

Search Frictions and Wage Dispersion*

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Abstract

Using the on-the-job search model we show theoretically how match-specific productivities can be identified. Second, we propose a way to empirically measure the contribution of match-specific productivities to wages. Having an independent measurement of match productivities we show how the returns to tenure and experience can be estimated in an unbiased way. This provides a possible resolution to the classic identification problem.

We apply the proposed methodology to measure the contribution of search frictions to the level of wage dispersion observed in the data. We find relatively small returns to firm tenure and a small variance of match qualities. In a counterfactual experiment we show that eliminating all search frictions will have only a modest impact on wage dispersion.

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1 Introduction

A robust finding in countless empirical studies is that over a half of the observed wage variation cannot be accounted for by the observable worker characteristics that are supposed to measure productivity differences between them. Yet, understanding the reasons for why observationally similar workers are paid differently is crucial for understanding the functioning of the labor market. First, wage dispersion is an important contributor to the dispersion of consumption. Labor income accounts for around two-thirds of aggregate income and for a much larger share of income of most households. Second, the assertion that differences in wages are not accounted for by differences in productivity raises questions about economic efficiency. In particular, if differences in wages are driven by different productivity of jobs or matches between workers and firms, output can be increased by reallocating workers from bad to good matches. Moreover, large differences in wages of similar workers suggest the presence of substantial rents and it is important to determine the size and allocation of these rents to understand the labor market.

A theory that appears to hold a great promise in accounting for the unexplained part of wage dispersion is based on labor market search (Mortensen (2005) articulates this view). One line of research (e.g., Butters (1977), Burdett and Judd (1983), Mortensen (1991), Burdett and Mortensen (1998)) has determined conditions under which dispersion in wage policy is the only equilibrium outcome in the labor market with search frictions even if all employers and employees are identical in their productivity. In another class of models with labor market search (e.g., Dimond (1982), Mortensen (1982), Pissarides (1985, 2000), Mortensen and Pissarides (1994)) workers and firms bargain over match surplus implied by search frictions and this implies wage dispersion given productivity dispersion across worker-employer matches.

In this paper we directly measure in the data the extent of wage dispersion attributable to search frictions in models with on-the-job search. To do so we must decompose wages in the data into components due to accumulation of general human capital accumulated with labor market experience and transferable across employers, employer-specific human capital accumulated with firm tenure, and changes in the quality of matches between workers and

firms.¹ Unfortunately, only two of these components can be identified using the wage data. This is a classic identification problem in labor economics. Clearly tenure and experience grow one-to-one while a worker is employed. Thus, to tell apart their contribution to wage growth on the job one must study the changes in wages when workers change jobs. To the extent that wages go down, a worker lost some employer-specific skills. To the extent that wages continue to grow the worker preserved his general skills accumulated with labor market experience. However, the quality of job matches changes when workers change jobs. A systematic relationship between job quality and labor market experience or firm tenure will thus cause a severe bias in estimating the returns to tenure and experience (Altonji and Shakotko (1987), Topel (1991)).² Even if this bias could be overcome, so that the returns to tenure and experience could be measured in an unbiased way, one could only measure job match qualities as a residual, implicitly assigning the complete residual wage dispersion across jobs to differences in match qualities attributable to search frictions.

Thus, to assess the contribution of search frictions to wage inequality one needs a direct measure of frictional wage dispersion. One possible solution is to develop and estimate a dynamic model of on-the-job search, which however has to be parsimonious in many respects and thus cannot account for the typical complexity of wage regressions (Eckstein and Wolpin (1989)). We propose a simpler strategy. We show that a dynamic model of on-the-job search implies that the expected match-specific component of the wage is a function of the expected number of offers. Number of offers received is not observable. However, in standard search models the probability of receiving an offer at a point in time is proportional to the labor market tightness. The labor market tightness is observable. Thus, we can use the data on labor market tightness to measure the probability to receive an offer each period. This allows us to measure the expected number of offers through the sum of labor market tightness during the employment spell. This new variable is our estimate of the worker's match-specific productivity. Next, we show that controlling for match quality through the sum of labor market tightness eliminates the identification problems in estimating the

¹In the labor literature, the empirical importance of job mobility and on-the-job search for wage growth has been recognized at least since the work of Topel and Ward (1992).

²Dustmann and Meghir (2005) are an exception as they assume that workers displaced because of a firm closure are a random sample of the workforce.

returns to seniority and delivers an unbiased estimate of the returns to tenure.

Having obtained the measures of returns to labor market experience and firm tenure as well as the measures of job match qualities we can conduct a hypothetical counterfactual experiment of eliminating search frictions. We assume that in the absence of search frictions all workers will work in the most productive job match eliminating the contribution of the dispersion of match qualities to the dispersion of wages. Moreover, the distribution of firm tenure will also change when search frictions are eliminated. We use the measured returns to tenure to evaluate the contribution of this change to wage dispersion as well. We find the both effects are quite small. Thus, we conclude that only a small fraction of residual wage dispersion is attributable to search frictions.

Why are then observationally similar workers paid differently? The jury is still out. Maybe because of unobserved productivity differences. Or maybe more sophisticated models can be constructed. This is very important issue to understand in future work.

The results of this paper shed also light on a more general issue. In the last thirty years or so economists have increasingly used search theory to model the labor market. It has then been established that search theory can address a wide variety of issues that standard frictionless (Walrasian) models have had difficulties addressing. Whereas this work is theoretically very successful the quantitative success is less clear. It is still an open question whether deviations from a Walrasian world, due to search frictions, are quantitatively significant. One possibility to answer this question, pursued in this paper, is to measure the amount of heterogeneity in the quality of jobs that a worker accepts or is potentially willing to accept. If this distribution is not very dispersed, all jobs have almost the same quality and the worker is happy to accept every job. Search frictions are not very important in this case since the gains from search would be tiny and the allocation of jobs is close to the Walrasian allocation where workers immediately locate the best job available. If on the other hand, the heterogeneity of acceptable job is large, workers may end up in bad jobs, at least for some time, although better jobs are available. Search frictions prevent workers from finding them leading to significant efficiency losses. The results of this paper that wage dispersion is only modest suggest that search frictions are not large and that the efficiency gains from eliminating them are expected to be small.

We can also use our method to answer further related questions. We can account for the evolution of wage dispersion over the lifecycle caused by search frictions. Are search frictions more important early in a worker's career and do they become smaller as career progresses? Moreover, workers might differ along two important dimensions. First, they might have permanent productivity differences. Second, they might have permanent differences in the frequency with which they receive job offers (for example, this might depend on the line of work these individuals are in). Our method also allows us to separately measure these two contributors to wage inequality.

A different method to assess the extent of wage inequality due to search frictions has been recently proposed by Hornstein, Krusell, and Violante (2009). They argue that the structure of the search model implies tight restrictions between the amount of wage dispersion the model can generate and the magnitude of labor market flows that can be directly measured. Their argument is strongest in models where workers cannot search on the job. In particular, they show that substantial wage dispersion due to search frictions is inconsistent with the observations that unemployment durations are very short in U.S. data. Given the plausible range of values for the disutility of being unemployed, short unemployment durations directly imply that the option value of waiting for a good offer is small.

Their approach is less powerful if workers are able to search on the job. And indeed, job-to-job flows are large in the data suggesting that this is an important restriction.³ If on-the-job search is as efficient as search while unemployed, the duration of unemployment contains little information about the distribution of potential match qualities because unemployed workers take the first draw that dominates the value of non-market activity and continue searching while employed. More generally, the more efficient is on-the-job search, the less informative is the duration of unemployment for the distribution of matches. There is some information contained in the frequency of observed job-to-job moves but how this translates into the probability to receive offer is strongly model-dependent. As a result, Hornstein, Krusell, and Violante (2009) show that the observed job-to-job moves imply fairly wide

³Based on the monthly Current Population Survey (CPS) data, Fallick and Fleischman (2004), for example, estimate that in the U.S. about 2.7% of employed workers move job-to-job every month. Moscarini and Thomsson (2007) show that a different treatment of missing observations in monthly CPS data raises this estimate to 3.2%.

bounds on the extent of wage dispersion that may be attributed to search frictions. For example, wage dispersion is relatively small in the model of Burdett and Mortensen (1998) on the one hand and large in the models of Postel-Vinay and Robin (2002) and Cahuc, Postel-Vinay, and Robin (2006) on the other hand.

The paper is organized as follows. In Section 2 we present an on-the-job search model. In Section 3 we describe the data used in the empirical analysis. Section 4 contains the results of the empirical evaluation of the contribution of match qualities, tenure and experience to wages. In Section 5 we calibrate our theoretical model and use it to evaluate the performance of our method in measuring job match qualities and the returns to tenure and seniority. We also contrast its performance with various alternative proposed in the literature for estimating the returns to tenure and experience. Finally, we conduct counterfactual experiments to study the effects on wage distribution from hypothetically eliminating search frictions. Section 6 concludes.

2 Model with On-the-job search

A continuum of risk-neutral workers of measure one participates in the labor market.⁴ At a moment in time, each worker can be either employed or unemployed. An unemployed worker faces a probability λ_θ of getting a job offer. This probability depends exogenously on a business cycle indicator θ and is increasing in θ . For example, a high level of θ (say, a high level of market tightness or low level of unemployment rate) means that it is easy to find a job, since λ_θ is high as well. Employed workers also face a probability q_θ of getting a job offer, which also depends monotonically on θ . A worker who accepts the period t offer, starts working immediately for the new employer in period $t + 1$. The unemployment rate in period t is denoted u_t . The business cycle indicator θ_t is a stochastic process which is drawn from a stationary distribution.

