

# Adverse Selection, Risk Sharing and Business Cycles\*

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**Abstract:** I consider a real business cycle model in which agents have private information about an idiosyncratic shock to their value of leisure. I consider the mechanism design problem for this economy and describe a computational method to solve it. This is an important contribution of the paper since the method could be used to solve a wide class of models with heterogeneous agents and aggregate uncertainty. Calibrating the model to U.S. data I find a striking result: That the information frictions that plague the economy have no effects on business cycle fluctuations.

**Keywords:** Adverse selection, risk sharing, business cycles, private information, incentives, optimal contracts, computational methods, heterogeneous agents.

**JEL:** C63, C68, D31, D82, E32

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## 1. Introduction

There is a long literature analyzing dynamic efficient contracts under private information (e.g. Phelan and Townsend [10], Spear and Srivastava [11], Thomas and Worrall [12], etc.). The basic setting consists of a risk neutral principal who seeks to design a contract for a risk averse agent who has private information about his type or actions. The principal faces a nontrivial trade-off between insurance and incentives: Given the risk aversion of the agent he would like to insure him completely, but if he does he would be giving the agent no incentives to reveal his private information. Balancing these conflicting goals, the principal seeks to provide as much insurance as possible while inducing the agent to behave truthfully.

In macroeconomics these ideas have been used, for example, to study optimal consumption inequality (e.g. Atkeson and Lucas [1], Green [6], etc.), optimal unemployment insurance (e.g. Hopenhayn and Nicolini [7], Kocherlakota [8], etc.), and taxation (e.g. Golosov et al. [4], Fahri and Werning [3], etc.). However, any interactions with aggregate fluctuations have been mostly neglected (Phelan [9] is a notable exception). The purpose of this paper is to close this gap by analyzing the cyclical behavior of efficient contracts under private information. Of particular interest are the implications of efficient contracts for the cyclical behavior of consumption and employment heterogeneity, for the cyclical behavior of the intertemporal and consumption-labor wedges emphasized by the new public finance literature, and for the aggregate business cycle properties of the economy.

The model that I use is a simple real business cycle model with private information. Agents value consumption and leisure, and receive idiosyncratic shocks about their value of leisure. These shocks, which are i.i.d. over time and across individuals, are private information. The production technology is standard. Output, which can be consumed or invested, is produced with capital and labor using a Cobb-Douglas production function subject to an aggregate productivity shock. The aggregate shock follows a standard AR(1) process.

Following the literature, contracts are given a recursive representation in which the state of the contract is a promised value to the agent. Given this current state, the contract specifies current consumption, current hours worked and next-period promised values as a function of the value of leisure reported by the agent. Since the model has a large number of agents subject to this type of contracts, it is necessary to keep track as a state variable the whole distribution of promised values across individuals. The high dimensionality of this object makes computations difficult. An

important contribution of the paper is to present a computational strategy that makes this problem tractable. In fact, the computational strategy described here is not only applicable to the model in this paper but to a wide class of economies with heterogeneous agents and aggregate uncertainty. In particular, the method should be applicable to any model in which agents have smooth decision rules and are subject to idiosyncratic uncertainty, and in which the aggregate shocks are described by autorregressive processes.

The computational approach that I use is to parametrize individual decision rules as spline approximations and to keep long histories of these parameters as state variables. Starting from the deterministic steady state distribution, the history of decision rules implied by these parameters is then used to obtain the current distribution of individuals across individual states. In practice, this is done performing a large number of Monte Carlo simulations. I then linearize the first order conditions with respect to the coefficients of the spline approximations and solve the resulting linear rational expectations model using standard methods

Applying this computational method to a calibrated version of the business cycle model described above provides a striking result: The presence of private information has no effects whatsoever on aggregate business cycles dynamics. The business cycle behavior of aggregate output, consumption, investment, hours worked and capital is exactly the same as in a public information version of the model economy. Moreover, there is no cyclical variation in the amount of consumption and leisure heterogeneity over the business cycle: The distributions of consumption, leisure and promised values shift horizontally in response to the aggregate shocks but maintain their shapes exactly.

The paper is organized as follows. Section 2 describes the economy. Section 3 describes the mechanism design problem. Section 4 characterizes the deterministic steady state and provides an algorithm to compute it. Section 5 describes the computational method for solving the mechanism design problem with aggregate fluctuations. Section 6 describes the intertemporal wedges and consumption-labor wedges present in the model. Section 7 calibrates the model to U.S. observations. Finally, Section 8 presents the results.

## **2. The economy**

The economy is populated by a unit measure of agents subject to stochastic lifetimes. Whenever an agent dies he is immediately replaced by a newborn, maintaining the aggregate population level

constant. The preferences of an individual born at date  $T$  are given by

$$E_T \left\{ \sum_{t=T}^{\infty} \beta^{t-T} \sigma^{t-T} [\ln c_t + s_t \ln(1 - h_t)] \right\}, \quad (2.1)$$

where  $c_t$  is consumption,  $h_t$  is hours worked,  $s_t$  is the idiosyncratic value of leisure,  $\sigma$  is the survival probability, and  $0 < \beta < 1$ . The idiosyncratic value of leisure  $s_t$  is assumed to take two possible values:  $s_L$  and  $s_H$ , where  $s_L < s_H$ . Realizations of  $s_t$  are i.i.d. across time and across individuals and are distributed according to the distribution function  $\psi = (\psi_L, \psi_H)$ . A key assumption maintained throughout the paper is that  $s_t$  is private information of the individual.

Output, which can be consumed or invested, is produced with the following aggregate production function:

$$Y_t = e^{z_t} K_{t-1}^\gamma H_t^{1-\gamma}$$

where  $Y_t$  is output,  $z_t$  is aggregate productivity,  $K_{t-1}$  is capital and  $H_t$  is hours worked. The aggregate productivity level  $z_t$  follows a standard AR(1) process given by:

$$z_{t+1} = \rho z_t + \varepsilon_{t+1},$$

where  $0 < \rho < 1$  and  $\varepsilon_{t+1}$  is normally distributed with mean zero and standard deviation  $\sigma_\varepsilon$ .

Capital is accumulated using a standard linear technology given by

$$K_t = (1 - \delta) K_{t-1} + I_t,$$

where  $I_t$  is gross investment and  $0 < \delta < 1$ .

### 3. Mechanism Design Problem:

In this section I provide a recursive formulation to the problem of a social planner that seeks to maximize utility subject to incentive compatibility and resource feasibility constraints. In order to do this it will be important to distinguish between two types of agents: young and old. A young agent is one that has been born at the beginning of the current period. An old agent is one that has been born in some previous period.

The social planner decides recursive plans for both types of agents. The state of a recursive plan is the value (i.e. discounted expected utility) that the agent is entitled to at the beginning

of the period. Given this promised value, the recursive plan specifies consumption, current hours worked and next period promised values as functions of the value of leisure currently reported by the agent.<sup>1</sup> A key goal of the social planner is to structure the recursive plans in such a way that the agents truthfully reveal their private information. Another, conflicting goal, is to structure the plans so that they provide as much insurance as possible. Throughout the paper I will assume that the social planner is fully committed to the recursive plans that he chooses and that the agents have no outside opportunities available.

A key difference between the young and the old is in terms of promised values. Since during the previous period the social planner has decided on some recursive plan for a currently old agent, he is restricted to delivering the corresponding promised value during the current period. On the contrary, the social planner is free to deliver any value to a currently young agent since this is the first period that he is alive. Reflecting this difference, I will specify the individual state of an old agent to be his promised value  $v$  and his current value of leisure  $s$ . His current consumption, hours worked and next-period promised values are then be denoted by  $c_s^o(v)$ ,  $h_s^o(v)$  and  $w_s^o(v, z')$ , respectively.<sup>2</sup> In turn, the individual state of a young agent is solely given by his current value of leisure  $s$ . His current consumption, hours worked and next-period promised values are denoted by  $c_s^y$ ,  $h_s^y$  and  $w_s^y(z')$ , respectively.

The aggregate state of the economy is given by  $(z, K, \mu)$ , where  $z$  is the aggregate productivity level,  $K$  is the stock of capital, and  $\mu$  is a measure describing the number of old agents across individual promised values  $v$ . The social planner seeks to maximize the weighted sum of welfare levels of current and future generations of young agents.<sup>3</sup> The welfare weight that the social planner puts on the generation to be born  $t$  periods into the future is  $\theta^t$ , where  $0 < \theta < 1$ . In recursive form, the social planner problem is described by the following Bellman equation:

$$V(z, K, \mu) = \max \left\{ (1 - \sigma) \sum_s [\ln c_s^y + s \ln(1 - h_s^y) + \beta \sigma E_z (w_s^y(z'))] \psi_s + \theta E_z V(z', K', \mu_{z'}) \right\} \quad (3.1)$$

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<sup>1</sup>Observe that I am focusing on direct revelation mechanisms.

<sup>2</sup>Observe that next-period promised values are allowed to be contingent on the realization of next-period aggregate productivity  $z'$ .

<sup>3</sup>Observe that the welfare level of old agents are predetermined by their promised values at the beginning of the period.

subject to:

$$(1 - \sigma) \sum_s c_s^y \psi_s + \int \sum_s c_s^o(v) \psi_s d\mu + I \leq e^z K^\gamma H^{1-\gamma}, \quad (3.2)$$

$$H \leq (1 - \sigma) \sum_s h_s^y \psi_s + \int \sum_s h_s^o(v) \psi_s d\mu, \quad (3.3)$$

$$\ln c_s^y + s \ln(1 - h_s^y) + \beta \sigma E_z [w_s^y(z')] \geq \ln c_s^y + s \ln(1 - h_s^y) + \beta \sigma E_z [w_s^y(z')], \quad (3.4)$$

$$\ln c_s^o(v) + s \ln(1 - h_s^o(v)) + \beta \sigma E_z [w_s^o(v, z')] \geq \ln c_s^o(v) + s \ln(1 - h_s^o(v)) + \beta \sigma E_z [w_s^o(v, z')], \quad (3.5)$$

$$v = \sum_s \{ \ln c_s^o(v) + s \ln(1 - h_s^o(v)) + \beta \sigma E_z [w_s^o(v, z')] \} \psi_s, \quad (3.6)$$

$$K' = (1 - \delta) K + I \quad (3.7)$$

$$\mu'_{z'}(B) = \sigma \sum_s \int_{\{(v,s): w_s^o(v,z') \in B\}} \psi_s d\mu + (1 - \sigma) \sigma \sum_{s: w_s^y(z') \in B} \psi_s, \quad (3.8)$$

$$z' = \rho z + \varepsilon'. \quad (3.9)$$

Equation (3.2) describes the aggregate feasibility constraint for the consumption good. It states that the total consumption of young and old agents, plus aggregate investment cannot exceed aggregate output.<sup>4</sup> Equation (3.3) is the aggregate labor feasibility constraint. It states that the input of hours into the production function cannot exceed the total hours worked by young and old agents. Equation (3.4), which holds for every  $(s, \hat{s})$ , is the incentive compatibility constraint of young agents. It states that the value of truthfully reporting  $s$  provides a higher utility level than reporting the alternative  $\hat{s}$ . Similarly, equation (3.5) is the incentive compatibility constraint for old agents. Equation (3.6) is the promise keeping constraint. It states that the recursive plan for an old agent with promised value  $v$  must provide him an expected utility equal to that promised value. Equation (3.7) is the law of motion for the stock of capital. Equation (3.8) is the law of motion for the measure of old agents across promised values. It states that the number of old agents that at the beginning of the following period will have a promised value in the Borel set  $B$  is given by the sum of two terms. The first term sums all currently old agents that receive a next-period promised value in the set  $B$  and do not die. The second term does the same for all currently young agents. Observe that since next-period promised values  $w_s^o(v, z')$  and  $w_s^y(z')$  are contingent on the realization of next-period aggregate productivity  $z'$ , that the same is true for the

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<sup>4</sup>Observe that, given the constant probability of dying  $1 - \sigma$  and the immediate replacement with newborns, the number of young agents in the economy is always equal to  $1 - \sigma$ .

measure  $\mu'_{z'}$ . Finally, equation (3.9) describes the stochastic process for aggregate productivity.

The incentive compatibility constraints (3.4) and (3.5) introduce non-convexities to the social planner's feasible set. For this reason, it will be convenient to redefine variables as follows:

$$\begin{aligned}x_s^y &= \ln c_s^y, \\n_s^y &= \ln(1 - h_s^y), \\x_s^o(v) &= \ln c_s^o(v), \\n_s^o(v) &= \ln[h_s^o(v)],\end{aligned}$$

i.e. instead of using consumption and hours worked directly I will consider the utility levels that they provide. With this change of variables the social planner problem becomes the following:

$$V(z, K, \mu) = \max \left\{ (1 - \sigma) \sum_s [x_s^y + sn_s^y + \beta\sigma E_z(w_s^y(z'))] \psi_s + \theta E_z V(z', K', \mu'_{z'}) \right\} \quad (3.10)$$

subject to:

$$(1 - \sigma) \sum_s e^{x_s^y} \psi_s + \int \sum_s e^{x_s^o(v)} \psi_s d\mu + I \leq e^z K^\gamma H^{1-\gamma}, \quad (3.11)$$

$$H \leq (1 - \sigma) \sum_s (1 - e^{n_s^y}) \psi_s + \int \sum_s [1 - e^{n_s^o(v)}] \psi_s d\mu, \quad (3.12)$$

$$x_s^y + sn_s^y + \beta\sigma E_z[w_s^y(z')] \geq x_s^y + sn_s^y + \beta\sigma E_z[w_s^y(z')] \quad (3.13)$$

$$x_s^o(v) + sn_s^o(v) + \beta\sigma E_z[w_s^o(v, z')] \geq x_s^o(v) + sn_s^o(v) + \beta\sigma E_z[w_s^o(v, z')], \quad (3.14)$$

$$v = \sum_s \{x_s^o(v) + sn_s^o(v) + \beta\sigma E_z[w_s^o(v, z')]\} \psi_s, \quad (3.15)$$

and equations (3.7)-(3.9). Observe that the objective function has become linear and that equations (3.11)-(3.15) now define a convex feasible set.

#### 4. Deterministic steady state

In this section I characterize the deterministic steady state of the economy, in which the aggregate productivity level  $z$  has been permanently set to zero, and provide an algorithm for computing it. It turns out that the steady state can be computed as the solution to two simple planning problems plus a side condition. One planning problem solves the allocations of old agents. The other one,

the allocation of the young. Steady state conditions for the deterministic version of the economy are provided in Appendix A.

