Occupational Choice, Human Capital, and Financing Constraints *

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Abstract

We develop a framework which allows us to study the effect of financing constraints jointly for firm-level investment decisions, and household-level schooling decisions. We characterize the joint determination of occupational choices, educational outcomes, and production decisions. We first evaluate the role of financial frictions in distorting resource allocation. We find significant departures from efficiency, from misallocation of talent into occupations, to production and investment/schooling distortions. We then (i) ask whether our model helps understand observed cross-country variation in outcomes, and (ii) quantify the full effect of financing frictions for economic development, and in particular whether our framework produces an amplification of the output and productivity effects of financing frictions compared to standard models without schooling investments.

Keywords: Economic Development, Financing Frictions, Entrepreneurship, Human Capital.

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1 Introduction

We investigate how schooling and occupational choice decisions are jointly determined in an environment where financial frictions affect both human capital and firm-level investments. We build a model which integrates two literatures/frameworks. One is a model of occupational choice and entrepreneurship with credit constraints, along the lines of Buera and Shin (2013), Buera *et al.* (2011) and Midrigan and Xu (2013), among others. The other one is a model of human capital accumulation along the lines of Erosa *et al.* (2010) and Manuelli and Seshadri (2014).[1]

Integrating entrepreneurship and schooling decisions into a single framework leads to important and novel insights. We find that entrepreneurship plays a crucial role in understanding of schooling decisions. The standard effect of credit constraints on education is that they generate a disincentive for individuals to make schooling investments. They force individuals to reduce current consumption, and thereby increase current schooling costs. As a result, individuals decrease schooling investments. Now think that individuals might decide to become entrepreneurs in the future. Think also that this activity entails running a firm for which capital needs to be raised, and that raising capital is subject to a collateral constraint. In addition to the standard effect, two new effects contribute to reducing schooling investments: one is that constrained entrepreneurs run their firms at inefficiently small scale, and this lowers the marginal return of education; the other is that, facing the possibility of being constrained in the future, individuals will want to build collateral rather than invest in education. We find that taking entrepreneurship into account is essential for understanding schooling outcomes. Our theory predicts large effects of financing frictions on the returns to schooling and schooling outcomes, not so much of workers, but fundamentally of entrepreneurs.

Conversely, we also find that schooling decisions play a crucial role in understanding entrepreneurial and production outcomes. As constrained entrepreneurs under-invest in schooling and reduce their human capital, firm productivity declines. Schooling provides

[1]Although these papers do not feature credit constraints, there is a vast literature studying the effects of credit constraints on schooling investments. Recent examples are Lochner and Monge-Naranjo (2011) and Córdoba and Ripoll (2013).

an amplification mechanism of the effect of financing frictions on production outcomes.

We also analyze the role of endogenous schooling decisions for the extent of resource misallocation. Most models of misallocation take the distribution of firm-level productivity as given. This distribution, however, is likely to depend on the extent of resource misallocation itself. Our model provides a mechanism along these lines. Once again, in an attempt to overcome the effect credit frictions, financially constrained entrepreneurs sacrifice schooling investments, thereby reducing firm-level productivity. The most constrained entrepreneurs in the model experience the largest reductions in productivity, which in turn mitigates the effects of resource misallocation.

1.1 Related Literature

Our paper is related to two large strands of literature. First, several recent papers try to understand the role of financing frictions and resource misallocation for production outcomes. Examples are Castro *et al.* (2004, 2009), Erosa and Hidalgo-Cabrillana (2008), Amaral and Quintin (2010), Buera *et al.* (2011), Greenwood *et al.* (2013), Midrigan and Xu (2013), Moll (2014), and Moll *et al.* (2013). Our model shares with this literature the mechanism by which limited access to external funding limits creation and growth of entrepreneurial firms and makes the allocation of resources across firms dependent on entrepreneurs' wealth in addition to their talent. In contrast to these studies, we incorporate human capital accumulation which allows us to study cross-country differences in schooling and explicitly take into account their effects on countries' levels of output per capita.

Second, there is a recent literature emphasizing the role of cross-country TFP differences for schooling decisions. Examples are Manuelli and Seshadri (2014) and Erosa *et al.* (2010). Our model shares with these papers the feature that, in addition to time, expenditure in goods (or education quality) is also a key input into the human capital accumulation process. Allowing a role for adjustments in the quality of education is an important way to generate large cross-country differences in educational attainment. In parallel to this human capital investments literature, there is also a literature studying the role of credit constraints in affecting educational decisions. An early example is Galor and Zeira (1993), and more recent developments are contained in Lochner and Monge-Naranjo (2012) and Córdoba and Ripoll

(2013). As in these papers, the presence of credit constraints in our model forces poor individuals to reduce current consumption in order to invest in schooling, and therefore increases the cost of education. In contrast to these papers, we introduce occupational choice decision and a collateral constraint limiting external financing of entrepreneurial firms. This allows us to study the effects of financing frictions on schooling of future entrepreneurs. Finally, Bhattacharya *et al.* (2013) consider the possibility that entrepreneurs invest in managerial skills over time in a setting with distortions on firm size. As in their paper, the distribution of firm-level measured total factor productivity in our model arises endogenously from entrepreneurs investing in human capital. In contrast to their framework, in our model distortions in firm size arise endogenously as a consequence of an interaction between a collateral constraint and saving decisions of the entrepreneurs. In addition, we take into account human capital accumulation by workers.

Our paper is also related to the resource misallocation literature, namely Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). These authors examine the aggregate productivity consequences of misallocation generated by firm-specific taxes and subsidies. These taxes and subsidies are effectively stand-in, generic distortions, meant to capture deeper allocative problems. Our model concentrates on one such allocative problem: malfunctioning credit markets. We provide an explicit mapping between fundamental distortions coming out of our model, and the stand-in taxes and subsidies that are typically considered in this literature. We are therefore able to apply Hsieh and Klenow's (2009) tools for measuring resource misallocation to our model economy. We find that the extent of resource misallocation is large, both in the production and in the schooling sector.

1.2 Motivating Facts

We now briefly summarize several key stylized facts that motivate our study and against which we will evaluate our quantitative model.

1. *In the U.S.*

   (a) Positive correlation between family income and schooling. In the early 1980s income didn't play a large role on schooling independently from ability and family background, however this effect is much larger in the early 2000s. Wealth seems to play a role similar to income (Lochner and Monge-Naranjo, 2011, Section 3.1).

2. *Cross-country*

   (a) Positive correlation between capital-output ratios, TFP, and per capita incomes (Hsieh and Klenow, 2010).

   (b) Positive correlation between schooling and per capita incomes (Hsieh and Klenow, 2010, Figure 7).

   (c) Negative correlation between rate of entrepreneurship and per capita incomes (Gollin, 2002, Figure 3).

   (d) Returns to schooling are uncorrelated with per capita incomes (Banerjee and Duflo, 2005).

   (e) Firm-size distribution is right-skewed in poor countries (mass concentrated on the left) (Hsieh and Klenow, 2010, Figure 9).

   (f) Marginal products of capital and labor are more dispersed in poor countries (Hsieh and Klenow, 2009).

2 Model

2.1 The Environment

Consider an economy with measure one of altruistic dynasties that value stochastic aggregate household consumption streams according to

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right). \tag{1}$$

The period utility function u is of class C^2, is strictly increasing, strictly concave, and satisfies the usual Inada conditions. An individual lives for 2 periods, childhood and adulthood. We call childhood the period when schooling and investment decisions are made, and adulthood the period when the individual's main economic activity is carried out.

The household, composed of a child and an adult parent, is the decision unit. In anticipation of our recursive formulation, we use primes to denote variables which pertain to the next generation, whereas those without primes refer to the current one. The household starts the period with wealth ω, and a draw of the child's abilities, current learning ability z and future entrepreneurial ability x. The inter-generational ability transmission is governed by a first-order Markov chain with transition probabilities $\pi(z', x'|z, x)$.

Given the state (ω, z, x), the household makes four decisions. First, it decides today's investment in the child's education, by choosing years schooling s and schooling expenditures e to produce human capital according to

$$h = z\left(s^{\eta} e^{1-\eta}\right)^{\xi}, \tag{2}$$

with $\eta \in [0,1]$ and $\xi \in [0,1)$.[2] We follow Erosa *et al.* (2010) and Manuelli and Seshadri (2014) in considering expenditures as an input to human capital accumulation in addition to time. This will imply that an individual's education increases with wages, since higher wages effectively reduce the relative price of the goods input, therefore increasing the net return on schooling investments.

Second, it decides today's saving for next period, by purchasing bonds in net amount q at unit price $1/(1+r)$.[3] Third, it decides the child's occupation for next period, whether to become an entrepreneur or a worker. Fourth, if the decision is to become an entrepreneur next period then the household also needs to raise capital, possibly relying in part on external funds, and hire labor in order to run the firm.

All production is carried out by entrepreneurs according to

$$y = Axh^{\theta}\left(k^{\alpha} l^{1-\alpha}\right)^{\gamma}, \tag{3}$$

with $\alpha, \gamma, \theta \in (0,1)$, where k and l denote physical capital and labor inputs, and A is an

[2]A more general formulation would be $h = z\left(s^{\eta} e^{1-\eta}\right)^{\xi} \bar{h}_0^{\zeta} + (1-\mu)\bar{h}_0$. Here we assume full depreciation ($\mu = 1$). The component $\bar{h}_0^{\zeta}$ gets incorporated into mean learning ability in our specification. Intergenerational transmission of human capital is captured by persistence in learning ability.

[3]Given our assumption on the resolution of uncertainty, saving is contingent upon x, the child's entrepreneurial ability for next period. This timing simplifies the analysis by, at least partially, allowing us characterize household investment decisions via simple non-arbitrage conditions.

aggregate TFP component common to all entrepreneurs. Entrepreneurial productivity is given by xh^{θ}, of which x is determined by luck, and h is the accumulated human capital. Physical capital depreciates at rate $\delta \in (0,1)$.

2.2 Household's Problem

We focus on stationary equilibria, in which prices and the distribution over individual states are invariant. Denote by w the wage rate (unit price of human capital) and by r the real interest rate. We begin by formulating the household's problem conditional on the child's occupational choice. We find it convenient to consider the occupational choice before the remaining decisions. Since all uncertainty is resolved at the start of an individual's life, there is no loss in doing so.

