Non Neutrality of Money in Dispersion: Hume Revisited

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Abstract

This paper seeks to explore non neutrality of money in the dispersion of transition process following an unanticipated money injection. It examines the responses of the output and nominal price to shocks. We show that a certain class of money injection schemes will induce quantitatively significant and persistent response in output, sluggish price adjustment, and a short-run negatively-sloped Phillips curve. The short-run trade-off between output and inflation is not exploitable in the long run.

JEL Classification Number: C73, D82, E40

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1 Introduction

From the classical essay *Of Money,* Hume [10] offered a brilliant thought regarding the short-run non neutrality of money following a money injection.

> [T]hough the high price of commodities be a necessary consequence of the encrease of gold and silver, yet it follows not immediately upon that encrease; but some time is required before the money circulates through the whole state, and makes its effect be felt on all ranks of people. ...[I]t is only...between the acquisition of money and rise of prices, that the encreasing quantity of gold and silver is favourable to industry. When any quantity of money is imported into a nation, it is not at first dispersed into many hands; but is confined to the coffers of a few persons, who immediately seek to employ it to advantage.

In his Nobel lecture, Lucas [13] reviewed a long historical line of thinking on the short-run non neutrality that starts from Hume and ends with his own prize-winning work [11]. As stressed in [13], any coherent theory on the short-run non neutrality must account for questions in the following line.

> If everyone understands that prices will ultimately increase in proportion to the increase in money, what force stops this from happening right away?

The key that prevents instantaneous price adjustment in [11] is that the public has imperfect information of money injection. As a popular alternative approach, the short-run non neutrality follows if it is assumed that old prices cannot change immediately, as in the sticky-price models.[1]

Here we explore a third approach to non neutrality that is built on a simple logic—the initial distributional effect of money injection leads to a non neutral dispersion process. Specifically, there exists the initial distributional effect when the post-injection distribution of money differs from the pre-injection distribution (after adjusted by the total quantity of money). Given this effect, it may take time for the distribution of money to disperse to or regain its pre-injection shape; money is non neutral in the dispersion/transition process and it is neutral when the pre-injection shape is attained.

This simple logic may have been in Hume's mind when he talked about the dispersion and circulation of injected money across the state. In modern time this logic has been recognized at least from Friedman.[2] This logic, however, has received not much attention from the literature. This might be attributed to that, as recognized by Friedman and echoed by Lucas, it is analytically challenging to say much about a dispersion/transition process.[3]

[1]These models appeal to the costs to adjust prices, typically referred to as menu costs due to Mankiw [8], to justify the assumption. For example, it is standard to motivate the pricing schemes of Taylor [16] and Calvo [5] by a large cost for one to change a price outside a preset slot. Menu costs have more broad interpretation than the physical costs (e.g., the amount of ink) to reset a price on a menu (cf. Ball and Mankiw [1]).

[2]In the context of non evenly distributed helicopter drop of money, Friedman [9] notes that "The existence of the initial distributional effect has, however, one substantive implication: the transition can no longer, even as a conceptual possibility, be instantaneous, since it involves more than a mere bidding up of prices."

[3]When discussing money injection in a cash-in-advance economy, Lucas [12] observes that "This seems to me to mirror exactly Friedman's statement, in a very similar context, that while "it is easy to see what the final position [following a change in M] will be ... it is much harder to say anything about the transition."

Indeed, it is far from evident that non neutrality in dispersion is even relevant for the observed movement patterns of price, output, and employment.

By way of numerical methods, we test relevance of the above logic in an off-the-shelf matching model. This model provides a natural environment in which "some time is required before the money circulates through the whole state, and makes its effect be felt on all ranks of people." The model is in a stationary equilibrium before an unanticipated money injection. We test a few different money injection schemes and find that a certain way of money injection can induce quantitatively significant and persistent response in output and for some parameter values the output response follows a hump-shaped pattern. Following the injection, the price adjustment is sluggish, there is a short-run negatively-sloped Phillips curve. The short-run trade-off between output and inflation is not exploitable in general. The one-shot money injection increases aggregate output by exerting a positive force on the extensive margin of aggregate output (i.e., change the distribution of different buyer-seller pairs by increasing the numbers of high-output pairs and decreasing the numbers of low-output pairs). However, once the authority institutionalizes the injection, the permanently increased inflation will lower the value of holding money, and hence the realized output in every buyer-seller pair. When inflation rate is high enough, such negative effect on the intensive margin outweighs the positive effect on the extensive margin, and the aggregate output fall as a result, explaining the breakdown of the Phillips curve.

To our knowledge, Williamson [18, 19] gives the only study on properties of non neutrality in dispersion. To get around analytical difficulty, he introduces a special large household structure that consists of both selfish and unselfish household members, and a special market structure that ties one's preference over goods to whether he receives a money transfer. While some of our findings are consistent with his, some are not. For example, we find a short-run negatively-sloped Phillips curve and a more significant response in output; he does not.

2 The benchmark model

2.1 Environment

The basic model we set up here is the standard matching model à la Shi [15] and Trejos and Wright [17].

Time is discretely dated as $t \geq 0$, and the horizon is infinite. There are $N \geq 3$ types of infinitely lived agents , as well as N types of nonstorable and divisible consumption goods. The preferences are such that a type n agent consumes only type $n+1$ good and produces only type n good (modulo N). Each agent maximizes his expected utilities with a discount factor $\beta \in (0,1)$. If a type n agent consumes $y_{n+1} \geq 0$ (when he is buyer) and produces $y_n \geq 0$ (when he is a seller), his realized utility in that date is given by $u(y_{n+1}) - c(y_n)$, where the functions u and c satisfy $u', c' > 0$, $u'' < 0$, $c'' \geq 0$, $v(0) = c(0) = 0$, and $u'(0) = \infty$.

In this economy, there exists an intrinsically useless good, which we shall refer to as fiat money. Money is costlessly storable but not perfectly divisible. We normalize its smallest unit as unity. Each agent is allowed to hold no more than B unit of money, and B is sufficiently large. The initial total stock of money is M.

At each date, each agent meets another agent at random. So he will meet someone able to produce what he want with probability $1/N$, or someone willing to consume what he produce with probability $1/N$, but not both. During each pairwise meeting, one can only observe each other's money holdings and specialization types, but not past trading histories, which rules out credits between the two agents. Under such a setting, any production must be accompanied by transferring money from the potential consumer to the potential producer. In particular, we assume that in a pairwise meeting, the potential consumer (which we shall refer to as the buyer) makes a take-it-or-leave-it offer to the producer (seller). We follow Berentsen, Molico and Wright [2] to allow such offers to include lotteries on monetary transfers, so as to mitigate the limitation of the indivisible money and to introduce additional pairwise divisibility[4]. Formally, when a seller with $i \in \{0, 1, \ldots, B-1\}$ units of money meets a buyer with $j \in \{1, 2, \ldots, B\}$ units of money, the trade offer suggested by the buyer is represented by the pair (y_{ij}, σ_{ij}), where

[4]One can assume that the trade offer includes lotteries on goods transfer as well as on money transfer. However, given our preference settings, it is easy to show that only lotteries which are degenerate on output are in the pairwise core.

$y_{ij} \in \mathbb{R}_+$ is the output and the monetary lottery σ_{ij} is a probability measure on $\{0, 1, \ldots, K(i,j)\}$ with $K(i,j) \equiv \min(j, B-i)$.

