Heterogeneity in Decentralized Asset Markets*

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Abstract

We consider a decentralized market for an asset (or durable good) where investors' valuations are heterogeneous and drawn from an arbitrary distribution. We provide a full characterization of the equilibrium, in closed form, both in and out of steady state. We find that investors with moderate valuations tend to specialize in intermediation, so that a "core-periphery" trading network emerges endogenously. This has important implications for both individual trading patterns–such as the expected amount of time it takes for each type of investor to trade–as well as aggregate outcomes–such as the degree of misallocation in the economy, the total volume of trade, and the amount of price dispersion. We also characterize the equilibrium in the limiting economy as trading frictions vanish. We show that price dispersion vanishes relatively quickly, price levels converge to their frictionless counterpart relatively slowly, and trading volume does not converge at all; that is, in the limit, the volume of trade is infinite.

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1 Introduction

We construct a search and bargaining model of an over-the-counter (OTC) market in which investors with heterogeneous valuations for the asset are periodically and randomly matched in pairs and given the opportunity to trade. Importantly, we allow investors' valuations to be drawn from an arbitrary distribution of types. We characterize the unique equilibrium, in closed form, both in and out of steady state. We find that *heterogeneity matters*, i.e., allowing for an arbitrary distribution of types has important implications for the patterns of trade, the nature and extent of misallocation, trading volume, and prices. In particular, when there are many types of investors, assets are reallocated through chains of inframarginal trades in which investors with near-marginal valuations tend to specialize in intermediation and account for most of the trading volume. As a result, a "core-periphery" trading structure emerges endogenously, with misallocation less pronounced among investors with extreme valuations in the periphery, and more concentrated among investors with near-marginal valuations in the core. This trading structure not only generates excess trading volume, but also has important effects on prices. In environments with many different types of investors, we show that frictions typically have larger effects on bilateral prices than when there are just a few types, and they have larger effects on the *level* of bilateral prices than on their *dispersion.* Thus, according to the model, assessing the impact of frictions using the amount of observed price dispersion is misleading, since dispersion can appear small while the impact of frictions on bilateral price levels can be much larger.

The motivation for our work is that many assets trade in markets that are not well-described by the Walrasian paradigm, in which trade occurs instantaneously at a single price. Instead, many assets—along with many durable goods—trade in decentralized markets with two important features: first, investors have to *search* in order to find a counterparty; and second, once they find a potential trading partner, they must attempt to determine the terms of trade *bilaterally*.[1] A recent literature has emerged that attempts to capture these important features using search and bargaining models, starting with Duffie, Gârleanu, and Pedersen (2005) (henceforth DGP). However, in order

[1]Examples of assets that trade in OTC markets include corporate and government bonds, emerging-market debt, mortgage-backed securities, and foreign exchange swaps, to name a few. However, as noted above, our analysis applies more generally to a variety of decentralized markets where the object being traded is durable, such as capital or real estate.

to keep these models simple and parsimonious, almost all of these papers have restricted attention to the case where investors switch between two types of valuation (e.g., high and low). While these models have certainly generated a number of important insights, the restriction to two valuations comes at a cost. In particular, since there is only one type of trade in these models, all trades occur at the same price and at the same frequency. Hence, models with this restriction are silent on heterogeneity across individual investors' trading experiences—i.e., the likelihood they are holding a sub-optimal portfolio, the time it takes them to find a suitable counterparty for trade, and the price at which they tend to buy or sell—as well as the implications for these different trading experiences for aggregate outcomes, such as the structure of the trading network, trading volume, misallocation, and price dispersion. Our model, with arbitrary heterogeneity, offers a unified framework to address all of these issues, and many more.

To facilitate comparison with the existing literature, our model adopts the main building blocks of DGP. We assume that there is a fixed measure of agents in the economy and a fixed measure of indivisible shares of an asset. Agents have heterogeneous valuations for this asset, which can change over time, and they are allowed to hold either zero or one share. Each agent is periodically and randomly matched with another agent, and a transaction ensues if there are gains from trade, with prices being determined by Nash bargaining. Our main point of departure from the existing literature is that we allow agents' valuations, which we denote by δ, to be drawn from an arbitrary distribution.

Our primary results are as follows. First, even though the network of meetings is fully random in our model, the network of trades that emerge in equilibrium are not: agents with "near-marginal" valuations tend to specialize in intermediation, buying from agents with lower valuations and selling to those with higher valuations. Moreover, these transactions involving agents with near-marginal valuations are relatively frequent, compared with transactions between agents with more extreme valuations. Hence, a "core-periphery" trading network emerges endogenously, which is consistent with the empirical evidence documented by Bech and Atalay (2010) and Li and Schürhoff (2012) in two major OTC markets. Moreover, these patterns of trade have important implications for both individual trading experiences and aggregate outcomes.

At the individual level, the expected time it takes for an investor to buy an asset is decreasing in

δ, while the expected time to sell an asset is increasing in δ. The expected trading price, however, is increasing in δ for both buyers and sellers. In our model this occurs for two reasons: first, agents with higher valuations have higher reservation prices; and second, agents with higher valuations tend to trade with other agents with high valuations, who also have high reservation prices. In contrast, note that trading times are *constant* across investors in environments in which either investors trade through a centralized exchange or investors trade with dealers who have access to a centralized exchange. Moreover, in either of these two alternative settings, prices only depend on an investor's own valuation and not on the valuation of the investor with whom he ultimately trades. Hence, in contrast to these alternatives, our model implies non-trivial, testable predictions about the relationships between, e.g., the amount of time an asset is "on the market," the price at which it trades, and the expected duration of ownership that follows. Indeed, Ashcraft and Duffie (2007) provide empirical evidence that is consistent with these predictions regarding the relationship between an investor's valuation, time to trade, and prices.

At the aggregate level, we show that misallocation is concentrated within the group of near-marginal investors: assets are passed quickly into this cluster of traders through a chain of infra-marginal trades, but they are slow to leave the cluster, as there is a dearth of agents with high valuations that haven't already found an asset to own. As the asset is passed along this chain of trades, excess volume is created, relative to a frictionless market. Moreover, trades can occur at prices that are quite far from the Walrasian benchmark, and they can be either above or below this benchmark, depending on the level of search frictions, the supply of the asset, and the relative bargaining powers of investors on either side of a transaction.

Finally, we also study the steady-state equilibrium as the rate at which agents contact each other, λ, approaches infinity. This is a relevant case to consider for two reasons. First, from a purely theoretical perspective, it allows us the address the classical question of whether outcomes in search markets become approximately Walrasian, and at which speed, as frictions vanish (see, e.g., Rubinstein and Wolinsky (1985) and Gale (1986)). Second, this is an empirically relevant case to consider, as technological developments continue to make trading speed in financial markets faster and faster. Our main results are that heterogeneity slows down the speed of convergence to the Walrasian benchmark, and that different equilibrium objects converge at different speeds. Namely, when the distribution of valuations is discrete, convergence occurs generically very quickly for all equilibrium objects, at a rate in order $1/\lambda$. When the distribution of valuations is continuous, convergence occurs more slowly: it occurs in order $1/\sqrt{\lambda}$ for bilateral price levels, and in order $\log(\lambda)/\lambda$ for price dispersion and for the welfare cost of misallocation. We emphasize two implications. First, frictions tend to have stronger effects in a model in which the distribution of valuation is continuous, than in earlier models when this distribution is discrete. Second, price dispersion converges faster than price levels. Thus, according to the model, quantifying the impact of search frictions using observed price dispersion can be misleading, since frictions can have substantial effects on price levels even when dispersion appears very small. In addition, we find that trading volume does not converge at all: in the limit, the volume of trade is infinite, and essentially all of this volume is accounted for by near-marginal investors who intermediate trades.

The paper is organized as follows. After providing a brief review of the related literature below, Section 2 provides a formal description of the environment, and also characterizes equilibrium outcomes in an alternative setting in which investors have access to a centralized, frictionless exchange, which provides a convenient benchmark. Section 3 then returns to the model with search frictions and provides a closed form characterization of the unique equilibrium, in and out of steady state, for an arbitrary distribution of valuations. Note that this is a potentially formidable task, as the state of the economy includes two (potentially) infinite-dimensional objects: the distributions of valuations among agents that hold zero and one assets, respectively, over time. Hence, part of our contribution is providing a methodology to solve for equilibrium in closed form despite these challenges. Then, in Section 4, we exploit our characterization of equilibrium to analyze how trading frictions affect the patterns of trade, allocations, and prices. We first study these effects when the intensity of meetings is finite, and then we study what happens as trade gets *fast* and frictions vanish. Section 5 concludes.

1.1 Related Literature

Our paper builds off of a recent literature that uses search models to study asset prices and allocations in OTC markets. Many of these papers are based on the basic framework developed in Duffie et al. (2005), who study how search frictions in OTC markets affect the bid-ask spread set by marketmakers who have access to a competitive interdealer market.[2] The current paper is closer in spirit to Duffie et al. (2007), who study a purely decentralized market—i.e., one without any such marketmakers—where investors with one of two valuations meet and trade directly with one another. Purely decentralized trade captures important features of reality; for example, Li and Schürhoff (2012) show that, in the municipal bond market, even the inter-dealer market is bilateral, with intermediation chains that typically involve more than two transactions. In the literature, the model of purely decentralized trade has been used to explore a number of important issues related to liquidity and asset prices; see, for example, Vayanos and Wang (2007), Weill (2008), Vayanos and Weill (2008), Afonso (2011), Gavazza (2011, 2013), and Feldhütter (2012).[3] However, all of these papers have maintained the assumption of only two valuations, and hence cannot be used to address many of the substantive issues that are analyzed in our paper.[4]

To the best of our knowledge, there are very few papers that consider purely decentralized markets and allow for more than two types of agents. Perhaps the closest to our work is Afonso and Lagos (2012), who develop a model of purely decentralized exchange to study trading dynamics in the Fed Funds market. In their model, agents have heterogenous valuation because they are heterogeneous with respect to their asset holdings. Several insights from Afonso and Lagos feature prominently in our analysis. Most importantly, Afonso and Lagos highlight the fact that agents with moderate asset holdings play the role of "endogenous intermediaries," buying from

[2]Other early papers that used search theory to analyze asset markets include include Gehrig (1993), Spulber (1996), Hall and Rust (2003), and Miao (2006).

[3]For example, Vayanos and Wang (2007) and Vayanos and Weill (2008) show that these models can generate different prices for identical assets, which can help explain the "on-the-run" phenomenom in the Treasury market; Weill (2008) explores how liquidity differentials, which emerge endogenously in these models, can help explain the cross-sectional returns of assets with different quantities of tradeable shares; Gavazza (2011, 2013) uses these models to explain price differences in the market for commercial aircraft; and Feldhütter (2012) uses these models to generate a measure of selling pressure in the OTC market for corporate bonds.

[4]The framework of Duffie et al. (2005), with a predetermined set of marketmakers, has also been extended in a number of directions; see, e.g., Weill (2007). Lagos and Rocheteau (2009) and Gârleanu (2009) show how to accommodate additional heterogeneity in this framework, as they allow agents to choose arbitrary asset holdings. Lagos et al. (2011) extend this framework even further to study market crashes. As we discuss below, allowing for various types of heterogeneity in an environment in which investors trade with intermediaries is simpler than our exercise, in some ways, since the expected time to trade is constant across all agents on one side of the market, and the price they pay does not depend on the distribution of valuations of agents on the other side of the market. Lester et al. (2014) consider a model in the spirit of Lagos and Rocheteau (2009) with *directed*, instead of random search, and show how this framework can generate a relationship between an investor's valuation, the price he pays, and the time it takes to trade.

agents with excess reserves and selling to agents with few. As we discuss at length below, similar agents specializing in intermediation emerge in our environment, and have important effects on equilibrium outcomes. However, our work is quite different from Afonso and Lagos in a number of important ways, too. For one, since our focus is not exclusively on a market in which payoffs are defined at a predetermined stopping time, we characterize equilibrium both in and out of steady state when the time horizon is infinite. Moreover, our methodology allows us to characterize most equilibrium objects in closed form. This tractability allows us to perform analytical comparative statics and, in particular, derive a number of new results about the patterns of trade, prices, and allocations as the trading speed tends to infinity.[5]

Our paper is also related to an important, growing literature that studies equilibrium asset pricing and exchange in exogenously specified trading networks. Examples include Gofman (2010), Babus and Kondor (2012) and Malamud and Rostek (2012). In these network models, intermediation chains arise somewhat mechanically; indeed, when investors are exogenously separated by network links, the only feasible way to re-allocate assets towards investors who value them most is to use an intermediation chain. In a dynamic search model, by contrast, intermediation chains arise by choice. In particular, all investors have the option to search long enough in order to trade directly with their best counterparty. In equilibrium, however, they find it optimal to trade indirectly, through intermediation chains. Hence, even though all contacts are random, the network of actual trades is not random, but rather exhibits a core-periphery-like structure that is typical of many OTC markets in practice.