Conditional on the child becoming a worker next period, the *worker-household*'s problem can be written recursively as:

$$v^{w}(\omega, z, x) = \max_{c,e,s,q} \left\{ u(c) + \beta \sum_{z',x'} \pi(z', x' | z, x)\, v(\omega', z', x') \right\} \tag{Pw}$$

subject to (2) and

$$c + ws\bar{l} + e + \frac{1}{1+r} q = w\bar{h}_0 (1-s) + \omega \tag{4}$$

$$s \leq 1 \tag{5}$$

$$q \geq -\phi\omega \tag{6}$$

$$\omega' \equiv wh + q. \tag{7}$$

Equation (4) is the budget constraint. The term $ws\bar{l} + e$ is the direct cost of investing in the child's education, composed of tuition fees $ws\bar{l}$ ($s\bar{l}$ is total teacher time) and expenditures in education quality e. On the right-hand-side, $w\bar{h}_0(1-s)$ is the child's labor earnings, where $\bar{h}_0$ is the child's initial human capital. Equation (5) is the child's time constraint (our assumptions on preferences and human capital technology allow us to ignore the non-negativity constraints on consumption, time, and schooling expenditures since they will never bind). Equation (6) is the inter-period *borrowing constraint*. Households can only borrow up to a multiple $\phi \geq 0$ of their wealth.[4] When $\phi = 0$ no borrowing is allowed, and investment must be completely funded out of the household's wealth; when $\phi = \infty$ financial markets work perfectly. Finally, equation (7) defines the initial wealth of the next household in the dynastic line, conditional on the fact that next period's parent will be a worker.

Similarly, conditional on the child becoming an entrepreneur next period, the *entrepreneur-household*'s problem reads:

$$v^e(\omega, z, x) = \max_{c,e,s,q} \left\{ u(c) + \beta \sum_{z',x'} \pi(z', x'|z, x)\, v(\omega', z', x') \right\} \tag{Pe}$$

subject to (2), (4), (5), (6) and a new definition of household's wealth which is now based on entrepreneurial profits

$$\omega' \equiv \Pi(q, h, x) + q, \tag{8}$$

where

$$\Pi(q, h, x) = \max_{k,l \geq 0} \left\{ Axh^{\theta} \left(k^{\alpha} l^{1-\alpha}\right)^{\gamma} - (r + \delta)\, k - wl \right\} \tag{Pf}$$

subject to

$$k \leq \lambda q, \tag{9}$$

with $\lambda \geq 1$. Entrepreneurs hire capital and labor to maximize profits, subject to an intra-period *capital constraint*. The maximum level of capital an entrepreneur can use in production is given by a multiple λ of the household's second period wealth, which acts as collateral.[5] When $\lambda = 1$ no external funding is allowed, and capital is solely determined by

[4]This constraint can be motivated by a simple static limited enforcement problem. Suppose an individual borrows $-q > 0$ and then decides whether to default. The only penalty is that financial intermediaries may seize a fraction $\nu \in [0, 1]$ of total wealth, including the amount just borrowed. Intermediaries then require that the gain from defaulting does not exceed the cost, that is $-(1 - \nu)q \leq \nu\omega$. This yields (6) with $\phi \equiv \nu/(1 - \nu) \geq 0$. The main advantage from using this simple specification is tractability. It shares with self-enforcing limits based on dynamic incentives (Kehoe and Levine, 1993) the key feature that richer individuals are able to borrow more.

[5]Constraint (9) implies that households that borrow today will not be able to run a firm tomorrow. As a result, only children from a significantly wealthy background can ever become entrepreneurs. A less severe constraint would allow entrepreneurs to also pledge a fraction of their profits as collateral. Similarly to (6), the constraint (9) may be motivated by a simple static limited enforcement problem. As in Buera and Shin (2013), suppose individuals borrow k from financial intermediaries against collateral q, and then have a decision whether to default. The only penalty is that intermediaries may seize the entire collateral, plus a

internal funds. When $\lambda = \infty$ financial markets work perfectly, and capital is not constrained by wealth.

Financing frictions affect the model via (6) and (9). We let a different parameter control the degree of frictions in each different credit market, to reflect the possibility that seizing wealth upon default, for example, might be easier in one situation compared to the other. An economy is unambiguously subject to more frictions only when both ϕ and λ are lower.

We finally consider the household's occupational choice for the child next period, which is given by:

$$v\left(\omega, z, x\right) = \max\left\{v^{w}\left(\omega, z, x, \right), v^{e}\left(\omega, z, x\right)\right\}. \tag{10}$$

2.3 Competitive Equilibrium

Definition 1. A stationary recursive competitive equilibrium is a set of value functions $v^{w}\left(\omega, z, x\right)$, $v^{e}\left(\omega, z, x\right)$, and $v\left(\omega, z, x\right)$, together with the associated decision rules, a set of entrepreneurial households B, prices w and r, and an invariant distribution over household states Ψ such that given prices,

- $v^{w}\left(\omega, z, x\right)$ and $v^{e}\left(\omega, z, x\right)$ solve problems (Pw) and (Pe), respectively, and $v\left(\omega, z, x\right)$ solves (10),

- the set of entrepreneur-households is defined by the optimal occupational choice rule:

$$B = \left\{\left(\omega, z, x\right) \in S \mid v^{e}\left(\omega, z, x\right) > v^{w}\left(\omega, z, x\right)\right\}, \tag{11}$$

  where $S \equiv \mathbb{R}^{3}_{+}$ is the individual household's state space,[6]

- market for labor clears:

$$\int_{B} l d\Psi + \int s\bar{l} d\Psi = \int_{S\backslash B} h d\Psi + \int \left(1 - s\right) \bar{h}_{0} d\Psi, \tag{12}$$

fraction of κ of k. No default requires $(1-\kappa)k \leq q$, which yields (9) with $\lambda \equiv 1/(1-\kappa) \geq 1$. Related work using identical collateral constraints include Evans and Jovanovic (1989), Moll (2014) and Moll *et al.* (2013).

[6] The credit constraint guarantees that ω is always positive and thus the household's state space can be restricted to $\mathbb{R}^{3}_{+}$. Since $\omega' \equiv \max\left\{\Pi\left(x, h, q\right), wh\right\} + q$, the borrowing constraint $q \geq -\phi\omega$ implies a lower bound of $\max\left\{\Pi\left(x, h, q\right), wh\right\} / \left(1 - \phi\right) > 0$ for ω.

- market for capital clears:

$$\int_B kd\Psi = \int \frac{q}{1+r} d\Psi, \tag{13}$$

- market for goods clears:

$$\int cd\Psi + \int ed\Psi + \delta \int_B kd\Psi = \int_B Axh^\theta \left(k^\alpha l^{1-\alpha}\right)^\gamma d\Psi, \tag{14}$$

- distribution Ψ is invariant:

$$\Psi\left(\hat{S}\right) = \int_S P\left(X, \hat{S}\right) d\Psi\left(X\right) \text{ for all } \hat{S} \in \mathcal{B}_S, \tag{15}$$

  where $P : S \times \mathcal{B}_S \to [0,1]$ is a transition function generated by the decision rules and the stochastic processes for z and x, and $\mathcal{B}_S$ is the Borel σ-algebra of subsets of S.

3 Analysis

In this section we present an analysis of individual optimal decisions. We start with entrepreneurial production decisions and proceed by backward induction to schooling and savings decisions. We finish with the occupational choice.

Our analysis proceeds under the following assumption on the production and human capital accumulation technologies:

Assumption 1. $\xi\theta < 1 - \gamma$.

This assumption ensures that decreasing returns to schooling expenditures set in fast enough so that the optimization problem is concave in e even in the case capital-unconstrained entrepreneur-households have increasing returns to human capital accumulation. This guarantees the human capital accumulation problem to have a well-defined optimum.

3.1 Production

Given their human capital, entrepreneurs hire labor and capital to maximize their profits. The presence of the capital constraint implies that the profit function will differ for constrained and unconstrained entrepreneurs:

$$\Pi(q,h,x)=\begin{cases}\Pi^{*}(h,x) & \text{if } q\geq q^{*}(h,x) \quad \text{(unconstrained)}\\ \Pi^{c}(q,h,x) & \text{else} \quad \text{(constrained)},\end{cases} \tag{16}$$

where

$$\begin{aligned}
q^{*}(h,x) &= k^{*}/\lambda\\
k^{*} &= \left[\frac{(1-\alpha)(r+\delta)^{1-\frac{1}{(1-\alpha)\gamma}}}{\alpha w}\right]^{(1-\alpha)\frac{\gamma}{1-\gamma}}(\alpha\gamma A x)^{\frac{1}{1-\gamma}}\,h^{\frac{\theta}{1-\gamma}}\\
l^{*} &= \frac{(1-\alpha)(r+\delta)}{\alpha w}k^{*}\\
y^{*} &= Axh^{\theta}\left((k^{*})^{\alpha}(l^{*})^{1-\alpha}\right)^{\gamma}\\
\Pi^{*}(h,x) &= y^{*}-wl^{*}-(r+\delta)k^{*}\\
&\equiv X_{1}(x)\,h^{\frac{\theta}{1-\gamma}}
\end{aligned} \tag{17}$$

with $X_1(x)$ a function of production function parameters and factor prices, and

$$\begin{aligned}
k^{c} &= \max\{\lambda q,0\}\\
l^{c} &= \left[\frac{\gamma(1-\alpha)A(k^{c})^{\alpha\gamma}}{w}xh^{\theta}\right]^{\frac{1}{1-(1-\alpha)\gamma}}\\
y^{c} &= Axh^{\theta}\left((k^{c})^{\alpha}(l^{c})^{1-\alpha}\right)^{\gamma}\\
\Pi^{c}(q,h,x) &= y^{c}-wl^{c}-(r+\delta)k^{c}\\
&\equiv B_{2}(q,x)\,h^{\frac{\theta}{1-(1-\alpha)\gamma}}-B_{1}q
\end{aligned} \tag{18}$$

with $B_2(q,x)$ and B_1 also functions of production function parameters and factor prices.[7] The constrained profit function is increasing in accumulated assets since higher q allows to raise more capital and increase the scale of the entrepreneurial firm closer to its optimal level.

[7] The expressions for $X_1(x)$, $B_2(q,x)$, and B_1 can be found in Appendix A.

3.2 Schooling/Saving Decisions

In our framework, it is natural to think about schooling and savings as investments into two different assets (human and physical capital). Our timing assumption allows us to characterize different investment opportunities in terms of simple non-arbitrage conditions that transpire from the first-order optimality conditions for problems (Pw) and (Pe) with respect to s, e, and q. Financing frictions will show up as wedges in these conditions.

