Sovereign Debt Crises and International Financial Contagion: Estimating Effects in an Endogenous Network

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Abstract

In an integrated global financial system, a sovereign default raises concerns of financial contagion to other countries. We develop and estimate an equilibrium model featuring a network of international borrowing and lending. In the model, the network structure of borrowing-lending relationships arises endogenously and results in the propagation of financial shocks across countries. We estimate the model using data on foreign claims among a network of 20 countries over six years. Simulating counterfactual experiments from the estimated model, we find a non-trivial role for financial contagion. The default of a sovereign in the network has a noticeable effect on the borrowing costs and default probabilities of other network members.

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1 Introduction

The recent sovereign debt crisis in Europe has renewed concerns of financial contagion. In an integrated global financial system, where countries are simultaneously borrowing and lending from one another, the default of one country can directly result in the subsequent default of its lenders. This linkage introduces the possibility of financial contagion, whereby the default of one sovereign borrower can impair the financial position of its counterparties, leading to a clustering of defaults. Such a crisis can have damaging effects on the real economic activity of both the defaulting and non-defaulting countries.

In this paper we develop and estimate an equilibrium model of international borrowing and lending in a network of large economies. In the model, countries finance a portion of their capital investment by borrowing from other countries. These bilateral lending-borrowing relationships between countries comprise an endogenous network where countries' financial conditions are interconnected. In such a network, contagion risk arises naturally as a shock to one sovereign can propagate through the network to generate a sovereign debt crisis affecting multiple countries in the network.

Using data on foreign claims from the Bank for International Settlements, we estimate the model for a network of 20 countries. In contrast to most of the existing literature on contagion that takes financial networks as exogenous, the network structure in our model arises endogenously as an equilibrium outcome. The lending-borrowing network evolves over time in response to local economic shocks and the global supply and demand for capital.

With the estimated model parameters, we also conduct a series of counterfactual experiments. These experiments provide us with an estimate of the degree of contagion risk present in the existing network of international borrowing and lending. In particular, we do this by simulating the default of a given country and examining the impact on the default probabilities of other countries in the network. As a starting point, we consider a global economy with risk-neutral agents in which countries' output shocks are uncorrelated. Simulating counterfactual defaults in the estimated network, we observe contagion effects following the default of a sovereign in the network. The fact that the estimated model is able to produce these effects under this simplified environment suggests a significant role for contagion in sovereign debt markets, even in the absence of an explicit amplification mechanism.

Our approach makes possible further analyses that will be added in an upcoming version that extends this preliminary draft. First, we will simulate outcomes under counterfactual network structures. That is, we will simulate the estimated model under alternative network topologies and examine how the degree of contagion risk varies. Second, we will simulate longer-run effects on investment and output, by solving for the complete lending-borrowing network and projecting its evolution over time. In addition, if needed we can extend the current empirical specification to allow for risk aversion and other nonlinearities by adopting a simulation-based estimator.

Our paper is related to two existing strands of literature. The first is a relatively new literature that studies financial networks and contagion. The second studies sovereign debt, default, and credit risk, typically in the context of a dynamic general equilibrium model of a small open economy.

The role of networks in financial contagion has been the subject of a growing literature that mainly began with the theoretical work of Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000). Both papers model an interbank market where liquidity shocks from consumers in different locations are the reason for interbank deposits and transfers. They consider the possible contagion of insolvency throughout the system if one bank fails, and both models indicate that more connected networks are more resilient against this contagion. Allen and Babus (2009) provide a survey of the subsequent theoretical and empirical literature. As they note, several authors have found evidence of contagion in interbank networks, although the network structures are taken as exogenous in these analyses.

Two recent papers attempt to endogenize the network structure, with certain restrictions. Babus (2009) models a network of interbank deposits among banks in one region with common liquidity shocks, given pre-existing links with banks in another region that has the opposite liquidity shocks. Assuming each bank is linked to every bank in the opposite region, a network forms in the home region that minimizes the risk of contagion. Cohen-Cole, Patacchini, and Zenou (2011) include a simple model of network formation in an empirical study of interbank loans. They assume that banks move individually in an exogenous sequence, and must either add or remove one link to maximize a myopic change in payoffs.[1] To draw a distinction with these papers, our model endogenizes the complete network of international loans and treats this network in a unified fashion. Agents move simultaneously, and there is no imposition of myopic beliefs.[2]

Relatively little work has considered the role of financial networks in spillovers from a sovereign debt crisis, or other international contexts for financial contagion.[3] The most relevant to us is Bolton and Jeanne (2011), who develop a model that links government

[1]The use of an exogenous sequence of moves, and the myopic beliefs, are similar to Christakis et al. (2010) and Mele (2010). These assumptions function as equilibrium selection rules, with unclear consequences.

[2]Dynamic considerations do not impact behavior in our model, but this arises naturally from the primitives.

[3]Allen and Gale (2000) note that the different regions in their model could be interpreted as countries.

debt to the financial sector (where it is used as collateral for interbank loans). Sovereign risk affects the reserves of domestic and foreign banks that hold this debt. With a simple, two-country network, they show that banks diversify their debt portfolios, and so a sovereign default in one country reduces liquidity, and hence investment and output, in both countries. Empirically, a handful of recent papers provide evidence on the international transmission of financial shocks during the recent financial crisis. De Haas and Van Horen (2011) and Giannetti and Laeven (2011) examine international lending by large banks, and find that those with greater losses or less access to credit made greater reductions in their cross-border lending. Similarly, Cetorelli and Goldberg (2010) and Popov and Udell (2010) find that the supply of loans fell in emerging markets due to reductions in cross-border lending from international banks and local lending from banks with foreign parents, as well as reductions in local lending by domestic banks affected by the international interbank market.

Our paper is also related to a literature that studies sovereign borrowing and lending. This strand of literature builds on the seminal work of Eaton and Gersovitz (1981) to study sovereign debt and default in a dynamic general equilibrium model. These papers study sovereign borrowing with endogenous default in dynamic general equilibrium models with incomplete markets. Arellano (2008) uses a quantitative model to study the interaction of sovereign credit risk with output fluctuations and interest rates. A calibrated version of the model is successful in matching Argentina's business cycles statistics and the country's 2001 default. Aguiar and Gopinath (2006) use a similar framework to study the quantitative predictions of a model of debt and default in a small open economy. They find a quantitatively important role for a stochastic trend in growth rates for emerging economies. Arellano and Ramanarayanan (2008) extend this framework of a sovereign borrower to allow for the choice of debt maturity. They find a quantitatively important role for a sovereign's choice of maturity structure as it trades off the hedging benefits of long-term debt against the repayment incentives induced by short-term debt. Borri and Verdelhan (2011) study the importance of a risk averse lender in a model of sovereign borrowing. They show the importance of a risk-averse lender, in this case with habit formation preferences, for matching the sovereign bond yields observed in the data.