Finally, our paper is also related to the literatures that use search-theoretic models to study monetary theory and labor economics. For example, a focal point in the former literature is under-

[5]Several other papers also deserve mention here. First, in an online Appendix, Gavazza (2011) proposes a model of purely decentralized trade with a continuum of types in which agents have to pay a search cost, c, in order to meet others. The optimal strategy is then for an agent with zero (one) asset to search only if his valuation is above (below) a certain threshold R_b (R_s). He focuses on a steady state equilibrium when c is large, so that $R_b > R_s$. Focusing on this case simplifies the analysis considerably, since all investors with the same asset holdings trade at the same frequency, and they trade only once between preference shocks. However, this special case also abstracts from many of the interesting dynamics that emerge from our analysis about trading patterns, the network structure, misallocation, and prices (both in and out of steady state). Many of these insights were derived independently earlier in two working papers—Hugonnier (2013) and Lester and Weill (2013)—which were later combined to form the current paper. Finally, in a recent working paper, Shen and Yan (2014) exploit a methodology related to ours in an environment with two assets to study the relationship between liquid assets (that trade in frictionless markets) and less liquid assets (that trade in OTC markets).

standing how the price of one particular asset—fiat money—depends on its value or "liquidity" in future transactions; for a seminal contribution, see Kiyotaki and Wright (1993).[6] Naturally, understanding such liquidity premia is central to our analysis as well. A key issue in the latter literature is understanding how workers move from firm to firm through the process of on-the-job search; see, e.g., Burdett and Mortensen (1998) and Postel-Vinay and Robin (2002). The dynamics of these worker flows across firms, and the corresponding distributions and measures of misallocation, share much in common with the way that assets move across investors in our model.

2 The model

2.1 Preference, endowment, and matching technology

We consider a continuous-time, infinite-horizon model, where time is indexed by $t \geq 0$. There is a fixed probability space $(\Omega, \mathcal{F}, P)$, as well as an information filtration satisfying the usual conditions (Protter, 1990). The economy is populated by a unit measure of infinitely-lived and risk-neutral investors who discount the future at the same rate $r > 0$. There is one indivisible, durable asset in fixed supply, $s \in (0, 1)$, and one perishable good, which we treat as the numéraire.

Investors can hold either zero or one unit of the asset. The utility flow an investor receives from holding a unit of the asset, which we denote by δ, differs across investors and, for each investor, changes over time. In particular, each investor receives i.i.d. preference shocks that arrive according to a Poisson process with intensity γ, whereupon the investor draws a new utility flow δ' from a cdf $F(\delta')$. We assume that the support of $F(\cdot)$ is a compact interval, and we make the interval sufficiently large so that $F(\delta)$ has no mass points at its boundaries. For simplicity, we normalize this interval to $[0, 1]$, and assume only that $F(\cdot)$ is non-decreasing, right-continuous, and satisfies $F(0) = 1 - F(1^-) = 0$. Thus, at this point, we place very few restrictions on the distribution of utility types; our specification of preferences includes discrete distributions (such as the two point distribution of Duffie, Gârleanu, and Pedersen, 2005), continuous distributions, and mixtures of the two.

[6]This literature has recently incorporated assets into the workhorse model of Lagos and Wright (2005) in order to study issues related to financial markets, liquidity, and asset pricing; see, e.g., Lagos (2010), Geromichalos et al. (2007), Lester et al. (2012), and Li et al. (2012).

Investors interact in a decentralized or *over-the-counter* market in which each investor initiates contact with another randomly selected investor according to a Poisson process with intensity $\lambda/2$. If two investors are matched and there are gains from trade, they bargain over the price of the asset. The outcome of the bargaining game is taken to be the Nash bargaining solution where the investor with asset holdings $q \in \{0, 1\}$ has bargaining power $\theta_q \in (0, 1)$, with $\theta_0 + \theta_1 = 1$.

An important object of interest throughout our analysis will be the joint distribution of utility types and asset holdings. We let $\Phi_{q,t}(\delta)$ denote the measure of investors at time $t \geq 0$ with asset holdings $q \in \{0, 1\}$ and utility type less than δ. Assuming that initial types are randomly drawn from $F(\delta)$ at $t = 0$, the following accounting identities must hold for all $t \geq 0$:[7]

$$\Phi_{0,t}(\delta) + \Phi_{1,t}(\delta) = F(\delta) \tag{1}$$

$$\Phi_{1,t}(1) = s. \tag{2}$$

Equation (1) highlights that the cross-sectional distribution of utility types in the population is always equal to $F(\delta)$, which follows from the assumption that the initial distribution of types is drawn from $F(\cdot)$, along with the assumption that investors' new utility types are independent from their previous utility types. Equation (2) highlights that the total measure of investors who own the asset must equal the total measure of assets in the economy, s. Given our previous assumptions, note that this implies $\Phi_{0,t}(1) = 1 - s$ for all $t \geq 0$.

2.2 The Frictionless Benchmark: Centralized Exchange

Consider a frictionless environment in which there is a competitive, centralized market where investors can buy or sell an asset instantly at some price $p(t)$. We conjecture, and confirm below, that there exists an equilibrium in which the price is constant, so that $p(t) \equiv p$ for all t. In this environment, the objective of an investor is to choose an asset holding process $q(t) \in \{0, 1\}$, of

[7] Most of our results extend to the case where the initial distribution is not drawn from $F(\delta)$, though the analysis is slightly more complicated. See Section **??** in the Appendix.

bounded variation and predictable with respect to the filtration generated by $\delta(t)$, that maximizes

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\delta(t)q(t)\,dt - \int_0^\infty e^{-rt}p(t)dq(t)\right] = pq(0) + \mathbb{E}\left[\int_0^\infty e^{-rt}\left[\delta(t) - rp\right]q(t)\,dt\right]. \quad (3)$$

The equality in (3) follows from integration by parts. The interpretation is that, in a frictionless market, one can imagine that an investor sells all of his holdings at $t = 0$, which yields $pq(0)$, and at each subsequent time purchases $q(t)$ units of the asset at the beginning of each time interval $[t, t + dt)$ and sells them at the end of the time interval. The net cost of buying $q(t)$ at time t and reselling it at $t + dt$ is $rp\,dt$. This representation of an investor's objective makes it clear that, at each time t, optimal holdings satisfy:

$$q(t) = \begin{cases} 0 & \text{if } \delta(t) < rp \\ 0 \text{ or } 1 & \text{if } \delta(t) = rp \\ 1 & \text{if } \delta(t) > rp. \end{cases}$$

It then follows immediately that, in equilibrium, the asset is allocated at each time to the investors who value it most. The "marginal" utility type—i.e., the investor who has the lowest valuation among all owners of the asset—is defined by

$$\delta^\star = \inf\{\delta \in [0, 1] : 1 - F(\delta) \leq s\}.$$

Hence, for all t, the distribution of types among investors who own a unit of the asset is

$$\Phi_1^\star(\delta) = \max\{0, F(\delta) - (1 - s)\},$$

which implies that the distribution of types among non-owners is $\Phi_0^\star(\delta) = \min\{F(\delta), 1 - s\}$ because of (1). The price of the asset has to equal the present value of the marginal investor's utility flow for the asset, so that

$$p^\star = \frac{\delta^\star}{r}.$$

Finally, we consider the total volume of trade that occurs at each instant, which we denote by

$\vartheta^\star$. Note that this variable is not uniquely defined: for instance, one can always assume that some investors engage in instantaneous round-trip trades, even if they do not have strict incentives to do so. This leads us to focus on the *minimum* trading volume necessary to accommodate all investors who have strict incentives to trade.

Consider first the case when there is a point mass at $\delta^\star$, so that $F(\delta^\star) > F(\delta^\star_-)$. The set of non-owners that strictly prefer to buy an asset at price $p^\star$ is equal to the flow of investors with zero asset holdings who draw a preference shock $\delta' > \delta^\star$. Similarly, the set of owners that strictly prefer to sell are those that draw a preference shock $\delta' < \delta^\star$. Hence, the volume has to be at least as large as the maximum of these two flows, so that the minimum volume is:

$$\vartheta^\star = \gamma \max\{sF(\delta^\star_-), (1-s)[1-F(\delta^\star)]\}. \tag{4}$$

Notice that, in the continuous case, $1 - F(\delta^\star) = s$, so that the the minimum volume reduces to:

$$\vartheta^\star = \gamma s(1-s). \tag{5}$$

3 Equilibrium with Search Frictions

In this section, we turn to the economy with search frictions. Our solution method goes beyond previous work by characterizing the equilibrium both in and out of steady state, for arbitrary distribution of types, in closed form.

Our characterization proceeds in two steps. First, we derive the reservation value of an investor with asset holdings $q \in \{0, 1\}$ and utility type $\delta \in [0, 1]$ at any time t, taking as given the evolution of the joint distribution of asset holdings and utility types of potential trading partners that such an investor might meet. Importantly, we establish that reservation values are strictly increasing in δ, which implies that an owner of type δ' and a non-owner of type δ have strict gains from trade if $\delta > \delta'$, weak gains from trade if $\delta = \delta'$, and no gains from trades if $\delta < \delta'$. The resulting patterns of exchange allow us to completely characterize the joint distribution of asset holdings and utility types for all t. Given sharp characterizations of both reservation values and distributions, we are able to construct the unique equilibrium and establish that it converges to the steady-state from any initial allocation satisfying (1) and (2) at $t = 0$.

3.1 Reservation values: recursive formulation

We now solve for $V_{q,t}(\delta)$, the maximum attainable utility of an investor with $q \in \{0, 1\}$ units of the asset and utility type $\delta \in [0, 1]$ at time $t \geq 0$. This value is defined for any $\delta \in [0, 1]$, even though the distribution $F(\delta)$ may assign zero probability to some subset of this interval. To state the Bellman equation for $V_{q,t}(\delta)$, it is helpful to define the time at which a preference shock arrives, as well as the time at which meetings arrive with other investors who own q units of the asset. Let τ_γ be an exponentially distributed time with intensity γ, corresponding to the arrival time of a preference shock. Similarly, let $\tau_{\lambda,0}$ and $\tau_{\lambda,1}$ be exponentially distributed with intensities λs and $\lambda(1 - s)$, respectively, corresponding to the arrival rates of buying and selling opportunities. Finally, let $\tau = \min\{\tau_\gamma, \tau_{\lambda,0}, \tau_{\lambda,1}\}$. Then, for an investor who owns an asset, we obtain by an application of Bellman's principle of optimality:

$$V_{1,t}(\delta) = \mathbb{E}_t\Bigg[\int_t^\tau e^{-r(u-t)}\delta\, du + e^{-r(\tau-t)}\Bigg(\mathbf{1}_{\{\tau=\tau_\gamma\}}\int_0^1 V_{1,\tau}(\delta')dF(\delta') + \mathbf{1}_{\{\tau=\tau_{\lambda,1}\}}V_{1,\tau}(\delta) + \mathbf{1}_{\{\tau=\tau_{\lambda,0}\}}\int_0^1 \max\Big\{V_{1,\tau}(\delta), V_{0,\tau}(\delta) + P_\tau(\delta,\delta')\Big\}\frac{d\Phi_{0,\tau}(\delta')}{1-s}\Bigg)\Bigg],$$

where $\mathbf{1}_{\{\cdot\}}$ is the indicator function and $P_\tau(\delta, \delta')$ denotes the solution to the bargaining problem at time τ between an owner of type δ and a non-owner of type δ'. In words, at time τ, there are three possible events for the investor: he receives a preference shock (i.e., $\tau = \tau_\gamma$), in which case a new utility type δ' is randomly drawn from $F(\cdot)$; he meets another owner (i.e., $\tau = \tau_{\lambda,1}$) in which case there are no gains from trade and his continuation utility is $V_{1,\tau}(\delta)$; or he meets a non-owner (i.e., $\tau = \tau_{\lambda,0}$), who is of utility type δ' with probability $d\Phi_{0,\tau}(\delta')/(1 - s)$, in which case the non-owner will purchase the asset at price $P_\tau(\delta, \delta')$ if the payoff from doing so exceeds the payoff from continuing to search.