The first-order conditions read:[8]

$$-\mu + w\left(\bar{l} + \bar{h}_0\right) u'(c) = \beta \sum_{z',x'} \pi\left(z', x' | z, x\right) v_1\left(\omega', z', x'\right) \omega_2'\left(q, h, x\right) \eta \xi \frac{h}{s} \tag{19}$$

$$u'(c) = \beta \sum_{z',x'} \pi\left(z', x' | z, x\right) v_1\left(\omega', z', x'\right) \omega_2'\left(q, h, x\right) (1-\eta) \xi \frac{h}{e} \tag{20}$$

$$\nu + \frac{1}{1+r} u'(c) = \beta \sum_{z',x'} \pi\left(z', x' | z, x\right) v_1\left(\omega', z', x'\right) \omega_1'\left(q, h, x\right), \tag{21}$$

where μ and ν are the Lagrange multipliers of the time constraint (5) and the borrowing constraint (6), respectively. These equations apply conditional on either occupation, only the derivatives of future wealth with respect to saving and human capital, respectively ω_1' and ω_2', differ across worker and entrepreneur households.

Combining (19) and (20) allows us to obtain schooling time as a function of schooling expenditures:

$$s(e) = \min\left\{ \frac{\eta}{(1-\eta) w \left(\bar{l} + \bar{h}_0\right)} e, 1 \right\} \tag{22}$$

and inserting this into the human capital production function we have

$$h(z, e) = z \left(s(e)^{\eta} e^{1-\eta}\right)^{\xi}. \tag{23}$$

Combining (20), (21), and (23) gives us a non-arbitrage condition equating the returns to

[8] Notice that v_1 is always defined at the optimum. Even though v has a kink in the wealth dimension induced by the occupational choice, the optimum will never occur at this kink. It follows that, at the optimum, v_1 is either equal to v_1^w or to v_1^e. Notice also that, with sufficient smoothness introduced by the ability shocks, which we assume, the first-order conditions are not only necessary but also sufficient for an optimum. See Clausen and Strub (2013) for a formal discussion.

physical and human capital accumulation:

$$\nu + (1+r)\,\omega_1'\left(q, h\left(z,e\right), x\right) = (1-\eta)\,\xi \frac{h\left(z,e\right)}{e}\omega_2'\left(q, h\left(z,e\right), x\right). \tag{24}$$

Specializing equation (24) for each occupation allows us to characterize the optimal schooling decisions for worker and entrepreneur households. In either case, Assumption 1 is sufficient to ensure that a unique well-defined solution for schooling expenditures exists.

3.2.1 Worker-Household

For a worker-household we have

$$\omega_1'\left(q,h,x\right) = 1 \text{ and } \omega_2'\left(q,h,x\right) = w. \tag{25}$$

As workers, all individuals have the same constant returns to human capital accumulation since wages are linear in worker's human capital. If the borrowing constraint does not bind ($q < -\phi\omega$ and $\nu = 0$), then we can substitute (25) in (24) and solve for the optimal schooling expenditures:

$$e^w\left(z\right) = \begin{cases} \left[\frac{w(1-\eta)\xi z}{1+r}\left(\frac{\eta}{(1-\eta)w\left(\bar{l}+\bar{h}_0\right)}\right)^{\xi\eta}\right]^{\frac{1}{1-\xi}} & \text{if } s\left(e\right) < 1, \\ \left[\frac{w(1-\eta)\xi z}{1+r}\right]^{\frac{1}{1-\xi(1-\eta)}} & \text{if } s\left(e\right) = 1. \end{cases} \tag{26}$$

The condition $\xi < 1$ ensures that the second-order conditions for a maximum are satisfied and (26) is a well-defined solution.

If instead the borrowing constraint binds ($q = -\phi\omega$ and $\nu > 0$), then the worker-household's optimal $e^w\left(z,\omega\right)$ solves:

$$\begin{aligned} \max_e \Bigg\{ u\left(\omega - e - w\left(s(e)\bar{l} - \bar{h}_0\left(1-s(e)\right)\right) + \frac{1}{1+r}\phi\omega\right) \\ + \beta \sum_{z',x'} \pi\left(z',x'|z,x\right) v\left(wh(z,e) - \phi\omega, z', x'\right) \Bigg\} \end{aligned} \tag{27}$$

where $s(e)$ and $h(z,e)$ are given respectively by (22) and (23). Notice that, in contrast to the unconstrained case, the optimal schooling expenditures of a credit-constrained worker-household depends on current wealth ω.

3.2.2 Entrepreneur-Household

The capital constraint (9), together with the condition that $k \geq 0$, implies that entrepreneur-households will always have $q > 0$ and therefore will never be credit-constrained. We have:

$$\omega_1' (q, h, x) = \begin{cases} 1 + \Pi_1^c (q, h, x) & \text{if } q \in (0, q^* (h, x)), \\ 1 & \text{if } q \geq q^* (h, x), \end{cases} \tag{28}$$

and

$$\omega_2' (q, h, x) = \begin{cases} B_2 (q, x) \frac{\theta}{\alpha\gamma + 1 - \gamma} h^{\frac{\theta}{\alpha\gamma+1-\gamma} - 1} & \text{if } q \in (0, q^* (h, x)), \\ X_1 (x) \frac{\theta}{1-\gamma} h^{\frac{\theta}{1-\gamma} - 1} & \text{if } q \geq q^* (h, x). \end{cases} \tag{29}$$

From (28) and (29) we can deduce how the marginal returns to physical and human capital accumulation vary with the entrepreneur-household's saving q. We obtain following results.

Proposition 1. *Given h, capital-constrained entrepreneur-households (with $q < q^* (x, h)$) face a higher marginal return to physical capital accumulation and a lower marginal return to human capital accumulation than unconstrained entrepreneur-households (with $q \geq q^* (x, h)$).*

Proof. The first part of the proposition follows from (28), the fact that $\Pi_1^c (q, h, x)$ is decreasing in q, and that $\Pi_1^c (q^* (h, x) h, x) = 0$. The second part follows from (29), the fact that $B_2 (q, x)$ is increasing in q, and that $B_2 (q^* (h, x), x) \frac{\theta}{\alpha\gamma+1-\gamma} h^{\frac{\theta}{\alpha\gamma+1-\gamma} - 1} = X_1 (x) \frac{\theta}{1-\gamma} h^{\frac{\theta}{1-\gamma} - 1}$.

□

The intuition behind the first part of Proposition 1 is that, for capital-constrained entrepreneur-households, saving relaxes the capital constraint and allows them to expand their firms closer to the optimal unconstrained scale. The second part holds because human capital and physical capital are complementary in production. Capital-constrained entrepreneur-households employ less capital, making their human capital less productive.

Proposition 1 shows how the capital constraint distorts saving and schooling decisions of entrepreneur-households. These households have an incentive to save more and invest less in education compared to unconstrained entrepreneur-households.

Substituting ω_1' and ω_2' for the case of $q \geq q^*(h,x)$ into (24) yields the optimal schooling expenditures for capital-unconstrained entrepreneur-households:

$$e^e(z,x) = \begin{cases} \left[\frac{X_1(x)}{1+r} \frac{\theta}{1-\gamma} (1-\eta)\, \xi z^{\frac{\theta}{1-\gamma}} \left(\frac{\eta}{(1-\eta) w \left(\bar{l} + \bar{h}_0 \right)} \right)^{\frac{\eta\xi\theta}{1-\gamma}} \right]^{\frac{1}{1-\frac{\xi\theta}{1-\gamma}}} & \text{if } s(e) < 1, \\ \left(\frac{X_1(x)}{1+r} \frac{\theta}{1-\gamma} (1-\eta)\, \xi z^{\frac{\theta}{1-\gamma}} \right)^{\frac{1}{1-\frac{(1-\eta)\xi\theta}{1-\gamma}}} & \text{if } s(e) = 1. \end{cases} \tag{30}$$

Analogously, we may replace ω_1' and ω_2' for the case of $q \in (0, q^*(h,x))$ into (24) to obtain the optimal schooling expenditures for capital-constrained entrepreneur-households, $e^e(q,z,x)$. This is the solution to a nonlinear equation for which an explicit form is not available.[9] Contrary to the unconstrained case, optimal spending in education now depends on household wealth, via saving q. This is so because higher wealth (and saving) relaxes the capital constraint and reduces the associated distortions in the investment and schooling decisions.

3.3 Occupational Choice

We find it analytically convenient to characterize the occupational choice conditional on the child's entrepreneurial ability x, human capital h, and saving q. An advantage of this characterization is that we can establish a direct contrast with the existing literature on entrepreneurship with credit constraints, namely Evans and Jovanovic (1989) and Buera and Shin (2013). In the latter work, the occupational choice is determined by q and x alone. Conditional on x, financing frictions imply that only sufficiently wealthy individuals decide to become entrepreneurs. In Evans and Jovanovic (1989), workers are also heterogeneous in their labor productivity, which is independent from entrepreneurial ability.

Our setting adds a nontrivial dimension to the occupational choice decision. Conditional on entrepreneurial ability, it depends not only on financial wealth, but also on human capital. The substantive difference between human capital and entrepreneurial ability for selection is that a higher human capital level benefits both occupations, whereas higher entrepreneurial

[9] Assumption 1 again ensures that a unique solution exists to the nonlinear equation.

ability only benefits entrepreneurship. Our next proposition summarizes our characterization of the occupational choice decision as a function of (h,q), for given x.

Proposition 2. *Occupational choice is characterized by two cut-off functions $\bar{h}(x)$ and $\bar{q}(h,x)$ defined by:*

$$\Pi^*\left(\bar{h}(x),x\right) = w\bar{h}(x),$$
$$\Pi^c\left(\bar{q}(h,x),h,x\right) = wh.$$

The particular pattern of selection into entrepreneurship depends on the curvature of the production function. Entrepreneur-households have states (ω,z,x) such that

$$\begin{cases} h(\omega,z,x) \geq \bar{h}(x) & \text{if } 1-\gamma \leq \theta, \\ h(\omega,z,x) \leq \bar{h}(x) & \text{if } 1-\gamma \geq \theta, \end{cases}$$

and

$$q(\omega,z,x) \geq \bar{q}\left(h(\omega,z,x),x\right).$$

Proof. We prove the characterization for the case $1-\gamma < \theta$, the complementary one is entirely analogous. First, if $h(\omega,z,x) < \bar{h}(x)$, then $wh \geq \Pi^*(h,x) \geq \Pi^c(q,h,x)$. The first inequality follows from the fact that when $1-\gamma<\theta$, $\Pi^*(h,x)$ is increasing and convex in h. The second inequality follows from the fact that $\Pi^*(h,x)$ is the unconstrained maximum profit. Thus, being an entrepreneur in this region leads to lower income than being a worker, and cannot be optimal.

Second, if instead $h(\omega,z,x) \geq \bar{h}(x)$ and $q(\omega,z,x) \geq q^*(h(\omega,z,x),x)$, then $\Pi^*(h,x) > wh$ and the household becomes an (unconstrained) entrepreneur.