2.2 Equilibrium

To define equilibrium, let π_t denote a probability measure on $\{0, 1, \ldots, B\}$, with $\pi_t(m)$ representing the fraction of agents with money holding m at the beginning of period t. Let v_t denote a value function on $\{0, 1, \ldots, B\}$, with $v_t(m)$ representing the expected discounted utility for an agent with money holding m at the end of period t.

The trade in a pairwise meeting between a seller with with m^s units of money and a buyer with m^b units of money can be described as follows. Given v_t, the problem for the buyer can be formulated as

$$f\left(m^b, m^s, v_t\right) = \max_{y,\sigma} u(y) + \sum_{d=0}^{K\left(m^s,m^b\right)} \left(\sigma(d)\, v_t\left(m^b - d\right)\right) \tag{1}$$

$$\text{s.t.} \quad \sum_{d=0}^{K\left(m^s,m^b\right)} \left(\sigma(d)\, v_t\left(m^s + d\right)\right) - c(y) \geqslant v_t\left(m^s\right) \tag{2}$$

Denote the solution as $\left(y_{m^s,m^b}, \sigma_{m^s,m^b}\right)$.

Given v_{t+1} and π_{t+1}, v_t satisfies

$$v_t(m) = \frac{N-1}{N}\beta v_{t+1}(m) + \frac{1}{N}\beta \sum_{m_s=0}^{B-1} \pi_{t+1}(m_s)\, f(m, m_s, v_{t+1}) \tag{3}$$

Given π_t, π_{t+1} satisfies

$$\pi_{t+1}(m) = \sum \delta_t(m', m)\, \pi_t(m') \tag{4}$$

where $\delta_t(m', m)$ is the proportion of agents with m' units of money who leave with m after the random matching in period t. Note that $\delta_t(\cdot,\cdot)$ is derived from the solution to 1. Specifically, we have

$$\delta_t(m, m+d) = \frac{1}{N}\sum_{j=1}^{B}\pi_t(j)\,\sigma_{m,j}(d)\,,\text{ for } d \in \{1, 2, ..., B-m\}$$

$$\delta_t(m, m-d) = \frac{1}{N}\sum_{i=0}^{B-1}\pi_t(i)\,\sigma_{i,m}(d)\,,\text{ for } d \in \{1, 2, ...m\}$$

$$\delta_t(m, m) = \frac{N-2}{N} + \frac{1}{N}\sum_{j=1}^{B}\pi_t(j)\,\sigma_{m,j}(0) + \frac{1}{N}\sum_{i=0}^{B-1}\pi_t(i)\,\sigma_{i,m}(0)$$

where σ is the solution to the problem 1.

The relevant definitions of equilibria are now in order.

Definition 1 *Given π_0, a sequence $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ is an equilibrium in the economy if it satisfies* (1)-(4) . *An equilibrium is a monetary equilibrium if $\pi_t(0) < 1$ for some t. A pair (v, π) is a steady state if $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ with $v_t = v$ and $\pi_t = \pi$ for all t is an equilibrium.*

Proposition 1 *(i) For any given π_0 there exists a Definition-1 monetary equilibrium $\{v_t, \pi_{t+1}\}_{t=0}^{\infty}$ such that v_t is concave, all t. (ii) There exists a Definition-1 monetary steady state (v, π) such that v is concave.*

Proof. All proofs are in the appendix. ■

3 Money injection

In this section we introduce money injections into the basic model. At the beginning of each period before the pairwise meetings, the monetary authority may inject outside money into the economy by allowing individual agents to receive some sort of helicopter drop of money. In the numerical analysis to be conducted in the next section, we shall consider several different types of money injection scheme. The first type is the uniform lump-sum injection, where all agents receives same amount of money regardless of his money holding. Specifically, each agent receive x_1 units of money with probability p_1, with $p_1 \in (0, 1]$.

In the second type, we consider non-uniform injection schemes where agents with different money holdings are affected differently by the policy.

In particular, we work with injection schemes in which it is costly for agents to receive the helicopter drop of money, and the cost may be either in the form of disutility cost, or in the form of fiat money. For the former case, we assume that in order to receive the helicopter drop of money, one has to exert an effort that incur a disutility of ξ. Once the effort is spent, the agent will receive x_2 units of money with probability p_2, with $p_2 \in (0, 1]$. Since the marginal value of money is different to agents holding different amount of money, only a set of agents, which is endogenously determined, will choose to receive the money. For the latter case, we assume that one has to pay κ units of money to be eligible for the helicopter drop of money. If an agent pays κ, the helicopter drop of money he will receive is random, with probability p_3 the receiver getting x_3 units of money, and with probability $1 - p_3$ he getting nothing, where $p_3 \in (0, 1)$ If an agent does not pay the cost κ, he will not receive any money. This formulation is to roughly capture the idea that only those connected to financial markets are on the receiving end, and that such connections usually comes at a monetary cost or depends on monetary wealth.

As a way of normalization, we assume that immediately after the injected money is received by the agents, each unit of money in the economy (held by the agents) will automatically disintegrate with probability δ. The value of δ is such that after the disintegration the aggregate money stock just returns to its pre-injection level. Such a drop-disintegrate policy is the indivisible-money equivalent of the policy of injection followed by a proportional deduction of money holding in divisible money models[5], which implies that the government finance the money injection by inflation tax imposed on all money holders.

In our numerical exercises, we first examine the real effect of such expansionary monetary policies when conducted in a one-shot fashion. Specifically, we set the economy in the benchmark-case stationary equilibrium before an unanticipated one-shot money policy takes place at the beginning of period 1. And there will be no more injection in future periods and the environment is the same as before the injection. As a result the economy will converge back to its pre-injection steady state after an initial response in period 1. And the dynamic path of transition will be studied. Next, we consider the long run effect of such monetary policies when they are conducted every period before

[5]See Deviatov and Wallace [7], who first introduces such policy into indivisible money models.

the pairwise meeting. And the resulting steady states will then be compared against the benchmark cases.

4 Numerical analysis

In this section we use numerical methods to analyze the effect of money dispersion process following different schemes of money injections. We first parameterize the model and then proceed to computations.

4.1 Parameterization

To begin with, we set the total money stock $M = 30$, so that the divisibility level in our model is 1/30. Under such a divisibility level, the effect of indivisible money on aggregate variables and money distributions is negligible. For the upper bound on money holding, we find that $B = 70$ works fine and making it larger will not change the result much. The number of specialization types N, is 3. We set the length of per period as a quarter, so we get $\beta = 1/(1 + 0.1)$ which implies an annual discount rate of 4%. In terms of the preferences, we follow the standard money matching literature (such as in [15]) to work with the utility function $u(y) = y^{1-\sigma}$ with $\sigma = 0.4$ and the cost function is $c(y) = y$.[6]

For the part of money injections, we set the relevant parameter that determines the amount of money received by agents as $x_1 = 1$,$x_2 = 1$, $\kappa = 1$ and $x_3 = 2$, i.e., we choose the smallest possible unit available. As for ξ, p_1, p_2, p_3, we will vary their values to match different money growth rates and examine their different effects.

