Firing Costs and Labor Market Fluctuations:  
A Cross-Country Analysis

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October 12, 2014

Abstract

We document large differences across OECD countries in fluctuations of the intensive and extensive margin of labor supply over the business cycle. Countries with larger fluctuations in employment relative to hours per worker tend to display larger fluctuations in total hours worked. These facts appear to be related to policies that impede the dismissal of workers. We then present a quantitative framework that features both margins of labor supply as well as costs to the adjustment of employment. Cross-country differences in dismissal costs can account for a large fraction of the patterns observed in the data.

*The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.
1 Introduction

Motivated by the observation that aggregate labor market outcomes differ significantly across OECD countries, a large literature has emerged that seeks to assess the effects of policies on such outcomes. Much of this literature has focused on either steady-state differences or differences in secular trends.\(^1\) This paper continues the effort to understand the connection between labor market policies and labor market outcomes, but instead focuses on cross-country differences in business cycle fluctuations in the labor market.

Specifically, this paper is motivated by three facts derived from the recently constructed cross-country quarterly dataset on employment, hours per worker and total hours in Ohanian and Raffo (2011). First, there are substantial differences in the relative volatility of hours per worker (intensive margin) and employment (extensive margin) over the business cycle across countries. Second, countries that have lower business cycle fluctuations in employment relative to output also have lower fluctuations in total hours worked relative to output. Third, countries with relatively higher fluctuations in employment relative to hours per worker also exhibit employment fluctuations that are more highly correlated with output.

One plausible candidate that could generate these differences is policies that affect the size of costs associated with worker dismissal, since a large literature has documented significant differences in these costs across countries. Intuitively, holding all else constant, higher firing costs would be expected to lead to relatively more fluctuations along the intensive margin and less along the extensive margin. In fact, there is a positive correlation between measures of firing costs that exist in the literature and the magnitude of intensive margin fluctuations relative to extensive margin fluctuations. In this paper we assess the extent to which differences along this one dimension can quantitatively account for the nature of differences in

labor market fluctuations across countries.

We carry out this quantitative assessment using a relatively standard real business cycle model extended to allow for labor supply fluctuations along both the intensive and extensive margin. Previous quantitative analyses of firing costs have typically assumed that labor is indivisible, implicitly specifying that all adjustment takes place on the employment margin, with no response on the intensive margin. We calibrate the model so that it is consistent with a standard set of first moments for the US economy as well as the nature of fluctuations along the extensive and intensive margins. We then vary the magnitude of dismissal costs and examine how this affects labor market fluctuations. Specifically, we ask whether differences along this one dimensions can quantitatively generate the differences that we document in the cross country data. We note that while it is intuitive that dismissal costs could account for some of the differences, there is no presumption that dismissal costs are the sole factor responsible for these differences, or even that it is the most important.

Our main finding is that differences in dismissal costs can account for a large share of the differences found in the cross country data. Interestingly, our results also indicate that abstracting from the presence of an intensive margin when evaluating the effects of firing costs is extremely important quantitatively. In particular, if we assume that there are no costs associated with employment adjustment in the US, we find that even very small firing costs relative to those observed in the cross-country data will virtually shut down fluctuations in employment, with effectively all fluctuations taking place along the intensive margin. Previous work that did not include an intensive margin severely underestimated the size of the effect on employment fluctuations precisely because they did not allow an alternative margin of adjustment.

Even if the US is assumed to have no legislated firing costs, one might think that it is
still reasonable to think that there are technological costs associated with adjustments in the number of workers employed. We then carry out a second calibration that incorporates one such cost—the cost of hiring a worker—for which the literature has produced estimates. Using this benchmark we then revisit the issue of how cross country differences in firing costs affect business cycle fluctuations. We find that our calibrated model that includes a hiring cost as part of the production technology quantitatively captures the salient features of the data referred to above.