To determine whether trade occurs between an owner of type δ and a non-owner of type δ', note that the total surplus of a trade can be written as $\Delta V_t(\delta') - \Delta V_t(\delta)$, where

$$\Delta V_t(\delta) \equiv V_{1,t}(\delta) - V_{0,t}(\delta).$$

We refer to $\Delta V_t(\delta)$ as a type-δ investor's *reservation value*, as it represents the most an investor of type δ would be willing to pay in order to acquire a unit of the asset. The gains from trade are positive if $\Delta V_t(\delta') - \Delta V_t(\delta) \geq 0$ and, with Nash Bargaining, the price $P_t(\delta, \delta')$ is equal to

$$P_t(\delta, \delta') = \theta_0 \Delta V_t(\delta) + \theta_1 \Delta V_t(\delta'). \tag{6}$$

Equation (6) is intuitive: for example, when the owner has a lot of bargaining power, $\theta_1 \simeq 1$ and $\theta_0 \simeq 0$, and he manages to sell at a price close to the reservation of the non-owner, $P_t(\delta, \delta') \simeq \Delta V_t(\delta')$.

Substituting this price-setting equation back into the Bellman equation and rearranging, we obtain:

$$\begin{aligned} V_{1,t}(\delta) = \mathbb{E}_t \Bigg[\int_t^\tau e^{-r(u-t)} \delta \, du + e^{-r(\tau-t)} \Bigg(& V_{1,\tau}(\delta) + \mathbf{1}_{\{\tau=\tau_\gamma\}} \int_0^1 \left(V_{1,\tau}(\delta') - V_{1,\tau}(\delta) \right) dF(\delta') \\ & + \mathbf{1}_{\{\tau=\tau_{\lambda,0}\}} \theta_1 \int_0^1 \max\left\{0, \Delta V_\tau(\delta') - \Delta V_\tau(\delta)\right\} \frac{d\Phi_{0,\tau}(\delta')}{1-s} \Bigg) \Bigg]. \end{aligned} \tag{7}$$

Proceeding similarly with the value of a non-owner, we find that

$$\begin{aligned} V_{0,t}(\delta) = \mathbb{E}_t \Bigg[e^{-r(\tau-t)} \Bigg(& V_{0,\tau}(\delta) + \mathbf{1}_{\{\tau=\tau_\gamma\}} \int_0^1 \left(V_{0,\tau}(\delta') - V_{0,\tau}(\delta) \right) dF(\delta') \\ & + \mathbf{1}_{\{\tau=\tau_{\lambda,1}\}} \theta_0 \int_0^1 \max\left\{0, \Delta V_\tau(\delta) - \Delta V_\tau(\delta')\right\} \frac{d\Phi_{1,\tau}(\delta')}{s} \Bigg) \Bigg]. \end{aligned} \tag{8}$$

Subtracting (8) from (7), one obtains a Bellman equation for the reservation value:

$$\begin{aligned} \Delta V_t(\delta) = \mathbb{E}_t \Bigg[\int_t^\tau e^{-r(u-t)} \delta \, du + e^{-r(\tau-t)} \Bigg(& \Delta V_\tau(\delta) + \mathbf{1}_{\{\tau=\tau_\gamma\}} \int_0^1 \left(\Delta V_\tau(\delta') - \Delta V_\tau(\delta) \right) dF(\delta') \\ & + \mathbf{1}_{\{\tau=\tau_{\lambda,0}\}} \theta_1 \int_0^1 \max\left\{0, \Delta V_\tau(\delta') - \Delta V_\tau(\delta)\right\} \frac{d\Phi_{0,\tau}(\delta')}{1-s} \\ & - \mathbf{1}_{\{\tau=\tau_{\lambda,1}\}} \theta_0 \int_0^1 \max\left\{0, \Delta V_\tau(\delta) - \Delta V_\tau(\delta')\right\} \frac{d\Phi_{1,\tau}(\delta')}{s} \Bigg) \Bigg]. \end{aligned} \tag{9}$$

Intuitively, when an investor of type δ acquires an asset, he enjoys flow utility δ from owning the asset until either a preference shock arrives or an opportunity arises to sell the asset. In the latter case, the investor receives a fraction θ_1 of any gains from trade. Finally, since an investor can hold only one asset at a time, there is an opportunity cost of acquiring the asset to begin with, captured by the last term in equation (9): this is the expected value of forgone buying opportunities, where the investor would have captured a fraction θ_0 of any gains from trade. An application of the Contraction Mapping Theorem to (9) yields the following result.

Lemma 1. *There exists a unique, uniformly bounded function $\Delta V : \mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$ satisfying* (9)*. Moreover, this function is differentiable with respect to t, and it is continuous and strictly increasing in δ.*

Note that these properties hold regardless of the distribution dynamics, $\Phi_{0,t}(\delta)$ and $\Phi_{1,t}(\delta)$, generated by other investors' trading behavior. In particular, the strict monotonicity in δ implies that, when an owner of type δ meets a non-owner of type $\delta' > \delta$, they will always agree to trade. Indeed, these two investors face the same distribution of future trading opportunities, and the same distribution of future utility types. Thus, the only relevant difference between the two is their utility types, which implies that the type-δ' investor has a strictly higher reservation value than the type-δ investor. As will be clear in the next section, this monotonicity property greatly simplifies the derivation of distribution dynamics.

Differentiating (9) with respect to time, and using the fact that $\Delta V_t(\delta)$ is strictly increasing to simplify the maximum operators, we arrive at the familiar Hamiltonian-Jacobi-Bellman equation

$$\begin{aligned} r\Delta V_t(\delta) \quad = \quad & \delta + \gamma \int_0^1 [\Delta V_t(\delta') - \Delta V_t(\delta)] dF(\delta') + \lambda\theta_1 \int_\delta^1 [\Delta V_t(\delta') - \Delta V_t(\delta)] d\Phi_{0,t}(\delta') \\ & -\lambda\theta_0 \int_0^\delta [\Delta V_t(\delta) - \Delta V_t(\delta')] d\Phi_{1,t}(\delta') + \Delta\dot{V}_t(\delta). \end{aligned} \tag{10}$$

Let us now assume that the reservation value is differentiable with respect to δ, and that its derivative is a bounded function of time.[8] Then

$$\sigma_t(\delta) = \frac{\partial \Delta V_t(\delta)}{\partial \delta}$$

measures the *local surplus* at δ, since $\sigma_t(\delta)\, d\delta$ represents the gains from trade between an owner

[8] At this stage, these assumptions entail no loss of generality. Their purpose is to construct a closed form solution of the functional equation (9). Since Lemma 1 established uniqueness, our constructed solution must be equal to $\Delta V_t(\delta)$, and so $\Delta V_t(\delta)$ must be differentiable with a bounded derivative.

of type δ and a non-owner of type $\delta + d\delta$. Differentiating equation (10) with respect to δ reveals that the local surplus, $\sigma_t(\delta)$, satisfies

$$R_t(\delta)\sigma_t(\delta) = 1 + \dot{\sigma}_t(\delta),$$

where

$$R_t(\delta) \equiv r + \gamma + \lambda\theta_1 \left[1 - s - \Phi_{0,t}(\delta)\right] + \lambda\theta_0\Phi_{1,t}(\delta). \tag{11}$$

The local surplus is then the unique bounded solution of this ODE, obtained by calculating the present value of flow gains from trades using the effective discount rate, $R_t(\delta)$:

$$\sigma_t(\delta) = \int_t^\infty e^{-\int_t^u R_v(\delta)\,dv} du. \tag{12}$$

This effective discount rate not only accounts for investor' time preference, but also for the fact that gains from trade disappear when investors change utility types and when they find alternative counterparties. Using this expression for $\sigma_t(\delta)$, we can change the order of integration in equation (10) and simplify to get

$$\begin{aligned} r\Delta V_t(\delta) = \delta - &\int_0^\delta \sigma_t(\delta')[\gamma F(\delta') + \lambda\theta_0\Phi_{1,t}(\delta'))]d\delta' \\ &+ \int_\delta^1 \sigma_t(\delta')[\gamma(1 - F(\delta')) + \lambda\theta_1(1 - s - \Phi_{0,t}(\delta'))]d\delta' + \Delta\dot{V}_t(\delta), \end{aligned} \tag{13}$$

which is an ordinary differential equation in t for any $\delta \in [0, 1]$. Since $\Delta V_t(\delta)$ is a uniformly bounded function, there is a unique solution to (13), which we derive explicitly in the following proposition.

Proposition 1. *Given any distributions $\Phi_{0,t}(\delta)$ and $\Phi_{1,t}(\delta)$ that are right-continuous and increasing in δ and TBA, the unique, uniformly-bounded reservation value satisfies*

$$\begin{aligned} \Delta V_t(\delta) = \int_t^\infty e^{-r(u-t)} \Bigg[\delta + &\int_0^\delta \sigma_u(\delta')[\gamma F(\delta') + \lambda\theta_0\Phi_{1,u}(\delta'))]d\delta' \\ &- \int_\delta^1 \sigma_u(\delta')[\gamma(1 - F(\delta')) + \lambda\theta_1(1 - s - \Phi_{0,u}(\delta'))]d\delta'\Bigg] du, \end{aligned} \tag{14}$$

where $\sigma_t(\delta)$ is fully described by (11) *and* (12).

3.2 Reservation values: sequential formulation

Our derivation implies two alternative sequential formulations of reservation values. These formulations are useful to derive natural comparative static. Later, in Section **??**, these formulation will also provide intuitions about the behavior of prices when $\lambda \to \infty$.

Market-adjusted valuation process. First, from (10), one sees clearly that the an investor's reservation value can be seen as the present discounted value of some "market-adjusted" valuation process:

$$\Delta V_t(\delta) = \mathbb{E}\left[\int_t^\infty e^{-r(u-t)}\hat{\delta}_u \, du \,\middle|\, \hat{\delta}_t = \delta\right]. \tag{15}$$

The process $\hat{\delta}_t$ not only takes into account investors' physical changes of types, but also their future trading opportunities. To describe this process, suppose that valuation at time t is δ. Then, during $[t, t+dt]$, the market-adjusted valuation can change for three reasons. With intensity γ, there is a change of type, in which case the new type is drawn according $F(\delta')$. With intensity $\lambda\theta_0\Phi_{1,t}(\delta)$, there is a purchase opportunity. Conditional on the arrival of a purchasing opportunity, the new type is drawn from the support $[0, \delta]$ according to the CDF $\Phi_{1,t}(\delta')/\Phi_{1,t}(\delta)$. Note that the market-adjusted valuation process accounts for future purchasing opportunities by creating transitions towards lower types. Indeed, as we noted before, the opportunity to purchase the asset in the future from lower types tends to reduce an investor's willingness to pay for the asset. Note also that the market adjustment scales down the physical intensity of future purchase opportunities by the investor's bargaining power θ_0. Finally, with intensity $\lambda\theta_1\left[1 - s - \Phi_0(\delta)\right]$, there is a sale opportunities. Conditional on the arrival of a sale opportunity, the new type is drawn from the support $[\delta, 1]$ according to the CDF $\left[\Phi_{0,t}(\delta') - \Phi_{0,t}(\delta)\right] / \left[1 - s - \Phi_{0,t}(\delta)\right]$. This time, the market-adjusted valuation process accounts for future sale opportunities by creating transitions towards *higher* types. Indeed, the opportunity to re-sell the asset in the future from higher types tends to increase willingness to pay for the asset. Note also that the market adjustment scales down the physical intensity of future purchasing opportunity by the investor's bargaining power θ_1.