Third, if $h(\omega,z,x) \geq \bar{h}(x)$ and $q(\omega,z,x) < q^*(h(\omega,z,x),x)$, then profits are given by $\Pi^c(q,h,x)$. In that case, if $q(\omega,z,x) < \bar{q}(h(\omega,z,x),x)$, then $wh > \Pi^c(q,h,x)$, which follows from the fact that $\Pi^c(q,h,x)$ is strictly increasing in q in this region (higher q relaxes the capital constraint). Thus, being an entrepreneur leads to lower income than being a worker, and cannot be optimal. $\square$

Given x, the occupational choice decision can be plotted in the (h,q) space. In that space,

$\bar{h}(x)$ is a vertical line, whereas $\bar{q}(h,x)$ is U-shaped. It is also convenient to plot $q^*(h,x)$, which gives the threshold level of saving that allows for unconstrained entrepreneurial operation. The pattern of the selection into entrepreneurship will be determined by two parametric restrictions.

First, the comparison between θ and $1-\gamma+\alpha\gamma$ determines the shape of $\bar{q}(h,x)$. If $\theta < 1-\gamma+\alpha\gamma$, then $\Pi^c(q,h,x)$ is concave in h, implying a convex and eventually increasing $\bar{q}(h,x)$. The intuition is that, for a given q, higher h households have a comparative advantage in working for wage, otherwise their firms would be run on an inefficiently small scale. If instead $\theta > 1-\gamma+\alpha\gamma$, then $\Pi^c(q,h,x)$ is convex in h implying a concave and eventually decreasing $\bar{q}(h,x)$. In that case, households with higher h have a comparative advantage in entrepreneurship.

Second, as stated in Proposition 2, the comparison between θ and $1-\gamma$ determines on which side of the $\bar{h}(x)$ line households prefer entrepreneurship, as well as the shape of $q^*(h,x)$. If $1-\gamma < \theta$, then $\Pi^*(h,x)$ is convex in h, implying a convex $q^*(h,x)$, and households with $h < \bar{h}(x)$ always choose working for wage. The intuition is that the returns to investment in h are increasing if the household becomes a capital-unconstrained entrepreneur, but they are constant if it becomes a worker. If instead $1-\gamma > \theta$, then $\Pi^*(h,x)$ is concave in h, implying a concave $q^*(h,x)$, and households with $h > \bar{h}(x)$ always choose working for wage. The returns to investment in h are now decreasing for capital-unconstrained entrepreneurs.

In Figure 1, and in the rest of the paper, we work with the case $1-\gamma < \theta < 1-\gamma+\alpha\gamma$, which we conjecture is the relevant one empirically.[10] Under these restrictions the production function has increasing returns with respect to (h,k,l), but decreasing returns with respect to (h,l). The former is relevant when capital is free to adjust (the entrepreneur operates at the optimal scale), whereas the latter applies when capital is fixed (the entrepreneur is capital-constrained). However, all the mechanisms exposed in this and the following sections are robust to the type of restriction we impose on these parameters.

Refer to Figure 1.[11] In the shaded area above $\bar{q}(h,x)$ and to the right of $\bar{h}(x)$, households

[10] Our parameterization is to be determined by our calibration procedure. Under this particular parameterization, our characterization of selection for a given distribution of h across individuals is in fact similar to Evans and Jovanovic's (1989).

[11] A more detailed description of the construction of Figure 1 can be found in Appendix B.

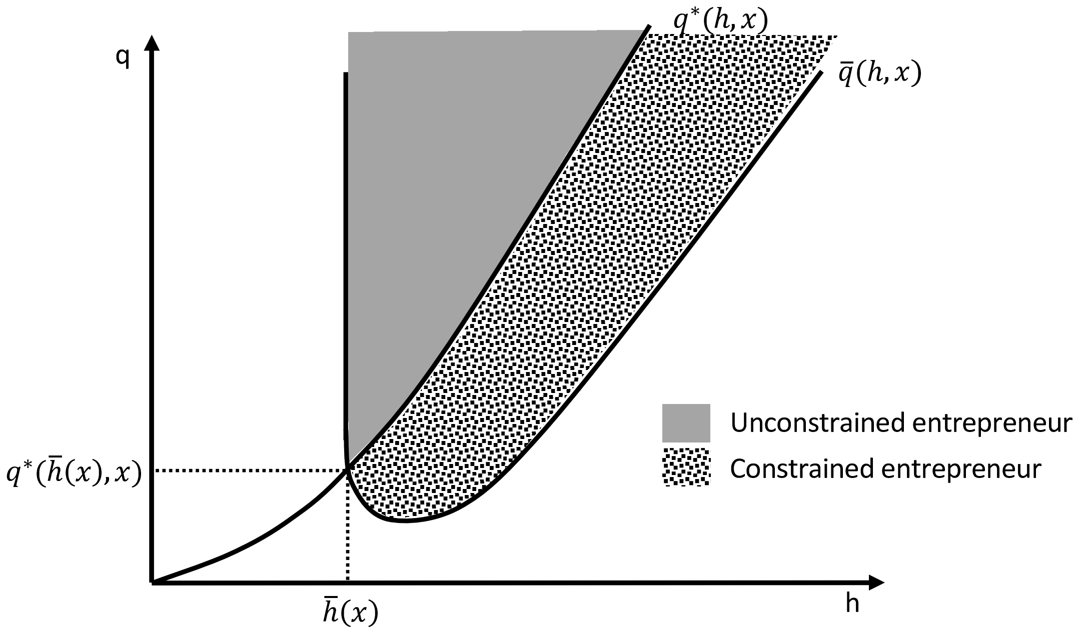


Figure 1: Occupational Choice

choose entrepreneurship. They are unconstrained in the area above $q^*(h,x)$, and constrained in the area below. Below $\bar{q}(h,x)$ and to the left of $\bar{h}(x)$, households choose to be workers. Due to the capital constraint, entrepreneurship is profitable only when investments in human capital are matched by adequate investments in physical capital.

It is worth considering how Figure 1 would look like in an economy without financing frictions. In this case q would not affect occupational choice, and the only relevant threshold would be the full vertical line $\bar{h}(x)$. Households with $h \geq \bar{h}(x)$ become entrepreneurs, else they become workers. We analyze the frictionless benchmark in more detail next.

3.4 The Frictionless Benchmark

In this section we summarize the main features of our economy under the benchmark of no financial frictions. Every household in this economy behaves in the way prescribed by the unconstrained decisions we obtained in the previous sections. It follows that schooling and production decisions depend solely on abilities, with wealth playing no role. Similarly, occupational choice decisions are also based on ability differences alone. Selection into entrepreneurship in fact admits a simple, explicit characterization.

To facilitate the exposition, concentrate on the parametric case in which the time constraint is not binding. It is easy to show that workers choose

$$h(z) = C_w z^{\frac{1}{1-\xi}},$$

and that entrepreneurial profits are

$$\Pi^*(z, x) = C_e x^{\frac{1}{1-\gamma}} z^{\frac{1-\gamma}{1-\gamma-\xi\theta}},$$

where $C_w, C_e > 0$ are functions of parameters and equilibrium prices.

The ability types indifferent between the two occupations thus satisfy

$$x = \left(\frac{wC_w}{C_e}\right)^{1-\gamma} z^{(1-\gamma)\left(\frac{1}{1-\xi} - \frac{1}{1-\xi\frac{\theta}{1-\gamma}}\right)}.$$

The sign of the exponent on learning ability depends critically on the term $\theta/(1-\gamma)$. This terms governs the nature of the returns to human capital accumulation for unconstrained entrepreneurs, as equation (17) makes clear. Profits are linear in human capital when $\theta = 1-\gamma$, and the returns to human capital accumulation are therefore constant for entrepreneurs. Since the same is true for workers, it follows that learning ability z is not a determinant of occupational choice. Households with entrepreneurial ability $x \geq (wC_w/C_e)^{1-\gamma}$ choose entrepreneurship, else they'll work for a wage.

When $\theta > 1-\gamma$, there are increasing returns to human capital accumulation. In this case, households with higher learning ability have a comparative advantage in becoming entrepreneurs, and z intervenes in the occupational choice. Households with higher z now have a lower cutoff x for becoming entrepreneurs. This favors the selection of higher learning ability types into entrepreneurship. Since data is consistent with better educated individuals selecting into entrepreneurship (Dunn and Holtz-Eakin, 2000; Hurst and Lusardi, 2004), our analysis emphasizes this parametric case.

4 Calibration

Our strategy is similar to Buera and Shin's (2013). We first calibrate the benchmark economy to the U.S. and then vary the financial friction parameters, holding the remaining parameters constant, in order to display the observed cross-country variation in the ratio of external finance to output.

We illustrate some of the model's implications for a given set of parameter values. We do not have a full calibration of the model at this stage. Our parameters are described in Table 1.

The first-order Markov chain governing abilities is obtained from the discretization of a VAR(1) in logs where

$$\ln z_{t+1} - \bar{z} = \rho_z \left(\ln z_{t+1} - \bar{z}\right) + \varepsilon^z_{t+1}$$
$$\ln x_{t+1} - \bar{x} = \rho_x \left(\ln x_{t+1} - \bar{x}\right) + \varepsilon^x_{t+1}$$

and the disturbances have variance-covariance matrix

$$\mathbf{\Sigma} = \begin{pmatrix} \sigma_z^2 & \sigma_{zx} \\ \sigma_{zx} & \sigma_x^2 \end{pmatrix}.$$

We employ the procedure described by Tauchen and Hussey (1991), with 15 states for entrepreneurial ability and 4 states for learning ability.

We take one model period to be 30 years. Individuals start life at age 6. The period going from age 6 until age 36 (young age) is where schooling and early working in the labor market take place. The period going from age 36 until retirement age 66 (old age) is where the main economic activity, entrepreneurship or working for a wage, takes place.

Some parameters are calibrated externally to the model. These are in the top block of Table 1. We normalize average ability to 1. The coefficient of relative risk aversion belongs to the interval of available estimates, and is a standard value in quantitative analysis. The rate of depreciation is also standard. The parameters governing the income share of capital (α) and the income share of entrepreneurial income (γ) are also standard in models of entrepreneurship (see for example Atkeson and Kehoe, 2005, Restuccia and Rogerson, 2008, and Buera and Shin, 2013). Finally, we set the autocorrelation coefficient of learning ability to the intergenerational correlation coefficient of IQ scores reported by Bowles and Gintis (2002) for the correlation between the average parental IQ score and the average offspring IQ score.

