Aggregate Issuance and Savings Waves*

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Abstract

We document the fact that at both the aggregate and the firm level, corporations tend to simultaneously raise external finance and accumulate liquid assets. For all but the very largest firms, the aggregate correlation between external finance raised and liquidity accumulation is 0.6, and the average firm level correlation is 0.2. Conditioning on firms that raise external finance, the aggregate correlation increases to 0.74. We also show that firms' decisions in the *cross-section* about their sources and uses of funds can be useful for identifying the *aggregate* level of the cost of external finance. Specifically, we measure the cross-sectional correlation between external finance and liquidity accumulation at each date, and show that the time series of this cross-sectional correlation is highly correlated with traditional measures of the cost of external finance. We explore the extent to which a simple dynamic model with constant costs of external finance can be improved upon by a model which features a shock to the cost of external finance in matching the empirical moments which describe the joint dynamics of internal and external finance. For its simplicity, the simple dynamic model performs surprisingly well. However the model with a stochastic cost generates more realistic levels of liquidity accumulation, breaks the stochastic singularity of the single shock model, and produces the observed relationship between the cost of external finance and firms' financing and investment decisions in the cross section. Finally, we use our dynamic model and a set of cross-sectional moments describing firms' internal and external financing decisions to estimate a time series for shocks to the cost of external finance in the US time series 1980-2010.

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I. Introduction

We document the fact that at both the aggregate and the firm level, corporations tend to simultaneously raise external finance and accumulate liquid assets. For all but the very largest firms, the aggregate correlation between external finance raised and liquidity accumulation is 0.6, and the average firm level correlation is 0.2. Conditioning on firms that raise external finance, the aggregate correlation increases to 0.74. We also show that firms' decisions in the *cross-section* about their sources and uses of funds can be useful for identifying the *aggregate* level of the cost of external finance. Specifically, we measure the cross-sectional correlation between external finance and liquidity accumulation at each date, and show that the time series of this cross-sectional correlation is highly correlated with traditional measures of the cost of external finance. We explore the extent to which a simple dynamic model with constant costs of external finance can be improved upon by a model which features a shock to the cost of external finance in matching the empirical moments which describe the joint dynamics of internal and external finance. For its simplicity, the simple dynamic model performs surprisingly well. However the model with a stochastic cost generates more realistic levels of liquidity accumulation, breaks the stochastic singularity of the single shock model, and produces the observed relationship between the cost of external finance and firms' financing and investment decisions in the cross-section. Finally, we use our dynamic model and a set of cross-sectional moments describing firms' internal and external financing decisions to estimate a time series for shocks to the cost of external finance in the US time series 1980-2010.

Understanding the role of a potentially time varying cost of external finance is important for several reasons. First, studying whether shocks to the cost of external finance are important for firm financing, liquidity accumulation, and investment dynamics may help to uncover the role of the financial sector in business cycles. Relatedly, it is important to understand whether policies aimed at reducing external finance costs will result in a potentially desirable increase in investment and output. We model firms' optimal accumulation of physical capital and liquid assets and relate these decisions to a portfolio choice with a riskless and a risky asset. We show that if investment is low because investment opportunities are absent, then policies which reduce external finance costs are likely to mainly lead to higher cash balances. From a theoretical standpoint, many models featuring endogenous variation in the cost of external finance in business cycles feature a tight link between variation in fundamentals such as productivity and variation in the informational frictions which make external finance costly. In contrast, shocks to the cost of external finance are allowed to vary independently in recent DSGE models with costly external finance. By using a dynamic model as a filter on the US time series for firm financing, liquidity accumulation, and investment, we aim to provide implied estimates of the cost of external finance at each date.

The empirical literature on business cycles has only recently begun to include quantities describing the financing of corporations. Jermann and Quadrini (forthcoming), and Covas and Den Haan (2011a) both document that debt issuances are highly procyclical, and Covas and Den Haan also report procyclical equity issuances. We are the first to incorporate data on firms' liquidity accumulation, as well as their investment, in order to consider the role of pure financing shocks vs. shocks to productivity in explaining firm level and aggregate investment and financing activities. We argue that looking at the joint dynamics of liquidity accumulation and external finance is useful for examining the role of shocks to the cost of external finance, since how firms use funds may help to disentangle financing shocks from shocks that drive investment opportunities.

First, we document the stylized facts regarding the joint dynamics of internal and external finance in the aggregate, as well as at the firm level. We exclude the very largest firms, as in Covas and Den Haan (2011a), since, as detailed in their paper, one-off financing activities at these firms can greatly affect aggregate correlations. At the aggregate level, we describe the aforementioned strong, positive correlation between liquidity accumulation and external finance. Importantly, the strong, positive correlation between liquidity accumulation and external finance at the aggregate level is not simply due to offsetting positions across firms. We show that this same positive correlation describes firm level behavior. The aggregate correlation increases when one conditions on firms which are raising external finance. When conditioning on firm size, we find that small firms exhibit stronger positive correlations between the accumulation of internal and external finance.

Our second main stylized fact is about the information in the cross-section of firms' financing and investment decisions about the aggregate state of the cost of external finance. Intuitively, firms should not raise external finance only to invest in low return liquid assets if external finance is costly. Supporting this idea, we find that the time series of the cross-sectional correlation between liquidity accumulation and external finance (XSrho) and the default spread is -0.64, and the correlation between XSrho and lending standards is 0.58.

We then build a dynamic model of firm level internal and external finance in which firms are subject to both idiosyncratic and aggregate shocks. We simulate our model, form a panel of data, and evaluate the model's ability to match both individual firm and aggregate moments for the corporate sector. Though our model is partial equilibrium, studying the aggregate implications is still useful for exploring how the behavior of individual firms subject to common shocks affects the aggregate time series for liquidity accumulation, external finance, and investment. Our focus on the joint dynamics of internal and external finance is new relative to the literature on the business cycle properties of debt and equity issuances.[1]

We first study the implications for external finance issuance and liquidity accumulation from a baseline model of corporate savings with three key features. These are: persistent productivity shocks, a fixed cost of external finance, and convex adjustment costs for investment. With these three features, when investment opportunities are good, firms raise more external finance than they need for current investment in order to

[1] One exception is Eisfeldt and Rampini (2009), which builds an aggregate model of internal and external finance to study the implications of corporate liquidity demand for the observed low return on liquid assets. Covas and Den Haan (2011a) focus on debt and equity issuances, but they do note that, empirically, firms tend to both accumulate financial assets and invest when they issue external finance.

avoid again paying the fixed cost of external finance. The additional funds raised are accumulated as liquid assets, and used to smooth investment over time. The baseline model is able to generate the observed aggregate correlation between external finance and liquidity accumulation of 0.6 using a fixed cost which implies that aggregate costs of external finance are only 1% of total proceeds raised.[2] Similarly, the investment adjustment costs are less than 1% on average. In fact, although the convex adjustment cost is key to the correlation we focus on, the firm level autocorrelation of investment in the model data is actually lower than that in Compustat data. Thus, this baseline model is a useful starting point because even without stochastic costs of external finance, it can generate a positive correlation between external finance and liquidity accumulation. The baseline model is a robust "straw man," which directs us where to look for unique predictions of the model with stochastic costs.

We compare the data from the baseline model to a version in which the costs of external finance feature an aggregate stochastic component. One important improvement that the stochastic cost model provides is to the level of liquidity accumulation. This is because the stochastic cost is an additional risk and thus increases firms' precautionary demand. Moreover, because capital is more productive than liquid assets, it provides a much higher return on average. Thus, if the only reason to accumulate liquidity is to hedge investment opportunities arising from productivity shocks, firms have a strong incentive to use capital itself to hedge. By contrast, the stochastic cost of external finance has a greater effect on liquidity accumulation on average.

The second improvement of the stochastic cost model over the baseline model is in reducing the baseline model's relatively high correlation between external finance and investment, and investment and liquidity accumulation, especially at the firm level. Although all of these cross correlations are positive at the firm and aggregate level in Compustat data, the baseline model with one shock generates higher correlations than the data show (i.e. the model displays 'stochastic singularity'). Because the stochastic cost induces independent variation in the relative returns to liquidity accumulation and investment, the model with stochastic cost helps to lower the correlation between investment and the financing variables.

We argue that the strongest support of the stochastic cost model comes from its ability to replicate the relationship between the correlation between external finance and liquidity in the cross-section, and the level of the shock to the cost of external finance. The model produces a correlation between XSrho and the level of the cost shock which is 0.8.

There are a few caveats to the success of the model with stochastic costs, however. First, it is perhaps not surprising that a two shock model performs better than a one shock model. Second, firms' only ultimate use of funds in the model is for capital investment. In practice, firms may raise funds in order to fund labor or other operating expenses. Thus, we expect the empirical correlation between external finance and investment to be considerably lower than in our model. Finally, the empirical correlations are based on flow

[2] Cummins and Nyman (2004), and Bazdresch (2005) both argue that fixed costs can help to match the lumpy nature of issuances, and note that such lumpiness can cause firms to save some of the proceeds from issuance.

measures over fiscal years, whereas the model data describes decisions at each model date. This timing issue may also reduce some of the empirical correlations, especially at the firm level.

Finally, we document the usefulness of using firms' financing and liquidity accumulation decisions in identifying aggregate conditions for the cost of external finance. Financing variables related to the stochastic cost of external finance in our model are empirically correlated with measures of the cost of external finance such as the default spread, and loan officers' reports of lending standards. In the model, the cost of external finance is strongly negatively related to the percentage of firms raising external finance, with a correlation of -0.7. Empirically, the correlation between the default spread and the percentage of firms raising external finance is -0.59, and the correlation between loan officers' reports of lending standards and the percent of firms raising external finance is -0.40.

This relationship between measures of the cost of external finance and issuance activity is intuitive, but perhaps not surprising. Thus, we turn to a measure which incorporates the information in firms' policies for both how much external finance they raise, and how those funds are used. We show that the cross-sectional correlation between the amount of external finance which firms raise, and the liquidity they accumulate, is strongly negatively correlated with the cost of external finance in the model and in the data. States in which external finance is less costly are states in which it is advantageous for firms to raise external finance and then save the proceeds as cash. When external finance is more costly, the cost of liquidity accumulation exceeds the benefit, and firms tend not to issue and save. Consistent with this intuition from the model, in the data the correlation between the cross-sectional correlation between external finance and liquidity accumulation and the default spread is -0.64. The correlation with loan officers reports of tightening lending standards is -0.58. Thus, we argue that using the information in the cross-section regarding firms' use of external finance may be useful for making inferences about aggregate costs of external finance.

Lastly, we turn to an estimation of the cost of external finance as implied by the model by using a set of simulated moment conditions. Specifically, at each date in time we choose the value of the stochastic cost that generates simulated cross-sectional moments as close as possible to those in the data. The estimated series indicates a high cost of external finance during episodes such as the recession of 1981, the stock market crash of 1987, the Asian financial crisis of 1997, the crash of the tech boom in 2001, and the recent financial crisis.

II. Related Literature

Several recent papers develop models which use a shock which originates in the financial sector to better match business cycle facts.[3] Jermann and Quadrini (forthcoming) show how a model with an endogenous credit limit and a shock to capital liquidity can generate realistic business cycles as well as matching the

[3]These papers build on the seminal contributions of Bernanke and Gertler (1989), Kiyotaki and Moore (1997) and Carlstrom and Fuerst (1997) on the role of financial market conditions on firm investment and business cycle dynamics.

procyclical debt issuance and countercyclical equity issuance which they document using US Flow of Funds data. Covas and Den Haan (2011a) show that in Compustat data both debt and equity issuance are procyclical. In Covas and Den Haan (2011b), they develop a model in which countercyclical equity issuance costs are useful for generating both procyclical equity issuance and a countercyclical default rate.[4] Khan and Thomas (2011) build a quantitative business cycle model in which credit shocks drive aggregate productivity down by inhibiting productive investment reallocation across firms. Hugonnier *et al.* (2011) build a search theory of external finance and show how idiosyncratic external finance risk affects corporate savings, investment, and payout policy. Bolton *et al.* (2011) develop a dynamic theory of firm finance and risk management with stochastic financing costs, and show analytically that such costs can increase savings and can delink external finance from investment at the firm level in a model with constant investment opportunities. Our model confirms these effects in a calibrated, quantitative model with stochastic investment opportunities, and we document their empirical relevance. We also use our model to estimate the cost of external finance in the US time series. Thus, our paper is most closely related to Jermann and Quadrini (forthcoming), with two key differences. First, Jermann and Quadrini (forthcoming) focus on the distinction between debt vs. equity in their estimation, whereas we do not distinguish between sources of external finance and instead incorporate information regarding how fund are used into our estimation strategy. Second, Jermann and Quadrini (forthcoming) use an assumed binding constraint to identify their shock. While cannot solve our model for the cost of external finance shock in closed form, we think that the use of a cross-sectional moments and an SMM like procedure to identify a hidden aggregate state is a methodology with other potential uses.

Despite this renewed interest, the fact that financial constraints, or shocks originating in the financial sector, are important for either firm level investment, or business cycle dynamics, is not a foregone conclusion amongst economists. While Ivashina and Scharfstein (2010), Duchin *et al.* (2010), Campello *et al.* (2010), Matvos and Seru (2011), and Almeida *et al.* (2009) provide evidence that the financial crisis hindered external finance and investment activity at the firm level, Paravisini *et al.* (2011) find only small effects of credit supply shocks on trade. Moreover, Chari *et al.* (2008) argue that aggregate data do not support the occurrence of a credit crunch and question the appropriateness of government interventions aimed at improving access to external finance.[5]

Another striking empirical fact is that aggregate corporate investment closely tracks aggregate corporate internal funds.[6] Moreover, aggregate investment rarely exceeds internal funds. Interestingly, this observation has been used both to motivate theories of costly external finance, such as the pecking order (Myers (1984) and Donaldson (1961)), and conversely to argue that perhaps frictions between the household and corporate sector are unimportant for corporate investment (Chari *et al.* (2007)). Chari, et. al. do, however,

[4]Choe *et al.* (1993), and Korajczyk and Levy (2003) also study issuances over the business cycle. Both find that equity issuance is procyclical. Korajczyk and Levy (2003) report countercyclical debt issuance.

[5]Likewise, Chari *et al.* (2007) use business cycle accounting to argue that shocks to the cost of installing capital, or to the return on capital, are only of tertiary importance for explaining the US fluctuations output, investment, and employment. However, papers such as Justiniano *et al.* (2010), and Christiano *et al.* (2010), assert that such shocks explain a large fraction of business cycle fluctuations.

[6]See Eisfeldt and Rampini (2009).

acknowledge that reallocation of funds within the corporate sector, and frictions therein, may play a role.[7] We show using our calibrated, dynamic model, that costly external finance is consistent with aggregate shortfalls being rare. In both our baseline model and in the stochastic cost version, the corporate sector as a whole is rarely raising external finance, although the likelihood for individual firms raising external finance is more than an order of magnitude higher than the aggregate likelihood.

It is important to note that even if costly external finance is an important driver of investment over the business cycle, it does not necessarily follow that government policies aimed at lowering such costs in recessions are useful. Gomes *et al.* (2006) point out that the shadow cost of external finance is procyclical in a standard business cycle model with agency costs of external finance. Gomes *et al.* (2006) estimate an aggregate production based asset pricing model in which the stochastic discount factor varies with the default premium, and find that the estimated shadow cost of funds is procyclical.[8] This makes sense if the shocks which drive firms' demand for external funds are procyclical. In our model with investment in both liquid assets and physical capital, lowering the cost of external finance without affecting the relative returns to liquid and physical capital will not spur investment in physical capital since firms can instead save funds for when investment opportunities improve. That this may be empirically relevant was evident in the financial crisis when government subsidized funding was provided to banks, and banks responded by hoarding the funds instead of by making more new loans.

Our paper is also related to papers which develop dynamic models of corporate saving. Kim *et al.* (1998) develop a three date model and show that cash accumulation is increasing in the cost of external finance, the variance of future cash flows, and the return on future investment opportunities, but decreasing in the return differential between physical capital and cash.[9] Almeida *et al.* (2004) study the cash flow sensitivity of cash and empirically document a link between the propensity to save out of cash flow and financial constraints.[10] Riddick and Whited (2009) construct a fully dynamic model of corporate savings and emphasize the importance of uncertainty for determining corporate savings, and argue that in such a model, the propensity to save is not an accurate measure of financial constraints. Thus, the link between financial constraints and investment in financial assets is also unresolved.[11] Our paper focuses on learning about the role of costly external finance in aggregate issuance and savings waves using both aggregate and cross-sectional moments for external finance, liquidity accumulation, and investment. A contemporaneous paper with a related focus is Warusawitharana and Whited (2011), which uses simulated method of moments

[7] See also Shourideh and Zetlin-Jones (2012).