A match between worker i and a job/employer/firm j at date t is characterized by an idiosyncratic productivity level ϕ_{ij} . Each time a worker meets a new employer, a new value

⁴Workers thus maximize expected discounted income if the market interest rate r and the discount factor β satisfy $\frac{1}{1+r} = \beta$. If instead the worker is risk averse and markets are complete, then maximizing expected utility and maximizing expected income are also equivalent (Rogerson, Shimer, and Wright (2005)).

of ϕ is drawn, according to a distribution function F with support $[\underline{\phi}, \bar{\phi}]$, density f and expected value μ_ϕ . A worker switches if and only if the present value of wages (and thus lifetime utility) increases. The level of ϕ and thus productivity remain unchanged as long as the worker does not switch.⁵

The wage in the model is consistent with the standard specifications in empirical literature.⁶ The accumulation of firm-specific and general human capital increase wages. Wages increase by $e^{\beta_1 X_{ijt}}$ if total labor market experience equals X_{ijt} reflecting the accumulation of general human capital. Wages also increase by $e^{\beta_2 T_{ijt}}$ if tenure equals T_{ijt} reflecting the accumulation of firm-specific human capital.⁷ The wage also has a fixed individual specific component, μ_i , and a fixed job match specific component, ϕ_{ij} , and thus equals

$$e^{\beta_1 X_{ijt} + \beta_2 T_{ijt}} \mu_i \phi_{ij}, \quad (1)$$

so that the log wage equals

$$W_{ijt} = \beta_1 X_{ijt} + \beta_2 T_{ijt} + \log(\mu_i) + \log(\phi_{ij}). \quad (2)$$

We abstract from time effects and from nonlinear terms in experience and tenure. Later we will allow for nonlinearity and show that is inconsequential for our results. To model time effects we could add the unemployment rate in period t to the model. Again this would not change our results since we allow for time dummies in the empirical implementation.

An employed worker faces an exogenous probability s of getting separated and becoming unemployed. Finally, we define the two variables which, as we will show, measure unobserved match quality. For every worker who left unemployment in period 0 and has worked continuously since then we first define an employment cycle. Assume that the worker

⁵This assumption just simplifies the theoretical analysis. We could, for example, add a temporary i.i.d. productivity shock η_{ijt} which is specific to the worker.

⁶E.g., Altonji and Shakotko (1987), Abraham and Farber (1987), and Topel (1991).

⁷To be precise, X years of experience increase general human capital by $\beta_1 x$ and T years of tenure increase firm specific human capital by $\beta_2 T$. Note that we assume that all benefits from human capital accrue to workers. For general human capital this seems reasonable as these skills are transferable across employers. For specific human capital we could instead assume that only a fraction χ accrues to workers and that the $\beta_2 = \chi \tilde{\beta}_2$, where $\tilde{\beta}_2$ is the true return to specific human capital. We could do the same for general human capital. All our results would remain unchanged.

switched employers in periods $1 + S_1, 1 + S_2, \dots, 1 + S_k$, so that this worker stayed with his first employer between periods 0 and S_1 , with the second employer between period $1 + S_1$ and S_2 and with employer i between period $1 + S_{j-1}$ and S_j . For such an employment cycle, a sequence $\theta_0, \dots, \theta_{S_j}$ of business cycle indicators and coefficients $\{\rho_t\}_{t=1, \dots}$, define $q_t^{HM} = \rho_1 q_{1+S_{j-1}} + \dots + \rho_{S_j - S_{j-1}} q_{S_j}$ for $1 + S_{j-1} \leq t \leq S_j$ and $q_t^{EH} = q_{S_1}^{HM} + \dots + q_{S_{j-1}}^{HM}$ for $1 + S_{j-2} \leq t \leq S_{j-1}$. The variable q_t^{HM} is constant within every job spell and equals the sum of q 's from the start of the current job spell until the the last period of this job spell. The variable q_t^{EH} summarizes the employment history in the current employment cycle until the start of the current job spell. The idea is that q^{HM} controls for selection effects from the current job spell whereas q^{EH} controls for the employment history. Note that q^{HM} and q^{EH} , receiving an offer and the switching dates S_j are individual specific and should have a superscript i (which we omitted for simplicity).

2.1 Implications

Our objective is to investigate how we can use q^{HM} and q^{EH} to control for the unobserved match quality ϕ . To this end, we have to consider how the value of ϕ is related to q^{HM} and q^{EH} . This unobserved component of the wage is the only contributor which is subject to selection effects in our model.

We first consider a worker who in his first job, so that the amount of tenure is positive but initial experience at the beginning of this job in period 0 is zero. We also temporarily ignore the possibility of exogenous and endogenous separations into unemployment. A worker who start a new job at time 0 draws a ϕ from the exogenous distribution F . We now want to derive the cdf of ϕ for such a worker with tenure T who has received N offers on the job.

As a first step we therefore have to derive worker's switching behavior. Suppose a worker in a match with idiosyncratic productivity ϕ_{ij} encounters another potential match with idiosyncratic productivity level $\tilde{\phi}$. In the appendix we show that a worker switches if and only if $\tilde{\phi} > \phi_{ij} e^{\beta_2 T_{ij} t}$, that is only if the productivity is higher in the new job than in the current one. Let (the random variable) \tilde{N}_t^i be the number offers of individual i in

period t :

$$\tilde{N}_t^i = \begin{cases} 1 & \text{with prob. } q_t \\ 0 & \text{with prob. } 1 - q_t. \end{cases}$$

A type $\hat{\phi}$ rejects an period t offer with probability

$$F(\min(\hat{\phi}e^{\beta_2 T_{ijt}}, \bar{\phi})). \quad (3)$$

The probability to have a value of ϕ less than or equal to $\hat{\phi}$ in period t then equals:

$$G_t^i(\hat{\phi} | \tilde{N}_0^i, \dots, \tilde{N}_t^i) = \frac{1}{\mu_t(\tilde{N}_0^i, \dots, \tilde{N}_t^i)} \int_{\underline{\phi}}^{\hat{\phi}} \prod_{s=0}^t F(\min(\phi e^{\beta_2 T_{ijs}}, \phi))^{\tilde{N}_s^i} f(\phi) d\phi, \quad (4)$$

where tenure $T_{ijs} = s$ and

$$\mu_t(\tilde{N}_0^i, \dots, \tilde{N}_t^i) = \int_{\underline{\phi}}^{\bar{\phi}} \prod_{s=0}^t F(\min(\phi e^{\beta_2 T_{ijs}}, \phi))^{\tilde{N}_s^i} f(\phi) d\phi, \quad (5)$$

so that G is a probability measure (as $G(\bar{\phi}) = 1$). Receiving an offer in period s ($\tilde{N}_s^i = 1$) shifts the distribution by First-order-stochastic dominance and thus leads to a higher expected value of ϵ . The reason is that a worker who has rejected more offers likely drew a higher match quality ϕ at the beginning of the current job.

Equation (4) contains valuable information about the value of ϕ_{ij} , which is constant during each job spell. The best predictor of ϕ , using the information available at date t , equals

$$E_t(\phi | \tilde{N}_0^i, \dots, \tilde{N}_t^i) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \phi dG_t^i(\phi | \tilde{N}_0^i, \dots, \tilde{N}_t^i). \quad (6)$$

Since ϕ is constant for $0 \leq s \leq S_1$, we use the predictor which contains the most information about this ϕ , the expectation at S_1 :

$$E_{T_1}(\phi | \tilde{N}_0^i, \dots, \tilde{N}_{S_1}^i) := \int_{\underline{\epsilon}}^{\bar{\epsilon}} \phi dG_{S_1}^i(\phi | \tilde{N}_0^i, \dots, \tilde{N}_{S_1}^i). \quad (7)$$

The expected value of ϵ then equals

$$E(\phi) = \sum_{s=0}^{S_1} E_{S_1}(\phi | \tilde{N}_0^i, \dots, \tilde{N}_{S_1}^i) P_s(N_s^i), \quad (8)$$

where $P_s(N_s^i) = q_s$ is the probability to receive an offer in period s .

We now extend these derivations and to allow for job-to-job moves. A worker in period t could be in job k instead of being in the first job after unemployment. In particular, the labor market experience at the beginning of the job is not trivial. Therefore, labor market experience or more generally the history of previous aggregate labor market conditions is a determinant of the current type ϕ and thus of wages. The following derivations are similar to the previous one except that the fact that the worker switched provides additional information (to just observing q) about the number of offers received. In particular, we know that a switcher received at least one offer. We now take these aspects into account and apply the theory separately to the jobs during an employment cycle.

Suppose the value of the idiosyncratic productivity level equals ϕ_{k-1} in the $(k-1)^{\text{th}}$ job before the worker switched to the k^{th} job in period $1 + S_{k-1}$. Conditional on this we compute now the expected value of ϕ_k in this new job. The expected value of ϕ_k in period $1 + S_{k-1} \leq t \leq S_k$ for a worker who is still employed in period t and has received \tilde{N}_s offers in period s equals

$$E_t(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_t) = \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} \phi d\tilde{G}_t(\phi | \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_t), \quad (9)$$

where $\tilde{\phi}_{k-1} = \phi_{k-1} e^{\beta_2 T_{S_{k-1}}}$ and

$$\tilde{G}_t(\hat{\phi} | \tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i) = \frac{1}{\mu_t(\tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i)} \int_{\tilde{\phi}_{k-1}}^{\hat{\phi}} \prod_{s=1+S_{k-1}}^t F(\min(\phi e^{\beta_2 T_s}, \bar{\phi}))^{\tilde{N}_s} f(\phi) d\phi, \quad (10)$$

where again setting

$$\mu_t(\tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i) = \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} \prod_{s=1+S_{k-1}}^t F(\min(\phi e^{\beta_2 T_s}, \bar{\phi}))^{\tilde{N}_s} f(\phi) d\phi \quad (11)$$

renders \tilde{G} a probability measure. The main difference between \tilde{G} and G is that we now truncate at $\tilde{\phi}_{k-1}$ because a worker switches if and only if $\tilde{\phi}_{k-1} \leq \phi_k$.