Finally, recent work in the finance literature has examined the time series properties and comovement of sovereign credit risk by examining sovereign CDS spreads. Longstaff et al. (2011) find that global factors explain a large portion of the common variation in sovereign CDS prices. Dieckmann and Plank (2011) study CDS spreads for a sample of developed countries and find strong comovement that increased significantly following the financial crisis beginning in 2008. Pan and Singleton (2008) find that the spreads of Mexico, Turkey, and Korea have a high correlation with volatility in the U.S. stock market as measured by the VIX.

The remainder of the paper is organized as follows. Section 2 presents the network model of international borrowing and lending. We discuss our empirical approach to estimating the model in Section 3 and the data used in the estimation in Section 4. In Section 5 we present the results from the estimated model. Finally, in Section 6 we use the estimated model to simulate counterfactual sovereign defaults and examine the contagion effects of these shocks propagating through the network of borrowing and lending relationships.

2 Model

We develop a representative agent model of macroeconomic investment and production, international borrowing and lending, and solvency. Our model shares many features with the existing literature, particularly Allen and Gale (2000) , Bolton and Jeanne (2011), and Arellano (2008). However unlike some previous work on financial contagion or sovereign debt crises, there is no separate banking sector or government sector. Instead, the agents in our model function jointly as consumers, producers, and lenders.

The global economy consists of countries $i = 1, \ldots, N$, which are of similar size.[4] Each country has a continuum of households, and these households have access to "projects" that can convert an investment of the consumption good in one period into output in the next period. Time is measured in discrete periods ($t = 0, 1, 2, \ldots$), with no terminal date. Given an investment I in period t, a project yields $f(I)$ in period $t+1$, with $f' > 0$ and $f'' < 0$. There is no other storage technology.

Households can make loans to invest in projects at other households, both in their own country and in other countries. However they cannot invest in their own projects. This restriction generates lending and links financial activity to aggregate production in a relatively simple macroeconomic model.[5] Similar restrictions appear in other work, such as Bolton and Jeanne (2011) where only some bankers have access to a project. A loan, l, made in one period is paid back in the next period for a total return of rl, unless the recipient defaults

[4] We can instead allow countries to have different measures of households, μ_i, but this is omitted to simplify the exposition. However no country is "small" relative to the global economy.

[5] It would be possible to remove this restriction, at the cost of other complications. For example, if households were risk averse there would be an incentive to diversify by lending to other countries. Or we could introduce competitive firms and have them manage the projects. These firms would need to distribute their profits back to the households each period.

in which case there is some exogenous recovery rate (described later). Loans are made from one particular household in country i to another household in country j (perhaps the same country). However, we will restrict attention to symmetric equilibria, so we can usually describe behavior in terms of one representative household in each country. In that case a loan from country i to country j in period t is denoted l_{ij}^t.

There is a cost of making loans to each country, given by a function c. It depends on the total loans made by an individual household (h) to households in country j, l_{hij}^t, and the per capita amount of loans made by *all households in the same country* to country j in the previous period: $\bar{l}_{ij}^{t-1}$. The cost is increasing and convex in the individual household's current loans (l_{hij}^t), which represents the difficulty of finding a good projects to invest in. However there is a negative cross-partial with the per capita amount of loans made in the previous period ($\bar{l}_{ij}^{t-1}$). This captures the idea that it is easier to find good projects in some country if more households in your own country have invested there before. We use the per capita amount rather than the household's own lagged amount for convenience, because it avoids a dynamic incentive to invest in order to reduce future costs. Still this assumption can be motivated, in part, because good projects will appear at different households over time, and so there may be a benefit in learning about these projects from other investors in your country. To summarize, the cost of making loans to country j for some household h in country i is $c(l_{hij}^t, \bar{l}_{ij}^{t-1})$, with partial derivatives $c_1 > 0$, $c_{11} > 0$, $c_{12} < 0$. In a symmetric equilibrium we will have $l_{hij}^t = l_{ij}^t$ and $\bar{l}_{ij}^{t-1} = l_{ij}^{t-1}$, so this becomes $c(l_{ij}^t, l_{ij}^{t-1})$

Each period a household has revenues from its latest project and the return on its loans. The total investment in the project at the representative household in country i in period $t-1$ is $I_i^{t-1} = \sum_j l_{ji}^{t-1}$. Given this investment, the project yields $f(I_i^{t-1})$ in period t. The loan repayment the household receives from country j in period t is denoted y_{ij}^t, which equals $r_j^{t-1} l_{ij}^{t-1}$ unless there is a default in country j. Total revenue from loan repayments is $\sum_j y_{ij}^t$. There is also an exogenous revenue shock each period, X_i^t, which is the same for all households in a country. This represents economy-wide productivity shocks, government fiscal shocks, etc., and is subtracted from the other revenue. Thus the available revenue for the representative household in country i in period t is $f(I_i^{t-1}) + \sum_j y_{ij}^t - X_i^t$. In addition, the household has obligations in the amount of $r_i^{t-1} I_i^{t-1}$, to pay back the loans it received in the previous period.

If revenues exceed obligations, the representative household in country i is solvent ($s_i^t = 1$) and can use its remaining money to make loans. In this case the budget constraint is

$$f(I_i^{t-1}) + \sum_j y_{ij}^t - X_i^t - r_i^{t-1} I_i^{t-1} - \sum_j [l_{ij}^t + c(l_{ij}^t, l_{ij}^{t-1})] \ \geq \ 0.$$

The household defaults ($s_i^t = 0$) iff its debt is greater than its revenues:

$$s_i^t = 0 \iff f(I_i^{t-1}) + \sum_j y_{ij}^t - X_i^t - r_i^{t-1} I_i^{t-1} < 0.$$

A default only lasts for the current period. When in default, a household cannot make loans or receive loans to invest in its project, and its consumption is zero. For simplicity we assume that its creditors are paid back an exogenous proportion of their loans, $\delta \in [0, 1]$, so that they receive $\delta r_i^{t-1} l_{ji}^{t-1}$ if country i defaults. We set $\delta = 0.4$. Finally, to account for whether a country is solvent or not, the loan repayments are defined as $y_{ij}^t = r_j^{t-1} l_{ij}^{t-1} [\delta + (1 - \delta) s_j^t]$.