2 Cross-Country Patterns in Labor Market Fluctuations

In this section we document three novel facts about labor market fluctuations across countries. In a recent contribution, Ohanian and Raffo (2011) construct a dataset for total hours worked at the quarterly frequency that covers 14 OECD countries and spans, in most cases, the period 1960 to 2010. This dataset is particularly useful because, to date, analysis of fluctuations across countries omitted the intensive margin and focused on employment as a summary statistics of labor market outcomes over the business cycle. This limitation has affected not only the conventional view about cyclical changes in labor, but also the study of the implications associated with different policies. For instance, the seminal work of Hopenhayn and Rogerson (1993) about the effects of hiring and firing costs suggested that European labor markets represent a laboratory for evaluating and quantifying the impact of these policies, but lack of data on the intensive margins severely limited the potential for these analysis.

We briefly summarize here the main elements of Ohanian and Raffo (2011) dataset and invite the reader to consult the original article for a more detailed description of their methodology. Data on employment, working age population, and output are obtained from the OECD. These authors then construct series on quarterly hours worked per worker implementing the
following three steps. First, they obtain from the Conference Board Total Economy Database (TED) annual data on hours worked per worker that are comparable across countries and are consistent with national accounts.\textsuperscript{2}. This dataset has been the benchmark in the literature for analysis on long trends in total hours worked (see Rogerson (2006), Ohanian \textit{et al} (2008), Rogerson and Shimer (2010)). Ohanian and Raffo then build a dataset of indicators for hours per worker for all the 14 countries considered drawing on a variety of international sources, including data from national statistical offices, establishment surveys, and household surveys. Finally, they adjust the quarterly indicators of hours worked per worker so that they conform with the annual TED series following the methodology proposed in Denton (1971).

Table 1 presents summary statistics for the standard deviations of total hours, employment and hours per worker relative to the standard deviation of output, as well as the correlation for each of the series with output. As is standard in the business cycle literature, statistics refer cyclical components of each series obtained applying a HP-filter with smoothing parameter equal to 1600 to the (log of the) series.

[Insert Table 1 Here]

We start by noting three basic facts that are apparent from this table. First, there is significant variation in the extent of volatility relative to output in all of the individual series across countries. Total hours are almost as volatile as output in Canada, Australia, and U.S., but less volatile in Germany, Japan, and Austria, with the median value of this ratio around 0.90. Second, there are substantial fluctuations along the intensive margin in all countries. This is important because it suggests that abstracting from fluctuations along

\footnotesize{\textsuperscript{2}Data contained in the TED are adjusted to reflect most sources of cross-country variation in hours worked, such as statuatory holidays and sick days.}
the intensive margin necessarily eliminates an important difference in cross-country patterns in labor market fluctuations. Third, total hours and output are highly correlated in all countries, consistent with the typical business cycle facts reported in the literature using the extensive margin only.

We next note the patterns in the data that serve to motivate the analysis in this paper. Using the values in Table 1, one can construct the ratio of the volatility of labor supply along the intensive margin to volatility of labor supply along the extensive margin. This ratio turns out to vary quite dramatically across countries, from a low of 0.39 in the U.S. to a high of 1.35 in Japan. The variation in the relative volatility along the two labor supply margins is correlated to the overall magnitude of fluctuations. Figure 1 plots the ratio of the standard deviations of total hours and output against the ratio of the standard deviations of hours per worker and total employment.

[Insert Figure 1 Here]

The figure shows a strong negative relationship, with a correlation coefficient of -0.7. This suggests that countries in which there is more adjustment along the extensive margin relative to the intensive margin tend to also display a higher volatility of total hours relative to output.

In a standard real business cycle model with technology shocks as the sole driving force of business cycles, the volatility of total hours relative to output is largely determined by the elasticity of labor supply. In a model that features variation along both the intensive and extensive margins, the volatility along the two margins would reflect labor supply elasticities along the two margins. In view of this, one interpretation of the evidence just presented is that labor supply elasticities along the two margins differ across countries. To the extent
that one is reluctant to ascribe differences in business cycle fluctuations across countries to differences in preference parameters, however, one is naturally led to consider alternative explanations.