#### 4.1. Planning problem for old agents

Consider the problem of maximizing the expected discounted “social profits” of providing a recursive plan to an old agent, subject to incentive compatibility and promise keeping constraints. Given a promised value to the old agent  $v$ , this planning problem is described by the following Bellman equation:

$$P(v) = \max \left\{ \sum_s [q(1 - e^{n_s^o}) - e^{x_s^o} + \theta\sigma P(w_s^o)] \psi_s \right\} \quad (4.1)$$

subject to:

$$x_s^o + sn_s^o + \beta\sigma w_s^o \geq x_{\hat{s}}^o + sn_{\hat{s}}^o + \beta\sigma w_{\hat{s}}^o \quad (4.2)$$

$$v = \sum_s [x_s^o + sn_s^o + \beta\sigma w_s^o] \psi_s \quad (4.3)$$

where  $q$  is the social value of labor. Observe that “social profits” are given by the social value of the hours worked by the old agent, net of the consumption goods that are transferred to him. Also observe that the planner discounts future social profits using the social discount rate  $\theta$  and the survival probability  $\sigma$ . Letting  $\xi_{s,\hat{s}}^o(v)$  and  $\eta(v)$  be the Lagrange multipliers for constraints (4.2) and (4.3), respectively, the first order conditions to this problem are given by equations (9.1)-(9.6) in Appendix A.<sup>5</sup>

A direct consequence of equation (4.2) is that

$$n_H^o \geq n_L^o \quad (4.4)$$

and

$$x_H^o + \beta\sigma w_H^o \leq x_L^o + \beta\sigma w_L^o, \quad (4.5)$$

where variables with subscript  $L$  and  $H$  now correspond to those with subscripts  $s_L$  and  $s_H$ , respectively. Moreover, these inequalities are strict if either the upward or downward incentive compatibility constraints is satisfied with strict inequality.

From the inequalities (4.4)-(4.5), the first order conditions (9.1)-(9.6), and the fact that  $P$  is

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<sup>5</sup>Observe that the envelope condition  $P'(v) = -\eta(v)$  is implicitly used in equation (9.3).



strictly concave (and therefore that the the Lagrange multiplier  $\eta$  is strictly increasing), it follows that<sup>6</sup>

$$x_H^o(v) + s_H n_H^o(v) + \beta \sigma w_H^o(v) > x_L^o(v) + s_H n_L^o(v) + \beta \sigma w_L^o(v) \quad (4.6)$$

and

$$x_L^o(v) + s_L n_L^o(v) + \beta \sigma w_L^o(v) = x_H^o(v) + s_L n_H^o(v) + \beta \sigma w_H^o(v). \quad (4.7)$$

That is, at the optimal plan, the incentive compatibility constraint of an old agent with high (low) value of leisure is always slack (binding).<sup>7</sup>

Given these results, the planning problem for old agents can be simplified to the following:

$$P(v) = \max \{ [q(1 - e^{n_H^o}) - e^{x_H^o} + \theta \sigma P(w_H^o)] \psi_H + [q(1 - e^{n_L^o}) - e^{x_L^o} + \theta \sigma P(w_L^o)] \psi_L \} \quad (4.8)$$

subject to:

$$x_L^o + s_L n_L^o + \beta \sigma w_L^o = x_H^o + s_L n_H^o + \beta \sigma w_H^o \quad (4.9)$$

$$v = [x_H^o + s_H n_H^o + \beta \sigma w_H^o] \psi_H + [x_L^o + s_L n_L^o + \beta \sigma w_L^o] \psi_L \quad (4.10)$$

The first order conditions to this problem are the following:

$$0 = -e^{x_H^o(v)} \psi_H - \xi^o(v) + \eta(v) \psi_H \quad (4.11)$$

$$0 = -e^{x_L^o(v)} \psi_L + \xi^o(v) + \eta(v) \psi_L \quad (4.12)$$

$$0 = -q e^{n_H^o(v)} \psi_H - s_L \xi^o(v) + \eta(v) s_H \psi_H \quad (4.13)$$

$$0 = -q e^{n_L^o(v)} \psi_L + s_L \xi^o(v) + \eta(v) s_L \psi_L \quad (4.14)$$

$$0 = -\theta \sigma \eta [w_H^o(v)] \psi_H - \beta \sigma \xi^o(v) + \eta(v) \beta \sigma \psi_H \quad (4.15)$$

$$0 = -\theta \sigma \eta [w_L^o(v)] \psi_L + \beta \sigma \xi^o(v) + \eta(v) \beta \sigma \psi_L \quad (4.16)$$

$$x_L^o(v) + s_L n_L^o(v) + \beta \sigma w_L^o(v) = x_H^o(v) + s_L n_H^o(v) + \beta \sigma w_H^o(v) \quad (4.17)$$

$$v = [x_H^o(v) + s_H n_H^o(v) + \beta \sigma w_H^o(v)] \psi_H + [x_L^o(v) + s_L n_L^o(v) + \beta \sigma w_L^o(v)] \psi_L \quad (4.18)$$

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<sup>6</sup>For a proof, since the Technical Appendix.

<sup>7</sup>From equation (4.6) it follows that the inequalities in equations (4.4) and (4.5) are strict.

where  $\xi^o$  and  $\eta$  are the Lagrange multipliers for constraints (4.9) and (4.10), respectively, and where the envelope condition

$$P'(v) = -\eta(v) \quad (4.19)$$

has been used in equations (4.15) and (4.16).

## 4.2. Planning problem for young agents

Now consider the problem of maximizing the social surplus of a young agent's recursive plan, subject to incentive compatibility constraints. The problem is the following:

$$\max \left\{ \sum_s \left[ x_s^y + sn_s^y + \beta\sigma w_s^y - \lambda e^{x_s^y} + \lambda q \left( 1 - e^{n_s^y} \right) + \theta\lambda\sigma P(w_s^y) \right] \psi_s \right\} \quad (4.20)$$

subject to

$$x_s^y + sn_s^y + \beta\sigma w_s^y \geq x_{\hat{s}}^y + sn_{\hat{s}}^y + \beta\sigma w_{\hat{s}}^y, \quad (4.21)$$

where  $\lambda$  is the shadow value of consumption. Observe that the social surplus is the utility level of the young agent, minus the social cost of the consumption provided to the young agent, plus the social value of the hours worked by the young agent, plus the discounted "social profits" of providing a promised value to the young agent at the beginning of the following period. These discounted social profits are the solution to the planning problem for old agents. Letting  $\xi_{s,\hat{s}}^y$  be the Lagrange multipliers for constraint (4.21), the first order conditions to this problem are given by equations (9.7)-(9.11) in Appendix A.

Equation (4.21) gives rise to similar inequalities as those in equations (4.4)-(4.5). These inequalities, the first order conditions (9.7)-(9.11), and the strict concavity of  $P$  imply that, at the solution, the incentive compatibility constraint of a young agent with high (low) value of leisure is always slack (binding).<sup>8</sup> Thus, the planning problem for young agents can be simplified to the following:

$$\begin{aligned} \max \left\{ \left[ x_H^y + s_H n_H^y + \beta\sigma w_H^y - \lambda e^{x_H^y} + \lambda q \left( 1 - e^{n_H^y} \right) + \theta\lambda\sigma P(w_H^y) \right] \psi_H \right. \\ \left. + \left[ x_L^y + s_L n_L^y + \beta\sigma w_L^y - \lambda e^{x_L^y} + \lambda q \left( 1 - e^{n_L^y} \right) + \theta\lambda\sigma P(w_L^y) \right] \psi_L \right\} \quad (4.22) \end{aligned}$$

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<sup>8</sup>For a proof, see the Technical Appendix.

subject to

$$x_L^y + s_L n_L^y + \beta \sigma w_L^y = x_H^y + s_L n_H^y + \beta \sigma w_H^y. \quad (4.23)$$

The first order conditions to this problem are given by equation (4.23) and the following:

$$0 = \psi_H - \lambda e^{x_H^y} \psi_H - \lambda \xi^y \quad (4.24)$$

$$0 = \psi_L - \lambda e^{x_L^y} \psi_L + \lambda \xi^y \quad (4.25)$$

$$s_H \psi_H - \lambda q e^{n_H^y} \psi_H - \lambda s_L \xi^y = 0 \quad (4.26)$$

$$s_L \psi_L - \lambda q e^{n_L^y} \psi_L + \lambda s_L \xi^y = 0 \quad (4.27)$$

$$0 = \beta \sigma \psi_H - \lambda \beta \sigma \xi^y - \theta \lambda \sigma \psi_H \eta(w_H^y) \quad (4.28)$$

$$0 = \beta \sigma \psi_L + \lambda \beta \sigma \xi^y - \theta \lambda \sigma \psi_L \eta(w_L^y) \quad (4.29)$$

where  $\xi^y$  is the Lagrange multiplier for constraint (4.23) and where the envelope condition (4.19) has been used in equations (4.28) and (4.29).

### 4.3. Computational algorithm

This section provides an algorithm for computing the deterministic steady state of the economy. While the method is fully standard, it will introduce objects and notation that will be needed in the next section.

In order to compute a deterministic steady state it is first important to realize that the shadow value of labor  $q$  in equations (4.8) and (4.20) is known. In particular it is given by<sup>9</sup>

$$q = (1 - \gamma) \left\{ \frac{1}{\gamma} \left[ \frac{1}{\theta} - 1 + \delta \right] \right\}^{\frac{\gamma}{1-\gamma}}.$$

Given this value of  $q$ , the planning problem for old agents in equations (4.8)-(4.10) can be solved using standard methods. I find it convenient to use cubic splines approximations and iterate with the first order conditions to this problem, given by equations (4.11)-(4.18). In order to do this, I first restrict promised values to lie on a closed interval  $[v_{\min}, v_{\max}]$  and define an equidistant vector of grid

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<sup>9</sup>See equations (9.14) and (9.15) in Appendix A.

points  $(v_j)_{j=1}^J$ , with  $v_1 = v_{\min}$  and  $v_J = v_{\max}$ .<sup>10</sup> Given the function  $\eta$  from the previous iteration, which is used to value next period promised values in equations (4.15) and (4.16), the values of  $[x_L^o(v_j), x_H^o(v_j), n_L^o(v_j), n_H^o(v_j), w_L^o(v_j), w_H^o(v_j), \xi^o(v_j), \eta(v_j)]_{j=1}^J$  that satisfy equations (4.11)-(4.18) are then solved at the grid points  $(v_j)_{j=1}^J$ . Once these values are found, the functions are extended to the full domain  $[v_{\min}, v_{\max}]$  using natural Hermite cubic splines. The iterations continue until the values for  $[x_L^o(v_j), x_H^o(v_j), n_L^o(v_j), n_H^o(v_j), w_L^o(v_j), w_H^o(v_j), \xi^o(v_j), \eta(v_j)]_{j=1}^J$  converge. Observe, that this solution does not depend on any other endogenous values, so it forms part of the steady state.

Given the steady state solution for  $\eta$  the planning problem for young agents can be solved next. This problem is static and has a finite number of decision variables. However, it has the complication is that it depends on the shadow price of consumption  $\lambda$ , which is an endogenous variable. Thus, conditional on a value for  $\lambda$ , equations (4.23)-(4.29) can be solved for  $(x_L^y, x_H^y, n_L^y, n_H^y, w_L^y, w_H^y, \xi^y)$ , but later on I will have to provide the condition that  $\lambda$  must satisfy for this to form part of the steady state.

Equation (9.12) describes the recursion that the invariant  $\mu$  has to satisfy. This equation corresponds to the case of a continuum of agents. However, I will find it convenient to work with a large, but finite number of agents, and perform the recursion for this case. In particular, consider a large but finite number of agents  $I$  and endow them with promised values in the interval  $[v_{\min}, v_{\max}]$ . Using the functions  $w_L^o$  and  $w_H^o$  obtained from the planning problem for old agents and the values  $w_L^y$  and  $w_H^y$  obtained from the planning problem for young agents, simulate the evolution of the promised values of these  $I$  agents and their descendants for a large number of periods  $T$ . To be precise, if agent  $i$  was promised a value  $v$  at the beginning of the current period (conditional on being alive), then his promised value (or his descendants, in case the agent dies) at the beginning of the following period will be given by:

$$v' = \begin{cases} w_L^o(v), & \text{with probability } \sigma\psi_L, \\ w_H^o(v), & \text{with probability } \sigma\psi_H, \\ w_L^y, & \text{with probability } (1-\sigma)\psi_L, \\ w_H^y, & \text{with probability } (1-\sigma)\psi_H. \end{cases} \quad (4.30)$$

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<sup>10</sup>When restricting promised values to lie in the interval  $[v_{\min}, v_{\max}]$ , the first order conditions (4.15)-(4.16) and (4.28)-(4.29) change by incorporating inequalities that check for the extreme values.

Simulating the  $I$  agents for  $T$  periods using equation (4.30) we obtain a realized distribution  $(\bar{v}_i)_{i=1}^I$  of promised values (conditional on being alive) across the  $I$  agents. Observe that the last iteration of equation (4.30) also gives the corresponding realized values of leisure  $(\bar{s}_i)_{i=1}^I$  across the  $I$  agents. The joint realized distribution of promised values and values of leisure  $(\bar{v}_i, \bar{s}_i)_{i=1}^I$  can then be used to compute statistics under the invariant distribution. In particular, aggregate consumption can be obtained as:

$$C = \sigma \frac{1}{I} \sum_{i=1}^I e^{x_{s_i}^o(\bar{v}_i)} + (1 - \sigma) \sum_s e^{x_s^y} \psi_s. \quad (4.31)$$

To understand this expression, suppose that we are at the beginning of period  $T + 1$ . The joint realized distribution  $(\bar{v}_i, \bar{s}_i)_{i=1}^I$  now corresponds to agents that were alive in the previous period, and thus a fraction  $\sigma$  of them will have survived and a fraction  $(1 - \sigma)$  of them will have died. The first term in equation (4.31) corresponds to those who have survived. It averages the consumption of these agents and multiplies the result by the probability of surviving  $\sigma$ . The second term corresponds to those who have died and thus have been replaced by young agents. It averages the consumption of young agents and multiplies the result by the probability of dying  $(1 - \sigma)$ .

Aggregate hours worked can be similarly computed as

$$H = \sigma \frac{\sum_{i=1}^I [1 - e^{n_{s_i}^o(\bar{v}_i)}]}{I} + (1 - \sigma) \sum_s (1 - e^{n_s^y}) \psi_s. \quad (4.32)$$

Observe that by a law of large numbers equations (4.31) and (4.32) will become arbitrarily good approximations to equations (9.13) and (9.16) as  $I$  and  $T$  tend to infinity.

Given aggregate hours worked, aggregate capital can be then obtained from the fact that the social planner equates the marginal productivity of capital to its shadow price. In particular, from equation (9.14) we have that aggregate capital is given by

$$K = \left( \frac{\gamma}{\frac{1}{\theta} - 1 + \delta} \right)^{\frac{1}{1-\gamma}} H. \quad (4.33)$$

Then, aggregate investment is

$$I = \delta K. \quad (4.34)$$

The last equation that needs to be satisfied is the feasibility condition for the consumption

good,

$$C + I = K^\gamma H^{1-\gamma}. \tag{4.35}$$

This is the side condition mentioned above for the shadow value of consumption  $\lambda$ . The shadow value of consumption determines the consumption, hours worked and promised values of young agents, and therefore each of the variables in equation (4.35). Therefore, it must be changed until equation (4.35) holds. In practice, this is done using a bisection root finding method.