[8] A related finding in Chari *et al.* (2007) is that using business cycle accounting it actually appears that financial frictions improved during the great depression.

[9] For a model which focuses on the value of the flexibility of cash for adjusting net leverage instead of funding investment, see Gamba and Triantis (2008).

[10] See also Faulkender and Wang (2006) for evidence that cash is more valuable when held by financially constrained firms. Harford *et al.* (2011) argue that firms save to insure against refinancing risk and document an inverse relationship between debt maturity and cash holdings which is stronger when credit market conditions are tighter.

[11] For a model which instead focuses on the value of the flexibility of cash for adjusting net leverage, see Gamba and Triantis (2008).

to show that equity misvaluation shocks can help explain firm level corporate issuance and savings policies.

Finally, our paper is related to dynamic models of capital structure. The fact that firms tend to simultaneously raise external finance and accumulate liquidity is at odds with standard static pecking order intuition. Static pecking order theories based on Myers (1984) predict that firms will first draw down cash balances and only once these are exhausted will they seek external finance. Thus, such theories predict a counterfactually negative correlation between external finance and liquidity accumulation. Both of our dynamic models feature a pecking order in the sense that internal funds are less costly than external funds, and both models generate the observed positive correlation between external finance and liquidity accumulation. This result is similar to the implications of the models in Hennessy and Whited (2005) and Strebulaev (2007) for the trade off theory of capital structure. Those papers show that data which appear to be inconsistent with static trade-off theories of capital structure can be generated by dynamic models in which firms' objectives are based precisely on the trade-off between the tax benefits and distress costs of debt.

III. Stylized Facts

A. Main Facts

We document two new stylized facts describing aggregate issuance and savings waves. First, the aggregate time series correlation between external finance raised and liquidity accumulation is strongly positive. For all but the top 10% of Compustat firms, the aggregate correlation is 0.60 and is statistically significant at the 5% level. Figure 1 plots cash flows to liquid assets vs. cash flows to external finance at this aggregated level and clearly illustrates our first stylized fact. This aggregate correlation is higher (0.74) if one conditions on firms that are currently raising external finance, so the positive aggregate correlation does not seem to be driven by some firms saving, and other firms issuing external finance. The aggregate correlation is also higher when one excludes more of the largest firms. For the top half of firms, the correlation between aggregated external finance raised and liquidity accumulated is 0.84. This is in contrast to conditioning on other measures of financial constraints, such as whether a firm pays no dividends, or has no credit rating, in which case we find correlations close to that for the larger sample (0.68 and 0.56 respectively). This could be due to the importance of fixed costs in accessing external financial markets, or it could be that size is simply a better proxy for financial constraints. Finally, we also find a positive correlation using flow of funds data. Table I displays our main aggregate issuance and savings stylized facts. The Subsection B. and the Data Appendix contain details of our data and variable construction.

Second, we show that in the cross-section, firms are more likely to raise external finance and save the proceeds when the default spread is low, and when lending standards are less tight. Thus, we argue that using firms' revealed preferences regarding issuance and savings decisions is informative about the aggregate state of external financing costs. When financing costs are high, we argue that firms are unlikely to raise costly external finance only to save the proceeds as low-return, liquid, assets. At each date, we compute the cross-sectional correlation between aggregate net external finance raised and liquidity accumulation (each normalized by lagged book assets), and construct a time series of this cross-sectional correlation, which we call XSrho. We then show that the correlation between XSrho and the negative of the Baa-Aaa default spread is 0.64. Similarly, the correlation between XSrho and the negative of the fraction of banks reporting tighter lending standards is 0.58. Both correlations are statistically significant at the 5% level. Figure 3 illustrates the strong relationship between XSrho, the default spread, and lending standards by plotting the time series for XSrho along with the negative of the default spread and lending standards. Although all the series are highly correlated, there is independent information in XSrho; only XSrho shows a low cost of external finance in the boom of 1986 since the default spread was not particularly low then.Finally, we show that, by contrast, XSrho is less correlated with TFP (0.48). These facts together motivate our estimation exercise in section VI.. Building on the idea of combining the information in both firms' *sources* and their *uses* of funds to learn about the cost of external finance, we use firms' financing, liquidity accumulation, and investment decisions in the cross-section to make inferences about the aggregate cost. Specifically, we use a version of SMM to estimate a time series in US data for the hidden external finance cost shock state using our model along with cross-sectional moments such as XSrho.

In sum, we present two new stylized facts, namely the strong positive correlation between aggregate issuance and savings, and the strong positive relation between issuance and savings in the cross-section and traditional measures of the cost of external finance. In Sections IV., V. and VI. we explore the role of shocks to the cost of external finance in generating these and related stylized facts about the joint dynamics of internal and external finance, and provide our estimate of the time series of the cost of external finance in the US time series 1980-2010.

B. Data Description

Our main data set consists of annual firm level data from Compustat from 1980-2010. We focus on Compustat data since we are able to analyze firm level, as well as aggregate, facts. Thus, our sample selection criterion closely follows that in Covas and Den Haan (2011a). When matching the aggregate facts, we show the results obtained using Flow of Funds data are qualitatively similar. The Data Appendix gives a detailed description of the construction of our data.

We use firm level cash flow statements to track corporate flows. We define liquidity accumulation as changes in cash and cash equivalents.[12] We define net external finance raised as the negative of the sum of net flows to debt and net flows to equity. We define flows to debt as debt reduction plus changes in current debt plus interest paid, less debt issuances, and flows to equity as purchase of common stock plus

[12]We do not use the balance sheet measure of cash since the stock measure is affected by acquisitions. Covas and Den Haan (2011a) instead remove firms involved in mergers which increase sales by more than 50%. We have checked that our findings are similar using stock measures and the non-merger sample.

dividends less sale of common stock. Following Covas and Den Haan (2011a), and Fama and French (2005), we also consider using the negative of the change in total liabilities as flows to debt and negative changes in book equity as flows to equity. We find similar results using these stock measures. We focus on the flow measures in the interest of brevity, and since our model does not feature issuances which are not truly "external" like those related to mergers or employee compensation which are emphasized in Fama and French (2005). Finally, we also verify that the results are similar if we just focus on issuances of debt and equity, rather than the total flow. We define investment (in physical capital) as capital expenditures. We do not include acquisitions in our investment measure. Firm level acquisitions are very lumpy, which can bias the correlations we compute. We verify that including acquisitions does not change our aggregate results, since the aggregate series smooths out individual firm lumpiness.

When computing most aggregate and firm level moments, we normalize firm level variables by current total book assets. When computing aggregate correlations, we instead normalize by the lag of book assets, to avoid inducing spurious correlation between the series. Book assets are slow moving and fairly acyclical and thus shouldn't induce any trends in our data. Our results are robust to alternative normalizations, such as aggregate output or aggregate gross-value added from the corporate sector. We use the Hodrick and Prescott (1997) filter to remove any remaining series trends when computing aggregate correlations, since, for example cash holdings have trended upwards as a share of assets over our sample (Bates *et al.* (2009)).

As in Covas and Den Haan (2011a), our main analysis drops the top 10% of firms by asset size. There are several reasons to do this. First, the very largest firms present unique measurement issues. More of the investment for these firms falls under the accounting category "other investments". These other investments are typically long term receivables to unconsolidated subsidiaries. Thus, a large firm may raise funds on behalf of a smaller subsidiary, which in turn may use the funds to build a new factory, or may store the funds as liquid assets. Since we are not able to measure the funds' ultimate use, we are not able to identify accumulated liquidity vs. physical investment, the main goal of this paper. Second, the largest firms tend to have a much larger share of foreign earnings. Cash accumulation for firms with large foreign earnings may be influenced by tax motives involving repatriation timing. Third, as Covas and Den Haan (2011a) point out, external finance for the largest firms is not representative of the rest of the sample. They show in particular that one incidence of AT&T raising equity during a recession in 1983 has implications for the cyclicality of *aggregate* equity issuance. They advocate dropping the top firms because they have an unusually large influence on the aggregate series. Fourth, it is possible that the very largest firms face little or no financial constraints. Finally, we note that in the type of stationary model we study, the distribution of firm sizes will be much less skewed than that in the data. Although the model will generate the decreasing correlation between external finance and liquidity as firm size increases, aggregate model data will not be as heavily driven by the activities of a few large firms.

For the Flow of Funds data, we normalize each series by the trend in gross-value added of the corporate sector (computed using the hp-filter), though using total gdp is similar. Here, we face a new data issue because the Flow of Funds data do not do a good job of identifying liquid assets. If we very narrowly define liquid assets as the net acquisition of financial assets minus trade receivables minus miscellaneous assets, we find a correlation between external finance and liquidity accumulation of 0.33. However, this definition clearly lacks a large part of investment in marketable securities, since the flow of funds data display a counterfactual decrease over time in this series for liquid assets held within the corporate sector.[13] Therefore, we also compute the correlation including 1/3 of miscellaneous other assets as liquid and find a correlation of 0.38 which is statistically significant.[14] Overall, the Flow of Funds data corroborates our main finding of a positive correlation.

IV. Model

We present a dynamic model of internal and external finance in the presence of financing costs and physical investment adjustment costs. The model clearly illustrates the challenges involved with identifying shocks to the cost of external finance. In particular, a baseline version of the model with a single shock to total factor productivity can generate issuance and savings waves purely due to variation in investment opportunities in the presence of constant physical and financial adjustment costs. In fact, for its parsimony, the baseline model performs surprisingly well. However, the baseline model falls short on a few important dimensions, and we use these shortcomings, along with evidence from the empirical corporate finance literature, to motivate introducing a stochastic financing cost. In particular, we show that the model with a stochastic cost generates higher liquidity accumulation and breaks the baseline model's tight, and counterfactual, correlation between external finance and investment.[15] We also introduce a new measure of the cost of external finance based on firms' actual decisions regarding the source and use of corporate financing. As documented in section III., this measure is highly correlated with traditional measures of the cost of external finance, but is not significantly correlated with estimated TFP shocks. This finding is consistent with the existence of a second shock. Our measure of the cost of external finance is based on the intuitive idea that firms will not raise costly external finance only to accumulate low return assets (liquidity) if financing costs are high. Supporting this idea, we show analytically that in a two-period version of our model, liquidity accumulation is much more sensitive to the cost of external finance than investment is. Accordingly, we show that in our calibrated model, the correlation in the cross-section between external finance and liquidity accumulation is highly correlated in the time series with the level of the shock driving the cost of external finance. Building on these findings from the model, in section VI. we use a version of SMM to infer the hidden external finance cost shock state using this and related cross-sectional moments.

[13] See Bates *et al.* (2009).

[14] The decision to use 1/3 of other miscellaneous assets was based on personal communication with staff at the Board of Governors. Their rough estimate using recent IRS data is that about 1/3 of miscellaneous other assets were marketable securities.

[15] See also Warusawitharana and Whited (2011) for a contemporaneous study with similar findings in the context of equity misvaluation at the firm level.

A. Setup

The economy consists of a continuum of heterogeneous, risk-neutral firms, which differ in terms of their current idiosyncratic productivity shock, and their current stocks of physical capital and liquid assets. The model is partial equilibrium. Firms face a common aggregate productivity shock, and take the exogenous risk-free rate as given. Each firm chooses its investment in physical and liquid assets in order to maximize the discounted value of its net payouts, subject to financing and investment adjustment costs.

Firms produce output or cash flows using physical capital according to:

$$y = zk^{\theta}$$

where z is the level of the firm's productivity.

Capital evolves according to the standard law of motion:

$$k' = (1-\delta)k + i_k,$$

and investment is subject to adjustment costs $\phi_i(i_k, k)$ given by:

$$\phi_i(i_k, k) = ck\Phi_i + \frac{a}{2}\left(\frac{i_k}{k}\right)^2 k.$$

Investment adjustment costs have both a fixed and convex component, governed by the parameters c and a, respectively. We specify that, $\Phi_i = 0$ whenever $i_k = \delta$ and $\Phi_i = 1$ otherwise.

Liquid assets evolve according to:

$$l' = (l + i_l)((1 + r(1-\tau))$$

Following the recent corporate finance literature on firm dynamics, corporate payouts are motivated by the tax wedge, as in Riddick and Whited (2009). We note, however, that in practice payout policy is likely to also be driven by agency and asymmetric information considerations, as in Eisfeldt and Rampini (2009).

Pre-financing cost, after tax, net payouts are then internal cash flows minus investment in physical capital and liquidity accumulation, less investment adjustment costs. We have:

$$e \equiv zk^{\theta}(1-\tau) - i_l - i_k - \phi_i(i_k, k), \tag{1}$$

where z is the firm's productivity level, k is its capital stock, and investment in physical capital and liquidity accumulation are denoted i_k and i_l respectively. Investment adjustment costs are denoted by $\phi_i(i_k, k)$. If $e > 0$ the firm is paying out funds and if $e < 0$ the firm is raising external finance. Intuitively, the firm raises external finance if after tax operating profits do not cover the firms' total investment in physical and liquid assets, net of physical adjustment costs.

Firms maximize this net payout, less their financing costs. Following Riddick and Whited, Hennessy and Whited (2005), and Hennessy and Whited (2007), we assume the following functional form for the cost of external finance

$$\phi_e(e,\xi) = \Phi_e \xi(-\lambda_0 + \lambda_1 e - \frac{1}{2}\lambda_2 e^2),$$

where Φ_e is an indicator that takes the value 1 when $e < 0$ and 0 otherwise and ξ denotes aggregate state for external financing costs. The cost of external finance consists of three components: a fixed, linear and a quadratic cost, governed by the parameters $\lambda_0, \lambda_1, \lambda_2$, respectively. Notice that for $\lambda_0, \lambda_1, \lambda_2 > 0$, $\phi_e(e) < 0$ if and only if $e < 0$.

In recursive form, the firm's problem is:

$$V(k,l,z,\xi) = \max_{k',l'} \left(e + \phi_{e,\xi}\left(e\left(k,k',l,l',z\right),\xi\right) + \frac{1}{1+r} E_t\left[V(k',l',z',\xi')\right]\right) \tag{2}$$

where we denote the firm's value function by V. Each firm's state is given by their individual stocks of physical capital k, and liquid assets l, along with their productivity z and the current state of external financing costs ξ.

Each firm's productivity z is the product of an idiosyncratic shock z_i,and an aggregate shock z_{agg}. The aggregate productivity level, and each idiosyncratic productivity level, follow AR(1) processes with identical persistence parameters. However, we allow for the idiosyncratic and aggregate processes to have different volatilities. We discuss these choices further when we detail our calibration. These assumptions allow us to construct each firm's productivity level as follows:

$$\begin{aligned} z &= z_i z_{agg} && (3)\\ ln(z_i') &= \rho ln(z_i) + \varepsilon_i' && (4)\\ ln(z_{agg}') &= \rho ln(z_{agg}) + \varepsilon_{agg}' && (5)\\ ln(z) &= \rho ln(z) + \varepsilon_i' + \varepsilon_{agg}'. && (6) \end{aligned}$$

For the aggregate state of external financing costs ξ, we consider two cases. First, we consider a constant cost; i.e. we set $\xi = 1$ always. Second, we consider shocks to the cost of external finance. Specifically, we specify that ξ follows a log-normal AR(1), $ln(\xi') = c + \gamma \ln(\xi) + \eta'$, where $c = \mu_\xi(1-\gamma)$. We refer to the latter as the stochastic cost (SC) version of the model.

B. Analytical Intuition

Investment Returns

We solve our model numerically, however we describe the basic intuition for firm investment and liquidity accumulation policies by describing the investment returns to each. We show that each return is equal to its "physical return" times an external finance discount factor. That is, physical capital and liquid assets each have a physical return, but the value of this physical return varies with how financially constrained the firm is. If the firm would otherwise be accessing funds through costly issuances, it places a particularly high value on funds generated internally through production or savings. In the model with only productivity shocks, these shocks alone drive the returns to capital and liquidity accumulation. With shocks to ξ, we show that, away from the optimum, the returns to liquidity accumulation in particular vary with the cost of external finance. In fact, in a two period version of our model, only liquidity accumulation depends on ξ. Capital investment is pinned down its productivity and is independent of ξ. This supports the use of information in the cross-sectional correlation between external finance and liquidity accumulation to uncover the aggregate cost of external finance.

