Career Length: Effects of Curvature of Earnings Profiles, Earnings Shocks, and Social Security*

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Abstract

The high labor supply elasticity in an indivisible-labor model with employment lotteries emerges also without lotteries when individuals must instead choose career lengths. The more elastic are earnings to accumulated working time, the longer is a worker's career. Negative (positive) unanticipated earnings shocks reduce (increase) the career length of a worker holding positive assets at the time of the shock, while the effects are the opposite for a worker with negative assets. Government provided social security can attenuate responses of career length to earnings profile slope and earnings shocks by inducing a worker to retire at an official retirement age.

KEY WORDS: Career length, indivisible labor, earnings profile, earnings shocks, taxes, social security, labor supply elasticity.

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1 Introduction

The influential framework for aggregate macroeconomic analysis advocated in Prescott's (2005) Nobel lecture features a high labor supply elasticity founded on Rogerson's (1988) aggregation theory that handles a labor supply indivisibility with employment lotteries and complete insurance markets. Skeptics have expressed doubts about the microeconomic realism of employment lotteries and complete markets for insuring consumption.[1] But advocates of a high aggregate labor supply elasticity can now appeal to another aggregation theory not based on complete markets and employment lotteries.

Ljungqvist and Sargent (2006) showed that a high aggregate labor supply elasticity emerges if individual workers manage the indivisible labor choice by choosing fractions of their lifetimes to work while trading a single risk-free asset to smooth consumption over time.[2,3] That analysis pinpoints the source of the high labor supply elasticity to be a high disutility of labor and not the Rogerson aggregation theory stressed by Prescott (2005).[4]

[1] Voicing a common criticism of complete-market employment-lottery models, Browning et al. (1999, p. 602) argue that "the employment allocation mechanism strains credibility and is at odds with the micro evidence on individual employment histories." Ríos-Rull (2008, p. 126) concurs and asks: "Where are those lotteries? There is no evidence of any such arrangement, at least for the labor market. I think very strongly that observed arrangements should carry the same weight as observed allocations and should be regarded as data."

[2] After showing that a particular distribution function of (insurable) ex-ante heterogeneity in an indivisible-labor complete-market model can render the aggregate labor supply isomorphic to that of a representative-agent model with divisible labor, Mulligan (2001, p. 13) suggested that the elimination of employment lotteries and complete markets for consumption claims from the former model might not make much of a quantitative difference, based on the following reasoning: "The smallest labor supply decision has an infinitesimal effect on lifetime consumption and the marginal utility of wealth in the [divisible-labor] model, and a small-but-larger-than-infinitesimal effect on the marginal utility of wealth in the [indivisible-labor] model – as long as the effect on lifetime consumption is a small fraction of lifetime income *or* the marginal utility of wealth does not diminish too rapidly." Abstracting from ex-ante heterogeneity, Ljungqvist and Sargent (2006) offered a first equivalence result for indivisible-labor models with and without employment lotteries in continuous time, and pursued the substantive implications for how life-cycle career decisions affect aggregate labor supply and its response to flat rate income taxes.

[3] Jones (2008, p. 13) (originally written in 1988) anticipated the equivalence result of Ljungqvist and Sargent (2006) when he wrote: "A natural question to raise here is that if the time horizons we are considering are sufficiently divisible, why cannot timing perform the same function as lotteries?" In the context of indivisible consumption goods, Jones showed how timing could replace lotteries in a particular example with the crucial assumption that preferences are such that an agent does not care *when* he consumes the good, just how often.

[4] Prescott (2006a) advocated the Ljungqvist and Sargent (2006) life-cycle framework with indivisible labor as "the initiation of an important research program ... to derive the implications of labor indivisibility for lifetime labor supply." While Prescott's (2005) original Nobel lecture was devoted to the complete-market representative-agent framework, a subsequent version (Prescott 2006b) contains an added section on "The Life Cycle and Labor Indivisibility."

Moving from lotteries to time averaging refocuses attention away from how a representative family chooses the fraction of its members to send to work and instead toward individual workers' decisions about career lengths.

Ljungqvist and Sargent (2006) demonstrated an *exact* equivalence between employment lotteries and time averaging that breaks down with human capital accumulation. When peak earnings materialize only after a lengthy career, retirement comes at the expense of foregoing those higher earnings, implying that a positively sloped earnings-experience profile faces a worker with what Ljungqvist and Sargent (2006) call a 'mother of all indivisibilities.' A collection of workers who choose career lengths and smooth consumption by personal saving while confronting positive earnings-experience profiles end up supplying *more* lifetime labor per capita than would a representative family that uses employment lotteries. Nevertheless, the insight of Ljungqvist and Sargent (2006, 2008a) survives that, at an *interior* solution for career length, the elasticity of lifetime labor supply can be high in a time averaging model.[5]

This paper extends our Ljungqvist and Sargent (2006) time averaging model to study how planned career length is affected by the shape of an experience-earnings profile, unanticipated earnings shocks, and government provided social security. We find that, with a commonly used specification of preferences that assures balanced growth, the more elastic are earnings to accumulated working time, the longer is a worker's career. This result suggests the possibility that it is a higher *slope* of the wage-experience profile of high wage workers, and not the *level* of the wage *per se*, that explains why people with higher wages and higher educations are more likely to retire later in life. For unanticipated permanent earnings shocks, we find that negative (positive) shocks reduce (increase) the career length of a worker with positive asset holdings at the time of the shock, while the effects are the opposite for a worker with negative asset holdings. In light of the increased labor income variability observed to have confronted individual workers for both transitory and permanent components of earnings, our finding that negative earnings shocks shorten careers for workers in mid- and late-age having positive life cycle savings can help explain the increased incidence of early retirement in recent decades. But our analysis of social security highlights countervailing forces because implicit taxation can result in a *corner* solution for career length at an official retirement age, contracting the labor supply elasticity.

Section 2 reviews how the time averaging model has reformed the microfoundations for

[5] For another time-averaging model that focuses on the extensive margin of labor supply, see Chang and Kim (2006). In their model, agents are infinitely lived and, hence, the life-cycle dimension of careers is absent, but they enrich the analysis by studying heterogeneous two-person households who choose labor supplies for both partners.

the high labor supply elasticity. Section 3 describes a lifetime labor supply problem in which a finitely lived worker confronts a labor supply indivisibility, chooses when to work, and smooths consumption by trading a risk-free bond.[6] How career lengths are affected by the shape of an experience-earnings profile, unanticipated earnings shocks, and social security are studied in sections 4, 5 and 6, respectively. Implications for social security reform are discussed in section 7. Section 8 offers some concluding remarks on the important shift in the labor market paradigm to be used for aggregate analysis after the Rogerson (1988) aggregation theory based on employment lotteries and complete insurance markets is abandoned and replaced by the time averaging model.

2 Reformed microfoundations for that high aggregate labor supply elasticity

Rogerson (1988) formulated a static model in which it is feasible for each of a continuum of people to supply either all or none of her time endowment as labor. He showed how to improve allocations by using employment lotteries and markets for lottery-outcome-contingent claims to consumption. With ex ante identical workers whose preferences are separable in consumption and leisure, the optimal allocation awards each person the same consumption and sends a fraction of them to work.

Hansen (1985) imported Rogerson's indivisible labor model with lotteries and complete markets in consumption claims into a dynamic stochastic real business cycle model, thereby attaining a representative family with preferences over consumption and employment being ordered by the expected value of $\sum_{t=0}^{\infty} \beta^t \big(\log(c_t) - BN_t\big)$, where $\beta \in (0,1)$, c_t is consumption allocated to each member of the family at t and $N_t \in [0,1]$ is the fraction of the family's members sent to work in period t. The real business cycle literature commonly calibrates the disutility of work B to make the equilibrium value of N_t match an economy's employment-population ratio.

A key finding in models of indivisible labor and employment lotteries is that any setting of B associated with an interior solution $N_t \in (0,1)$ delivers a high labor supply elasticity. In the words of Prescott (2005, p. 385), "the aggregate elasticity of labor supply is infinite up to the point that the fraction of employed is one." Prescott attributes this outcome to employment lotteries: "Rogerson's aggregation result is every bit as important as the

[6] Appendix A provides a comparison of our time averaging model to a corresponding employment lottery model with complete markets.

one giving rise to the aggregate production function. In the case of production technology, the nature of the aggregate production function in the empirically interesting cases is very different from that of the individual production units being aggregated. The same is true for the aggregate or a stand-in household's utility function in the empirically interesting case. ... *the aggregate labor supply elasticity is much greater than the individual labor supply elasticity.*" (Prescott (2005, p. 385), author's italics).

Ljungqvist and Sargent (2006) compared a continuous-time, life-cycle version of Prescott's representative family that chooses a fraction *of its members* to send to work with the problem that would face a worker with the same preferences as Prescott's individual workers but who has to choose what fraction *of her lifetime* to work while trading a single risk-free asset to smooth consumption across periods of working and not working. When the subjective discount factor equals the market rate of return in a nonstochastic setting, these two fractions turn out to be identical. An interior solution for the employment-population ratio in the employment lotteries model corresponds to an interior solution for career length in the time-averaging model. The same high aggregate labor supply elasticity characterizes an interior solution for both models.

Prescott et al. (2009) adopted the Ljungqvist and Sargent (2006) time-averaging model,[7] then extended it by adding an intensive margin to the worker's labor supply decision. When they returned to the tax analysis of Ljungqvist and Sargent (2006), Prescott et al. (2009, p. 31) found that the labor supply response to taxes was unchanged from what Ljungqvist and Sargent had found because "all of the adjustment in hours takes place along the extensive margin, i.e., the fraction of life devoted to work."[8] Prescott et al. (2009) suggested that governments might impose quantity constraints either directly on the intensive margin (interpreted as 'constraints on length of workweek') or on the extensive margin (interpreted as 'constraints on working life'). In the latter case, workers would adjust the intensive margin to partly offset the impact of the constraint on total hours worked over a lifetime.[9]

The change of focus from the fraction of the labor force sent to work in a Rogerson (1988) – Hansen (1985) employment-lottery model to the fraction of an individual worker's lifetime devoted to work in a Ljungqvist and Sargent (2006) time-averaging model refocuses attention to heterogeneity in the situations of individual workers. For example, in their model

[7] Compare section 3 of Prescott et al. (2009) with section 3 of Ljungqvist and Sargent (2006).

[8] Compare section 4.1 of Prescott et al. (2009) with section 4.1 of Ljungqvist and Sargent (2006).

[9] Rogerson and Wallenius (2009) also introduced human capital, but instead of making human capital endogenous as in Ljungqvist and Sargent (2006), they assumed that workers face an exogenously given age-specific labor productivity that captures the hump-shaped earnings curve observed over their lifetimes.

augmented with an intensive margin, Prescott et al. (2009) postulate that occupations differ in set-up costs represented in terms of a function that maps lengths of workweeks to labor services. They find that a larger set-up cost implies a longer optimal workweek length. They suggest that this finding can rationalize variations in weekly hours observed across occupations.

In this paper, we revert to Ljungqvist and Sargent's (2006) exclusive focus on the extensive margin. Our analyses of the elasticity of the earnings-experience profile and unanticipated earnings shocks in sections 4 and 5 preserve the high labor supply elasticity at an interior solution for career length. But a much *lower* labor supply elasticity comes with our social security analysis of section 6, which identifies a possible *third* nonconvexity (beyond (1) the original indivisible $[0,1]$ labor supply choice and (2) the career indivisibility brought by an upward-sloping earnings-experience profile). Specifically, social security tax and benefit rules put a kink into a worker's budget set. Such a policy-induced nonconvexity can lead to a corner solution for career length at an official retirement age that extinguishes the high labor supply elasticity that would be associated with an interior solution. In section 7, we briefly indicate how the constellation of forces identified by our experiments may have balanced out in ways that can help explain variations in labor market outcomes across time and space.