## 5. Computing business cycle fluctuations

The method to compute the solution to the stochastic economy will be completely standard. It will require linearizing first order conditions around the deterministic steady state and solving the resulting linear rational expectations model. The novelty stems from the set of variables chosen to represent the decision variables and the state of the economy in order to break the “curse of dimensionality”. As we will see, we will end up with a large but finite number of variables. Linearizing the system of equations defined in terms of these variables will require performing a massive number of Monte Carlo simulations. Moreover, solving the resulting linear rational expectations model will require performing linear algebra with matrices of high dimensionality. While these are non-trivial difficulties, the computational turns out to be completely feasible given current technology.

### 5.1. Linearization

In what follows, it will be convenient to switch from a recursive representation of variables, in which variables are written as a time invariant function of the state of the economy, to a dated representation. The convention that I will follow is that a variable dated  $t$  is one that becomes known at date  $t$ .<sup>11</sup> Appendix B provides the first order conditions to the problem defined by equations (3.10)-(3.15) in terms of these dated variables.<sup>12</sup> Since these equations present different types of issues in the linearization, I will classify them into different categories.

The first category is composed of equations that only entail real variables. Equations (10.10)-

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<sup>11</sup>This is the same convention used in Uhlig [13].

<sup>12</sup>See the Technical Appendix for their derivation.

(10.13), (10.16), and (10.19)-(10.21) fall into this category. For example, consider equation (10.10):

$$0 = \psi_H - \lambda_t e^{x_{H,t}^y} \psi_H - \lambda_t \xi_t^y.$$

This equation is a function of  $\{\lambda_t, x_{H,t}^y, \xi_t^y\}$ , which are all real numbers. Linearizing this equation around the deterministic steady state values  $\{\bar{\lambda}, \bar{x}_H^y, \bar{\xi}^y\}$  poses no difficulty.<sup>13</sup>

The second category is composed of a continuum of equations that only entail real variables. Equations (10.1)-(10.4) and (10.7)-(10.9) fall into category. Consider, for example, equation (10.1):

$$0 = -e^{x_{H,t}(v)} \psi_H - \xi_t^o(v) + \eta_t(v) \psi_H.$$

This equation depends on  $\{x_{H,t}^o(v), \xi_t^o(v), \eta_t(v)\}$  which are all real numbers. The problem is that there is one of these equations for every value of  $v$  in the interval  $[v_{\min}, v_{\max}]$ . In this case the ‘‘curse of dimensionality’’ is solved by considering this equation only at the grid points  $(v_j)_{j=1}^J$  that were used in the computation of the deterministic steady state. It is now straightforward to linearize each of these  $J$  equations with respect to  $\{x_{H,t}^o(v_j), \xi_t^o(v_j), \eta_t(v_j)\}$  at their deterministic steady state values  $\{\bar{x}_H^o(v_j), \bar{\xi}^o(v_j), \bar{\eta}(v_j)\}$ . Extending  $\{x_{H,t}^o(v), \xi_t^o(v), \eta_t(v)\}$  to the full domain  $[v_{\min}, v_{\max}]$  using spline approximations will make equation (10.1) hold only approximately outside of the grid points  $(v_j)_{j=1}^J$ . The quality of this approximation will obviously depend on how many grid points  $J$  we work with.

The third category is composed of equations that entail both real variables and functions. Equations (10.14) and (10.15) fall in this category. For example, consider equation (10.14):

$$0 = \beta \sigma \psi_H - \lambda_t \beta \sigma \xi_t^y - \theta \lambda_{t+1} \sigma \psi_H \eta_{t+1} \left( w_{H,t+1}^y \right).$$

This equation depends on  $\lambda_t, \xi_t^y, \lambda_{t+1}, w_{H,t+1}^y$  and on the function  $\eta_{t+1}$ , which is a high dimensional object. In this case the ‘‘curse of dimensionality’’ is broken by considering that  $\eta_{t+1}$  is a spline approximation and, therefore, is completely determined by the finite set of values  $\{\eta_{t+1}(v_j)\}_{j=1}^J$ , i.e. the value of the function at the grid points. The equation can then be linearized with respect to  $\left[ \lambda_t, \xi_t^y, \lambda_{t+1}, w_{H,t+1}^y, \{\eta_{t+1}(v_j)\}_{j=1}^J \right]$  at the deterministic steady state values  $\left[ \bar{\lambda}, \bar{\xi}^y, \bar{\lambda}, \bar{w}_H^y, \{\bar{\eta}(v_j)\}_{j=1}^J \right]$ .

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<sup>13</sup> Although in this case derivatives can be taken analytically, throughout the section derivatives are assumed to be numerically obtained.

The fourth category is a combination of the previous two: it is composed of a continuum of equations that entail both real variables and functions. Equations (10.5) and (10.6) fall in this category. For example, consider equation (10.5),

$$0 = -\lambda_t \beta \sigma \xi_t^o(v) + \lambda_t \eta_t(v) \beta \sigma \psi_H - \theta \lambda_{t+1} \sigma \psi_H \eta_{t+1} [w_{H,t+1}^o(v)].$$

Similarly to the third category, this equation depends on the real numbers  $\lambda_t$ ,  $\xi_t^o(v)$ ,  $\lambda_{t+1}$ ,  $w_{H,t+1}^o(v)$  and on the function  $\eta_{t+1}$ . Similarly to the second category there is one of these equations for every value of  $v$  in the interval  $[v_{\min}, v_{\max}]$ . Given these similarities we can use the same strategies. In particular, we can consider this equation only at the grid points  $(v_j)_{j=1}^J$  and linearize each of these  $J$  equations with respect to  $\left[ \lambda_t, \xi_t^o(v_j), \lambda_{t+1}, w_{H,t+1}^o(v_j), \{\eta_{t+1}(v_k)\}_{k=1}^J \right]$  at the deterministic steady state values  $\left[ \bar{\lambda}, \bar{\xi}^o(v_j), \bar{\lambda}, \bar{w}_H^o(v_j), \{\bar{\eta}(v_k)\}_{k=1}^J \right]$ .

The fifth category is much more involved. It is composed of equations that entail real numbers and integrals of variables with respect to the distribution  $\mu_t$ . Equations (10.18) and (10.22) fall in this category. For example, consider equation (10.22):

$$\begin{aligned} 0 = & (1 - \sigma) e^{x_{L,t}^y} \psi_L + (1 - \sigma) e^{x_{H,t}^y} \psi_H + \int e^{x_{L,t}^o(v)} \psi_L d\mu_t \\ & + \int e^{x_{H,t}^o(v)} \psi_H d\mu_t + I_t - e^{z_t} K_{t-1}^\gamma H_t^{1-\gamma}. \end{aligned}$$

This equation depends on the real numbers  $x_{L,t}^y$ ,  $x_{H,t}^y$ ,  $I_t$ ,  $z_t$ ,  $K_{t-1}$ , and  $H_t$ , and on the integrals  $\int e^{x_{L,t}^o} d\mu_t$  and  $\int e^{x_{H,t}^o} d\mu_t$ . To make progress it will be important to represent these integrals with a convenient finite set of variables. In order to do this, I will follow a strategy that is closely related to the one that was used in Section 4.3 for computing statistics under the invariant distribution. In particular, consider the same large but finite number of agents  $I$  that was used in that section and endow them with the realized distribution of promised values  $(\bar{v}_i)_{i=1}^I$  that was obtained when computing the steady state. Now, assume that these agents populated the economy  $M$  time periods ago and consider the history

$$\left\{ w_{L,t-m}^o, w_{H,t-m}^o, w_{L,t-m}^y, w_{H,t-m}^y \right\}_{m=0}^M,$$

which describes the allocation rules for next-period promised values that were chosen during the last  $M$  periods (where  $t$  is considered to be the current period). Observe that, given that  $w_{L,t-m}^o$  and  $w_{H,t-m}^o$  are spline approximations, this history can be represented by the following finite list



of values:

$$\left\{ [w_{L,t-m}^o(v_j)]_{j=1}^J, [w_{H,t-m}^o(v_j)]_{j=1}^J, w_{L,t-m}^y, w_{H,t-m}^y \right\}_{m=0}^M. \quad (5.1)$$

Using the history of allocation rules for next-period promised values, we can simulate the evolution of promised values for the  $I$  agents and their descendants during the last  $M$  time periods to update the distribution of promised values from the initial  $(\bar{v}_i)_{i=1}^I$  to a current distribution  $(v_{i,t})_{i=1}^I$ .

In particular, we can initialize the distribution of promised values at the beginning of period  $t - M - 1$  as follows:

$$v_{i,t-M-1} = \bar{v}_i,$$

for  $i = 1, \dots, I$ . Given a distribution of promised values at the beginning of period  $t - m - 1$ , the distribution of promised values at period  $t - m$  is then obtained as follows:

$$v_{i,t-m} = \begin{cases} w_{L,t-m}^o(v_{i,t-m-1}), & \text{with probability } \sigma\psi_L, \\ w_{H,t-m}^o(v_{i,t-m-1}), & \text{with probability } \sigma\psi_H, \\ w_{L,t-m}^y, & \text{with probability } (1-\sigma)\psi_L, \\ w_{H,t-m}^y, & \text{with probability } (1-\sigma)\psi_H, \end{cases} \quad (5.2)$$

for  $i = 1, \dots, I$ . Proceeding recursively for  $m = M, M - 1, \dots, 0$ , we obtain a realized distribution of promised values  $(v_{i,t})_{i=1}^I$  at the beginning of period  $t$ .

Observe that the last iteration of equation (5.2) also gives the corresponding realized values of leisure  $(s_{it})_{i=1}^I$  across the  $I$  agents. The joint realized distribution of promised values and values of leisure  $(v_{it}, s_{it})_{i=1}^I$  can then be used to compute statistics under the distribution  $\mu_t$ . In particular, equation (10.22) can be re-written as:

$$0 = (1 - \sigma) \left[ e^{x_{L,t}^y \psi_L} + e^{x_{H,t}^y \psi_H} \right] + \sigma \frac{1}{I} \sum_{i=1}^I e^{x_{s_{it}}^o(v_{it})} + I_t - e^{z_t} K_{t-1}^\gamma H_t^{1-\gamma}. \quad (5.3)$$

Since  $x_{L,t}^o$  and  $x_{H,t}^o$  are splines approximations, they can be summarized by their values at the grid points  $(v_j)_{j=1}^J$ . Therefore, equation (5.3) can be linearized with respect to

$$\begin{aligned} & I_t, z_t, K_{t-1}, H_t, x_{L,t}^y, x_{H,t}^y, [x_{L,t}^o(v_j)]_{j=1}^J, [x_{H,t}^o(v_j)]_{j=1}^J, \\ & \left\{ [w_{L,t-m}^o(v_j)]_{j=1}^J, [w_{H,t-m}^o(v_j)]_{j=1}^J, w_{L,t-m}^y, w_{H,t-m}^y \right\}_{m=0}^M \end{aligned} \quad (5.4)$$

at their steady state values

$$\begin{aligned} & \bar{I}, 0, \bar{K}, \bar{H}, \bar{x}_L^y, \bar{x}_H^y, [\bar{x}_L^o(v_j)]_{j=1}^J, [\bar{x}_H^o(v_j)]_{j=1}^J, \\ & \left\{ [\bar{w}_L^o(v_j)]_{j=1}^J, [\bar{w}_H^o(v_j)]_{j=1}^J, \bar{w}_L^y, \bar{w}_H^y \right\}_{m=0}^M. \end{aligned}$$

Observe that equation (5.4) provides a large but finite list of variables. In particular one directly verifies that there are  $M(2J + 2)$  variables in the second line of equation (5.4). Taking numerical derivatives with respect to each of these variables requires simulating  $I$  agents over  $M$  periods. As a consequence, linearizing equation (5.3) requires performing a massive number of Monte-Carlo simulations. While this seems a daunting task it is easily parallelizable. Thus, using massively parallel computer systems can play an important role in reducing computing times and keeping the task manageable.

The last category of equations has only one element: equation (10.17), which describes the law of motion for the distribution  $\mu_t$ . While frightening at first sight, this equation is greatly simplified by our approach of representing the distribution  $\mu_t$  using the history of values given by equation (5.1). In fact, updating the distribution  $\mu_t$  is merely reduced to updating this history. In particular, date- $(t + 1)$  histories can be obtained from date- $t$  histories using the following equations

$$\left[ w_{L,(t+1)-m}^o(v_j) \right]_{j=1}^J = \left[ w_{L,t-(m-1)}^o(v_j) \right]_{j=1}^J \quad (5.5)$$

$$\left[ w_{H,(t+1)-m}^o(v_j) \right]_{j=1}^J = \left[ w_{H,t-(m-1)}^o(v_j) \right]_{j=1}^J \quad (5.6)$$

$$w_{L,(t+1)-m}^y = w_{L,t-(m-1)}^y \quad (5.7)$$

$$w_{H,(t+1)-m}^y = w_{H,t-(m-1)}^y \quad (5.8)$$

for  $m = 1, \dots, M$ . Observe that the values for  $\left[ w_{L,(t+1)-0}^o(v_j) \right]_{j=1}^J$ ,  $\left[ w_{H,(t+1)-0}^o(v_j) \right]_{j=1}^J$ ,  $w_{L,(t+1)-0}^y$ ,  $w_{H,(t+1)-0}^y$  cannot be obtained from the date- $t$  history because they are jump variables that will become known at date  $t + 1$ . Also observe that the law of motion described by equations (5.5)-(5.8) is already linear, so no further linearization is needed. Finally, observe that the variables that are  $M$  periods old in the date- $t$  history become dropped from the date- $(t + 1)$  history. Thus, the law of motion described by equations (5.5)-(5.8) introduces a truncation. However, the consequences of this truncation are expected to be negligible. The reason is that the truncation only affects the agents that had survived for  $M$  consecutive periods, and given a sufficiently small survival probability  $\sigma$  and/or a sufficiently large  $M$  there will be very few of these agents.

## 5.2. Linearized system

Define the vector of endogenous state variables as follows:

$$x_{t-1} = \left( \Delta \ln K_{t-1}, \left\{ \Delta w_{L,t-m}^y, \Delta w_{H,t-m}^y, [\Delta w_{L,t-m}^o(\bar{v}_j)]_{j=1}^J, [\Delta w_{H,t-m}^o(\bar{v}_j)]_{j=1}^J \right\}_{m=0}^M \right),$$

and the vector of decision variables and Lagrange multipliers as follows:

$$\begin{aligned} y_t = & \left( \Delta w_{L,t+1}^y, \Delta w_{H,t+1}^y, [\Delta w_{L,t+1}^o(\bar{v}_j)]_{j=1}^J, [\Delta w_{H,t+1}^o(\bar{v}_j)]_{j=1}^J \right. \\ & \Delta x_{L,t}^y, \Delta x_{H,t}^y, \Delta n_{L,t}^y, \Delta n_{H,t}^y, \Delta \ln \xi_t^y, \Delta \ln \lambda_t, \Delta \ln q_t, \\ & [\Delta \ln \eta_t(\bar{v}_j)]_{j=1}^J, [\Delta x_{L,t}^o(\bar{v}_j)]_{j=1}^J, [\Delta x_{H,t}^o(\bar{v}_j)]_{j=1}^J, [\Delta n_{L,t}^o(\bar{v}_j)]_{j=1}^J, \\ & \left. [\Delta n_{H,t}^o(\bar{v}_j)]_{j=1}^J, [\Delta \ln \xi_t^o(\bar{v}_j)]_{j=1}^J, \Delta \ln H_t, \Delta \ln I_t \right), \end{aligned}$$

where  $\Delta x_t$  denotes the difference between the variable  $x_t$  and its steady state value  $\bar{x}$ .