Note that, in a Walrasian market, the market-adjusted valuation process is $\hat{\delta}_t = \delta^\star$: it is equal to the valuation of the marginal investor at all times, regardless of an investor's current type. In our decentralized market, the market-adjusted valuation process differs for two reasons. First, investors cannot trade instantly and so must enjoy his private utility flow until he finds counterparty. Second, conditional on finding a counterparty, investors are not trading against the same "marginal type". Instead, the terms of trade are random and depends on the distributions of potential counterparties, $\Phi_{0,t}(\delta)$ and $\Phi_{1,t}(\delta)$.

Expected valuation under discounted occupation measure. One can further simplify expression (15) into

$$\Delta V_t(\delta) = \frac{\int_0^1 \delta' d\Psi_t\left(\delta' \mid \delta\right)}{r} \tag{16}$$

where the CDF $\Psi_t\left(\delta' \mid \delta\right)$ is defined as:

$$\Psi_t\left(\delta'|\delta\right) \equiv \mathbb{E}\left[\int_t^\infty re^{-r(u-t)}1_{\{\hat{\delta}_u \leq \delta'\}}du \middle| \hat{\delta}_t = \delta\right]. \tag{17}$$

In words, the CDF $\Psi_t(\delta' \mid \delta)$ is a discounted occupation measure: it is the discounted amount of time that the market-adjusted valuation process, $\hat{\delta}_t$, spends visiting valuation less than δ'.

To derive comparative statics, assume that the distributions $\Phi_{1,t}(\delta)$ and $\Phi_{0,t}(\delta)$, are time invariant (as will be the case in a steady state equilibrium). In this case, direct comparison with the equation for resevation values, $\Delta V(\delta)$, shows that:

$$\Psi\left(\delta'|\delta\right) = \begin{cases} \frac{\gamma F(\delta')+\lambda\theta_0\Phi_1(\delta')}{r+\gamma+\lambda\theta_0\Phi_1(\delta')+\lambda\theta_1[1-s-\Phi_0(\delta')]} & \text{if } \delta' < \delta \\ \frac{r+\gamma F(\delta')+\lambda\theta_0\Phi_1(\delta')}{r+\gamma+\lambda\theta_0\Phi_1(\delta')+\lambda\theta_1[1-s-\Phi_0(\delta')]} & \text{if } \delta' \geq \delta. \end{cases} \tag{18}$$

Note in particular that the distribution has an atom at δ: indeed, the market-adjusted valuation process will stay at δ for some non-negligible amount of time, until the investor switches type or receives a new opportunity to trade. We also obtain the following natural comparative statics:

Proposition 2. *Suppose that the distributions $\Phi_0(\delta)$ and $\Phi_1(\delta)$ are time invariant. Then reservation values decrease with the bargaining power of non-owners, θ_0, and increase with the bargaining power of owners, θ_1. They increase with first-order stochastic dominance (FOSD) shifts in both*

$\Phi_0(\delta)$ *and* $\Phi_1(\delta)$.

Suppose we increase the bargaining power of non-onwers, θ_0. Then, non-owners can buy at lower prices, and so all reservation values decrease. To see this effects at work in the occupation measure formula, note that increasing θ_0 increases $\Psi\left(\delta' \,|\, \delta\right)$, i.e., it creates a negative first-order-stochastic dominance shift in the discounted occupation measure. Thus, all reservation values decrease. Symmetrically, when we increase the bargaining power of owners, θ_1, they can sell at higher prices and so all reservation values increase.

Finally, suppose we increase the valuations of non-owners. Then, owners can sell at higher prices and so this increases all reservation values. To see this effect at work in the occupation measure formula, let us engineer a positive FOSD shift in the distribution of non-owners' valuation by decreasing $\Phi_0(\delta)$. Then, this decreases $\Psi\left(\delta' \,|\, \delta\right)$ and so creates a postive FOSD shift in the discounted occupation measure. Thus, it increases all reservation values. The effect of a positive FOSD shift in $\Phi_1(\delta)$ is symmetric.

3.3 The Joint Distribution of Asset Holdings and Types

In this section, we provide a closed-form characterization of the joint distribution of asset holdings and utility types that prevail in equilibrium for any $t \geq 0$. We then establish that this distribution converges strongly to the steady-state distribution from any initial conditions satisfying (1) and (2).

Since $\Delta V_t(\delta)$ is strictly increasing in δ, trade occurs between two investors if one of them is an owner of the asset with utility type δ' and the other does not own the asset and has utility type $\delta'' > \delta'$.Investors with the same δ are indifferent between trading or not, but whether or not they trade is irrelevant for the distribution since the owner and the non-owner effectively exchange type: the owner of type δ becomes a non-owner of type δ and vice versa. As a result, the change in the measure of owners with utility type less than δ can be written

$$\dot{\Phi}_{1,t}(\delta) = \gamma\left[s - \Phi_{1,t}(\delta)\right]F(\delta) - \gamma\Phi_{1,t}(\delta)\left[1 - F(\delta)\right] - \lambda\Phi_{1,t}(\delta)\left[1 - s - \Phi_0(\delta)\right]. \tag{19}$$

The first term in equation (19) is the inflow due to type-switching: at each instant, a measure $\gamma\left[s - \Phi_{1,t}(\delta)\right]$ of owners with utility type greater than δ draw a new utility type, which is less than or equal to δ with probability $F(\delta)$. Similar logic can be used to understand the second term, which is the outflow due to type-switching. The third term is the outflow due to trade. In particular, a measure $(\lambda/2)\Phi_{1,t}(\delta)$ of investors who own the asset and have utility type less than δ initiate contact with another investor, and with probability $1 - s - \Phi_0(\delta)$ that investor is a non-owner with utility type greater than δ, so that trade ensues. The same measure of trades occur when non-owners with utility type greater than δ initiate trade with owners with utility type less than δ, so that the sum equals the third term in (19). Note that trading generates *no net inflow* into this set of owners with utility type less than δ. Indeed, such inflow from trade requires that a non-owner with utility type $\delta' \leq \delta$ meets an owner with an even lower utility type $\delta'' < \delta'$. By trading, the previous owner of type δ'' leaves the set but the new owner of type δ' enters the same set, resulting in no net inflow.

Using equation (1), equation (19) can be re-written as

$$\dot{\Phi}_{1,t}(\delta) = \Phi_{1,t}(\delta)^2 + \Phi_{1,t}(\delta)\left[1 - s - F(\delta) + \frac{\gamma}{\lambda}\right] - s\frac{\gamma}{\lambda}F(\delta). \tag{20}$$

Note that we are not imposing any assumptions about continuity with respect to δ; the ODE in (20) works *for each* δ. Proposition 3 below provides an explicit expression for the unique solution to this Ricatti equation and shows that it converges to the steady state. To state the result, let

$$\Phi_1(\delta) \equiv \frac{-\left[1 - s + \frac{\gamma}{\lambda} - F(\delta)\right] + \Lambda(\delta)}{2} \tag{21}$$

denote the steady state distribution of owners with utility type less than δ, i.e., the unique, strictly positive solution to $\dot{\Phi}_{1,t}(\delta) = 0$, with

$$\Lambda(\delta) \equiv \sqrt{\left(1 - s + \frac{\gamma}{\lambda} - F(\delta)\right)^2 + 4s\frac{\gamma}{\lambda}F(\delta)}. \tag{22}$$

Proposition 3. *The measure of asset owners with utility type less than δ at time t is given by*

$$\Phi_{1,t}(\delta) = \Phi_1(\delta) + \frac{\left[\Phi_{1,0}(\delta) - \Phi_1(\delta)\right]\Lambda(\delta)}{\Lambda(\delta) + \left[\Phi_{1,0}(\delta) - \Phi_1(\delta) + \Lambda(\delta)\right]\left(e^{\lambda\Lambda(\delta)t} - 1\right)} \tag{23}$$

and converges strongly to the steady state distribution $\Phi_1(\delta)$ from any initial conditions satisfying

(1) *and* (2) *at* $t = 0$.

To illustrate the convergence of the equilibrium distributions to the steady state, Figure **??** plots the equilibrium distributions among owners and non owners at various points in time in a simple environment where utility types are uniformly distributed among the whole population and $\Phi_{1,0}(\delta) = sF(\delta)$. As can be seen from the figure, the distribution of types among owners is initially uniform but as time passes this distribution moves down and to the right, indicating that assets are gradually allocated towards investors with higher valuations. Similarly, the distribution of utility types among non owners gradually shifts up and to the left, indicating that low valuation investors are less and less likely to hold the asset as time passes.

3.4 Equilibrium

An *equilibrium* is a collection of reservation values $\Delta V_t(\delta)$, prices $P_t(\delta, \delta')$, and distributions, $\Phi_{0,t}(\delta)$ and $\Phi_{1,t}(\delta)$ such that, for $t \geq 0$ and $(\delta, \delta') \in [0,1]^2$: the distribution $\Phi_{1,t}(\delta)$ satisfies (23) and $\Phi_{0,t}(\delta) = F(\delta) - \Phi_{1,t}(\delta)$; the reservation value $\Delta V_t(\delta)$ satisfies (14), given $\Phi_{0,t}(\delta)$ and $\Phi_{1,t}(\delta)$; and prices $P_t(\delta, \delta')$ satisfy (6) given $\Delta V_t(\delta)$. Given our results above, Theorem 1 follows immediately.

Theorem 1. *There exists a unique equilibrium.*

Note that uniqueness follows from the fact that we proved reservation were strictly increasing directly, given arbitrary time paths for $\Phi_{0,t}(\delta)$ and $\Phi_{1,t}(\delta)$, rather than guessing and verifying that such an equilibrium exists.

4 Analysis

We now use the characterization from Section 3 to study the effects of trading frictions on equilibrium outcomes. We break our analysis into two parts: in Section 4.1, we study the trading patterns and resulting allocations that arise in equilibrium; and then, in Section 4.2, we study the implications for asset prices. In both sections, we derive results when λ is finite and search frictions are present, and then we study what happens as λ approaches infinity.

4.1 Trading Patterns and Allocations

In this section we study the process through which bilateral trades re-allocate assets towards those investors who value it most. Since trade is decentralized and takes time, the asset cannot be allocated perfectly. However, even though search is random, the patterns of trades and misallocation are not: assets are passed along quickly from those with very low valuations, and then exchanged more slowly as the owner's valuation increases. We show how this trading pattern implies that misallocation tends to cluster around investors with valuations near the marginal type. Since these investors are often holding the "wrong" portfolio, they trade relatively frequently, which has two important implications. First, trading volume is augmented by these "infra-marginal trades" that would not occur in centralized markets. Second, as this cluster of traders tend to account for a large fraction of trading volume, a core-periphery network structure emerges endogenously. We show that these patterns become more pronounced as friction vanish: all misallocation clusters near the marginal type, volume goes to infinity and is, for the most part, accounted for by infra-marginal trades.

4.1.1 Time to Trade.

Consider an asset that is initially owned by an investor who has a very low utility type, say $\delta \simeq 0$. As there are many non-owners who value the asset more than δ, the asset will likely be sold relatively quickly to an investor with a higher utility type $\delta' > \delta$: in particular, the intensity with which the owner of type δ will find a profitable trade with a non-owner of type $\delta' > \delta$ is equal to $\lambda \left[1 - s - \Phi_0(\delta)\right]$. Next, absent a preference shock, the investor of type δ' will continue to search until he meets a non-owner with valuation $\delta'' > \delta'$. This trade will likely take longer to materialize than the first one, since there are fewer non-owners with utility type greater than δ': indeed, since $\delta' > \delta$, the intensity with which this second trade occurs

$$\lambda \left[1 - s - \Phi_0(\delta')\right] < \lambda \left[1 - s - \Phi_0(\delta)\right]$$

is lower than for the first trade. Absent any preference shocks, this process will continue and the asset will be allocated to investors with higher and higher utility types, at a slower and slower speed.