3 A lifetime labor supply problem

A worker's preferences are ordered by

$$\int_0^1 \Big[\log(c_t) - Bn_t\Big]\,dt, \qquad B > 0, \tag{1}$$

where $c_t \geq 0$ and $n_t \in \{0,1\}$ are consumption and labor supply at time t, respectively. That $n_t \in \{0,1\}$ reflects that labor supply is indivisible. A worker with past employment spells totaling $h_t = \int_0^t n_s ds$ has the opportunity to work at earnings

$$w_t = W h_t^{\phi}, \qquad W > 0, \;\; \phi \in [0,1]. \tag{2}$$

Because the worker can borrow and lend at a zero interest rate, she faces the life-time budget constraint $\int_0^1 c_t dt \leq \int_0^1 w_t n_t dt$.[10] An optimal plan prescribes a constant consumption path

[10]We retain the assumption of Ljungqvist and Sargent (2006) that the worker's subjective discount rate and the market interest rate are equal. And, for expositional simplicity, we assume that both rates are equal

and a fraction $T \in [0,1]$ of a lifetime devoted to work.

The worker is indifferent about the timing of her labor supply. Therefore, we are free to assume that the worker frontloads work at the beginning of life, so that the present value of labor income for someone who works a fraction T of her lifetime is

$$\int_0^T W t^{\phi}\, dt = W \frac{T^{\phi+1}}{\phi+1} \equiv W\, e(T;\phi). \tag{3}$$

Equating the present value of labor income $W\, e(T;\phi)$ from (3) to the present value of consumption $\int_0^1 c_t dt$, imposing the consumption-smoothing outcome that $c_t = \bar{c}$ for $t \in [0,1]$, and solving for $\bar{c}$ shows that $c_t = \bar{c} = We(T,\phi)$. Therefore, the optimal career length $\bar{T}$ solves

$$\max_{T\in[0,1]} \Big\{ \log[W\, e(T;\phi)] - BT \Big\}, \tag{4}$$

so that

$$\bar{T} = \bar{T}(\phi) = \min\left\{ \frac{\phi+1}{B}\,,\ 1 \right\}. \tag{5}$$

Notice that $\bar{T}(\phi)$ increases in the curvature parameter ϕ but is independent of the level parameter W. (For a generalization to a class of power utility functions that are consistent with balanced growth, see appendix B.)

4 Effect of earnings profile elasticity on career length

An elasticity parameter $\phi = 0$ means constant earnings, $w_t = W$, while $\phi > 0$ indicates an earnings profile that increases in cumulated time worked h_t, but at a decreasing rate (except for the linear specification, $\phi = 1$). A higher value of ϕ implies a slower relative decay in the slope of the earnings profile with respect to time worked. Evidently, a worker confronting a higher ϕ responds by working longer.

As an illustration, for a disutility of work $B = 1.6$, figure 1 depicts two earnings profiles with elasticity parameters $\phi = 0.3$ and $\phi = 0.5$, respectively, with the optimal fraction of lifetime spent working, $\bar{T}(\phi)$, marked by a circle on each profile. As a normalization, we set level parameters $W = 1$ and $W = e(\bar{T}(0.3), 0.3)/e(\bar{T}(0.3), 0.5)$, respectively, so both

to zero. If instead they were both strictly positive, Ljungqvist and Sargent (2006) show that the worker would prefer to shift her labor supply to the end of life. Why? Because at a given lifetime disutility of work, working later in life would mean spending more total time working. That would push the worker further up the experience-earnings profile and thereby increase the present value of lifetime earnings.

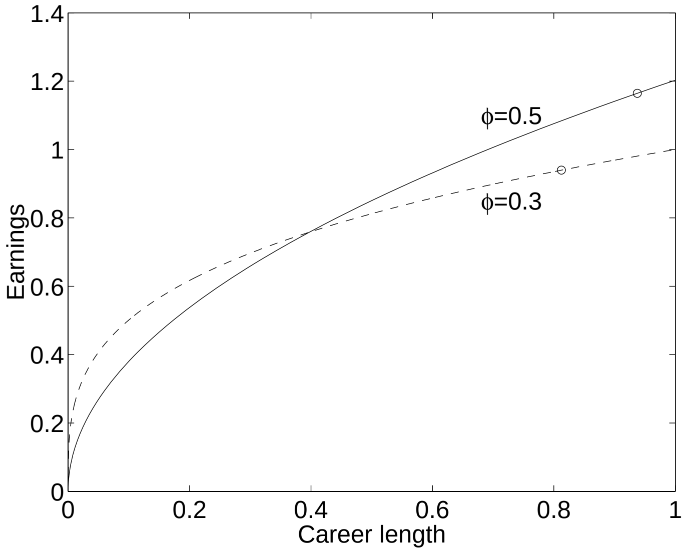

Figure 1: Two earnings profiles with $\phi = 0.3$ (dashed line) and $\phi = 0.5$ (solid line), respectively. For a disutility of work $B = 1.6$, the circle on each profile denotes the optimal career length $\bar{T}(\phi)$.

earnings profiles yield the same present value of labor income when the same fraction $\bar{T}(0.3)$ is devoted to work. But while that choice is optimal for a worker with profile $\phi = 0.3$, the agent with the higher $\phi = 0.5$ will choose to work a bigger fraction of her lifetime.

Viewed as a model of self-financed retirement, the streamlined model with the interior solutions presented here asserts that workers who retire later are those with earnings profiles that are more elastic to accumulated working time. In the remaining sections of this paper, we discuss how this outcome is modified when other features affect the worker's budget set, such as unanticipated earnings shocks and social security. But first we discuss a possible piece of empirical evidence.

4.1 Empirical evidence?

Eckstein and Wolpin (1989) estimated a dynamic model of married women's labor force participation for a specification that posits that wages depend on past work experience. After estimating their model, they performed counterfactual experiments by perturbing the slope of the wage-experience profile away from their estimated value and found the following outcomes:

> Halving the slope of the log wage-experience profile implies that for a woman

with ten years of experience at age 39, the expected additional number of years of work to age 60 will fall from 16.7 to 1.2. Doubling the coefficient implies that all women will work in every year subsequent to age 39 independent of work experience at age 39. (Eckstein and Wolpin 1989, p. 388)

We can interpret the simulation results of Eckstein and Wolpin in terms of responses of an interior solution for $\bar{T}(\phi)$ to the earnings-experience curvature parameter ϕ in (5) in our time-averaging setting. But to do so, we have to resort to a misspecification analysis because the forces driving outcomes in our model differ substantially from those in Eckstein and Wolpin's. In contrast to us, Eckstein and Wolpin (i) assume that households can neither save nor borrow, and (ii) allow the disutility of work to vary with work experience and estimate that it actually increases with experience. Workers' inability to borrow or save in Eckstein and Wolpin's model completely disarms the mechanism at work in our time-averaging model, whereby workers use the credit market to smooth consumption and to 'convexify' the indivisiblity in their instantaneous labor supply opportunities by choosing fractions of their lifetimes to work. This is not the force that drives career length outcomes in Eckstein and Wolpin (1989). Instead, their effect rests on an estimated schedule of disutilities of work that increases with past work experience. But we can reinterpret their result in terms of a specification analysis in which our model generates life-cycle employment and wage data that we mistakenly use to estimate Eckstein and Wolpin's model. We would estimate an increasing disutility of work, but that would be an artifact of misspecified preferences and mistaken exclusion of a credit market. Specifically, the estimate of an increasing disutility of work would truly reflect a falling marginal value of additional savings for retirement in the time-averaging model. (For a formal exposition of our misspecification analysis, see appendix C.)

4.2 Robustness to taxes and social security

A simple system of taxation and social security does not overturn the finding that the more elastic are earnings to accumulated working time, the longer is a worker's career.[11] Specifically, we assume that a worker pays a flat tax rate τ on labor income and after retiring at an age of her choice is entitled to social security at a replacement rate ρ of her average labor earnings. The present value of net-of-tax labor income and social security income for someone who works a fraction $T > 0$ of her lifetime is $(1-\tau)We(T,\phi)+(1-T)\rho We(T,\phi)/T$.

[11] We thank Gianluca Violante for suggesting this robustness check.

After imposing the optimal outcome of constant consumption over the lifetime, the worker's optimization problem is[12]

$$\max_{T\in(0,1]} \left\{ \log \left[W\, e(T;\phi) \left(1 - \tau + \rho \frac{1-T}{T} \right) \right] - BT \right\}. \tag{6}$$

The optimal career length is either a corner solution with $T = 1$, or determined by the first-order condition at equality,

$$T = \frac{\phi + 1 - \dfrac{\rho}{(1-\tau)T + \rho(1-T)}}{B}. \tag{7}$$

The right hand side of (7) is a concave function with intercept ϕ/B at $T = 0$ and hence, for $\phi > 0$, there is at most one intersection with the left hand side of (7), a 45-degree line with intercept zero at $T = 0$.[13] Since a higher ϕ raises the intercept of the right hand side of (7) without affecting the curvature, it follows that a higher ϕ increases the optimal career length.

In section 6, we revisit the effects of taxes and social security on career length, first by adopting Prescott's (2002) assumption that all tax receipts are distributed lump-sum to workers, and then by studying social security arrangements with realistic features such as an earliest age of eligibility.

5 Effect of earnings shock on career length

To bring out salient substitution and wealth effects, this section puts our model to work by studying how an unanticipated mid career earnings shock affects a worker's lifetime

[12]We have excluded $T = 0$ from the choice set to enable division by T in the computation of average labor earnings in the social security formula. This exclusion is irrelevant because a worker with preferences described by (1) would never choose zero consumption if that can be avoided.

[13]The following analysis applies when $\phi = 0$. While the intercept of the right hand side of (7) is then zero at $T = 0$, this is not a solution because, as mentioned in footnote 12, a worker would not choose zero consumption. Instead as long as $1 - \tau > \rho$, the right hand side of (7) is a strictly concave, upward-sloping function and therefore, there exists at most one intersection with the left hand side of (7) and that $T > 0$ would be the optimal career length. But if $1 - \tau \leq \rho$, the right hand side of (7) is nonincreasing and the optimal solution is one with limiting behavior in which the worker works an infinitesimal amount of time in order to live on social security for the rest of her life. Such limiting behavior does not characterize a worker with $\phi > 0$ who would choose to work a well-defined $T > 0$ even if $\rho \geq 1 - \tau$, in order to increase the average labor earnings upon which social security is calculated.

labor supply.[14] Throughout this section, we assume as above that the worker frontloads her working time. We find that when $\phi > 0$, the sign of the effect on career length of a permanent unanticipated upward or downward shift in the earnings profile depends on the sign of the stock of assets that the worker had accumulated when the unanticipated earnings shock arrives. This outcome reflects how wealth and substitution effects combine to shape the worker's continuation plan for career length and consumption.[15]

We let $\bar{T}$ be the optimal fraction of her lifetime that the worker intends to devote to work before the realization of the unanticipated earnings shock, and $\hat{T}$ be the optimal fraction after the earnings shock. We assume that ϕ, B are such that, before the earnings shock, (5) implies that $\bar{T} \in (0,1)$, which is equivalent to imposing $B > \phi + 1$. With frontloaded working time, before the earnings shock, the original optimal savings profile for $t \leq \bar{T}$ is

$$A_t = \int_0^t \left[W s^\phi - W e(\bar{T};\phi)\right] ds = \frac{W\,t}{\phi+1}\left[t^\phi - \left(\frac{\phi+1}{B}\right)^{\phi+1}\right] \equiv W a(t;\phi), \tag{8}$$

where we have used $\bar{T}(\phi) = (\phi+1)/B$. For $\phi > 0$, there exists a cutoff value $\bar{t}(\phi)$ such that accumulated assets are negative for $t \in (0, \bar{t}(\phi))$ and positive for $t > \bar{t}(\phi)$. (Workers who expect rising earnings borrow when young, repay when older, then lend when even older.) We can solve (8) for $\bar{t}(\phi)$ to get

$$\bar{t}(\phi) = \left(\frac{\phi+1}{B}\right)^{\frac{\phi+1}{\phi}} \in \left(0, \bar{T}(\phi)\right). \tag{9}$$

The limit point of $\bar{t}(\phi)$ in (9) is zero as $\phi \to 0$, so we define $\bar{t}(0) = 0$. Thus, with a front loaded lifetime labor supply, asset holdings are always nonnegative for a worker with a flat $\phi = 0$ earnings profile.