As in the derivation of equation (7), we use the predictor which contains the most information about ϕ , so that the expectation of ϕ_k at $1 + S_{k-1} \leq t \leq S_k$ equals

$$E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) = \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} \phi d\tilde{G}_{S_k}(\phi | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) d\phi. \quad (12)$$

The expected value of ϕ then equals

$$E_{S_k}(\phi_k | \tilde{\phi}_{k-1}) = \sum_{s=1+S_{k-1}}^{S_k} E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) P_s(N_s^i), \quad (13)$$

where $P_s(N_s^i) = q_s$ is the probability to receive an offer in period s .

2.2 Linear Approximation

To make our estimator $E_{S_k}(\phi_k | \tilde{\phi}_{k-1})$ applicable for our empirical implementation, we linearize (13) and relate it to an observable (to the econometrician) variable. We first approximate the integral (12). It equals (integration by parts):

$$E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) = \bar{\phi} - \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} \tilde{G}_{S_k}(\phi | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) d\phi. \quad (14)$$

Receiving an offer in period s ($\tilde{N}_s = 1$) increases this value, so that

$$\begin{aligned} \Delta_s &= E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s = 1, \dots, \tilde{N}_{S_k}) \\ &\quad - E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s = 0, \dots, \tilde{N}_{S_k}) \end{aligned} \quad (15)$$

is positive. More generally, interpreting \tilde{N} as a continuous variable, we show that

$$\begin{aligned} \Delta_s(\epsilon) &= E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s + \epsilon, \dots, \tilde{N}_{S_k}) \\ &\quad - E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s, \dots, \tilde{N}_{S_k}) \end{aligned} \quad (16)$$

is positive for all $\epsilon > 0$.⁸ We now want to approximate $E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k})$ as a linear function of \tilde{N} and $\tilde{\phi}_{k-1}$. As we have just established, receiving an offer increases the expected value and this increase is lower at higher tenure levels.⁹ We therefore obtain the following approximation,

$$E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) \approx \gamma_0 + \sum_{T=1}^{S_k - S_{k-1}} \rho_T \tilde{N}_{S_{k-1}+T} + \gamma_1 \tilde{\phi}_{k-1}, \quad (17)$$

for a constant γ_0 and positive values $\rho_T > 0$. We also show that the expectation of ϕ_k is increasing in $\tilde{\phi}_{k-1}$, so that $\gamma_1 > 0$. All coefficients are evaluated at their steady state levels.

⁸We can also show that this difference is smaller for higher tenure levels if $\frac{f_x}{F}$ is decreasing in x . This condition is stronger than F being *log-concave* ($\log(F)$ is concave), so that we do not necessarily expect this property, $\Delta_s(\epsilon) \leq \Delta_r(\epsilon)$, for $s > r$, holds.

⁹The latter result holds only for log-concavity. We do not impose any such sign restrictions in the empirical implementation.

The expected value of ϕ_k conditional on $\tilde{\phi}_{k-1}$, $E_t(\phi_k|\tilde{\phi}_{k-1})$ can then be simplified to:¹⁰

$$E_t(\phi_k|\tilde{\phi}_{k-1}) \approx \gamma_0 + \sum_{T=1}^{S_k-S_{k-1}} \rho_T \tilde{N}_{S_{k-1}+T} P(\tilde{N}_{S_{k-1}+T} = 1) + \gamma_1 \tilde{\phi}_{k-1}. \quad (18)$$

Since the probability to receive an offer in period t and thus the expected number of offers in period equal q_t , the expected value of ϕ_k , conditional on $\tilde{\phi}_{k-1}$ for $1 + S_{k-1} \leq t \leq S_k$ can be expressed as

$$E_t(\phi_k|\tilde{\phi}_{k-1}) \approx c_0 + \sum_{T=1}^{S_k-S_{k-1}} \rho_T q_{S_{k-1}+T} + c_1 \tilde{\phi}_{k-1}. \quad (19)$$

It thus holds for the unconditional expectation

$$E_t(\phi_k) \approx c_0 + q_{S_k}^{HM} + c_1 E_{S_{k-1}}(\tilde{\phi}_{k-1}). \quad (20)$$

Since $\tilde{\phi}_{k-1} = \phi_{k-1} e^{\beta_2 T_{S_{k-1}}}$, we now want to first relate $E_t(\phi_k)$ to q^{EH} . Similar derivations as before show that $E_{T_{k-1}}(\phi_{k-1})$ can be approximated through $\sum_{N_{T_i}} E_t(\phi | N_{T_i}) P_{T_i}(N_{T_i})$, which equals by the same arguments as above

$$\sum_{N_{T_{k-1}}} E_t(\phi | N_{T_i}) P_{T_i}(N_{T_i}) = \sum_{l=1}^{k-1} q_{S_l}^{HM} = q_{S_{k-1}}^{EH}. \quad (21)$$

Using that $\tilde{\phi}_{k-1} = \phi_{k-1} e^{\beta_2 T_{S_{k-1}}}$ we have that

$$E_t(\phi_k) \approx c_0 + q_{S_k}^{HM} + c_2 q_{S_{k-1}}^{EH} e^{\beta_2 T_{S_{k-1}}}. \quad (22)$$

Alternatively we could approximate $E_{S_{k-1}}(\phi_{k-1})$ by applying the derivation for ϕ_k to ϕ_{k-1} . This yields the expected value of $E_t(\phi_{k-1})$, for $1 + S_{k-2} \leq t \leq S_{k-1}$ conditional on $\tilde{\phi}_{k-2}$:

$$E_t(\phi_{k-1}) \approx c_0 + q_{S_{k-1}}^{HM} + c_1 E_{S_{k-2}}(\tilde{\phi}_{k-2}), \quad (23)$$

so that for $1 + S_{k-1} \leq t \leq S_k$

$$E_t(\phi_k) \approx c_0 + q_{S_k}^{HM} + c_1 e^{\beta_2 T_{S_{k-1}}} \{c_0 + q_{S_{k-1}}^{HM} + c_1 E_{T_{k-2}}(\tilde{\phi}_{k-2})\}. \quad (24)$$

One possibility is again to approximate $E_{S_{k-2}}(\tilde{\phi}_{k-2})$ by $q_{S_{k-2}}^{EH} e^{\beta_2 T_{S_{k-2}}}$ so that

$$E_t(\phi_k) \approx c_0 + c_0 c_1 e^{\beta_2 T_{S_{k-1}}} + q_{S_k}^{HM} + c_1 q_{S_{k-1}}^{HM} e^{\beta_2 T_{S_{k-1}}} + c_1 c_1 q_{T_{k-2}}^{EH} e^{\beta_2 (T_{S_{k-1}} + T_{S_{k-2}})}. \quad (25)$$

¹⁰Note that the expectation w.r.t. N_{T_k} only affects the N -term since ϕ_{k-1} is constant in job spell k and σ_t^k is an aggregate variable.

Alternatively, iterating these substitutions for $\phi_{k-2}, \phi_{k-3}, \dots$ shows that for any $0 \leq m \leq k-1$, $E_t(\phi_k)$ can be approximated as a function of $q_{S_k}^{HM}$ and $q_{S_{k-1}}^{HM} e^{\beta_2 \sum_{j=k-1}^{k-1} T_{S_j}} \dots q_{S_{k-m}}^{HM} e^{\beta_2 \sum_{j=k-m}^{k-1} T_{S_j}}, q_{S_{k-m-1}}^{EH} e^{\beta_2 \sum_{j=k-m-1}^{k-1} T_{S_j}}$. In the extreme case, for $m = k-1$, $E_t(\phi_k)$ is a function of q^{HM} and $q^{HM} e^{\beta_2 \sum T}$ only. However, this inflates the number of regressors and we will find that this renders many of them insignificant. We therefore use only three regressors, $q_{S_k}^{HM}, q_{S_{k-1}}^{EH} e^{\beta_2 T_{S_{k-1}}}$ as implied by equation (22) and show that this parsimonious specification yields the same results as richer specification which use more regressors.

Finally, we approximate

$$\log(\phi) \approx \tilde{c}_0 + \tilde{c}_1 \log(q^{HM}) + \tilde{c}_2 \log(q^{EH}) + \tilde{c}_3 T_{S_{k-1}}, \quad (26)$$

for coefficients \tilde{c}_i .

2.3 Nonlinearity of Tenure and Experience

In general, the switching rule of a worker is time dependent and depends on all other state variables $\tilde{\phi}_t(\phi, T, X)$. In the special case with linear tenure and linear experience (as in the previous sections), we established that the switching rule takes a simple form, $\tilde{\phi} = \min(\phi e^{\beta_2 T}, \bar{\phi})$. Nothing in the derivation of this result depends on the assumption that the returns to experience are linear. However, if the returns to tenure are not linear, things are not that easy. Whereas the decision to stay or to switch does not affect the increase in experience, it does affect the accumulation of tenure. By definition, tenure is reset to zero if a workers switches but keeps accumulating in case the worker stays. The worker then faces a potentially nontrivial intertemporal decision problem. A worker with positive tenure may decide to switch to job with a higher level $\hat{\phi}$ than the current level ϕ but that pays less in the current period than what the worker would earn if he stayed (because he loses tenure by switching). However, if tenure accumulates faster in the new job than it would have in the old job, the current period wage may eventually dominate the current period wage in the old job. The worker thus has to weigh the early losses against the later gains, what renders the problem truly dynamic. This intertemporal dimension becomes relevant only if tenure accumulates faster in the new job than it would have in the old job. This is for

example the case if the returns to tenure are quadratic. It does not apply if the returns to tenure are linear, explaining why we can derive a simple rule in this case. With linear tenure, a reversal of the ranking of wages if staying and wages if switching never occurs. If one wage is higher than the other in the first period after switching, it will also be higher in the future.