A natural alternative is the possibility that the observed differences in labor market fluctuations reflect differences in labor market policies or institutions that influence how different economies react to the same shock. A long literature has pursued this idea to understand the very different secular evolutions of labor market outcomes across OECD economies (See, for example, Bruno and Sachs (1985), Krugman (1994), Blanchard and Wolfers (2000), Ljungqvist and Sargent (1998) and Hornstein et al (2007)). In this paper, we pursue this in the context of business cycle fluctuations. A leading candidate policy that may help to understand the patterns just documented is employment protection policies that increase the costs associated with worker dismissals. Intuitively, making it more costly to adjust labor input along the extensive margin will both reduce the overall extent of labor input fluctuations, and increase the use of the intensive margin relative to the extensive margin. Figure 2 provides additional motivation for pursuing this idea more rigorously; it plots the volatility of labor supply along the extensive margin relative to the volatility of labor supply along the intensive margin versus an index of employment protection legislation (EPL) obtained from the OECD.

[Insert Figure 2 Here]

Figure 2 shows that there is a strong negative correlation between the two variables: economies with a higher EPL relay more heavily on the intensive margin rather than the extensive margin for labor input adjustment over the business cycle. The remainder of this
paper assesses this more formally in the context of a relatively standard real business cycle model.

3 Model

The model is a version of the standard neoclassical growth model extended in a simple and flexible way to allow for labor supply decisions along both the intensive and extensive margin. There is a representative household that consists of a large number of members. The household has preferences defined by:

\[ E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, e_t, h_t) \]

where \( c_t \) is average consumption per household member at time \( t \), \( e_t \) is the fraction of individuals that are employed in period \( t \), and \( h_t \) is the amount of work per employed member in period \( t \). In our quantitative work we assume that the period utility function \( U(c_t, e_t, h_t) \) takes the following form:

\[ U(c_t, e_t, h_t) = \log c_t - \frac{A}{1 + \gamma} h_t^{\gamma+1} e_t - \frac{B}{1 + \eta} e_t^{\eta+1} \]

where \( \gamma > 0 \), \( \eta > 0 \) and \( A \) and \( B \) are positive constants. As we will see below, this utility function can easily generate interior solutions for labor supply along both the intensive and extensive margins in steady state, and will imply that labor supply will adjust along both margins in response to shocks. We generate these outcomes by embedding them directly into preferences. Cho and Cooley (1994) motivated this form of household utility function by positing a distribution of fixed costs across household members associated with employment along the extensive margin. Other papers in the literature (see, e.g., Kydland and Prescott (1991), French (2005), Rogerson and Wallenius (2009) and Erosa et al (2012)) instead start with preferences in which individuals care only about total hours worked, and generate allo-
ations in which labor supply adjusts along the intensive and extensive margins by positing non-convexities in the market technology and heterogeneity among household members. We view these two approaches as complementary, and have chosen this one because it offered a more tractable and transparent model for our purposes.

As is standard in the real business cycle literature, there is an aggregate production function that uses capital and labor to produce output and is subject to persistent productivity shocks. Although we distinguish between employment and hours in the household’s utility function, we assume that employment and hours per worker are perfect substitutes in terms of producing output. However, motivated by the desire to consider the effects of employment protection policies, we will allow for adjustment costs associated with changes in employment between consecutive periods. Formally, we have:

$$y_t = z_t k_{t-1}^\theta N_t^{1-\theta} - F \cdot \max\{e_{t-1} - e_t, 0\}$$

where $z_t$ is the technology shock in period $t$, $k_t$ is input of capital in period $t$, and $N_t = e_t h_t$ is total hours in period $t$, and $F \geq 0$ is the per worker cost associated with reductions in employment, measured in units of output. We assume that $z$ follows an AR(1) process in logs:

$$\log z_{t+1} = \rho \log z_t + \varepsilon_{t+1}$$

where $\varepsilon_t$ is an iid normally distributed innovation with mean $\mu_{\varepsilon}$ and standard deviation $\sigma_{\varepsilon}$.