Consider an unanticipated mid career earnings shock at time $\hat{t} \in (0, \bar{T}]$. In particular, for $t < \hat{t}$, we assume that the worker had conformed to an optimal plan associated with W, ϕ, B. At time $\hat{t}$, the earnings profile unexpectedly jumps from Wt^ϕ to $\hat{W}t^\phi$ for $t \in [\hat{t}, 1]$. Subject

[14] Alternatively, we could have modeled an explicit stochastic earnings process which would have entailed the study of precautionary savings dynamics over the worker's life cycle or more precisely, over her working career. But such added complications would have made less transparent the income and substitution effects of large earnings shocks that we highlight here in closed forms.

[15] To abstract from the effects on career length of different values of ϕ, as studied in section 4, we assume that ϕ stays constant and only W changes at the earnings shock. According to (5), career length does not respond to a shift in W at the very beginning of a lifetime, which reflects that preferences in (1) are consistent with balanced growth. An unanticipated shift later in life is another matter.

to the asset stock $Wa(\hat{t};\phi)$ that had been accumulated under the old plan, the wage jump from W to $\hat{W}$ prompts the worker to maximize the remainder of her lifetime utility

$$\int_{\hat{t}}^{1} \Big[\log(\hat{c}_t) - B\hat{n}_t\Big]\, dt \tag{10}$$

by choosing new values $\hat{c}_t \geq 0$ and $\hat{n}_t \in \{0,1\}$ of consumption and labor supply, respectively, for $t \in [\hat{t},1]$. The worker's revised optimal plan prescribes a constant consumption path over the interval $[\hat{t},1]$ and a fraction $\hat{T} \in [\hat{t},1]$ of her lifetime devoted to work.[16]

For the worker who after the unanticipated wage shock at $\hat{t}$ chooses to work a fraction $T \in [\hat{t},1]$ of her total lifetime, the sum of the financial assets already accumulated at time $\hat{t}$, $Wa(\hat{t};\phi)$, and the present value of future labor income becomes

$$\begin{aligned} Wa(\hat{t};\phi) + \int_{\hat{t}}^{T} \hat{W} s^{\phi}\, ds \;&=\; \hat{W}\left\{ \left(\frac{W}{\hat{W}} - 1\right) a(\hat{t};\phi) + \frac{1}{\phi+1}\left[T^{\phi+1} - \hat{t}\,\left(\frac{\phi+1}{B}\right)^{\phi+1} \right] \right\} \\ &\equiv\; \hat{W}\, \hat{e}(T;\hat{t}, W/\hat{W}, \phi), \end{aligned} \tag{11}$$

where the first equality is obtained by adding and subtracting $\hat{W}a(\hat{t};\phi)$. This time $\hat{t}$ present value of financial plus non-financial wealth must equal the present value of consumption over the period $[\hat{t},1]$, so it follows that $\hat{W}\hat{e}(T;\hat{t},W/\hat{W},\phi)/(1-\hat{t}\,)$ is the constant consumption rate over the remaining lifetime $1-\hat{t}$.

The worker's optimal lifetime labor supply thus solves

$$\max_{T\in[\hat{t},1]} \left\{ (1-\hat{t}\,)\log\left[\frac{\hat{W}\hat{e}(T;\hat{t},W/\hat{W},\phi)}{1-\hat{t}} \right] - B(T-\hat{t}\,) \right\}. \tag{12}$$

The first-order condition for T is

$$\frac{(1-\hat{t}\,)T^{\phi}}{\left(\frac{W}{\hat{W}} - 1\right) a(\hat{t};\phi) + \frac{1}{\phi+1}\left[T^{\phi+1} - \hat{t}\,\left(\frac{\phi+1}{B}\right)^{\phi+1}\right]} - B \begin{cases} <0, & \text{corner soln } \hat{T} = \hat{t}; \\ =0, & \text{interior soln } \hat{T} \in [\hat{t},1]; \\ >0, & \text{corner soln } \hat{T} = 1; \end{cases} \tag{13}$$

where $\hat{T}$ is the optimal lifetime labor supply after the earnings shock at time $\hat{t}$. We let $\hat{T}(\hat{t},W/\hat{W},\phi)$ denote an interior solution that is determined implicitly by (13) at equality,

[16] We impose the implicit parameter restriction that any negative asset holdings at time $\hat{t}$ are strictly less than the present value of future labor income if the worker works for the rest of her lifetime.

i.e.,

$$(1-\hat{t}\,)\hat{T}^{\phi}=B\left\{\left(\frac{W}{\hat{W}}-1\right)\,a(\hat{t};\phi)+\frac{1}{\phi+1}\left[\hat{T}^{\phi+1}-\hat{t}\,\bar{T}^{\phi+1}\right]\right\}, \tag{14}$$

where we have invoked $(\phi+1)/B=\bar{T}(\phi)$. An interior solution for the post-shock career length $\hat{T}$ relates to the original career length $\bar{T}$ in the following way:[17]

$$\hat{T}(\hat{t},W/\hat{W},\phi)\left\{\begin{array}{l}<\bar{T}(\phi)\\ =\bar{T}(\phi)\\ >\bar{T}(\phi)\end{array}\right\}\text{ if }\left(\frac{W}{\hat{W}}-1\right)\,a(\hat{t};\phi)\left\{\begin{array}{l}>0;\\ =0;\\ <0;\end{array}\right. \tag{15}$$

Evidently, the sign of the revision $\hat{T}-\bar{T}$ to an unanticipated earnings shock depends (i) on whether $\hat{W}>W$ or $\hat{W}<W$, and (ii) on whether the worker's asset holdings at the time of the shock, $A_{\hat{t}}$, are positive or negative. In response to a *negative* earnings shock, $\hat{W}<W$, the worker reduces (increases) her lifetime labor supply if her time $\hat{t}$ asset holdings are positive (negative), i.e., if $a(\hat{t};\phi)>0$ ($a(\hat{t};\phi)<0$) which means that the shock occurs at a time $\hat{t}>\bar{t}(\phi)$ ($\hat{t}<\bar{t}(\phi)$), where $\bar{t}(\phi)$ is defined in (9). In contrast, in response to a *positive* earnings shock, $\hat{W}>W$, the worker increases (decreases) her lifetime labor supply if her current asset holdings are positive (negative).

In the case of a flat $\phi=0$ earnings profile and a frontloaded lifetime labor supply, asset holdings are always nonnegative in the initial plan, and, hence, the worker's labor supply response depends only on the sign of the earnings shock. Specifically, when $\phi=0$, we can rewrite first-order condition (14) at an interior solution as

$$\hat{T}=\bar{T}-(1-\bar{T})\left(\frac{W}{\hat{W}}-1\right)\hat{t}\;\left\{\begin{array}{ll}<\bar{T} & \text{if }\;\hat{W}<W,\\ =\bar{T} & \text{if }\;\hat{W}=W,\\ >\bar{T} & \text{if }\;\hat{W}>W.\end{array}\right. \tag{16}$$

As could be anticipated from (15), a worker with a flat earnings profile will reduce (increase) her lifetime labor supply in response to a negative (positive) earnings shock.

[17] Suppose that $(W/\hat{W}-1)\,a(\hat{t};\phi)>(<)0$ but, contrary to (15), lifetime labor supply satisfies $\hat{T}\geq(\leq)\bar{T}(\phi)$. According to (14), this would imply

$$(1-\hat{t}\,)\hat{T}^{\phi}>(<)B\frac{1}{\phi+1}\left[\hat{T}^{\phi+1}-\hat{t}\,\hat{T}^{\phi+1}\right],$$

which leads to the contradiction that $\hat{T}<(>)(\phi+1)/B=\bar{T}(\phi)$. When $(W/\hat{W}-1)\,a(\hat{t};\phi)=0$, the equality $\hat{T}=\bar{T}$ can be confirmed by plugging that solution into (14) to verify that $\hat{T}=(\phi+1)/B=\bar{T}(\phi)$.

5.1 Interpretation of wealth and substitution effects

For a worker with positive asset holdings at $\hat{t}$, a *negative* earnings shock means that returns to working fall relative to the marginal value of her wealth. That induces the worker to enjoy more leisure because doing that has now become relatively less expensive. But with negative asset holdings at $\hat{t}$, a negative earnings shock compels the worker to supply more labor both to pay off time $\hat{t}$ debt and to moderate the adverse effect of the shock on her future consumption.

With a *positive* earnings shock, leisure becomes more expensive, causing the worker to substitute away from leisure and toward consumption. This force makes lifetime labor supply increase for a worker with positive wealth. But why does a positive earnings shock lead to a *reduction* in life-time labor supply when time $\hat{t}$ assets are negative?

In the case of a positive earnings shock and negative time $\hat{t}$ assets, consider a hypothetical asset path that would have prevailed if the worker had enjoyed the higher earnings profile associated with $\hat{W}$ from the beginning starting at $t = 0$. Along that hypothetical path, the worker would have been even further in debt at $\hat{t}$ (since assets would be scaled by $\hat{W}$ rather than W in (8)). So at $\hat{t}$, the worker actually finds herself *richer* at $\hat{t}$ than she would have in our hypothetical scenario. Because there is less debt to be repaid at $\hat{t}$, the worker chooses to supply less labor than she would have in the hypothetical scenario.

To construct another revealing hypothetical path in the case of a positive earnings shock and negative time $\hat{t}$ assets, suppose instead that the worker had known her actual earnings profile *including* the positive earnings shock at $\hat{t}$ from time $t = 0$ on. That would have induced her to choose a higher consumption level prior to time $\hat{t}$. That would leave her *more* in debt at time $\hat{t}$. We conclude that in the actual situation with a positive earnings shock and negative asset holdings at $\hat{t}$, it is not optimal to make up for what would have been past underconsumption relative to our hypothetical path. Instead, the worker chooses to enjoy more leisure because she has relatively less debt at $\hat{t}$ than she would along the hypothetical path.

6 Effects of taxes and social security on career length

We now introduce a government that taxes labor income and runs a balanced budget either by returning the tax receipts lump sum to workers or by using the revenues to finance a social security system. Newborn workers enter the economy at a rate that keeps the population and age structure constant over time. Our focus is not on the determination of intertemporal prices in this overlapping generations environment with its possible dynamic inefficiencies,[18] so we retain our small open economy assumption of an exogenously given interest rate. All workers have the same preference parameter B and the same earnings profile parameters W and ϕ.