To construct investment returns using firms' marginal rates of transformation, we combine firms' first order conditions with their envelope conditions as in Cochrane (1991) and Cochrane (1996). Thus, in what follows, we analyze the solution for firms with interior investment and financing policies at each date. Due to the fixed costs of investment and external finance, in general the solution will exhibit regions of action and inaction.

The first order condition with respect to k' is

$$\left(1+a\frac{i_k}{k}\right)(1+\Phi_e\xi(\lambda_1-\lambda_2 e)) \quad = \quad \frac{1}{1+r}E_t\left(\frac{\partial V'}{\partial k'}\right) \tag{7}$$

Using the envelope condition $\frac{\partial V}{\partial k}=\frac{\partial e}{\partial k}(1+\phi_e'(e))$, we have

$$\begin{aligned}&\left(1+a\left(\frac{i_k}{k}\right)\right)(1+\Phi_e\xi(\lambda_1-\lambda_2 e)) \\ = \quad &\frac{1}{1+r}E_t\left(\left((1-\tau)\theta z'k'^{\theta-1}+(1-\delta)-c\Phi_i+a\left(\frac{i_k'}{k'}\right)\left(\frac{1}{2}\frac{i_k'}{k'}+1-\delta\right)\right)\left(1+\Phi_e'\xi'(\lambda_1-\lambda_2 e')\right)\right)\end{aligned} \tag{8}$$

Rearranging, we have the familiar pricing equation for a risk-neutral investor

$$1=\frac{1}{1+r}E_t\left(R_k\right) \tag{9}$$

where R_k, the return on capital, is given by

$$R_k=\frac{\left((1-\tau)\theta z'k'^{\theta-1}+(1-\delta)\left(1+a\left(\frac{i_k'}{k'}\right)\right)-c\Phi_i'+a\left(\frac{i_k'}{k'}\right)\left(\frac{1}{2}\frac{i_k'}{k'}\right)\right)}{\left(1+a\frac{i_k}{k}\right)}\frac{(1+\Phi_e'\xi'(\lambda_1-\lambda_2 e'))}{(1+\Phi_e\xi(\lambda_1-\lambda_2 e))} \tag{10}$$

We can understand the return on capital by thinking of the marginal benefit of increasing capital one unit today relative to the marginal cost. Define R_k = (marginal benefit / marginal cost) as the return from this strategy. The marginal benefit to increasing capital by one unit is: a marginal increase in output, the value of the additional depreciated capital, and a lower convex cost less the higher fixed costs of investment in the following period. This is the physical return to capital. The total return is the physical return multiplied by how much a dollar will be worth inside the firm tomorrow, namely, the marginal cost of funds in the following period. Thus, additional capital is more valuable if internal funds are expected to be scarce, i.e. if the firm will be raising funds externally. The physical marginal cost of increasing capital by one unit is a dollar, plus adjustment costs. Again, the total marginal cost is the physical cost multiplied by the shadow value of internal funds today.

Since the conditional expected return on capital in equation (9) is a constant, quantities must adjust for this asset pricing Euler equation to hold. The return to capital is decreasing in investment and increasing in productivity. Thus, the firm will increase investment in high productivity states until this optimality condition holds.

The first order condition with respect to l' is

$$\frac{1}{1+r(1-\tau)}(1+\Phi_e\xi(\lambda_1-\lambda_2 e)) \quad = \quad \frac{1}{1+r}E_t\left(\frac{\partial V'}{\partial l'}\right) \tag{11}$$

Using the envelope condition for l, $\frac{\partial V}{\partial l} = 1+\xi\Phi_e(\lambda_1-\lambda_2 e)$, and rearranging yields

$$1 = \frac{1}{1+r}E_t\left(R_l\right) \tag{12}$$

where R_l the return on liquid assets, is given by

$$R_l = (1+r(1-\tau))\,\frac{1+\Phi'_e\xi'(\lambda_1-\lambda_2 e')}{1+\Phi_e\xi(\lambda_1-\lambda_2 e)} \tag{13}$$

The return on liquid assets is made up of two components. The first is simply the risk-free rate earned by liquid assets (the risk-free rate less any taxes paid). The second piece gives the marginal value of a dollar of internal funds tomorrow versus today. The return on savings will be high when a dollar of internal funds is more valuable tomorrow than it is today.

It is convenient to define the *external finance discount factor* that governs firms' state-pricing as follows

$$\mathcal{F} = \frac{1+\Phi'_e\xi'(\lambda_1-\lambda_2 e')}{1+\Phi_e\xi(\lambda_1-\lambda_2 e)} \tag{14}$$

Intuitively, the discount factor is the ratio of a firm's marginal value of funds tomorrow versus today. Assets that pay off when the firm is raising costly external finance are more valuable since they provide internal funds. Notice that $\Phi_e(\lambda_1-\lambda_2 e) > 0$ if and only if $e < 0$. When the firm is not raising external finance at either date, the marginal value of a dollar inside the firm is the same as it is outside the firm. In contrast, when a firm is currently raising more external finance, the marginal value of a dollar inside the firm is greater than one and is increasing in the amount of external finance raised. A similar effect would appear in a model with a constraint on funds raised; the marginal value of a dollar would increase with the tightness of that constraint.

The external finance discount factor implies that although the firm is risk neutral, it can behave as if it is risk averse. Define $\widehat{R}_l$, $\widehat{R}_k$, as the returns to capital and liquidity without external financing costs (the neoclassical case). Then, we can write the firm's investment return moments as

$$1 = \frac{1}{1+r} E_t \left(\mathcal{F} \, \widehat{R}_l \right) \tag{15}$$

$$1 = \frac{1}{1+r} E_t \left(\mathcal{F} \, \widehat{R}_k \right) \tag{16}$$

showing that indeed $\mathcal{F}$ acts as a type of external finance induced stochastic discount factor.

Looking at 10 and 13, one can see that while the return to physical capital depends directly on TFP shocks, the return to liquidity accumulation varies directly with ξ only (but depends on TFP through e and the shadow value of funds at different dates.) Intuitively, both external finance raised and investment in liquid assets are more sensitive to the cost of external finance than investment in capital is. One can see this by considering the following perturbation argument: At the optimum, all returns are equated, and equal to $(1+r)$. Starting at the optimal policies at date t for a firm with positive investment and liquidity accumulation, consider perturbing ξ to $\xi - \varepsilon$. This lowers the marginal cost of an additional dollar and the firm will raise more external finance. The firm will also invest in capital and liquidity until those returns equal the new marginal cost. Due to the concavity of the production function, the marginal product of capital declines with greater investment. The true return on liquidity accumulation is likely to be concave, due to the hedging benefits which are tied to the value of funds used in production, however it is intuitive that the payoff is less concave than that for capital. Thus, liquidity accumulation should respond more strongly to the decrease in ξ. In the next section we fully characterize a simpler, two period version of our model in order to analytically link between the cost of external finance and the cross-sectional correlation between external finance and liquidity accumulation.

Two Period Example

We present a two period model of corporate financing and investment in order to analytically characterize the relationship between the cost of external finance, the amount of external finance raised, and investment in capital and liquid assets. We show how the optimality conditions for financing, investment, and liquidity accumulation in the two date model motivate the use of the cross-sectional correlation between liquidity accumulation and external finance in identifying the level of the cost of external finance. In particular, we show why, empirically, the cross-sectional correlation between liquidity accumulation and external finance, XSrho, should be a decreasing function of the cost of external finance.

There are two dates, zero and one. At date zero the firm receives internal funds y and chooses how much to invest in both physical capital (i_k) and liquid assets (i_l). Liquid assets produce r_l at date one, while physical capital produces output according to zi_k^{θ}. External finance, $e<0$ is raised at a cost $\xi\frac{1}{2}(e)^2$ where ξ is interpreted as the current "level" of the cost of external finance. Both assets depreciate fully between date zero and one, however we assume that both z and r_l are greater than one which in essence incorporates less than full depreciation. The firm does not discount the future. We can motivate r_l larger than one despite a unit discount rate by considering that liquid assets may provide a hedge for investment opportunities at date one, as in the more dynamic model.

The firm's objective is:

$$\max_{i_k,i_l}\left\{zi_k^{\theta}+r_l i_l-\mathbb{I}_{(e<0)}\xi\left(\frac{1}{2}e^2\right)-i_l-i_k+y\right\}$$
$$s.t.\quad e=y-i_l-i_k$$
$$i_l\geq 0.$$

The marginal cost of the first small ε of external finance is $\varepsilon\xi$, and as long as y is not too large, the marginal benefit will exceed this marginal cost, For simplicity, we also assume that liquidity accumulation will be positive. As will become clear, this will be true as long as the level of investment at which the marginal product of capital equals the marginal cost of an additional dollar of external finance implies a marginal product of capital that is less than the return on liquid assets. Intuitively, as we we show, this will be true as long as the level of the cost of external funds ξ is low enough, which is consistent with the idea that firms raising finance just to save the funds in liquid assets indicates that the cost of external finance is low. In this case, the firm raises funds and invests in capital and liquid assets until the marginal product of capital, and the marginal cost of an additional dollar of funds, are both equal to the marginal return on liquid assets, which is fixed due to the linear return on liquid assets.

In the case that the constraint $i_l\geq 0$ is not binding, we can write the firm's problem as:

$$\max_{i_k,i_l}\left\{zi_k^{\theta}+r_l i_l-\xi\left(\frac{1}{2}(y-i_l-i_k)^2\right)-i_l-i_k\right\}.$$

The first order condition with respect to investment in liquid assets, i_l, is:

$$r_l=1-\xi(y-i_l-i_k)$$

The first order condition with respect to capital investment, i_k, is:

$$\theta z (i_k)^{\theta-1} = 1 - \xi (y - i_l - i_k)$$

These first order conditions imply the following optimal financing and investment policies:

$$i_l = \frac{r_l - 1}{\xi} - \frac{r_l^{\frac{1}{\theta-1}}}{\theta z}$$

$$i_k = \frac{r_l^{\frac{1}{\theta-1}}}{\theta z}$$

$$-e = \frac{r_l - 1}{\xi}$$

Thus, the amount of external finance raised, and the amount of liquidity accumulated are both decreasing in ξ. By contrast, capital investment is independent of ξ and is instead pinned down by productivity and the other return and production function parameters. Formally, we have the following comparative statics:

$$\frac{\partial i_k}{\partial \xi} = 0$$

$$-\frac{\partial e}{\partial \xi} = \frac{1 - r_l}{\xi^2}$$

$$\frac{\partial i_l}{\partial \xi} = \frac{1 - r_l}{\xi^2}$$

$$-\frac{\partial i_l}{\partial e} = -\frac{\partial i_l}{\partial \xi}\frac{\partial \xi}{\partial e} = 1$$

At the optimum, capital investment is independent of ξ. On the other hand, both external finance and liquidity accumulation have the same, negative, partial derivative with respect to the level of the cost of external finance, ξ. At the optimum, the partial derivative of liquidity accumulation with respect to external finance raised is equal to one. Thus, as long as liquidity accumulation is positive, any additional dollar will be invested in liquid assets. Thus, at the margin, liquidity accumulation and external finance will increase one for one if ξ decreases since all additional funds raised will be used to augment cash balances.

Figure 16 illustrates the firm's investment and financing decisions decisions graphically by plotting the net marginal benefit of capital investment and investment in liquid assets, along with the marginal cost of external finance. Note that e is negative when the firm raises external finance. The graph depicts the case in which both types of investment are strictly positive. First, the firm uses its internal funds, y, to invest in physical capital. As the firm invests more, the marginal product declines. When the firm runs out of internal funds for investment, it raises external funds, and the marginal cost of external funds increases linearly in the amount of funds raised. Once the firm invests enough such that the marginal product of capital declines to the level of the marginal product of liquid assets, which is constant, the firm begins to invest in liquid assets.

The firm then raises external finance and invests in liquid assets until the marginal cost of external funds rises linearly to equal the marginal return on liquid assets. Clearly, there will always be some investment in physical capital. As long as y is not too large, the firm will also raise external finance. Then, if r_l is high enough (the level of the $r_l - 1$ line is high enough) or if ξ is low enough (the slope of the $\xi(-e)$ line is shallow enough), then the firm will also accumulate liquid assets. Thus, for liquidity accumulation to be positive, we need that

$$\theta z \hat{i}_k^{\theta-1} = r_l > 1 + \hat{e}\xi \tag{17}$$

where $\hat{i}_k$ and $\hat{e}$ are the levels of capital investment and external finance that set $\theta z \hat{i}_k^{\theta-1} = r_l$. Note that the figure clearly illustrates that investment in physical capital is independent of the cost of external finance in the region with positive liquidity accumulation; as one changes the slope of $\xi(-e)$ in this region, only i_l is affected. Note also that in this simple model, the amount of external finance is independent of internal funds in the region with positive liquidity accumulation. A higher y shifts the $\xi(-e)$ line to the right, but the amount of external finance will still be pinned down by setting $\xi(-e)$ equal to $r_l - 1$.

The comparative statics show that external finance and liquidity accumulation move one for one together at the margin if the firm is raising external finance and has positive liquidity accumulation. We further motivate the use of the correlation between external finance and liquidity accumulation in the cross-section to uncover the aggregate level of the cost of external finance by examining this correlation directly in the two date economy. For $y = 0$ and $i_l > 0$, the correlation between liquidity accumulation and external finance is given by:

$$corr(-e, i_l) = corr\left(\frac{r_l - 1}{\xi}, \frac{r_l - 1}{\xi} - \frac{r_l^{\frac{1}{\theta-1}}}{\theta z}\right)$$

Clearly, when the cost of external finance, ξ, is low, external finance and liquidity accumulation will both be dominated by the $\frac{r_l-1}{\xi}$ term, and as a result these two flows will be more correlated. We argue that even in an economy in which this two period model is repeated over time and across many firms with different levels of productivity, that the measured correlation between liquidity accumulation and external finance will be higher when ξ is low. This intuition is corroborated by the high correlation between XSrho and the Baa-Aaa default spread (0.64) and the fraction of banks reporting tighter lending standards (0.58) in the data. It is also supported by the high correlation between XSrho and ξ in our calibrated solution to the infinite horizon model.

We note also that equation 17 shows that the ξ which induces positive liquidity accumulation is decreasing in z. Thus, if policy interventions lower ξ when z is low the effect will be stronger on liquidity than on capital investment. This is because, at low z, firms are likely to satisfy equation (17) with a low i_k. Thus, if policy interventions decrease ξ, firms will respond mainly by increasing liquidity accumulation until the new marginal cost of funds equals the marginal return on liquid assets.

Finally, we show that although our dynamic model endogenizes the return to liquidity through its value as a hedge for future investment opportunities, as well as incorporating taxes, adjustment costs and the more dynamic return to capital, the investment returns in the two date economy are conceptually analogous. Just as in our infinite horizon model, each return is the physical return times the external finance discount factor. The external finance discount factor is high when the firm is raising a lot of external finance ($e << 0$), and this high cost reduces the return to investment and liquidity accumulation. Specifically, we can compare the returns in (10) and (13) to their two date counterparts:

$$R_k = \frac{\theta z i_k^{\theta-1}}{1} \frac{1}{1-\xi e},$$

where again we can interpret the first term as the physical return and the second term as the external finance discount factor. For the return on liquidity accumulation, we have:

$$R_l = \frac{r_l}{1} \frac{1}{1-\xi e}.$$

As discussed, if liquidity accumulation is positive, then $\frac{r_l}{1-\xi e}$ is also the return to an additional dollar of external finance, since that dollar will be invested in liquid assets. At the optimum, all returns are equated, and equal to one since there is no discounting. Starting at this optimum, consider perturbing ξ to $\xi - \varepsilon$. Quantities must adjust so that returns equate to one under the new cost of external finance. The lower ξ lowers the marginal cost of an additional dollar and so the firm will raise more external finance. The firm will also invest in capital and liquidity until those returns equal the new marginal cost. Due to the concavity of the production function, the marginal product of capital declines with greater investment, but the marginal benefit of liquidity accumulation does not since its return is linear. Thus, liquidity accumulation will respond more strongly to the decrease in ξ. In fact, we showed that at the margin around the optimum, liquidity accumulation will increase one for one with external finance. In the infinite horizon model, the production function is again concave, and the pecuniary return to liquidity accumulation is linear. The true return on liquidity accumulation is likely to be concave, due to the hedging benefits which are tied to the value of funds used in production, however it is intuitive that the payoff is less concave than that for capital. Thus, we conjecture that a similar result holds in the infinite horizon model.