Finally, the resource constraint posits that output can be used for either consumption or investment, with capital depreciating at rate $\delta$

$$c_t + k_t - (1 - \delta)k_{t-1} = y_t$$

Note that we have assumed that the costs associated with employment reductions are associated with output losses. In our quantitative work below we will use the adjustment
cost function to capture both costs that are part of the technology associated with workforce adjustments, as well as those that reflect costs associated with employment protection. To the extent that employment protection legislation imposes administrative burdens on employers, this representation is consistent. Alternatively, much of the literature has studied firing taxes as a way to focus on the distortionary effects of legislated severance payments, and our formulation is consistent with the approach if we assume that the government is using the revenues to finance spending that enters utility in a separable way. Because our main goal is to focus on the distortion between labor supply adjustments along the two margins we do not think that this is an important issue.

4 Calibration

Our general strategy is to calibrate the model to match some key features of the US economy, and then consider how business cycle fluctuations are affected when we vary the level of costs associated with employment reductions, as captured by the parameter $F$. We will consider two different calibrations for the US. In the first calibration, we will set the value of $F$ in the US equal to zero. This specification is consistent with the view that $F$ is purely associated with employment protection legislation and that these provisions do not exist in the US. This view figures prominently in the literature that uses explicit models to assess the effects of employment protection on aggregate labor market outcomes (see, for example, Hopenhayn and Rogerson (1993), Alvarez and Veracierto (1999), and Veracierto (2001, 2008)). In the second calibration we will assume that the value of $F$ is positive in the US. This calibration is consistent with the view that that even if the US economy does not have legislation that imposes adjustment costs, there may still be costs associated with employment adjustment that reflect technology rather than policy. A sizeable literature documents the presence of
hiring and firing costs for individual establishments. In a business cycle context with a representative firm, hiring and firing costs are broadly similar in their effect, and so rather than separately modeling the two components we bundle these costs into the firing cost parameter $F$. In this second specification we calibrate this parameter to be consistent with the estimated hiring costs in Hagedorn and Manovskii (2006), which is 4.5% of quarterly wages.

In each of the two different calibrations, the remaining parameters are chosen according to the same procedure. As is standard, we calibrate the values of $\beta$, $\theta$, $\delta$, $A$ and $B$ so that the model’s steady state matches some first moments from the US data. Specifically, the moments that we match are values for capital’s share of income, the investment to output ratio, the steady state interest rate and the levels of the two labor supply margins, employment and hours per worker.

The essence of our analysis concerns how variation in the level of employment reduction costs influence the optimal response of labor supply along the intensive and extensive margins in reaction to aggregate shocks. As noted in the previous section, the only shock that we consider is an aggregate technology shock. This choice is motivated by the fact that it is well known that technology shocks are able to generate business cycle fluctuations that capture many of the properties of business cycle fluctuations in the data. However, we suspect that our findings would also hold in the presence of other aggregate shocks such as shocks to government spending or tax rates, assuming that we consider the optimal response of labor supply to these shocks. Consistent with most of the literature, we consider a process for technology shocks that is quite persistent, with the value of $\rho$ set to .98. One of the key issues in the business cycle literature is the magnitude of fluctuations, which in the context of technology shock driven business cycles is intimately related to the standard deviation of
the innovations, which we denoted by $\sigma_\varepsilon$. The business cycle statistics that we focus on in our analysis of cross country patterns will all be measured relative to fluctuations in output. For this reason the actual size of fluctuations in output turns out not to be of key importance in our study. In particular, our cross-country comparisons do not need to take a stand on whether the shocks that hit different economies have different magnitude. We will, however, assume that the persistence of shocks is the same across economies.

The two remaining parameters are the two preference parameters that are associated with labor supply elasticities: $\gamma$ and $\eta$. These two parameters will heavily influence both the overall magnitude of fluctuations in total hours of work as well as how these fluctuations are divided along the intensive and extensive margin. Understanding the underlying foundations for these labor supply parameters is an important issue, but is not the focus of our study. Rather, our goal is to understand the extent to which a specific labor market policy can help us understand cross-country differences in the nature of labor market fluctuations. For this exercise we want to start with a model that captures the nature of fluctuations in the US economy, and then examine how the nature of fluctuations is affected by changes along one dimension. For this reason we follow the somewhat non-standard procedure of calibrating the two labor supply elasticity parameters $\gamma$ and $\eta$. Specifically, following Cho and Cooley (1994), we set $\eta$ at 0.5 and choose $\gamma$ to match the volatility of employment relative to hours per worker. One of the advantages of our specification of the utility function is that it sufficiently flexible to allow us to match properties of labor market fluctuations.

