}_g}{\mathcal{N}_g+\mathcal{N}_\pi}$	0.20
Credit market:	Σ	2%	real wage	W/P	0.67

of selling firms over producing firms, namely $\mathcal{N}_\pi/(\mathcal{N}_g+\mathcal{N}_\pi)$. Given $\frac{\mathcal{N}_\pi}{\mathcal{N}_g}=\frac{(1-s)\lambda}{(s+\tau-s\tau)}$ we have $\frac{(1-s)\lambda}{(s+\tau-s\tau)}=\frac{\sigma}{1-\sigma}$which implies that

$$\lambda=\frac{\varphi}{1-\varphi}\left(\tau+\frac{s}{1-s}\right)$$

at least if the right hand side is below 1 (otherwise $\lambda=1$). For instance, assuming $\varphi=0.8$ and $s=\tau=0.1$ we have $\lambda=0.84$. A higher value of φ raises the calibrated value of λ. The intuition is that a higher sale/production ratio requires a more efficient good market. Conversely, if $\sigma=0.5$ (half the production is depreciated), this implies a relatively inefficient good market ($\lambda=0.21$). We set the goods market exit rate τ to 0.05 : on average, consumers change taste every 20 quarters.[11] Finally, we need to specify the consumer search effort cost funciton $\sigma(e)$ to determine the steady state level of effort and the elasticity of the cost funciton $\varepsilon_\sigma=\frac{\partial\sigma(e)}{\partial e}\frac{e}{\sigma(e)}$. We assume this function takes the form $\sigma(e)=\chi_\sigma e^{\varepsilon_\sigma}$ with $\chi_\sigma>0$ and $\varepsilon_G\geq 1$. We enther the level parameter into our numerical search procedure and assume an elasticity 1, perform a series of sensitivity analyses below.

We target a of ratio of sales to production - which can be thought as a proxy for capacity utilization - of 80% in order to set the values of the level parameter is set to χ_G and the elastcitity η_G.

Table 2 reports the implied steady state values for the baseline calibration. For example, we obtain a average quarterly job finding rate of 0.75. On the good market, the probability for a firm of finding a consumer is 0.70 while the probability for a consumer finding a good to purchase is 0.92. Thus average search duration on both sides of the goods market is relatively short in the baseline calibration.

4.2 Propagation under credit, labor and goods market frictions

The models are eveluated with a set of impulse responses to an expansionary productivity shocks, and computing a set second-order moments.

[11] We don't have any good justification for this number and simply argue that it reflects the average duration of a brand, becoming unfashionable after a while: 5 years being an aeverage across all types of good : taste on clothing probably change faster, taste on durables such as housing or washing machines etc. probably longer. Our results are not very sensitive to the exact value of τ.

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Figure 1: Credit, labor and goods market friction and the dynamics of labor market tightness

4.2.1 Credit, labor and goods market friction and labor maket tightness

Recall from equation (26) that deviations of labor market tightness from the steady are given by

$$\widehat{\theta}_t = \frac{1}{\eta_L}\frac{S_g}{S_g - K(\phi)}\mathbb{E}_t\widehat{S}_{g,t+1}$$

Figure 1plots this equation following a positive shock to productivity under three scenarios in order to focus on the contributions of the difference market imperfections. The dashed and stared responses represent the contribution of friction credit markets to the dynamics of the labor market: the increase in labor market tightness on impact is amplified in the presence of credit market frictions by a financial accelarator inversely related to the size of the firm surplus $(S_g - K(\phi))$.

The solid line plots the response of labor market tightness when credit markets are perfect but there are frictions on the goods market. In this case, deviations of labor market tightness follow $\widehat{\theta_t^G} = \frac{1}{\eta_L}\mathbb{E}_t\widehat{S}_{g,t+1}$ and differ from the canonical model with labor market frictions only by the dynames of the value of recruiting a worker $\mathbb{E}_t\widehat{S}_{g,t+1}$. This last difference is essential for the propagation of productiivity shocks: the response of labor market tightness is hump-shaped, reach a peak around 12 periods after the innovation. In the next section, when presenting the results for the full model, we will show how this pattern stem from the changes in congestion on the goods market: selling goods, or moving from state g to state π for a firm slowly increase the value a hire worker even after the increase in productivity has begun to decay:

$$\widehat{S}_{g,t} = -\frac{w}{S_g}\widehat{w}_t + \frac{1-s}{1+r}\mathbb{E}_t\left\{\lambda\left(\frac{S_\pi}{S_g}\widehat{S}_{\pi,t+1} + \frac{S_\pi - S_g}{S_g}\widehat{\lambda}_t\right) + (1-\lambda)\widehat{S}_{g,t+1}\right\}$$