V. Calibrated Solution

We build on the intuition provided in the previous section by calibrating our model and solving it numerically. We study how far a baseline model with constant costs of external finance can go in matching the joint dynamics of internal and external finance, and compare the results to a model with stochastic costs of external finance.

A. Numerical Methods and Calibration

We solve the model using standard discrete state space dynamic programming techniques. Specifically, we use the value function iteration method in Ljungqvist and Sargent (2004). Given the policy functions implied by our model solution, along with values for the model state variables and the stochastic processes for the exogenous states, we can simulate a panel of firms, and analyze the data from that panel as in Gomes (2001). We construct two panels. The first uses calibrated processes to study the implications of the model for aggregate and firm level moments. The second uses the actual TFP shocks from US time series in order to assess the performance of the model over specific US episodes graphically.

To discretize the state space, we approximate the realization of the productivity shock using standard Gauss-Hermite quadrature techniques (see Tauchen and Hussey (1991)). We choose six possible realizations of the productivity shock. For the stochastic external financing cost model, we specify two states for ξ. For capital and liquid assets, we choose a large enough grid such that the stationary probabilities of being at the upper bound of the grid are negligible, something we verify ex-post.

Table II displays our calibration and compares our parameters choices to those in the literature. The column EM displays parameters for the baseline model with constant costs and the EMSC column shows parameters for stochastic costs model. The last two columns offer a comparison to the parameters in Riddick and Whited (2009), (RW), and the benchmark real business cycle (RBC) parameters from Cooley and Prescott (1995).

We begin by describing the calibration of the baseline model. For the tax rate, we choose 10%. In our model, liquid assets are only accumulated in order to hedge investment opportunities in physical capital. Moreover, firms can simply over-accumulate physical capital and hedge via the additional cash flows that capital produces. Thus, if the tax rate is too high, firms do not accumulate any liquidity. In practice, most firms do not pay the 34% statutory rate. Moreover, we do not include a major benefit of cash in practice, which is that cash insures against costs of financial distress. Similarly, our model firms do not experience shortfalls from operating or financial leverage as in Eisfeldt and Rampini (2009). We use a standard RBC value for depreciation, 8%. For the production function curvature parameter, we specify 0.65, which is consistent with evidence in Cooper and Haltiwanger (2006). Higher curvature parameters, i.e. production functions closer to linear, imply too large investment volatilities and disinvestment frequencies. We calibrate the persistence of both idiosyncratic and aggregate productivity shocks to be 0.66, which allows us to conserve on one state variable since firms only care about total productivity in our partial equilibrium model. This value is equal to that used by RW for the firm level. Khan and Thomas (2008), page 407, contains a detailed discussion of the disagreement in the literature about this parameter, however based on our reading 0.66 is a good modal value. Moreover, we also found the average industry level persistence in the data from Basu *et al.* (2006) to be close to this value (0.65). Finally, using two trend breaks, as advocated in Fernald (2007), we find an aggregate persistence of about 0.62 using the data from Fernald (2009). We set the total volatility of firm level productivity equal to the value in RW, which is also near the value used by Khan and

Thomas (2011). We then specify that aggregate volatility is about one third to one fourth of idiosyncratic volatility as in Khan and Thomas (2008) and Cooper and Haltiwanger (2006). We set the risk free rate to 0.04 as in the RBC literature, and in RW.

Finally, we choose the parameters governing the cost of external finance and the investment adjustment costs in order to best match the aggregate correlation between liquidity accumulation and external finance of 0.60. In particular, this implies higher values for the fixed cost of external finance and the quadratic costs of investment adjustment than used by RW. RW uses the estimates from Hennessy and Whited (2007) for the costs of external finance, however they do not incorporate aggregate moments as we do. In general, different moments lead to different parameter values. Our calibration implies an average percentage issuance cost of 1.7%, which is well within empirical estimates.[16] Relative to RW, our calibration features a higher fixed cost of raising external funds and lower linear and quadratic components. The higher fixed cost is consistent with the importance of the variation in the extensive margin of external finance over the business cycle, as well as with the evidence in Bazdresch (2005) and Cummins and Nyman (2004) which emphasize the importance of lumpy external finance. Our model does produce an average fraction of firms raising external finance which is low relative to the data, but this may be due to small issuances in the data such as drawdowns on lines of credit which do not incur costs.

For investment, our calibration features a smaller fixed cost and higher quadratic cost than RW. We note that the smaller fixed cost of investment is consistent with our lower curvature parameter since this leads to smaller firm sizes. Moreover, the higher quadratic adjustment cost is consistent with the relatively high amount of autocorrelation in investment in the data, see Eberly *et al.* (forthcoming). We match exactly the autocorrelation of aggregate investment in our sample of 0.38. Firm level investment is less autocorrelated in our baseline model (0.17) than it is in our Compustat sample (0.29). The average cost of investment is less than 1% of investment dollars spent.

Our calibration of the model with stochastic costs of external finance (EMSC) retains most of the baseline calibration. However, in this version we drop the fixed cost of external finance in order to provide a contrast to the baseline model where the fixed cost is crucial for generating issuance and savings waves. We also computed the model with a stochastic fixed cost. The results for most all moments fall between that of the baseline and the presented SC model so we do not show them here. For the linear and quadratic costs in the SC model, we use the parameters in RW. Finally, we calibrate the stochastic process for ξ. We choose the persistence of the stochastic cost to match the persistence of the annual Baa-Aaa default spread over our sample period. We assume the stochastic cost is uncorrelated with the aggregate productivity shock, an assumption consistent with our empirical estimates. The empirical correlation between innovations in the default spread and TFP shocks is -0.2 and is not statistically significantly different from zero. Similarly, the empirical correlation between innovations to the expected default adjusted excess bond spread from Gilchrist and Zakrajsek (2012) and TFP shocks is -0.1 and is again not statistically significantly different

[16] For example, Asquith and Mullins (1986) find that abnormal stock returns around secondary equity offerings are about 3%.

from zero. The autocorrelation of innovations this series based on annualized quarterly data is 0.35, so this is also consistent with our persistence parameter for ξ of 0.4. We choose the volatility of the ξ shock to match the standard deviation of external finance raised. At the firm level, this is 0.17, and we match this in our SC model. Finally, given this volatility, we choose the mean of the ξ shock to generate an average credit shock (exp $\left(\mu - \frac{\sigma^2}{2}\right)$) of about one so that on average the costs equal the cost under the RW parameters. For the SC model, the average issuance cost is 8.5% of proceeds. This is higher than our baseline model, but is roughly consistent with evidence in Hennessy and Whited (2007) who estimate that firms face an issuance cost of 8.3% on the first million dollars raised.

B. Aggregate Analysis

We begin with the aggregate implications of our model, since we are mainly interested in aggregate issuance and savings waves. With our numerical solution in hand, we can construct a panel of firm data. Specifically, we simulate 1,000 idiosyncratic productivity processes, 1 aggregate productivity process, and 1 aggregate stochastic cost process following the persistence and volatility given in Table II. We then create 1,000 total firm productivity shocks by summing each firm specific and aggregate productivity series and taking the exponential. We simulate 600 years of data, throwing away the first 100 years to avoid any initial dependencies. We then aggregate across firms to form aggregate corporate flows, analogous to our procedure in Compustat. To compare the output from our model to the actual US time series 1980-2010 graphically, we perform an additional simulation using the actual TFP shock realizations from Fernald (2009) for our aggregate shock. This way, we can examine the performance of the model over specific US episodes, such as the recent financial crisis, the recession of the early 1980s, and the tech boom and bust. For the simulation using actual TFP shock realizations, we use a TFP series which consists of 500 years of calibrated shocks, plus the actual shocks realized for the 60 years 1950-2010. Then, we discard all simulated data except that corresponding to the last 30 years, 1980-2010.

Table III presents a comparison of aggregate issuance, savings, and investment moments in the data and in the model. Both the baseline model and the stochastic cost model come close to exactly matching the overall correlation between liquidity accumulation and external finance for our main sample of 0.60. TFP shocks alone apparently generate realistic issuance and savings waves in the presence of reasonably calibrated physical capital adjustment costs, and constant costs of external finance. This is because given the fixed cost of external finance and persistent investment opportunities, when hit by a positive productivity shock firms raise more external finance than they currently need. Moreover, the convex cost of investment leads firms to smooth investment over time, accumulating funds not used for current investment as liquid assets. In the stochastic cost model, as characterized in the two period model, when the cost of external finance is low and the firm's physical capital stock is large enough, the return to liquid assets exceeds both the return to physical capital and the cost of external finance until the firm raises enough external finance to equate the marginal cost to the approximately linear return on liquid assets. Thus, time varying costs of external finance generate issuance and savings waves in that model.

Table IV displays conditional moments from the data and models. Both models also generate the observed negative correlation between firm size bins and the correlation between external finance and liquidity accumulation. However, neither model replicates the extreme importance of the largest 10% of firms in driving the aggregate moments in Compustat data. In both models and in the data the top 10% of firms display a much lower correlation between external finance and liquidity accumulation. However, only in the Compustat data does including those firms change the total aggregate correlation by more than an order of magnitude. This is because the model is not designed to match the extreme empirical skewness in firm size. Thus, we compare the model data to the firms in the bottom 90% of the Compustat sample as in Covas and Den Haan (2011a). Conditional on raising external finance, the correlation between liquidity accumulation and external finance is stronger both in the data (0.74), and in the models (0.91 and 0.82 respectively). Similarly, conditional on paying out funds, this correlation is lower in the data and in both models. Thus, both the baseline model with a constant cost, and the model with a stochastic cost generate aggregate issuance and savings waves with the observed conditional properties.

To illustrate the baseline model's ability to generate realistic time series for external finance and liquidity accumulation from TFP shocks alone, we plot the aggregate flows to liquid assets against flows to external finance in the data and model in figures 1 and 2, respectively. For this simulation, we use the empirical TFP shocks from 1980-2010. Compared to the actual data, our model using actual TFP matches fairly well. The early 1980s recession is associated with relatively low external finance and liquidity accumulation both in actual and simulated data. The mid 1980s was a period of relatively high productivity and the data and model show relatively high external finance and liquidity accumulation. The early 1990s are recessionary, and again financing and liquidity accumulation are low. In the mid to late 1990s, the model predicts less external finance and liquidity accumulation than were observed empirically. Similarly, the simulated data to not capture the strong decline in external finance and liquidity accumulation after the tech bust in 2001. However, the simulated data do seem to capture the financing and savings wave observed in the mid 2000s, as well as the observed decline during the financial crisis. We also display the empirical and simulated data for the percentage of firms raising external finance in Figures 5 and 6. Since the fixed cost of external finance in the baseline model drives the external margin for raising external finance, it makes sense to assess its performance along this dimension. In terms of matching the empirical timing of issuance waves, the baseline model again performs well with the exception of the mid-late 1990s and 2001. However, table III shows that this model does imply an unconditional probability for raising funds which is too low (0.15) relative to that observed in Compustat data (0.43). On the other hand, we note that the high empirical probability of issuance may be due to small issuances with low costs, such as drawdowns on lines of credit. Consistent with this, Bazdresch (2005) provides evidence that a small fraction of observations account for most of firms' external financing activity.

Thus, the baseline model performs surprisingly well for its simplicity. Table III also shows that, in addition to matching the empirical correlation between external finance and liquidity accumulation, the baseline model's moments are close to their empirical counterparts for the standard deviation of liquidity accumulation, expected investment and the standard deviation of investment, the serial correlation of investment, and the standard deviation of external finance. The baseline model also matches the fact that the aggregate shortfalls, i.e. years when the corporate sector is a net receiver of funds, are rare. The probability of an aggregate shortfall is 5% in the data, and 1% in the baseline model.

We now turn to the improvements offered by the SC model. The second panel of Table III shows that the baseline model replicates the fact that the correlation between liquidity accumulation and investment is higher when one conditions on firms that are raising external finance. However, even conditioning on firms raising external finance, the baseline model implies a correlation between liquidity accumulation and investment which is too high (0.79 in the model vs. 0.37 in the data). Unconditionally, the baseline model's performance in matching the correlation between liquidity accumulation and investment is even poorer; in the data this correlation is 0.12 and in the model it is 0.45. The last column of Table III shows that the stochastic cost model performs better in terms of matching the lower correlation between liquidity accumulation and investment, both conditionally and unconditionally. The SC model retains the success of the baseline model in terms of the aggregate issuance and savings waves moments, as well as moments describing the standard deviation of liquidity accumulation, expected investment and the standard deviation of investment, the serial correlation of investment, and the standard deviation of external finance. The SC model also generates a more realistic average liquidity balance (0.06 vs. 0.15 in the data and 0.01 in the baseline model).[17]

In comparing our baseline model to the SC version, it is important to emphasize that the baseline model has only one shock; that is, all variables are likely to be highly correlated due to stochastic singularity. Figures 7 and 8 confirm this by plotting investment along with external finance and liquidity accumulation in our Compustat sample and in the baseline model simulation using actual TFP shocks, repsectively. Although all three series appear procyclical in the data when one considers the plotted NBER recessions, measured correlations are clearly not as high as in the simulated data. The same fact can be seen this in the third panel of Table III, which displays the correlation of corporate flows with gdp. In the data, investment is the most procyclical series, with a correlation of 0.47, as compared to 0.97 in the baseline model. Likewise, in the baseline model, the lowest correlation is between liquidity accumulation and gdp and that correlation is 0.41 (as opposed to zero in the data).

An additional shock certainly has the potential to improve the model's ability to match the aggregate correlations we study since an additional shock will break stochastic singularity. There are many shocks to choose from in the large literature on business cycles. We use evidence from the fraction of firms raising

[17] These findings are consistent with those in the contemporaneous paper, Warusawitharana and Whited (2011). That paper focuses only on equity financing at the firm level, and explores the ability of equity misvaluation shocks to explain firm level financing, saving, and investment behavior.

external finance, as well as from the relationship between the time series of the cross-sectional correlation between liquidity accumulation and external finance to motivate considering a shock to the cost of external finance. We note also that that other candidate shocks may have counterfactual implications for issuance and savings waves. A simple shock to costs (eg. operating leverage) may lead to countercyclical issuances since cash flows are low in recessions. Similarly, uncertainty shocks may lead to higher savings for precautionary reasons in bad times. More importantly, carefully identified studies of differences in changes in investment across firms that are more and less dependent on external finance show that credit supply shocks do affect firm level investment, suggesting an important role for stochastic variation in the cost of external finance.[18] Figure 9 displays investment for firms which realize each of the twelve states in our SC model. One can see the challenge in identifying the effects of a financing cost shock in the data from these model policy functions. The only firms materially affected by the cost shock are those in one of the two highest productivity states. Thus, our model supports the importance of methods such as those in the papers cited in order to identify such shocks. Moreover, the policy functions in Figure 9 are consistent with the findings in those papers.

As discussed in Section B., the cross-sectional correlation between external finance and liquidity (XSrho) should be high when the cost of external finance is low. Figures 3 and 4 provide support for this idea by plotting the empirical time series for XSrho against the empirical proxiy for ξ, and the simulated time series for XSrho against the ξ shock in the SC model, respectively. The bottom panel of Table I shows that the aggregate correlation between XSrho and TFP is lower (0.48) than the correlation between XSrho and the negative of the Baa-Aaa default spread (0.64) and the fraction of banks tightening lending standards (0.58). The bottom panel of table III uses the negative of the Baa-Aaa default spread as our empirical proxy for ξ shocks and compares the correlation between XSrho and ξ to the correlation between XSrho and TFP in the data and model. The SC model reproduces the fact that XSrho is more correlated with ξ than with TFP. However, the SC model undershoots on the correlation between ξ and TFP, producing a correlation of zero. This may be because even though we find no statistically significant negative correlation between TFP shocks and the negative of the default spread, TFP estimates may truly be negatively correlated with external finance cost shocks. We argue that XSrho contains useful information about the cost of external funds from firms' decisions about not only how much external finance they raise, but how they use those funds. This information is different from that in the default spread, which may reflect changes in firms' riskiness, and lending standards, which may reflect changes in bank balance sheets more than the true costs of external finance.[19] The bottom panel of Table III provides additional evidence for the SC model. The last two rows show that the fraction of firms raising external finance in the data and the SC model is more correlated with the ξ shock than with TFP. This is consistent with the analytical results regarding firms' market timing with

[18]See Ivashina and Scharfstein (2010), Duchin *et al.* (2010), Campello *et al.* (2010), Almeida *et al.* (2009). See also Matvos and Seru (2011) for evidence of financing shocks from their estimation of a structural model comparing resource allocation by diversified and undiversified firms during the financial crisis.