To formalize this intuition, let $\eta_q(\delta)$ denote the expected amount of time before a trade occurs for an investor with $q \in \{0, 1\}$ units of the asset and utility type δ. Lemma 2, below, characterizes η_q in closed form when $F(\delta)$ is continuous, and offers several natural comparative statics.

Lemma 2. *Suppose $F(\delta)$ is continuous. Then*

$$\eta_q(\delta) = \left[\phi \log\left(\frac{\phi}{1+\phi}\right) + \left(1 - \frac{1+\phi}{\Phi_q(\delta^e)}\right) \log\left(1 - \frac{\Phi_q(\delta^e)}{1+\phi}\right)\right]^{-1} \frac{1}{\gamma + \lambda_q(\delta)}, \tag{24}$$

where $\phi = \gamma/\lambda$, $\delta^e = \int_0^1 \delta dF(\delta)$, and $\lambda_q(\delta) = \lambda q(1 - s - \Phi_0(\delta)) + \lambda(1-q)\Phi_1(\delta)$. The steady state expected time to trade is decreasing in δ for non-owners, increasing in δ for owners, and decreasing in λ for both owners and non-owners.

Figure 1 plots $\eta_0(\delta)$ and $\eta_1(\delta)$ for various values of λ, holding γ fixed. Note that trading times are S-shaped with an inflection point near the marginal type so that, e.g., owners with a valuation below this inflection point typically sell fast, while those with valuations above the inflection point sell much slower.

Insert figure 1 here.

4.1.2 Misallocation.

The trading patterns described above imply that misallocation peaks near the marginal type, $\delta^\star$. To see this, note first that misallocation has two symptoms: some assets are owned by the "wrong" investors, with type $\delta < \delta^\star$, and some of the "right" investors, with type $\delta > \delta^\star$, do not own an asset. These two symptoms are captured by the following measures of cumulative misallocation, respectively:

$$\begin{aligned} \delta < \delta^\star &: M_1(\delta) = \Phi_1(\delta) \\ \delta > \delta^\star &: M_0(\delta) = 1 - s - \Phi_0(\delta). \end{aligned}$$

To measure misallocation at a specific utility type $\delta < \delta^\star$, one can simply calculate $\frac{dM_1(\delta)}{dF(\delta)}$, which is the fraction of type-δ investors who own an asset in the equilibrium with search frictions, but would not own an asset in the frictionless benchmark. Likewise, for $\delta > \delta^\star$, $-\frac{dM_1(\delta)}{dF(\delta)}$ measures the fraction of type δ investors who do not own an asset in our equilibrium, but would own one in the absence of search frictions. Since the speed with which an owner trades is decreasing in δ, the likelihood that an investor owns an asset—and hence $d\Phi_1(\delta)$—is increasing in δ By the same reasoning, an investor with a higher δ is less likely to be a non-owner. Thus, for $\delta > \delta^\star$, $-\frac{dM_0(\delta)}{dF(\delta)}$ is decreasing with δ. The following result follows immediately.

Lemma 3. *The functions* $\frac{dM_1(\delta)}{dF(\delta)}$ *and* $-\frac{dM_0(\delta)}{dF(\delta)}$ *achieve their maximum at* $\delta^\star$*, over their respective domains.*

While the slopes of $\eta_0(\delta)$ and $\eta_1(\delta)$ are sufficient to understand why misallocation peaks at $\delta^\star$, the *shape* of these functions tell us even more about the patterns of misallocation. In particular, as λ increases, trading intensities are substantially smaller for owners in a neighborhood of $\delta^\star$ than they are for of owners with valuations that are signficantly less than $\delta^\star$ (the opposite is true for non-owners). As Figure 2 illustrates, this implies that misallocation is concentrated in a cluster around the marginal type.

Insert Figure 2 here.

Note that this pattern of misallocation arises because trade is fully decentralized, and would not arise in a model in which investors trade through dealers. Indeed, in such an environment, all investors would contact dealers and trade with the same intensity, so that the measure of misallocation described above would be constant across types.

4.1.3 Volume and the Structure of the Trading Network.

The discussion above highlights the fact that, in a purely decentralized market, assets are reallocated over time through chains of bilateral and infra-marginal trades, a phenomenon that has been pointed out before in both the search and the network literatures. To see the contribution of these trades to volume, note that we can express trading volume as:

$$
\begin{aligned}
\vartheta &= \int_0^1 \lambda\Phi_1(\delta)d\Phi_0(\delta) = \lambda\int_0^{\delta^\star} d\Phi_0(\delta)\Phi_1(\delta) + \lambda\int_{\delta^\star}^1 d\Phi_0(\delta)\Phi_1(\delta) \\
&= \lambda\Phi_1(\delta^\star)\left[1 - s - \Phi_0(\delta^\star)\right] + \lambda\int_0^{\delta^\star} d\Phi_0(\delta)\Phi_1(\delta) + \lambda\int_{\delta^\star}^1 d\Phi_1(\delta)\left[1 - s - \Phi_0(\delta)\right], \qquad (25)
\end{aligned}
$$

where the second line follows from integration by parts. The first term is the volume that is due to trades between owners of type $[0, \delta^\star)$ and non-owners of type $[\delta^\star, 1]$. In a Walrasian model, or in a model in which agents trade through dealers, these would be the only trades taking place in equilibrium. Indeed, the price in these models would be set by the marginal type, so an investor below $\delta^\star$ would only find it optimal to sell, and an investor above $\delta^\star$ would only find it optimal to buy. In our environment with search frictions, there are additional trades because any investor may end up buying or selling depending on who she meets. These additional trades are captured by the second and third terms in (25): the second term is the measure of trades between investors of type $\delta < \delta^\star$ who purchase the asset when they meet owners with lower types; and the third term is the measure of trades between investors of type $\delta > \delta^\star$ who sell the asset when they meet non-owners with higher types.

Lemma 4. *If $F(\delta)$ is strictly increasing and continuous, then the steady state trading volume is*

$$
\vartheta = \gamma s(1-s)\left[\left(1 + \frac{\gamma}{\lambda}\right)\log\left(1 + \frac{\lambda}{\gamma}\right) - 1\right], \qquad (26)
$$

is strictly increasing in λ and γ.

Equation (26) offers some natural comparative statics: trading volume increases in λ because investors can find counterparties faster, and it increases in γ because investors change type more often. Also note that, when the distribution of types is continuous, the trading volume is actually independent of the distribution. This is because, in our model, trading patterns are purely ordinal: they only depend on investors' rank in the distribution, not on the intensity of their taste for holding assets.[9]

[9]When the distribution of types is discrete, measuring volume requires specifying whether investors with identical valuations trade with each other. With a continuous distribution, such meetings have zero measure and so do not matter for volume. In Appendix **??**, we show that, if investors never trade when their utility types are the same, then trading

As noted above, as λ increases, misallocation begins to vanish at extreme values of δ, and clusters around the marginal type, $\delta^{\star}$. Therefore, since investors outside of this cluster rarely have an occasion to trade, the share of trading volume accounted for by investors within this cluster grows. Hence, a core-periphery network structure emerges endogenously: over any time interval, if one created a connection between every pair of investors who trade, the network would exhibit what (Jackson, 2010, p. 67) describes as a "core of highly connected and interconnected nodes and a periphery of less-connected nodes." To illustrate this phenomenon, figure 3a plots the share of sales accounted for by investors of each type, while Figure 3b plots the intensity of sales by each owner-nonowner pair.

Figure 3 here.

4.1.4 Trading patterns near the frictionless limit

We now consider the frictionless limit, $\lambda \to \infty$, and show that the trading patterns discussed above become much more pronounced: while the level of misallocation goes to zero, its distribution clusters more and more near the marginal type, resulting in larger and larger trading volume. This trading volume is, for the most part, accounted for by trades between investors in the cluster. To derive these results we first establish some basic results about allocations as frictions vanish.

Lemma 5. *The steady state distribution, $\Phi_1(\delta)$, is decreasing in λ with $\lim_{\lambda \to 0} \Phi_1(\delta) = sF(\delta)$ and $\lim_{\lambda \to \infty} \Phi_1(\delta) = \Phi_1^{\star}(\delta)$. Correspondingly, the measures of misallocation, $M_1(\delta)$ and $M_0(\delta)$, decrease with λ and converge to zero as $\lambda \to \infty$.*

As $\lambda \to 0$, the frequency of trading opportunities becomes arbitrarily small relative to the frequency of preference shocks, and the distribution of asset holdings and valuations become independent: thus, $\Phi_1(\delta)$ is equal to the probability of holding an asset, s, multiplied by the probability of drawing a type below δ. As trading opportunities become more frequent, λ increases, there is a first order stochastic dominant shift in $\Phi_1(\delta)$, which implies that the asset is being allocated to agents with higher valuations more efficiently. Finally, in the limiting case of $\lambda \to \infty$, the allocation coincides with that of the frictionless benchmark.

volume is strictly smaller than ϑ. If they always trade, then it is strictly greater. Furthermore, there exits a trading rule such that volume is exactly equal to ϑ.

Misallocation. Next, we derive asymptotic misallocation. For this, one needs to go back to the inflow-outflow steady-state equation:

$$\gamma F(\delta)\left[s-\Phi_1(\delta)\right]=\gamma\Phi_1(\delta)\left[1-F(\delta)\right]+\lambda\Phi_1(\delta)\left[1-s-\Phi_0(\delta)\right]. \tag{27}$$

When $\delta < \delta^\star$, it follows from Lemma 5 that $\Phi_1(\delta) \to 0$ and $1-s-\Phi_0(\delta) \to F(\delta^\star) - F(\delta)$. This implies that the left-hand side is $\gamma s F(\delta) + o(1)$, and the right-hand side is $o(1) + \lambda\left[F(\delta^\star) - F(\delta) + o(1)\right]\Phi_1(\delta)$. Therefore:

$$\text{for } \delta < \delta^\star: \; M_1(\delta) = \Phi_1(\delta) = \frac{1}{\lambda}\frac{\gamma F(\delta) s}{\left[F(\delta^\star) - F(\delta)\right]} + o\left(\frac{1}{\lambda}\right).$$

Thus, the measure of asset owners with type less than $\delta < \delta^\star$ vanishes at rate $\frac{1}{\lambda}$. However, note that, as δ approaches the marginal type $\delta^\star$, the multiplicative coefficient of $\frac{1}{\lambda}$ becomes larger and larger, meaning that the convergence is slower and slower. This reflects the fact that the asset takes longer and longer to change hands as δ approaches $\delta^\star$: when $\lambda \to \infty$, the asset is almost perfectly allocated and so it becomes very difficult to find investors with type $\delta > \delta^\star$ who do not already own assets.

By symmetry, we obtain:

$$\text{for } \delta > \delta^\star: \; M_0(\delta) = \frac{1}{\lambda}\frac{\gamma\left[1-F(\delta)\right](1-s)}{F(\delta) - F(\delta^\star)} + o\left(\frac{1}{\lambda}\right).$$

Finally, we derive what happens at the point $\delta^\star$. For this we observe that $M_0(\delta^\star) = 1-s-\Phi_0(\delta^\star) = 1-s-F(\delta^\star)+\Phi_1(\delta^\star) = M_1(\delta^\star)$, since $1-s = F(\delta^\star)$ and $\Phi_1(\delta^\star) = M_1(\delta^\star)$. Plugging this back into (27) shows that $\lambda\Phi_1(\delta^\star)^2$ has a non-zero limit, which can be written as:

$$M_1(\delta^\star) = M_0(\delta^\star) = \sqrt{\frac{\gamma s(1-s)}{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right).$$

Taken together, these calculations show that, while misallocation becomes arbitrarily small as $\lambda \to \infty$, it clusters more and more near the marginal type. This can be seen clearly by calculating the distribution of misallocated assets across utility type $[0, \delta^\star]$, whose cdf is $\frac{M_1(\delta)}{M_1(\delta^\star)}$. According to the above calculation, this distribution converges to a Dirac distribution at $\delta^\star$.