4.2.2 Impulse response to a productivity shocks in full model

We begin by examining the the response of equilibrium labor and goods markets tightness to a 1% expansionary shock to labor productivity fori n Figure (2). The same figure also show the reponse of the measures of market tighness in the absence of either goods or credit market imperfections.

In the presence of frictions on all three markets, the general reponse of labor market tightness of on positive productivity shock is and incease of a duration close to that of the innovation. The three reponses in the first panel however, differ greatly in the degree of amplification and persistence of the response affected by the difference market friction. Compared to the canonical model, the presence of goods and credit market imperfection leads to *both* measures of propagation. Moreover, we see that frictions on the credit market are source of amplification and frictions of goods markets both persistence and amplification.

The following panel plots the reponse of the tightness of goods markets, for the full model and without credit market fricton. In both cases, ξ drops initialy, with a lag, as firms enter the market with their products before there is any change in the stock of unmatched consumers. These dynamics shed light on the response of labor market tightness which declines as the goods market deteriorates for producers after the intial jump. As consumer income catches up and raise effective demand for new goods, goods market tighness increases improving the expected benefit of a job, and so does, labor market tightness increases again. The second effect pushes θ beyond the initial response to the technology shock, giving rise to a large, hump-shaped response.

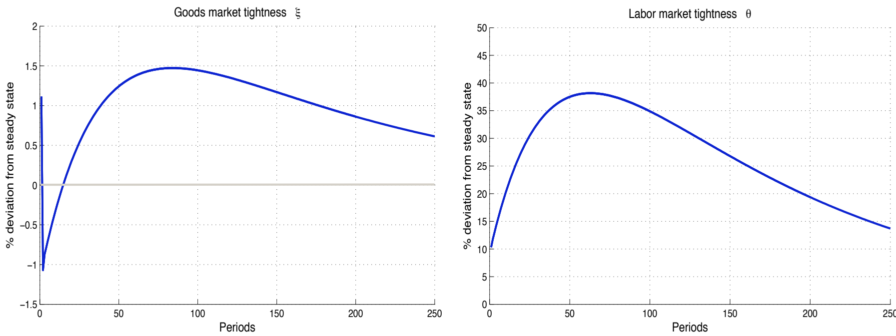

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Figure 2: IRFs of labor and good markets tightness and of labor and goods market hazard rates

4.2.3 Stocks and GDP

The next figure (3) reports the response to the same innovation of the stocks of consumers and firms, which, along with the evolution of hazard rates on the labor and goods markets reported above , sheds some light on the dynamics of the goods market. As seen in the lower left quadrant of Figure 3, there is an influx of goods search for a shrinking customer base. This is seen in the decline of the stock of consumers $\mathscr{C}_0$ and the rise of the stock $\mathscr{C}_1$.

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Figure 3: IRFs of stocks of consumers on firms on goods market

Finally, Figure (4) plots the evolution of aggregate production and aggregate sales, where sales are $C_{1,t} \times Y_t$. Thus the friction on the goods market induces greater elasticity in aggregate expenditure based measure of GDP to productivity shocks.

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Figure 4: IRFs of aggregate production and sales

4.2.4 Second moments

The following table presents the second moments for the principle model variables and their empirical counterparts.

The first columns of Table 3 report the standard deviation of the business cycle relative to GDP of the main labor market variables in the model and consumption. The last rows reports the autocorrelation in the growth rates of GDP per capita and labor market tightness for the U.S.. Both sets of moments summarized the empirical shortcomings of the canonical search model of unemployment in explain short run fluctuations on labor markets. This first concern the well know lack of amplificiation of productivity shocks: labor market tighness is nearly 15 time more volatile than GDP over the business cycle whereas the model generates of relative volatility of 3. This shortcoming extends to the relative volatility of unemployment and job vacancies. The second concerns persistence, measured of autocorrelations in growth rates. Labor market tightness is very persistent in the data, much more so than GDP. Just as the real business cycle model failes to deliver on the persistence in GDP, so to does the labor search model on the persistence of labor market tightenss. In fact, the model generates no persistence: θ follows exactly the shock process.