The resulting parameter values for our two different calibrations are summarized in Table 2.
5 Results

We begin by presenting results for the calibration in which the US economy is assumed to have no adjustment costs. Table 3 presents the properties of business cycle fluctuations for both the $F = 0$ economy as well as several other values of $F$. We measure $F$ in units of (quarterly) wages, so that $F = .33$ represents a firing tax equal to one month of wages, and $F = 1$ corresponds to firing costs equal to one quarter of wages.

[Insert Table 3]

The patterns found in Table 3 confirm the intuition that we expressed earlier: increasing dismissal costs leads to less adjustment along the employment margin and more adjustment along the intensive margin, though the volatility of total hours decreases. However, there some additional findings in the table that we want to note. First, a point that we will return to later is that most of the effect of dismissal costs is realized for relatively small values of the cost. To see this, note that almost all of the variation in this table occurs as we move from the first column ($F = 0$) to the second column ($F = .33$). Specifically, when the dismissal cost is set to one month of wages, the volatility of employment is reduced by almost three quarters, falling from .69 to .17. Although this value continues to drop as dismissal costs are increased further, the decrease when we move from dismissal costs of one month to dismissal costs of one quarter is only .07, as compared with the drop of .52 when we move from costs of zero to one month of wages. Similarly, almost all of the increase in the volatility of hours per worker occurs when we increase dismissal costs from zero to one month, with further increases being quite modest. Given these two results, it follows that virtually all of the drop in aggregate hours is realized by the introduction of dismissal costs equal to one month
of wages. Because the policy only affects adjustment along the extensive margin, once the adjustment on the extensive margin becomes sufficiently small, it is intuitive that further increases in the dismissal cost will not have substantial effects on the economy.

Another pattern of interest is that the correlation between employment and output falls quite significantly when dismissal costs are introduced. Lastly, while not our focus in this paper, we also note that dismissal costs have relatively small impacts on the volatility of consumption and investment.

[Insert Figure 3 here]

Next we ask whether the magnitudes of the effects documented in Table 3 regarding volatility of total hours and its composition in terms of intensive and extensive margins are in line with the magnitudes of differences found in the cross country data presented earlier. We focus on the relationship between the volatility of total hours relative to output and the volatility of total hours relative to employment, since these variables capture two of the facts that we documented in the data. Figure 3 plots the relationship between these values in the data as well as the relationship implied by the model as we increase dismissal costs from zero. Note that in the figure we have normalized both ratios to one for the US economy, so that we are focusing on the values of these ratios relative to the US.

The solid line in the figure is the fitted curve based on a log regression using the actual values. The other line is the curve implied by the calibrated model when the only variation across countries is the size of the dismissal cost. While both curves are negatively sloped, the line from the data is much more steeply sloped. Additionally, although it is not shown on the figure, the range of dismissal costs that are represented in the figure is very small, with a maximum value of 0.04. This property has important implications for the previous literature.
on evaluating the consequences of dismissal costs. The norm in the previous literature is to assume that the US economy has zero adjustment costs on employment, and that the extensive margin is the only margin through which labor input can be adjusted. Both of these features are present in the recent analysis of Veracierto (2008), for example. In these other studies, increasing dismissal costs has a significant but much more modest effect on employment fluctuations. Perhaps not surprisingly, this result turns out to be extremely sensitive to whether one allows for adjustment of labor input along the intensive margin. If the extensive margin is the only margin of adjustment then it is used even when it becomes costlier to do so. But when one introduces a second margin that is not subject to the dismissal cost, the response in employment fluctuations is dramatically increased. A key message from the above analysis is that abstracting from the intensive margin when assessing the impact of dismissal costs on labor market fluctuations has very significant implications for the quantitative analysis.