4.2 The benchmark case: steady state

Now we compute the steady state equilibrium of the benchmark model without money injections, as in Definition-1. The algorithm, which is essentially an iteration on the mappings implied by (1)-(4), is described in the appendix. Figure 1 illustrates the distribution of money holdings and value function in

[6] Molico [14], who also studies the money matching model using numerical methods, adopts a different setting of preferences, with a utility function defined on domain $[0, +\infty)$ and a cost function on $[0, \bar{y}]$. In the appendix, we discuss the difference between the two settings and their effect on the results.

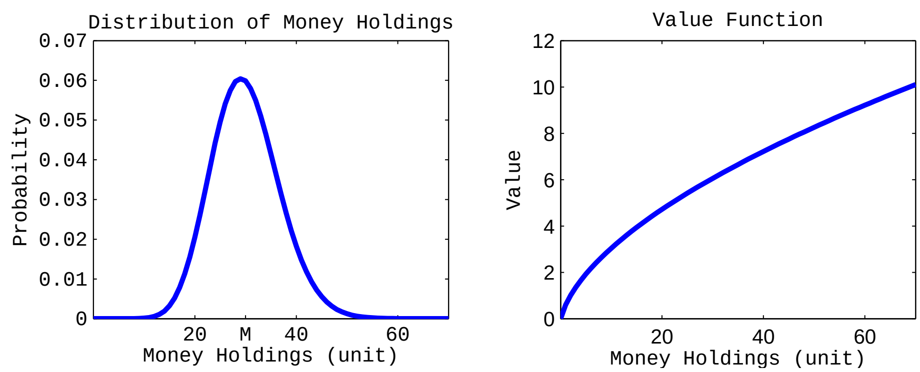

Figure 1: Distribution and Value Function in the Steady State Equilibrium

the steady state. The distribution is non-degenerate and its shape resembles a normal distribution, while the value function displays concavity. Note that both functions show great smoothness despite their discreteness, this is owing to the adoption of lottery trade which introduces additional pairwise divisibility into our model of indivisible money[7]. In Figure 2, we plot y_{m^b,m^s}, the pairwise output between a seller with money m^s and a buyer with m^b. As is expected, y increases in m^b and decreases in m^s. The intuition is that rich seller has lower marginal value of money and hence is less willing to produce, while rich buyer has lower marginal value of money and hence is more willing to spend money which tends to elicit higher output from the seller. When m^b is close to m^s, the margin value for buyer is close to that for seller, therefore the resulting output y_{m^b,m^s} is close to the ex-ante optimal output y^* that satisfies $u'(y^*) = c'(y^*)$[8] . When m^b is very small and m^s very large, the resulting y_{m^b,m^s} is very small and close to zero. However, when m^b is very large and m^s very small, y_{m^b,m^s} remarkably larger than the other areas. As is shown in Figure 2, y_{m^b,m^s} can reach as high as 2.5, approximately nine times y^*. Such a shape implies that given the shape of y_{m^b,m^s}, the aggregate output will be higher when the distribution of money holdings is more dispersed. On the other hand, a more dispersed distribution will lead to a lower ex-ante welfare, which is given by the inner product of the value function

[7]Berentsen, Camera and Waller [3] is the first to observe that by adding randomized monetary (lottery) trade into indivisible money matching model, one can generate aggregate distributions which match those observed in numerically simulated economies with fully divisible money.

[8]With our parameter choice, $y^* = (1-\sigma)^{1/\sigma} = 0.2789$.

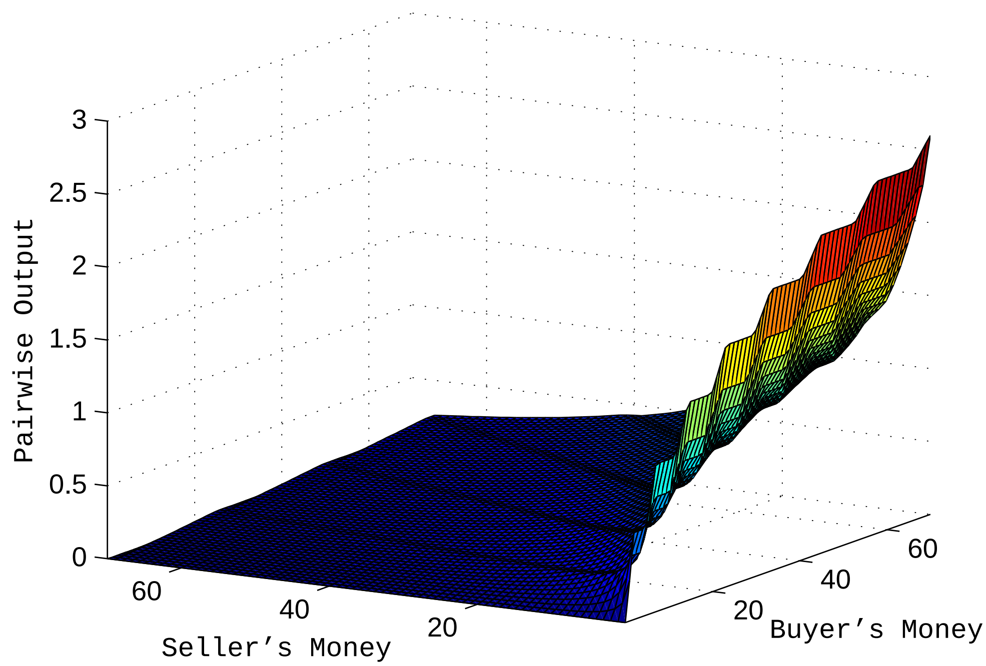

Figure 2: Amount of Goods Traded Between Different Pairs in the Steady State Equilibrium

and the distribution, because of the concavity of the value function showed in Figure1.

4.3 Lump-sum injections

One-shot injection

We let the pre-injection economy be in the above-computed Proposition-1(ii) steady state and the dynamic process following the injection is a Proposition-1(i) equilibrium whose initial distribution π_0 is implied by the specific injection scheme under study. We design our algorithm by assuming that the computed Proposition-1(i) equilibrium converges to the Proposition-1(ii) steady state after the initial response to the shock. For computation purpose, we approximate the process by assuming that the equilibrium path reach the

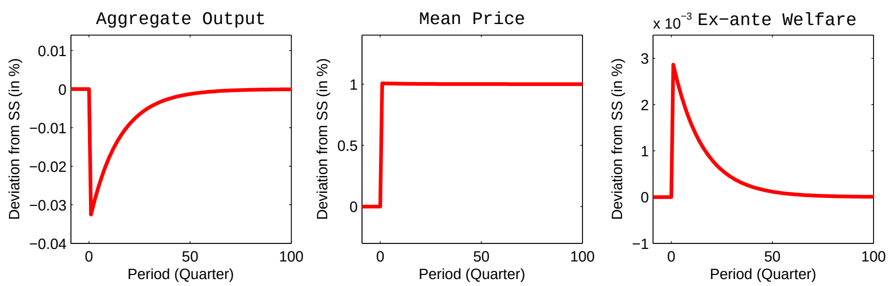

Figure 3: Transition paths after a one-shot lump-sum injection with 1% inflation rate.

steady state after T periods. In our exercises, we find that $T = 200$, or 50 years, is good enough as an approximation; details about the algorithm can be found in the appendix.