Linear tenure, although easy to analyze, may not be the most appropriate description of the accumulation of firm specific human capital. We therefore extend our analysis by allowing for curvature in tenure and experience and show that, after linearizing, we obtain basically the same result. The necessary modifications resort to replacing the simple switching rule by the general switching rule. The returns to tenure are now described by a function $\beta_2(T)$ and the returns to experience are described by a function $\beta_1(X)$. The expected value of ϕ_k in period $1 + S_{k-1} \leq t \leq S_k$ for a worker who is still employed in period t and has received \tilde{N}_s offers in period s now equals

$$E_t(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_t) = \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} \phi d\tilde{G}_t(\phi | \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_t), \quad (27)$$

where $\tilde{\phi}_{k-1} = \tilde{\phi}_{T_{S_{k-1}}}(\phi_{k-1}, T_{S_{k-1}}, X_{S_{k-1}})$ and

$$\tilde{G}_t(\hat{\phi} | \tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i) = \frac{1}{\mu_t(\tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i)} \int_{\tilde{\phi}_{k-1}}^{\hat{\phi}} \prod_{s=1+S_{k-1}}^t F(\min(\tilde{\phi}_s(\phi, T_s, X_s), \bar{\phi}))^{\tilde{N}_s} f(\phi) d\phi, \quad (28)$$

where again setting

$$\mu_t(\tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i) = \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} \prod_{s=1+S_{k-1}}^t F(\min(\tilde{\phi}_s(\phi, T_s, X_s), \bar{\phi}))^{\tilde{N}_s} f(\phi) d\phi \quad (29)$$

Following the same step as before we get the following approximation,

$$E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) \approx \gamma_0 + \sum_{T=1}^{S_k-S_{k-1}} \rho_T \tilde{N}_{S_{k-1}+T} + \gamma_1 \tilde{\phi}_{k-1}, \quad (30)$$

for a constant γ_0 and positive values $\rho_T > 0$ and γ_1 .

It thus holds for the unconditional expectation

$$E_t(\phi_k) \approx c_0 + q_{S_k}^{HM} + c_1 E_{S_{k-1}}(\tilde{\phi}_{k-1}). \quad (31)$$

Since $\tilde{\phi}_{k-1} = \phi_{k-1} e^{\beta_2 T_{S_{k-1}}}$, we now want to first relate $E_t(\phi_k)$ to q^{EH} . Similar derivations as before show that $E_{T_{k-1}}(\phi_{k-1})$ can be approximated through $\sum_{N_{T_{k-1}}} E_t(\phi | N_{T_i}) P_{T_i}(N_{T_i})$, which equals by the same arguments as above

$$\sum_{N_{T_{k-1}}} E_t(\phi | N_{T_i}) P_{T_i}(N_{T_i}) = \sum_{l=1}^{k-1} q_{S_l}^{HM} = q_{S_{k-1}}^{EH}. \quad (32)$$

We then approximate $\tilde{\phi}_s(\phi, T_s, X_s)$ as a linear function of ϕ and T_s ,

$$\tilde{\phi}_s(\phi, T_s, X_s) \approx d_0 \phi + d_1 T_s, \quad (33)$$

where both d_0 and d_1 are positive as is shown in the appendix. We could also add age s and experience X_s . However adding initial experience to a regression that also includes tenure and experience would lead to an identification problem since experience equals the sum of initial experience and tenure. Since age is highly correlated with actual experience X similar arguments apply. We therefore do not include them in the regression. We thus have that

$$E_t(\phi_k) \approx c_0 + q_{S_k}^{HM} + c_2 q_{S_{k-1}}^{EH} + c_3 T_{S_{k-1}}. \quad (34)$$

Finally, we again approximate

$$\log(\phi) \approx \tilde{c}_0 + \tilde{c}_1 \log(q^{HM}) + \tilde{c}_2 \log(q^{EH}) + \tilde{c}_3 T_{S_{k-1}}, \quad (35)$$

for coefficients \tilde{c}_i .

2.4 The hazard rate

Our objective is to investigate how we can use q^{HM} and q^{EH} to determine the probability to move to another (better) job. To this end, we can very much follow the derivations used to derive how the value of ϕ is related to q^{HM} and q^{EH} .

We immediately start the analysis for the case where we allow for job-to-job moves. A worker in period t could be in job k instead of being in the first job after unemployment. In particular, the labor market experience at the beginning of the job is not trivial. Therefore, labor market experience or more generally the history of previous aggregate labor market conditions is a determinant of the current type ϕ and thus of wages.

Suppose the value of the idiosyncratic productivity level equals ϕ_{k-1} in the $(k-1)^{\text{th}}$ job before the worker switched to the k^{th} job in period $1 + S_{k-1}$. Conditional on this we compute now the expected value to switch in this new job (which of course depends on ϕ_k). The expected probability not to switch in period $1 + S_{k-1} \leq t \leq S_k$ for a worker who is still employed in period t and has received \tilde{N}_s offers in period s equals

$$E_t(F(\phi)^{\tilde{N}_t} | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_t) = \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} F(\phi)^{\tilde{N}_t} d\tilde{G}_t(\phi_k | \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_t) d\phi, \quad (36)$$

where $\tilde{\phi}_{k-1} = \phi_{k-1} e^{\beta_2 T S_{k-1}}$ and \tilde{G}_t is the same distribution as above. Note that $F(\phi_k)^{\tilde{N}_t}$ is the probability that a type ϕ_k declines \tilde{N}_t offers (what happens if the offers he receives are all worse than the current type ϕ_k). We have to integrate this probability over all possible values of ϕ_k which are distributed according to the distribution \tilde{G}_t .

As before we use the predictor which contains the most information about ϕ , so that the expectation of $F(\phi_k)^{\tilde{N}_t}$ at $1 + S_{k-1} \leq t \leq S_k$ equals

$$E_{S_k}(F(\phi_k)^{\tilde{N}_t} | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) = \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} F(\phi_k)^{\tilde{N}_t} d\tilde{G}_{S_k}(\phi | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) d\phi. \quad (37)$$

The expected value of $F(\phi_k)^{\tilde{N}_t}$ then equals

$$E_{S_k}(F(\phi_k)^{\tilde{N}_t} | \tilde{\phi}_{k-1}) = \sum_{s=1+S_{k-1}}^{S_k} E_{S_k}(F(\phi_k)^{\tilde{N}_t} | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) P_s(N_s^i), \quad (38)$$

where $P_s(N_s^i) = q_s$ is the probability to receive an offer in period s .

2.4.1 Linear Approximation

To make our estimator applicable for our empirical implementation, we linearize (38) and relate it to an observable (to the econometrician) variable. The only difference between the derivation for the expected value of ϕ_k and the expected probability of not switching is that we now replace ϕ_k by $F(\phi_k)^{\tilde{N}_t}$. Thus the marginal effects for all variables except \tilde{N}_t are the same. As a result only difference between the two approximations is the additional derivative w.r.t \tilde{N}_t . We therefore obtain the following approximation,

$$E_{S_k}(F(\phi_k)^{\tilde{N}_t} | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) \approx \nu_0 + \sum_{T=1}^{S_k-S_{k-1}} \chi_T \tilde{N}_{S_{k-1}+T} + \nu_1 \tilde{\phi}_{k-1} + \nu_2 \tilde{N}_t, \quad (39)$$

for a constant ν_0 and where $\chi_T > 0$. We also show that the expectation of ϕ_k is increasing in $\tilde{\phi}_{k-1}$, so that $\nu_1 > 0$. The last term $\nu_2 \tilde{N}_t$ is due to the additional derivative w.r.t \tilde{N}_t , where we expect $\nu_2 < 0$. All coefficients are evaluated at their steady state levels.

The expected value of $F(\phi_k)^{\tilde{N}_t}$ conditional on $\tilde{\phi}_{k-1}$, $E_t(F(\phi_k)^{\tilde{N}_t} | \tilde{\phi}_{k-1})$ can then be simplified to:¹¹

$$E_t(F(\phi_k)^{\tilde{N}_t} | \tilde{\phi}_{k-1}) \approx \nu_0 + \sum_{T=1}^{S_k - S_{k-1}} \chi_T \tilde{N}_{S_{k-1}+T} P(\tilde{N}_{S_{k-1}+T} = 1) + \nu_1 \tilde{\phi}_{k-1} + \nu_2 \tilde{N}_{S_{k-1}+t} P(\tilde{N}_{S_{k-1}+t} = 1). \quad (40)$$

Since the probability to receive an offer in period t and thus the expected number of offers in period t equal q_t , the expected value of $F(\phi_k)^{\tilde{N}_t}$, conditional on $\tilde{\phi}_{k-1}$ for $1 + S_{k-1} \leq t \leq S_k$ can be expressed as

$$E_t(F(\phi_k)^{\tilde{N}_t} | \tilde{\phi}_{k-1}) \approx \nu_0 + \sum_{T=1}^{S_k - S_{k-1}} \chi_T q_{S_{k-1}+T} + \nu_1 \tilde{\phi}_{k-1} + \nu_2 q_{S_{k-1}+t}. \quad (41)$$

It thus holds for the unconditional expectation

$$E_t(F(\phi_k)^{\tilde{N}_t}) \approx \gamma_0 + \gamma_1 q_{T_k}^{HM} + \gamma_2 E_{T_{k-1}}(\tilde{\phi}_{k-1}) + \gamma_3 q_{S_{k-1}+t}. \quad (42)$$

We have thus established that the expected value of ϕ is a function of q^{HM} and $q_{S_{k-1}+t}$, the probability of receiving an offer in the period of interest. The remainder of the analysis is as before since it only concerns $\tilde{\phi}_{k-1}$. We thus have that

$$E_t(F(\phi_k)^{\tilde{N}_t}) \approx \gamma_0 + \gamma_1 q_{T_k}^{HM} + \gamma_3 q_{S_{k-1}+t} + \gamma_4 q_{T_{k-1}}^{EH}. \quad (43)$$

Finally, again we approximate the probability not to receive an offer

$$\log(F(\phi_k)^{\tilde{N}_t}) \approx \tilde{\gamma}_0 + \tilde{\gamma}_1 \log(q^{HM}) + \tilde{\gamma}_2 \log(q^{EH}) + \tilde{\gamma}_4 \log(q_t), \quad (44)$$

for coefficients $\tilde{\gamma}_i$.