Parameter	Value	Target	Data	Model
External calibration				
σ	1.0	direct estimates		
δ	0.844	yearly depreciation rate of 6%		
α	1/3	capital income share		
$\bar{x}$	1.0	normalization		
γ	0.85	direct estimates		
ρ_z	0.72	intergenerational correlation of IQ scores		
Internal calibration				
β	0.215	yearly real interest rate	0.04	0.041
θ	0.17	average years of schooling among entrepreneurs	14	19.543
ξ	0.85	average years of schooling among workers	13	7.014
η	0.6	output share of schooling expenditures	0.045	0.056
$\bar{l}$	0.025	output share of teacher and staff compensation	0.05	0.072
$\bar{h}_0$	0.001	average labor earnings at age 46 over average at age 25	1.75	69.26
$\bar{z}$	1.15	entrepreneurship rate	0.1	0.032
σ_z	0.01	standard deviation of years of schooling	2.92	5.11
ρ_x	0.9	intergenerational correlation of entrepreneurship	0.32	0.68
σ_x	0.01	coefficient of variation of establishment size	6.1	1.33
σ_{xz}	0.0	ratio of avg entrepreneurial income to avg worker income	7.67	1.66
λ	3.3	ratio of external finance to output	2.5	7.59
ϕ	1.0	intergenerational correlation of years of schooling	0.46	0.76
		total wealth share of entrepreneurs	0.418	0.205
		employment share of top 5% establishments	0.517	0.285

Table 1: Calibration

The remaining 13 parameters are chosen in order to minimize the sum of squared deviations of 15 data moments from their model analogues. The bottom block of Table 1 shows the values for these parameters, as well as the models success in matching the data moments.[12]

[12]At this stage we have not yet attempted to minimize the distance between the data moments and the model's.

As is common in this type of analysis, we identify each parameter with a moment which we believe is particularly helpful in identifying it, although in the end all parameters are jointly determined through a fairly complex system of nonlinear equations.

We comment on each of the moments we have selected. A yearly real interest rate of 4% is roughly between the real return on riskless bonds and the real return on equity over a long horizon. To compute the average years of schooling of entrepreneurs and workers, we convert the evidence reported by Hurst and Lusardi (2004) for highest grade attained. More specifically, assuming that less than high-school corresponds to 10 years of school on average, high-school to 12 years, some college to 14 years, and college to 16 years, we obtain that entrepreneurs spend approximately 14 years in school, whereas workers spend approximately 13 years. For the output share of (public and private) schooling expenditures we use the same number as Manuelli and Seshadri (2014), and for the output share of teacher and staff compensation we use the same number as Erosa *et al.* (2010). The ratio of average labor earnings at age 50 over the ratio at age 25 comes from Figure 1 of Kambourov and Manovskii (2009), and refers to the cohort entering the labor market in 1968. Guner *et al.* (2008) suggest a rate of entrepreneurship between 5% and 10%, with the former being their preferred estimate. We take a value in between, consistent with estimates reported elsewhere (Fairlie and Robb, 2008; Hurst and Lusardi, 2008). The cross-sectional standard deviation of years of schooling was computed by Erosa *et al.* (2010). The intergenerational correlation of years of schooling is also from Erosa *et al.* (2010), based on a study by Hertz *et al.* (2008), whereas the intergenerational correlation of entrepreneurial occupation is reported by Dunn and Holtz-Eakin (2000). The coefficient of variation of establishment size in the aggregate for the U.S. economy comes from Henly and Sanchez (2009). The ratio of total external finance (including private credit) to output in the U.S. is the same number used by Buera and Shin (2013), which is based on data from Beck *et al.* (2000). The last two moments are the wealth share of entrepreneurs, taken from Hurst and Lusardi (2008), and the employment share of the top 5% establishments from Henly and Sanchez (2009).

Throughout our quantitative analysis, we follow Erosa *et al.* (2010) in concentrating on a measure of output in the model which excludes teacher's salaries. As in their case, our motivation is to avoid the complication of having to select a proper international wage to evaluate schooling costs in the model. In the data, we also compute output net of teacher's salaries.[13]

5 Results

5.1 Allocative Consequences of Financing Constraints

Our first set of results concern the effect of financing constraints on the allocation of resources. In the absence of financing frictions, the wealth distribution should play no role in resource allocation, which should depend only on the distribution of abilities. With financing frictions, the wealth distribution interferes with resource allocation. We now illustrate different types of distortions and discuss their origins.

In what follows, we display model outcomes for a coarse partition of the state space. We focus on two learning ability intervals defined by the median level, on three entrepreneurial ability intervals defined by the distribution terciles, and on four wealth intervals defined by the distribution quartiles.

5.1.1 Misallocation of Talent

Entrepreneurship rates in the two model versions, with and without financing frictions, are shown in Figure 2. We concentrate on the subset of our partition the state-space where entrepreneurship rates per cell are strictly positive. This subset happens to be the same in both model versions.

Start with the economy without frictions. Only individuals with sufficiently high entrepreneurial ability become entrepreneurs. Among these, individuals with higher learning ability are more likely to be entrepreneurs. Recall that our parameterization imposes $1 - \gamma < \theta$ which, as highlighted in Section 3.3, means that returns to human capital accumulation are increasing for unconstrained entrepreneurs. This implies that individuals with higher z have a comparative advantage at becoming entrepreneurs in the economy without frictions.

[13] Our moments in Table 1, however, are based on a measure of output which includes teachers' salaries, to make them comparable to the data.

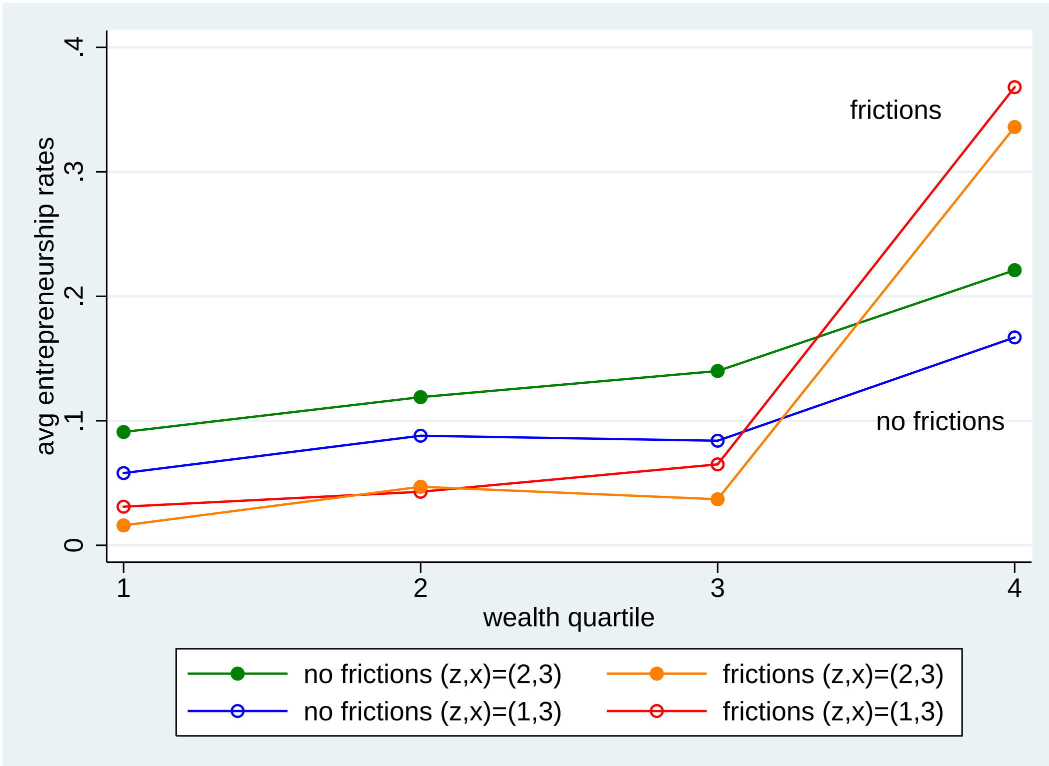


Figure 2: Entrepreneurship rate across ability and wealth

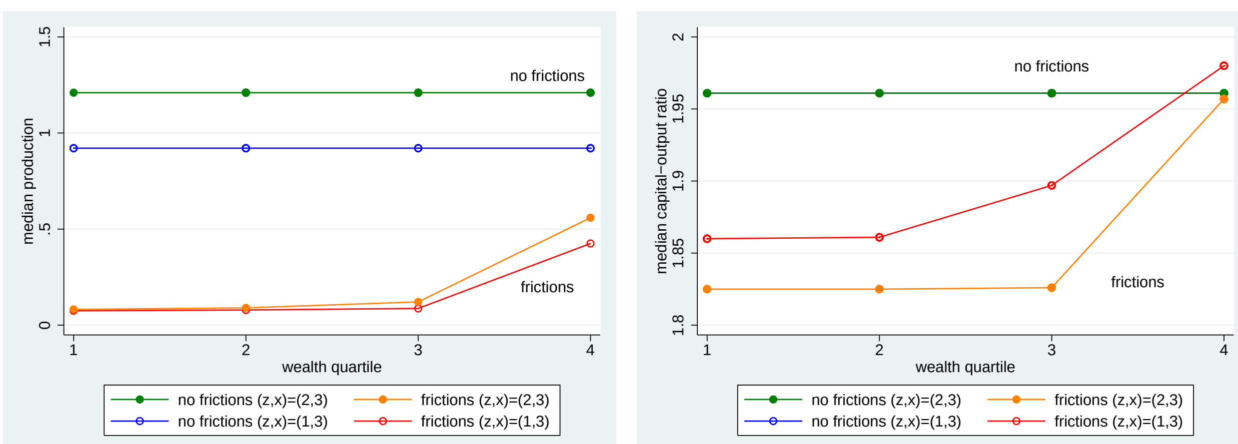
Entrepreneurship rates are actually shown to be slightly increasing in wealth in the frictionless economy. This is due to composition effects: because of ability persistence, individuals with higher abilities tend to be wealthier, which affects the aggregation into our partition of the state-space. Wealth doesn't otherwise impact the selection into entrepreneurship in the frictionless economy, in the sense that all individuals with a given ability pair make exactly the same occupational choice decisions irrespective of initial wealth. This point is illustrated in the figure by relatively flat entrepreneurship rates across wealth.

The economy with frictions is very different. Wealth now exerts a strong influence on occupational choice decisions. This manifests itself in two ways. First, the entrepreneurship rate among the poorest households is now lower, whereas it is significantly larger among the richer ones. Some highly productive but poor types now choose to become workers, whereas some less productive but wealthy types now prefer entrepreneurship. The latter change is induced by general equilibrium price effects: production inputs become cheaper with frictions, making entrepreneurship profitable for less talented people. The net effect is an increase in the aggregate rate of entrepreneurship, from 2.3% in the frictionless economy to 3.2% in the economy with frictions.