Without loss of generality, we work with the case of $p_1 = 0.3$, which correspond to a money growth rate of 1%. Figure 3 shows the responses of aggregate output, mean price and ex-ante welfare to the unanticipated monetary shock. Following the 1% one-shot injection, aggregate output first drops by about 0.03% and then gradually returns to its pre-injection steady state level. The mean price immediately adjust, in roughly the same proportion to the increase of total money stock. The ex-ante welfare, with its i-th element being the inner product of the value function and the distribution at the beginning of period i, displays a initial increase, but with very small magnitude.

To understand, note that because of the one-shot nature of the policy, people's expectation about the future gains in pairwise money-goods trades changes remains almost the same. Hence the forward-looking value function v changes very little throughout the transition process. So does y_{m^b,m^s}, since it depends only on v. On the other hand, the impact of the one-shot lump-sum injection on the distribution are not so trivial. During the injection phase, everyone receives one unit of injected money regardless of their initial money holdings. But when the disintegration occurs, those with more money will lose more. This implies that such a policy essentially serves to decrease the dispersion of the distribution. As a result, the aggregate output experiences an initial decrease while the ex-ante welfare an initial increase, and both with small magnitudes since the change of distribution is also very small. In addition, for both output and welfare, the transition paths exhibits a persis-

Value of p_1	0.25	0.50	0.75	1
Money growth rate	0.83%	1.67%	2.50%	1.33%
Avg payment	229.09%	337.34%	431.11%	514.44%
Aggregate output	90.50%	81.32%	72.64%	64.58%
Ex-ante welfare	97.97%	95.47%	92.49%	89.15%

Table 1: Steady states with lum-sum injections; output and welfare are expressed in relative to those of the benchmark model.

tent pattern. Due to the decentralized nature of trade, it takes many rounds of exchange, hence a long time, for the injected money to disperse across the economy and the distribution to restore its shape before the injection. And the process of dispersion of money is accompanied by the long dynamic processes of output and welfare as we observe in Figure 3. It takes about 50 periods, or quarters, for most of the impacts on output or welfare to die away, as it is required for the distribution to regain its pre-injection shape. Finally, note that because of the difference between the magnitudes of growth rate and of output response, the price exhibits an immediate adjustment, instead of a sluggish one, as is showed in Figure 3.

Permanent injection: steady state

Now we consider the case where the monetary authority conducts the above mentioned lump-sum money injection every period. We compute for the steady states with different values of p_1 and compare them against the steady state of the benchmark model. In Table 1, we illustrate our results. First note that the average money traded in pairwise meetings increases significantly with lump-sum injections. Expecting to receive money in the future, buyers are more willing to part with money when trading. Also note that although one-shot lump-sum injection induces a negative response in output and a positive response in welfare, here when the injection is implemented in every period, both output and welfare are lower than that of the benchmark case with no injection. Moreover, both output and welfare decrease in p. To understand this, we need to delve into further details. In Figure 4, we plot the steady-state distributions and value functions with no injection, $p_1 = 0.25$ and $p_1 = 0.75$.

Unlike the one-shot case, here with lump-sum money injections, higher

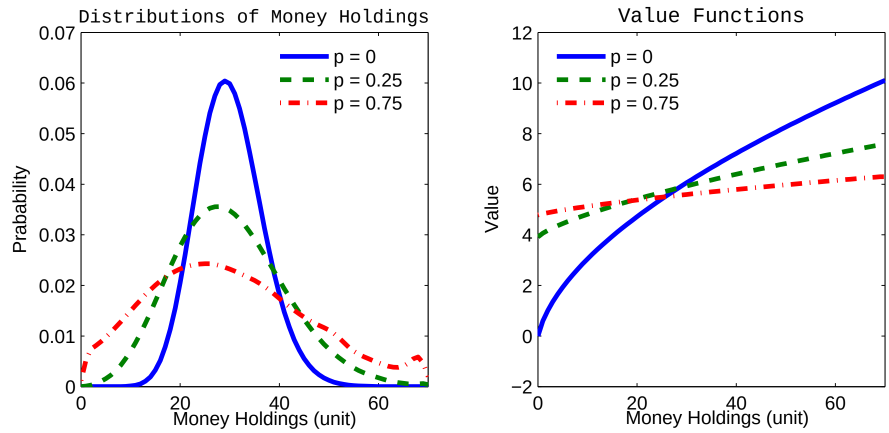

Figure 4: Distributions and Value Functions in the Steady State with Lump Sum Injections.

money growth tends to increase the dispersion of the steady state money holding distribution, because on average buyers are paying more money in pairwise trades. According to the shape of y_{m_b,m_s} as in Figure 2, such increase in dispersion should lead to increase in aggregate output, if other things equal. However, that is not the case here. Higher money growth also changes the shape of value function by making it flatter, which leads to lower marginal value of money. As a result, output in each pairwise meeting will be different, and y_{m_b,m_s} will no longer retain the shape as in the case of benchmark model without injections. Table 2 shows the effect of such changes in value functions on pairwise output. For illustrative purpose, we do so by picking sellers and buyers with money holdings of $M/2$ units, M units, and $2M$ units, and we compare the benchmark case against the inflationary case with $p = 0.5$. For most of seller-buyer pairs, the pairwise output falls significantly. The intuition is straightforward, with money losing value to inflation (or disintegration) every period, sellers in general are now having less incentive to produce goods to get money. The only exceptions are those pairs with poor buyers and rich sellers, whose output increases rather than decreases. Anticipating that he will receive injected money every period while hardly bearing the cost of losing money to inflation (disintegration), poor buyers are hence more willing to spend money than they are without injections. Although rich buyers are also less willing to produce, but it is offset by the increase in poor buyers willingness to spend money. As a result, the

		No Injection					Injection with $p = 0.5$	
		Buyer's Money					Buyer's Money	
		$M/2$	M	$2M$			$M/2$	M	$2M$
Seller's	$M/2$	0.2248	0.4489	0.7770		$M/2$	0.1728	0.2707	0.4249
Money	M	0.1227	0.2437	0.4809	$\Longrightarrow$	M	0.1229	0.2007	0.3159
	$2M$	0.0612	0.1231	0.2550		$2M$	0.0795	0.1288	0.1859

Table 2: Pairwise outputs in steady states without and with lump-sum injections.

pairwise output between these pairs increase. However, note that the output between these pairs is already close to zero, therefore this increase is negligible and dominated by the decrease of output between other pairs. Therefore, despite that lump-sum injection generates more dispersed distribution that favors higher output, the decrease in individual pairwise output is even more significant and eventually lead to a lower total output.

Finally, note that this result is different from Molico [14], who concludes that small inflation tends to increase both output and welfare. We argue that this difference stems from the difference in preference settings, and if we instead adopt his functional forms for consumption utility and production disutility but use a different parameter set. Our result is qualitatively the same. We refer the readers to the appendix for more details.