¹¹Note that the expectation w.r.t. N_{T_k} only affects the N -term since ϕ_{k-1} is constant in job spell k and σ_t^k is an aggregate variable.

2.5 Measurement of Wage Dispersion

To measure wage dispersion we have to take into account that there could be permanent differences in receiving offers between individuals, so that $q^i = \chi^i q$. To identify χ^i we use the two equations we can estimate in the data, for wages and for the probability not to switch (ignoring tenure, experience etc.).

$$\log(w_t^i) = \log(u^i) + \tilde{c}_0 + (\tilde{c}_1 + \tilde{c}_2) \log(\chi^i) + \tilde{c}_1 \log(q^{HM}) + \tilde{c}_2 \log(q^{EH}) + \tilde{c}_3 u^{max} \quad (45)$$

$$\begin{aligned} Prob(\text{no switch}) = \tilde{\gamma}_0 &+ (\tilde{\gamma}_1 + \tilde{\gamma}_2 + \tilde{\gamma}_4) \log(\chi^i) \\ &+ \tilde{\gamma}_1 \log(q^{HM}) + \tilde{\gamma}_2 \log(q^{EH}) + \tilde{\gamma}_3 u^{max} + \tilde{\gamma}_4 \log(q_t), \end{aligned} \quad (46)$$

where u^i are permanent productivity differences between individuals. These affect wages but not the decision to switch. This is how we identify χ^i from these two equations. Estimating these two equations gives us the coefficients multiplying the various q variables $\tilde{c}_1, \tilde{c}_2, \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_4$ and estimates for the fixed effects:

$$\log(u^i) + (\tilde{c}_1 + \tilde{c}_2) \log(\chi^i) \quad (47)$$

and

$$(\tilde{\gamma}_1 + \tilde{\gamma}_2 + \tilde{\gamma}_4) \log(\chi^i). \quad (48)$$

Using the estimated coefficients for $\tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_4$ allows us to compute $\log(\chi^i)$ from the hazard rate equation. Using then \tilde{c}_1 and \tilde{c}_2 allows us to compute the individual q^i multiplied with its marginal effect on wages:

$$(\tilde{c}_1 + \tilde{c}_2) \log(\chi^i) + \tilde{c}_1 \log(q^{HM}) + \tilde{c}_2 \log(q^{EH}). \quad (49)$$

The dispersion of this variable is our measure of wage inequality accounted for by search frictions.

There is one potential caveat in this logic. Our measure of wage dispersion ignores the approximation error. Suppose $N(q, \nu)$ is the number of offers received as a function of q (ignore for now q^{HM} and q^{EH}). The stochastic component ν reflects the uncertainty in

receiving offers conditional on q . The theory implies that the expected value of the job quality ϕ is a function of N and we assume that this function is linear. We thus get that

$$\phi = c_1 N(q, \nu) + \eta(N), \quad (50)$$

where the expectation of the error η is zero, $E(\eta) = 0$. The next step approximates N as a linear function of q , so that the error of this approximation

$$\omega = N(q, \nu) - c_2 q \quad (51)$$

(q also includes the marginal effect on wages, so one can think of it as $\tilde{c}q$) is independent from q . Since the expected value of N equals $c_2 q$, we have $E(\omega) = 0$. Note that both errors η and ω are purely random since they are equal to the realization of a random variable minus its expected value. The error term η is the deviation of the realized value of ϕ from its expected value and ω is the deviation of the realized value of N from its expected value. However the variance of these shocks depends on N or q respectively. For the overall approximation error $\delta(q)$ we have

$$\delta(q) = \phi - c_1 c_2 q = \eta + c_1 \omega. \quad (52)$$

In particular, the expected value of ϕ equals $c_1 c_2 q$ and the error term δ has mean zero, $E(\delta) = 0$. Note that the coefficient $c_1 c_2$ can be recovered from a wage regression that includes q . Suppose that the wage $w = \phi + \dots$, where all other variables are observable. It then holds that

$$Cov(w, q) = Cov(\phi, q) = c_1 c_2 Var(q). \quad (53)$$

The regression coefficient of w on q then equals $\frac{Cov(w, q)}{Var(q)} = c_1 c_2$.¹²

¹²Using the variable q is rather like an IV approach than measurement error. If a variable X^* is measured with error so that we observe only $X = X^* + \epsilon$ then we have an attenuation bias. Suppose X^* would have the role of ϕ and X the role of q . We can write $X^* = X - \epsilon$ and this looks similar to $\phi = c_1 c_2 q + error$. One difference however is that ϵ and X are correlated but q and the error are not. Therefore if for example we regress X^* on X we get a biased estimate whereas a regression of ϕ on q delivers an unbiased estimate of $c_1 c_2$. The same logic then applies if we add X (instead of X^* or q (instead of ϕ) to the regression.

Let us now turn to how the approximation error affects our estimate of the variance of ϕ . The overall variance of wage inequality accounted for by search frictions equals

$$\begin{aligned} Var(\phi) &= Var(c_1 N) + Var(\eta) \\ &= Var(c_1 c_2 q) + Var(c_1 \omega) + Var(\eta) \\ &= Var(c_1 c_2 q) + Var(\delta). \end{aligned} \tag{54}$$

The previous equation ignores the second component $Var(\delta(q))$ and we now investigate whether this is an issue. Suppose the wage of individual i at time t equals

$$w_t^i = X_t^i + \mu_t^i + N(q_t^i, \nu_t^i) + \epsilon_t^i, \tag{55}$$

where X_t^i captures the contribution of observables, μ_t^i is a permanent (random walk) productivity shock with innovation ξ_t^i (which is independent from the job) and ϵ_t^i is measurement error (transitory shocks are indistinguishable from measurement error). Our theory implies that we obtain an unbiased estimates of X , so let

$$\tilde{w}_t^i = w_t^i - X_t^i \tag{56}$$

be the wage residual. Define the residual within-job wage growth

$$g_t^i = \Delta \tilde{w}_t^i = \mu_t^i - \mu_{t-1}^i + \Delta \epsilon_t^i = \xi_t^i + \Delta \epsilon_t^i. \tag{57}$$

The variance of this variable is σ_w^2 . Define now the wage growth for job switchers (job-to-job only, because we subsequent periods):

$$\begin{aligned} \gamma_t^i &= \xi_t^i + \Delta \epsilon_t^i + N(q_t^i, \nu_t^i) - N(q_{t-1}^i, \nu_{t-1}^i) \\ &= \xi_t^i + \Delta \epsilon_t^i + q_t^i - q_{t-1}^i + \delta(q_t^i) - \delta(q_{t-1}^i) \end{aligned} \tag{58}$$

In our previous notation this is equal to $w_{1+S_k} - w_{S_k}$ for some job k . Computing the variance using that all variables are uncorrelated. We use the following properties of the approximation error. Since the error δ is purely random. In particular, δ 's for different jobs are independent of each other.

$$Var(\gamma_t^i) = \sigma_w^2 + 2\sigma_q^2 - 2Cov(q_t^i, q_{t-1}^i) + 2\sigma_\delta^2. \tag{59}$$

As we can measure all variables and their variance and the autocorrelation of q directly except for δ , we use these variances to compute the variance of δ , σ_δ^2 :

$$\sigma_\delta^2 = \frac{Var(\gamma_t^i) - \sigma_w^2}{2} - \sigma_q^2 + Cov(q_t^i, q_{t-1}^i). \quad (60)$$

This allows us to assess the importance of variation of the approximation error δ across time and individuals.

3 Data

3.1 National Longitudinal Survey of Youth Data

The NLSY79 is a nationally representative sample of young men and women who were 14 to 22 years of age when first surveyed in 1979. We use the data up to 2006. NLSY is convenient because it allows to measure all the variables we are interested in. In particular, it contains detailed work-history data on its respondents in which we can track employment cycles. Each year through 1994 and every second year afterward, respondents were asked questions about all the jobs they held since their previous interview, including starting and stopping dates, the wage paid, and the reason for leaving each job.

The NLSY consists of three subsamples: A cross-sectional sample of 6,111 youths designed to be representative of noninstitutionalized civilian youths living in the United States in 1979 and born between January 1, 1957, and December 31, 1964; a supplemental sample designed to oversample civilian Hispanic, black, and economically disadvantaged non-black/non-Hispanic youths; and a military sample designed to represent the youths enlisted in the active military forces as of September 30, 1978. Since many members of supplemental and military samples were dropped from the NLSY over time due to funding constraints, we restrict our sample to members of the representative cross-sectional sample throughout.

We construct a complete work history for each individual by utilizing information on starting and stopping dates of all jobs the individual reports working at and linking jobs across interviews. In each week the individual is in the sample we identify the main job as the job with the highest hours and concentrate our analysis on it. Hours information is

missing in some interviews in which case we impute it if hours are reported for the same job at other interviews. We ignore jobs that in which individual works for less than 15 hours per week or that last for less than 4 weeks.¹³

We partition all jobs into employment cycles following the procedure in Barlevy (2008). We identify the end of an employment cycle with an involuntary termination of a job. In particular, we consider whether the worker reported being laid off from his job (as opposed to quitting). We use the workers stated reason for leaving his job as long as he starts his next job within 8 weeks of when his previous job ended, but treat him as an involuntary job changer regardless of his stated reason if he does not start his next job until more than 8 weeks later.¹⁴ If the worker offers no reason for leaving his job, we classify his job change as voluntary if he starts his next job within 8 weeks and involuntary if he starts it after 8 weeks. We ignore employment cycles that began before the NLSY respondents were first interviewed in 1979.