The simple tax and social security arrangement in section 4.2 with optimal career length given by (7) could constitute an equilibrium in this environment, but only so long as total tax receipts are at least equal to the social security payments, with any surplus being used to finance public expenditures that are assumed not to affect a worker's optimization problem. For example, when all tax receipts are used for such public expenditures and there are no social security payments ($\rho = 0$), expression (7) shows that labor supply is not affected by taxation.[19] But if instead all tax receipts are rebated lump sum to workers, then labor supply responds in ways that we study next.

6.1 Labor tax receipts handed back lump sum

Following Prescott (2002), we assume that the government levies a flat tax rate $\tau \in [0, 1)$ on labor income and that the tax receipts are handed back as equal lump-sum transfers to all workers. The present value of lump-sum transfers that each worker receives over her lifetime, call it x, is determined by the government budget constraint

$$\tau W \, e(T^\star; \phi) = x, \tag{17}$$

where $T^\star$ is the equilibrium career length. Note that given a zero interest rate and a lifetime of unit length, x is the instant-by-instant per capita lump-sum transfer that satisfies the government's static budget constraint (17) as well as the present value of total lump-sum transfers paid to a worker over her lifetime.

A worker again chooses a constant consumption path, but now her optimal career length solves

$$\max_{T \in [0,1]} \Big\{ \log[(1-\tau) W \, e(T; \phi) + x] - BT \Big\}. \tag{18}$$

Substituting (17) into the first-order condition for this problem shows that the equilibrium

[18] For a treatment of overlapping generations models, see e.g. Ljungqvist and Sargent (2004).

[19] Prescott (2002, p. 7) noted that "If [labor tax] revenues are used for some public good or are squandered, private consumption will fall, and the tax wedge will have little consequence for labor supply."

career length is

$$T^{\star}(\tau) = \min\left\{\frac{(1-\tau)(\phi+1)}{B}\,,\ 1\right\}. \tag{19}$$

Hence, at an interior solution for career length, the economy's elasticity of aggregate labor supply with respect to the net-of-tax rate, $[(1-\tau)/T]\,\partial T/\partial(1-\tau)$, equals one.[20] As shown in appendix A, this high elasticity is the same as in a corresponding with employment lotteries and complete markets in contingent claims to consumption. An important ingredient of this high elasticity is that the government rebates taxes lump sum to workers.

6.2 Social security with minimum age of eligibility

Instead of returning all tax receipts lump sum to workers, we now assume that all revenues are used to finance a social security system in which workers are eligible to retire and collect benefits after an official retirement age R. Only those labor earnings accruing *before* R are subject to a flat rate social security tax $\tau \in (0,1)$. Benefits *after* the worker's chosen retirement date T, which may or may not equal R, are computed as a replacement rate ρ times a worker's average earnings prior to R. Thus, labor earnings after R are not taxed; neither do they affect the base for calculating benefits. Workers who choose to retire after R collect no benefits until they actually retire.[21]

To construct an equilibrium, we set the two parameters R and τ of the social security system, and then solve residually for a replacement rate ρ that is consistent with a balanced

[20]Note that we have computed a labor supply elasticity with respect to the net-of-tax rate $(1-\tau)$ rather than to disposable wage income *per se*. As pointed out above and emphasized in footnote 19, what matters for the effect of taxes on labor supply is how wage income is split into two parts: one that goes directly to the worker as disposable wage income, another that is first paid to the government as taxes, but then returned to the worker in the form of lump-sum transfers.

[21]While our specification of social security taxes and benefits is overly simple, it captures key features of some real-world programs. The assumption that the replacement rate is a function of average earnings but not career length, is a good approximation to programs that compute benefits on the basis of fewer years than a primary worker's normal choice of career length, a feature that makes the first-order condition with respect to career length reflect a worker's marginal rather than inframarginal lifetime labor supply. (We elaborate on this point in section 7.2.) As an example, U.S. social security benefits are computed based on the average of a worker's highest 35 years of earnings. As for our assumption that someone who works beyond the official retirement age R receives no social security benefits until she actually retires, Schulz (2001, pp. 141-2) describes how this was the situation in the U.S. social security system between 1950 and 1972, after the repeal in 1950 of an earlier provision of a 1 percent increase in benefits for each year of delay. After 1972, a delayed retirement credit was reintroduced, but it is only with rules that recently became effective that the compensation is high enough for there to be no loss in the actuarial value of a worker's lifetime benefits. We consider implications of those recent major policy changes in the U.S. from the perspective of our framework in section 7.3.

government budget. To simplify the task of characterizing equilibria, we restrict attention to policies with $R \in (0.5, 1)$, and we bound the disutility of work from above:

$$B \leq \frac{\phi + 1 - \tau}{1 - R}. \tag{20}$$

We shall show that these parameter restrictions deliver two equilibrium outcomes. First, the equilibrium career length, denoted $\tilde{T}$, is *longer* than the official retirement period, i.e., $\tilde{T} > 1 - R$. Second, workers strictly prefer to supply their labor *before* rather than after the official retirement age, i.e., $\int_0^R n_t dt = \min\{\tilde{T}, R\}$.[22] These outcomes simplify the task of characterizing an equilibrium while also being consistent with empirical facts about how primary workers distribute work over their lives, since the unit length of lifetime refers to a worker's adulthood.

With our parameter restrictions and these conjectured equilibrium outcomes, the government budget constraint is[23]

$$\tau W \min\{e(\tilde{T}; \phi),\, e(R; \phi)\} = \left(1 - \max\{R, \tilde{T}\}\right) \frac{\rho}{\min\{\tilde{T}, R\}} W \min\{e(\tilde{T}; \phi),\, e(R; \phi)\}, \tag{21}$$

where the left side is tax revenues and the right side is social security benefits. The first (second) argument of the max and min operators in (21) presumes an equilibrium outcome in which workers retire before (after) the official retirement age. That is, if the equilibrium career length $\tilde{T}$ is shorter (longer) than the official retirement age R, tax revenues are $\tau We(\tilde{T}; \phi)$ ($\tau We(R; \phi)$) and social security pays a benefit of $\rho We(\tilde{T}; \phi)/\tilde{T}$ ($\rho We(R; \phi)/R$) over the eligible nonworking period that lasts $1 - R$ ($1 - \tilde{T}$). Note that the unit length of a lifetime implies that an age interval corresponds both to a fraction of a worker's lifetime and also to a fraction of the population within that age interval at any point in time. From (21)

[22] As shown in appendix D, the key to having workers prefer to supply their labor before rather than after the official retirement age is that the part of the equilibrium career length during which social security taxes are paid be longer than the part of the equilibrium retirement period during which benefits are collected, an outcome ensured by parameter restrictions (20) and $R \in (0.5, 1)$. This outcome makes the social security tax τ needed to balance the government's budget be lower than the social security replacement rate ρ. When $\rho > \tau$, a worker would not want to try to avoid the social security tax by postponing labor supply until after the official retirement age: lost social security benefits would outweigh tax savings.

[23] Division by $\min\{\tilde{T}, R\}$ in (21), as well as division by $(1 - \max\{R, \tilde{T}\})$ in (22), is permissible since $R \in (0.5, 1)$ and equilibrium career length can be neither $\tilde{T} = 0$ (see footnote 12) nor $\tilde{T} = 1$ as discussed below.

we can solve for the replacement rate,

$$\rho = \frac{\min\{R, \tilde{T}\}}{1 - \max\{R, \tilde{T}\}}\, \tau. \tag{22}$$

Again, with our parameter restrictions and conjectured equilibrium outcomes, a worker's optimal career length solves[24]

$$\max_{T\in(0,1]} \Big\{ \log\Big[(1-\tau)W \, \min\{e(T;\phi),\, e(R;\phi)\} + W \, \max\{0,\, e(T;\phi) - e(R;\phi)\} \\ + \rho W \, \min\{(1-R)e(T;\phi)/T,\, (1-T)e(R;\phi)/R\} \Big] - BT \Big\}, \tag{23}$$

where inside the max and min operators working on the three components comprising consumption (i.e., the argument of the log function), arguments appear in the same order as in (21) and (22), i.e., the first (second) argument refers to the case when the worker chooses to work shorter (longer) than the official retirement age.

Case with $\tilde{T} \leq R$

In the case of an optimal career length $T \leq R$, the first-order condition of (23) at an interior solution (with respect to $T \leq R$) becomes

$$\frac{\phi + 1}{T} - \frac{\rho(1-R)/T}{(1-\tau)T + \rho(1-R)} - B = 0. \tag{24}$$

By government budget balance in (22), $\rho = \tau\tilde{T}/(1-R)$, which can be substituted into (24) to yield an expression for equilibrium career length,

$$\tilde{T} = \frac{\phi + 1 - \tau}{B} \equiv R^{+}(\tau). \tag{25}$$

Given an equilibrium with $\tilde{T} \leq R$, equilibrium expression (25) implies $R \geq R^{+}(\tau)$. If $R^{+}(\tau) \in (0.5, 1)$, it can be verified that $R^{+}(\tau)$ is the lowest possible official retirement age $R \in (0.5, 1)$ for which equilibrium expression (25) holds, namely, $\tilde{T} = R$ for $R = R^{+}(\tau)$.

Case with $\tilde{T} \geq R$

In the case of an optimal career length $T \geq R$, the first-order condition of (23) at an interior

[24]Regarding our exclusion of $T = 0$ from the choice set, see footnote 12.

solution (with respect to $T \geq R$) becomes

$$\frac{-\dfrac{\rho}{R}\dfrac{R^{\phi+1}}{\phi+1}+T^{\phi}}{\left[\rho\dfrac{1-T}{R}-\tau\right]\dfrac{R^{\phi+1}}{\phi+1}+\dfrac{T^{\phi+1}}{\phi+1}}-B\geq 0, \tag{26}$$

which holds with equality except under a binding corner solution with $T=1$. However, such a corner solution can be ruled out as an equilibrium because government budget balance in (22) would imply that the replacement rate goes to infinity; hence, it must be optimal for a worker to retire prior to the end of her lifetime. After substituting $\rho=\tau R/(1-\tilde{T})$ into (26) at equality, we obtain an expression for equilibrium career length,

$$\tilde{T}=\frac{\phi+1-\tau\dfrac{R}{1-\tilde{T}}\left(\dfrac{R}{\tilde{T}}\right)^{\phi}}{B}. \tag{27}$$

Given an equilibrium with $\tilde{T}\geq R$, equilibrium expression (27) implies

$$R\leq\frac{\phi+1-\tau\dfrac{R}{1-R}}{B}, \tag{28}$$

where the right side is an upper bound for the right side of (27), attained at $\tilde{T}=R$ because the right side of (27) is a decreasing function in $\tilde{T}$.[25] Next, we implicitly define $R^{-}(\tau)$ as a fixed point of (28) at equality,

$$R^{-}(\tau)=\frac{\phi+1-\tau\dfrac{R^{-}(\tau)}{1-R^{-}(\tau)}}{B}. \tag{29}$$

Over the interval $[0,1]$, there exists a unique fixed point $R^{-}(\tau)\in(0,1)$, since the left side of (29) is a straight line with intercept zero and a positive slope, while the right side is a strictly decreasing function that starts at $(\phi+1)/B>0$ and has minus infinity as the limit

[25] The derivative of the right side of (27) with respect to $\tilde{T}$ is

$$-\frac{\tau R^{\phi+1}}{\tilde{T}^{\phi}(1-\tilde{T})B}\left[\frac{1}{1-\tilde{T}}-\frac{\phi}{\tilde{T}}\right]<0,$$

where the strict inequality follows from $\phi\in[0,1]$ and $\tilde{T}\in[R,1)$, where $R\in(0.5,1)$.

when $R^{-}(\tau) \to 1$. If $R^{-}(\tau) \in (0.5, 1)$, it can be verified that $R^{-}(\tau)$ is the highest possible official retirement age $R \in (0.5, 1)$ for which equilibrium expression (27) holds, namely, $\tilde{T} = R$ for $R = R^{-}(\tau)$. Moreover, if $R^{-}(\tau) \in (0.5, 1)$, it follows from (25) and (29) that $R^{-}(\tau) < R^{+}(\tau)$.[26]

We can now state a proposition that describes how the retirement age $\tilde{T}$ chosen in equilibrium depends on the official social security retirement age. The proof appears in appendix D.