Trading intensity. We focus for brevity on owners. Using the above asymptotic expansions, we obtain that:

$$\lambda_1(\delta) = \begin{cases} \lambda\left[F(\delta^\star) - F(\delta)\right] + o\left(\lambda\right) & if\ \delta < \delta^\star \\ \sqrt{\lambda s(1-s)} + o\left(\sqrt{\lambda}\right) & if\ \delta = \delta^\star \\ \frac{\gamma[1-F(\delta)]}{F(\delta)-F(\delta^\star)} + o(1) & if\ \delta > \delta^\star. \end{cases} \tag{28}$$

The case of non-owners is symmetric. The formula reveals that, when an investor below the marginal type owns an asset, he will sell very quickly, at an intensity going to infinity in order λ. When the asset arrives in the hands of an investor near the marginal type, the trading intensity remains very large but decreases sharply, as it is now going to infinity in order $\sqrt{\lambda}$. When the asset crosses the marginal type and is held by $\delta > \delta^\star$, it continues being traded, but at finite intensities. Indeed, owners with $\delta > \delta^\star$ meet new counterparties at rate λ, but only a vanishingly small fraction of these counterparties, in order $1/\lambda$, are willing to trade. In the limit, these two effects cancel out exactly and the trading intensity is finite.

Volume. As shown in (25), trading volume has three terms. The first term represents trades between owners of type $[0, \delta^\star)$ and non-owners of type $[\delta^\star, 1]$. From the expansion of $M_1(\delta^\star)$, this term converges to $\gamma s(1-s)$, which is the Walrasian volume.

The second and the third term captures the volume created by the inframarginal trades arising because of the OTC market structure. One sees that the asymptotic volume created by these trades will be determined by two forces going in opposite directions. Consider for instance an owner of type $\delta > \delta^\star$. Because misallocation vanishes, the measure of profitable infra-marginal trading opportunities, $1 - s - \Phi_0(\delta)$, goes to zero in order $1/\lambda$, which tends to reduce volume. At the same time, this investor finds trading opportunities faster and faster, with intensity λ, which tends to increase volume. However, for any fixed $\delta > \delta^\star$, these two forces cancel out exactly: the rate at which the investor finds an infra-marginal trading opportunity goes to a finite limit, as shown in equation (28). The asymptotic infra-marginal volume contributed by owners of type $\delta > \delta^\star$ is:

$$\lim_{\lambda\to\infty} \lambda d\Phi_1(\delta)\Phi_0(\delta) = dF(\delta)\,\frac{\gamma\left[1 - F(\delta)\right]}{F(\delta) - F(\delta^\star)}.$$

But one sees that this asymptotic infra-marginal volume contributed by owners $\delta > \delta^\star$ becomes arbitrarily large as δ approaches $\delta^\star$. This is, again, an implication of the fact that non-owners of type $\delta > \delta^\star$ tend to cluster near the marginal type. Moreover, when one adds up all these contributions to volume, one actually finds that the asymptotic volume is infinite. This can be immediately seem from equation (26), which shows that volume goes to infinity as λ goes to infinity.

Corollary 1. *As λ goes to infinity, volume goes to infinity. Moreover, for any $\varepsilon > 0$:*

$$\lim_{\lambda\to\infty} \frac{\lambda \int_{\delta^\star-\varepsilon}^{\delta^\star} d\Phi_0(\delta)\Phi_1(\delta) + \lambda \int_{\delta^\star}^{\delta^\star+\varepsilon} d\Phi_1(\delta)\left[1 - s - \Phi_0(\delta)\right]}{\vartheta} = 1,$$

that is, most of the volume is accounted for by the infra-marginal trades of investors near $\delta^\star$.

4.2 Prices

In this section, we study the implications of the model for prices. We show two main results. First, as $\lambda \to \infty$, all bilateral prices converge to the Walrasian limit. With a continuum of utility types, convergence occurs at a slow rate, in order $\frac{1}{\sqrt{\lambda}}$. With a discrete number of utility types, by contrast, the convergence rate is generically much faster, in order $\frac{1}{\lambda}$. Second, price dispersion converges to zero at a much faster rate, in order $\frac{\log(\lambda)}{\lambda}$. Therefore, our model suggests that it would be misleading to infer the price impact of OTC market frictions based on the level of price dispersion. Price dispersion can be very low, while the impact of frictions on the level of all prices can be much larger.

Market-adjusted valuation with small frictions. Recall from equation (15), in Section 3.2, that reservation values can be viewed as the present discounted value of a "market-adjusted" valuation process $\hat{\delta}_t$. We now analyze the behavior of this process when $\lambda \to \infty$.

If $\hat{\delta}_t < \delta^\star$ then, as $\lambda \to \infty$, the selling intensity $\lambda \left[1 - s - \Phi_0(\hat{\delta}_t)\right]$ goes to infinity, but the buying intensity $\lambda\Phi_1(\hat{\delta}_t)$ has a finite limit. That is, because the asset is almost perfectly allocated, it is much faster to find a buyer than a seller. As a result, the market-adjusted valuation process, $\hat{\delta}_t$, switches up very quickly. Similarly, if $\hat{\delta}_t > \delta^\star$ then, as $\lambda \to \infty$, the selling intensity $\lambda\left[1-s-\Phi_0(\hat{\delta}_t)\right]$ has a finite limit and the buying intensity $\lambda\Phi_1(\hat{\delta}_t)$ goes to infinity, so that $\hat{\delta}_t$ switches down very quickly. Taken together, these observations imply that, as $\lambda \to \infty$, the market-adjusted valuation process is attracted towards $\delta^\star$. This implies that:

Proposition 4. *As $\lambda \to \infty$, reservation values and bilateral prices all converge towards the Walrasian price, $\frac{\delta^\star}{r}$.*

Next, we analyze the speed of convergence towards the Walrasian price. Since the market-adjusted valuation process, $\hat{\delta}_t$, is attracted very quickly towards $\delta^\star$, the asymptotic behavior of reservation values is ultimately determined by the behavior of $\hat{\delta}_t$ near $\delta^\star$. Moreover, we have shown that, because the asset is almost perfectly allocated, it is harder and harder to find trading opportunities as $\hat{\delta}_t$ approaches $\delta^\star$: buyers become very scarce as $\hat{\delta}_t \uparrow \delta^\star$, and sellers become very scarce as $\hat{\delta}_t \downarrow \delta^\star$. As a result, the speed of convergence of the market-adjusted valuation process, $\hat{\delta}_t$, is much slower near $\delta^\star$: it is roughly in order $\sqrt{\lambda}$. This is the valuation counterpart of the observation that misallocation and volume are very large near $\delta^\star$.

Since the market-adjusted valuation process, $\hat{\delta}_t$, converges to $\delta^\star$ at a speed in order $\sqrt{\lambda}$, we study:

$$\sqrt{\lambda}\left[\Delta V(\delta)-\frac{\delta^\star}{r}\right]=\mathbb{E}\left[\int_0^\infty e^{-rt}\sqrt{\lambda}\left(\hat{\delta}_t-\delta^\star\right)dt\,\middle|\,\hat{\delta}_0=\delta\right]=\mathbb{E}\left[\int_0^\infty e^{-rt}\hat{x}_t\,|\,x_0=\sqrt{\lambda}\,(\delta-\delta^\star)\right]$$

where $\hat{x}_t=\sqrt{\lambda}\left(\hat{\delta}_t-\delta^\star\right)$ is a scaled version of the market-adjusted valuation process. Our main result is:

Proposition 5. *As $\lambda \to \infty$, the discounted occupation measure for the scaled market-adjusted valuation process, $\hat{x}_t=\sqrt{\lambda}\left(\hat{\delta}_t-\delta^\star\right)$, converges pointwise to:*

$$\Psi^\star(x)=\frac{\theta_0 g(x)}{\theta_0 g(x)+\theta_1 g(-x)},$$

with support $(-\infty,\infty)$, and where $g(x)$ is the positive solution of $g(x)^2-g(x)F'(\delta^\star)x-\gamma s(1-$

$s) = 0$. *Moreover, reservation values admit the approximation:*

$$\Delta V(\delta) = \frac{\delta^\star}{r} + \frac{\int_{-\infty}^{\infty} x d\Psi^\star(x)}{r} + o\left(\frac{1}{\sqrt{\lambda}}\right)$$

$$\text{where } \int_{-\infty}^{\infty} x d\Psi^\star(x) = \frac{\pi}{F'(\delta^\star)}\left(\frac{1}{2} - \theta_0\right)\left(\frac{\gamma s(1-s)}{\theta_0\theta_1}\right)^{\frac{1}{2}}\frac{1}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right).$$

The first term of the expansion follows from Proposition 4. The second term of the expansion determines how reservation values and prices deviate from their Walrasian limits. It has three components. The first component is the average time with which the marginal type find counterparties, which we know from the study of asymptotic trading intensities is in order $\frac{1}{\sqrt{\lambda}}$. The second component is the size of trading gains in a typical match. We know that the distribution of misallocation converges towards a Dirac at $\delta^\star$: this means that a marginal type is most likely to trade with investors in the cluster near $\delta^\star$. The size of the corresponding trading gains is, in turns, determined by the heterogeneity of valuation between investors in the cluster, which is measured by $\frac{1}{F'(\delta^\star)}$. For example, $F'(\delta^\star)$ is very large, then the density of investors concentrates at $\delta^\star$ and so investors' valuations in the cluster are all very close to $\delta^\star$, and trading gains are small. Finally, the relative bargaining power of investors, θ_0 and θ_1, determines whether the asset is sold at a discount or at a premium. If $\theta_0 > \frac{1}{2}$, then buyers have most of the bargaining power and so, in all bilateral meetings, the asset is sold at a discount relative to the Walrasian price, and vice versa if $\theta_0 < \frac{1}{2}$.

To further emphasize the role of valuation heterogeneity, consider what happens when the continuous distribution approximates a discrete distribution (ADD FIGURE). Then, at the marginal type, the cdf $F(\delta)$ will be a smooth approximation of a step function, with $F'(\delta^\star) \simeq \infty$, and so the deviation from the Walrasian price will be much smaller. Indeed, if we work out the asymptotic expansion with discrete instead of continuous types, then the rate of convergence is much faster, in order $\frac{1}{\lambda}$ instead $\frac{1}{\sqrt{\lambda}}$.

Finally, the Proposition shows that to a first-order approximation, there is no price dispersion: the coefficient multiplying the converge at rate, $\frac{1}{\sqrt{\lambda}}$, is independent of δ. To obtain further results about price dispersion, one needs to work out higher order terms:

Proposition 6. *Suppose that the distribution of utility type, $F(\delta)$, is twice continuously differen-*

tiable with a strictly positive derivative. Then, price dispersion admits the asymptotic expansion:

$$\Delta V(1) - \Delta V(0) = \int_0^1 \sigma(\delta)\, d\delta = \frac{1}{2\theta_0\theta_1 F'(\delta^\star)} \frac{\log(\lambda)}{\lambda} + O\left(\frac{1}{\lambda}\right). \tag{29}$$

Price dispersion vanishes at a rate $\frac{\log(\lambda)}{\lambda}$, an order of magnitude faster rate than the price discount or premium. Empirically, this means that inferring the impact of the search frictions based on the observable level of price dispersion can be misleading. Frictions can have a very small impact on dispersion, while at the same time have a much larger impact on the price discount or premium.

5 Conclusion

To be added.

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A Proofs

A.1 Proof of Proposition 2

An increase in θ_0 increases $\Psi\left(\delta' \mid \delta\right)$ and an increase in θ_1 decreases $\Psi\left(\delta' \mid \delta\right)$. Together with the restriction that $\theta_1 = 1 - \theta_0$, this implies that an increase in θ_0 induces a negative first-order stochastic dominance shift in the discounted occupation measure, and so decreases all reservation values. The effect of an increase in θ_1 is symmetric.

A positive first-order stochastic dominance shift in $\Phi_0(\delta)$ results in a decrease in $\Phi_0(\delta)$, decreases $\Psi\left(\delta' \mid \delta\right)$, induces a positive first-order stochastic dominance shift in the discounted occupation measure, and so increases all reservation values. The effect of a positive first-order stochastic dominance shift in $\Phi_1(\delta)$ is symmetric.