2.1 Solution

A symmetric equilibrium is assumed. Given interest rates $r_j^t, j = 1 \ldots N$, households in country i choose loan amounts ($l_{ij}^t, j = 1 \ldots N$) and a level of debt and investment (I_i^t) to maximize consumption each period:

$$C_i^t = s_i^t \cdot \left(f(I_i^{t-1}) + \sum_j y_{ij}^t - X_i^t - r_i^{t-1} I_i^{t-1} - \sum_j [l_{ij}^t + c(l_{ij}^t, \bar{l}_{ij}^{t-1})] \right).$$

Let $p_i^{t+1} = \mathrm{E}_t s_i^{t+1}$ be the equilibrium probability that the households in country i will be solvent in the next period. Because the individual households are small, they cannot affect default probabilities with their investment decisions, and so these probabilities are taken as given as well. Expected consumption in the next period is

$$\mathrm{E}_t C_i^{t+1} = p_i^{t+1} \mathrm{E}_t \left(f(I_i^t) + \sum_j y_{ij}^{t+1} - X_i^{t+1} - r_i^t I_i^t - \sum_j [l_{ij}^{t+1} + c(l_{ij}^{t+1}, \bar{l}_{ij}^t)] | s_i^{t+1} = 1 \right),$$

as consumption is zero when $s_{i,t+1} = 0$. The optimization problem in period t is thus

$$\max_{I_{it}, (l_{ijt})} C_{it} + \mathrm{E}_t \sum_{s=1}^{\infty} \rho^s C_i^{t+s} \tag{1}$$
$$s.t.\ f(I_i^{t-1}) + \sum_j y_{ij}^t - X_i^t - r_i^{t-1} I_i^{t-1} - \sum_j [l_{ij}^t + c(l_{ij}^t, \bar{l}_{ij}^{t-1})] \geq 0.$$

(This includes a subjective discount rate $\rho < 1$ so that the objective is well defined.) To solve this problem we only need to consider consumption in periods t and $t+1$. This is because an individual household cannot affect the per capita loan amounts, $\bar{l}_{ij}^t$, that affect costs and hence lending in future periods. As a result, the optimization problem is in essence static.

The FOCs for loans involve the probability of default in the next period, both for the lender and the debtor, because a loan is fully repaid and the lender benefits from this money only if both are solvent. This can be seen from the expected benefit of a loan to country j, which is

$$\mathrm{E}_t[\rho s_i^{t+1} y_{ij}^{t+1}] = \rho p_i^{t+1} \mathrm{E}_t[r_j^t l_{ij}^t [\delta + (1-\delta) s_j^{t+1}] | s_i^{t+1} = 1] = \rho p_i^{t+1} r_j^t l_{ij}^t [\delta + (1-\delta) p_{j|i}^{t+1}]$$

where $p_{j|i}^{t+1} = \mathrm{E}_t[s_j^{t+1} | s_i^{t+1} = 1]$ is the probability that the households in country j are solvent conditional on those in i being solvent. Thus the marginal benefit is $\rho p_i^{t+1} r_j^t [\delta + (1-\delta) p_{j|i}^{t+1}]$ while the marginal cost of a loan is $1 + c_1(l_{ij}^t, \bar{l}_{ij}^{t-1})$, and so the FOC is

$$\rho p_i^{t+1} r_j^t [\delta + (1-\delta) p_{j|i}^{t+1}] = 1 + c_1(l_{ij}^t, \bar{l}_{ij}^{t-1}) \tag{2}$$

For loans within the same country, the expression is simpler because $p_{i|i}^{t+1} = 1$ (intuitively, either all households will default or none will). So the expected benefit of a domestic loan is $p_i^{t+1} r_i^t l_{ii}^t$, and hence the FOC is $\rho p_i^{t+1} r_i^t = 1 + c_1(l_{ii}^t, \bar{l}_{ii}^{t-1})$.

The solvency probabilities, p_i^{t+1}, satisfy a system of equations:

$$p_i^{t+1} = \int_Y F_X \left(f(I_i^t) - r_i^t I_i^t + \sum_j y_{ij}^{t+1} \right) \mathrm{d}F_Y(y_{i1}^{t+1} \dots y_{iN}^{t+1}) \tag{3}$$

where F_X is the CDF of X and F_Y is the joint distribution of (y_{ij}^{t+1}). The latter is derived from the joint distribution of $(X_j^{t+1})_{j=1}^N$ because $y_{ij}^{t+1} = r_j^t l_{ij}^t \cdot [\delta + (1-\delta) s_j^{t+1}]$, and s_j^{t+1} is determined as

$$s_j^{t+1} = 1 \left\{ f(I_j^t) - r_j^t I_j^t + \sum_k y_{jk}^{t+1} - X_j^{t+1} \geq 0 \right\}. \tag{4}$$

As a result, computing (3) involves solving for s conditional on X and integrating over the joint distribution of X.

Finally, the FOC for the borrowing and investment amount is

$$r_i^t = f_i'(I_i^t) \tag{5}$$

Then the equilibrium is completed with one other aggregate condition: investment equals the sum of loans, $I_i^t = \sum_j l_{ji}^t$.

3 Empirical Approach

For the preliminary analysis, we focus on the estimation of equation (3) which gives the solvency probabilities, using data on CDS prices, aggregate investment, interest rates, and foreign claims. With this we can simulate the short-term consequences of a default and of alternative network structures. The further analysis will add equation (2), the FOC for cross-country loans, in order to recover the remaining structural parameters. With the model fully estimated we will then be able to simulate the long-term consequences on investment, output, and risk.

To derive the probabilities in (3) we must consider solutions to (4). First we show that solutions exist where within-country loans (l_{ii}^t) can drop out of this equation. Substituting in for $-r_i^t I_i^t$ and y_{ij}^{t+1}, we have

$$s_i^{t+1} = 1\left\{ f(I_i^t) - \sum_j r_i^t l_{ji}^t + \sum_j r_j^t l_{ij}^t [\delta + (1-\delta) s_j^{t+1}] - X_i^{t+1} > 0 \right\}.$$

Let V_i^t represent all the terms within the brackets above, except for $-r_i^t l_{ii}^t$ and $+r_i^t l_{ii}^t[\delta + (1 - \delta) s_i^{t+1}]$. There is a unique solution for s_i^{t+1} except when $V_i^t > 0$ but $V_i^t - (1-\delta) r_i^t l_{ii}^t < 0$. In this case, the country would default if its own households fail to pay each other back, but would be solvent if they do pay each other back. If we select the equilibrium in which the households pay each other back, this means the terms $-r_i^t l_{ii}^t$ and $+r_i^t l_{ii}^t[\delta + (1-\delta)s_i^{t+1}]$ will always cancel and can be removed from the expression. To gain further tractability, we impose an additional equilibrium selection rule. Suppose that, given realizations of X^{t+1}, there are two solutions for s_i^{t+1} and s_j^{t+1}: either both countries default or both are solvent. In what follows, we always select the equilibrium where i and j remain solvent and pay each other back. We apply this selection rule in determining $p_{j|i}^{t+1}$, $p_{k|i,j}^{t+1}$ and so on. In principle these expressions can be derived all the way to $p_{z|-z}^{t+1}$, which is the probability that country z will be solvent conditional on all the other countries being solvent. However in the implementation we truncate the expansion used to compute p_i^{t+1} at three steps (i.e., up to $p_{m|i,j,k}^{t+1}$). This is a reasonable approximation because each new step is discounted by $\frac{1-\delta}{\sigma}$, so further steps would have little impact on the value of p_i^{t+1}.