The second effect of wealth on occupational choice decisions is in making entrepreneurship especially attractive for individuals with low learning ability. This happens because our parameterization also imposes $\theta < 1 - \gamma + \alpha\gamma$, implying that the returns to learning ability in entrepreneurship decline more rapidly when capital constraints bind.[14] This is because the firm needs to be run at an inefficiently small scale, as is the case for most entrepreneurs in our calibration. Under frictions, relatively poor households with higher learning ability will therefore prefer to work for a wage rather than be constrained entrepreneurs.

5.1.2 Production Distortions

Figure 3a displays median production within each cell of our partition of the state-space where production is strictly positive.[15] In the economy with no frictions, production is independent from wealth. Household-entrepreneurs with higher learning ability accumulate more human capital, thereby increasing firm-level productivity xh^θ and producing more as a result.



(a) Production (y)

(b) Capital-output ratios (k/y)

Figure 3: Production and capital-output ratios across ability and wealth

[14] The fact that entrepreneurship is less attractive for households with higher learning ability corresponds to the situation depicted in Figure 1, in which the threshold $\bar{q}(h, x)$ is increasing in h. The higher z, and h are, the higher the wealth threshold that entices an household to choose (constrained) entrepreneurship over working for a wage.

[15] Notice that by focusing on median instead of average production, we minimize the effects of selection. These effects were instead present in the discussion surrounding Figure 2

The economy with financing frictions behaves again very differently. The equilibrium allocation is clearly very far from the first-best, with wealth having a very large effect on production. First, there is much less production, independently from wealth. Second, wealth exerts a significant effect, with the drop in production being significantly larger among the less wealthy. Third, production differences among ability types become less significant with frictions. The reason is that high learning ability types tend to have higher firm-level productivity, thus larger efficient firm size, and are therefore more capital-constrained for given wealth. Their levels of production suffer relatively more under frictions compared with low ability types.

Production is affected not just because overall project size is constrained to be lower, but also because of input distortions. Figure 3b illustrates these input distortions. Without frictions, capital-output ratios are independent from wealth, and also independent from entrepreneurial ability.

In the economy with financing frictions, the input distortions become apparent. The less wealthy entrepreneurs employ much less capital per output than the more wealthy ones. As financing constraints are relaxed, which happens for some entrepreneurs as we move up the wealth distribution, capital-output ratios rise significantly. In the top quartile, capital-output ratios actually increase above the frictionless level, once again because the interest rate is lower in the economy with frictions.

Capital-output ratios are also seen to depend now on learning ability z. Consistently with the discussion surrounding Figure 3a, higher ability entrepreneurs are forced to employ lower capital-output ratios compared to low ability entrepreneurs.

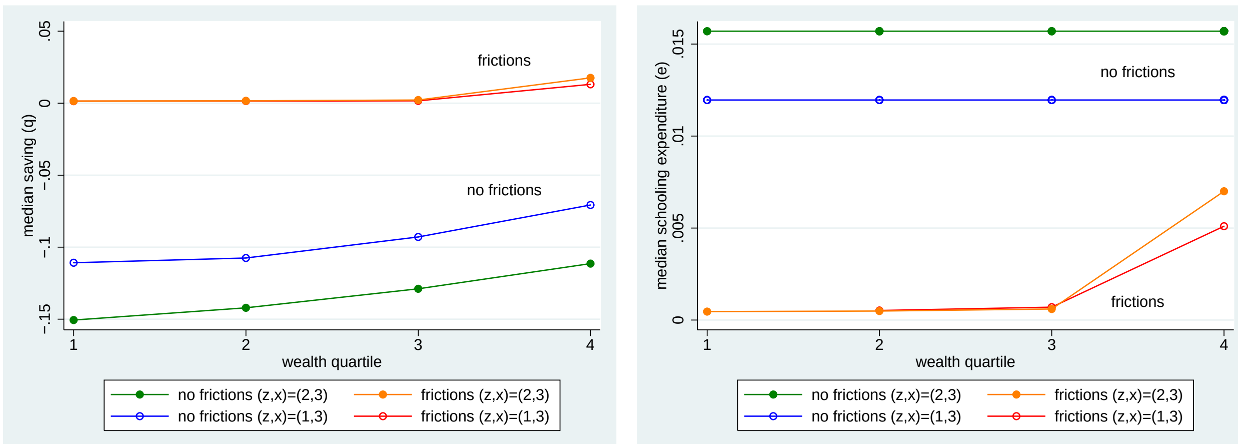
Labor-output ratios, not shown in Figure 3b, do not get distorted instead. Labor can adjust freely. These ratios depend only on entrepreneurial ability in both model versions, and increase from a level of 1.98 without frictions to 2.06 with frictions. This is due to a lower wage rate under frictions.

5.1.3 Investment Distortions

Households have two ways to transfer resources to future periods, either by saving (q) or by investing in child's education (h, via s and e).

We display the median saving and schooling expenditure decisions of entrepreneurs and workers in Figures 4 and 5.

Start with the entrepreneurs. Without frictions, schooling expenditures depend only on ability, not wealth. Individuals with higher learning ability invest more. Entrepreneurs tend to borrow, with higher ability types borrowing more. This is not only to help finance the larger schooling investment, but also a larger level of capital.



(a) Saving (q)

(b) Schooling expenditures (e)

Figure 4: Saving and schooling expenditures of entrepreneurs

With frictions, entrepreneurs need to save a strictly positive amount in order to meet the capital constraint $k \leq \lambda q$. The counterpart to the higher saving is that schooling investments drop. Figure 4b shows that schooling expenditures decline significantly relative to the frictionless case, particularly among the poorest, more constrained entrepreneurs.

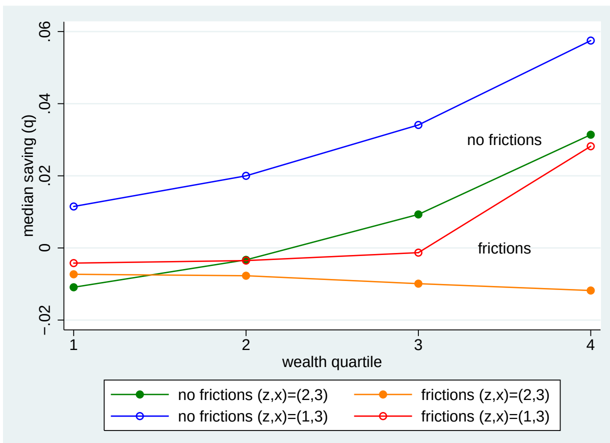
With frictions, entrepreneurial investments now depend on learning ability too. This is related to the discussion of the previous section surrounding production outcomes. Higher ability entrepreneurs, who are more credit-constrained, sacrifice schooling investments a lot in order to prevent capital-output ratios from declining so much.[16] By doing so, they succeed at increasing saving above the level of low ability entrepreneurs.[17]

[16] Although it's not visible from Figure 4b, schooling expenditures under frictions are actually lower for higher learning ability households compared to low ability ones, except for those in the top quartile. Constrained entrepreneurs with high ability are more willing to sacrifice schooling investments in order to save more and relax the capital constraint. This gets reversed in the top quartile because of a lower incidence of the credit constraint.

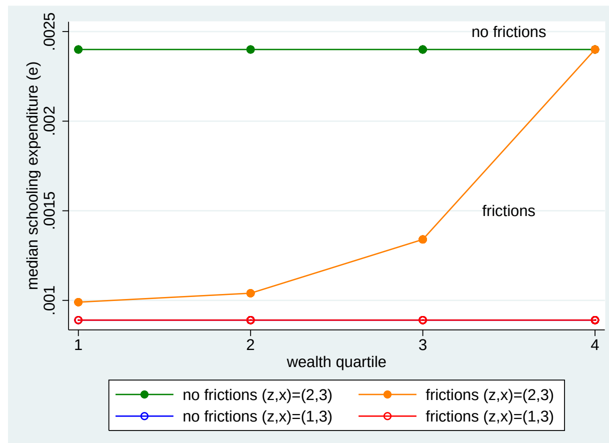
[17] Saving of high learning ability entrepreneurs is larger than that of low ability ones, even for low wealth levels. In Figure 4a this is only noticeable for wealthier households.

The behavior of workers is more standard. For expositional ease, we concentrate only on workers in the subset of our partition featuring entrepreneurs - workers actually span the whole partition.

In the frictionless economy, the poorest workers, as well as those with the highest learning ability, are the ones saving the least. See Figure 5a. This allows schooling expenditures to depend only on learning ability, as in Figure 5b.[18]



(a) Saving (q)



(b) Schooling expenditures (e)

Figure 5: Saving and schooling expenditures of workers

When workers are subject to the borrowing constraint $q \geq -\phi\omega$, their borrowing is forced to track wealth. In our current parameterization, only high learning ability workers are constrained - and all of them are, except for those in the top wealth quartile. Then, for high ability workers, the relationship between borrowing and wealth gets inverted, with the more wealthy borrowing more.[19] This implies, as Figure 5b shows, that schooling expenditures for these workers increase with wealth.

To summarize, endogenous human capital accumulation works to mitigate some of the effects of credit constraints. Although higher ability households do tend to be more credit-constrained, in our model they are able to respond by sacrificing schooling investments. This allows them to accumulate more wealth thereby relaxing credit constraints. Capital-output

[18]Notice that the lines for schooling expenditures of low learning ability types under frictions and no frictions sit (almost) on top of each other.

[19]Poor workers of low ability still save less than richer ones, since they are unconstrained. However, saving is now negative. The reason is that, under frictions, average wealth is lower for individuals in these states. This is in part due to selection, as wealthier households now prefer entrepreneurship.

ratios do not decline as much as they would otherwise.[20]

An additional source of mitigation of credit constraints is that, at the same time that capital declines less, firm-level productivity also decreases. The latter is due to lower schooling investments. This attenuates the tendency for high-productivity firms to operate at a small scale when constrained. We should therefore expect a lower degree of resource misallocation when schooling investments and productivity are allowed to respond to credit frictions.

On the other hand, the reduction in schooling investments by constrained entrepreneurs, as well as the misallocation of talent, work to significantly reduce the overall productivity of the economy. Ultimately, these are the most significant effects of financing frictions on aggregate productivity.

5.2 Measuring Misallocation

We characterize misallocation as in Hsieh and Klenow (2009). They focus on revenue productivity (TFPR) as a measurement tool, following Foster *et al.* (2008). This notion of total factor productivity is obtained by dividing nominal production revenue by an appropriate measure of production inputs. As Hsieh and Klenow (2009) show, the distribution of TFPR across production units is particularly informative about misallocation. In the benchmark case of no frictions, this distribution is degenerate, as TFPR reflects only factors which are common across production units, such as market prices and common technological parameters. Importantly, TFPR does not reflect differences in real, or physical productivity across units (TFPQ in the literature's terminology), it only reflects unit-specific deviations from marginal product equalization. The higher the degree of misallocation frictions, the more dispersed the TFPR distribution should become.