A.2 Proof of Proposition 4 and 5

After evaluating (14), at $\delta = \delta^\star$ and at the steady state, we obtain that

$$r\Delta V(\delta^\star) = \delta^\star - \frac{D(\lambda)}{\sqrt{\lambda}} + \frac{P(\lambda)}{\sqrt{\lambda}},$$

where

$$D(\lambda) = \sqrt{\lambda}\int_0^{\delta^\star} \sigma(\delta)\left\{\gamma F(\delta) + \lambda\theta_0\Phi_1(\delta)\right\}\, d\delta = \sqrt{\lambda}\int_0^{\delta^\star} \frac{\gamma F(\delta) + \lambda\theta_0\Phi_1(\delta)}{r+\gamma+\lambda\theta_0\Phi_1(\delta)+\lambda\theta_1\left[1-s-\Phi_0(\delta)\right]}$$
$$P(\lambda) = \sqrt{\lambda}\int_{\delta^\star}^{1} \sigma(\delta)\left\{\gamma\left[1-F(\delta)\right] + \lambda\theta_1\left[1-s-\Phi_0(\delta)\right]\right\}\, d\delta = \sqrt{\lambda}\int_{\delta^\star}^{1} \frac{\gamma\left[1-F(\delta)\right] + \lambda\theta_1\left[1-s-\Phi_0(\delta)\right]}{r+\gamma+\lambda\theta_0\Phi_1(\delta)+\lambda\theta_1\left[1-s-\Phi_0(\delta)\right]}\, d\delta.$$

To study the asymptotic behavior of $D(\lambda)$ and $P(\lambda)$, we make the change of variable $x = \sqrt{\lambda}\,(\delta - \delta^\star)$. We obtain:

$$D(\lambda) = \int_{-\delta^\star\sqrt{\lambda}}^{0} \frac{\frac{\gamma}{\sqrt{\lambda}}F\left(\delta^\star + x/\sqrt{\lambda}\right) + \theta_0 g_1(x)}{\frac{r+\gamma}{\sqrt{\lambda}} + \theta_0 g_1(x) + \theta_1 g_0(x)}\, dx$$
$$P(\lambda) = \int_{0}^{\delta^\star\sqrt{\lambda}} \frac{\frac{\gamma}{\sqrt{\lambda}}\left[1 - F\left(\delta^\star + x/\sqrt{\lambda}\right)\right] + \theta_1 g_0(x)}{\frac{r+\gamma}{\sqrt{\lambda}} + \theta_0 g_1(x) + \theta_1 g_0(x)}\, dx$$

where

$$g_1(x) \equiv \sqrt{\lambda}\Phi_1\left(\delta^\star + x/\sqrt{\lambda}\right) \text{ and } g_0(x) \equiv \sqrt{\lambda}\left[1 - s - \Phi_0\left(\delta^\star + x/\sqrt{\lambda}\right)\right].$$

To apply the Dominated Convergence Theorem, we first study the limit of the integrand. For this we use the following Lemma:

Lemma 6. *Assume that $F(\delta)$ is differentiable at $\delta^\star$. Then, as $\lambda \to \infty$, $g_1(x) \to g(x)$ and $g_0(x) \to g(-x)$ where, for any $x \in \mathbb{R}$, $g(x)$ is the positive solution of $g(x)^2 - g(x)F'(\delta^\star)x - \gamma s(1-s) = 0$.*

Proof. Equation (20), evaluated at steady state, implies that $g_1(x)$ is the unique positive root of the quadratic equation:

$$g_1(x)^2 + g_1(x)\left\{\frac{\gamma}{\sqrt{\lambda}} + \sqrt{\lambda}\left[F(\delta^\star) - F\left(\delta^\star + x/\sqrt{\lambda}\right)\right]\right\} - \gamma s F\left(\delta^\star + x/\sqrt{\lambda}\right) = 0. \tag{30}$$

Because this quadratic equation is positive at $g = 1$, it follows that $g_1(x) \leq 1$. Thus, $g_1(x)$ has at least one accumulation point, $g(x)$, as $\lambda \to \infty$. Going to the limit in the equation show that this accumulation point is the unique positive solution of $g(x)^2 - g(x)F'(\delta^\star)x - \gamma s(1-s)$. Since $g_1(x)$ has a unique accumulation point, it must converge to it. Next, we note that $g_0(x) = g_1(x) + \sqrt{\lambda}\left[F(\delta^\star) - F\left(\delta^\star + x/\sqrt{\lambda}\right)\right]$. Plugging this back in to the quadratic equation for $g_1(x)$, we obtain that $g_0(x)$ is the positive solution of:

$$g_0(x)^2 + g_0(x)\left\{\frac{\gamma}{\sqrt{\lambda}} - \sqrt{\lambda}\left[F(\delta^\star) - F\left(\delta^\star + x/\sqrt{\lambda}\right)\right]\right\} - \gamma(1-s)\left[1 - F\left(\delta^\star + x/\sqrt{\lambda}\right)\right], \tag{31}$$

and the result follows from the same arguments as before. ■

Lemma 7. *Suppose that $F'(\delta)$ is continuous and strictly positive. Then, there is some $K \geq 0$ such that:*

$$g_1(x) \leq -\frac{K}{x} \quad \textit{if } x \leq 0, \qquad \textit{and} \qquad g_0(x) \leq \frac{K}{x} \quad \textit{if } x \geq 0.$$

Moreover, for any $\bar{x} > 0$, there is some $k > 0$ such that, for all λ large enough:

$$g_1(x) \geq kx \quad \textit{if } x \geq 0, \qquad \textit{and} \qquad g_0(x) \geq -kx \quad \textit{if } x \leq 0.$$

Proof. Recall that $g_1(x)$ is the positive root of (30). Thus $g_1(x) \leq -K/x$ if and only if the second-order polynomial on the left-hand side of (30) positive when evaluated at $-K/x$, that is if and only if:

$$\frac{K^2}{x^2} - \frac{K}{x}\left(\frac{\gamma}{\sqrt{\lambda}} + \sqrt{\lambda}\left[F(\delta^\star) - F\left(\delta^\star + x/\sqrt{\lambda}\right)\right]\right) - \gamma s F\left(\delta^\star + x/\sqrt{\lambda}\right) \geq 0.$$

For this inequality to hold for all $x \leq 0$ and all $\lambda > 0$, it is sufficient that:

$$-\frac{K}{x}\sqrt{\lambda}\left[F(\delta^\star) - F\left(\delta^\star + x/\sqrt{\lambda}\right)\right] - \gamma \geq 0.$$

Using Taylor Theorem, we can write $F(\delta^\star) - F(\delta^\star + x/\sqrt{\lambda}) = -x/\sqrt{\lambda}F'\left[\hat{\delta}(x)\right]$ for some $x \in [\delta^\star + x/\sqrt{\lambda}, \delta^\star]$. Plugging this back into the above inequality we obtain that $g_1(x) \leq -K/x$ if $KF'\left[\hat{\delta}(x)\right] - \gamma \geq$

0, and so we can choose

$$K \geq \frac{\gamma}{\min_{\delta \in [0,1]} F'(\delta)}.$$

Note that the right-hand side is less than infinity because $F'(\delta)$ is assumed to be strictly positive for all $\delta \in [0,1]$. One obtain the same constant K when applying the same calculations to $g_0(x)$ and $x \geq 0$.

Now let us turn to the second part of the proposition and fix some $\bar{x} > 0$. As before, we recall that $g_1(x)$ is the positive root of (30). Thus $g_1(x) \geq kx$ if and only if the above second-order polynomial on the left-hand side of (30) is negative when evaluated at kx, that is if and only if:

$$\begin{aligned}
& k^2x^2 + kx\left\{\frac{\gamma}{\sqrt{\lambda}} + \sqrt{\lambda}\left[F(\delta^\star) - F\left(\delta^\star + x/\sqrt{\lambda}\right)\right]\right\} - \gamma s F\left(\delta^\star + x/\sqrt{\lambda}\right) \leq 0 \\
\Leftrightarrow \quad & k^2x^2 + kx\left\{\frac{\gamma}{\sqrt{\lambda}} - xF'\left[\hat{\delta}(x)\right]\right\} - \gamma s F\left(\delta^\star + x/\sqrt{\lambda}\right) \leq 0
\end{aligned}$$

where we move from the first to the second line using Taylor Theorem as before, and where $\hat{\delta}(x) \in [0,1]$. Dividing through by x^2, we obtain that a sufficient condition for this inequality to hold for all $x \geq \bar{x}$ is:

$$k^2 - k \min_{\delta \in [0,1]} F'(\delta) + \frac{\gamma}{\sqrt{\lambda}\bar{x}} \leq 0.$$

If we pick any $k < \min_{\delta \in [0,1] F'(\delta)}$, then the above inequality holds for all λ large enough. One obtain the same constant k when applying the same calculations to $g_0(x)$ and $x \leq -\bar{x}$. ■

Using the above preliminary results, we obtain:

Lemma 8. *As $\lambda \to \infty$:*

$$D(\lambda) \to \int_{-\infty}^{0} \frac{\theta_0 g(x)\,dx}{\theta_0 g(x) + \theta_1 g(-x)} \quad \textit{and} \quad P(\lambda) \to \int_{0}^{\infty} \frac{\theta_1 g(-x)\,dx}{\theta_0 g(x) + \theta_1 g(-x)}.$$

Proof. The integrand of $D(\lambda)$ can be written:

$$\mathbf{1}_{\left\{x \in \left[-\delta^\star\sqrt{\lambda}, 0\right]\right\}} \frac{\frac{\gamma}{\sqrt{\lambda}} F\left(\delta^\star + x/\sqrt{\lambda}\right) + \theta_0 g_1(x)}{\frac{r+\gamma}{\sqrt{\lambda}} + \theta_0 g_1(x) + \theta_1 g_0(x)}.$$

By Lemma 6, this integrand converges pointwise to $\frac{\theta_0 g(x)}{\theta_0 g(x) + \theta_1 g(-x)}$. Now fix any $\bar{x} > 0$ and λ large enough. On the interval $[-\bar{x}, 0]$, we can bound the integrand above by 1. On the interval $\left[-\delta^\star\sqrt{\lambda}, -\bar{x}\right]$, we use Lemma 7 to bound the integrand above by:

$$\frac{\frac{\gamma}{\sqrt{\lambda}} - \frac{\theta_0 K}{x}}{-\frac{\theta_0 K}{x} - \theta_1 kx} = \frac{-\frac{\gamma x}{\sqrt{\lambda}} + \theta_0 K}{\theta_0 K + \theta_1 kx^2} \leq \frac{\gamma\delta^\star + \theta_0 K}{\theta_0 K + \theta_1 kx^2},$$

where we used that $x \geq -\delta^\star\sqrt{\lambda}$. On the interval $(-\infty, -\delta^\star\sqrt{\lambda}]$, the integrand is zero and so the previous bound holds as well. Taken together, we have bounded the integrand above by a positive function, integrable over $(-\infty, 0]$. This allows to apply the Dominated Convergence Theorem (ADD REFERENCE), and the result follows. The result for $P(\lambda)$ follows from identical calculations. ■

After making the change of variable $y = -x$ in the limit of $D(\lambda)$, we obtain

$$\lim_{\lambda\to\infty} D(\lambda) = \int_0^\infty \frac{\theta_0 g(-x)}{\theta_0 g(-x) + \theta_1 g(x)}\, dx.$$

Therefore:

$$\begin{aligned}\lim_{\lambda\to\infty} P(\lambda) - D(\lambda) &= \int_0^\infty \frac{\theta_1 g(-x)}{\theta_0 g(x) + \theta_1 g(-x)}\, dx - \int_0^\infty \frac{\theta_0 g(-x)}{\theta_0 g(-x) + \theta_1 g(x)}\, dx \\ &= \int_0^\infty \frac{(1-2\theta_0) g(x) g(-x)\, dx}{[\theta_0 g(x) + \theta_1 g(-x)]\,[\theta_0 g(-x) + \theta_1 g(x)]}.\end{aligned}$$

Now solving the quadratic equation for $g(x)$ we obtain that:

$$g(x) = \frac{1}{2}\left[F'(\delta^\star)x + \sqrt{4\gamma s(1-s) + (xF'(\delta^\star))^2}\right].$$