At each interview the information is recorded for each job held since the last interview on average hours, wages, industry, occupation, etc. Thus, we do not have information on, e.g., wage changes in a given job during the time between the two interviews. This leads us to define the unit of analysis, or an observation, as an intersection of jobs and interviews. A new observation starts when a worker either starts a new job or is interviewed by the NLSY and ends when the job ends or at the next interview, whichever event happens first. Thus, if

¹³We have also experimented with the following more complicated algorithm with no impact on our conclusions. (1) Hours between all the jobs held in a given week are compared and the job with the highest hours is assigned as the main job for that week. (2) If a worker has the main job *A*, takes up a concurrent job *B* for a short period of time, then leaves job *B* and continues with the original main job *A*, we ignore job *B* and consider job *A* to be the main one throughout (regardless of how many hours the person works in job *B*). (3) If a worker has the main job *A*, takes up a concurrent job *B*, then leaves job *A* and continues with job *B*, we assign job *B* to be the primary one during the period the two jobs overlap (regardless of how many hours the person works in job *B*).

¹⁴As Barlevy (2008) notes, most workers who report a layoff do spend at least one week without a job, and most workers who move directly into their next job report quitting their job rather than being laid off. However, nearly half of all workers who report quitting do not start their next job for weeks or even months. Some of these delays may be planned. Yet in many of these instances the worker probably resumed searching from scratch after quitting, e.g. because he quit to avoid being laid off or he was not willing to admit he was laid off.

an entire job falls in between of two consecutive interviews, it constitutes an observation. If an interview falls during a job, we will have two observations for that job: the one between the previous interview and the current one, and the one between the current interview and the next one (during which the information on the second observation would be collected). Consecutive observations on the same job broken up by the interviews will identify the wage changes for job-stayers. Following Barlevy (2008), we removed observations with an reported hourly wage less than or equal to \$0.10 or greater than or equal to \$1,000. Many of these outliers appear to be coding errors, since they are out of line with what the same workers report at other dates, including on the same job. We also eliminated employment cycles where wages change by more than a factor of two between consecutive observations.

To each observation we assign a unique value of worker’s job tenure, labor market experience, race, marital status, education, smsa status, and region of residence, and whether the job is unionized. Since the underlying data is weekly, the unique value for each of these variables in each observation is the mode of the underlying variable (the mean for tenure and experience) across all weeks corresponding to that observation. The educational attainment variable is forced to be non-decreasing over time.

We merge the individual data from the NLSY with the aggregate data on unemployment and vacancies. Seasonally adjusted unemployment, u , is constructed by the Bureau of Labor Statistics (BLS) from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index, v , is constructed by the Conference Board. Both u and v are quarterly averages of monthly series. The ratio of v to u is the measure of the labor market tightness.

We use the underlying weekly data for each observation (job-interview intersection) to construct aggregate statistics corresponding to that observation. The current unemployment rate for a given observation is the average unemployment rate over all the weeks corresponding to that observation. Similarly, the current labor market tightness for a given observation is the average labor market tightness over all the weeks corresponding to that observation.

Finally we set up the variables $q_{S_k}^{HM} = \sum_{T=1}^{S_k-S_{k-1}} \rho_T q_{S_{k-1}+T}$ and $q_{S_{k-1}}^{EH} = \sum_{l=1}^{k-1} q_{S_l}^{HM}$, which we use to measure match quality. The key feature of these variables is that receiving

offers at different tenure levels has different effects. For the empirical implementations we have to split jobs into periods where receiving an offer has the same effect (q is multiplied with the same ρ_T). We use the following four periods (1) $\leq 52 \cdot 3$ weeks, (2) $> 52 \cdot 3$ but $\leq 52 \cdot 6$ weeks, (3) $> 52 \cdot 6$ but $\leq 52 \cdot 9$ weeks, and (4) $> 52 \cdot 9$ weeks. For each job spell let q_i^{HM} , $i = 1, \dots, 4$ be the sum of the q in the respective period. Thus q_1^{HM} is the sum of q 's received in the first $52 \cdot 3$ weeks, q_2^{HM} is the sum of q 's received between weeks $52 \cdot 3 + 1$ and $52 \cdot 6$, q_3^{HM} is the sum of q 's received between weeks $52 \cdot 6 + 1$ and $52 \cdot 9$ and finally q_4^{HM} is the sum of q 's received after week $52 \cdot 9$. The sum of these four variables (weighted by the estimated regression coefficients ρ_T) for a given job yields the value of q^{HM} for that job (and each observation in it). Similarly, the sum of these four variables (weighted by the estimated regression coefficients) over all jobs in the employment cycle preceding the current job yields the value of q^{EH} for that job (and each observation in it).

All empirical experiments that we conduct are based on the individual data weighted using custom weights provided by the NLSY which adjust both for the complex survey design and for using data from multiple surveys over our sample period. In practice, we found that using weighted or unweighted data has no impact on our substantive findings.

4 Empirical Results

The estimated coefficients of the wage equation are presented in Table 1. For comparison, we report the standard OLS estimates that do not account for match quality and the estimates implied by the on-the-job search model. Implied returns to tenure are summarized in Table 2. We find that the returns to firm tenure are fairly small, less than 10% after 10 years.

We now ask what fraction of wage variance is accounted for by the variability of estimated match qualities. To do so we compute σ_q^2 which equals $Var(\tilde{c}_1 \log(q^{HM}) + \tilde{c}_2 \log(q^{EH}))$ in notation of Equation (45). The coefficients \tilde{c}_1 and \tilde{c}_2 can be found in Table 1. We find $\sigma_q^2 = 0.006558$. Given the overall residual wage variance of .15 in our sample, this estimate implies that the variance of match qualities accounts for less than 5% of the residual wage dispersion.

This calculation does not take into account permanent differences across individuals in

their ability to find jobs. To assess the extent of these differences we estimate the hazard rate of switching job-to-job as in Equation (46). Using the estimated coefficients we calculate $\sigma_q^2 \equiv \text{Var}((\tilde{c}_1 + \tilde{c}_2) \log(\chi^i) + \tilde{c}_1 \log(q^{HM}) + \tilde{c}_2 \log(q^{EH}))$ in notation of Equation (45). We find $\sigma_q^2 = 0.0119$. The comparison between σ_q^2 and σ_w^2 implies that there are substantial permanent differences across individuals in the ability to receive job offers. However, even taking these differences into account, results in only about 8% of overall wage dispersion being accounted for by the dispersion in job match qualities.

Finally, we compute the variance of the approximation error δ across time and individuals using Equation (60). We find the variance of the wage growth for job stayers $\sigma_w^2 = 0.034$ and for job switchers $\text{Var}(\gamma_t^i) = 0.059$. We also find $\text{Cov}(q_t^i, q_{t-1}^i) = 0.0026$. Given the estimated $\sigma_q^2 = 0.0119$ reported above we obtain

$$\sigma_\delta^2 = \frac{\text{Var}(\gamma_t^i) - \sigma_w^2}{2} - \sigma_q^2 + \text{Cov}(q_t^i, q_{t-1}^i) \quad (61)$$

$$= \frac{0.059 - 0.034}{2} - 0.0119 + 0.0026 \quad (62)$$

$$= 0.0032 \quad (63)$$

The overall wage dispersion due to search frictions equals

$$\text{Var}(\phi) = \text{Var}(c_1 c_2 q) + \text{Var}(\delta) \quad (64)$$

$$= 0.0119 + 0.0032 \quad (65)$$

$$= 0.0151. \quad (66)$$

Thus the approximation error is small relative to the dispersion of wages due to search frictions as it accounts for only about 20% of the variance of ϕ .

The fraction of overall wage dispersion explained by search frictions thus equals

$$\frac{0.0151}{0.15} = 0.1. \quad (67)$$

5 Model Simulations

This section is very incomplete.

The objective of this section is to assess the performance of our method to measure job match quality and to perform counterfactual experiments. To do so we start by calibrating the model of on-the-job search presented above. Most of the targets are directly computed in the data (e.g., the process for the labor market tightness and labor market flows). The remaining targets, such as the returns to tenure, labor market experience, the dispersion of match qualities, and permanent differences across individuals in their ability to generate offers, are implied by the estimates presented above. Having calibrated the model, we first ask whether the calibrated model is a good laboratory in which to conduct quantitative experiments by assessing whether the model generates a number of untargeted predictions that are consistent with the data (for example, we verify whether the the ols wage regression that does not control for job match qualities estimated in the model generated data has the same coefficients as the one estimated in the data). Second, we assess whether our method to measure job match quality recovers the true match qualities and yields unbiased estimates of the returns to tenure and experience. Finally, we use the calibrated model to assess the effects of eliminating search frictions on wage dispersion.

Since we are only interested in how wages are set given aggregate labor market conditions, the model is partial equilibrium. This means that the stochastic driving force is an exogenous process instead of being the result of a general equilibrium model with optimizing agents.¹⁵ However, since we have to match the model to the data, we have to take a stand on what the driving force is. We choose market tightness, since this variable determines the probability to receive offers, which in turn determines the evolution of unemployment.