Our goal is precisely to compute a model-based distribution of a summary measure of distortion similar to TFPR to characterize misallocation, both in the schooling and in the production sectors. This approach has the advantage of providing an explicit mapping between deeper financing frictions and misallocation.

[20] This feature is related to Midrigan and Xu (2013), who emphasize the role of self-financing in mitigating the effect of credit frictions. Although we also allow for self-financing in our model, the present mechanism is an additional one related to changing the composition of the investment portfolio.

5.2.1 Basic Model Wedges

Financing frictions distort decisions by introducing wedges in the optimality conditions that would emerge in frictionless case. We call them basic distortions, or basic model wedges. We first identify these basic wedges, and then show how they map into proxy, or stand-in schooling and production misallocation wedges. The latter are featured in much of the misallocation literature, for example Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) among many others. Our result is that financing frictions do nicely map into the stand-in distortions considered in these papers.

The worker-household's optimality conditions under frictions derived in Section 3.2 can be re-written as:

$$\begin{aligned} u'(c) &= \beta\left(1+\tau_q^w\right)(1+r)\sum_{z',x'}\pi\left(z',x'|z,x\right)v_1\left(\omega',z',x'\right), \\ u'(c) &= \beta(1-\eta)\,\xi\frac{h}{e}w\sum_{z',x'}\pi\left(z',x'|z,x\right)v_1\left(\omega',z',x'\right), \\ \sum_{z',x'}\pi\left(z',x'|z,x\right)v_1\left(\omega',z',x'\right) &= \left(1+\tau_{env}^w\right)\sum_{z',x'}\pi\left(z',x'|z,x\right)u'\left(c'\right), \end{aligned} \tag{31}$$

where the last equation obtains from the envelope condition. The first two equations may be combined to yield

$$(1+r)\left(1+\tau_q^w\right) = (1-\eta)\,\xi\frac{h}{e}w. \tag{32}$$

Equations (31) and (32) define two basic wedges for the worker-household as a function of the current state (ω, z, x).

The wedge $\tau_q^w \geq 0$ acts like a subsidy to saving, capturing the effect of this period's inter-temporal borrowing constraint. It is strictly positive whenever $q = -\phi\omega$. According to equation (32), τ_q^w is also an indirect tax on the return to schooling expenditures.

The wedge $\tau_{env}^w \geq 0$ also acts like a subsidy to saving, capturing the effect of next period's inter-temporal borrowing constraint. It is strictly positive whenever saving more today helps relax next period's borrowing constraint in some future states.

Similarly, the entrepreneur-household's optimality and envelope conditions can be written as:

$$
\begin{aligned}
u'(c) &= \beta(1+r)\left(1+\tau_q^e\right)\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x'),\\
u'(c) &= \beta(1-\eta)\,\xi\frac{h}{e}\left(1-\tau_h^e\right)\frac{\theta}{1-\gamma}X_1 h^{\frac{\theta}{1-\gamma}-1}\\
&\quad\times\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x'),\\
\sum_{z',x'}\pi(z',x'|z,x)\,v_1(\omega',z',x') &= \left(1+\tau_{env}^e\right)\sum_{z',x'}\pi(z',x'|z,x)\,u'(c').
\end{aligned}
$$

These equations define three basic wedges for the entrepreneur-household as a function of the current state. From the entrepreneur's optimality conditions under frictions (19)-(21), the first two may be rewritten as

$$
\tau_q^e = \begin{cases} \partial\Pi^c(q,h,x)/\partial q \text{ if } q\in(0,q^*(h,x)) \\ 0 \text{ else} \end{cases} \tag{33}
$$

$$
(1+r)\left(1+\tau_q^e\right) = \left(1-\tau_h^e\right)\frac{\theta}{1-\gamma}X_1 h^{\frac{\theta}{1-\gamma}-1}(1-\eta)\,\xi\frac{h}{e} \tag{34}
$$

The wedge $\tau_{env}^e \geq 0$ acts like a subsidy to saving in a way entirely analogous to the one for the worker-household.

The wedge $\tau_q^e \geq 0$ also acts like a subsidy to saving, capturing the fact that whenever the capital constraint binds, an increase in saving today relaxes that constraint and increases profits tomorrow. That is, for constrained entrepreneurs, $\Pi_1^c(q,h,x) > 0$.

The wedge $\tau_h^e \geq 0$ acts like a tax on the returns to schooling, capturing the fact that human capital is less productive for constrained entrepreneurs. This is because physical capital is lower, and is complementary with human capital.

Note that decisions in the economy with frictions are not only affected by distortions but also by general equilibrium price effects.

5.2.2 Production Wedges

We recast the firm's problem as in Hsieh and Klenow (2009):

$$\Pi = \max_{k,l \geq 0} \left\{ (1-\tau_y)\, pAx\, (h^*)^{\theta} \left(k^{\alpha} l^{1-\alpha}\right)^{\gamma} - (1+\tau_k)(r+\delta)\, k - wl \right\}, \tag{Pf$'$}$$

where h^* is the human capital level that would emerge if the entrepreneur was not subject to frictions (but still subject to the prices under frictions), and p is the output price, which may be normalized to 1 in our setup.

We label τ_y and τ_k proxy wedges, in the sense that they are generic wedges standing-in for the fundamental distortions affecting the economy. τ_y captures a distortion along the marginal product vs cost margin, whereas τ_k captures a distortion along the capital vs labor margin.

The production technology underlying the stand-in problem (Pf$'$) is $y^* \equiv Ax\,(h^*)^{\theta}\,(k^{\alpha} l^{1-\alpha})^{\gamma}$. We follow Hsieh and Klenow (2009) in defining a firm's summary measure of distortions as[21]

$$SMD \equiv \frac{py^*}{k^{\alpha} l^{1-\alpha}}.$$

The optimality conditions are

$$\begin{aligned}
(1-\tau_y)\,\gamma(1-\alpha)\left(\frac{k}{l}\right)^{\alpha} SMD &= w \\
(1-\tau_y)\,\gamma\alpha\left(\frac{k}{l}\right)^{\alpha-1} SMD &= (1+\tau_k)(r+\delta),
\end{aligned}$$

and so

$$\frac{k}{l} = \frac{\alpha}{1-\alpha}\frac{w}{(1+\tau_k)(r+\delta)}.$$

The summary measure of distortions is therefore

$$SMD \propto \frac{(1+\tau_k)^{\alpha}}{1-\tau_y}.$$

[21] In Hsieh and Klenow (2009) the summary measure of distortions is akin to firm's revenue productivity (TFPR). In our set-up, this interpretation would be inexact, because the wedge τ_y is directly affecting not only firm's revenue but also the actual quantity of physical output.

Our task is now to infer proxy wedges from basic wedges. The proxy firm problem (Pf′) yields the same solution as the original firm problem (Pf) when

$$
\begin{aligned}
1-\tau_y &= \left(\frac{h}{h^*}\right)^{\theta} \\
1+\tau_k &= 1+\frac{\frac{d\Pi}{dq}}{\lambda\left(r+\delta\right)}.
\end{aligned}
$$

Using (33) and (34) in these two condition we obtain the mapping between proxy distortions and basic distortions:

$$
\begin{aligned}
1-\tau_y &\propto \left(\frac{1+\tau_q^e}{1-\tau_h^e}\frac{e}{e^*}\right)^{1-\gamma} \\
1+\tau_k &= 1+\frac{\tau_q^e}{\lambda\left(r+\delta\right)},
\end{aligned}
$$

where e^* is the level that would prevail if the firm was not subject to frictions.

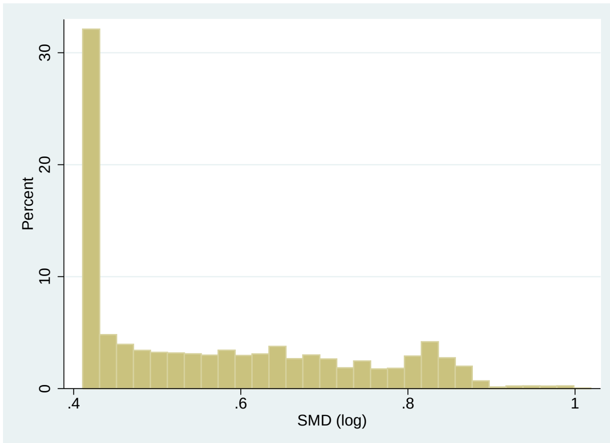
These expressions allow us to interpret the proxy wedges in terms of our financing frictions model. τ_y amounts to a positive tax on firm's physical output. The reason is that the capital constraint decreases firm-level productivity xh^{θ}, by reducing the incentives for schooling investments. The composite distortion $(1+\tau_q^e)/(1-\tau_h^e)$ captures the total disincentive to investing in human capital. It amounts to a positive net tax since (i) capital-constrained entrepreneurs run smaller firms, reducing the returns to investing in human capital, and (ii) for these households, accumulating wealth relaxes the capital constraint, and therefore commands a higher return compared to investing in human capital. The term e/e^* simply converts the gap in human capital levels into a gap in the returns to schooling investments.

τ_k amounts to a positive tax on capital. The reason is that the capital constraint increases the shadow rental price of capital.

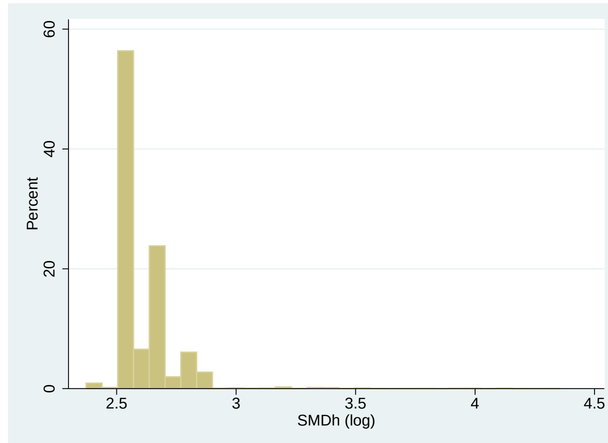
Absent frictions, $\tau_q^e = \tau_h^e = \tau_q^e = 0$ and $e = e^*$. Therefore $\tau_y = \tau_k = 0$, and the distribution of SMD is degenerate. Figure 6a plots the distribution of SMD in our model.[22] The degree of production misallocation in the economy with frictions is large. The summary measure of

[22] Letting SMD^* be the level absent frictions, it follows that $SMD \geq SMD^*$. Financing frictions generate only the equivalent of firm-specific taxes, not subsidies. Note also that a larger value for SMD is indicative of more distortions.

distortions can be 60% larger for constrained entrepreneurs relative to unconstrained ones. The standard deviation of log SMD is 15.7%.