4.4 Injections costly to receive: disutility cost

In reality, monetary policies rarely resemble the above discussed lump-sum injections scheme under which everyone in the economy is equally affected by the policy. Rather, only a subset of economic agents are at the receiving end of the money injection, as is observed by Hume. In this subsection, we consider a type of non-uniform money injection scheme. Namely, one has to exert an effort that incur a disutility of ξ, in order to receive the helicopter drop of 1 units of money with probability p_2. Given the concave value function, it is straightforward that there exist a threshold $\widetilde{m} \geq 0$ such that for all agents holding $m \leq \widetilde{m}$ units of money will choose to exert the

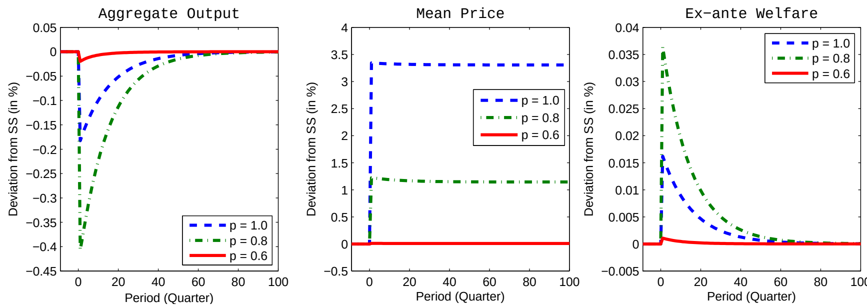

Figure 5: Transition Paths after injection. Agents need to exert some effort to receive the injected money.

effort and receive the money.

One-shot injection

Again we let the economy be in the steady state before the injection, and the algorithm is similar to the one we used with lump-sum injection. Without loss of generality, we fix $\xi = 0.1$ and try three values of p_2: 0.6, 0.8, and 1. We compute the dynamic process for these different values of p_2 separately. When $p_2 = 0.6$, the temporary money growth rate is 0.008% and only 0.40% of the agents exert the effort to receive the money injection. When $p_2 = 0.8$, the money growth rate is 1.144% and 42.90% of the agents choose to receive the money injection. When $p_2 = 1.0$, the money growth rate is 3.306% and 99.18% of the agents choose to receive the money injection. The computed transition paths for output, price and welfare are depicted in Figure 5.

There are two remarks now in order. First, all three variables respond qualitatively the same as in the case with one-shot lump-sum injection. Namely, output decreases and welfare increases, both effects persistent; price adjusts immediately, with slight over-shooting. This is because this injection scheme, by letting a fraction of poorest agents earn the injected money, also serves to reduce the dispersion of money holding distribution. Second, the significance of the response is not monotone in the aggressiveness of the money injection (i.e. value of p_2). Rather, the responses of output and welfare are the most significant in the middle case with $p_2 = 0.8$, i.e. when 42.90% of the agents seek to receive the injection. In response to a 1.144% increase in money stock, output decreases by about 0.4%. This output response is more significant than the other two cases, and more significant than

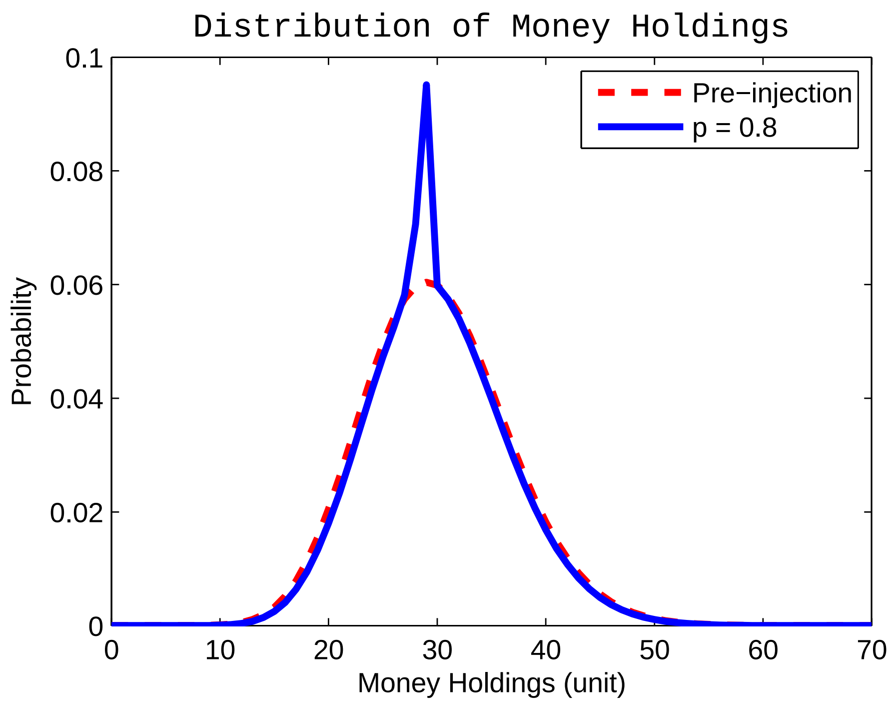

Figure 6: Distribution before and after the one-shot injection. $p_2 = 0.8$

under the lump-sum injection scheme. The reason is that the dispersion of distribution falls by the greatest degree in this case, as is showed in Figure 6. After the injection, there is a spike around the mean value of the distribution, while the population of poorest agents and richest agents both declines, the former due to the reception of injection and the latter due to the inflationary tax (money disintegration). For both $p_2 = 0.6$ and $p_2 = 1.0$, dispersions decrease only slightly because in the former case a small fraction of poorest agents receive money and in the latter case almost all agents receive money just as in the case of one-shot lump-sum injection. The resulting distribution is therefore very similar to the pre-injection distribution and therefore not plotted here.

Permanent injection: steady state

Now we study the case when such an injection scheme is permanently institutionalized. Again we compute for the steady states for cases with p_2 equal 0.6, 0.8 and 1.0. The results is shown in Table 3. For all three cases, the percent-

Value of p_2	0.6	0.8	1.0
Participation rate	0.001%	0.021%	0.125%
Money growth rate (Quarterly)	0.00002%	0.0006%	0.0042%
Avg payment	129.77%	150.91%	174.05%
Aggregate output	99.87%	99.81%	99.64%%
Ex-ante welfare	100.03%	100.00%	99.81%

Table 3: Steady states with money injections requiring disutility cost to receive; output and welfare are expressed in relative to those of the benchmark model.

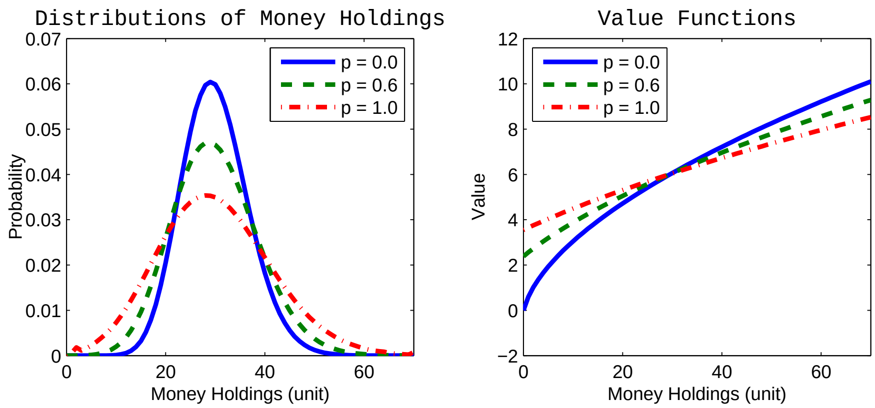

Figure 7: Distributions and value functions in the steady state with injections.

age of agents willing to spend the effort to receive the injections is remarkably small in the inflationary steady states, and the resulting money growth rate is close to zero. Yet these low money growth lead to some non-neutrality in the long run. The output is always lower than in the non-inflationary steady state, while welfare is slightly improved for $p_2 = 0.6$ and 0.8, but deteriorates once p_2 is large enough. In Figure 7 we plot the steady-state distributions and value functions with no injection, $p_2 = 0.6$ and $p_2 = 1.0$ (we leave out the case with $p_2 = 0.8$ only for illustrative convenience). Such injection schemes increase the dispersion of money holding distributions and flatten the value functions. The two effects have opposite impacts on total output, just like in the scenario of lump-sum injections we discussed previously. And here money injections also lead to lower output than in the benchmark case.