Then, using the approach described in the previous section, the linear approximation to equations (10.1)-(10.22) can be written as follows:

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t, \quad (5.9)$$

$$0 = Gx_t + Jy_{t+1} + Ky_t + Lz_{t+1}, \quad (5.10)$$

$$z_{t+1} = Nz_t. \quad (5.11)$$

where I have applied the certainty equivalence principle.

We seek a linear solution to equations (5.9)-(5.11) of the following form:

$$x_t = Px_{t-1} + Qz_t \quad (5.12)$$

$$y_t = Rx_{t-1} + Sz_t \quad (5.13)$$

Equations (5.9)-(5.13) have exactly the same structure as in Uhlig [13], so his methods can be directly applied. Alternatively, one could iterate with equations (5.12)-(5.13) as follows. Suppose that at iteration  $j$  we have that

$$x_t = P^j x_{t-1} + Q^j z_t, \quad (5.14)$$

$$y_t = R^j x_{t-1} + S^j z_t. \quad (5.15)$$

and that we want to find

$$x_t = P^{j+1}x_{t-1} + Q^{j+1}z_t, \quad (5.16)$$

$$y_t = R^{j+1}x_{t-1} + S^{j+1}z_t, \quad (5.17)$$

for iteration  $j + 1$ . Substituting equations (5.14)-(5.17) into equations (5.9)-(5.10), it is easy to show that  $P^{j+1}$ ,  $Q^{j+1}$ ,  $R^{j+1}$ , and  $S^{j+1}$  are the solution to the following system of linear equations:

$$\begin{bmatrix} A & C \\ (JR^j + G) & K \end{bmatrix} \begin{bmatrix} P^{j+1} & Q^{j+1} \\ R^{j+1} & S^{j+1} \end{bmatrix} = - \begin{bmatrix} B & D \\ 0 & (JS^j + L)N \end{bmatrix}, \quad (5.18)$$

which can be solved using a LU decomposition. Iterating with equation (5.18) until convergence is an alternative way of obtaining the solution  $P$ ,  $Q$ ,  $R$  and  $S$  that we seek.

The important thing, is that whatever method one chooses to solve it, the linear rational expectations model given by equations (5.9)-(5.13) is completely standard. The only difficulty is its high dimensionality. Once equations (5.12)-(5.13) are obtained, they can be used to simulate the economy.

## 6. New Public Finance

This section explores the implications of the model for the distortions emphasized by the New Public Finance literature (e.g. Golosov et al. [5]) since, in principle, it would be interesting to characterize their optimal cyclical behavior. I first discuss the “intertemporal wedges” present in the model and then I turn to the “consumption-labor wedges”.

### 6.1. Intertemporal wedges

From equations (10.1), (10.2), (10.5), (10.6), (10.19) and the fact that  $u(c) = \ln c$  we get that

$$\frac{1}{u'(c_{s,t}^o(v))} = \frac{1}{\beta E_t \left\{ \left( e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right) \left( \sum_{s'} \left[ u'(c_{s',t+1}^o [w_{s,t+1}^o(v)]) \right]^{-1} \psi_{s'} \right)^{-1} \right\}}, \quad (6.1)$$

for  $s = s_L, s_H$ , a condition that is known in the literature as the “inverse Euler equation”.

Observe that in the case that  $s_L = s_H$  we have that:

$$u'(c_t^o(v)) = \beta E_t \left\{ \left( e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right) u'(c_{t+1}^o [w_{t+1}^o(v)]) \right\},$$

so the regular Euler equation holds. Otherwise, applying Jensen's inequality to equations (6.1) we get that:

$$u'(c_{s,t}^o(v)) < \beta E_t \left\{ \left( e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right) \sum_{s'} u'(c_{s',t+1}^o[w_{s,t+1}^o(v)]) \psi_{s'} \right\},$$

and the regular Euler equation does not hold

Following the literature, the intertemporal wedge (distortion)  $\tau_{s,t}^o(v)$  can then be defined as:

$$1 - \tau_{s,t}^o(v) = \frac{u'(c_{s,t}^o(v))}{\beta E_t \left\{ \left( e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right) \sum_{s'} u'(c_{s',t+1}^o[w_{s,t+1}^o(v)]) \psi_{s'} \right\}} < 1,$$

a distortion that is sometimes interpreted as a tax on capital.

Intertemporal wedges for young agents can be similarly defined. They are given by:

$$1 - \tau_{s,t}^y = \frac{u'(c_{s,t}^y)}{\beta E_t \left\{ \left( e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right) \sum_{s'} u'(c_{s',t+1}^o[w_{s,t+1}^y]) \psi_{s'} \right\}} < 1,$$

for  $s = s_L, s_H$ .

## 6.2. Consumption-labor wedges

From equations (10.1), (10.3) and (10.20) we have that

$$e^{z_t} K_{t-1}^{\gamma} (1 - \gamma) H_t^{-\gamma} e^{n_{H,t}^o(v)} = \frac{(s_H - s_L) \xi_t^o(v)}{\psi_H} + s_H e^{x_{H,t}^o(v)} > s_H e^{x_{H,t}^o(v)}$$

and, therefore, that

$$1 - \kappa_{H,t}^o(v) = \frac{s_H e^{x_{H,t}^o(v)}}{e^{z_t} K_{t-1}^{\gamma} (1 - \gamma) H_t^{-\gamma} e^{n_{H,t}^o(v)}} < 1. \quad (6.2)$$

We see from equation (6.2) that the full information efficiency condition that agents equate the marginal product of labor to the marginal rate of substitution between consumption and leisure does not hold in this case. The consumption-labor wedge  $\kappa_{H,t}^o(v)$  can then be defined as one minus the ratio in equation (6.2). This consumption-labor wedge is positive and often interpreted as a labor tax.

On the contrary, from equations (10.2), (10.4) and (10.20) we have that

$$1 - \kappa_{L,t}^o(v) = \frac{s_L e^{x_{L,t}^o(v)}}{e^{z_t} K_{t-1}^\gamma (1 - \gamma) H_t^{-\gamma} e^{n_{L,t}^o(v)}} = 1. \quad (6.3)$$

That is, when old agents have a low value of leisure, they equate the marginal product of labor to the marginal rate of substitution between consumption and leisure. Thus the consumption-labor wedge  $\kappa_{L,t}^o(v)$  is equal to zero in this case and labor supply is undistorted.

Similar derivations for young agents give that

$$1 - \kappa_{H,t}^y = \frac{s_H e^{x_{H,t}^y}}{e^{z_t} K_{t-1}^\gamma (1 - \gamma) H_t^{-\gamma} e^{n_{H,t}^y}} < 1, \quad (6.4)$$

$$1 - \kappa_{L,t}^y = \frac{s_L e^{x_{L,t}^y}}{e^{z_t} K_{t-1}^\gamma (1 - \gamma) H_t^{-\gamma} e^{n_{L,t}^y}} = 1. \quad (6.5)$$

That is, the labor supply of young agents gets distorted in the high leisure valuation state but not in the low valuation state.

## 7. Calibration

Except for the private information, the basic structure of the model corresponds to a standard real business cycle model. In fact, with the log-log preferences of equation (2.1) the basic structure of the model is identical to the one in Cooley and Prescott [2]. For this reason, I will treat their model as a benchmark and calibrate all parameters associated with the neoclassical growth model to the same observations as theirs. In order to simplify computations, the model time period is selected to be one year.

Following Cooley and Prescott [2] the labor share parameter  $1 - \gamma$  is set to 0.60, the depreciation rate  $\delta$  is chosen to reproduce an investment-capital ratio  $I/K$  equal to 0.076, and the social discount factor  $\theta$  is chosen to reproduce a capital-output ratio  $K/Y$  equal to 3.32. The values of leisure  $s_L$  and  $s_H$  are chosen to satisfy two criteria: that aggregate hours worked  $H$  equal to 0.31 (another observation from Cooley and Prescott [2]) and that the hours worked by old agents with the high valuation of leisure and the highest possible promised value  $n_H^o(v_{\max})$  be a small but positive number. The rationale for this second criterion is that I want to maximize the relevance of the information frictions while keeping an internal solution for hours worked. The probability of drawing a high value of leisure  $\psi_H$  is chosen to maximize the standard deviation of the invariant distribution

promised values. It turns out that a value of  $\psi_H = 0.50$  achieves this. The survival probability  $\sigma$  is chosen to generate an expected lifespan of 40 years. In turn, the individual discount factor  $\beta$  is chosen to be the same as the social discount factor  $\theta$ . In terms of the parameters for the aggregate productivity stochastic process,  $\rho$  is chosen to be 0.95 (since Cooley and Prescott report that aggregate productivity is close to a random walk) and the variance of the innovations to aggregate productivity  $\sigma_\varepsilon^2$  is chosen to be  $4 \times 0.007^2$  (another estimate from Cooley and Prescott [2]).

While the above parameters are structural, there are a number of computational parameters to be determined. The number of grid points in the spline approximations  $J$ , the total number of agents simulated  $I$ , the length of the simulations for computing the invariant distribution  $T$ , and the length of the histories kept as state variables when computing the business cycles  $M$  are all chosen to be as large as possible, while keeping the computational task manageable and results being robust to non-trivial changes in their values. The lower and upper bounds for the range of possible promised values  $v_{\min}$  and  $v_{\max}$  in turn were chosen so that the fraction of agents in the intervals  $[v_1, v_2]$  and  $[v_{J-1}, v_J]$  are each less than 0.1%. Thus, truncating the range of possible values at  $v_{\min}$  and  $v_{\max}$  should not play an important role in the results.

Table 1 and 2 describe all parameter values. It turns out that under the computational parameters specified in the second table the dimensionality of the linear system described by equations (5.9)-(5.10) is about  $12,000 \times 12,000$ , a large system indeed.

## 8. Results

Before turning to the business cycle dynamics, I will describe different features of the model at its deterministic steady state.

Figure 1 shows the invariant distribution of promised values across the  $J-1$  intervals  $[v_j, v_{j+1}]_{j=1}^{J-1}$  defined by the grid points of the spline approximations. While it is hard to see at this coarseness level, the distribution is approximately symmetrical. More importantly, we see that the invariant distribution puts very little mass at extreme values. As a consequence, in what follows I will report allocation rules only between the 7th and 15th ranges of the histogram. The reason is not only that there are very few agents at the tails of the distribution for them to matter, but being close to the artificial bounds  $v_{\min}$  and  $v_{\max}$  greatly distorts the shape of the allocation rules.

Figure 2 reports the log of consumption for old agents  $x_L^o(v)$  and  $x_H^o(v)$  across promised values  $v$ , as well as those of young agents  $x_L^y$  and  $x_H^y$  (which are independent of  $v$ ). We see that, in all cases

consumption is higher when the value of leisure is low. Both  $x_L^o$  and  $x_H^o$  are strictly increasing in the promised value  $v$ , are linear and parallel to each other. Moreover, the vertical difference between  $x_L^o$  and  $x_H^o$  is the same as between  $x_L^y$  and  $x_H^y$ . Figure 3 reports the log of leisure for old agents  $n_L^o(v)$  and  $n_H^o(v)$  across promised values  $v$ , as well as those of young agent  $n_L^y$  and  $n_H^y$ . We see that in all cases leisure is lower when the value of leisure is low. Both  $n_L^o$  and  $n_H^o$  are strictly increasing in the promised value  $v$ , are linear and parallel to each other. Moreover, the vertical difference between  $n_L^o$  and  $n_H^o$  is the same as between  $n_L^y$  and  $n_H^y$ . In turn, Figure 4 reports the next-period promised values for old agents  $w_L^o(v)$  and  $w_H^o(v)$  across promised values  $v$ , as well as those of young agent  $w_L^y$  and  $w_H^y$ . We see that in all cases next-period promised values are higher when the value of leisure is low. Both  $w_L^o$  and  $w_H^o$  are strictly increasing in the promised value  $v$ , are linear and parallel to each other. Moreover, the vertical difference between  $w_L^o$  and  $w_H^o$  is the same as between  $w_L^y$  and  $w_H^y$ .

Taken together Figures 2-4 given a clear picture of how the social planner deals with the trade-off between insurance and incentives. When an agent reports a low value of leisure he is put to work more hours, which is the efficient thing to do under perfect information. However, in order to induce truth-telling these agents must be compensated with a mix of higher consumption and higher next-period promised value, thus full insurance is not achieved. In fact, when agents get a stream of low values of leisure they get pushed upwards in the distribution of promised values, with persistent effects on their consumption and leisure levels. This creates persistent heterogeneity across individuals.

Actually, in Figure 4 we see that the 45 degrees line passes right between  $w_L^o$  and  $w_H^o$ , and that the three lines are parallel. This gives rise to a random walk behavior with equal upwards and downwards increments as the low and high values of leisure are realized. This leads to a well known result in Atkeson and Lucas [1]: That inequality grows over time without bound. The difference with Atkeson and Lucas [1] is that in this paper the immiserizing result is obtained only within a same cohort of individuals. However, as these individuals die, new agents are born with next-period promised values at the center of the distribution which produces a strong reversion to the mean. In fact, the existence of an invariant distribution of promised values when agents have a constant survival probability has been previously shown by Phelan [9].<sup>14</sup>

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<sup>14</sup> Another reason why this paper has a well defined invariant distribution is the presence of lower and upper bounds for the range of possible promised values.



Figure 5 shows the intertemporal wedges  $\tau_L^o(v)$  and  $\tau_H^o(v)$  across promised values  $v$ . We see that these wedges are roughly constant. The constancy of the intertemporal wedges weakens quite substantially as we get closer to the extremes but this is a consequence of the numerical approximation. What is important to realize in Figure 5 is the scale of the vertical axis. These intertemporal wedges are absolutely negligible. Figure 6 in turn shows the consumption-labor wedge  $\kappa_{H,t}^o(v)$  (recall that  $\kappa_{L,t}^o$  was shown to be identical to zero). We see that this wedge is also roughly constant and, while not being negligible, it is definitely small: only 0.145%. Both Figures 5 and 6 show that, at least by this metric, the economy is close to being full information efficient.

The discussion of business cycle dynamics that follows will be centered around the analysis of the impulse responses of different variables to a one standard deviation increase in aggregate productivity.

Figure 7 shows the impulse responses of the (log of) consumption levels of young agents  $x_L^y$  and  $x_H^y$ . We see that both impulse responses coincide, which means that the amount of consumption heterogeneity of young agents across  $s_L$  and  $s_H$  is constant over time. Moreover, we see that the shape of both impulse responses qualitatively resembles the one for aggregate consumption in a standard RBC model.