We assume that the X_i shocks are independent across countries are independent across countries and follow a conditional normal distribution. To account for hetereogeneity across sovereigns, we assume that the standard deviation of a country's shock scales in the size of its economy, Z_i. Specifically, each country's shock, X_i, is conditionally normally distributed as

$$X_i^{t+1}|X_i^t \sim \mathcal{N}(-\beta_0 - \beta_1 X_i^t, \;\; Z_i^2\sigma^2)$$

Additionally, we allow for an intercept term equal to $\beta_0 Z_i$. In what follows, we set Z_i to be a country's (seasonally adjusted) GDP in the fourth quarter of 2004, the quarter immediately preceding the beginning of our sample period. To complete the specification of equation (3), we use a quadratic for the production function: $f(I_i^t) = \alpha_1 I_i^t + \alpha_2 (I_i^t)^2$. This yields the following specification for a country's solvency probability:

$$p_i^{t+1} = \mathbb{E}\left[\mathbb{1}\left\{\alpha_1 I_i^t + \alpha_2 (I_i^t)^2 - \sum_{j\neq i} r_i^t l_{ji}^t + \sum_{j\neq i} r_j^t l_{ij}^t [\delta + (1-\delta) s_{j|i}^{t+1}] - X_i^{t+1} > 0\right\} \mid \mathbf{X}^t\right] \tag{6}$$

where $\mathbb{1}$ is an indicator function and $\mathbf{X}^t$ is the vector of all X_j^t (i inclusive). We obtain similar expressions for $p_{j|i}^{t+1}$ and the two further steps in the expansion.

If we suppose that there is only simple measurement error between the empirical default probabilities from transformed CDS spreads in the data and the "true" beliefs about solvency probabilities from the model, the observed values can be written as $\widehat{p}_i^{t+1} = p_i^{t+1} + \xi_i^{t+1}$. We can then recover the parameters α, β, and σ by minimizing the squared loss between the observed and predicted values:

$$\sum_{t=1}^{T} \sum_{i=1}^{N} \left(\widehat{p}_i^{t+1} - p_i^{t+1}\right)^2$$

For the preliminary analysis we use this as the objective function.[6]

4 Data

In this section we discuss the data used in estimating our structural model of an international borrowing and lending network. We begin by collecting data on international financial relationships provided by the Bank for International Settlements (BIS) and supplement these data with macroeconomic and financial variables from the OECD and IMF. Using these data, we construct a network of lending relationships consisting of 20 countries for the period from

[6] Clearly this ignores any possible sources of endogeneity bias and any dependence in X across countries. It would be better to include a "real" unobservable, such as common beliefs about country i's future solvency that are not known to the econometrician. We plan to do this in a future version.

2005Q1 to 2011Q3. Table I lists the countries included in our sample.

Figure 1 gives a representation of the lending network in 2011Q3, the last period in our data. The arrows represent financial claims that one country has on another. These amounts are normalized by the size of the economy of the lender country, using total GDP in 2004, to reflect their relative exposure. Darker arrows indicate larger proportional amounts, and claims worth less than one percent of the lender's 2004 GDP are not shown. The darkest arrows, which are from Switzerland (CH) to the United Kingdom (GB) and the United States (US), from the United Kingdom to the United States, and from Ireland (IE) to the United Kingdom (largely obscured), represent claims worth over 50% of the lender's 2004 GDP. Nearly all countries have claims on each other, and so arrows can be bidirectional such as between Austria (AT) and Italy (IT). Yet there are many unidirectional arrows because often one country out of any pair has a relatively small amount of claims on the other.

The algorithm places more strongly connected countries in the center and more weakly connected countries in the periphery.[7] The US and UK are very large debtors when claims are put relative to their creditors' economies, so they are made quite central. Switzerland is a very large lender relative to its own economy, France (FR) is both a moderately large lender and debtor in the normalized amounts, and Germany (DE) is a fairly large debtor. Other countries such as the Netherlands (NL) may serve as important "bridges" in the network structure.

In addition to the lending relationships, we collect data on countries' GDP, investment, yields on government-issued long-term bonds, and prices on sovereign credit default swaps.

[7]However this is not a unique representation of the network, as it is a projection of an N by N matrix into two dimensions. Different algorithms produce different visual representations, although the qualitative features are reasonably stable.

The GDP and investment series for each country come from the OECD's Quarterly National Accounts dataset. Specifically, we use quarterly GDP growth rates and gross fixed capital formation. The gross fixed capital formation is measured in fixed PPP and both series are seasonally adjusted. Additionally, we collect yields on 10-year government bonds from the OECD's Monthly Monetary and Financial Statistics database.

In Table II we report the time series mean and standard deviation of each country's GDP growth, investment growth, and bond yield during the sample period. All statistics are computed at a quarterly frequency over our sample period of 2005Q1 to 2011Q3. The first column of the table indicates that all countries in our sample have a positive average GDP growth over this period, however, there is significant heterogeneity in the average growth rate and volatility of growth across countries. The third and fourth columns of the table display the time series mean and standard deviation of the change in each country's investment. Again, we see significant heterogeneity in these values across the sovereigns in our sample. The last two columns display the mean and standard deviation of each country's 10-year government bond.

We collect CDS prices from Datastream for each country in our sample. The prices are all for 5-year CDS contracts referencing the sovereign entity, with all contracts denominated in US dollars. Table III displays summary statistics for the time series of CDS prices for each of the sovereigns in our sample. We report the time series mean and standard deviation, as well as the serial correlation of the monthly sovereign CDS prices. Comparing the first and second columns of Table III, we see that the time series standard deviation exceeds the mean for many sovereigns in our sample. In addition, the table shows that the sovereign CDS prices exhibit a high degree of serial correlation.

Figure 2 plots the time series of the cross-sectional mean and quartiles of the sovereign CDS prices in our sample. For each date, we compute the cross-sectional mean, median, 25th, and 75th percentiles of the sovereign CDS prices. Panel A displays the time series of the mean and Panel B presents the quartiles. Beginning in 2008, we observe a susbstantial rise in both the level and cross-sectional dispersion of CDS prices and this pattern continues through the remainder of our sample.

In Table IV we present the matrix of pairwise correlations of the CDS prices for our sample period. The table shows that nearly all of the sovereigns in our network have positively correlated CDS prices and for many pairs of countries this correlation is strikingly high. To further investigate the comovement of the sovereign CDS prices in our sample, we estimate the first two principal components for our CDS price series. Panel A of Table V reports the first two principal components estimated for the sovereign CDS prices for the countries in our sample. The first principal component explains 64% of the variation in the sovereign CDS prices and the first two principal components together explain 89% of the variation. As a point of reference, we estimate the first two principal components for the government long-term bond yields and GDP growth rates in our sample. These values are reported in Panels B and C of Table V. The first principal component explains 60% and 63% of the variation in government bond yields and GDP growth rates, respectively.

From these data, we construct the variables used in equation (6) as follows. Using the 5-year sovereign CDS spreads and the U.S. Treasury yield curve, we compute the time series of quarterly solvency probabilities for each sovereign.[8] For the gross returns on loans, r_i^t, we directly use the yields on 10-year government bonds. The measure of baseline output,

[8]Note that this transformation of CDS spreads to solvency probabilities assumes a 40% recovery rate and a discount factor derived from the current Treasury yield.