Proposition 1: Given an official retirement age $R \in (0.5, 1)$ and a tax rate $\tau \in (0, 1)$ that satisfy (20), the equilibrium career length $\tilde{T}(R, \tau)$ is unique and can be characterized in terms of $R^{+}(\tau)$ and $R^{-}(\tau)$, as defined in (25) and (29):

i) If $R \geq R^{+}(\tau)$, then $\tilde{T}(R, \tau) = R^{+}(\tau)$ (retirement *before* the official retirement age).

ii) If $R \leq R^{-}(\tau)$, then $\tilde{T}(R, \tau) \in [R^{-}(\tau), R^{+}(\tau))$, $\tilde{T}(R^{-}(\tau), \tau) = R^{-}(\tau)$ and $\partial\tilde{T}(R, \tau)/\partial R < 0$ (retirement *after* the official retirement age).

iii) Otherwise, $\tilde{T}(R, \tau) = R$ (retirement *at* the official retirement age).

Figure 2 displays the equilibrium career length as a function of R and τ, and figure 3 compares equilibrium outcomes in two economies with different values of ϕ. We proceed to explain the shapes of these functions when $\tilde{T} < R$ and when $\tilde{T} > R$.

6.3 Possible corner solution at the official retirement age

According to Proposition 1, there exist a range of official retirement ages sufficiently *high* that they induce equilibrium retirements *before* the official retirement age and a range of official retirement ages sufficiently *low* that they induce equilibrium retirements *after* the official retirement age. Between these two intervals there exists an intermediate interval of official retirement ages that induce equilibrium retirement *at* the official retirement age. In

[26] Given that $R^{-}(\tau) \in (0.5, 1)$, the following strict inequality holds

$$\Big(R^{-}(\tau) = \Big) \qquad \frac{\phi + 1 - \tau \dfrac{R^{-}(\tau)}{1 - R^{-}(\tau)}}{B} < \frac{\phi + 1 - \tau}{B} \qquad \Big(= R^{+}(\tau) \Big).$$

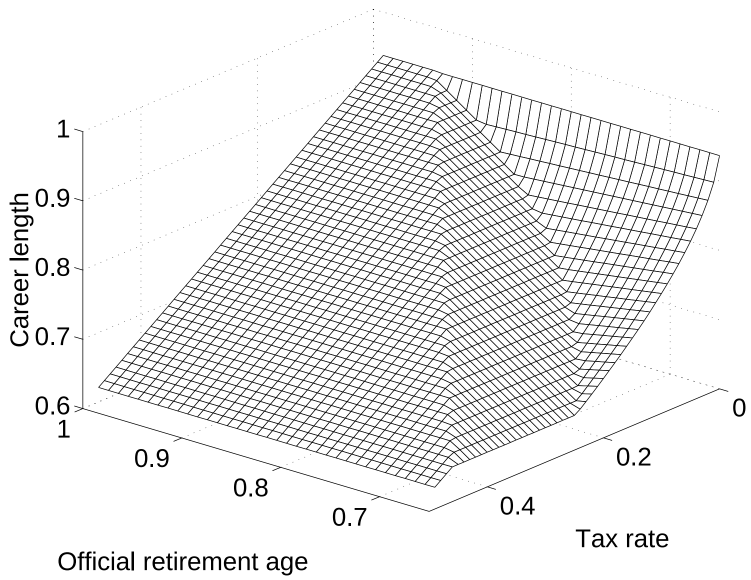

Figure 2: Equilibrium career length $\tilde{T}(R, \tau)$ as a function of the official retirement age R and the tax rate τ when the earnings profile parameter is $\phi = 0.5$ and the disutility of work is $B = 1.6$.

this middle range, the coincidence of official and actual retirement ages indicates a kink in implicit taxation that occurs at the official retirement age – a situation commonly said to describe actual social security arrangements.[27]

For an R in our high range in which $\tilde{T} = R^{+}(\tau) < R$ so that workers retire *before* the official retirement age, the effect of the social security tax on career length in (25) is quantitatively similar to the effect of a labor tax in (19) in the style of Prescott's (2002) analysis in which all tax receipts are handed back lump sum to workers. Indeed, as can be seen by comparing formula (19) for T^* with formula (25) for $\tilde{T}$, when $\phi = 0$ the lifetime labor supply effects are actually identical to the labor supply effects obtained by Prescott (2002). The reasons are that (a) under our assumption that average lifetime earnings alone determine the replacement rate without regard to career length, when $\phi = 0$ workers regard the social security contribution purely as a tax and perceive no extra benefits accruing to them from paying it, while (b) the present value of future social security payments operates

[27] In the empirical analysis of Rust and Phelan (1997), the peaks in the distribution of retirement in the U.S. at age 62 and 65 (the ages of early and normal eligibility for social security benefits, respectively) are rationalized as artifacts of particular details of the rules for social security and for public health insurance for the elderly (Medicare). Hairault et al. (2009) analyze how social security rules in France in conjunction with specific income support programs for workers between age 55 and 59, shape implicit taxation and cause French nonemployment to rise sharply even prior to age 60 when workers become eligible for social security.

like a lump sum transfer when optimal career length falls short of the official retirement age. When $\phi > 0$, the social security tax in (25) is less distorting than Prescott's (2002) labor tax, since longer careers now have the advantageous effect of increasing social security benefits due to the higher average lifetime earnings when a worker moves up along the earnings profile. But besides the longer career length $\tilde{T}$ in (25) as compared to T^* in (19) when $\phi > 0$, lifetime labor supply in the region with $\tilde{T} < R$, remains highly responsive to tax changes, as illustrated by the downward-sloping plane to the left in figure 2 (where, as explained above, the value of R has no effect).

For an R sufficiently low that $\tilde{T} = R^{-}(\tau) > R$ so that workers retire *after* the official retirement age, the marginal decision on career length is distorted by the loss of benefits incurred from working beyond the official retirement age. In this region with $\tilde{T} > R$, the tax rate and the official retirement age both affect $\tilde{T}$ through their effects on the equilibrium replacement rate as determined by $\rho = \tau R/(1 - \tilde{T})$ in (22). Specifically, for an unchanged career length $\tilde{T}$, the replacement rate rises in response to an increase in either τ or R. A worker who faces the resulting higher opportunity cost of retirement benefits foregone while working beyond the official retirement age would choose to reduce her career length. Thus, in the region with $\tilde{T} > R$, lifetime labor supply falls in response to an increase in either τ or R. This effect is depicted by the bowl-shaped surface at the far right in figure 2 where career length $\tilde{T}$ decreases with increases in both the tax rate and the official retirement age. The latter effect is easier to discern in figure 3 where the downward-sloping portions of both of the equilibrium career length functions refer to the region with $\tilde{T} > R$. These equilibrium forces underpin the analytical derivative shown above in case ii) of Proposition 1.

For an R within our intermediate range, there are no effects of the tax on lifetime labor supply so long as workers choose to remain at the corner solution highlighted in case iii) of Proposition 1. For example, at $R = 0.65$ in figure 2, any tax rate between 0.25 and 0.45 would induce equilibrium retirement *at* the official retirement age. For the parameterization in figure 2, such a corner solution prevails for a tax range that is wider than 15 percentage points up until an official retirement age of 0.80. Thereafter, the tax range associated with a corner solution narrows and eventually, at a high enough official retirement age, there exists only equilibria with equilibrium retirement before the official retirement age.

To provide another perspective, figure 3 depicts equilibrium career lengths $\tilde{T}(R, \tau)$ as functions of the official retirement age R in two economies with distinct earnings profile parameters $\phi = 0.3$ and $\phi = 0.5$, respectively. The two economies share the same tax

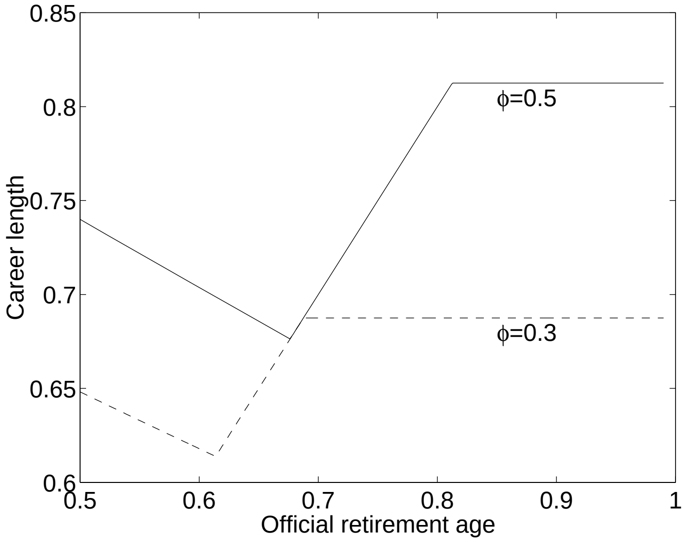

Figure 3: Equilibrium career length $\tilde{T}(R, \tau)$ as a function of the official retirement age R, in two economies with earnings profile parameter $\phi = 0.3$ (dashed line) and $\phi = 0.5$ (solid line), respectively. Both economies have the same tax rate $\tau = 0.2$ and a disutility of work $B = 1.6$.

rate $\tau = 0.2$ and the same disutility of work $B = 1.6$.[28] (We use the same preference and earnings profile parameters as in figure 1.) The figure illustrates how the presence of social security modifies but does not remove the tendency for workers with a higher earnings-curve elasticity parameter ϕ to retire at a later age, as studied in section 4. However, the existence of our intermediate range of official retirement ages in which equilibrium career length equals the official retirement age opens up the possibility that the equilibrium career lengths are identical across two economies with different ϕ's but identical τ's and R's. This is evidently the case in figure 3 when R is approximately two thirds of a worker's (adult) lifetime.

7 Career lengths, past and future, through the lens of our model

7.1 Diverse workers retiring at the same official retirement age

While the two equilibrium mappings in figure 3 refer to two distinct economies with the only difference in primitives being the earnings-profile parameter ϕ, we can imagine the

[28]The solid line in figure 3 is a slice of figure 2 at $\tau = 0.20$ (but for a somewhat wider range of R).

outcomes depicted there to refer to two groups of workers who live in the *same* economy, in particular, an economy in which the government runs a balanced social security budget for each group of workers, there being identical policy parameters τ and R across the two groups but different replacement rates ρ determined by (22)). This interpretation reminds us of a feature of real-world social security programs that tends to increase the range of official retirement ages for which an equilibrium would imply that heterogeneous situated workers all end up choosing identical career lengths by retiring at the official retirement age. Thus, real-world social security programs often redistribute from high to low income earners, and the former workers usually have more elastic (higher ϕ) earnings profiles than the latter workers. It follows that if the redistribution associated with social security payout rules ends up lowering and raising the implicit returns to work for high and low income workers, respectively, it tends to lower and raise the corresponding equilibrium mappings in figure 3 for high and low income workers, respectively. That would seem to widen the range of official retirement ages for which both groups of workers find it optimal to choose to retire at the official retirement age. To execute a precise analysis, we would need to specify the details of such a social security program and derive an equilibrium.