Figure 8 shows the impulse responses of the (log of) consumption of old agents with a low value of leisure  $x_L^o(v)$ , at each of the eleven grid points  $(v_j)_{j=6}^{16}$ . While the figure shows eleven impulse responses, only one of them is actually seen because they happen to overlap perfectly. This means that, in response to the aggregate productivity shock, the function  $x_L^o(v)$  depicted in Figure 2 shifts vertically over time. Figure 9, which does the same for  $x_H^o(v)$ , is identical to Figure 8. This means that  $x_H^o(v)$  also shifts vertically over time and that its increments are the same as those of  $x_L^o(v)$ . Since both  $x_L^o(v)$  and  $x_H^o(v)$  are linear, parallel to each other and are shifting vertically over time with identical increments, the variance of consumption levels across old agents will remain the same if the variance of promised values does. This will be true if the distribution of promised values shifts horizontally in response to aggregate productivity.

Figures 10-12 are analogous to Figures 7-9, except that they depict the behavior of the (log of) leisure. Figure 10 shows that the amount of leisure heterogeneity of young agents across  $s_L$  and  $s_H$  is constant over time, and that the impulse response of leisure for these individuals is qualitatively similar to that of aggregate leisure in a standard RBC model. Figures 11 and 12 indicate identical vertical shifts over time in the functions  $n_L^o(v)$  and  $n_H^o(v)$ . Since both  $n_L^o(v)$  and  $n_H^o(v)$  are linear, parallel to each other and are shifting vertically over time with identical increments, the variance

of leisure across old agents will remain constant if the variance of promised values does. Again, this result will be true if the distribution of promised values happens to shift horizontally in response to aggregate productivity.

Turning to promised values, Figure 13 depicts the behavior of  $w_L^y$  and  $w_H^y$ . We see that both impulse responses coincide, which indicates a constant amount of heterogeneity across  $s_L$  and  $s_H$  for young agents promised values. Figure 14 and 15 report the impulse response functions for  $w_L^o$  and  $w_H^o$ , respectively, across the eleven grid points  $(v_j)_{j=6}^{16}$ . The eleven impulse responses coincide in each of the figures, indicating vertical shifts in the  $w_L^o$  and  $w_H^o$  functions of Figure 4. Moreover, since Figures 14 and 15 are identical, the vertical shifts in  $w_L^o$  and  $w_H^o$  are the same. Comparing the impact responses of  $w_L^y$  and  $w_H^y$  in Figure 13 and of  $w_L^o$  and  $w_H^o$  in Figures 14 and 15, respectively, we see that all promised values shift horizontally to the right by the same amount. After this initial horizontal shift to the right in the distribution of promised values,  $w_L^o$  and  $w_H^o$  shift back but not all the way to their initial levels. In Figure 4 it is easy to see that as long as  $w_L^o$  and  $w_H^o$  remain above their initial levels, that their displacements relative to the 45 degrees line will generate an upward drift in promised values. This upward drift continues to shift the distribution of promised values to the right, mimicking the increments in  $w_L^y$  and  $w_H^y$  observed in Figure 13 after the initial period. Observe that when  $w_L^y$  and  $w_H^y$  start to retreat in Figure 13 is exactly when  $w_L^o$  and  $w_H^o$  drop below their initial values, now generating a negative drift in promised values. In fact all what this is indicating is that the distribution of promised values shifts horizontally on impact and continues to drift to the right for a while before reverting the sign of its drift and starting to shift back towards its initial position. A key implication is that shape of the distribution of promised values remains constant over time. From the above discussion, this means that the variance of log consumption and the variance of log leisure will remain unchanged.

Since the amount of consumption and leisure heterogeneity stays the same, one suspects that an aggregation result may be in place. To verify that this is the case I consider a version of the model with full information and solve its social planning problem, which is given by:

$$V(z, K) = \max \left\{ x + \sum_s s n_s \psi_s + \beta E_z [V(z', K')] \right\}$$

subject to:

$$e^x + K' - (1 - \delta) K \leq e^z K^\gamma H^{1-\gamma},$$

$$H \leq \sum_s [1 - e^{n_s}] \psi_s.$$

Calibrating this full information economy to the same observations as the benchmark economy requires exactly the same set of parameter values. Figure 16 shows the impulse responses for aggregate output, consumption, investment, hours and capital that correspond to this parametrization. Figure 17 does the same for the benchmark economy. Comparing Figures 16 and 17 we find that they are identical. Thus we verify that the information frictions that plague the benchmark economy have no implications whatsoever for aggregate business cycle dynamics.

## 9. Appendix A: Deterministic steady state conditions

Taking first order conditions to the problem defined by equations (3.10)-(3.15), assuming that the aggregate productivity level  $z$  is identical to zero, and imposing the condition that allocations are constant over time, we get the following steady state conditions:

$$0 = -e^{x_s^o(v)} \psi_s + \xi_{s,\hat{s}}^o(v) - \xi_{\hat{s},s}^o(v) + \eta(v) \psi_s \quad (9.1)$$

$$0 = -qe^{n_s^o(v)} \psi_s + s\xi_{s,\hat{s}}^o(v) - \hat{s}\xi_{\hat{s},s}^o(v) + \eta(v) s\psi_s \quad (9.2)$$

$$0 = \beta\sigma\xi_{s,\hat{s}}^o(v) - \beta\sigma\xi_{\hat{s},s}^o(v) + \eta(v) \beta\sigma\psi_s - \theta\sigma\psi_s\eta[w_s^o(v)] \quad (9.3)$$

$$x_s^o(v) + sn_s^o(v) + \beta\sigma w_s^o(v) \geq x_{\hat{s}}^o(v) + sn_{\hat{s}}^o(v) + \beta\sigma w_{\hat{s}}^o(v) \quad (9.4)$$

$$0 = \xi_{s,\hat{s}}^o(v) \{x_s^o(v) + sn_s^o(v) + \beta\sigma w_s^o(v) - [x_{\hat{s}}^o(v) + sn_{\hat{s}}^o(v) + \beta\sigma w_{\hat{s}}^o(v)]\} \quad (9.5)$$

$$v = \sum_s [x_s^o(v) + sn_s^o(v) + \beta\sigma w_s^o(v)] \psi_s \quad (9.6)$$

$$\psi_s - \lambda e^{x_s^y} \psi_s + \lambda \xi_{s,\hat{s}}^y - \lambda \xi_{\hat{s},s}^y = 0 \quad (9.7)$$

$$s\psi_s - \lambda q e^{n_s^y} \psi_s + \lambda s \xi_{s,\hat{s}}^y - \lambda \hat{s} \xi_{\hat{s},s}^y = 0 \quad (9.8)$$

$$0 = \beta\sigma\psi_s + \lambda\beta\sigma\xi_{s,\hat{s}}^y - \lambda\beta\sigma\xi_{\hat{s},s}^y - \theta\lambda\sigma\psi_s\eta(w_s^y) \quad (9.9)$$

$$x_s^y + sn_s^y + \beta\sigma w_s^y \geq x_{\hat{s}}^y + sn_{\hat{s}}^y + \beta\sigma w_{\hat{s}}^y \quad (9.10)$$

$$0 = \xi_{s,\hat{s}}^y \{x_s^y + sn_s^y + \beta\sigma w_s^y - [x_{\hat{s}}^y + sn_{\hat{s}}^y + \beta\sigma w_{\hat{s}}^y]\} \quad (9.11)$$

$$\mu(B) = \sigma \sum_s \int_{\{(v,s): w_s^o(v) \in B\}} \psi_s d\mu + (1 - \sigma) \sigma \sum_{s: w_s^y \in B} \psi_s \quad (9.12)$$

$$H = (1 - \sigma) \sum_s \left(1 - e^{n_s^y}\right) \psi_s + \int \sum_s \left[1 - e^{n_s^o(v)}\right] \psi_s d\mu \quad (9.13)$$

$$\gamma K^{\gamma-1} H^{1-\gamma} = \frac{1}{\theta} - 1 + \delta \quad (9.14)$$

$$q = K^\gamma (1 - \gamma) H^{-\gamma} \quad (9.15)$$

$$C = (1 - \sigma) \sum_s e^{x_s^y} \psi_s + \int \sum_s e^{x_s^o(v)} \psi_s d\mu \quad (9.16)$$

$$I = \delta K, \quad (9.17)$$

$$C + I = K^\gamma H^{1-\gamma} \quad (9.18)$$

where  $\lambda$ ,  $q$ ,  $\xi_{s,\hat{s}}^y$ ,  $\xi_{s,\hat{s}}^o(v)$  and  $\eta(v)$  are the Lagrange multipliers to constraints (3.11), (3.12), (3.13), (3.14) and (3.15), respectively.<sup>15</sup>

## 10. Appendix B: Optimality conditions for stochastic economy

Taking first order conditions to the problem defined by equations (3.10)-(3.15), and using the fact that equations (3.13) and (3.14) are reduced to equations (4.23) and (4.9), respectively, we get the following equations:

$$0 = -e^{x_{H,t}^o(v)} \psi_H - \xi_t^o(v) + \eta_t(v) \psi_H \quad (10.1)$$

$$0 = -e^{x_{L,t}^o(v)} \psi_L + \xi_t^o(v) + \eta_t(v) \psi_L \quad (10.2)$$

$$0 = -q_t e^{n_{H,t}^o(v)} \psi_H - s_L \xi_t^o(v) + \eta_t(v) s_H \psi_H \quad (10.3)$$

$$0 = -q_t e^{n_{L,t}^o(v)} \psi_L + s_L \xi_t^o(v) + \eta_t(v) s_L \psi_L \quad (10.4)$$

$$0 = -\lambda_t \beta \sigma \xi_t^o(v) + \lambda_t \eta_t(v) \beta \sigma \psi_H - \theta \lambda_{t+1} \sigma \psi_H \eta_{t+1} [w_{H,t+1}^o(v)] \quad (10.5)$$

$$0 = \lambda_t \beta \sigma \xi_t^o(v) + \lambda_t \eta_t(v) \beta \sigma \psi_L - \theta \lambda_{t+1} \sigma \psi_L \eta_{t+1} [w_{L,t+1}^o(v)] \quad (10.6)$$

$$0 = x_{L,t}^o(v) + s_L n_{L,t}^o(v) + \beta \sigma E_t [w_{L,t+1}^o(v)] - \{x_{H,t}^o(v) + s_L n_{H,t}^o(v) + \beta \sigma E_t [w_{H,t+1}^o(v)]\} \quad (10.7)$$

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<sup>15</sup>For a derivation of these steady state conditions, see the Technical Appendix.

$$0 = v - \{x_{L,t}^o(v) + s_L n_{L,t}^o(v) + \beta \sigma E_t [w_{L,t+1}^o(v)]\} \psi_L \quad (10.8)$$

$$- \{x_{H,t}^o(v) + s_H n_{H,t}^o(v) + \beta \sigma E_t [w_{H,t+1}^o(v)]\} \psi_H \quad (10.9)$$

$$0 = \psi_H - \lambda_t e^{x_{H,t}^y} \psi_H - \lambda_t \xi_t^y \quad (10.10)$$

$$0 = \psi_L - \lambda_t e^{x_{L,t}^y} \psi_L + \lambda_t \xi_t^y \quad (10.11)$$

$$0 = s_H \psi_H - \lambda_t q_t e^{n_{H,t}^y} \psi_H - \lambda_t s_L \xi_t^y \quad (10.12)$$

$$0 = s_L \psi_L - \lambda_t q_t e^{n_{L,t}^y} \psi_L + \lambda_t s_L \xi_t^y \quad (10.13)$$

$$0 = \beta \sigma \psi_H - \lambda_t \beta \sigma \xi_t^y - \theta \lambda_{t+1} \sigma \psi_H \eta_{t+1} \left( w_{H,t+1}^y \right) \quad (10.14)$$

$$0 = \beta \sigma \psi_L + \lambda_t \beta \sigma \xi_t^y - \theta \lambda_{t+1} \sigma \psi_L \eta_{t+1} \left( w_{L,t+1}^y \right) \quad (10.15)$$

$$0 = x_{L,t}^y + s_L n_{L,t}^y + \beta \sigma E_t [w_{L,t+1}^y] - \left\{ x_{H,t}^y + s_L n_{H,t}^y + \beta \sigma E_t [w_{H,t+1}^y] \right\} \quad (10.16)$$

$$0 = \sigma \sum_s \int_{\{(v,s): w_{s,t+1}^o(v) \in B\}} \psi_s d\mu_t + (1 - \sigma) \sigma \sum_{s: w_{s,t+1}^y \in B} \psi_s - \mu_{t+1}(B) \quad (10.17)$$

$$0 = (1 - \sigma) \left[ 1 - e^{n_{L,t}^y} \right] \psi_L + (1 - \sigma) \left[ 1 - e^{n_{H,t}^y} \right] \psi_H \\ + \int \left[ 1 - e^{n_{L,t}^o(v)} \right] \psi_L d\mu_t + \int \left[ 1 - e^{n_{H,t}^o(v)} \right] \psi_H d\mu_t - H_t \quad (10.18)$$

$$0 = -\lambda_t + \theta E_t \left\{ \lambda_{t+1} \left[ e^{z_{t+1}} \gamma K_t^{\gamma-1} H_{t+1}^{1-\gamma} + 1 - \delta \right] \right\} \quad (10.19)$$

$$0 = q_t - e^{z_t} K_{t-1}^\gamma (1 - \gamma) H_t^{-\gamma} \quad (10.20)$$

$$0 = K_t - (1 - \delta) K_{t-1} - I_t \quad (10.21)$$

$$0 = (1 - \sigma) e^{x_{L,t}^y} \psi_L + (1 - \sigma) e^{x_{H,t}^y} \psi_H + \int e^{x_{L,t}^o(v)} \psi_L d\mu_t \\ + \int e^{x_{H,t}^o(v)} \psi_H d\mu_t + I_t - e^{z_t} K_{t-1}^\gamma H_t^{1-\gamma} \quad (10.22)$$

For ease of comparison, these equations follow the same ordering as those in Appendix A.