Table 3: Second moments - data and model

	US data		Credit, labor and goods frictions		Credit & Labor		Labor & Goods		Labor only	
	a	b	a	b	a	b	a	b	a	b
Vacancies	8.88	0.89	6.88	0.91	2.47	0.84	7.08	0.90	2.16	0.85
Unemployment	7.98	-0.84	4.62	-0.74	1.60	-0.78	5.00	-0.77	1.40	-0.78
Labor tightness	16.34	0.90	9.93	0.97	3.35	0.99	10.26	0.99	2.93	0.99
Wage	0.69	0.55	0.44	0.95	0.45	0.99	0.44	0.95	0.45	0.99
Consumption	0.80	0.85	0.71	0.81	0.83	0.52	0.66	0.93	0.80	0.80
$\sigma(GDP)$	1.56		1.06		1.05		1.05		1.03	

Persistence:	GDP	θ	GDP	θ	GDP	θ	GDP	θ	GDP	θ
$corr(\Delta x_t, \Delta x_{t-1})$	0.44	0.72	0.12	0.30	0.15	-0.01	0.03	0.14	0.13	-0.01
$corr(\Delta x_t, \Delta x_{t-2})$	0.32	0.36	0.23	0.28	-0.03	0	0.25	0.08	-0.03	0
$corr(\Delta x_t, \Delta x_{t-3})$	0.18	0.09	0.08	0.27	-0.01	0	0.05	0.08	-0.01	0

Notes: (a): standard deviation relative to GDP; (b): contemporaneous correlation with GDP.

Data sources: B.E.A., B.L.S. and Conference Board, 1951:Q1 to 2006:Q2.

In this discussion, we mostly focus on the volatility and persistence of labor market tightness. In the ’pure’ labor search model, the volatility relative to GDP is 2.93, far below the numbers found in US data (16.34) as pointed out in Shimer (2005). Adding up good market frictions drastically change this perspective and the new volatility is augmented by a factor 3, reaching 10.26. Interestingly, the impact of financial frictions alone with perfect good markets leads in this set of calibration a much smaller additional volatility as what we obtained in previous calibration exercises (Petrosky-Nadeau and Wasmer, 2010). Finally, the introduction of frictions in all three markets leads to substantially higher volatitility as in the pure labor search and the pure labor/credit search model, and we reach a value of 9.93 relative to GDP.

The second set of results concerns the firt order autocorrelation of labor market tightness. Thus value is typically low or negative in all models with perfect good markets. However, adding up good market imperfections generates a significant qualitative improvmenet

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Figure 5: Response of expected next period's price, $\mathbb{E}_t \mathscr{P}_t$.

here : the autocorrelation is positive, equal to 0.14 with good and labor frictions, and 0.30 with all three market frictions. This is still far away from the observed persistence in the data (0.72) but this points out the right direction : margins of improvements of this statistics lies in good market imperfections. The main intuition for this statistics can be found in the discussion around equation (36) : price dynamics is complex, and includes both lagged terms (income), contemporaneous terms (productivity and production costs) and lead values (expected surplus growth). Out of the steady-state, the dynamics is therefore slow and complex only when prices are bargained due to consumer search frictions. The same discussion applies on second order auto-correlation terms on labor market tightness: they are positive only when good market frictions are introduced.