Value of p_3	0.5031	0.5036	0.5041
Participation rate	24.04%	51.06%	74.22%
Increase in money stock	0.004%	0.012%	0.020%

Table 4: Choices of different p_3 and corresponding participation rates

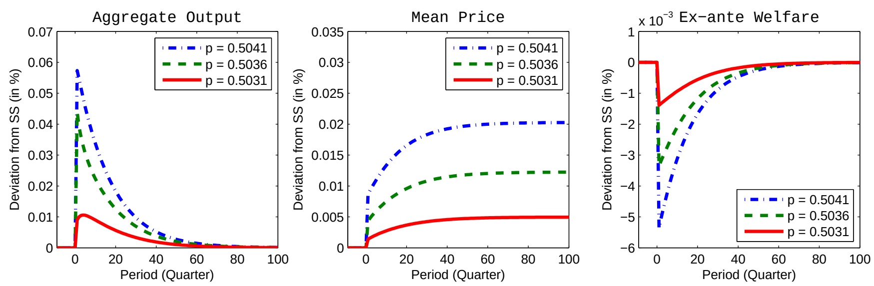

Figure 8: Transition Paths after injection. Agents need to pay some monetary cost to receive the injected money.

4.5 Injections costly to receive: monetary cost

One-shot injection

Next we focus on another type of non-uniform injection schemes. We assume that one has to some cost in form of money to be eligible for the helicopter drop of money. Therefore for those in for the money injection, with probability p_3 they receive $x - \kappa$ unit of money and with $1 - p_3$ they lose κ unit of money. The monetary cost κ (which we set to 1), once paid by the agents to the monetary authority, flows back into the economy as part of the injected money. Because of the concavity of the value function, such a lottery-like money injection is more attractive to rich agents than to poor agents. The number of agents who are willing to pay the cost is endogenously determined by and positively correlated with p_3. We select a set of different value of p_3, corresponding to different levels of participation. Since the participation rate is very sensitive to the value of p_3, we choose three different p_3 to roughly match participation rates of 25%, 50% and 75%, as is showed in Table 4.

In Figure 8, we document the computed response of output, price and welfare after a injection occurring at period 0.

In all scenarios, we find that after the injections aggregate output rises in response to the money injection. For money stock increase of 0.004%, 0.012% and 0.020%, output initially increases by 0.009%, 0.044% and 0.057% respectively. And the more aggressive is the injection (higher p_3), the more significant is the response.[9] In other word, there is a short-run relationship between inflation rate and aggregate output, as in the Phillips Curve. More interestingly, we find that when $p_3 = 0.5031$ (the solid line), the output response is hump-shaped with the peak occurs after three or four periods (quarters). This is consistent with the empirically based consensus among economists (e.g., Christiano, Eichenbaum and Evans [6]) that monetary policy shocks have a short-run effect on real economic activities which follows a hump-shaped pattern in which the peak impact is reached several quarters after the initial response and then gradually dies out. Next, note that although prices eventually rise in proportion to the increase in money, such adjustments are sluggish. In all scenarios, it takes several years for the mean prices to reach its long-run level after the injection. In other words, price adjustment displays some rigidity. To understand, note that the price here is implied by the terms of trade between a trading pairs. Since the injection increase both the amounts of goods traded and the quantity of money changed hand, the implied price will only rise in a smaller magnitude. Unlike the sticky-price models where sluggish price adjustment leads to non neutrality of money on output, here in our model the causality chain runs the other way around. Finally, the money injection has a negative, persistent but insignificant effect on welfare.

Why does this type of money injection can induce response so different, with such a significantly positive response, while the other injection schemes we considered in previous sections fail to do so? The answer lies again in the change of distributions brought about by the injections under study. With the current injection scheme, $1-p_3$ of the prospect recipients of money injection will end up with a net loss of $-\kappa$ from the injection, while the rest p_3 of them receive the injected money and leave with a net gain of $x-\kappa$. By making a fraction of a certain group of people poorer but the rest of them richer, the money injection scheme here effectively increases the dispersion of money holding distribution immediately. The distributions at period 1 and at period 3 are plotted in Figure 9, where both the case of $p_3 = 0.5031$ and

[9] When making this argument, we exclude very large values of p_3. Note that if p_3 is set very high, for instance $p_3 = 0.9$, such injection will induce a negative response of output. The reason is that, in this case the problem facing the agents regarding the decision of paying the κ is trivial. Everyone will pay κ, and the injection is very much similar to a lump-sum injection.

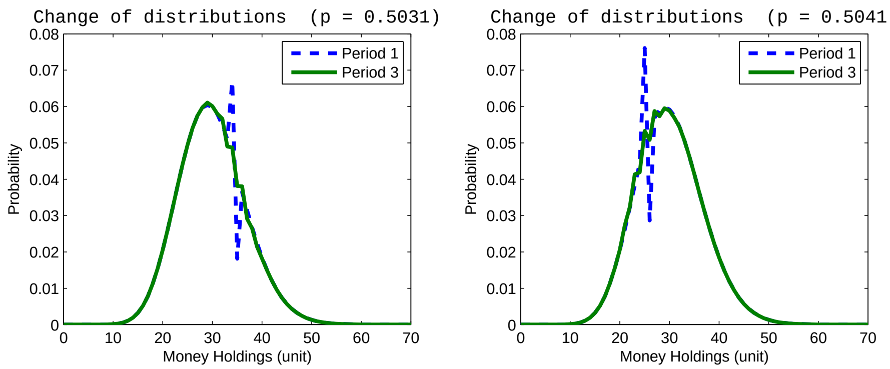

Figure 9: Distributions after money injection, for the case of $p_3 = 0.5031$ (left) and $p_3 = 0.5041$ (right).

the case of $p_3 = 0.5041$ are shown.

Permanent injection: steady state

The short-run relationship between inflation and aggregate output we observed in the one-shot experiment makes one wonder what will happen if the authority exploit such seemingly stable trade-off in the long-run by letting such injections last for ever. Will the short-run relationship breaks down in our new steady state just like how Phillips Curve dissolves in the 1970s? We compute the steady state equilibria with permanent injection for different value of p_3. The results, which are summarized in Table 5, suggest that such this short-run relationship is not exploitable. When p_3 is small, or the rate of monetary expansion is low, an increase in p_3 tends to increase aggregate output. However when p_3 is large enough, further raising p_3 will only lead to lower output. This coincides with the empirical evidences (e.g. Bullard and Keating [4]) of positive effects of inflation on output for low-inflation countries and negative effects for high-inflation countries. Nevertheless, note that such money injection, when implemented permanently, deteriorates rather than improve welfare, regardless of the rate of expansion or p_3.