Z_i, is a country's annual GDP in 2004, and investment, I_i^t, is quarterly gross fixed capital formation.[9] For the revenue shock, X_i^t, we use the quarterly GDP growth rate multiplied by the baseline output. In addition the growth rate is first detrended by subtracting the average quarterly growth rate from 1995 to 2004. For the amount of loans from one country to another, l_{ij}^t, we directly use the BIS data on foreign claims on an ultimate risk basis. While these claims mostly have maturities longer than three months, the existence of secondary markets means that claims do not have to be held until maturity. Lastly, as noted before, we fix the exogenous recovery rate at $\delta = 0.4$.

5 Results

The parameters recovered by our estimation procedure are listed in Table VI. The marginal effect of international debts on a sovereign's solvency probability is significant. A difference of one unit in the normalized debt load is large, meaning a difference in total foreign claims on a country equal to its annual GDP in 2004 (i.e., $\sum_{j \neq i} l_{ji}^t r_i^t / Z_i = 1$). However the data span this amount; for example, over our time period the total foreign claims on Austria, Ireland, and Portugal all range by more than their GDP. At a point in time, the range of the normalized debt load across countries is also greater than one in most quarters. The size of this marginal effect is large relative to the average default probability: a one unit increase in the debt load raises the quarterly default probability by approximately 105 bps. Alternatively, a net debt increase of 85%, which is equal to one standard deviation of the normalized distribution, amounts to a decrease of 89 bps in the sovereign's solvency probability in the next quarter.

Other factors have significant estimated effects as well. A one percentage point increase

[9] All series are in US dollars (converted via fixed PPP) and seasonally adjusted where appropriate.

in a sovereign's quarterly GDP growth results in a reduces the sovereign's probability of default in the next quarter by 40 bps. A one standard deviation change in the normalized distribution of investment (which amount to a reduction in investment of 5% of a country's 2004 Q4 GDP), results in a 82 bps decrease in the sovereign's solvency probability in the next quarter.

Figure 3 plots the observed and predicted solvency probabilities to illustrate the dispersion in the data and the model's fit. In particular, we plot each sovereign's observed solvency probability, computed from its 5-year CDS spread, against the model predicted solvency probability for each quarter in our sample. The values displayed are the probability that a sovereign does not default in the next quarter. We can see that for most of the observations, a country's observed and predicted solvency probability are both close to 1. However, the figure illustrates clear exceptions to this, most notably Greece, Ireland, and Portugal. Generally, the model predictions match their counterpart empirical values relatively well, as seen from the fact that most observations fall relatively close to the 45° line in the plot. The model predictions appear to overestimate the solvency probability of Portugal and Spain, relative to their empirical values, for some quarters, while it underestimates the solvency probability of Ireland.

In Table VII, we present additional results on the distribution of solvency probabilities as well as the fit of the model predicted probabilities with what is observed in the data for our 396 observations. Panel A presents quartiles as well as the 5th and 95th percentile of the distribution of quarterly solvency probabilities. The first column reports the empirical solvency probabilities computed from the sovereign CDS spreads and the second column presents the counterpart values predicted by the model under the estimated parameter values given in Table VI. Panel B reports the correlation of the observed and model predicted solvency probabilities as well as the sum of squared residuals. These values are 0.741 and 0.081, respectively.

6 Simulations

Using the estimated version of equation (6) we can simulate the short-run effect of a nation-wide default on the solvency of other countries. The equation gives the probability that each country will be solvent in the next quarter (i.e., period $t+1$). To simulate the default of some country j in that period, we replace the computed conditional probabilities $p_{j|i}^{t+1}$ with zero (and do the same for $p_{j|\cdots}^{t+1}$ in the second and third steps of the expansion). This makes it so that the other countries receive no endogenous payments from country j, although they still benefit from the exogenous recovery rate for a portion of their claims. The difference between the original predicted solvency probabilities (p_i^{t+1}) and the new predictions that result from this change ($\tilde{p}_i^{t+1}$) shows the increase in the default probabilities of the other countries in period $t+1$. This is a measure of the immediate contagious effect of a default in one country.

We do this exercise separately for a default of Italy and a default of Spain using the last period of our data (2011Q3), which means the simulated default occurs in 2011Q4. Table VIII shows the total claims each country had on these two countries in 2011Q3, normalized by the lender's GDP in 2004. The largest lenders to Spain, relative to their own economies, were the Netherlands and Portugal, both of which had claims worth over 10% of their annual output. To Italy, the largest relative lender was France with claims worth over 20% of its annual output.

The results of these simulations are in Tables IX and X. We focus our analysis on the effect that a default by Spain or Italy would have on the other European sovereigns in our sample. In the first column of each table, we present the absolute change, measured in basis points, a sovereign's default probability in the next quarter resulting from the default of Italy (Table IX) or Spain (Table X). This change is measured relative to the model predicted baseline in a case where the country does not default. The absolute magnitude of the baseline quarterly default probability, even at the end of 2011, is relatively small for most sovereigns. Therefore, the relative change in a sovereign's default probability is also of interest as a small absolute change in the default probability can still represent a significant relative change. To this end, the second column of each table reports the relative increase, measured in percent, in a sovereign's one quarter default probability resulting from the default of Italy or Spain. For both simulate default events, we observe significant heterogeneity in the response of other sovereigns. Additionally, we see that a given country can have a significantly different response to the two default events. For example, Portugal's response to a default by Spain is about an order of magnitude larger than its response to the default of Italy. Conversely, we see that France would be more affected by Italy's default than the default of Spain.

Table I:
Country List

This table lists the countries included in our sample.

Country	**Abbreviation**
Austria	AT
Australia	AU
Belgium	BE
Switzerland	CH
Chile	CL
Germany	DE
Denmark	DK
Spain	ES
Finland	FI
France	FR
United Kingdom	GB
Greece	GR
Ireland	IE
Italy	IT
Japan	JP
Netherlands	NL
Portugal	PT
Sweden	SE
Turkey	TR
United States	US

Table II:
Summary Statistics for Macroeconomic Variables

This table reports the mean, $\mu(\cdot)$, and standard deviation, $\sigma(\cdot)$, for each country's quarterly GDP growth (Δy), investment growth (Δi), and long-term government bond yield (r) over the sample period 2005Q1 - 2011Q3. Further details regarding the data are provided in Section 4.