[Insert Table 4]

Table 4 illustrates to what extent not allowing for an adjustment along the intensive margin affects the assessment of the impact of dismissal costs on labor market fluctuations. This table compares the moments reported in Table 3 with those generated by a model without the intensive margin.\(^3\) Without the intensive margin the increase in the dismissal cost (from zero to one month of wages) reduces the volatility of employment by almost two thirds (from 0.92 to 0.32), remarkably less than reduction in volatility of employment with the intensive margin. Not including the intensive margin also affects the assessment on the

\(^3\)To ease the comparison, the model without the intensive margin has the same steady state as the model with the intensive model. The only difference is that in the former, \(h_t = \bar{h} \land t\).
impact of dismissal costs on other properties of the business cycle. For instance, the fall in output volatility induced by an increase of the dismissal cost from zero to one month without the intensive margin is three times the fall with the intensive margin.

We now turn to the second calibration, in which we assumed that even in the US there is a significant cost associated with employment adjustment along the extensive margin, which we interpret to be indicative of technological costs associated with adjustment, rather than costs that are induced by policy. Note that we calibrate preference parameters in this case so that the model still generates an empirically reasonable amount of fluctuations along the two labor supply margins. Given the previous results, it is clear that if we kept the previous parameterization and increased $F$ to the value that we use in this specification for the US, employment fluctuations would fall dramatically. We now repeat the earlier analysis. Results are presented in Table 5.

[Insert Table 5]

This table conveys the same information of Table 3: dismissal costs lead to less adjustment along the extensive margin. Note however that the elasticity of the response in employment volatility to increases in dismissal costs is much smaller under this second calibration than under the first one. This feature is very intuitive: if the level of adjustment costs are high then a small increase due to policy represents a much smaller percent increase in costs and is likely to have smaller consequences.

We again ask whether variation in dismissal costs can capture the key variation in labor market fluctuations found across countries. Figure 4 repeats the same exercise that we previously presented in Figure 3.
To ease comparison with the earlier results, this figure also includes the curve implied by the model under the previous calibration. Interestingly, we see that the model generated curve now has a somewhat steeper slope than the data generated curve. We conclude that if we assume a small but positive cost of employment adjustment in the US, then the model can capture the patterns in the data, both qualitatively and quantitatively. It is also of interest to examine how this alternative calibration affects the responsiveness of hours to changes in the dismissal cost. In this regard, we emphasize that the new curve in Figure 4 now represents a range of dismissal costs running from 0.045 to 0.9.

6 Conclusion

Labor market fluctuations display very different quantitative patterns across OECD countries. Given the large differences in labor market policies and institutions across countries it is natural to ask to what extent these differences in fluctuations are the result of differences in specific policies and/or institutions. This paper represents a first step in this broader research agenda. Specifically, we present evidence showing that there are very substantial differences across countries in terms of both the relative volatility of hours compared to output, as well as the relative volatility of labor supply along the intensive and extensive margins. Intuitively, these patterns are qualitatively consistent with differences that one might expect from differences in dismissal costs.In this paper we introduce dismissal costs into an otherwise standard real business cycle model that also features labor supply adjustment along both the intensive and extensive margins. Our main finding is that this feature appears to be able to account for the patterns found in the cross country data, not only qualitatively but also quantitatively. Two additional findings that come out of our analysis are the following. First,
that it is very important to allow for an empirically reasonable response along the intensive margin when studying dismissal costs. Assuming that the only margin of adjustment is the extensive margin, as is common in virtually all studies of firing taxes, turns out to have very significant quantitative implications. Second, the assumption that the US labor market has zero adjustment costs associated with employment is not a harmless normalization. The level of this cost turns out to be critical in assessing the elasticity of the response in employment volatility to increases in dismissal costs. This feature is very intuitive: if the level of adjustment costs are high then a small increase due to policy represents a much smaller percent increase in costs and is likely to have smaller consequences.

7 References

References


8 Appendix A. Solving the Model

We solve for the equilibrium of the model by solving the associated Social Planner’s problem. One difficulty with this is that the adjustment cost function is not differentiable. To circumvent this problem, we replace the firing cost function with a differentiable function \( g(\cdot) \).