[19]See Atkeson *et al.* (2012) for a history of the financial soundness of banks.

stochastic costs in Bolton *et al.* (2011).

Finally, Table V displays the correlations between liquidity and investment with debt vs. equity separately. While we see that the correlation with liquidity accumulation is stronger for equity (0.69) then debt (0.16), both are positive. Conditional on firms raising external finance, we see both correlations increase to 0.77 and 0.33, respectively, and both are statistically significant. We also note that investment is more correlated with debt (0.60) than equity (-0.15). This fact has been pointed out by DeAngelo *et al.* (2010) who argue that debt might be used more frequently for investment. Also, we note that debt drives most of the variation in external finance, with a correlation with external finance of 0.77 vs 0.43 for equity. For parsimony, for almost all of our analysis we choose to focus on the overall correlation with external finance and abstract from debt vs equity. Although contract terms differ, Covas and Den Haan (2011b) document that both debt and equity issuances are procyclical for all but the very largest firms. Studying total external finance allows us to focus on what is new in our work, namely the relationship between external finance and liquidity accumulation at the aggregate level.

Given its simplicity, the baseline model does surprisingly well in replicating the joint dynamics of internal and external finance. However, the model with the stochastic cost improves the match with the data on important dimensions by generating higher liquidity accumulation, and de-linking liquidity accumulation and investment. Moreover, the SC model and the analytical characterization of the two-date model offer support for a new measure of the aggregate state for the cost of external finance, namely the cross-sectional correlation between liquidity accumulation and external finance, XSrho. This measure uses firms' decisions regarding external finance and its end use to make inferences about the aggregate state of external capital markets. In section VI. we build on these ideas and use a version of SMM to uncover the hidden ξ state.

C. Firm Level Analysis

Before turning to our estimation exercise, we provide additional intuition for our results by describing the numerical policy functions for firm level decisions, and by presenting firm level moments from simulated and empirical data for the baseline and SC models. We begin with firm level policy functions in the baseline model. Figures 10, 11, and 12 plot the policy functions for external finance, investment, and liquidity accumulation, respectively in the baseline model. In each panel, we fix productivity and plot the optimal policy as a function of current liquidity and capital (l,k). Productivity decreases from left to right across panels, thus the upper left gives the policy in the highest productivity state, while the lower right gives the policy in the lowest productivity state.

We highlight several key features of the model. First, in the higher productivity states the firm is more likely to raise external finance ($e<0$) for a given cash, capital pair. Intuitively, since productivity is persistent, high productivity states signal good investment opportunities and the firm therefore wants to increase its capital stock. We also see liquid assets increase, particularly in low cash and low capital states. Thus cash accumulations happen simultaneously with raising external finance, and in particular in states where capital is low. In low capital states, firms would like to invest more, but high quadratic adjustment costs prevent them from doing so. They therefore raise a large amount of funds, invest some today, and save the rest for the good (expected) investment opportunities tomorrow. The next panel, which plots investment and adjustment costs, mirrors this intuition.

Conversely, the lower panels of each figure display the policies for low productivity states. Here, we see that the firm raises funds only when capital is extremely low. Similarly, the firm almost always decumulates liquid assets. Investment is typically negative, as the firm looks to sell off unproductive capital and pay out funds. This view is consistent with Jermann and Quadrini (forthcoming) who find that firms pay down debt during contractions. Since large disinvestment is costly (due to large adjustment costs), and cash flows are scarce, the firm pays out by drawing down cash balances when possible.

The policy functions provide two main takeaways: in good times, when capital is productive and investment opportunities are high, firms borrow large amounts of funds to invest smoothly over subsequent periods. Cash balances allow them to smooth investment opportunities. In contrast, in bad times when investment opportunities are scarce, firms pay out to claim holders due to the tax motive and to discounting. Since cash flows are low, and large scale disinvestment is costly, firms fund payouts by decumulating cash balances if they can.

To further understand the model's implied correlations between internal and external finance, and investment, we plot the policy functions as a function of productivity z only. Specifically, we fix both current capital (k) and liquidity (l) and instead plot the policy function as a function only of productivity in Figure 13. We display three panels which have different fixed values of initial capital, thus giving intuition for differences in small, medium, and large firm behavior. In each case, we fix current liquid assets at their median value. Moving from the left to right (lower to higher productivity), we see that flows to external funds (e) decrease, liquid balances increase, and investment increases, showing again that both liquidity accumulation and investment are positively correlated with external finance.

Figure 14 gives the policy functions for investment, external finance, and liquidity accumulation for the model with the stochastic cost. Notably, investment is more sensitive to cash on hand when the cost of external finance is high. Thus, states where investment opportunities are high but raising external finance is costly are states where cash on hand is valuable to fund investment, providing the main role for cash holding. Considering the policy for external finance, we see that, as expected, the amount raised decreases significantly in the state where the cost of funds is high.

Next, we use the optimal policy functions for a given firm to compare the model generated firm level moments to those in the data. We report the relevant moments in Table VI. For firm level issuance and savings activity, we find a correlation between external finance (-e) and liquidity accumulation of 0.57 in the baseline model, 0.62 in the SC model, and 0.18 in the data. Both models overshoot on this correlation at the firm level, but get the aggregate correlation just right, since this aggregate moment was one of the main moments targeted by our calibration. We note that the lower correlation at the firm level may be due to noise, or timing issues at the firm level in the Compustat data.

We find a slightly positive correlation between liquidity and investment in the baseline model (0.22), while the correlation is near zero in the data (-0.06), and is zero in the SC model. Thus, the SC model matches this moment better than the baseline model at the firm level as well as in the aggregate. Although one might expect liquidity and investment to be negatively correlated even in the baseline model, we should note that there are two confounding effects. First, when firms raise funds and invest, they over-raise and accumulate liquid assets, creating a positive correlation between liquidity and investment. However, in subsequent periods the firm will avoid raising costly external funds and choose instead to fund investment with its large liquid balances, inducing a negative correlation. Which effect wins out depends on the parameters of the model, but we typically found a positive correlation in the baseline model. The SC model similarly helps to lower the correlation between external finance and investment somewhat. At the firm level, this correlation is 0.20 in the data, and 0.93 and 0.77 in the baseline model and the SC model, respectively. The SC model also leads to higher liquidity accumulation at the firm level. We find the average liquid to total assets ratio to be 1% in the model and 11% in the data. Again, the SC model leads to greater liquidity accumulation, implying average liquid to total assets of 6%. The (unconditional) probability of raising funds is about 15% in the baseline model and 17% in the SC model. These probabilities are roughly a third of the number in the data (43%), possibly for reasons we discussed with respect to the aggregate moments. Likewise, the average amount of external finance raised is larger in both models than in the data. Both models do well in matching the average level of investment and its volatility, and on the standard deviation of external finance. Importantly, we do not generate our results with overly large persistence in investment (0.17 and 0.18 in the baseline and SC models respectively, vs. 0.29 in the data) despite our high investment adjustment costs.

VI. Estimating The Cost of External Finance

Intuitively, if at a given date firms are simultaneously raising funds and saving them, it is likely that costs of raising external finance are low. Empirically we find correlations between the cross-sectional correlation between liquidity accumulation and external finance, XSrho, and the default spread and lending standards of -0.64 and -0.58, respectively. Analytically, we show in a two-date model that XSrho will be higher when ξ is lower, and we conjecture why this should hold in the infinite horizon as well. Results from our calibrated solution of the infinite horizon model confirm this intuition; in the model, the correlation between XSrho and ξ is 0.80. Thus, we argue that focusing on times when firms simultaneously raise funds and save them (as measured by the cross-sectional correlation) can be informative about aggregate credit conditions.

We next turn to a more formal estimation of the cost of external finance using the SC model. At each date in time, we use a version of SMM to infer the value of the stochastic cost that generates simulated cross-sectional moments as close as possible to the data. The difference between our estimation exercise and a typical SMM estimation is that we are looking to uncover a hidden state instead of estimating a parameter. This distinction matters since state variables, unlike parameters, influence the model's transition dynamics over time, not just through policy functions, but also directly. As moments, we choose the level of investment and the correlations between liquidity accumulation, external finance, and investment.

More specifically, we define the vector M_t as follows:

$$M_t = \begin{bmatrix} E_{N,\mathrm{mod}}\left[\frac{i_t}{k_t}(\xi_t)\right] - E_{N,data}\left[\frac{i_t}{k_t}\right] \\ \rho_{N,\mathrm{mod}}\left(\frac{liqacc}{TA}(\xi_t), \frac{extfin}{TA}(\xi_t)\right) - \rho_{N,data}\left(\frac{liqacc}{TA}, \frac{extfin}{TA}\right) \\ \rho_{N,\mathrm{mod}}\left(\frac{inv}{TA}(\xi_t), \frac{extfin}{TA}(\xi_t)\right) - \rho_{N,data}\left(\frac{inv}{TA}, \frac{extfin}{TA}\right) \\ \rho_{N,\mathrm{mod}}\left(\frac{liqacc}{TA}(\xi_t), \frac{inv}{TA}(\xi_t)\right) - \rho_{N,data}\left(\frac{liqacc}{TA}, \frac{inv}{TA}\right) \end{bmatrix}$$

where $\rho_{N,\mathrm{mod}}(x,y)$ represents the cross-sectional correlation between x and y in the model, which is a function of ξ_t, and $\rho_{N,data}(x,y)$ represents the empirical counterpart. We use N to emphasize that these are cross-sectional, rather than time-series moments.[20]

At every date t we choose the value ξ_t that minimizes deviations of the cross-sectional model implied moments and empirical moments. Specifically, we choose ξ_t to miminze the following objective fucntion

$$\min_{\{\xi_t\}} M_t' W M_t$$

where we set $W = I_{4x4}$ as the identity matrix which weights all moments equally.

We initialize the series by first starting capital and liquidity at their steady state values, and then feeding in the observed aggregate TFP and the default spread series beginning in 1951. We do this to ensure that the model distribution over capital and liquidity stocks reflects the history of TFP shocks in the US given calibrated ξ_t shocks. We use the empirical realizations of these series to proxy for $z_{t,agg}$ and ξ_t until 1980 when our Compustat sample begins. Thereafter, we estimate ξ_t date-by-date by setting the value of aggregate productivity to equal to our observed TFP series and choosing ξ_t to minimize our objective function. This allows us to estimate a rolling time series for the stochastic cost of external finance that firms most likely face in a given year using the empirical cross-sectional moment.

We plot our estimated series in Figure 15. Our estimation procedure appears to pick up events that our priors suggest might be associated with costly external finance, such as the recession of 1981, the stock market crash of 1987, the Asian financial crisis of 1997, the crash of the tech boom in 2001, and the recent financial crisis. On the other hand, the process clearly does not follow an AR(1) with persistence parameter 0.4 as specified in our calibration. This is because a single shock is being chosen to match several moments, and these moments may vary more in the data than in the model. For the same reason, the estimation procedure picks only extreme high or low values. We are currently working to refine our

[20]We have also estimated the series adding two adiditional moments: $(\xi_t - \mu_\xi) - \rho(\xi_{t-1} - \mu_\xi)$ and $\left((\xi_t - \mu_\xi) - \rho(\xi_{t-1} - \mu_\xi)\right)^2 - \sigma_\varepsilon^2$ which help ensure that the series ξ_t follows an AR(1) with the parameters we calibrated. The results are qualitatively similar.

estimation procedure, but argue that these preliminary results seem promising.

VII. Conclusion

We document the empirical regularity of aggregate issuance and savings waves. We show that a very simple dynamic model which features persistent productivity shocks, a fixed cost of external finance, and a convex investment adjustment cost, can generate empirically realistic aggregate time series for liquidity accumulation and external finance. This simple model generates the observed strong aggregate correlation between the two series, as well as the observed strongly procyclical series for the fraction of firms raising external finance.

In order to evaluate the contribution of a shock to the cost of external finance to the business cycle dynamics of liquidity accumulation, external finance, and investment, we compare our results from the baseline model to those from a model featuring a stochastic cost of external finance. The model with the stochastic cost replicates the success of the baseline model on most measures, and improves on the dimensions of liquidity accumulation, and the relatively low correlation between external finance and investment, and liquidity accumulation and investment.

We also use the model with stochastic financing costs to identify empirical variables which proxy for the stochastic cost of external finance in the model. We find that in the stochastic cost model, the percentage of firms raising external finance, and the cross-sectional correlation between external finance and liquidity accumulation are highly correlated with the cost of external finance. Consistent with this, we show that the time series for the the percentage of firms raising external finance, and the cross-sectional correlation between external finance and liquidity accumulation are highly correlated with standard measures of the aggregate state of the cost of external finance, such as the default spread and the fraction of banks tightening lending standards.

We argue that both the observed realization of the correlation between external finance and liquidty accumulation in the cross-section, and our model implied estimate of the level of the cost of external finance ξ in the US time series 1980-2010, are useful measures of the state of the aggregate level of the cost of external finance. Using firms' actual decisions about how much external finance to raise and how they use the proceeds from external finance is a revealed preference method of making inferences about the true cost of external finance. Such a measure might provide useful policy guidance as to the likely impact of interventions aimed at lowering the cost of external finance since the macroeconomic benefit of lowering the cost of external funds depends (amongst other things) on whether those funds will be used for investment, or accumulated as cash.

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VIII. Data Appendix

Data Appendix

Our data construction closely follows Covas and Den Haan (2011a). Our primary source of data is the Compustat fundamentals annual file. Our main results use data from 1980-2010. We exclude financials, utilities and firms with SIC codes starting with 9. We also exclude firms with missing assets, equity, debt, and those with missing or negative PPE and cash balances. As in Covas and Den Haan (2011a), we also remove GM, GE, Chrysler, and Ford, since these firms were the most affected by the accounting change in 1988 requiring firms to consolidate the balance sheets of their wholly owned subsidiaries.

Computstat Data

We first define liquidity accumulation, investment, and external finance as:

$Investment =$CAPEX (Capital Expenditures)

$Liquidity\ Accumulation =$ CHECH (Cash and cash equivalents, change)

$External\ Finance = -(CF_D + CF_E)$

For flows to debt and equity and operating cash flows we use the statement of cash flows:

For statments of cash flows:

$CF_O =$ Income before extra items (IBC) + Depreciation and amortization (DPC) + EI & Discontinued Oper (XIDOC) + Deferred Taxes (TXDC) + Equity in net loss (ESUBC) + Funds from operations: other (FOPO)+ Income taxes: accrued inc(dec) (TXACH) + Assets & Liab: other (net change) (AOLOCH) + Accounts receivable dec(inc) (RECCH)+ Inventory dec(inc) (INVCH) + Accounts payable inc(dec) (APALCH) + Interest paid (net) (XINT)

$CF_E =$ - Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

$CF_D =$ - Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

For statments by source and use of funds:

$CF_O =$ Income before extra items (IBC) + Depreciation and amortization (DPC) + EI & Discontinued Oper (XIDOC) + Deferred Taxes (TXDC) + Equity in net loss (ESUBC) + Funds from operations: other (FOPO)+ Interest expense (XINT)

$CF_E =$ - Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

$CF_D =$ - Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

For working capital statementes:

CF_O = Income before extra items (IBC) + Depreciation and amortization (DPC) + EI & Discontinued Oper (XIDOC) + Deferred Taxes (TXDC) + Equity in net loss (ESUBC) + Funds from operations: other (FOPO)+ Interest expense (XINT)

CF_E = - Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

CF_D = - Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

For cash statements by activity:

CF_O = Income before extra items (IBC) + Depreciation and amortization (DPC) + EI & Discontinued Oper (XIDOC) + Deferred Taxes (TXDC) + Equity in net loss (ESUBC) + Funds from operations: other (FOPO)+ Interest expense (XINT)

CF_E = - Sale of common and pref. stock (SSTK)+ Purchase of common and pref. stock (PRSTKC) + Cash dividends (DV)

CF_D = - Long-term debt issuance (DLTIS)+ Long-term debt: reduction (DLTR) + Changes in current debt (DLCCH) + Interest paid (net) (XINT)

Flow of Funds Data

We use annual data from the electronic ASCII flow of funds seasonally adjusted annual rates table F.102 available at
http://www.federalreserve.gov/Releases/z1/Current/data.htm.
Refer to the coded tables for definitions and relationships between entries. Codes appear in parentheses after variable names. Interest payments, not reported in table F.102, are from NIPA table 1.14 line 25 "Net interest and miscellaneous payments" for nonfinancial corporate business.