Country	$\mu(\Delta y)$	$\sigma(\Delta y)$	$\mu(\Delta i)$	$\sigma(\Delta i)$	$\mu(r)$	$\sigma(r)$
Australia	0.68	0.54	1.53	2.55	5.48	0.49
Austria	0.45	0.88	0.08	1.22	3.78	0.49
Belgium	0.34	0.75	0.33	2	3.87	0.44
Chile	1.06	1.27	2.24	5.32	5.29	2.3
Finland	0.35	1.75	0.37	3.85	3.68	0.53
France	0.23	0.62	0.18	1.52	3.71	0.47
Germany	0.4	1.16	0.66	2.92	3.47	0.58
Greece	0.06	1.22	-1.38	4.18	6.25	3.65
Ireland	0.14	2.01	-2.79	8.55	5.13	2.03
Italy	0.02	0.94	-0.35	1.77	4.28	0.48
Japan	0.12	1.41	-0.48	1.78	1.42	0.24
Netherlands	0.37	0.85	0.38	2.8	3.66	0.52
Portugal	0.06	0.82	-0.95	2.6	4.9	1.9
Spain	0.29	0.74	-0.73	2.52	4.16	0.58
Sweden	0.55	1.32	0.7	2.89	3.47	0.55
Switzerland	0.54	0.67	0.66	2.06	2.3	0.55
Turkey	1.2	2.59	1.57	5.61		
United Kingdom	0.19	0.94	0.04	3.05	4.19	0.62
United States	0.28	0.81	-0.42	2.46	3.87	0.74
Average	0.46	1.11	0.21	3.14	4.03	0.93

Table III:
Summary Statistics for Sovereign CDS Prices

This table presents summary statistics for prices of 5-year Credit Default Swaps referencing the sovereign entities in our sample. The columns labeled 'Mean' and 'Std Dev' display the time series mean and standard deviation of the CDS price series. The autocorrelation of the monthly CDS prices is given in the column labeled 'AC(1)'. The final column reports the number of monthly observations available for each sovereign's CDS price series. All CDS prices are taken from Datastream.

	Mean	**Std Dev**	**AC(1)**	**Min**	**Max**	**N**
Austria	47.8	58.9	0.95	1.5	235	100
Australia	35.2	30.1	0.88	2.5	158.3	112
Belgium	61.6	83.2	0.97	1.2	307.3	100
Chile	64.9	55	0.92	7.8	254.2	100
Germany	24	28.3	0.95	0.9	110.8	100
Denmark	32.8	38.9	0.90	1.6	142	104
Spain	115.6	134.4	0.97	2	462.5	85
Finland	39	20.6	0.86	7.3	85.3	48
France	48.5	57.4	0.98	1.5	214.7	81
United Kingdom	63.8	31.7	0.92	7	147.3	54
Greece	427.8	1020.3	0.92	5	6499.8	97
Ireland	175.5	253.3	0.99	2	882.1	105
Italy	97.6	127.3	0.97	5.6	485.4	100
Japan	38.5	39.8	0.95	3	142.7	100
Netherlands	36.6	35.9	0.91	1.2	120.8	80
Portugal	208.4	363.9	0.98	2	1601	100
Sweden	29.2	29.9	0.80	1.4	140	105
Turkey	238.1	94.2	0.89	124.7	605.2	96
United States	37.9	17.2	0.84	6.5	91.2	53
Average	95.9	132.65	0.92	9.7	864.1	91

Table IV:

Correlations of CDS Prices

This table presents pairwise correlations for the sovereign CDS prices. The data are sampled at a monthly frequency for the period January 2003 - April 2012. All CDS prices are taken from Datastream.

Variables	Austria	Australia	Belgium	Chile	Germany	Denmark	Spain	Finland	France	U.K.	Greece	Ireland	Italy	Japan	Netherlands	Portugal	Sweden	Turkey	U.S.
Austria	1.00																		
Australia	0.91	1.00																	
Belgium	0.86	0.70	1.00																
Chile	0.82	0.88	0.55	1.00															
Germany	0.95	0.83	0.96	0.70	1.00														
Denmark	0.89	0.83	0.79	0.76	0.87	1.00													
Spain	0.80	0.62	0.98	0.46	0.92	0.71	1.00												
Finland	0.95	0.83	0.77	0.57	0.93	0.97	0.67	1.00											
France	0.86	0.67	0.99	0.51	0.96	0.80	0.96	0.82	1.00										
U.K.	0.86	0.85	0.51	0.73	0.72	0.75	0.44	0.72	0.51	1.00									
Greece	0.63	0.44	0.81	0.31	0.76	0.63	0.74	0.64	0.86	0.30	1.00								
Ireland	0.77	0.65	0.95	0.47	0.89	0.70	0.94	0.60	0.90	0.43	0.71	1.00							
Italy	0.88	0.70	0.98	0.57	0.96	0.83	0.96	0.82	0.99	0.52	0.85	0.89	1.00						
Japan	0.90	0.77	0.92	0.63	0.95	0.78	0.91	0.75	0.91	0.67	0.73	0.88	0.91	1.00					
Netherlands	0.93	0.85	0.84	0.73	0.91	0.94	0.79	0.97	0.86	0.78	0.63	0.72	0.86	0.81	1.00				
Portugal	0.72	0.53	0.93	0.36	0.86	0.71	0.92	0.66	0.94	0.30	0.88	0.91	0.94	0.85	0.72	1.00			
Sweden	0.79	0.82	0.51	0.80	0.66	0.78	0.39	0.68	0.45	0.88	0.32	0.46	0.53	0.60	0.72	0.34	1.00		
Turkey	0.20	0.30	-0.03	0.50	0.09	0.22	0.02	0.44	0.19	0.38	0.01	-0.10	0.01	-0.02	0.45	-0.08	0.25	1.00	
U.S.	0.75	0.80	0.53	0.62	0.69	0.67	0.48	0.67	0.49	0.90	0.28	0.56	0.48	0.62	0.70	0.34	0.75	0.29	1.00

Table V:
Principal Components Analysis

This table presents the first two principal components for the CDS prices (Panel A), 10-year bond yields (Panel B), and GDP growth rates (Panel C), for the sovereigns in our sample. CDS prices are taken from Datastream, bond yields are from the International Financial Statistics tables from the IMF, and GDP growth rates are computed from the OECD's quarterly national accounts.

Panel A: CDS Prices		
	Proportion	Cumulative
Component 1	0.640	0.640
Component 2	0.248	0.888
Panel B: Bond Yields		
	Proportion	Cumulative
Component 1	0.599	0.599
Component 2	0.246	0.845
Panel C: GDP Growth Rates		
	Proportion	Cumulative
Component 1	0.630	0.630
Component 2	0.079	0.709

Table VI:
Parameter Estimates

This table presents our main estimates of equation (6). The data and construction of the variables are described in Section 4.

Parameter	Value
α_1	58.29
α_2	33.84
β_0	1.29
β_1	70.54
σ	6.03

Table VII:
Observed and Model Predicted Solvency Probabilities

This table compares the observed empirical solvency probabilities, computed from 5-year CDS spreads, to those predicted from the estimated model. Panel A reports percentiles from the observed and model-predicted distribution of solvency probabilities. Panel B reports the correlation of the observed and predicted probabilities, the sum of squared residuals from a regression of the observed on the predicted solvency probabilities for the 396 observations in our sample.