7.2 Retirement as a marginal decision off a corner solution

An important message of a life cycle model with indivisible labor is that a marginal labor supply decision is about the choice of retirement age. Hence, at an interior solution for career length, how taxes and social security affect actual retirement age are important determinants of the aggregate labor supply. To emphasize this point, consider an implicit tax and benefit system with the following characteristics. The system is such that a worker with earnings profile (2) chooses to supply labor at least during all of some initial phase of life $P \in (0,1)$ that we can call the 'prime of life'. This yields a present value of disposable lifetime income equal to m, after paying taxes and receiving government transfers including the discounted value of future social security benefits. When contemplating any additional old-age labor supply, $T > P$, we assume that the worker faces an 'effective' tax rate $\breve{\tau} \in [0,1)$ that incorporates both positive and negative effects that extra earnings might have on future social security benefits. Given a policy configuration that makes a prime-age labor supply of P optimal, the remainder problem that pins down the optimal career length is

$$\max_{T\in[P,1]} \Big\{ \log\left[(1-\breve{\tau})W\left(e(T;\phi) - e(P;\phi)\right) + m\right] - BT \Big\}. \tag{30}$$

The first-order condition is

$$\frac{(1-\breve{\tau})W\dfrac{\partial e(T;\phi)}{\partial T}}{(1-\breve{\tau})W\left(e(T;\phi)-e(P;\phi)\right)+m} \geq B. \tag{31}$$

The equilibrium career length $\breve{T}$ that satisfies this first-order condition depends on a worker's equilibrium consumption level. In our stationary economy with identical agents, we represent equilibrium consumption as a fraction of a worker's lifetime labor earnings

$$(1-\breve{\tau})W\left(e(\breve{T};\phi)-e(P;\phi)\right)+m=(1-\nu)e(\breve{T};\phi), \tag{32}$$

where the economy's resource constraint implies $\nu \in [0,1)$.[29] For example, ν would be strictly positive if the government uses some lifetime tax receipts to finance a public good that is not a perfect substitute with private consumption. In both section 6.1 and 6.2, we imposed $\nu = 0$ because all tax receipts were either handed back lump sum to workers or fully used to finance a social security system. More generally, let $\nu \in [0,1)$ and substitute (32) into (31) to arrive at the following expression for equilibrium career length:

$$\breve{T}(\breve{\tau},\nu)=\min\left\{\frac{(1-\breve{\tau})(\phi+1)}{(1-\nu)B},\ 1\right\}. \tag{33}$$

The equilibrium career length in (33) depends only on the tax rate $\breve{\tau}$ in old age and the equilibrium fraction ν of lifetime earnings of which the government deprives workers. Remarkably, the exact details of the tax and social security system during the prime-age period do not enter here at all.

Before concluding that an optimal tax policy would set the tax distortion for older workers to zero, recall our presumption about the implicit tax and social security system, namely, that the system is such that workers choose to supply labor P early, i.e., while they are prime aged. Purely age-related tax relief proposals targeted to older workers are subject to the objection that they would just inspire workers to postpone labor market participation in order to enjoy more favorable tax treatment over their lives. That would surely happen in the formal framework of this paper with its ample room for workers to engage in labor supply arbitrage over their life cycle.

[29] We rule out $\nu = 1$ so that workers consume something in an equilibrium so that their lifetime utilities that include the logarithm of consumption remain well-defined.

However, features omitted from our model could limit the extensive intertemporal substitution underlying the caveat made in the previous paragraph. Factors that should make workers reluctant strategically to postpone their lifetime labor supplies are incomplete markets and uncertainties about future health status and how various aspects of individual labor careers will play out. It is not only impediments to borrowing against future labor earnings that explain why young workers enter the labor market. There is also their interest in resolving uncertainties about their destinies in the labor market. Similarly, established workers are unlikely to put careers on hold and to engage in spells of temporary early retirement. Intermittent interruptions and returns are not good for careers. For these reasons, we still suspect that if the goal is to increase total labor supplied over the life cycle, well designed policies will feature tax and benefit reforms targeted at older workers

7.3 Implications of recent changes in U.S. social security rules

Recently, there have been major changes in the U.S. social security rules. The Full Retirement Age (FRA) is gradually being increased from 65 to 67. In 2000, the earnings test through age 69 for persons who choose to work beyond the FRA was removed. In addition, for us an important change is the gradual increase in the Delayed Retirement Credit (DRC) for someone who reaches the FRA in 2009 and who delays claiming benefits, which tops out at an annualized credit of 8 percent per each year of delay until age 70. Thus, the kink in a worker's budget set associated with the FRA has been smoothed out because, as pointed out by Schulz (2001, p. 142), for there to be no loss in the actuarial value of a worker's lifetime benefits, the benefit level needs to be increased by about 8 percent for each year of delay.

In terms of our analysis, these changes have de facto raised the *effective* official retirement age R to 70. So we should think of 70 as now being the location of the potential corner solution highlighted in part iii) of Proposition 1.[30] This makes it more likely that workers will now plan to retire *before* the raised effective R (i.e., in range i) identified in Proposition 1), in particular, at the value $\tilde{T}(R,\tau) = R^{+}(\tau) = (\phi + 1 - \tau)/B$ with R now being the effective age 70 that has been set by the recent reforms.

In terms of the implied aggregate labor supply elasticities, any reforms that move people from the corner case iii) to the interior case i) of Proposition 1 are very important. For

[30]Our claim that the new rules have de facto raised the official retirement age R to 70 rests on transforming the actual U.S. system in the following way. Think of the U.S. social security system as stipulating 70 to be the full retirement age but allowing for early retirement at what is now actually the FRA with an adjustment to keep the actuarial value of a worker's lifetime benefits unchanged.

as we pointed out in subsection 6.3, the lifetime labor supply elasticity at a case i) interior solution becomes almost as large as found with Prescott's (2002) labor tax with tax receipts handed back lump sum. Reforms that move significant measures of people from the corner to the interior would substantially raise aggregate labor supply elasticities. We discuss some implications of such changes in the next section.

7.4 Confronting observations about career lengths

When combined with our section 5 analysis of unanticipated earnings shocks, events that move substantial measures of workers between the interior solution of section 4 and the corner solution of section 6 open up variations in career length around the official retirement age. In particular, the disutility of work B can take such a value that while workers had originally planned to retire at an official retirement age, sufficiently large unforeseen earnings shocks can impel workers to reoptimize their planned career lengths, given their life cycle savings accumulated up to that point. It is intriguing to ask whether, by pushing workers on and off the corner solution associated with the official social security retirement age, an interplay among these forces can help explain the increased incidence of early retirement observed in the last few decades (see e.g. the country studies compiled by Gruber and Wise (2004)). More generally, nonemployment has risen especially among older workers in Europe – a key feature of the trans-Atlantic employment puzzle posed by Krugman (1987, p. 68): "no strong case exists that Europe's welfare states were much more extensive or intrusive in the 1970s than in the 1960s, and no case at all exists that there was more interference in markets in the 1980s than in the 1970s. Why did a social system that seemed to work extremely well in the 1960s work increasingly badly thereafter?" To address these observations, we suspect that it will be useful to combine the forces isolated in this paper with insights from empirical studies that have documented increased variability of both transitory and permanent components of individual workers' earnings (see e.g. the literature review of Katz and Autor (1999)), and thereby to extend and modify our earlier efforts to solve the problem posed by Krugman (Ljungqvist and Sargent (1998, 2008b)) by incorporating a more serious model of career length.[31]

[31] Kitao et al. (2008) pursue an analysis along those lines.

8 Macro-labor shifts paradigms

Important revisions that occurred between two versions of Prescott's (2005, 2006b) Nobel lecture herald a shift in the labor market paradigm to be used for aggregate analysis (see footnote 4). Between the 2005 and 2006 versions, Prescott abandoned his exclusive focus on the Rogerson (1988) aggregation theory based on employment lotteries and complete insurance markets, and embraced the time-averaging setup of Ljungqvist and Sargent (2006) that Prescott had discussed at an intervening NBER Macroeconomics Annual meeting. That labor market paradigm change has important ramifications. Instead of the vision of a representative family that uses employment lotteries to choose a fraction of its members to send to work, the time averaging model focuses attention on individual workers' choices of career lengths. This paradigm shift resolves a long-standing dispute between advocates and critics of an aggregation theory founded on Rogerson's employment lotteries (see footnote 1) in favor of the critics.

But the emerging consensus about the proper object of inquiry – individual workers' lifetime labor supply – leaves unresolved the still contentious issue of the magnitude of aggregate labor supply elasticities. Are labor supply elasticities as low as suggested by most microeconometric studies, or as high as Prescott uses to explain aggregate labor supply fluctuations over the business cycle and again when he attributes low European employment to high labor taxes? As shown by Ljungqvist and Sargent (2006), advocates of a high labor supply elasticity can now replace their earlier argument that had rested on an interior solution to the fraction of the population sent to work in an employment-lottery model with one that instead rests on an interior solution to individual workers' choice of the fraction of a lifetime devoted to work in a time-averaging model.[32]

Macroeconomists have long studied life-cycle dynamics in the overlapping generations framework. For example, Auerbach and Kotlikoff (1987) offered an early quantitative analysis with long-lived agents. Like Auerbach and Kotlikoff, subsequent studies have typically

[32]Rogerson and Wallenius' (2009) notions of "micro" and "macro" elasticities can be understood in terms of workers' choices of hours worked while employed and total lifetime labor supply (including variations in career lengths), respectively. In Ljungqvist and Sargent's (2006) original time-averaging model with only an extensive and no intensive margin, it is trivially true that in the Rogerson and Wallenius sense the 'micro' elasticity' is small (it is equal to zero), and that the 'macro' elasticity is large whenever workers adjust their lifetime labor supply by varying career lengths. Furthermore, the 'macro' elasticity is as large as in a corresponding employment-lottery model. The Ljungqvist and Sargent analysis has been repeated and reaffirmed by Prescott et al. (2009) and Rogerson and Wallenius (2009) who find that low 'micro' elasticities are consistent with high 'macro' elasticities in a model where individuals effectively face a fixed cost in their labor supply.

not considered indivisible labor from the time-averaging perspective, but have rather either modeled a period's labor supply as a continuous choice variable, in which case labor income varies linearly with labor supply, or formulated environments in which the preferred lifetime labor supply is at a corner solution at full employment. A standard assumption has been to impose a mandatory retirement age.[33] A possible justification for that assumption is that government provided social security programs have compelled workers to retire at an official retirement age. Substantial microeconomic analysis has been devoted to understanding this issue, a prominent example being the study of Rust and Phelan (1997) (see footnote 27). While recent microeconomic models of life-cycle dynamics have enriched the environment in various ways, for example, French (2005) and Low et al. (2009), the focus remains on how government welfare programs interact with changes in agents' earnings potential induced, for example, by deteriorating health conditions or human capital loss associated with permanent layoffs, to shorten agents' labor market careers. In contrast, earlier advocates of the employment-lottery framework might seek to unleash the high labor supply elasticity in a time-averaging framework when most people are modeled to be at an interior solution of lifetime supply regardless of any shocks, and therefore poised to alter their life-time labor supplies sensitively in response to changes in the marginal tax rate.