Temporary increase in p_3

Now, we examine whether the results we obtained regarding the short-run effect of one-shot money injections are robust to alternative versions of policy shocks. Instead of hitting a non-inflationary economy with a shock of an

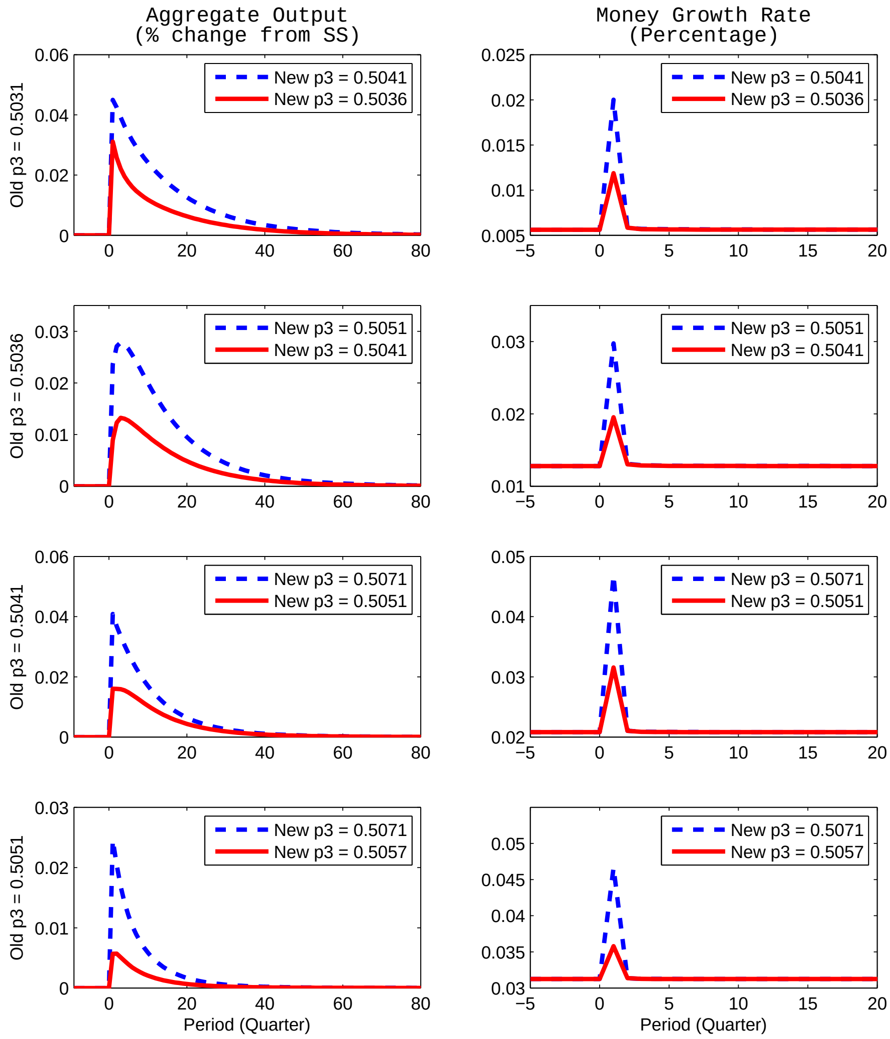

Figure 10: Output response when a policy shock raises p_3 temporarily.

Value of p_3	0.504	0.506	0.508	0.510	0.52	0.54
Money growth rate	0.019%	0.038%	0.053%	0.067%	0.133%	0.267%
Aggregate output	100.93%	101.34%	101.34%	101.23%	100.55%	99.01%
Ex-ante welfare	99.45%	99.13%	99.04%	98.99%	98.82%	98.57%

Table 5: Steady states with injections requiring monetary cost to receive; output and welfare are expressed in relative to those of the basic model

one-shot increase of money stock, here we hit an inflationary economy with a shock which temporarily increase p_3. To proceed, we set the economy in a steady state with some $p_3 > 0$ before the shock. And when the shock takes place at the beginning of period 1, it raises p_3 to $\tilde{p_3} > p_3$. From period 2 and onward, p_3 is resumed and the economy converges back to the pre-shock steady state equilibrium. We compute the transitional processes for different values of p_3 and p_3, and plot the result in Figure 10. For all the scenarios we consider, there are significantly positive and persistent responses in output. And for some scenarios, we again observe hump-shaped output responses. In other words, given the current money injection scheme, if the one-shot shock is a temporary increase in the monetary expansion, the output will still respond significantly, as in the case of one-shot increase of money stock via such injection.

5 Discussions

So far we have studied different types of unproportional money injection schemes. In all cases, one-shot money injection leads to a non-neutral response in output, because it changes the distributional of money holdings and hence the distribution of different seller-buyer pairs, which we identify as the extensive margin of the aggregate output. On the other hand, the individual output between each of these pairs, or the intensive margin of the aggregate output, is almost unaffected by the money injection because of its one-shot nature. Therefore, the distributional effect of money injection on aggregate output is about how the aggregate output is affected along the the extensive margin. Our numerical results suggested that pattern of pairwise output actually favors more dispersed distributions. Therefore, when the money injection is such that the recipients has to pay some monetary cost, it increases the dispersion of money holding distribution, which is translated into a positive and significant response in aggregate output immediately after the injection. And the output will return to its pre-injection level only when the pre-injection distribution of money holdings is resumed, which occurs only when the newly injected money disperse across the whole economy by way of transactions. The decentralized pattern of trade makes such dispersion a long process, therefore distributional effect induced by money injection is very persistent.

However, if such money injection is permanently implemented as higher inflation in the long run, there emerges a negative effect along the intensive margin of the aggregate output, because inflation erodes the value of money and hence suppresses individual output between every buyer-seller pair. If the inflation is high enough, the negative effect on the intensive margin dominates the positive effect on the extensive margin, and output is lower under such inflation rate.

6 Concluding Remarks

In this paper, we study the distributional effect of monetary policy in the form of different schemes of direct money injection. By applying numerical methods to a standard off-the-shelf money matching model, we find the distribution of money holdings plays a significant role. First, unproportional money injection change the distribution of money holdings. We show that under certain schemes of money injection will initially change the distribution or disperse the money in such a way that can immediately generate a significant response of aggregate output. Second, since it takes time for the distribution of money to disperse back to its pre-injection shape in the decentralized economy, such effect on output is very persistent. Moreover, we find that following the injection, the price adjustment is sluggish and there is a short-run negatively-sloped Phillips curve. However, such short-run relationship is not exploitable in the long run.

Note that the randomness of the centralized market plays an important role here. Therefore, a potential extension of the work is to include a phase of centralized market transaction between decentralized pairwise transactions, and to see whether the results still hold.

The framework developed in this paper, as well as the numerical approach to solve for it, can be utilized by adding other elements so as to answer other topics and questions regarding money. For example, what roles do illiquid bonds play in such a model? How monetary policy via raising or reducing the interest rate of such bonds can affect the economy?

Appendix

A. Proofs of Propositions 1

The proof is standard and follows directly from Zhu [20].

B. Numerical algorithms

In this section we describe the numerical algorithms we adopted to compute the steady state and transition paths of the models. The FORTRAN 90 codes for the algorithms, are available upon request.

B1. Computing steady states of the benchmark model

The algorithm is essentially an iteration on the mappings defined by the following steps.