Panel A: Distribution of Solvency Probabilities

Percentile	Observed	Predicted
5th	0.9599	0.9682
25th	0.9880	0.9895
50th	0.9955	0.9948
75th	0.9990	0.9985
95th	0.9998	1.0000

Panel B: Fit of Model-Predicted Solvency Probabilities

ρ(Observed, Predicted)	0.741
SSR	0.081
# obs	396

Table VIII:
Normalized Foreign Claims on Spain and Italy in 2011Q3

This table reports the normalized value of foreign claims from each country listed in the first column, on Spain in the second column and on Italy in the third column. Country abbreviations are listed in Table I. The claims data come from the BIS and are on an ultimate risk basis. The raw amounts are normalized to reflect the relative exposure of the creditor countries to Spain and Italy, by dividing by the creditor's annual GDP in 2004.

Creditor Country	Claims on Spain	Claims on Italy
AT	0.023	0.079
AU	0.001	0.002
BE	0.065	0.068
CA	0.003	0.005
CH	0.082	0.093
CL	0.000	0.000
DE	0.065	0.058
ES	- - -	0.031
FI	0.008	0.004
FR	0.080	0.207
GB	0.048	0.032
GR	0.001	0.002
IE	0.080	0.072
IT	0.018	- - -
JP	0.007	0.010
NL	0.127	0.069
PT	0.116	0.012
SE	0.010	0.003
TR	0.000	0.000
US	0.004	0.003

Table IX:
Simulated Effect of Italy Default in 2011Q4

This table presents the results of the simulation described in Section 6. Baseline default probabilities come from the model as originally estimated. Simulated default probabilities come from the model when we make the probability of repayment from Italy be zero. The relative change is the difference divided by the baseline. See the text for further details.

	Absolute Change (bps)	Relative change (%)
AT	7.8	2.2
BE	7.7	1.8
DE	0.8	0.9
ES	0.8	1.4
FI	0.7	0.1
FR	11.0	6.2
GB	3.8	0.6
IE	42.2	1.1
IT	–	–
NL	2.3	0.7
PT	3.6	0.3
SE	0.8	0.3

Table X:
Simulated Effect of Spain Default in 2011Q4

This table presents the results of the simulation described in Section 6. Baseline default probabilities come from the model as originally estimated. Simulated default probabilities come from the model when we make the probability of repayment from Spain be zero. The relative change is the difference divided by the baseline, which is the same for either the one-year or five-year probabilities. See the text for further details.

	Absolute Change (bps)	Relative change (%)
AT	2.3	0.7
BE	7.7	1.8
DE	0.8	0.9
ES	–	–
FI	0.8	0.1
FR	4.7	2.7
GB	4.6	0.7
IE	48.2	1.3
IT	2.3	0.6
NL	7.0	2.0
PT	36.7	3.0
SE	0.8	0.3

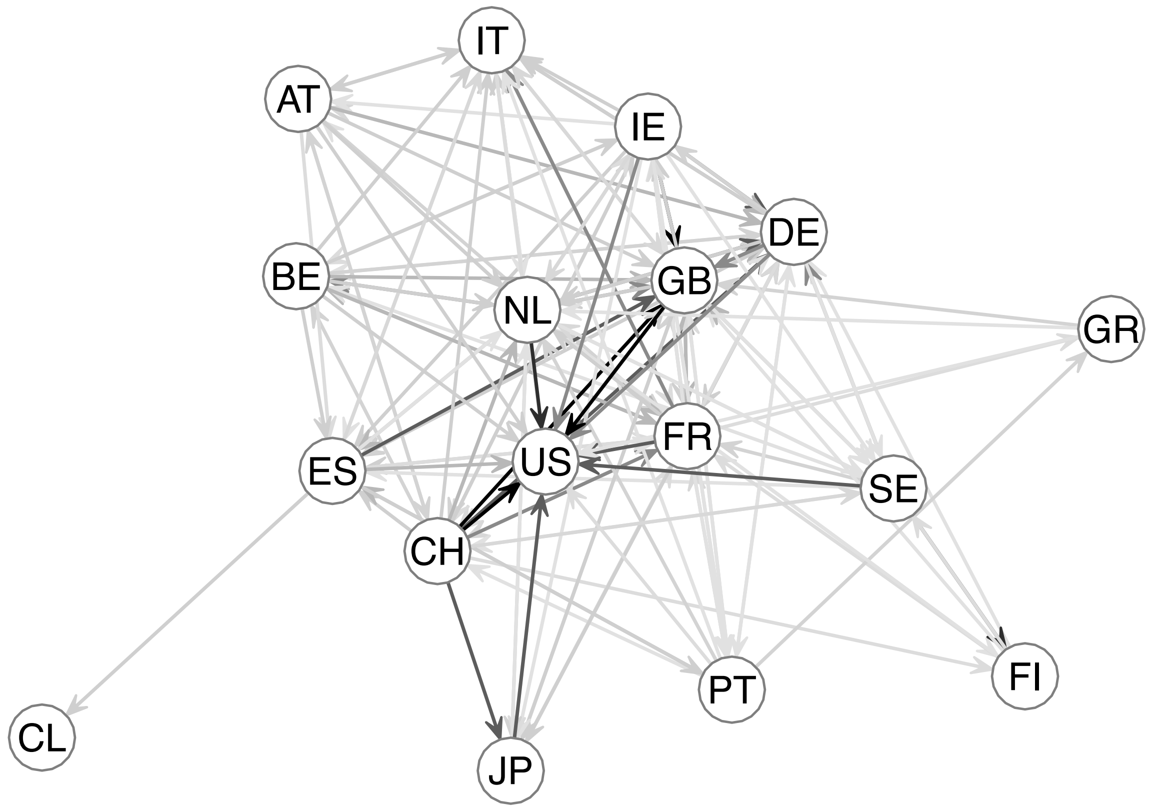
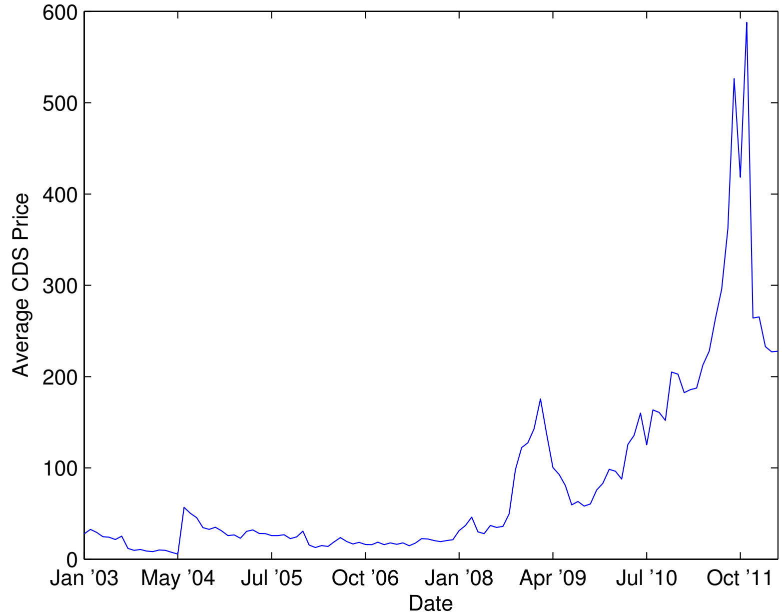


Figure 1: Network Graph. This figure displays the network structure of lending relationships in the third quarter of 2011. Countries are represented by their two letter abbreviation in Table I. Arrows represent claims that one country has on another, and darker arrows indicate larger amounts in proportion to the creditor country's GDP in 2004. For further description, see Section 4.