4.3 The role of price dynamics

In this section, we investigate the role of greater price rigidity by varying δ and an outright fully fixed price. The effects of the bargaining weight δ are best understood in the context of its importance for the dynamics of the price $\mathscr{P}_t$. In the baseline calibration, the price is countercyclical, with a contemporaneous correlation with GDP of -0.42. The next figure plots the response of the expected price for next period, to which firms reaction when making their job vacancy decisions, for the baseline calibration with $\delta = 0.5$ and an alternative in which consumers extract a smaller share of the surplus, $\delta = 0.25$. In the baseline calibration the price obtain if one manages to sell next period declines, limiting the incentive to hire workers to meet demand. However, the price level increases persistently such that even if productivity is declining and returning to its steady state, the return to selling goods, and hence hiring workers, continues to rise. This plays an important role in explaining the large hump-shape in the response of labor market tightness for the baseline calibration and in raising the persistence in its growth rate closer to the data. Indeed, when the bargaining weight is reduced to 0.25, the expected price no longer has an intial decline, and it subsequent growth is much more moderate. The results in a reduction in the short-run autocorrelation for labor market tightness of 50% at each lag. The correlation of the price with GDP is now 0.39, and the relative volatility declines from 0.55 to 0.30.

Note also from Table 4 that reducing δ contribute substantially to the amplification of productivity shock: the standard deviation of GDP increases from 1.06 to 1.23 and the relativie volatility of labor market tighness increases from 9.93 to 17.62, overshooting the data. The last columns of Table 4 completely shut down the price mechanism by assuming a fixed price. There a decline in the persistence of labor market tightness similar to when we reduce the consumer's bargaining. The main difference is in a small, and not large, increase in volatility. This argue for the importance of the rich dynamics of endgoneous prices in genrrate large degrees of propagation.

4.4 Robustness

Finally, we examine the effects of reducing the elasticity of the goods matching function from $\eta_G = 0.5$ to $\eta_G = 0.25$. This has little effect on the dynamics of the price, which has a contemporaneous correlation with GPD of -0.41 and a relative volatility of 0.51. The elasticity in part determines the response of congestion on goods market following a productivity shock, mostly affecting the persistence of adjustments on the labor market as a consequence.

Table 4: Sensitive to goods market parameters

Credit, labor and goods frictions	baseline		Consumer barg. $\delta = 0.25$		Matching elasticity $\eta_G = 0.25$		Fixed Price $\mathscr{P}_t = \mathscr{P}\ \forall t$	
	a	b	a	b	a	b	a	b
Vacancies	6.88	0.91	12.37	0.83	6.93	0.92	8.22	0.88
Unemployment	4.62	-0.74	8.21	-0.83	4.69	-0.73	5.47	-0.81
Labor tightness	9.93	0.97	17.62	0.97	10.08	0.97	11.77	0.99
Wage	0.44	0.95	0.38	0.86	0.45	0.95	0.42	0.91
Consumption	0.71	0.81	0.89	0.98	0.70	0.84	0.79	0.96
$\sigma(GDP)$	1.06		1.23		1.04		1.12	
Persistence:	GDP	θ	GDP	θ	GDP	θ	GDP	θ
$corr(\Delta x_t, \Delta x_{t-1})$	0.12	0.30	0.15	0.16	0.17	0.33	0.07	0.13
$corr(\Delta x_t, \Delta x_{t-2})$	0.23	0.28	0.45	0.13	0.16	0.30	0.30	0.10
$corr(\Delta x_t, \Delta x_{t-3})$	0.08	0.27	0.16	0.12	0.08	0.29	0.10	0.09

Notes: (a): standard deviation relative to GDP; (b): contemporaneous correlation with GDP.

Data sources: B.E.A., B.L.S.and Conference Board, 1951:Q1 to 2006:Q2.

5 Conclusion

Our paper shows the potential of good market frictions in macroeconomics. The qualitative features of labor market dynamics is very much affected by complex price dynamics arising from two complementary features of the economy : first, consumer search for goods and firm's search for consumers generate ex-post surpluses. Second, this surplus leads to price negociation where leads and lagged terms enter. In particular, lags arising from income shocks propagation raise the volatility of labor markets and generate a inverted U-shaped pattern.

Quantitative improvements of second moments also occur. Volatility of labor market relative to GDP approaches US data and first order autocorrelations are positive, not zero, even though a factor 2 is still missing. This is the subject of future research.

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Detailed Appendix to: The Propagation of Technology Shocks: Do Good, Labor and Credit Market Imperfections Matter and How Much?