Because the macro-labor paradigm shift from the employment lotteries to the time-averaging model means that both sides of the big-versus-low labor supply elasticity debate now focus on the same object of inquiry, namely, *individual* workers' lifetime labor supplies, we are optimistic about the further progress that will emerge from Browning et al.'s (1999) two-way street between macro and micro.[34]

[33] For recent examples, see Castaneda et al. (2003), Guvenen (2007), Heckman et al. (1998), and Storesletten et al. (2004).

[34] Browning et al. (1999, p. 625): "While dynamic general equilibrium models may suggest new directions for empirical macroeconomic research, it is essential to build the dynamic economic models so that the formal incorporation of microeconomic evidence is more than an afterthought. Macroeconomic theory will be enriched by learning from many of the lessons from modern empirical research in microeconomics. At the same time, microeconomics will be enriched by conducting research within the paradigm of modern dynamic general equilibrium theory, which provides a framework for interpretation and synthesis of the micro evidence across studies."

A Equivalence between employment lotteries and time averaging?

Ljungqvist and Sargent (2006) found that in models with indivisible labor, a high disutility of labor is the source of a high aggregate labor supply elasticity, not the Rogerson aggregation theory based on employment lotteries and complete markets. The time-averaging model with indivisible labor and a high disutility of labor yields a high aggregate labor supply elasticity for a variety of specifications, including ones in which experience affects earnings.

But an *exact* equivalence of aggregate outcomes under individual time-averaging, on the one hand, and employment lotteries with complete markets, on the other hand, hinges on work experience not affecting earnings. Ljungqvist and Sargent (2006, sections 3.5, 3.6) analyze an increasing experience-earnings profile that is a step function with two flat spots and show that the equivalence between the lotteries and time-averaging models breaks down. It also break down for the specification that we have adopted in this paper. An increasing earnings-experience profile creates a nonconvexity over careers and allows a representative family to achieve aggregate allocations with employment lotteries that individuals cannot attain by time averaging.

Thus, consider a representative family consisting of a continuum $j \in [0,1]$ of ex ante identical workers like those in section 3. The family chooses a consumption and employment allocation $c_t^j \geq 0$, $n_t^j \in \{0,1\}$ to maximize

$$\int_0^1 \int_0^1 \left[\log(c_t^j) - Bn_t^j\right] dt\, dj \tag{34}$$

subject to

$$\int_0^1 \int_0^1 \left[w_t^j n_t^j - c_t^j\right] dt\, dj \geq 0\,, \tag{35}$$

where w_t^j is the potential earnings of worker j at time t which depends on her past work experience, as described in (2).

As in Ljungqvist and Sargent (2006, section 3.6), the family solves this problem by setting $c_t^j = \bar{c}$ for all j, $t \in [0,1]$ and by administering a lifetime employment lottery once and for all before time 0 that assigns a fraction $N \in [0,1]$ of people to work always ($n_t^j = 1$ for all $t \in [0,1]$ for these unlucky people) and a fraction $1-N$ always to enjoy leisure ($n_t^j = 0$ for all $t \in [0,1]$ for these lucky ones). An individual who works throughout her lifetime generates present-value labor income equal to $We(1;\phi)$, as defined in (3). Thus, the family's optimal labor supply that solves

$$\max_{N\in[0,1]}\Big\{\log[N\,We(1;\phi)] - BN\Big\}, \tag{36}$$

is $\bar{N} = \min\{B^{-1}, 1\}$. Hence, members of the representative family on average work less than individuals who are left to 'time average', as characterized by (5).[35] The latter individuals confront a difficult choice between enjoying leisure and earning additional labor income at the peak of their lifetime earnings potential. This tension is not experienced by the individuals who follow the instructions of the family planner who uses lotteries to convexify the indivisibility brought by careers. Of course, in the special ($\phi = 0$) case when work experience does not affect earnings, the aggregate labor supplies are exactly the same across a Rogerson (1988) employment-lottery model and a Ljungqvist and Sargent (2006) time-averaging model, and people enjoy the same expected lifetime utilities.

We can concisely summarize the message of this appendix by comparing the responses of aggregate time spent employed to labor tax rate τ for the employment-lottery model,[36]

$$N^{\star}(\tau) = \min\left\{\frac{1-\tau}{B},\ 1\right\}, \tag{37}$$

and for the time-averaging model in (19). As noted above, individuals in the time-averaging model choose a longer career length than the average lifetime labor supply in the employment lottery model, at an interior solution. Therefore, if the equilibria without taxation are characterized by a corner solution, e.g. due to a binding official retirement age, successive increases in taxation will first reduce employment in the economy with employment lotteries while the labor supply in the economy with time averaging is more robust. However, when taxes are so high that the response of the individuals in the time-averaging model is to shorten their careers, the elasticity of aggregate labor supply with respect to the net-of-tax

[35] Ljungqvist and Sargent (2006, sections 3.5, 3.6) obtain a similar outcome in their model with an experience-earnings profile that has two flat spots. With time-averaging, those individuals who work enough to lift themselves beyond the lower flat part of the experience-earnings profile devote a fraction of their lifetimes to work that is higher than is the fraction of people working in the employment-lottery model. But for someone in the time-averaging model who chooses to work sufficiently little that she stays on the first flat segment of the experience-earnings profile, the optimal fraction of her lifetime devoted to work equals the fraction of people who work in the employment-lottery model. The latter outcome is consistent with the analysis here in the following sense. Under a flat experience-earnings profile, workers who 'time average' choose the same life-time labor supply as the average work in an employment-lottery model. For the employment-lottery model, equation (36) shows that the representative family chooses a fraction of family members who work that does not depend on whether the experience-earnings profile slopes upward.

[36] See Ljungqvist and Sargent (2006, sections 4.2, 4.3) for the same exercise in their model with an experience-earnings profile that has two flat spots.

rate, $[(1-\tau)/L]\,\partial L/\partial(1-\tau)$, is equal to one in both the employment-lottery ($L = N$) and time-averaging ($L = T$) models. So, yes, the exact equivalence between the models breaks down, but nevertheless with a high disutility of labor like those calibrated in the real business cycle literature, a high labor supply elasticity can still come through in both frameworks.

B Generalization to power utility functions

King et al. (1988) show that members of the following class of utility functions are consistent with balanced growth,

$$u(c_t, 1-n_t) = \frac{c_t^{1-\gamma}}{1-\gamma} v(1-n_t) \tag{38}$$

for $0 < \gamma < 1$ and $\gamma > 1$, while for $\gamma = 1$,

$$u(c_t, 1-n_t) = \log(c_t) + v(1-n_t), \tag{39}$$

where c_t and n_t are consumption and labor supply at time t, respectively. The total time endowment is normalized to one, so $1-n_t$ is leisure at time t. In the multiplicatively separable case of (38), the function $v(\cdot)$ is (i) increasing and concave if $\gamma < 1$ and (ii) decreasing and convex if $\gamma > 1$; with an additional condition on the second derivative of $v(\cdot)$ to assure overall concavity of $u(\cdot)$ (see King et al. (1988, p. 202)). In the additively separable case of (39), all that we require is that $v(\cdot)$ is increasing and concave.

Under our assumption of indivisible labor, the precise curvature of $v(\cdot)$ is not an issue because we evaluate the function only at two points, $n_t \in \{0, 1\}$. Hence, we can normalize $v(1) = 1$ and let $v(0) = B$, so that the pertinent generalization of our worker's preferences from section 3 is

$$\int_0^1 \left[\frac{c_t^{1-\gamma}}{1-\gamma} \max\{1-n_t,\, Bn_t\} \right] dt, \tag{40}$$

where for $0 < \gamma < 1$ ($\gamma > 1$), we require $0 < B < 1$ ($B > 1$) in order to satisfy the above conditions that make utility decrease in labor supply. For $\gamma = 1$, the worker's lifetime utility function is given by (1) (under the normalization $v(1) = 0$).

Since the subjective discount rate equals the market interest rate, the optimal consumption plan prescribes constant consumption when working, $c_t = \bar{c}$ when $n_t = 1$, and also when not working, but at a different level, $c_t = c$ when $n_t = 0$; and the marginal utilities of consumption should be the same across those two states so that

$$\bar{c}^{-\gamma} B = c^{-\gamma} \qquad \Longrightarrow \qquad \bar{c} = c\, B^{1/\gamma}. \tag{41}$$

The consumption plan must also satisfy the worker's present value budget constraint,

$$W\, e(T;\phi) = T\bar{c} + (1-T)c,$$

where $W\, e(T;\phi)$ is the present value of labor income, as defined in (3). Using (41), the present value budget constraint can be rearranged to become

$$c = \frac{W\, e(T;\phi)}{T\, B^{1/\gamma} + 1 - T}\,. \tag{42}$$

Using our characterization of an optimal consumption plan, i.e., using (41) and (42), we can express lifetime utility (40) in terms of the career length T,

$$T\, \frac{\bar{c}^{1-\gamma} B}{1-\gamma} + (1-T)\, \frac{c^{1-\gamma}}{1-\gamma} = \frac{\big[W\, e(T;\phi)\big]^{1-\gamma}}{1-\gamma}\, \big(T\, B^{1/\gamma} + 1 - T\big)^{\gamma}. \tag{43}$$

The worker maximizes lifetime utility with respect to $T \in [0,1]$. At an interior solution, the optimal career length is determined by the first-order condition at equality,

$$\bar{\bar{T}}(\phi) = \frac{(1-\gamma)(\phi+1)}{\big[(1-\gamma)(\phi+1)+\gamma\big]\big(1-B^{1/\gamma}\big)}\,, \tag{44}$$

with the second order condition calculated to be the negative of the inverse of $\bar{\bar{T}}(\phi)$. Hence, if there exists an interior solution, $\bar{\bar{T}}(\phi) \in (0,1)$, the second-order condition is trivially satisfied, $-1/\bar{\bar{T}}(\phi) < 0$.

At an interior solution (44), the optimal career length does not depend on the level parameter W but increases in the earnings-profile curvature parameter ϕ:

$$\frac{\partial \bar{\bar{T}}(\phi)}{\partial \phi} = \frac{(1-\gamma)\big(1-B^{1/\gamma}\big)\gamma}{\big[(1-\gamma)(\phi+1)+\gamma\big]^2\big(1-B^{1/\gamma}\big)^2} > 0\,. \tag{45}$$

where the strict inequality follows from the above parameter restrictions, i.e., if $0 < \gamma < 1$ $(\gamma > 1)$, then $0 < B < 1$ $(B > 1)$.