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Figure 2: Log of consumption levels

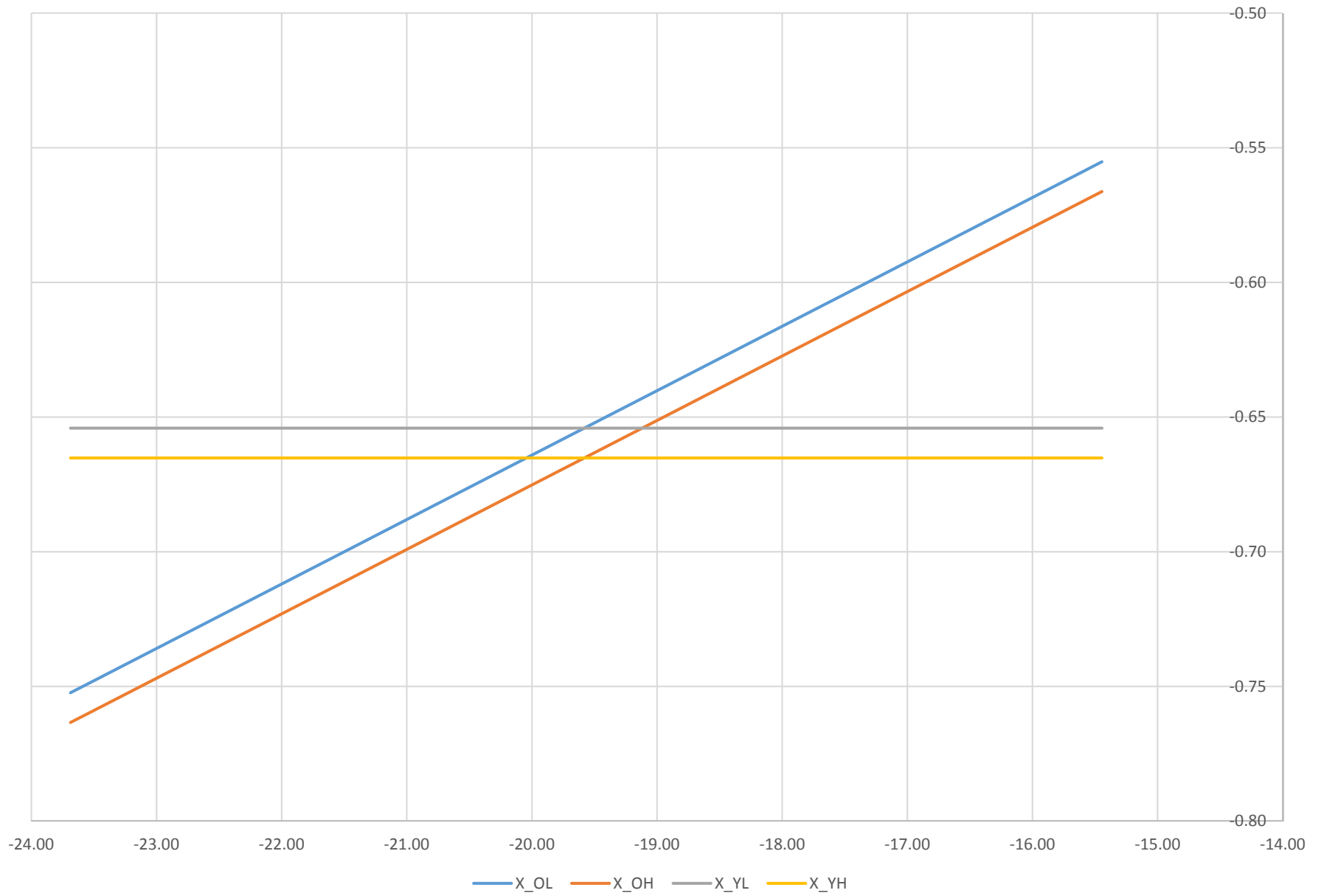


Figure 3: Log of leisure

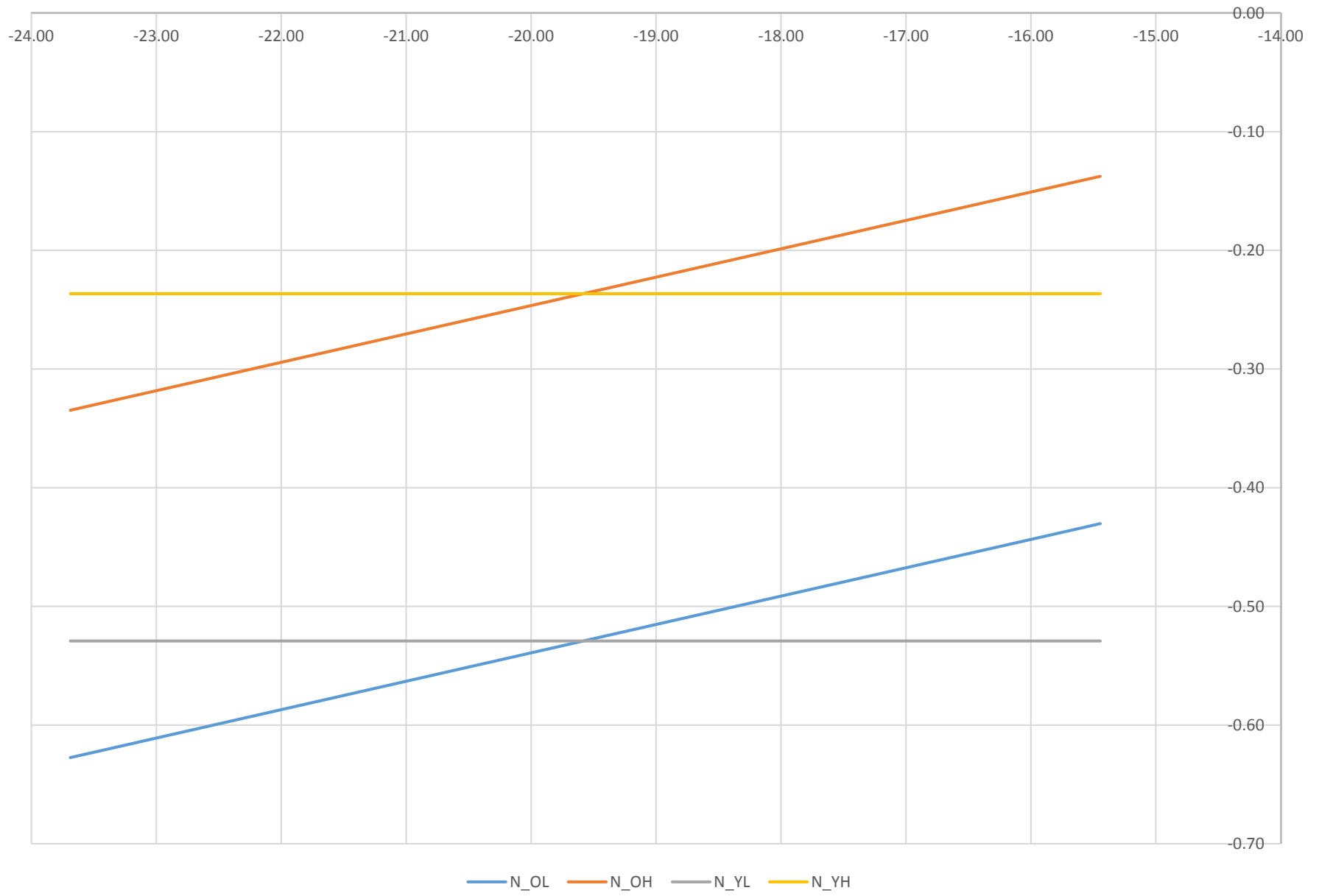


Figure 4: Promised values

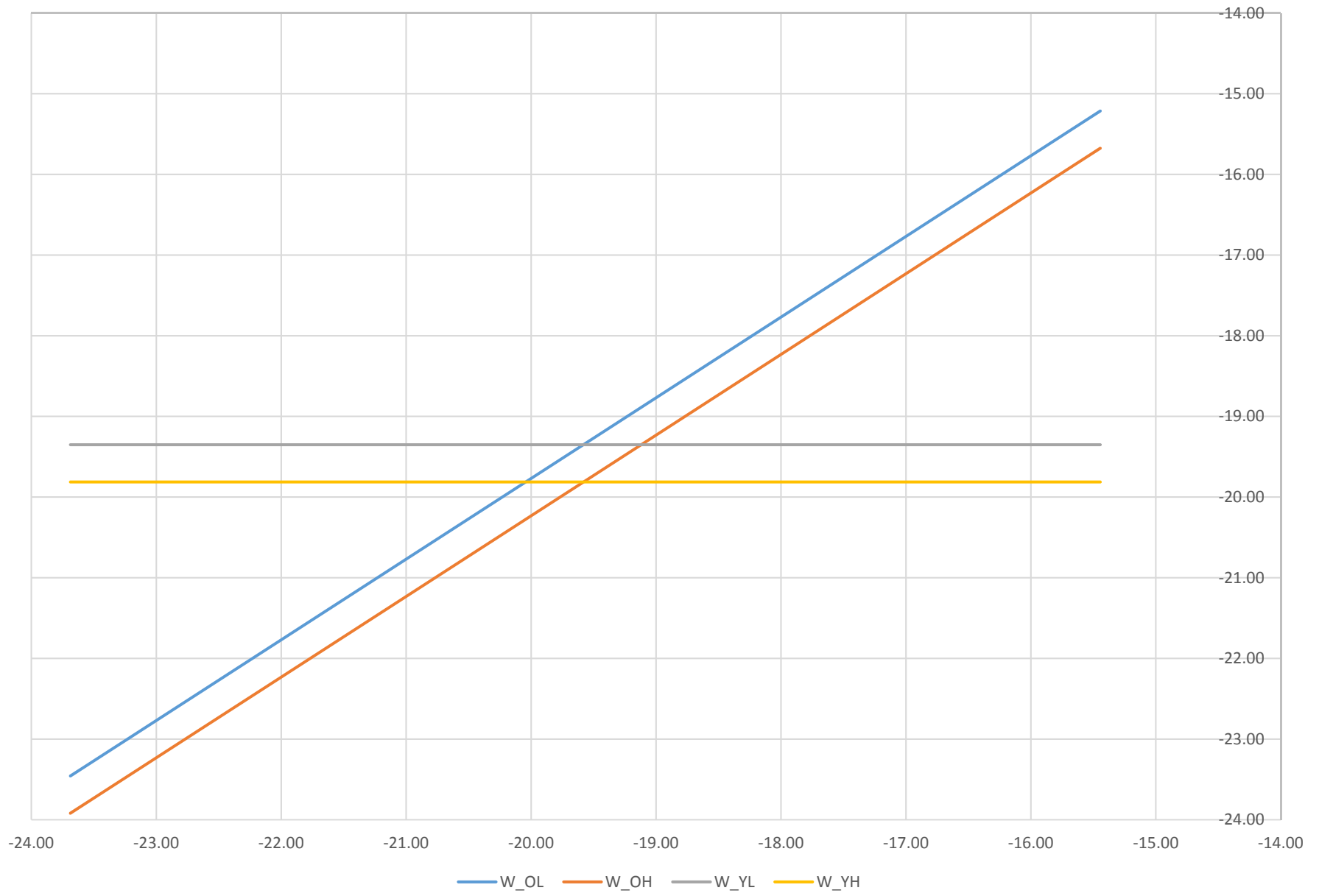


Figure 5: Intertemporal wedges

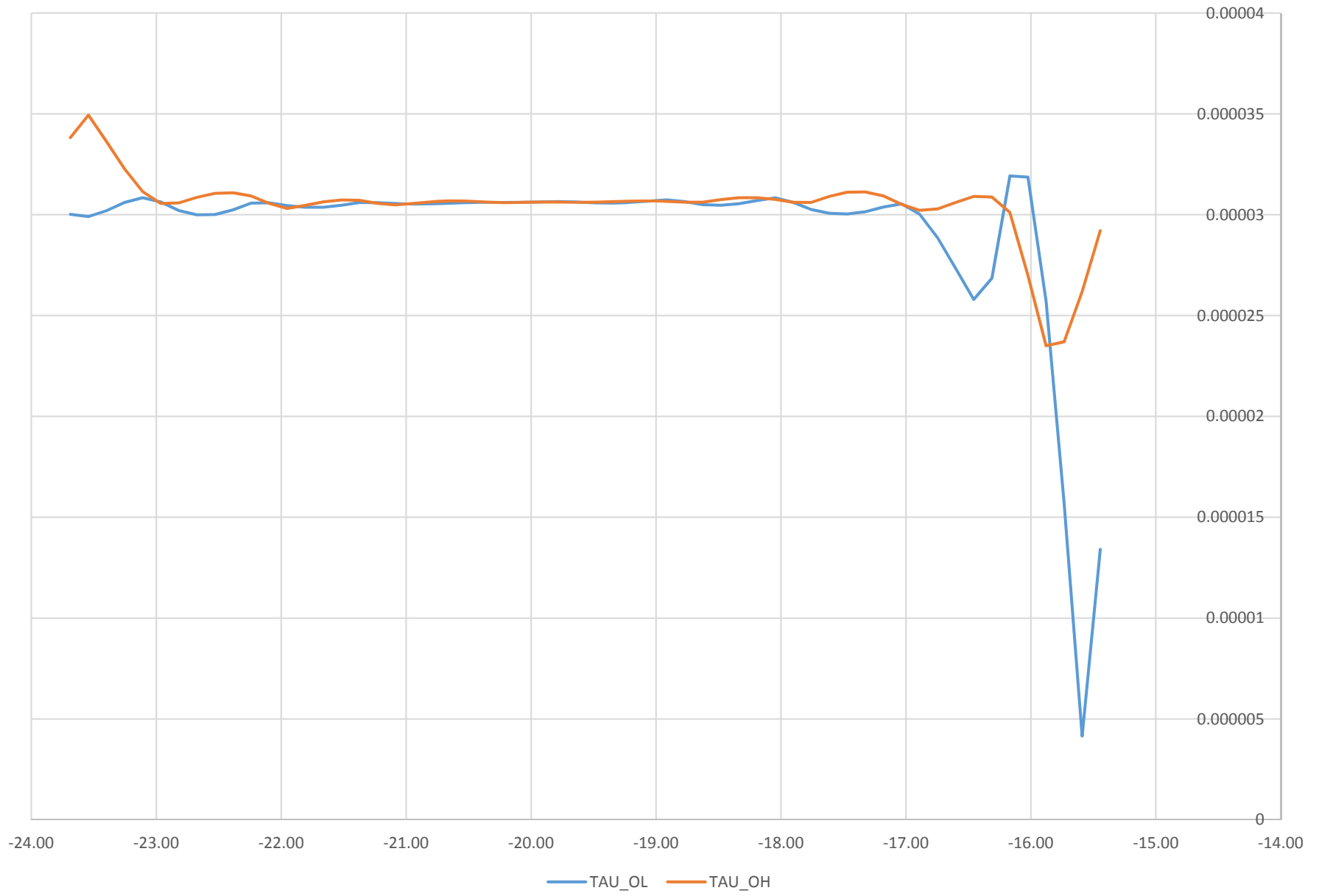