(TO BE COMPLETED)

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A Consumption good surplus

We derive a dynamic equation for the consumption good surplus using the Belman equation for consumers and firms, yielding

$$\begin{aligned} G_t &= \mathscr{P}_t y_t - \Omega + \frac{1-s}{1+r}\mathbb{E}_t\left[(1-\tau)S_{\pi,t+1} + \tau S_{g,t+1}\right] - \frac{1-s}{1+r}\mathbb{E}_t\left[\lambda_t S_{\pi,t+1} + (1-\lambda_t)S_{g,t+1}\right] \\ &+ v(y_t, Y_{t-1} - \mathscr{P}_t y_t) + \frac{1-s}{1+r}\mathbb{E}_t\left[\tau D_{0,t+1} + (1-\tau)D_{1,t+1}\right] + \frac{s}{1+r}\mathbb{E}_t D_{0,t+1} \\ &- v(0, Y_{t-1}) + \sigma(e_t) - \frac{1}{1+r}\mathbb{E}_t\left[e_t\tilde{\lambda}_t D_{1,t+1} + (1-\tilde{\lambda}_t e_t)D_{0,t+1}\right] \end{aligned}$$

$$\begin{aligned} G_t &= \mathscr{P}_t y_t - \Omega + v(y_t, Y_{t-1} - \mathscr{P}_t y_t) - v(0, Y_{t-1}) + \sigma(e_t) \\ &+ \frac{1-s}{1+r}\mathbb{E}_t\left[(1-\tau)S_{\pi,t+1} + \tau S_{g,t+1} + (1-\tau)D_{1,t+1} + \tau D_{0,t+1}\right] \\ &- \frac{1-s}{1+r}\mathbb{E}_t\left[\lambda_t S_{\pi,t+1} + (1-\lambda_t)S_{g,t+1}\right] + \frac{s}{1+r}\mathbb{E}_t D_{0,t+1} \\ &- \frac{1}{1+r}\mathbb{E}_t\left[e_t\tilde{\lambda}_t D_{1,t+1} + (1-\tilde{\lambda}_t e_t)D_{0,t+1}\right] \end{aligned}$$

$$\begin{aligned} G_t &= \mathscr{P}_t y_t - \Omega + v(y_t, Y_{t-1} - \mathscr{P}_t y_t) - v(0, Y_{t-1}) + \sigma(e_t) \\ &+ \frac{1-s}{1+r}\mathbb{E}_t\left[(1-\tau)S_{\pi,t+1} + \tau S_{g,t+1} + (1-\tau)D_{1,t+1} + \tau D_{0,t+1}\right] \\ &- \lambda_t\frac{1-s}{1+r}\mathbb{E}_t\left[S_{\pi,t+1} - S_{g,t+1}\right] - \frac{(1-s)}{1+r}\mathbb{E}_t D_{0,t+1} - \frac{(1-s)}{1+r}\mathbb{E}_t S_{g,t+1} \\ &- \frac{e_t\tilde{\lambda}_t}{1+r}\mathbb{E}_t\left[D_{1,t+1} - D_{0,t+1}\right] \end{aligned}$$

$$
\begin{aligned}
G_t &= \mathscr{P}_t y_t - \Omega + v(y_t, Y_{t-1} - \mathscr{P}_t y_t) - v(0, Y_{t-1}) + \sigma(e_t) \\
&\quad + \frac{1-s}{1+r}\left[(1-\tau) - (1-\delta)\lambda_t\right]\mathbb{E}_t G_{t+1} - \delta\frac{e_t\tilde{\lambda}_t}{1+r}\mathbb{E}_t G_{t+1}
\end{aligned}
$$