1. Begin with an initial guess $\{\pi^0, v^0\}$, where π^0 is consistent with the total money stock M.

2. Given v^i, we can solve for problem 1 for all pairs of $\left\{m^b, m^s\right\}$, which gives us $\left(y_{m^s,m^b}, \sigma_{m^s,m^b}\right)$[10], $f\left(m^b, m^s, v^i\right)$ and $\delta\left(m^b, m^s\right)$. By applying them together with π^i to 3 and 4, we get a new pair $\{\pi^{i+1}, v^{i+1}\}$

3. Repeat step 2 until the convergence criterion is satisfied: $\|v^{i+1} - v^i\| < 10^{-6}, \|\pi^{i+1} - \pi^i\| < 10^{-6}$.

4. Denote the final result $\{\pi^*, v^*\}$

B2. Computing transition paths following money injections

The computation for the transition path is essentially about iterations on the series of $\chi \equiv \{v_t, \pi_t\}_{t=1}^{T}$, where T is the number of periods it takes for the economy to reach a new steady state. Since the the transition path converges to the pre-injection steady state computed in B1, we set $v_T = v^*$. We also have to apply the effect of money injection on the pre-injection distribution π^* to get the distribution immediately after the injection. We denote this beginning distribution as π_1.

[10] To solve for the lottery σ_{m^s,m^b}, we can first solve for the money traded if no lottery is allowed. Denote the money traded in this case d_{m^s,m^b}. Then utilizing the concavity of v, we set the lottery space to be on $\left\{d_{m^s,m^b} - 1, d_{m^s,m^b}, d_{m^s,m^b} + 1\right\}$ and solve for σ_{m^s,m^b}.

1. Take an initial guess with $v_t^0 = v^*$ for all t, and $\pi_1^0 = \pi_1$.

2. Start from $t = 1$ and set $\pi_1^i = \pi_1$. Given π_t^i and v_t^i, solve the problem in (1), and get the solution as $\left(y_{m^s,m^b}^i(t), \sigma_{m^s,m^b}^i(t)\right)$ for all $\left\{m^s, m^b\right\}$. Then use the solution to derive π_{t+1}^i according to (4). Repeat this process until $t = T$, and we get $\left(y_{\cdot,\cdot}^i(t), \sigma_{\cdot,\cdot}^i(t)\right)$ for all t. Then use them backward, from period T to 1, to get an updated series of $\left\{v_t^{i+1}\right\}_{t=1}^T$.

3. Repeat step 2 until the convergence criterion is satisfied: $\max_t \left(\left\|\pi_t^{i+1} - \pi_t^i\right\|\right) < 10^{-6}$ and $\max_t \left(\left\|v_t^{i+1} - v_t^i\right\|\right) < 10^{-6}$.

B3. Computing steady states with injections

The computation for the inflationary steady state is similar to B1. But the algorithm is complicated by money injection and disintegration at the beginning of every period, especially when the recipients of the injection are endogenously determined. To proceed, in addition to π_t and v_t, we denote the distribution after the injection and disintegration but before pairwise meeting in period t as θ_t, and the value function after the injection but before the disintegration in period t as w_t.

1. Begin with an initial guess $\{\theta^0, w^0\}$, where θ^0 is consistent with the total money stock M.

2. Begin the $(i+1)$-th iteration with $\{\theta^i, w^i\}$. Given the value function after the injection w^i, we can solve for problem of agents deciding whether or not to receive the money injection. As a result we get the value function before the injection v^i, which is also the value function after pairwise meetings. Use v^i and θ^i, we can solve the problem in (1), and get π^i accordingly. With π^i, we solve for the set of agents seeking to receive the injection, and the disintegration probability δ needed to normalize the money stock. And we can update θ^{i+1} accordingly. Finally, we can build the transition matrices implied by the pairwise meetings and money injections, which allow us to update w^{i+1}.

3. Repeat step 2 until the convergence criterion is satisfied: $\|w^{i+1} - w^i\| < 10^{-6}, \|\theta^{i+1} - \theta^i\| < 10^{-6}$.

4. Denote the final result $\{\theta^*, w^*\}$, and which is accompanied by $\{\pi^*, v^*\}$.

C. Differences with the literature

In the literature, Molico [14] also employs numerical methods to examine the effect of inflation on output, but arriving at different results from ours. Specifically, he show that lump-sum money injection, when conducted permanently, can increase both the total output and ex-ante welfare, while in our model, lump-sum injection has just the opposite effect. There are some difference in the model setup between his and our works, for example, he assumes divisible money and uses approximations methods to compute distributions and value functions, while we assume indivisible money with lottery trade and directly compute distributions and value functions, but the different results is indeed due to the difference in preference settings. In our model, the utility function of consumption and disutility function of production follow the standard form in the literature: $u(y) = y^{1-\sigma}$ and $c(y) = y$, where y is allowed to take any value in $(0, +\infty)$

In Molico [14], the utility function of consumption and disutility function of production take the following forms:

$$\begin{aligned} u(y) &= A \cdot \log(1+y) \\ c(y) &= B\left(\frac{1}{\bar{y}-y} - \frac{1}{\bar{y}}\right), \text{ for all } y \in [0, \bar{y}], \end{aligned}$$

where $A, B \in \mathbb{R}^+$, $\bar{y} > 0$ and $A > B/\bar{y}^2$.

Note that under such setting, the ex-ante pairwise optimal quantity y^* such that $u'(y^*) = c'(y^*)$ (and $y^* \in [0, \bar{y}]$), is

$$\begin{aligned} y^* &= \bar{y} + \frac{B}{2A} - \sqrt{\left(\frac{B}{2A}\right)^2 + \frac{B}{A}(\bar{y}+1)} \\ &< \bar{y} \end{aligned}$$

Moreover, he take $A = 100$, $B = 1$, and $\bar{y} = 1$, which immediately gives us $y^* = 0.8635$, very close to $\bar{y}$. In other words, under such parameter values there is a very restrictive upper bound on pairwise output[11]. We argue that it is because of the specialty of such values of A and B ($A \gg B$) that lead

[11] We mention y^* here and compare it, instead of other possible value of y, with y^*, because y^* is close to the pairwise output between a buyer and a seller both with M units of money. Note that such a pair has the largest probability mass, and all our numerical results showed that the aggregate output is always close to y^*.

to the results in [14]. We hold $B = 1$, and try different values of A. We computes the steady state of the model with permanent lump-sum injection, where p_1 is the probability of an agent receiving injected money. The results are reported in Table 6. Note that as A decreases, the upper bound of $\overline{y}$ becomes less restrictive, the effect of lump-sum injection on output and welfare will diminish, and eventually becomes negative, just like in our model. In addition, for small values of A, the value function, distribution function, and pairwise output are all similar to what we have in this paper.

	$A=100$, $y^*=0.8635$		$A=28$, $y^*=0.75$		$A=6$, $y^*=0.5$		$A=2.222$, $y^*=0.25$		$A=1.358$, $y^*=0.1$	
p_1	Output	Welfare	Output	Welfare	Output	Welfare	Output	Welfare	Output	Welfare
0	100%	100%	100%	100%	100%	100%	100%	100%	100%	100%
0.05	100.90%	101.58%	100.16%	100.86%	99.67%	99.95%	98.73%	99.47%	96.16%	97.98%
0.15	100.88%	102.02%	99.83%	101.15%	98.83%	99.84%	96.17%	98.33%	88.62%	93.57%
0.30	100.70%	102.10%	99.38%	101.16%	97.40%	99.41%	92.33%	96.48%	77.99%	86.57%

Table 6: The long-run effect of lump-sum injection on output and welfare, when the preference functional forms follow Molico [14]. $B=1$, $\bar{y}=1$.

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