Figure 6: Consumption-labor wedge

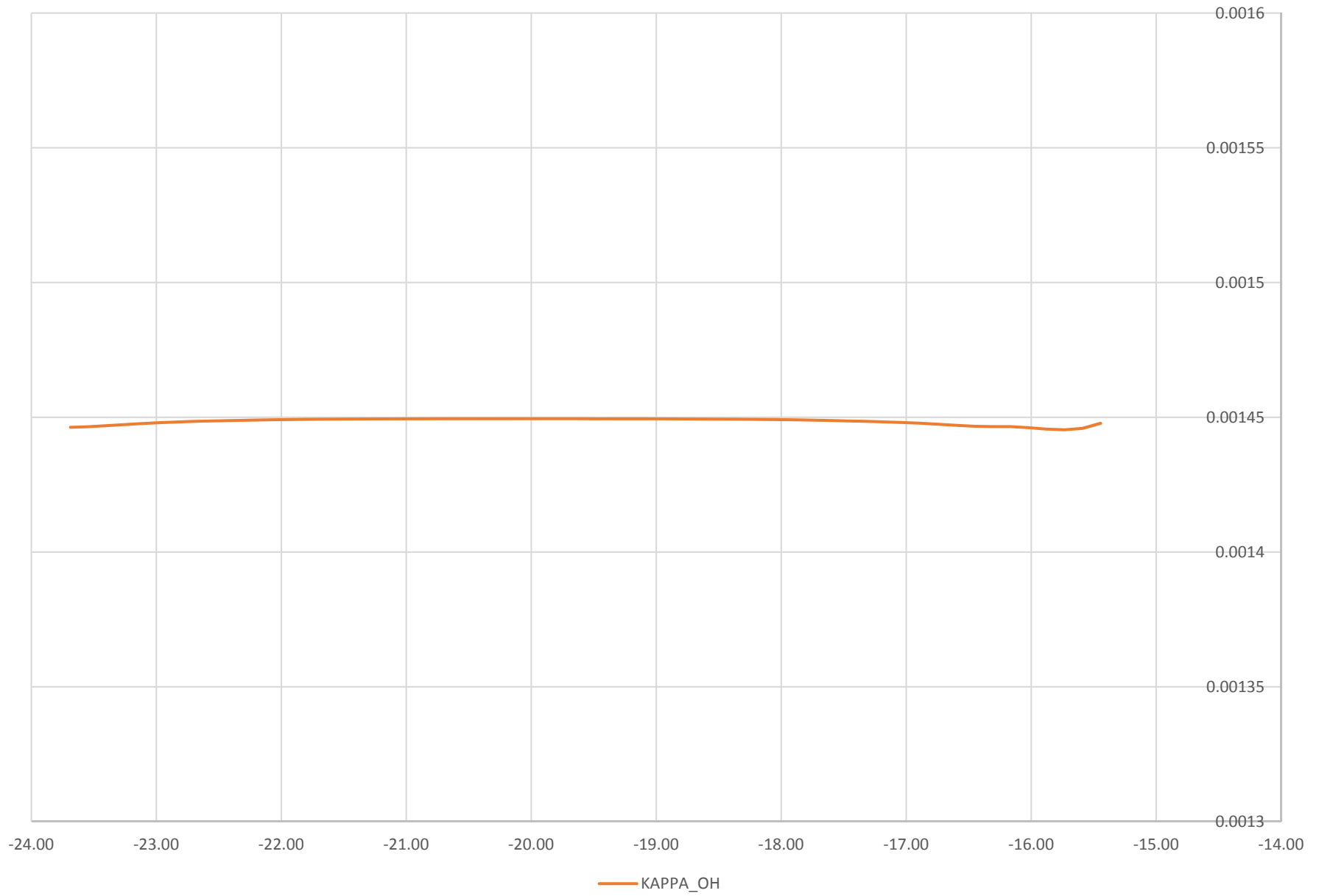


Figure 7: Log of consumption levels, young agents

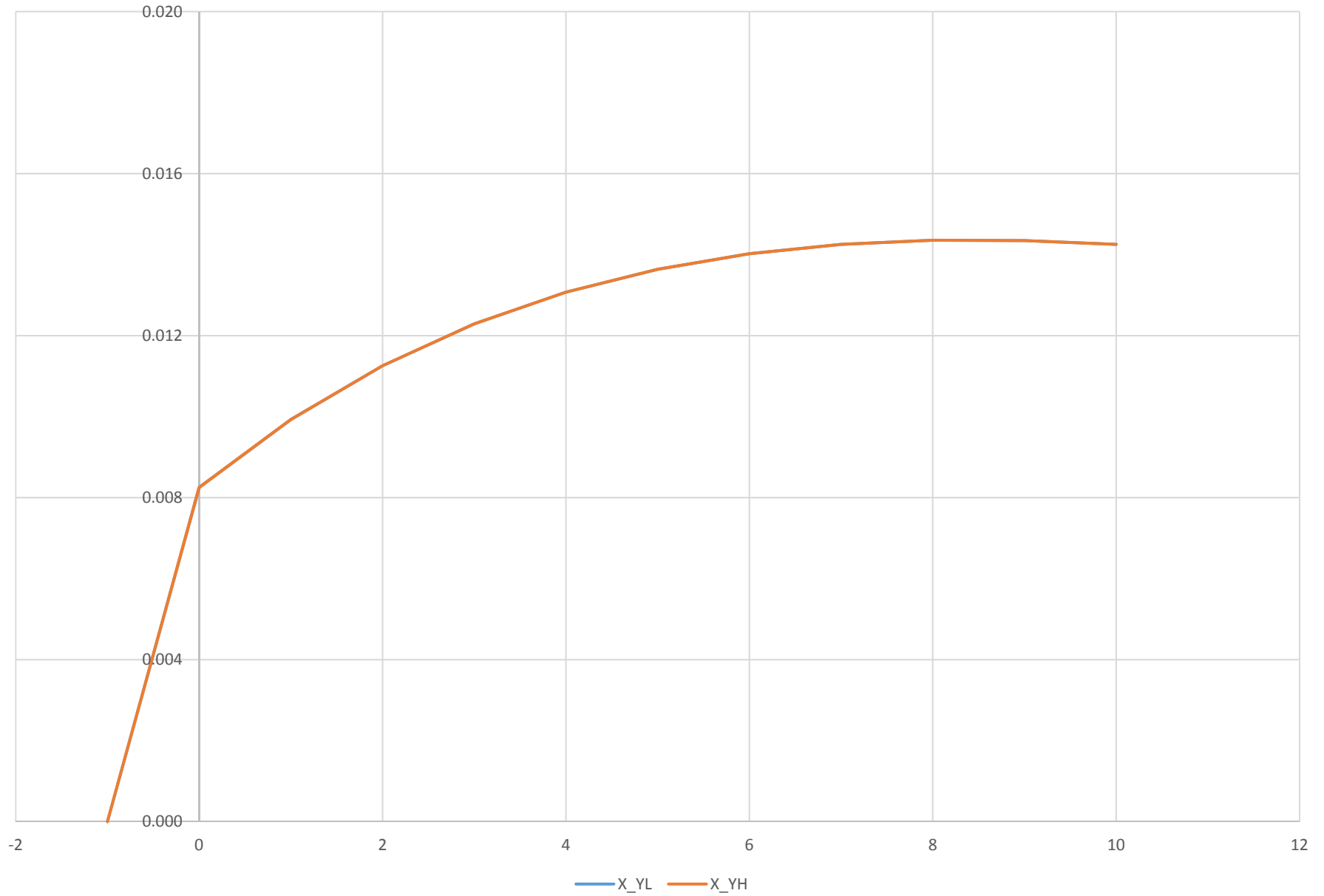


Figure 8: Log of consumption levels, old agents with low value of leisure L

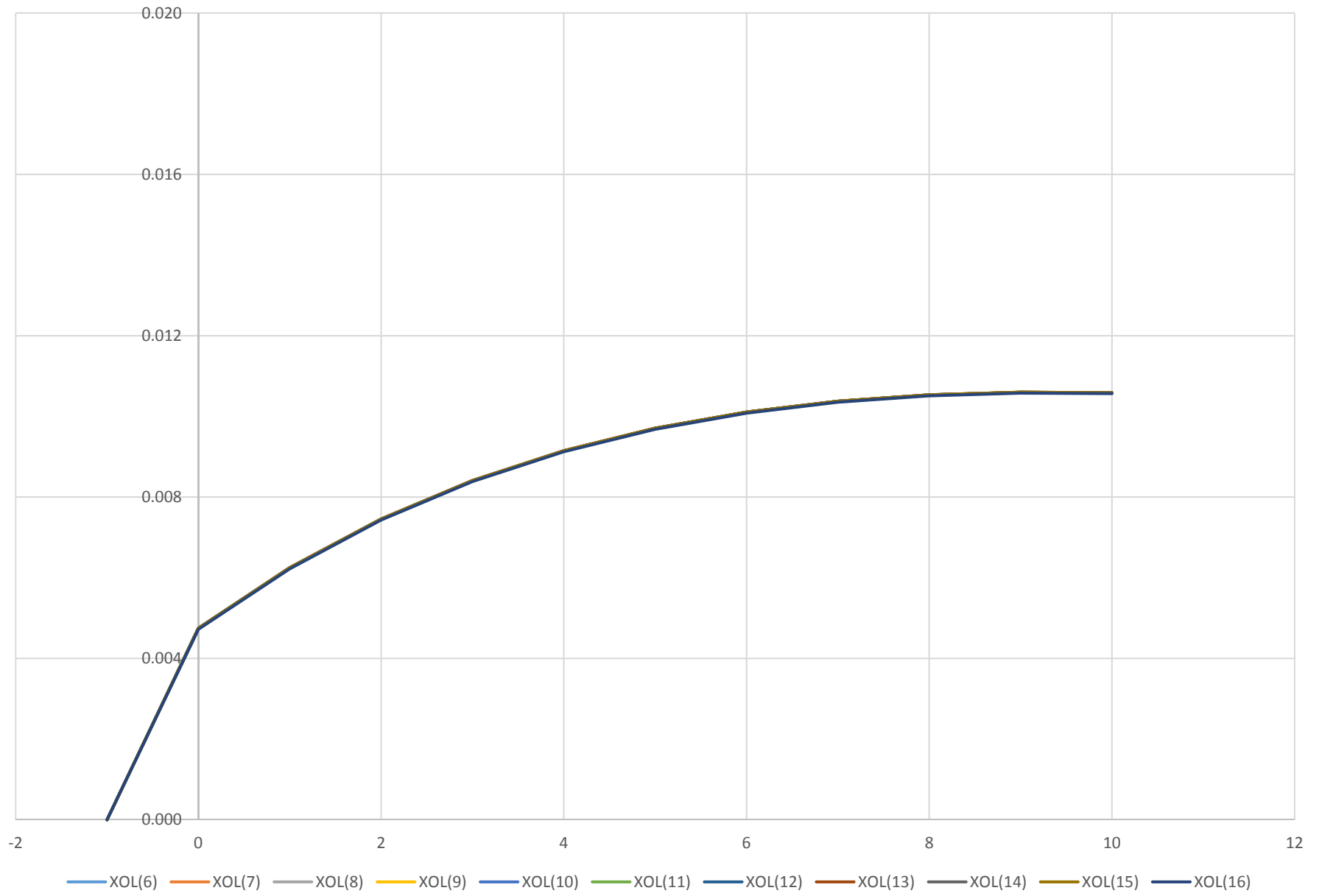
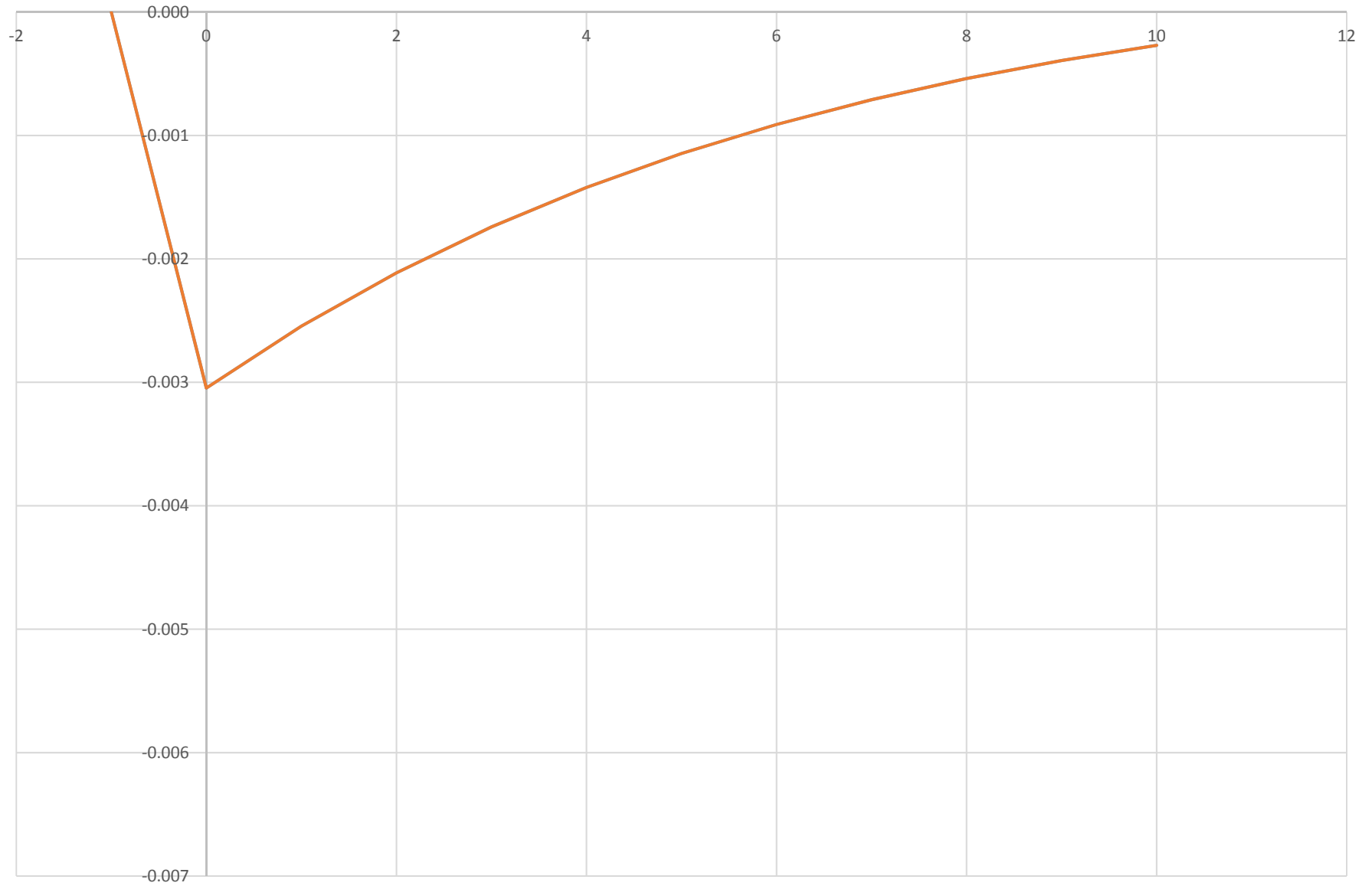