We also obtain the following general price equation

$$
\begin{aligned}
\mathscr{P}_t y_t &= \Omega + (1-\delta)G_t - \frac{1-s}{1+r}(1-\tau-\lambda_t)(1-\delta)\mathbb{E}_t G_{t+1} \\
&= \delta\Omega + (1-\delta)\left[v(y_t, Y_{t-1} - \mathscr{P}_t y_t) - v(0, Y_{t-1}) - \sigma(e_t) + \mathscr{P}_t y_t\right] \\
&\quad + (1-\delta)\frac{1-s}{1+r}\left[(1-\tau-\lambda_t) + \delta\lambda_t\right]\mathbb{E}_t G_{t+1} - \delta(1-\delta)\frac{e_t\tilde{\lambda}_t}{1+r}\mathbb{E}_t G_{t+1} \\
&\quad - \frac{1-s}{1+r}(1-\tau-\lambda_t)(1-\delta)\mathbb{E}_t G_{t+1} \\
\delta\mathscr{P}_t y_t &= (1-\delta)\left[v(y_t, Y_{t-1} - \mathscr{P}_t y_t) - v(0, Y_{t-1}) - \sigma(e_t)\right] + \delta\left(\Omega + \frac{(1-\delta)}{1+r}\left[(1-s)\lambda_t - e_t\tilde{\lambda}_t\right]\mathbb{E}_t G_{t+1}\right)
\end{aligned}
$$

B System of equations for multi-frictional economy

Asset values:

$$
\begin{aligned}
K(\phi^*) &= S_{l,t} && \text{(B.1)} \\
S_{l,t} &= -\gamma + \frac{1}{1+r}\mathbb{E}_t\left[q_t S_{g,t+1} + (1-q_t)S_{l,t+1}\right] && \text{(B.2)} \\
S_{g,t} &= -w + \frac{1-s}{1+r}\mathbb{E}_t\left[\lambda_t S_{\pi,t+1} + (1-\lambda_t)S_{g,t+1}\right] && \text{(B.3)} \\
S_{\pi,t} &= Py_t - w - \Omega_t + \frac{1-s}{1+r}\mathbb{E}_t\left[\tau S_{g,t+1} + (1-\tau)S_{\pi,t+1}\right] && \text{(B.4)}
\end{aligned}
$$

Stocks and flows:

$$
\begin{aligned}
\mathscr{C}_{0,t+1} &= (1-\tilde{\lambda}_t)\mathscr{C}_{0,t} + \tau C_{1,t} && \text{(B.5)} \\
\mathscr{C}_{1,t+1} &= (1-\tau)\mathscr{C}_{1,t} + \tilde{\lambda}_t\mathscr{C}_{0,t} && \text{(B.6)} \\
\mathscr{N}_{g,t+1} &= (1-s)(1-\lambda_t)\mathscr{N}_{g,t} + q(\theta_t)\mathscr{V}_t + (1-s)\tau N_{\pi,t} && \text{(B.7)} \\
\mathscr{N}_{\pi,t+1} &= (1-s)(1-\tau)\mathscr{N}_{\pi,t} + (1-s)\lambda_t\mathscr{N}_{g,t} && \text{(B.8)}
\end{aligned}
$$

$$
\begin{aligned}
u_{t+1} &= s(1-u_t) + (1-f(\theta_t))u_t && \text{(B.9)} \\
1-u_t &= \mathscr{N}_{g,t} + \mathscr{N}_{\pi,t} && \text{(B.10)}
\end{aligned}
$$

Credit, Labor and goods market tightness and hazard rates:

$$
\begin{aligned}
\theta_t &= \frac{\mathscr{V}_t}{u_t} && \text{(B.11)}\\
q(\theta_t) &= \chi_L \theta_t^{-\eta_L} && \text{(B.12)}\\
f(\theta_t) &= \chi_L \theta_t^{1-\eta_L} && \text{(B.13)}\\
\xi_t &= \frac{Y_{t-1}\mathscr{C}_{0,t}}{\mathscr{N}_{g,t}} && \text{(B.14)}\\
\lambda_t &= \chi_G \xi_t^{1-\eta_G} && \text{(B.15)}\\
\tilde{\lambda}_t &= \chi_G \xi_t^{-\eta_G} && \text{(B.16)}\\
\phi^* &= \frac{\kappa}{e}\frac{1-\beta}{\beta} && \text{(B.17)}\\
K(\phi^*) &= \frac{e}{p(\phi)} + \frac{\kappa}{\phi p(\phi)} && \text{(B.18)}\\
p(\phi^*) &= \chi_C \phi^{*-\eta_C} && \text{(B.19)}
\end{aligned}
$$

and income

$$
Y_t = P y_t \mathscr{N}_{\pi,t} - \gamma \mathscr{V}_t - \kappa \mathscr{B}_{ct} \tag{B.20}
$$

Variables: S_l, S_g, S_π, Which is a system of 20 equations for 20 variables.