C A reinterpretation of Eckstein and Wolpin (1989)

As discussed in section 4.1, if we resort to a misspecification analysis, we can interpret the simulation results of Eckstein and Wolpin in terms of our time averaging model. In particular, consider the alternative preference specification $\int_0^1 [c_t - \tilde{B}_t n_t]dt$ where $\tilde{B}_t = \tilde{b}h_t$, $\tilde{b} > 0$. A worker with these preferences would also be indifferent about the timing of her labor supply. Therefore, we continue to assume that the worker frontloads her work at the beginning of time, and the lifetime utility of consumption and lifetime disutility of labor for someone who works a fraction T of her lifetime are $W\, e(T;\phi)$ and $\int_0^T \tilde{b}t\, dt = \tilde{b}T^2/2$, respectively. Thus, the worker's optimal lifetime labor supply is the solution to

$$\max_{T\in[0,1]} \Big\{ W\, e(T;\phi) - \tilde{b}\frac{T^2}{2} \Big\}, \tag{46}$$

with a first-order condition at an interior solution,

$$WT^{\phi} - \tilde{b}T = 0, \tag{47}$$

and a second-order condition,

$$W\phi T^{\phi-1} - \tilde{b} < 0. \tag{48}$$

By substituting the interior solution $\bar{T}(\phi)$ from (5) into (47), we can solve for the parameter value $\tilde{b}^{\star} = W((\phi+1)/B)^{\phi-1}$, at which (47) would result in the same choice of labor supply as for our time-averaging model. Furthermore, by plugging the expressions for $\bar{T}(\phi)$ and $\tilde{b}^{\star}$ into (48), we find that the second-order condition reduces to $\phi - 1 < 0$, which holds for our assumptions (except for the borderline a linear specification with $\phi = 1$). Hence, we have shown that the optimal labor supply of our time-averaging model can be reproduced in an alternative model where utility is linear in consumption and the disutility of work increases with past work experience. In the alternative model with the market interest rate being equal to the worker's subjective discount rate, the worker would not regret that a credit market is absent.

D Proof of Proposition 1

The formulation of optimization problem (23) is predicated on workers preferring to supply their labor before rather than after the official retirement age, i.e., $\int_0^R n_t dt = \min\{\tilde{T},\, R\}$. We proceed as if this is true in the following equilibrium characterization, and then afterwards verify its correctness under parameter restriction (20) and $R \in (0.5, 1)$.

i) If $R \geq R^{+}(\tau)$, then $\tilde{T}(R, \tau) = R^{+}(\tau)$.

   For any $R \in (0.5, 1)$ that satisfies $R \geq R^{+}(\tau)$, the constant career length $\tilde{T} = R^{+}(\tau)$ is an equilibrium since it satisfies both the government budget constraint (21) and a worker's first-order condition (24) for the case with $T \leq R$, as summarized in equilibrium expression (25). It remains just to show that there cannot exist another equilibrium in which workers choose a career length longer than R, i.e., we will show that equilibrium expression (27) cannot hold when $R \geq R^{+}(\tau)$. For any $R \in (0.5, 1)$ that satisfies $R \geq R^{+}(\tau)$, it follows from the fact that (25) holds with equality that the right-hand side of (27), evaluated at $\tilde{T} = R$, must fall below the left-hand side of (27). Next, since the left-hand side of (27) is strictly increasing in $\tilde{T}$, while the right-hand side is decreasing in $\tilde{T}$ (see footnote 25), we can rule out the existence of any $\tilde{T} > R$ at which equilibrium expression (27) would hold.

ii) If $R \leq R^{-}(\tau)$, then $\tilde{T}(R, \tau) \in [R^{-}(\tau), R^{+}(\tau))$, $\tilde{T}(R^{-}(\tau), \tau) = R^{-}(\tau)$ and $\partial \tilde{T}(R, \tau)/\partial R < 0$.

   Since $R^{-}(\tau) \in (0, 1)$ as established in section 6.2, it follows that if there is any $R \in (0.5, 1)$ that satisfies $R \leq R^{-}(\tau)$, it must be that $R^{-}(\tau) \in (0.5, 1)$. Moreover, since $R^{-}(\tau)$ is the fixed point of (29), it follows that equilibrium expression (27) for an interior solution with $\tilde{T} \geq R$ holds for $\tilde{T} = R = R^{-}(\tau)$, i.e., $\tilde{T}\left(R^{-}(\tau), \tau\right) = R^{-}(\tau)$. Next, since the right-hand side of (27) is strictly decreasing in R, it follows that for $R < R^{-}(\tau)$, the right-hand side of (27) lies strictly above the left-hand side of (27) when evaluated at $\tilde{T} = R^{-}(\tau)$. Together with the fact that the right-hand side of (27) is strictly decreasing in $\tilde{T}$ (see footnote 25) while the left-hand side of (27) is strictly increasing, it follows that, for $R \in (0.5,\, R^{-}(\tau))$, the solution to (27) is unique and has $\tilde{T} > R^{-}(\tau)$. Note that the existence of an interior solution $\tilde{T} < 1$ is ensured since the right-hand side of (27) goes to minus infinity when $\tilde{T} \to 1$.

   To establish the upper bound $\tilde{T} < R^{+}(\tau)$, we show that $R^{+}(\tau)$ is strictly greater than the right-hand side of (27) for all $\tilde{T} \geq R \in (0.5, 1)$, i.e.,

$$\frac{\phi + 1 - \tau}{B} > \frac{\phi + 1 - \tau \dfrac{R}{1 - \tilde{T}} \left(\dfrac{R}{\tilde{T}}\right)^{\phi}}{B}, \tag{49}$$

which can be simplified to

$$R^{\phi+1} > (1 - \tilde{T})\tilde{T}^{\phi}. \tag{50}$$

Note that for the inadmissible values $R = 0.5$ and $\tilde{T} = 0.5$, the left- and right-hand side of (50) are equal. Next, since the left-hand side is strictly increasing in R while the right-hand side is strictly decreasing in $\tilde{T}$,[37] it follows that inequality (50) holds for all $\tilde{T} \geq R \in (0.5, 1)$.

Given the upper bound $R^{+}(\tau) > \tilde{T}$, it also follows that there cannot exist another equilibrium in which workers choose a career length shorter than R, i.e., equilibrium expression (25) cannot hold when $R \leq R^{-}(\tau)$. Specifically, for any $R \in (0.5, 1)$ that satisfies $R \leq R^{-}(\tau)$, we have shown the existence of an equilibrium with $\tilde{T} \geq R$ with an upper bound $R^{+}(\tau) > \tilde{T}$, and therefore, $R^{+}(\tau) > R$. The latter inequality rules out the existence of another equilibrium with career length shorter than R, because as shown in case i) above, the equilibrium career length in such an equilibrium would be $R^{+}(\tau)$ which now lies *above* rather than below R, i.e., a contradiction.

To establish that $\partial\tilde{T}(R,\tau)/\partial R < 0$, we form an implicit function for (27),

$$F(\tilde{T}, R) \equiv \frac{\phi + 1 - \tau \dfrac{R}{1-\tilde{T}} \left(\dfrac{R}{\tilde{T}}\right)^{\phi}}{B} - \tilde{T} = 0, \tag{52}$$

and use the implicit function theorem,

$$\frac{\partial\tilde{T}}{\partial R} = -\frac{\partial F(\tilde{T},R)/\partial R}{\partial F(\tilde{T},R)/\partial\tilde{T}} = -\frac{\tau(\phi+1)R^{\phi}}{\tau R^{\phi+1}\left\{\dfrac{1}{1-\tilde{T}} - \dfrac{\phi}{\tilde{T}}\right\} + B(1-\tilde{T})\tilde{T}^{\phi}} < 0, \tag{53}$$

where the strict inequality is assured by the nonnegativity of the expression in braces because $\phi \in [0, 1]$ and $\tilde{T} \in [R, 1)$, where $R \in (0.5, 1)$.

iii) Otherwise, $\tilde{T}(R, \tau) = R$.

For any $R \in (0.5, 1)$ that satisfies neither $R \geq R^{+}(\tau)$ nor $R \leq R^{-}(\tau)$, the equilibrium career length $\tilde{T}$ is characterized neither by expression (25) for an an interior solution

[37] The derivative of the right-hand side of (50) with respect to $\tilde{T}$ is

$$-\tilde{T}^{\phi-1}[\tilde{T} - (1-\tilde{T})\phi] < 0, \tag{51}$$

where the strict inequality follows from $\phi \in [0, 1]$ and $\tilde{T} \in [R, 1)$, where $R \in (0.5, 1)$.

with respect to $\tilde{T} \leq R$, nor by expression (27) for an interior solution with respect to $\tilde{T} \geq R$. Thus, the equilibrium career length is at a corner solution with $\tilde{T} = R$.

The range of official retirement ages for which the equilibrium career length is at a corner solution, is given by $R \in (\max\{0.5,\, R^{-}(\tau)\},\, \min\{1,\, \max\{0.5,\, R^{+}(\tau)\}\})$. This range reflects the fact, as shown above, that the equilibrium sets for case i) and ii) are disjoint in the policy space $(R \in (0.5, 1),\, \tau \in (0, 1))$. In particular, if $R^{-}(\tau) \in (0.5, 1)$, it follows from (25) and (29) that $R^{-}(\tau) < R^{+}(\tau)$ (see footnote 26).

Returning to the assertion underlying the formulation of optimization problem (23), namely, that workers prefer to supply their labor *before* rather than *after* the official retirement age, we now verify its correctness for the three cases above. In particular, we show that an infinitesimal shift of labor supply from before to after the official retirement age R reduces the present value of a worker's disposable income. (Note that we hold total labor supply constant in these perturbations so the disutility of work remains unchanged.)

i) Suppose that $\tilde{T}(R, \tau) < R$, when the worker under the solution above pays total taxes equal to $\tau W e(\tilde{T}; \phi)$ and collect total social security benefits equal to $(1-R)\rho W e(\tilde{T}; \phi)/\tilde{T}$. After an infinitesimal shift of labor supply from before to after the official retirement age, the worker saves on taxes at the rate $\tau W[\partial e(T; \phi)/\partial T]$ for $T = \tilde{T}$, but loses both on a shorter time of collecting social security, at the rate $-[\partial(1-T)/\partial T]\rho W e(\tilde{T}; \phi)/\tilde{T}$ for $T = R$, and on the lower benefit level caused by lower average labor earnings prior to the official retirement age, at the rate $(1-R)\rho W[\partial e(T; \phi)T^{-1}/\partial T]$ for $T = \tilde{T}$. The worker loses from such a shift in labor supply if the implied savings on taxes fall short of the implied losses on social security collection,

$$\tau W \tilde{T}^{\phi} < \rho W \frac{\tilde{T}^{\phi}}{\phi + 1} + (1 - R)\rho W \frac{\phi \tilde{T}^{\phi - 1}}{\phi + 1}. \tag{54}$$

After invoking (22), i.e., $\rho = \tau\tilde{T}/(1-R)$, this condition simplifies to $(1-R) < \tilde{T}$ which is indeed true for equilibrium career length (25) under parameter restriction (20).

ii) Suppose that $\tilde{T}(R, \tau) > R$, when the worker under the solution above pays total taxes equal to $\tau W e(R; \phi)$ and collect total social security benefits equal to $(1-\tilde{T})\rho W e(R; \phi)/R$. The condition corresponding to (54) becomes

$$\tau W R^{\phi} < \rho W \frac{R^{\phi}}{\phi + 1} + (1 - \tilde{T})\rho W \frac{\phi R^{\phi - 1}}{\phi + 1}. \tag{55}$$

After invoking (22), i.e., $\rho = \tau R/(1-\tilde{T})$, this condition simplifies to $(1-\tilde{T}) < R$ which is indeed true for equilibrium career length $\tilde{T}(R,\tau) > R$ and $R \in (0.5, 1)$.

iii) Suppose that $\tilde{T}(R,\tau) = R$ and hence, the calculation in (54) still applies. But now it follows immediately that condition $(1-R) < \tilde{T}$ is true for equilibrium career length $\tilde{T}(R,\tau) = R$ and $R \in (0.5, 1)$.

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