CF_O =(Total internal funds + IVA) (FA1060000105) - Discrepancy (FA107005005) + Net dividends (FA106120005) + Trade payables (FA103170005) + Taxes payable (FA103178000) + Miscellaneous liabilities (FA103190005) - Trade receivables (FA103070005) + NIPA interest

CF_D =Commercial paper (FA103169700) + Mortgages (FA103065003) - Credit market instruments (FA104104005) + NIPA interest

CF_E =Net dividends (FA106120005) - Net new equity issues (FA103164003)

Liquidity Accumulation = Net acquisition of financial assets - Commercial paper - Mortgages - Trade receivables - Other Assets

Investment =Capital expenditures

Other Data

The following series used can be found in the FRED database at the St Louis Fed website.

Gdp= Real gross domestic product

Default Spread= Difference between Moody's Seasoned Baa and Aaa yield. We use end of year values.

Lending Standards= Net Percentage of Domestic Respondents Tightening Standards for Commercial and Industrial Loans Large and Medium Firms (DRTSCILM). We use end of year values.

Finally, we obtain TFP data from John Fernald's website at: `http://www.frbsf.org/economics/economists/staff.php?jfernald`. We construct the log level series from the series of annual changes provided, and detrend the series with two breaks as in , which advocates breaks after 1974 and 1995. Shocks are then residuals from an AR(1) regression on the log level series.

IX. Figures, Tables. View color figures electronically.

Figure 1: We plot aggregate accumulation of liquid assets against aggregate external finance. Sample excludes largest 10% of firms. Data are normalized by lagged assets, HP-filtered, and then scaled to have unit variance. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). Correlation between plotted series is 0.6.

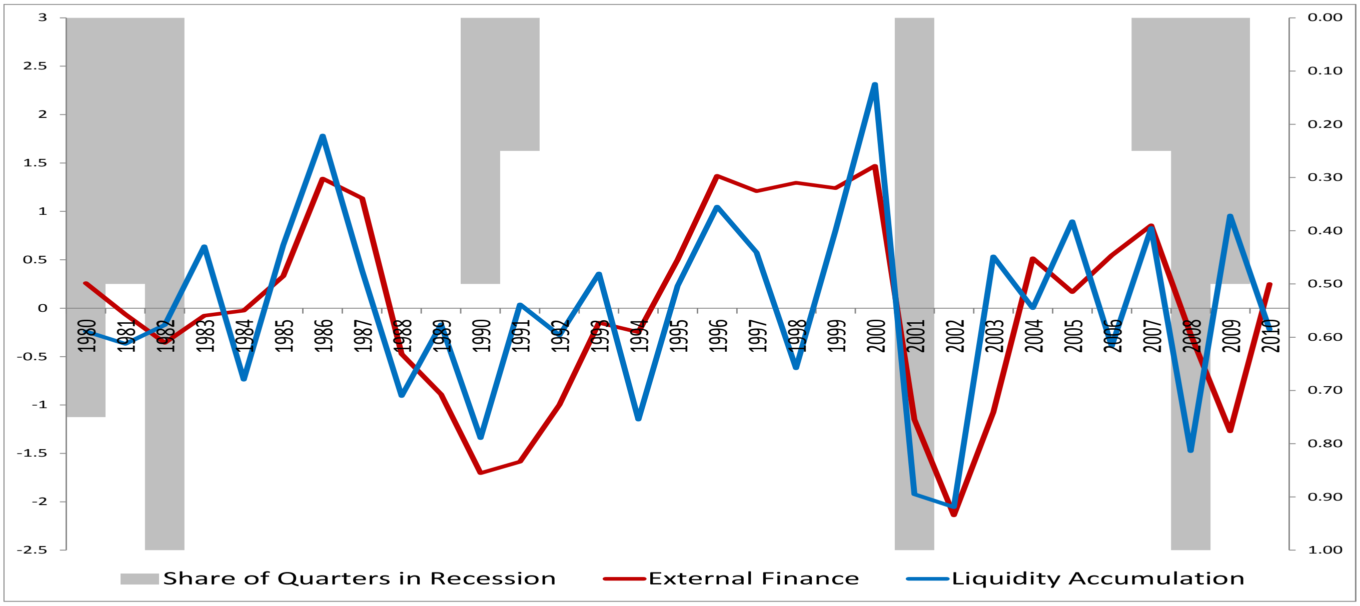

Figure 2: We plot aggregate accumulation of liquid assets against aggregate external finance from our baseline model simulation using empirical realized TFP shocks. Both series are normalized by total assets and scaled to have unit variance. Gray bars are the fraction of quarters the economy is in a recession in the given year (right axis).

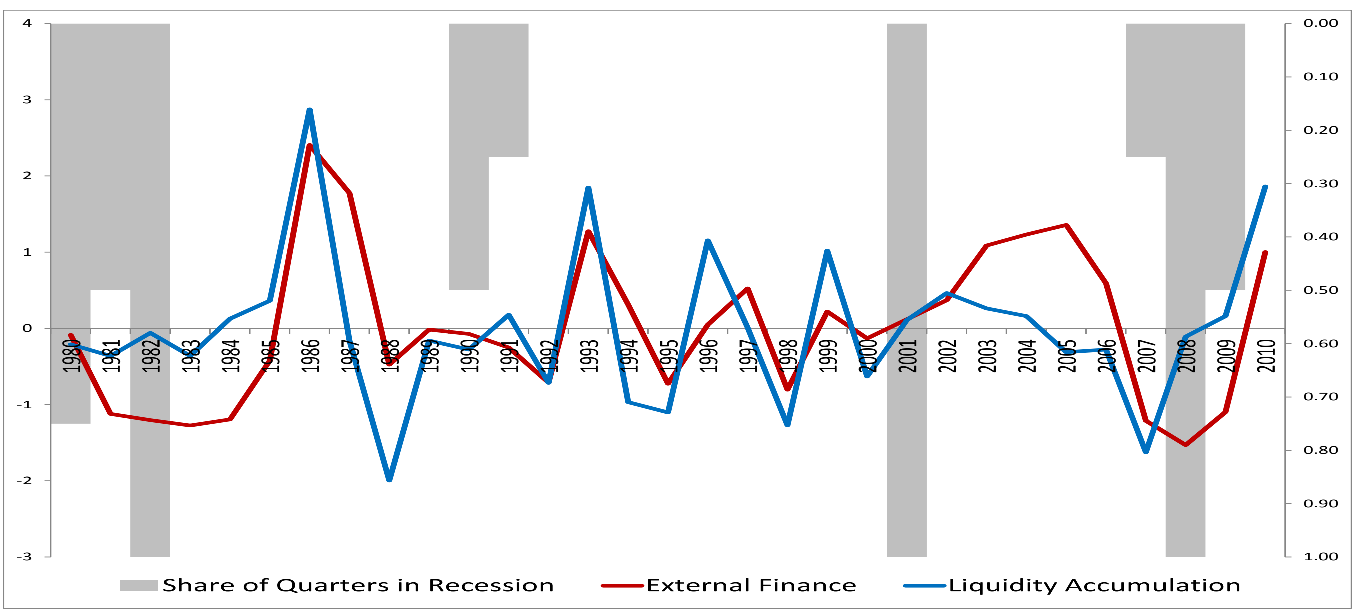

Figure 3: We plot the the cross-sectional correlation between liquidity accumulation and external finance and the negatives of the Moody's Baa-Aaa rate and the net % of banks tightening lending standards for large and medium firms. The correlation between XS rho and the negative of the Default spread and Lending Standards are 0.64 and 0.58, respectively. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis). Each series is standardized to have mean zero and unit variance.

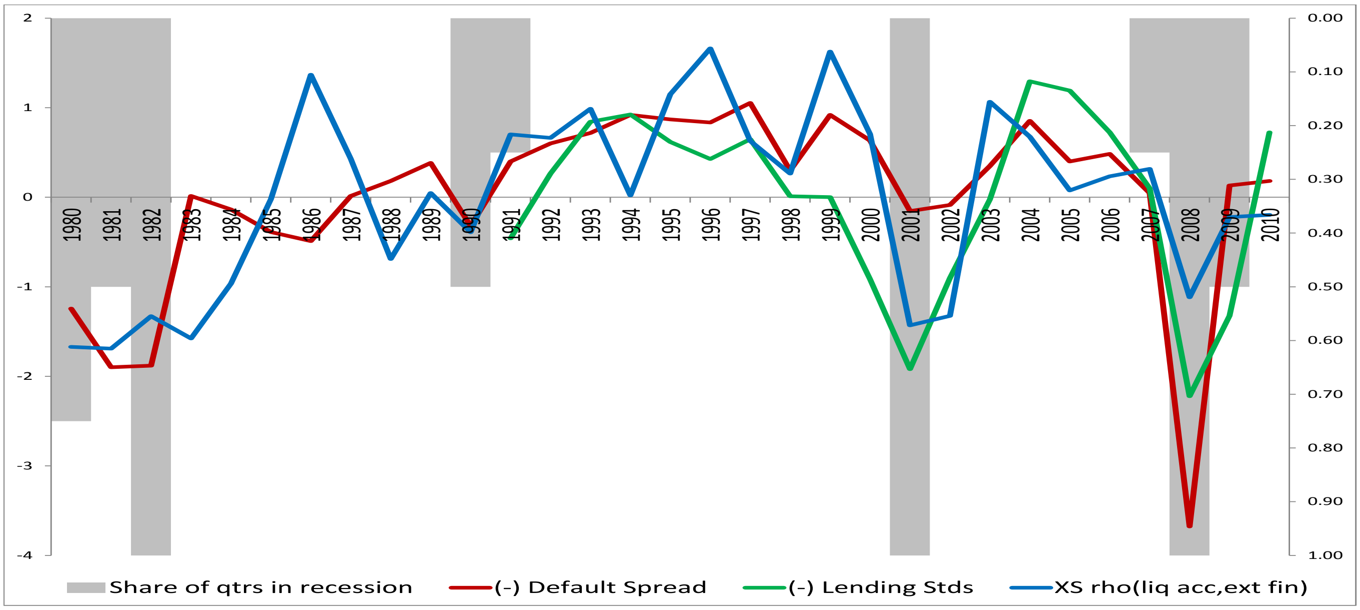

Figure 4: We plot the the cross-sectional correlation between liquidity and external finance (XS rho). In our model, this proxy reveals times when external finance is expensive, as measured by the negative of the stochastic cost of external finance (-ln(ξ)). The correlation between XS rho and the negative of the log cost (-ln(ξ)) is 0.80 in the model. Gray bars indicate when gdp growth falls below trend (right axis). Each series is standardized to have mean zero and unit variance.

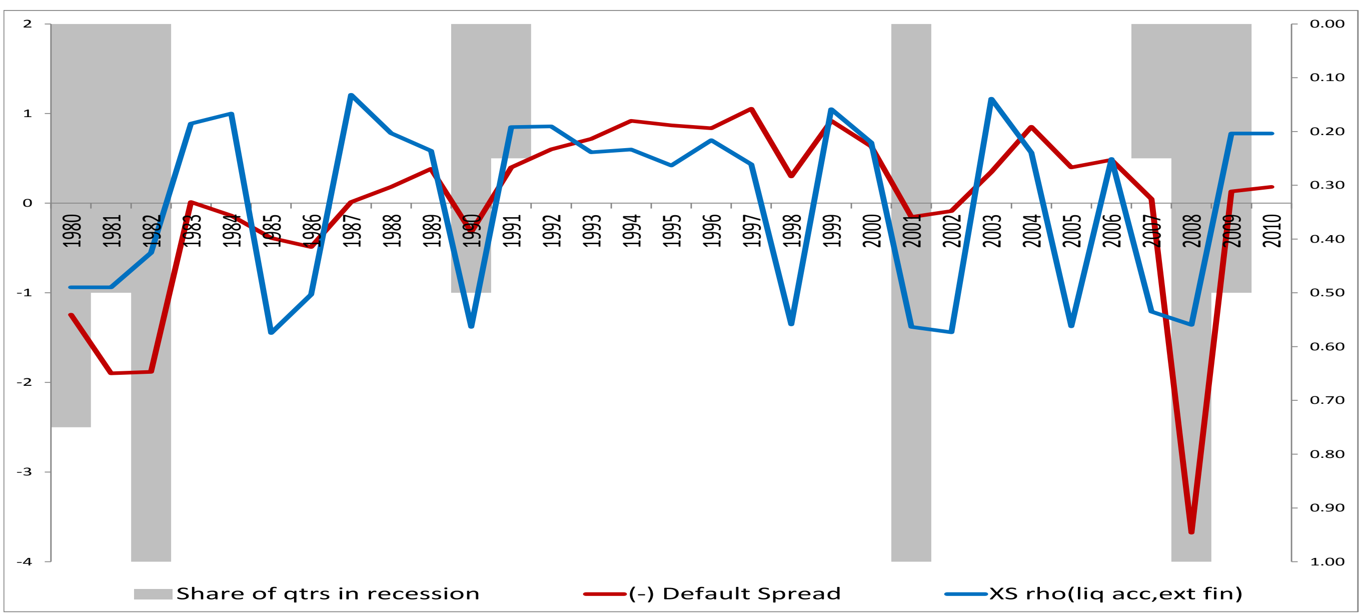

Figure 5: This figure plots the time-series of the percentage of firms raising external finance over the business cycle, measured as the growth rate of gdp. The firm level data are from Compustat. A firm is raising external finance if net flows to external finance are negative. Gray bars are the fraction of quarters the economy is in a recession in the given year (right axis). Each series is standardized to have mean zero and unit variance

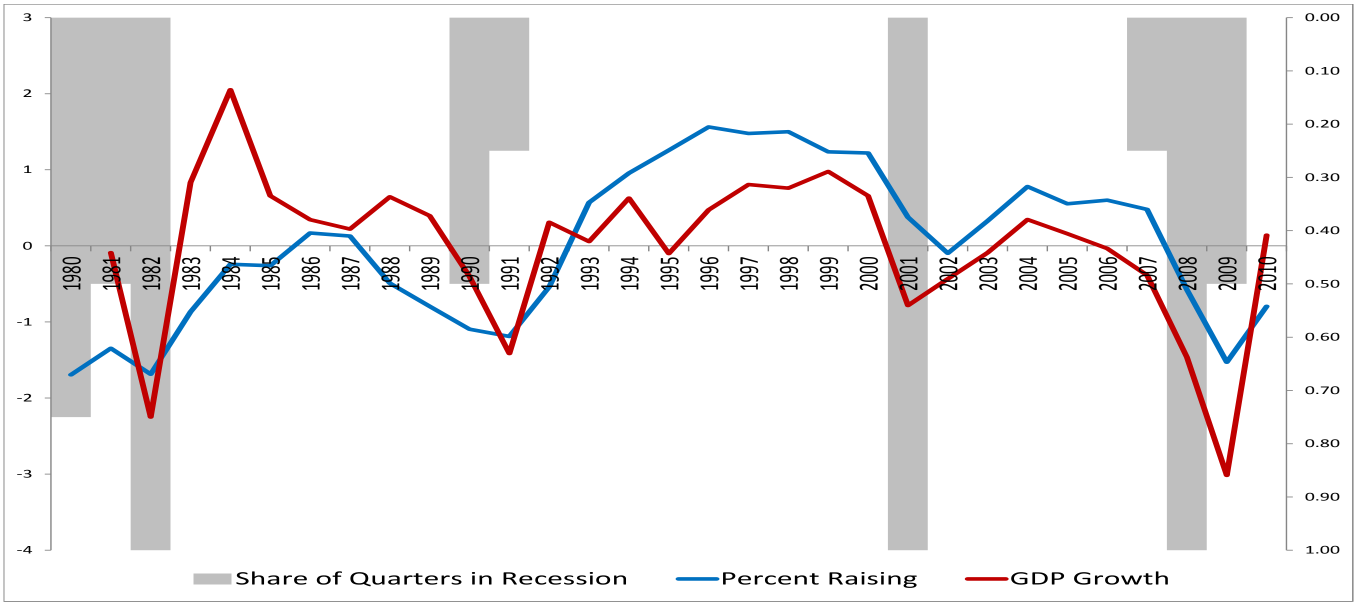

Figure 6: This figure plots the time-series of the percentage of firms raising external finance and the growth rate of gdp from our baseline model simulation using empirical realized TFP shocks. Gray bars are the fraction of quarters the economy is in a recession in the given year (right axis). Each series is standardized to have mean zero and unit variance.