B.1 Stocks : consumption and unemployment

The steady state stocks on labor and goods markets,

$$
\begin{aligned}
\mathscr{C}_0 &= (1-\tilde{\lambda})\mathscr{C}_0 + \tau C_1 \\
\mathscr{C}_1 &= (1-\tau)\mathscr{C}_1 + \tilde{\lambda}\mathscr{C}_0 \\
\mathscr{N}_g &= (1-s)(1-\lambda)\mathscr{N}_g + f(\theta)u + (1-s)\tau N_\pi \\
\mathscr{N}_\pi &= (1-s)(1-\tau)\mathscr{N}_\pi + (1-s)\lambda\mathscr{N}_g \\
\mathscr{N}_g + \mathscr{N}_\pi &= 1-u \\
u &= s(1-u) + (1-f(\theta))u
\end{aligned}
$$

lead to

$$
\begin{array}{cc}
\mathscr{C}_0 = \frac{\tau}{\tau+\tilde{\lambda}} & \mathscr{C}_1 = \frac{\tilde{\lambda}}{\tau+\tilde{\lambda}} \\
u = \frac{s}{s+f(\theta)} & \mathscr{N}_g + \mathscr{N}_\pi = \frac{f(\theta)}{s+f(\theta)}
\end{array}
$$

Then, noting that $\frac{\mathscr{N}_\pi}{\mathscr{N}_g} = \frac{(1-s)\lambda}{(s+\tau-s\tau)}$,

$$
\begin{aligned}
\mathscr{N}_\pi &= \frac{f(\theta)}{s+f(\theta)} \frac{\frac{(1-s)\lambda}{(s+\tau-s\tau)}}{1+\frac{(1-s)\lambda}{(s+\tau-s\tau)}} \\
\mathscr{N}_g &= \frac{f(\theta)}{s+f(\theta)} \frac{1}{1+\frac{(1-s)\lambda}{(s+\tau-s\tau)}}
\end{aligned}
$$

B.2 Steady-state equilibrium tightness

We already know that **credit market tightness** is given, both in the steady-state or out of the steady-state, by

$$\phi^* = \frac{\beta}{1-\beta}\frac{\kappa}{e}$$

We can calculate the steady-state **consumption tightness** ξ^* as the solution to:

$$\xi = \frac{Y\mathscr{C}_0}{\mathscr{N}_g} = Y\frac{\tau}{\tau+\tilde{\lambda}(\xi)}\frac{s+f(\theta)}{f(\theta)}\left(1+\frac{(1-s)\lambda(\xi)}{(s+\tau-s\tau)}\right)$$

B.3 Steady-state equilibrium

Before studying the dynamics properties of this multi-frictional economy, we can obtain some intuition by examining the properties of the steady state.

B.3.1 Stocks : consumption and unemployment

The steady state stocks of unmatched and matched consumers are simply the ratios

$$\mathscr{C}_0 = \frac{\tau}{\tau+\tilde{\lambda}} \quad and \quad \mathscr{C}_1 = \frac{\tilde{\lambda}}{\tau+\tilde{\lambda}}$$

while the steady state unemployment and employment rates are[12]

$$u = \frac{s}{s+f(\theta)} \quad and \quad \mathscr{N}_g + \mathscr{N}_\pi = \frac{f(\theta)}{s+f(\theta)}$$

Since the ratio of matched to unmatched firms on the goods market is given by $\frac{\mathscr{N}_\pi}{\mathscr{N}_g} = \frac{(1-s)\lambda}{(s+\tau-s\tau)}$, the steady state stocks are of firms in the final stages are

$$\begin{aligned}
\mathscr{N}_\pi &= \frac{f(\theta)}{s+f(\theta)} \frac{\frac{(1-s)\lambda}{(s+\tau-s\tau)}}{1+\frac{(1-s)\lambda}{(s+\tau-s\tau)}} \\
\mathscr{N}_g &= \frac{f(\theta)}{s+f(\theta)} \frac{1}{1+\frac{(1-s)\lambda}{(s+\tau-s\tau)}}
\end{aligned}$$

B.4 Numerical solution method

[12] See the appendix for details.