The function \( \max(0, x) \) can be written as follows:

\[
\max(0, x) = \frac{x}{2} + \frac{|x|}{2}
\]

where \(|\cdot|\) is the absolute value function. To approximate the maximum function with a differentiable function, \( g(x) \), we need to approximate the non-differentiable part. One alternative is to use:

\[
g(x) = \frac{x}{2} + \frac{\sqrt{x^2 + \kappa}}{2}
\]

where \( \kappa \) is positive real number determining the accuracy of the approximation: the lower \( \kappa \), the better the approximation. If \( \kappa = 0 \) and \( x > 0 \), then left hand side is equal to \( x \). For \( x < 0 \), the two sumands cancel each other out and \( g(x) = 0 \).

Hence, we approximate the firing cost function with the following specification.

\[
g(e_t, e_{t-1}; \kappa) = F \left( \frac{1}{2} (e_{t-1} - e_t) + \frac{1}{2} \sqrt{(e_{t-1} - e_t)^2 + \kappa} \right)
\]

which implies:

\[
\frac{\partial g(e_t, e_{t-1}; \kappa)}{\partial e_t} = -F \left( \frac{1}{2} + \frac{1}{2} \frac{(e_{t-1} - e_t)}{\sqrt{(e_{t-1} - e_t)^2 + \kappa}} \right)
\]

Setting \( \kappa \) close to zero always provides a perfect approximation to the firing cost function, but there is a trade-off because, the first derivatives of \( g(\cdot) \) go to infinity as \( \kappa \to 0 \).

Having introduced the function \( g(\cdot) \), we proceed to solve the social planner problem, which consist on maximizing the discounted expected utility of households subject to the
usual resource constraints. Replacing the function the firing cost by the function $g(\cdot)$, the f.o.c. of the problem can be written as:

$$
\beta E_t \left( \frac{c_{t+1}}{c_t} \right)^{\sigma-1} \left( \frac{\theta z_{t+1} k_t^\theta (e_{t+1} h_{t+1})^{1-\theta}}{k_t} \right) + (1 - \delta) = 1
$$

$$
Ac_t^{1-\sigma} h_t^\gamma e_t = (1 - \theta) z_t k_{t-1}^\theta (e_t h_t)^{1-\theta} h_t
$$

$$(1 - \theta) \frac{z_t k_{t-1}^\theta (e_t h_t)^{1-\theta}}{e_t} - \frac{\partial g(e_t, c_{t-1}; \kappa)}{\partial e_t} - \beta E_t \left( \frac{c_{t+1}}{c_t} \right)^{\sigma-1} \frac{\partial g(e_{t+1}, e_t; \kappa)}{\partial e_t} = \left( \frac{A}{1 + \gamma h_t^{1+\gamma} + B e_t^\theta} \right) e_t^{1-\sigma}
$$

The above equations, along with the following resource constraint\(^4\), characterize the solution,

$$
c_t + k_t - (1 - \delta) k_{t-1} = z_t k_{t-1}^\theta (e_t h_t)^{1-\theta} - \left( g(e_t, e_{t-1}; \kappa) - \frac{F \sqrt{\kappa}}{2} \right)
$$

Denoting $\bar{x}$ as the steady state of variable $x_t$, the steady state of the model is represented by the following set of equations,

$$
\bar{y} = \bar{k}^\theta (\bar{e} \bar{h})^{1-\theta}
$$

$$
\bar{e} + \delta \bar{k} = \bar{y}
$$

$$
1 = \beta \left( \frac{\theta \bar{y}}{k} + (1 - \delta) \right)
$$

$$
Ac_t^{1-\sigma} \bar{h}^\gamma \bar{e} = (1 - \theta) \frac{\bar{y}}{h}
$$

$$
(1 - \theta) \frac{\bar{y}}{\bar{e}} + \frac{F (1 - \beta)}{2} = \left( \frac{A}{1 + \gamma h_t^{1+\gamma} + B e_t^\theta} \right) \bar{e}^{1-\sigma}
$$