We choose the model period to be one month. The stochastic process for market tightness is assumed to follow an AR(1) process:

$$\log \theta_{t+1} = \rho \log \theta_t + \nu_{t+1}, \tag{68}$$

where $\rho \in (0, 1)$ and $\nu \sim N(0, \sigma_\nu^2)$.¹⁶ To calibrate ρ and σ_ν^2 , we consider quarterly averages

¹⁵We can thus not answer the question whether this process and the model's endogenous variables could be the mutually consistent outcome of a general equilibrium model. We leave this interesting question for future research.

¹⁶For notational convenience we will say that market tightness shocks follow a Markov process characterized by the transition function $Q(\theta, \cdot)$. The Markov process for θ possesses an invariant distribution ζ that satisfies $\zeta(\Theta) = \int Q(\theta, \Theta) \zeta(d\theta)$, where Θ denotes sets of tightness shocks.

of monthly market tightness and HP-filter (Prescott (1986)) this process with a smoothing parameter of 1600, commonly used with quarterly data. In the data we find an autocorrelation of 0.924 and an unconditional standard deviation of 0.206 for the HP-filtered process. However, at monthly frequency, there is no $\rho < 1$ which generates such a high persistence after applying the HP-filter. We therefore choose $\rho = 0.99$, since higher values virtually do not increase the persistence of the HP-filtered process in the simulation. For this persistence parameter we set $\sigma_\nu = 0.095$ in the model to replicate the observed volatility of market tightness. The mean of θ is normalized to one.

An unemployed worker receives up to M offers per period, each with probability λ , and an employed worker receives up to M offers per period, each with probability q . We assume that both λ and q are functions of the driving force θ :

$$\log \lambda_t = \log \bar{\lambda} + \kappa \log \theta_t \quad \text{and} \quad (69)$$

$$\log q_t = \log \bar{q} + \kappa \log \theta_t. \quad (70)$$

A job-holder receives k offers with probability $\binom{M}{k} q^k (1-q)^{(M-k)}$. However, not every received offer leads to a job-switch, since workers change jobs only if the new job features a sufficiently higher idiosyncratic productivity level ϕ^i . Thus the probability to switch jobs depends not only on q but also on the distribution of ϕ^i , which endogenously evolves over time. A new value of ϕ is drawn, according to a distribution function F , which is assumed to be normal, $F = \mathcal{N}(\mu_\phi, \sigma_\phi^2)$, and truncated at two standard deviations, so that the support equals $[\underline{\phi}, \bar{\phi}] = [\mu_\phi - 2\sigma_\phi, \mu_\phi + 2\sigma_\phi]$.¹⁷

The accumulation of firm-specific and general human capital increase wages. Wages increase by $e^{\beta_1 X_{ijt}}$, where X_{ijt} is a quartic in total labor market experience and β_1 is the associated vector of coefficients. Wages also increase by $e^{\beta_2 T_{ijt}}$, where T_{ijt} is a quartic in firm tenure and β_2 is the corresponding vector of coefficients. The wage also has a fixed individual specific and a fixed job match specific component and thus equals

$$e^{\beta_1 X_{ijt} + \beta_2 T_{ijt}} \mu_i \phi_{ij} e^{u_t}, \quad (71)$$

where u_t measures the impact of aggregate variables on individual wages.

¹⁷We are working on recovering the shape of the distribution, at least in some range, using our method. However, for now, we proceed with the assumption that the distribution is Normal.

The log wage equals

$$W_{ijt} = \beta_1 X_{ijt} + \beta_2 T_{ijt} + \log(\mu_i) + \log(\phi_{ij}) + u_t. \quad (72)$$

5.1 Calibration

5.2 Simulation Results

6 Conclusion

Table 1: Wage Regression Estimates. NLSY Data.

Variable	Specification	
	OLS (1)	Controlling for Match Quality (2)
1. Tenure	.0017798 (.0001031)	.000577 (.0000964)
2. Tenure ²	-4.97e-06 (4.82e-07)	-1.49e-06 (4.43e-07)
3. Tenure ³	5.80e-09 (7.52e-10)	1.72e-09 (6.99e-10)
4. Tenure ⁴	-2.28e-12 (3.61e-13)	-6.81e-13 (3.55e-13)
5. Experience	.0022535 (.0001122)	.002264 (.0002062)
6. Experience ²	-3.43e-06 (3.65e-07)	-3.23e-06 (5.26e-07)
7. Experience ³	2.88e-09 (4.44e-10)	2.74e-09 (5.95e-10)
8. Experience ⁴	-8.71e-13 (1.76e-13)	-8.69e-13 (2.27e-13)
9. q_1^{HM}	—	.0222944 (.0030437)
10. q_2^{HM}	—	.0088692 (.0014905)
11. q_3^{HM}	—	.0021324 (.0022521)
12. q_4^{HM}	—	.0052247 (.002571)
13. q_1^{EH}	—	.0194614 (.003789)
14. q_2^{EH}	—	.0062919 (.0028012)
15. q_3^{EH}	—	-.0006502 (.0042406)
16. q_4^{EH}	—	.0025581 (.0051192)

Note - Standard errors are in parentheses.

Table 2: Estimated Returns to Tenure. NLSY Data.

Variable	Specification	
	OLS (1)	Controlling for Match Quality (2)
1. 2 Years of Tenure	0.147576	0.046834
2. 5 Years of Tenure	0.244388	0.079543
3. 10 Years of Tenure	0.261124	0.093746
4. 15 Years of Tenure	0.318976	0.114562

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APPENDICES

I Proofs and Derivations

I.1 Optimal Switching Rule: Linear Tenure

We first derive the result for an agent who lives L periods. The corresponding infinite horizon result then follows if $L \rightarrow \infty$. Let $V_t(\phi, T, X)$ be the time t value function describing the solution to the sequence problem of an agent in our economy who is type ϕ , has tenure T and experience X . The function V thus describes the expected discounted present value of this worker's lifetime income (at the beginning of the period before uncertainty realizes) and thus satisfies (existence and uniqueness for sequence problems are guaranteed under very mild assumptions, see LS) :

$$\begin{aligned}
 V_t(\phi, T, X) = & sU(X) \\
 & + (1-s)E \max_{\chi \in \{0, \tilde{N}\}} \{ \chi [e^{\beta_1 X + \beta_2 0} \hat{\phi} \mu + \delta V_{t+1}(\hat{\phi}, 1, X+1)] \\
 & \quad + (1-\chi) [e^{\beta_1 X + \beta_2 T} \phi \mu + \delta V_{t+1}(\phi, T+1, X+1)] \}
 \end{aligned} \tag{A1}$$

where δ is the discount factor and $\hat{\phi}$ is the type associated with an offer. The value of unemployment equals $U(X)$ and the probability of displacement equals s , which realizes at the beginning of the period. If $\tilde{N} = 1$, the agent received an offer and thus has the option to decline the offer $\chi = 0$ (stay with type ϕ and continue with tenure $T+1$ next period) or to accept the offer $\chi = 1$ (switch to type $\hat{\phi}$ and continue with tenure 1 next period). The evolution of U is described through

$$U_t(X) = b + \delta [\lambda E(V_{t+1}(\hat{\phi}, 0, X)) + (1-\lambda)U_{t+1}(X)], \tag{A2}$$

but this is not important for our results (b is the monetary flow value of non-market activity).

Uniqueness of the optimal solution implies the existence of a threshold level of switching $\hat{\phi}_t(\phi, T, X)$ such that a worker of type ϕ , tenure T and experience X is indifferent between staying and switching to type $\hat{\phi}$ at time t . Since V is monotone in ϕ , T and X , the worker does not switch if the new job offers a type below $\hat{\phi}$ and switches if the new match quality

is above $\hat{\phi}$.¹⁸ We set $\hat{\phi} = \bar{\phi}$ in case the worker never switches independent how good the offer is.

We now show that

$$\hat{\phi}_t(\phi, T, X)\hat{\phi}_t(\phi, T) = \min(\phi e^{\beta_2 T}, \bar{\phi}). \quad (\text{A3})$$

For interior solutions $\hat{\phi} < \bar{\phi}$ this is equivalent to

$$e^{\beta_1 X + \beta_2 0} \hat{\phi}(\phi, T)\mu + \delta V_{t+1}(\hat{\phi}(\phi, T), 1, X + 1) = e^{\beta_1 X + \beta_2 T} \phi\mu + \delta V_{t+1}(\phi, T + 1, X + 1). \quad (\text{A4})$$

We now proceed by backward induction. For the last period, equation (A4) holds since $V_{L+1} \equiv 0$. We now show that the claim also holds for t given that it holds for $t + 1$. Since

$$\begin{aligned} & e^{\beta_1 X + \beta_2 0} \hat{\phi}(\phi, T)\mu + \delta V_{t+1}(\hat{\phi}(\phi, T), 1, X + 1) \\ &= e^{\beta_1 X + \beta_2 T} \phi\mu + \delta V_{t+1}(\hat{\phi}(\phi, T), 1, X + 1), \end{aligned} \quad (\text{A5})$$

we have to show that

$$V_{t+1}(\hat{\phi}(\phi, T), 1, X + 1) = V_{t+1}(\phi, T + 1, X + 1). \quad (\text{A6})$$

By induction we know that the claim holds for $t + 1$. In period $t + 1$, the type $\hat{\phi}(\phi, T)$ worker (who started a new job in period t) accepts an offer $\tilde{\phi}$ if and only if $\tilde{\phi} > \hat{\phi}(\hat{\phi}(\phi, T), 1) = \hat{\phi}(\phi, T + 1)$. The type ϕ worker (who stayed in the old job in period t) accepts an offer $\tilde{\phi}$ if and only if $\tilde{\phi} > \hat{\phi}(\phi, T + 1)$. Thus both workers show the same switching behavior and thus have the same continuation value in period $t + 2$ if they switch. If the type $\hat{\phi}(\phi, T)$ declines the offer,

$$\begin{aligned} V_{t+1}(\hat{\phi}(\phi, T), 1, X + 1) &= e^{\beta_1(X+1) + \beta_2} \hat{\phi}(\phi, T)\mu + \delta V_{t+2}(\hat{\phi}(\phi, T), 2, X + 2) \\ &= e^{\beta_1(X+1) + \beta_2(T+1)} \phi\mu + \delta V_{t+2}(\hat{\phi}(\hat{\phi}(\phi, T), 2), 0, X + 2) \\ &= e^{\beta_1(X+1) + \beta_2(T+1)} \phi\mu + \delta V_{t+2}(\hat{\phi}(\phi, T + 2), 0, X + 2). \end{aligned} \quad (\text{A7})$$

If the type ϕ declines the offer,

$$\begin{aligned} V_{t+1}(\phi, T + 1, X + 1) &= e^{\beta_1(X+1) + \beta_2(T+1)} \phi\mu + \delta V_{t+2}(\phi, T + 2, X + 2) \\ &= e^{\beta_1(X+1) + \beta_2(T+1)} \phi\mu + \delta V_{t+2}(\hat{\phi}(\phi, T + 2), 0, X + 2). \end{aligned} \quad (\text{A8})$$

Thus we have established that (A6) holds. Therefore the switching rule in (A3) solves the Bellman equation and thus describes the optimal decision rule.