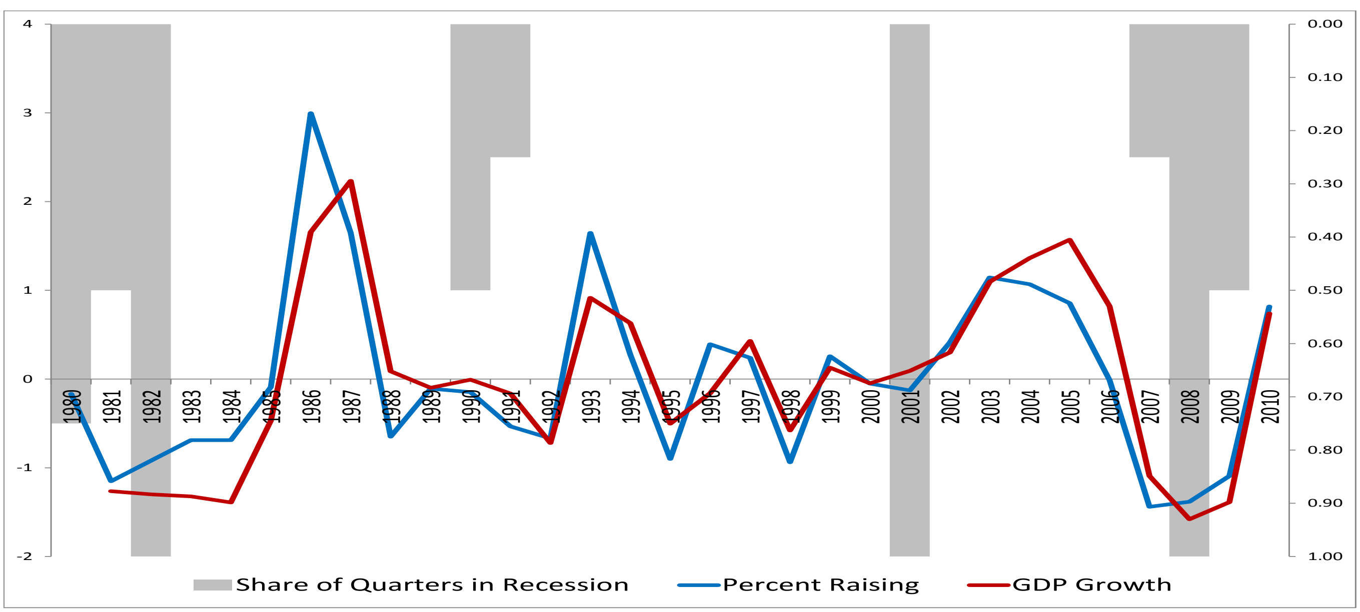

Figure 7: We plot aggregate investment, accumulation of liquid assets, and external finance. Sample excludes largest 10% of firms. Data are normalized by lagged assets, HP-filtered, and then scaled to have unit variance. Gray bars indicate fraction of quarters economy is in a recession in the given year (right axis).

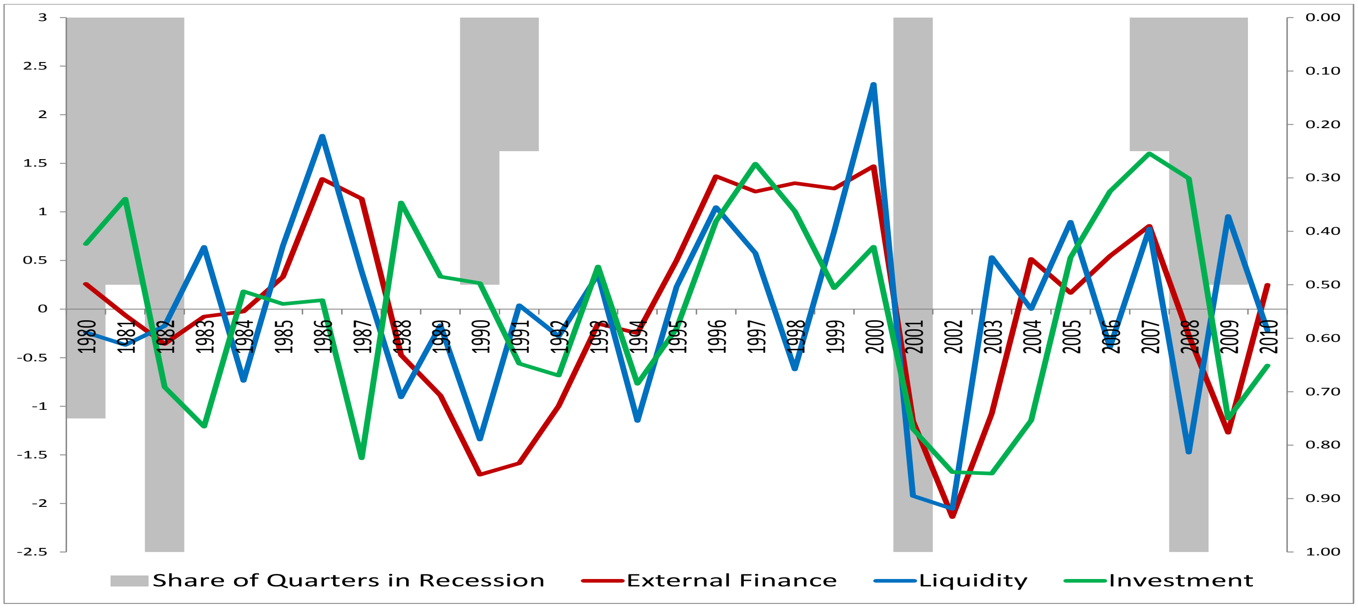

Figure 8: We plot aggregate accumulation of liquid assets against aggregate external finance from our baseline model simulation using empirical realized TFP shocks. Both series are normalized by total assets and scaled to have unit variance. Gray bars are the fraction of quarters the economy is in a recession in the given year (right axis).

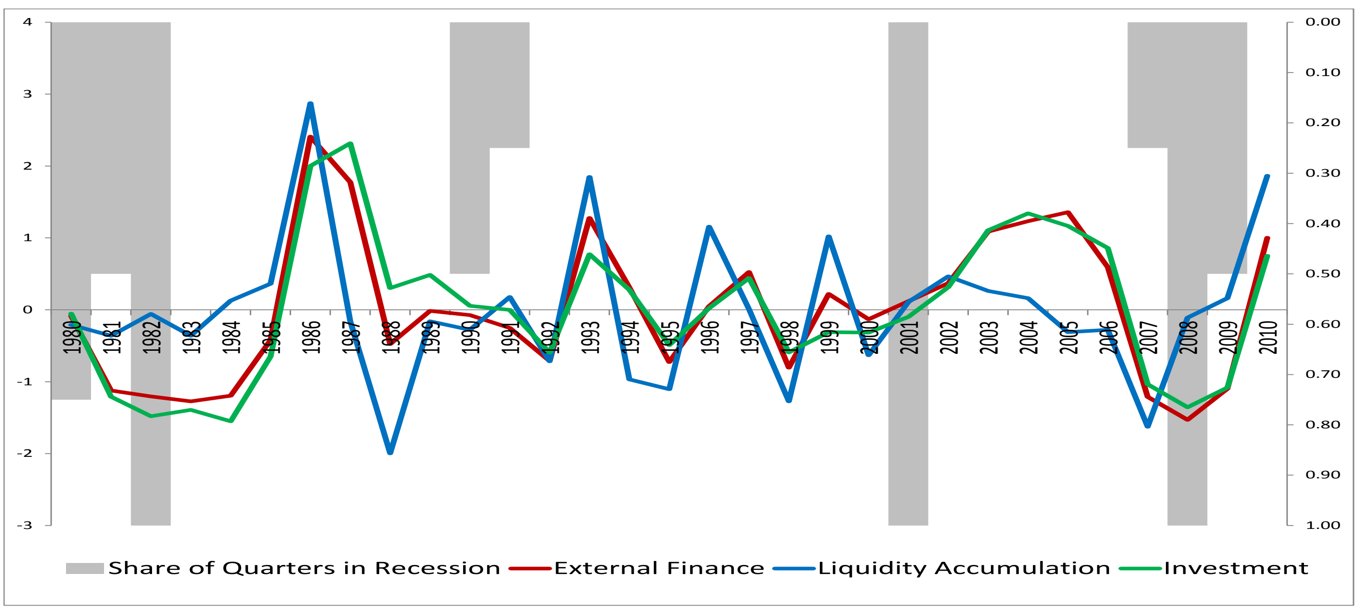

Figure 9: The figure plots investment policies as a function of firms' cash and capital stocks for various realizations of the six productivity and two stochastic cost shocks. From left to right and top to bottom, the twelve states in the SC model are ordered as follows: Top left state is the "best state", highest productivity and low cost of financing, next state is highest productivity, high cost of external finance. Third state is second highest productivity, low cost, fourth state is second highest productivity, high cost, and so on.

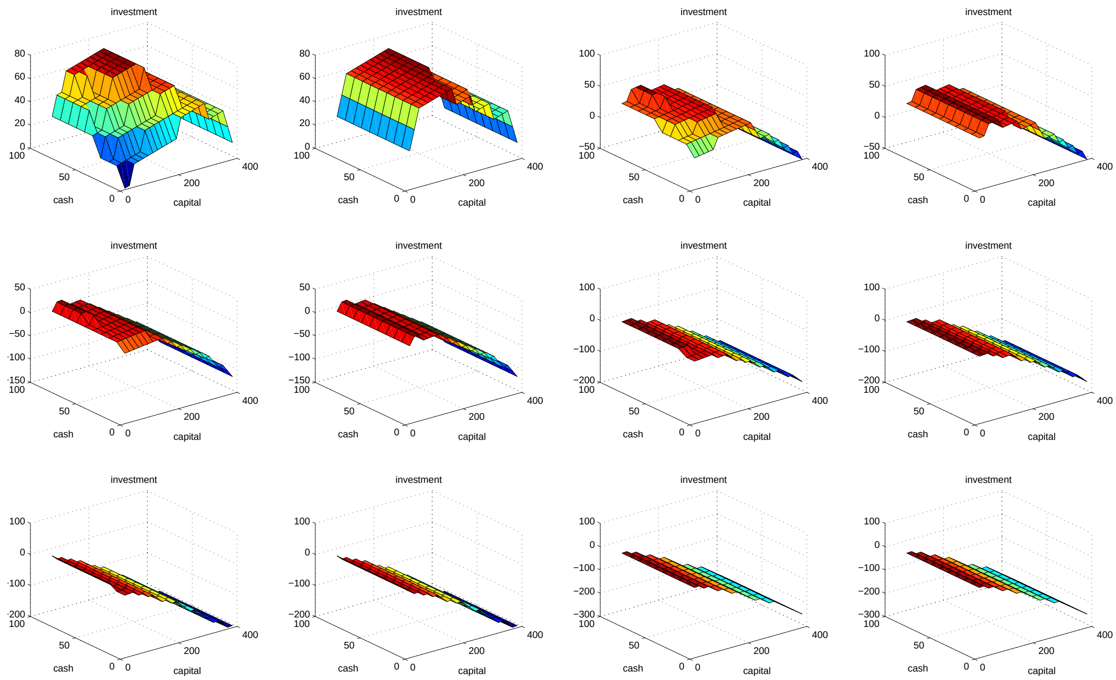

Figure 10: The figure plots external finance for various realizations of the productivity shock. When e is below zero, the firm is raising external finance, and paying out otherwise. The upper left panel is the highest productivity state, while the bottom right is the lowest.

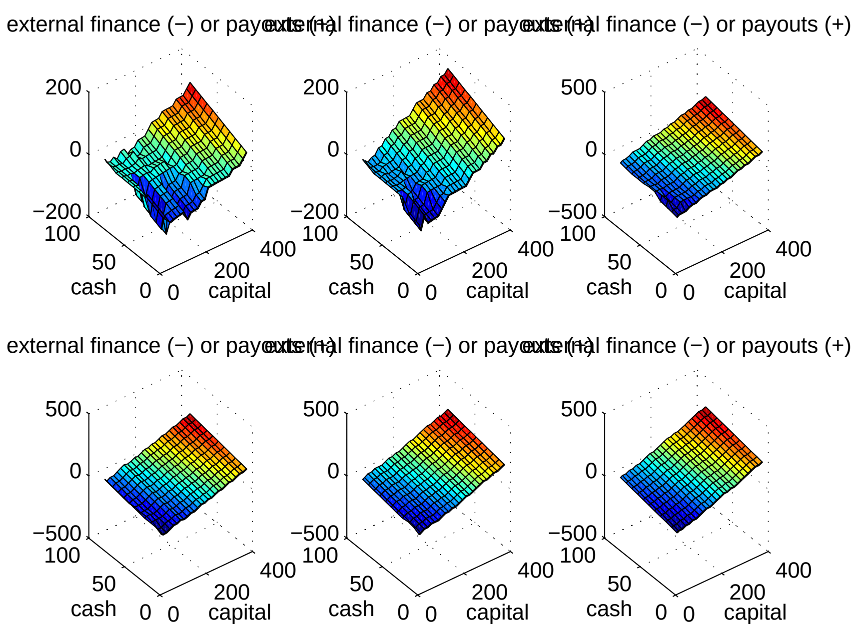

Figure 11: The figure plots investment for various realizations of the productivity shock. The upper left panel is the highest productivity state, while the bottom right is the lowest.

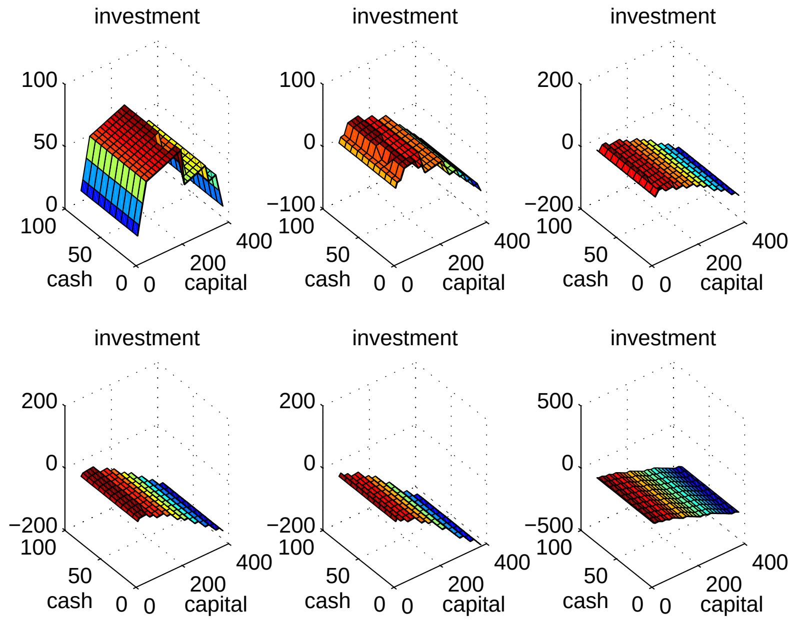

Figure 12: The figure plots liquidity accumulation for various realizations of the productivity shock. The upper left panel is the highest productivity state, while the bottom right is the lowest.

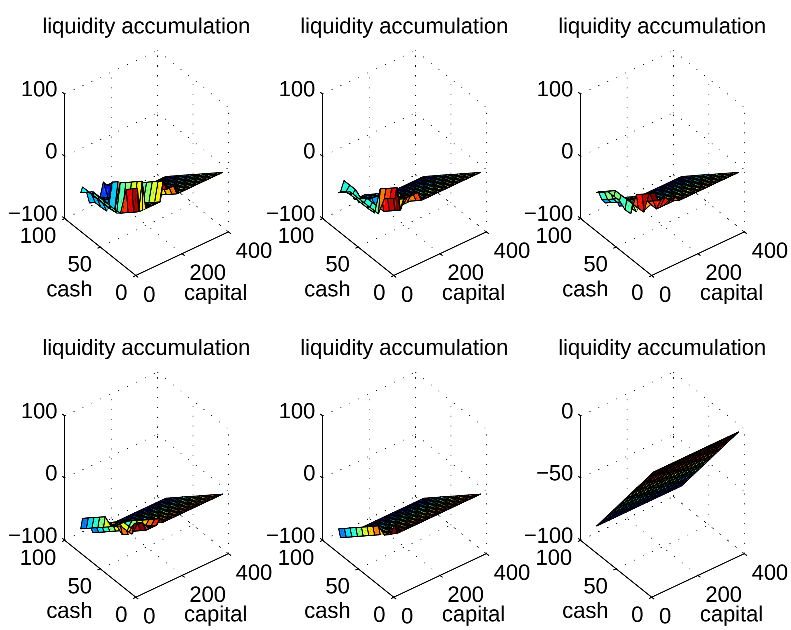

Figure 13: The figure plots the policy function for different current levels of capital. The x-axis represents the aggregate productivity state, increasing from left to right. For example, a downward sloping line means the series is counter-cyclical and vice versa. The three panels depict the policy functions for a small, medium, and large firm respectively.