A. Mean CDS Price



B. CDS Price Quartiles

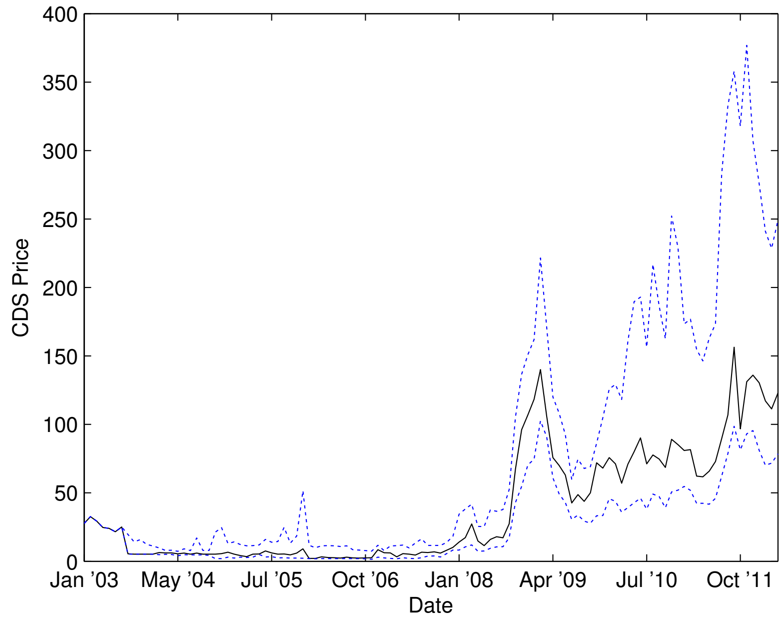


Figure 2: Mean and Quartiles for CDS Prices. This figure plots the mean, median, and first and third quartiles of the cross-section of the CDS prices for our sample of sovereigns. In Panel (A) the time series of the cross-sectional mean is plotted for January 2003-April 2012. Panel (B) plots the time series of the cross-sectional median CDS price (solid black line) along with the 25th and 75th percentiles (blue dotted lines). All CDS prices are for 5-year CDS contracts referencing the sovereign entity. The sovereigns comprising our sample are listed in Table I.

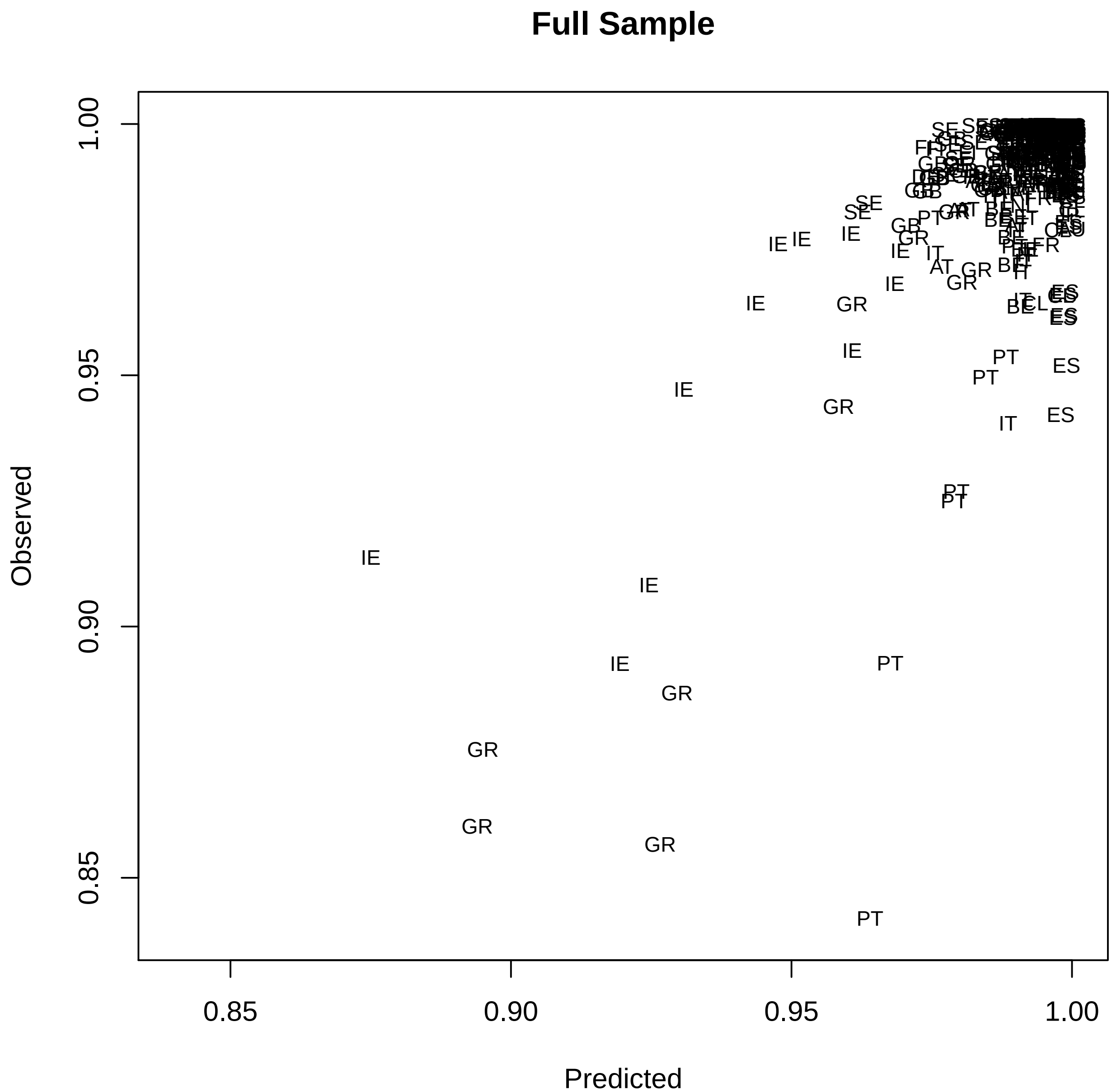


Figure 3: Predicted and Observed Solvency Probabilities. This figure plots the predicted and observed quarterly solvency probabilities for each country at each quarter in our sample. The observed solvency probabilities are obtained with a transformation of 5-year CDS contract prices, as described in Section 4. The predicted solvency probabilities are from the estimated version of equation (6). Country abbreviations are listed in Table I.

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