We solve for the business cycle properties of the model by log-linearizing the model around the deterministic steady state. Letting $\hat{x}_t \equiv \log \left( x_t / \bar{x} \right)$, the following equations constitute the log-linearized model.

$$
\hat{y}_t = \hat{z}_t + \theta \hat{k}_{t-1} + (1 - \theta) \left( \hat{e}_t + \hat{h}_t \right)
$$

\(^4\) The term $\frac{F \sqrt{\kappa}}{2}$ guarantees that in the steady state the resource constraint is not affected by the firing cost.
\[ \ddot{c}k_t + \ddot{k}k_{t-1} - (1 - \delta) \ddot{k}k_{t-1} = \ddot{y}y_t - g(\ddot{e}, \ddot{e}; \kappa) \hat{g}_t \]

\[ 0 = \beta \theta \frac{\ddot{y}}{k} E_t \left( \ddot{y}_{t+1} - \hat{k}_t \right) - E_t (\dot{c}_{t+1} - \dot{c}_t) \]

\[ \dot{c}_t + \gamma \hat{h}_t + \dot{e}_t = \dot{y}_t - \hat{h}_t \]

\[ (1 - \theta) \frac{\ddot{y}}{\ddot{e}} (\ddot{y}_t - \ddot{e}_t) - g_1(\ddot{e}, \ddot{e}; \kappa) \hat{g}_{1,t} - \beta g_2(\ddot{e}, \ddot{e}; \kappa) E_t \left\{ - (\dot{c}_{t+1} - \dot{c}_t) + \dot{g}_{2,t+1} \right\} \]

\[ = \frac{A}{1 + \gamma} \tilde{h}^{1+\gamma} \ddot{c} \left( 1 + \gamma \right) \hat{h}_t + \dot{c}_t \]

where:

\[ g(\ddot{e}, \ddot{e}; \kappa) \hat{g}_t = \frac{\ddot{e}F}{2} (\dot{e}_{t-1} - \dot{e}_t) \]

\[ g_1(\ddot{e}, \ddot{e}; \kappa) \hat{g}_{1,t} = -\frac{\ddot{e}F}{2\sqrt{\kappa}} (\dot{e}_{t-1} - \dot{e}_t) \]

\[ g_2(\ddot{e}, \ddot{e}; \kappa) \hat{g}_{2,t+1} = -\frac{\ddot{e}F}{2\sqrt{\kappa}} (\dot{e}_t - \dot{e}_{t+1}) \]
### Table 1. Business Cycle Statistics

<table>
<thead>
<tr>
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<th>Standard Deviation relative to Output</th>
<th>Correlation with Output</th>
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<td></td>
<td>Total Hours</td>
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</tr>
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<td>0.61</td>
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<tr>
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### Table 2. Calibration

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<td>Depreciation rate ($\delta$)</td>
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<td>Curvature in utility function ($\eta$)</td>
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<tr>
<td>Standard deviation of technology shock ($\sigma_\varepsilon$)</td>
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<tr>
<th>Alternative calibrations</th>
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<td>Curvature in utility function ($\gamma$)</td>
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<td>Utility scale parameter ($B$)</td>
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Table 3. Results, first calibration

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<th>1.50</th>
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<td>4.17</td>
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<table>
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<th>0.66</th>
<th>1.00</th>
<th>1.50</th>
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<tr>
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<tr>
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<td>1.00</td>
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Table 4. Comparison of results, first calibration

<table>
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<th>0.33</th>
<th>0.66</th>
<th>1.00</th>
<th>1.50</th>
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</thead>
<tbody>
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<td>1.49</td>
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<tr>
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<tr>
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Table 5. Results, second calibration

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<th>0.33</th>
<th>0.66</th>
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<th>1.50</th>
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<th>0.66</th>
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<td>1.00</td>
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</tr>
</tbody>
</table>
Figure 3. Results, first calibration
Figure 4. Results, second calibration

Data

Fitted (in logs)

Model (F=0)

Model (F>0)

Std(Hours)/Std(Employment)

Std(Total Hours)/Std(Output)

AUS

AUT

CAN

FIN

FRA

GER

IRE

JAP

KOR

SWE

UK

USA