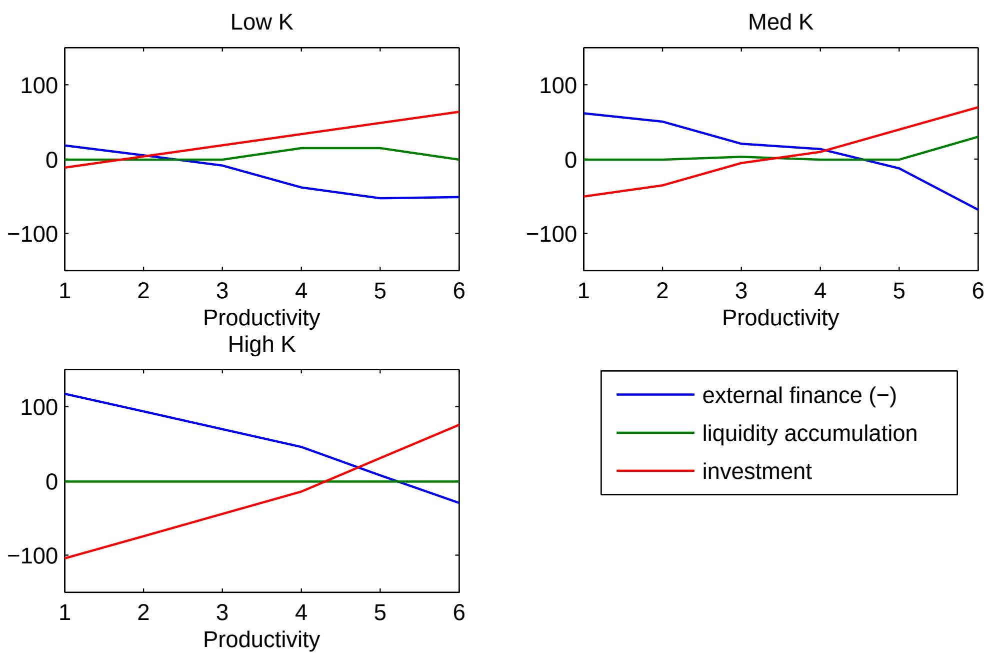

Figure 14: The figure plots investment, external finance, and liquidity accumulation for various realizations of the productivity and stochastic cost shock. Each row gives the policy function in the (highest productivity, lowest cost), (highest productivity, highest cost), (2nd highest productivity, lowest cost), and (2nd highest productivity, highest cost) respectively as you move from left to right. We only show relatively high productivity states since these are where the cost of finance shock most strongly impacts firm policies.

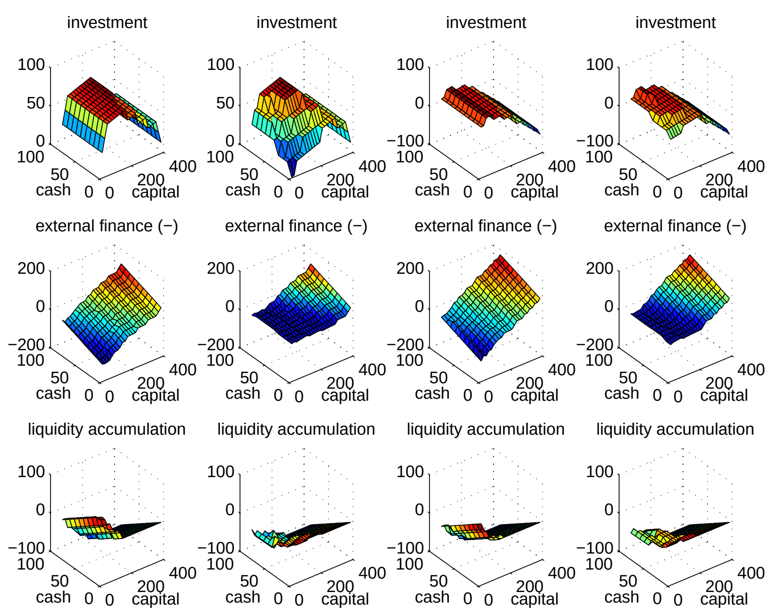

Figure 15: The figure plots the US time series for the aggregate level of the cost of external finance ξ estimated using cross-sectional moments and the estimation procedure described in Section VI..

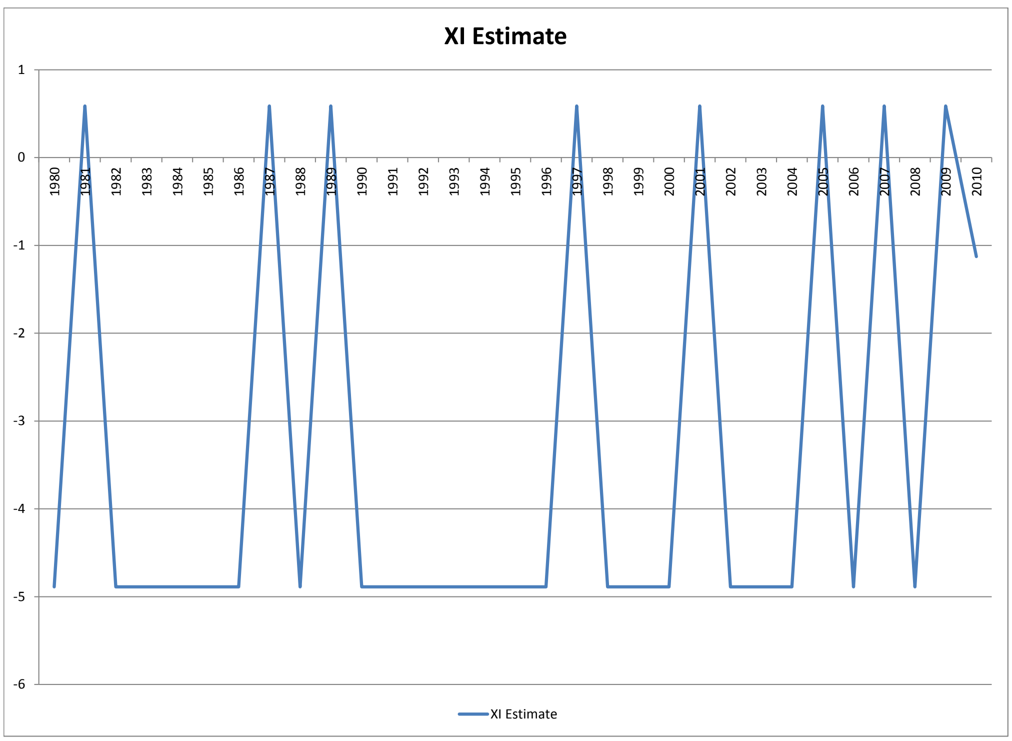

Figure 16: Two Period Model

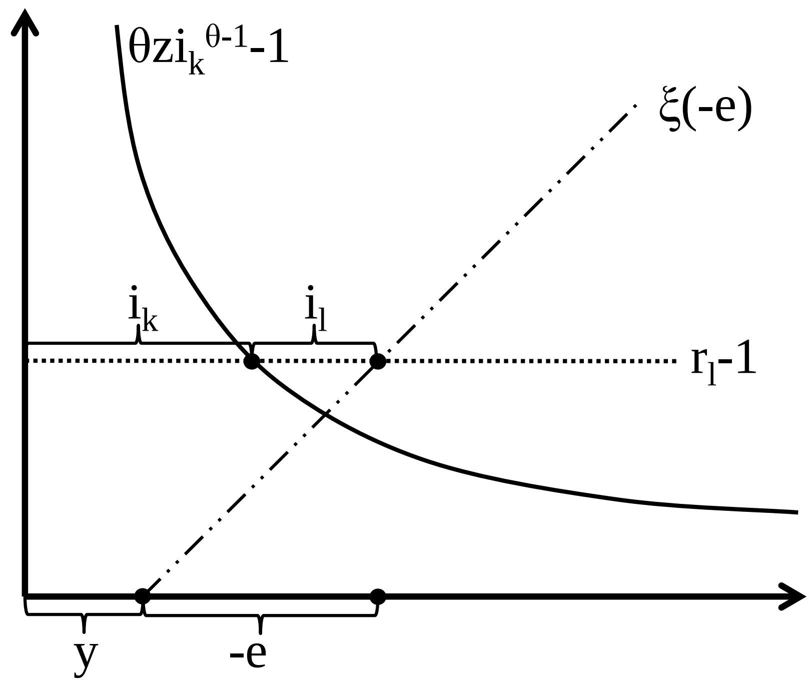

Table I: This table displays the main aggregate issuance and savings waves facts. Except where noted, we use annual Compustat data from 1980-2010. We normalize aggregate series by the lag of total assets and hp-filter. Size bins are determined by total asset size. The main results in the paper use the [0,90]% sample. Flow of funds data are normalized by the trend in gross value added for the corporate sector. Narrow liquidity is the net acquisition of financial assets minus trade receivables minus miscellaneous assets. Broader liquidity also includes 1/3 of miscellaneous other assets as liquid assets. * indicates significance at a 5% level. XSrho$_t$ is $\rho_t\left(\frac{ext_i}{TA_i}, \frac{liqacc_i}{TA_i}\right)$

Aggregate Issuance and Savings

$\rho\left(\frac{\Sigma ext_t}{\Sigma TA_{t-1}}, \frac{\Sigma liqacc_t}{\Sigma TA_{t-1}}\right)$	
[0,50]%	0.84*
[0,90]%	**0.60***
[0,100]%	0.12
Conditional on Raising funds: e<0	0.74*
No Dividends	0.68*
No Rating	0.56*
Flow of Funds: Narrow Liquidity	0.33
Flow of Funds: Broader Liquidity	0.38*

$\rho\left(\text{XSrho}_t, \text{Aggregate State}_t\right)$	
Minus Baa-Aaa Spread	0.64*
Minus Lending Standards	0.58*
TFP Shock	0.48

Table II: We give our calibrated parameters below along with those in Riddick and Whited (RW) and the standard business cycle literature (RBC). The label e.c.f. denotes external cost of finance and i.a. denotes investment adjustment costs. The lower panel gives the implied average costs of issuance and investment firms pay with the given parameters. For example, the implied average cost of issuance gives the average cost paid for a firm raising external finance as a fraction of the amount of funds raised.

Symbol	Description	EM	EMSC	RW	RBC
τ	tax rate	0.1	0.1	0.20	——
δ	depreciation	0.08	0.08	0.15	0.08
θ	curvature	0.65	0.65	0.75	0.33
ρ	persistence	0.66	0.66	0.66	0.9
σ	total vol of prod	0.121	0.121	0.121	——
σ_i	idiosyncratic vol	0.11	0.11	——	——
σ_{agg}	aggregate vol	0.03	0.03	——	0.022
λ_0	e.c.f. fixed	0.2334	0	0.389	——
λ_1	e.c.f. linear	0.004	0.053	0.053	——
λ_2	e.c.f. quad	0.00001	0.0002	0.0002	——
a	i.a. quad	0.147	0.147	0.049	——
c	i.a. fixed	0.01	0	0.039	——
r	risk-free	0.04	0.04	0.04	0.04
σ_η	vol cost of funds	——	3	——	——
γ	persistence	——	0.4	——	——
μ_ξ	mean credit	——	0.4	——	——

Symbol	Description	EM	EMSC
$E\left[\frac{\phi_e(e)}{e}\right]$	implied average issuance cost	0.017	0.085
$E\left[\frac{\phi_i(i_k,k)}{i_k}\right]$	implied average investment cost	0.009	0.009

Table III: This table displays aggregate moments from the model (both baseline and stochastic costs (SC) versions) using a simulated panel of firms. We compare these moments with those from annual Compustat data, 1980-2010. For correlations, we normalize each series by lagged assets and apply the hp-filter. All other series are normalized by current assets. XSrho_t is $\rho_t\left(\frac{ext_i}{TA_i}, \frac{liqacc_i}{TA_i}\right)$. TFP are TFP level shocks. We use the Baa-Aaa default spread as an empirical proxy for ξ. * indicates significance at 5% level.

Aggregate Moments

Moment	Data	Model	ModelSC
E[liq lev]	0.11	0.03	0.06
$\sigma(liq)$	0.03	0.05	0.01
$\sigma(liqacc)$	0.01	0.01	0.01
E[i]	0.07	0.08	0.08
$\sigma(inv)$	0.01	0.03	0.04
$\rho(inv_t, inv_{t-1})$	0.38*	0.38	0.32
E[ext]	-0.01	-0.10	-0.10
$\sigma(ext)$	0.03	0.03	0.04
$\rho(liqacc, ext)$	0.60*	0.59	0.58
$\rho(liqacc, inv)$	0.12	0.45	0.34
$\rho(ext, inv)$	0.46*	0.98	0.95
prob(raise ext)	0.43	0.15	0.17
prob(agg shortfall)	0.05	0.01	0.004

Aggregate Moments: Conditional on e<0

Moment	Data	Model	ModelSC
$\rho\,(liqacc, ext)$	0.74*	0.91	0.82
$\rho\,(liqacc, inv)$	0.37*	0.79	0.55

Aggregate Moments: Business Cycle Correlations

Moment	Data	Model	ModelSC
$\rho(liqacc, gdp)$	0.00	0.41	-0.07
$\rho(ext, gdp)$	0.28*	0.99	0.84
$\rho(inv, gdp)$	0.47*	0.97	0.70

TFP vs. ξ Moments

	Data:	Model	ModelSC:
$\rho(XSrho, \xi)$	0.64*	——	0.80
$\rho(XSrho, TFP)$	0.48*	0.72	0.00
$\rho(\%raise, \xi)$	0.59*	——	0.70
$\rho(\%raise, TFP)$	0.25	0.85	0.18

Table IV: This table gives the correlation between liquidity accumulation and external finance conditional on firm size and conditional on whether firms are raising external finance ($e < 0$). We use annual Compustat data from 1980-2010. Aggregate series are normalized by the lag of total assets and hp-filtered. Size bins are by total asset size. The main results in the paper focus on the [0,90]% bin. * indicates significance at 5% level.

$$\text{Corr}\left(\frac{LiqAcc}{TA}, \frac{ExtFin}{TA}\right)$$

Conditional on Size

Size	Data	Model	ModelSC
[0,25]%	0.59*	0.94	0.92
[0,50]%	0.84*	0.86	0.89
[0,75]%	0.76*	0.71	0.66
[0,90]%	0.60*	0.60	0.62
[0,100]%	0.12	0.59	0.58
[90,100]%	0.03	0.20	0.11

Conditional on e

	Data	Model	ModelSC
e<0	0.74*	0.91	0.82
e>0	0.15	0.15	0.45

Table V: This table displays moments for debt and equity separately using our Compustat sample. We normalize each aggregate series by lagged aggregate assets and apply the hp-filter. * indicates significance at 5% level.

Aggregate Compustat Moments: Debt vs. Equity

	Unconditional	Conditional on e<0
$\rho\,(liqacc, debt)$	0.16	0.33*
$\rho\,(liqacc, equity)$	0.69*	0.77*
$\rho\,(inv, debt)$	0.60*	
$\rho\,(inv, equity)$	-0.15	

Table VI: Firm Level Facts. The table gives firm level moments. In each case, we compute the relevant moment for the entire panel of firms and then take a median across firms. We use our simulated panel of data (Model column) and Compustat (Data column). We normalize the series by total book assets. * indicates significance at 5% level.

Firm Level Moments

Moment	Data	Model	ModelSC
E[liq lev]	0.15	0.01	0.06
$\sigma(liqacc)$	0.10	0.07	0.11
E[inv]	0.06	0.11	0.07
$\sigma(inv)$	0.07	0.16	0.12
$\rho(inv_t, inv_{t-1})$	0.29*	0.17	0.18
E[ext]	-0.02	-0.07	-0.11
$\sigma(ext)$	0.17	0.21	0.17
$\rho(liqacc, ext)$	0.18*	0.57	0.62
$\rho(liqacc, inv)$	-0.06*	0.22	-0.00
$\rho(ext, inv)$	0.20*	0.93	0.77
prob(raise funds)	0.43	0.15	0.17