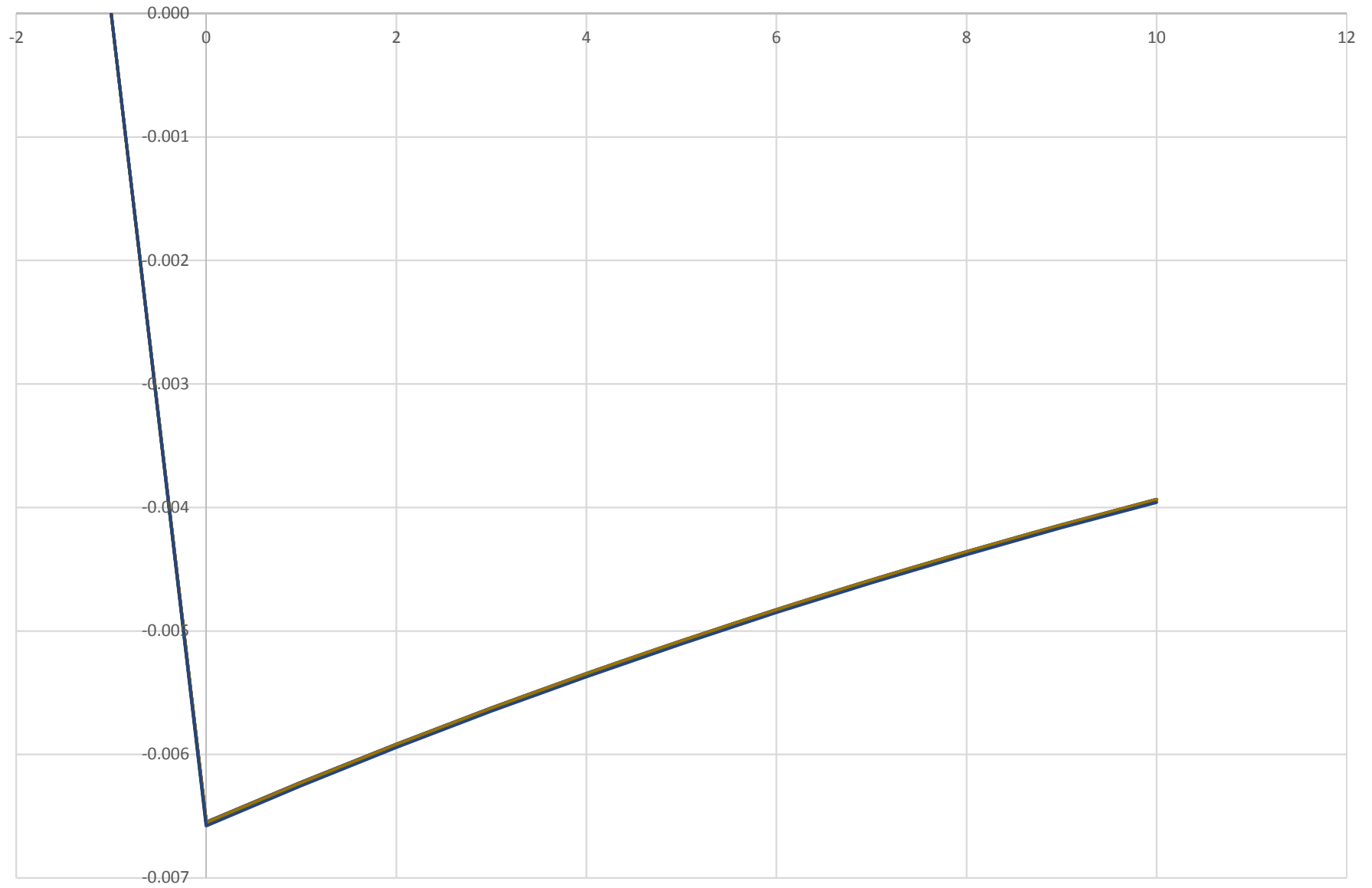


Figure 10: Log of leisure, young agents



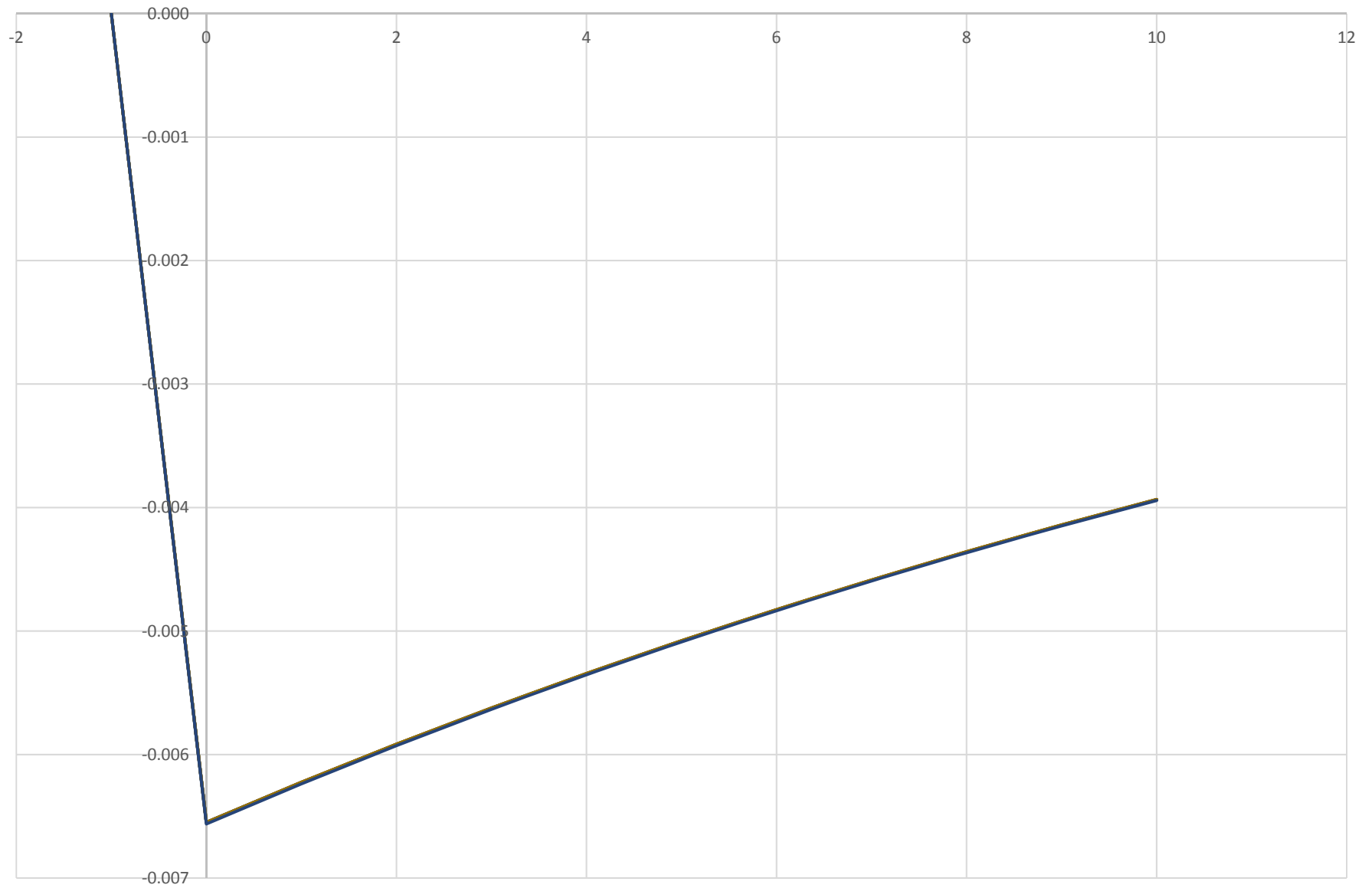
— N\_YL — N\_YH

Figure 11: Log of leisure, old agents with low value of leisure



NOL(6) NOL(7) NOL(8) NOL(9) NOL(10) NOL(11) NOL(12) NOL(13) NOL(14) NOL(15) NOL(16)

Figure 12: Log of leisure, old agents with high value of leisure



NOH(6) NOH(7) NOH(8) NOH(9) NOH(10) NOH(11) NOH(12) NOH(13) NOH(14) NOH(15) NOH(16)

Figure 13: Promised values to young agents

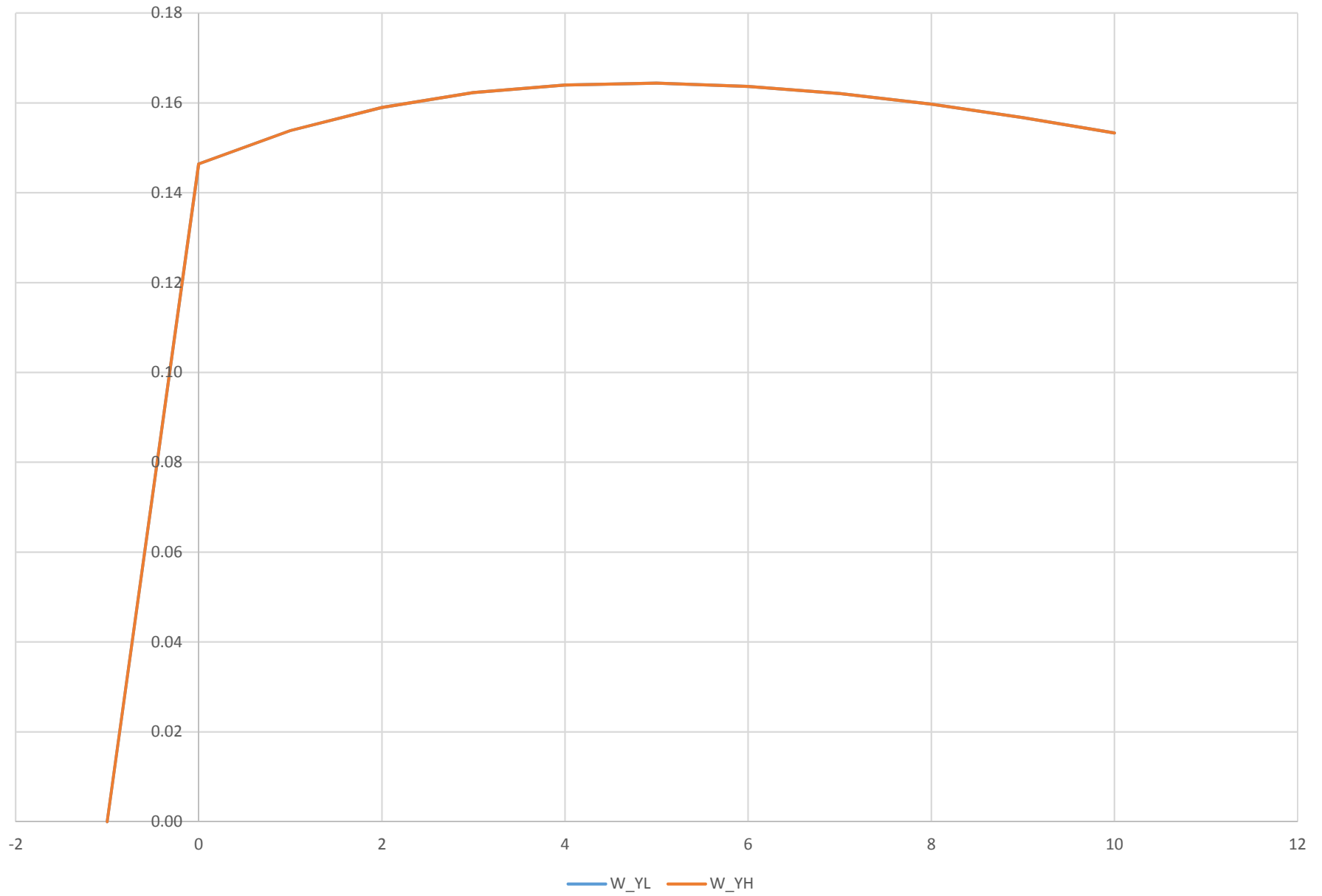






Figure 16: Impulse Responses Macro Variables  
(public info economy)

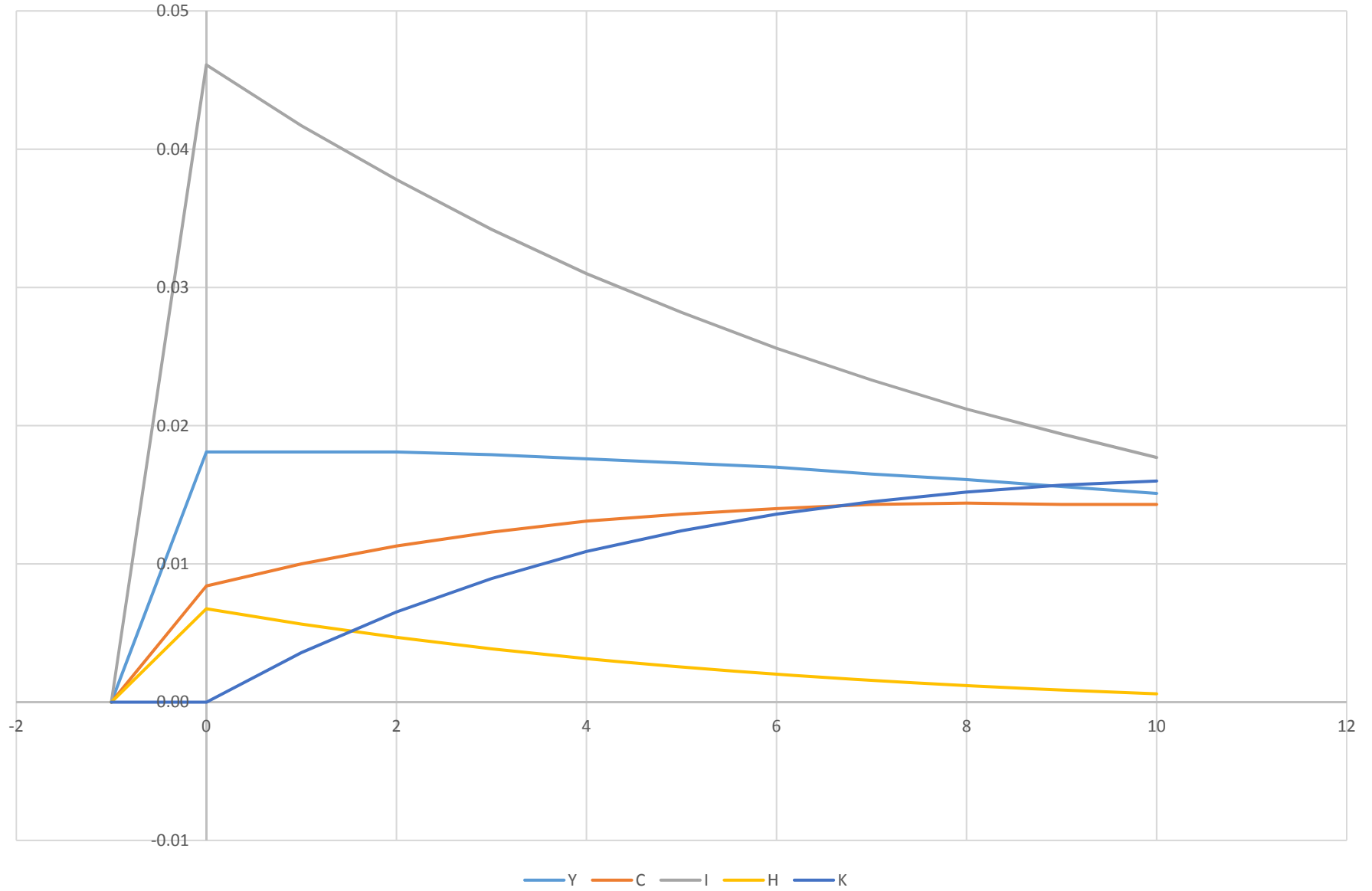


Figure 17: Impulse Responses Macro Variables  
(moral hazard economy)