Plugging this formula in the above and simplifying, we obtain that:

$$\begin{aligned}\lim_{\lambda\to\infty} P(\lambda) - D(\lambda) &= \int_0^\infty \frac{\gamma s(1-s)(1-2\theta_0)}{\gamma s(1-s) + \theta_0\theta_1 (xF'(\delta^\star))^2} dx \\ &= \frac{\pi\sqrt{\gamma}}{F'(\delta^\star)}\left(\frac{1}{2} - \theta_0\right)\left(\frac{s(1-s)}{\theta_0\theta_1}\right)^{\frac{1}{2}}\end{aligned}$$

since $\int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{2}$. Collecting the results we conclude that

$$\Delta V(\delta^\star) = \frac{\delta^\star}{r} + \frac{\pi/r}{F'(\delta^\star)}\left(\frac{1}{2} - \theta_0\right)\left(\frac{\gamma s(1-s)}{\theta_0\theta_1}\right)^{\frac{1}{2}} \frac{1}{\sqrt{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right).$$

This expansion applies to all reservation values and price because, by Proposition 6 show below:

$$\Delta V(\delta) = \Delta V(\delta^\star) + [\Delta V(\delta) - \Delta V(\delta^\star)] \text{ and } \lim_{\lambda\to\infty} \sqrt{\lambda}\,[\Delta V(\delta) - \Delta V(\delta^\star)] = 0.$$

A.3 Proof of Proposition 6

The first intermediate result is:

Lemma 9. *As λ goes to infinity:*

$$\lambda \int_0^{\delta^\star} \sigma(\delta)\, d\delta = \int_0^{\delta^\star} \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)(\delta^\star - \delta) + \Phi_1(\delta)} + O(1) \tag{32}$$

$$\lambda \int_{\delta^\star}^1 \sigma(\delta)\, d\delta = \int_{\delta^\star}^1 \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_0 F'(\delta^\star)(\delta^\star - \delta) + 1 - s - \Phi_0(\delta)} + O(1). \tag{33}$$

Proof. We start with (32), noting that:

$$\lambda\sigma(\delta) = \frac{\lambda}{r + \gamma + \lambda\theta_1 \left[1 - s - \Phi_0(\delta)\right] + \lambda\theta_0\Phi_1(\delta)} = \frac{1}{\frac{r+\gamma}{\lambda} + \theta_1 \left[F(\delta^\star) - F(\delta)\right] + \Phi_1(\delta)},$$

where we used that $\Phi_0(\delta) = F(\delta) - \Phi_1(\delta)$, and $F(\delta^\star) = 1 - s$. Using this expression to calculate the difference between the left- and the right-hand side of (32), we obtain:

$$\left| \int_0^{\delta^\star} \left(\lambda\sigma(\delta) - \frac{1}{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)(\delta^\star - \delta) + \Phi_1(\delta)} \right) d\delta \right| \leq \int_0^{\delta^\star} \frac{\theta_1 \left|F'(\delta^\star)(\delta^\star - \delta) - \left[F(\delta^\star) - F(\delta)\right]\right|}{\theta_1^2 F'(\delta^\star) \left[\delta^\star - \delta\right] \left[F(\delta^\star) - F(\delta)\right]} d\delta.$$

In the right-hand side integral, under our assumption that $F(\delta)$ is twice continuously differentiable, we can use the Taylor Theorem to extend the integrand by continuity at $\delta^\star$, with value $\frac{F''(\delta^\star)}{2\theta_1 F'(\delta^\star)}$. Thus, the integrand is bounded, establishing (32). Turning to equation (33), we first note that:

$$\lambda\sigma(\delta) = \frac{\lambda}{r + \gamma + \lambda\theta_1 \left[1 - s - \Phi_0(\delta)\right] + \lambda\theta_0\Phi_1(\delta)} = \frac{1}{\frac{r+\gamma}{\lambda} + \theta_0 \left[F(\delta) - F(\delta^\star)\right] + 1 - s - \Phi_0(\delta)},$$

where we used that $\Phi_1(\delta) = F(\delta) - F(\delta^\star) + F(\delta^\star) - \Phi_0(\delta) = F(\delta) - F(\delta^\star) + 1 - s - \Phi_0(\delta)$ since $F(\delta^\star) = 1 - s$. The rest of the proof is identical as the one for (32). ■

Next, we obtain a lower bound for the integral on the right-hand side of (32) by bounding $\Phi_1(\delta)$ above by $\Phi_1(\delta^\star)$:

Lemma 10. *As $\lambda \to \infty$:*

$$\int_0^{\delta^\star} \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)(\delta^\star - \delta) + \Phi_1(\delta)} \geq \frac{\log(\lambda)}{2\theta_1 F'(\delta^\star)} + O(1) \tag{34}$$

$$\int_{\delta^\star}^1 \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_0 F'(\delta^\star)(\delta - \delta^\star) + 1 - s - \Phi_0(\delta)} \geq \frac{\log(\lambda)}{2\theta_0 F'(\delta^\star)} + O(1). \tag{35}$$

Proof. For (34), this follows by noting that $\Phi_1(\delta) \leq \Phi_1(\delta^\star)$, and integrating directly:

$$\int_0^{\delta^\star} \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)(\delta^\star - \delta) + \Phi_1(\delta^\star)} = \left[-\frac{1}{\theta_1 F'(\delta^\star)} \log\left(\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)\left[\delta^\star - \delta\right] + \Phi_1(\delta^\star)\right)\right]_0^{\delta^\star}$$
$$= O(1) - \frac{1}{\theta_1 F'(\delta^\star)} \log\left[\sqrt{\frac{\gamma s(1-s)}{\lambda}} + o\left(\frac{1}{\sqrt{\lambda}}\right)\right] = \frac{\log(\lambda)}{2\theta_1 F'(\delta^\star)} + O(1),$$

where the second equality follows from plugging in the asymptotic expansion of $\Phi_1(\delta^\star)$ derived in Section 4.1.4. For (35), this follows from the same manipulation: first by noting that $1-s-\Phi_0(\delta) \leq 1-s-\Phi_0(\delta^\star)$, and integrating directly. ■

Next we establish the reverse inequality:

Lemma 11. *As $\lambda \to \infty$:*

$$\int_0^{\delta^\star} \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)(\delta^\star - \delta) + \Phi_1(\delta)} \leq \frac{\log(\lambda)}{2\theta_1 F'(\delta^\star)} + O(1) \tag{36}$$
$$\int_{\delta^\star}^1 \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_0 F'(\delta^\star)(\delta - \delta^\star) + 1 - s - \Phi_0(\delta)} \leq \frac{\log(\lambda)}{2\theta_0 F'(\delta^\star)} + O(1). \tag{37}$$

Proof. For (36), let us break down the integral into an integral over $\left[0, \delta^\star - 1/\sqrt{\lambda}\right]$, and an integral over $\left[\delta^\star - 1/\sqrt{\lambda}, \delta^\star\right]$. The first integral can be bounded above by:

$$\int_0^{\delta^\star - 1/\sqrt{\lambda}} \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)(\delta^\star - \delta)} = -\frac{1}{\theta_1 F'(\delta^\star)} \log\left(\frac{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)/\sqrt{\lambda}}{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)\delta^\star}\right) = \frac{\log(\lambda)}{2\theta_1 F'(\delta)} + O(1).$$

The second term can be bounded above by:

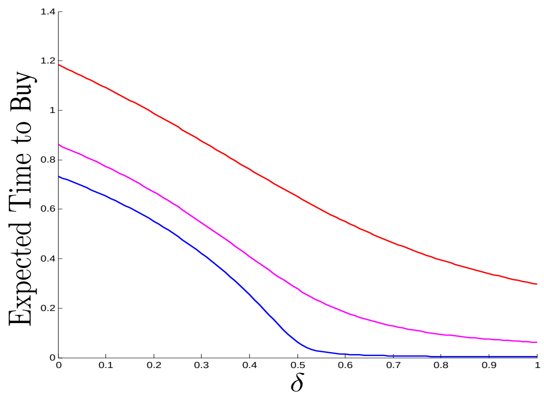
$$\int_{\delta^\star - 1/\sqrt{\lambda}}^{\delta^\star} \frac{d\delta}{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)(\delta^\star - \delta) + \Phi_1\left(\delta^\star - 1/\sqrt{\lambda}\right)} = \frac{1}{\theta_1 F'(\delta^\star)} \log\left(\frac{\frac{r+\gamma}{\lambda} + \theta_1 F'(\delta^\star)\alpha(\lambda) + \Phi_1\left(\delta^\star - 1/\sqrt{\lambda}\right)}{\frac{r+\gamma}{\lambda} + \Phi_1\left(\delta^\star - 1/\sqrt{\lambda}\right)}\right)$$
$$\to \frac{1}{\theta_1 F'(\delta^\star)} \log\left(\frac{\theta_1 F'(\delta^\star) + g(-1)}{g(-1)}\right) = O(1),$$

where $g(-1)$ is the limit of $\sqrt{\lambda}\Phi_1\left(\delta^\star - 1/\sqrt{\lambda}\right)$ as shown in Lemma 6. For (37), the result follows from identical algebraic manipulations. ■

B Figures

Figure 1: Expected Time to Buy and Sell Varying λ

(a) $\eta_0(\delta)$



(b) $\eta_1(\delta)$

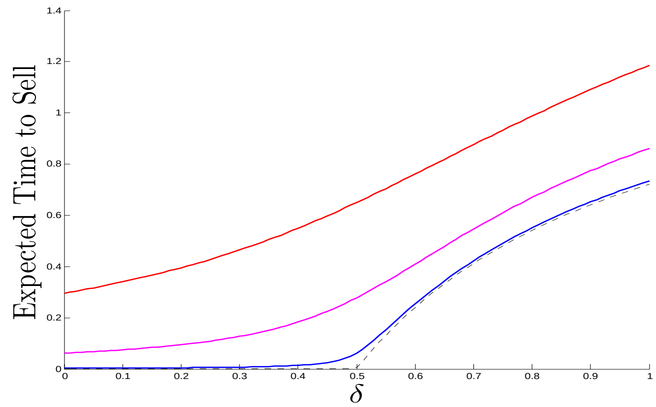


Figure 2: Misallocation by Type Varying λ

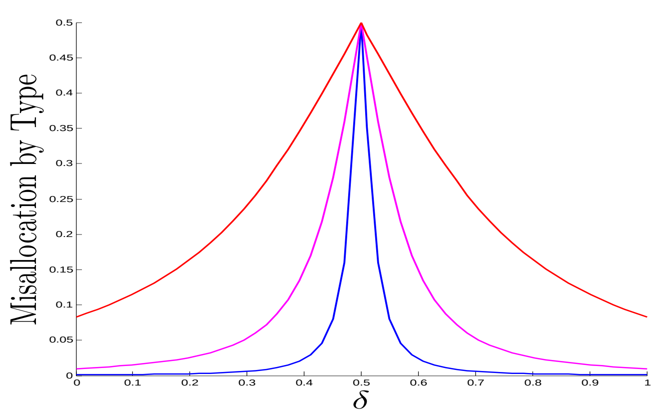
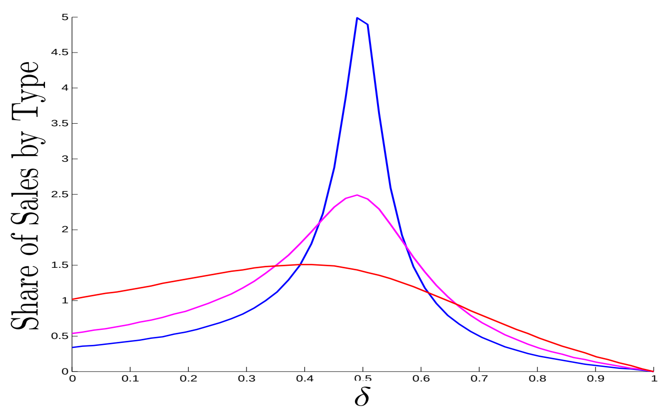


Figure 3: Measure of Trade by Type

(a) Fraction of Sales by Type Varying λ



(b) Fraction of Trades by (δ, δ') Pair