¹⁸Monotonicity follows from Bellman principle/ envelope theorem. Just compare $V() - V(x)$.

I.2 Effects of offers on match quality

We show that

$$\begin{aligned} \Delta_s(\epsilon) &= E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s + \epsilon, \dots, \tilde{N}_{S_k}) \\ &\quad - E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s, \dots, \tilde{N}_{S_k}) \end{aligned} \quad (\text{A9})$$

is positive for all ϵ , implying that, by setting $\tilde{N}_s = 0$ and $\epsilon = 1$

$$\begin{aligned} 0 &< E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s = 1, \dots, \tilde{N}_{S_k}) \\ &\quad - E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s = 0, \dots, \tilde{N}_{S_k}). \end{aligned} \quad (\text{A10})$$

Since

$$E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) = \bar{\phi} - \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} \tilde{G}_{S_k}(\phi | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) d\phi, \quad (\text{A11})$$

it is sufficient to show that

$$\begin{aligned} \tilde{\Delta}_s(\hat{\phi}, \epsilon) &= \tilde{G}_{S_k}(\hat{\phi} | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s + \epsilon, \dots, \tilde{N}_{S_k}) \\ &\quad - \tilde{G}_{S_k}(\hat{\phi} | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s, \dots, \tilde{N}_{S_k}) \end{aligned} \quad (\text{A12})$$

is negative for all ϵ . Since

$$\tilde{G}_t(\hat{\phi} | \tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i) = \frac{1}{\mu_t(\tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i)} \int_{\tilde{\phi}_{k-1}}^{\hat{\phi}} \prod_{s=1+S_{k-1}}^t F(\min(\phi e^{\beta_2 T_s}, \bar{\phi}))^{\tilde{N}_s} f(\phi) d\phi,$$

it follows that

$$\tilde{\Delta}_s(\hat{\phi}, \epsilon) = \int_{\tilde{\phi}_{k-1}}^{\hat{\phi}} \underbrace{\left(\frac{(F(\min(\phi e^{\beta_2 T_s}, \bar{\phi})))^\epsilon}{\tilde{\mu}_s(\epsilon)} - \frac{1}{\tilde{\mu}} \right)}_{:=\omega_s(\phi, \epsilon)} \prod_{r=1+S_{k-1}}^{S_k} F(\min(\phi e^{\beta_2 T_r}, \bar{\phi}))^{\tilde{N}_r} d\phi, \quad (\text{A13})$$

where $\tilde{\mu} = \mu_t(\tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k})$ and $\tilde{\mu}_s(\epsilon) = \mu_t(\tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s + \epsilon, \dots, \tilde{N}_{S_k})$.

Since \tilde{G} is a probability measure on $[\tilde{\phi}_{k-1}, \bar{\phi}]$, it holds that

$$\tilde{\Delta}_s(\bar{\phi}, \epsilon) = 0. \quad (\text{A14})$$

Since $\tilde{\Delta}_s(\tilde{\phi}_{k-1}, \epsilon) = 0$ and $\omega_s(\phi, \epsilon)$ is increasing in ϕ it follows that a $\hat{\phi}$ exists such that $\omega_s(\hat{\phi}, \epsilon) = 0$, $\omega_s(\phi, \epsilon) < 0$ for $\phi < \hat{\phi}$ and $\omega_s(\phi, \epsilon) > 0$ for $\phi > \hat{\phi}$. This implies that

$$\tilde{\Delta}_s(\hat{\phi}, \epsilon) \leq 0, \quad \text{for all } \tilde{\phi}_{k-1} \leq \hat{\phi} \leq \bar{\phi}. \quad (\text{A15})$$

I.3 Non-Linear Tenure and Experience

I.3.1 Optimal Switching Rule

Existence of a switching rule follows from the same principles as in the linear case. The only difference is that the switching rule depends on all state variables, $\tilde{\phi}_t(\phi, T, X)$. For interior solutions $\tilde{\phi} < \bar{\phi}$ the decision rule satisfies

$$V_t(\phi, T, X) = e^{\beta_1(X) + \beta_2(T)} \tilde{\phi}_t(\phi, T, X) \mu + \delta V_{t+1}(\tilde{\phi}_t(\phi, T, X), 1, X + 1) \quad (\text{A16})$$

$$= e^{\beta_1(X) + \beta_2(T)} \phi \mu + \delta V_{t+1}(\phi, T + 1, X + 1). \quad (\text{A17})$$

It is straightforward to see from this definition that $\tilde{\phi}$ is increasing in ϕ and T since V is increasing in ϕ and T . Thus the RHS is increasing in ϕ and T and the LHS is constant for a constant $\tilde{\phi}$. Thus to reestablish equality, $\tilde{\phi}$ has to increase.

I.3.2 Effects of offers on match quality

We show again that

$$\begin{aligned} \Delta_s(\epsilon) &= E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s + \epsilon, \dots, \tilde{N}_{S_k}) \\ &\quad - E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s, \dots, \tilde{N}_{S_k}) \end{aligned} \quad (\text{A18})$$

is positive for all ϵ , implying that, by setting $\tilde{N}_s = 0$ and $\epsilon = 1$

$$\begin{aligned} 0 &< E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s = 1, \dots, \tilde{N}_{S_k}) \\ &\quad - E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s = 0, \dots, \tilde{N}_{S_k}). \end{aligned} \quad (\text{A19})$$

Since

$$E_{S_k}(\phi_k | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) = \bar{\phi} - \int_{\tilde{\phi}_{k-1}}^{\bar{\phi}} \tilde{G}_{S_k}(\phi | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k}) d\phi, \quad (\text{A20})$$

it is sufficient to show that

$$\begin{aligned} \tilde{\Delta}_s(\hat{\phi}, \epsilon) &= \tilde{G}_{S_k}(\hat{\phi} | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s + \epsilon, \dots, \tilde{N}_{S_k}) \\ &\quad - \tilde{G}_{S_k}(\hat{\phi} | \tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s, \dots, \tilde{N}_{S_k}) \end{aligned} \quad (\text{A21})$$

is negative for all ϵ . Since

$$\tilde{G}_t(\hat{\phi} \mid \tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i) = \frac{1}{\mu_t(\tilde{\phi}_{k-1}, \tilde{N}_0^i, \dots, \tilde{N}_t^i)} \int_{\tilde{\phi}_{k-1}}^{\hat{\phi}} \prod_{s=1+S_{k-1}}^t F(\min(\tilde{\phi}_s(\phi, T_s, X_s), \bar{\phi}))^{\tilde{N}_s} f(\phi) d\phi,$$

it follows that

$$\tilde{\Delta}_s(\hat{\phi}, \epsilon) = \int_{\tilde{\phi}_{k-1}}^{\hat{\phi}} \underbrace{\left(\frac{(F(\min(\tilde{\phi}_s(\phi, T_s, X_s), \bar{\phi})))^\epsilon}{\tilde{\mu}_s(\epsilon)} - \frac{1}{\tilde{\mu}} \right)}_{:=\omega_s(\phi, \epsilon)} \prod_{r=1+S_{k-1}}^{S_k} F(\min(\tilde{\phi}_r(\phi, T_r, X_r), \bar{\phi}))^{\tilde{N}_r} d\phi \quad (\text{A22})$$

where $\tilde{\mu} = \mu_t(\tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_{S_k})$ and $\tilde{\mu}_s(\epsilon) = \mu_t(\tilde{\phi}_{k-1}, \tilde{N}_{1+S_{k-1}}, \dots, \tilde{N}_s + \epsilon, \dots, \tilde{N}_{S_k})$.

Since \tilde{G} is a probability measure on $[\tilde{\phi}_{k-1}, \bar{\phi}]$, it holds that

$$\tilde{\Delta}_s(\bar{\phi}, \epsilon) = 0. \quad (\text{A23})$$

Since $\tilde{\Delta}_s(\tilde{\phi}_{k-1}, \epsilon) = 0$ and $\omega_s(\phi, \epsilon)$ is increasing in ϕ (because $\tilde{\phi}$ is increasing in ϕ) it follows that a $\hat{\phi}$ exists such that $\omega_s(\hat{\phi}, \epsilon) = 0$, $\omega(\phi, \epsilon) < 0$ for $\phi < \hat{\phi}$ and $\omega(\phi, \epsilon) > 0$ for $\phi > \hat{\phi}$.

This implies that

$$\tilde{\Delta}_s(\hat{\phi}, \epsilon) \leq 0, \quad \text{for all } \tilde{\phi}_{k-1} \leq \hat{\phi} \leq \bar{\phi}. \quad (\text{A24})$$