C Dynamic Solution

C.1 Log-linear system of equations

First equation for labor market:

$$0 = q\left(S_g - K(\phi)\right)\widehat{q}_t + qS_g\mathbb{E}_t\widehat{S}_{g,t+1}$$

Second, equation for goods market:

$$S_g\widehat{S}_{g,t} = -w\widehat{w}_t + \frac{1-s}{1+r}\mathbb{E}_t\left\{\lambda S_\pi\widehat{S}_{\pi,t+1} + (1-\lambda)S_g\widehat{S}_{g,t+1} + \lambda\left[S_\pi - S_g\right]\widehat{\lambda}_t\right\}$$

Third, equation for sales stage, with constant operating costs $\Omega_t = \Omega$:

$$S_\pi\widehat{S}_{\pi,t} = Py\left[\widehat{P}_t + \widehat{y}_t\right] - w\widehat{w}_t + \frac{1-s}{1+r}\mathbb{E}_t\left\{\tau S_g\widehat{S}_{g,t+1} + (1-\tau)S_\pi\widehat{S}_{\pi,t+1}\right\}$$

$$\begin{aligned}
\mathscr{C}_0\widehat{\mathscr{C}}_{0,t+1} &= (1-\tilde{\lambda})\mathscr{C}_0\widehat{\mathscr{C}}_{0,t} + \tau\mathscr{C}_1\widehat{\mathscr{C}}_{1,t} - \tilde{\lambda}\mathscr{C}_0\widehat{\tilde{\lambda}}_t \\
\mathscr{C}_1\widehat{\mathscr{C}}_{1,t+1} &= (1-\tau)\mathscr{C}_1\widehat{\mathscr{C}}_{1,t} + \tilde{\lambda}\mathscr{C}_0\left[\widehat{\tilde{\lambda}}_t + \widehat{\mathscr{C}}_{0,t}\right] \\
\mathscr{N}_g\widehat{\mathscr{N}_{g,t}} &= (1-s)(1-\lambda)\mathscr{N}_g\widehat{\mathscr{N}_{g,t-1}} + q(\theta)\mathscr{V}\left[\widehat{q(\theta_t)} + \widehat{\mathscr{V}_t}\right] + (1-s)\tau N_\pi\widehat{N}_{\pi,t} - \lambda(1-s)\mathscr{N}_g\widehat{\lambda}_t \\
\mathscr{N}_\pi\widehat{\mathscr{N}_{\pi,t+1}} &= (1-s)(1-\tau)\mathscr{N}_\pi\widehat{\mathscr{N}_{\pi,t}} + (1-s)\lambda\mathscr{N}_g\left[\widehat{\lambda}_t + \widehat{\mathscr{N}_{g,t}}\right]
\end{aligned}$$

$$\begin{aligned}
u\widehat{u}_{t+1} &= (1-f(\theta)-s)u\widehat{u}_t - f(\theta))uf(\widehat{\theta}_t) \\
-u\widehat{u}_t &= \mathscr{N}_g\widehat{\mathscr{N}_{g,t}} + \mathscr{N}_\pi\widehat{\mathscr{N}_{\pi,t}}
\end{aligned}$$

Labor and goods market tightness and hazard rates:

$$\begin{aligned}
\hat{\theta}_t &= \hat{\mathscr{V}}_t - \hat{u}_t \\
q(\widehat{\theta}_t) &= -\eta_L\widehat{\theta}_t \\
f(\widehat{\theta}_t) &= (1-\eta_L)\widehat{\theta}_t \\
\hat{\xi}_t &= \widehat{Y}_{t-1} + \widehat{\mathscr{C}}_{0,t} - \widehat{\mathscr{N}_{g,t}} \\
\lambda(\widehat{\xi}_t) &= (1-\eta_G)\widehat{\xi}_t \\
\tilde{\lambda}(\widehat{\xi}_t) &= -\eta_G\widehat{\xi}_t
\end{aligned}$$

and income

$$Y\widehat{Y}_t = Py\mathscr{N}_\pi\left[\widehat{y}_t + \widehat{\mathscr{N}_{\pi,t}}\right] - \gamma\mathscr{V}\widehat{\mathscr{V}}_t$$

C.2 Numerical solution method