(a) Firm-level summary measure of distortions



(b) Household-level schooling revenue productivity

Figure 6: Summary measure of distortions distributions

5.2.3 Schooling Wedges

We now proceed in an analogous fashion for schooling. We recast the schooling investment problem conditional on either occupational choice as

$$\max_{e}(1-\tau_h^i)p_h h-(1+r)(1+\tau_q^i)e$$

subject to (2) and (22), for $i = w, e$. Households maximize earnings net of the (opportunity) cost of schooling expenditures, subject to individual and occupation-specific distortions. Here p_h is the unit skill price, possibly individual-specific. For workers $p_h = w$, whereas for entrepreneurs $p_h = X_1(x)h^{\theta/(1-\gamma)-1}\theta/(1-\gamma)$. The latter is the individual-specific skill price that would prevail for entrepreneurs absent frictions.

The mapping between basic wedges and proxy wedges is trivial in this case, they are equal to each other for both workers and entrepreneurs. Workers do not face an earnings wedge, $\tau_h^w = 0$. Their cost wedge τ_q^w arises because the presence of the borrowing constraint increases the shadow cost of current spending. For entrepreneurs, the cost wedge τ_q^e arises because the presence of the capital constraint increases the opportunity cost of schooling expenditures, relative to saving. The earnings wedge τ_h^e is due to the fact that capital-constrained entrepreneurs enjoy lower returns to human capital accumulation, due to a lower production scale.

Define the summary measure of distortions in the schooling sector as the revenue productivity[23]

$$SMD_h \equiv \frac{p_h h}{e}.$$

The optimality conditions for this stand-in problem are (32) for workers, and (34) for entrepreneurs. They are thus also the solution to the original problem. Using our definition of revenue productivity, we obtain

$$SMD_h \propto \begin{cases} 1+\tau_q^w & \text{for workers} \\ \frac{1+\tau_q^e}{1-\tau_h^e} & \text{for entrepreneurs,} \end{cases}$$

with the same constant of proportionality across occupations.

The distribution of revenue productivity in the schooling sector is also degenerate absent frictions. Figure 6b plots the distribution of SMD_h in our model.[24]

Revenue productivity in schooling is much more dispersed among entrepreneurs, which can be twice as large as the level absent frictions. The standard deviation of log SMD_h among entrepreneurs only is 50.2%. However, since the rate of entrepreneurship is low in our model economy, Figure 6b is dominated by workers. In the aggregate, the standard deviation of log SMD_h is still large however, 14.4%.

5.3 Misallocation and Aggregate Productivity

The final good sector admits an aggregate production function

$$Y = TFP\left(K^\alpha L^{1-\alpha}\right)^\gamma,$$

[23] To define an appropriate summary measure of distortions, notice that the time constraint might bind. We therefore compute productivity with respect to the freely adjustable input.

[24] Letting SMD_h^* be the level absent frictions, it follows once again that $SMD_h \geq SMD_h^*$. Also in the schooling sector, financing frictions generate only firm-specific taxes, not subsidies.

where $Y \equiv \int_S y d\Psi$, $K \equiv \int_S k d\Psi$, and $L \equiv \int_B l d\Psi$. See Appendix C.

Total factor productivity (TFP) is an aggregator of individual physical productivities and distortions. Defining the firm's physical productivity in absence of frictions as

$$TFPQ \equiv x\left(h^*\right)^{\theta},$$

we obtain the following expression for TFP

$$TFP = \frac{\int_B \left(TFPQ \frac{1-\tau_y}{(1+\tau_k)^{\alpha\gamma}}\right)^{\frac{1}{1-\gamma}} d\Psi}{\left[\int_B \left(TFPQ \frac{1-\tau_y}{(1+\tau_k)^{1-\gamma+\alpha\gamma}}\right)^{\frac{1}{1-\gamma}} d\Psi\right]^{\alpha\gamma} \left[\int_B \left(TFPQ \frac{1-\tau_y}{(1+\tau_k)^{\alpha\gamma}}\right)^{\frac{1}{1-\gamma}} d\Psi\right]^{(1-\alpha)\gamma}}. \tag{35}$$

In order to understand in more detail the impact of distortions on aggregate TFP it is useful to follow Hsieh and Klenow (2009) and consider an example with $TFPQ$, $(1-\tau_y)$, and $(1+\tau_k)$ jointly log-normally distributed among the firms. In that special case the logarithm of the aggregate TFP can be written as a simple function of the moments of the joint distribution:

$$\log TFP = (1-\gamma)\log ent + (1-\gamma)\log E\left[\left(TFPQ\left(1-\tau_y\right)\right)^{\frac{1}{1-\gamma}}\right] - \frac{1}{2}\frac{\alpha\gamma\left(1-\gamma+\alpha\gamma\right)}{1-\gamma} var\left(\tau_k\right), \tag{36}$$

where $ent \equiv \int_B 1\, d\Psi$ is the measure of the set of entrepreneur-households and the expectation is conditional on the states in this set. The first term in equation (36) is the TFP gain from specialization. Intuitively, because firm-level technology exhibits decreasing returns to scale, aggregate productivity rises when output is produced by a relatively higher number of relatively smaller firms. The following two terms in equation (36) show that the firm-level distortions reduce the aggregate TFP by two main channels. First, τ_y decreases the average physical productivity of firms in the economy by introducing a gap between the firms' potential physical productivities without frictions ($TFPQ$) and the actual physical productivities. This effect is entirely due to the endogenously lower human capital investments by the entrepreneur-households. Second, dispersion in τ_k reduces aggregate TFP by introducing dispersion in marginal products of capital across firms, which is the effect traditionally emphasized by the misallocation literature.

Decomposing further the second term in equation (36) allows us to identify five key moments that determine the total effect of financing frictions on aggregate TFP

$$
\begin{aligned}
\log TFP = & \underbrace{(1-\gamma)\log ent}_{\text{(Specialization)}} + \underbrace{(1-\gamma)\log E\left[TFPQ^{\frac{1}{1-\gamma}}\right]}_{\text{(Talent)}} + \underbrace{(1-\gamma)\log E\left[(1-\tau_y)^{\frac{1}{1-\gamma}}\right]}_{\text{(Human capital)}} \\
& + \underbrace{\frac{1}{1-\gamma} cov\left(TFPQ, 1-\tau_y\right)}_{\text{(Interaction)}} - \underbrace{\frac{1}{2}\frac{\alpha\gamma\left(1-\gamma+\alpha\gamma\right)}{1-\gamma} var\left(\tau_k\right)}_{\text{(Physical capital)}}. \qquad (37)
\end{aligned}
$$

The first term is again the gain from specialization. The second term is determined by the selection of households into entrepreneurship and thus by the misallocation of talent. The third term represents the effect of financing frictions on entrepreneur-households investments in human capital. The fourth covariance term stems from the interaction between selection into entrepreneurship and human capital investments of entrepreneur-households. Intuitively, higher covariance between $TFPQ$ and $(1-\tau_y)$ increases the aggregate TFP because it implies the less distorted physical productivities of highly talented entrepreneurs. Finally, the fifth term is the effect of the misallocation of physical capital.

5.4 Aggregate Consequences of Financing Frictions

We now consider two countries differing only in the extent of financing constraints. The country with frictions is parameterized according to Table 1. The country with no frictions shares the same parameters, except that ϕ and λ are large enough not to be binding. We then compare several macroeconomic variables across the two economies to investigate whether variation in financing frictions has a potential to explain the key stylized facts on economic development summarized in Section 1.2.

Model	Y	K/Y	TFP
No frictions	0.036	1.961	0.886
Frictions	0.024	1.930	0.817

Table 2: Macroeconomic aggregates

Table 2 shows that the implications of our model are qualitatively in line with the observed differences in the main macroeconomic aggregates across poor and rich countries. As in the data, the country with lower per capita aggregate output Y (2/3 of the frictionless level) has slightly lower capital-output ratio K/Y (98% of the frictionless level). This is in large part because of production distortions discussed in Section 5.1.2. The poor country also has lower TFP (92% of the frictionless level).[25]

The reduction in TFP comes from three main sources: misallocation of talent, lower human capital investments by entrepreneur-households, and dispersion in the marginal products of physical capital across producers. Table 3 shows the approximate decomposition of the loss in aggregate TFP into the five components identified in equation (37) and an approximation error.[26] Distortions in human capital investments ($(c)+(d)$) and misallocation of physical capital (e) are the most important components lowering together the approximate aggregate TFP by 30 percent.

Distortion type	Term	$\log \text{Term}_{nf} - \log \text{Term}_f$
Specialization gain	(a)	-0.048
Talent misallocation	(b)	0.019
Human capital investments	(c)	0.090
Interaction talent $\times$ human capital	(d)	0.037
Capital misallocation	(e)	0.150
Approximated $\log TFP$ loss	$(a)+(b)+(c)$ $(d)+(e)$	0.248
Approximation error	(f)	-0.167
Real $\log TFP$ loss	$(a)+(b)+(c)+$ $(d)+(e)+(f)$	0.081

Table 3: Aggregate TFP loss decomposition

Table 4 focuses on the most salient features of the firm-size distribution. In line with the observed stylized facts, our model generates more entrepreneurs ($\%\text{ent} \equiv \int_B d\Psi$), operating

[25]The aggregate production function can be written as $Y = TFP^{\frac{1}{1-\gamma}} \left(\left(\frac{K}{Y}\right)^{\alpha} \left(\frac{L}{Y}\right)^{1-\alpha} \right)^{\frac{\gamma}{1-\gamma}}$. Differences in TFP get amplified in accounting for differences in Y

[26]The approximation error stems from the fact that the decomposition in equation (37) is exact only if the potential physical productivities and the firm-level distortions are jointly log-normally distributed, whereas in our model this distribution is endogenously determined in equilibrium and is not necessarily a multivariate log-normal.

on average at a smaller scale (avg. firm size $\equiv \int_B l d\Psi$) in the poor country.[27] The firm-size distribution is not only shifted to the left, but also develops a right skew: there is now a large mass of very small firms, i.e. the median firm size declines more than the mean (size skew $\equiv$ mean(l)/median(l)).

Model	% ent	avg. firm size	size skew
No frictions	0.023	3.110	1.298
Frictions	0.032	1.534	1.755

Table 4: Entrepreneurship and firm size distribution

Finally, Table 5 looks at human capital accumulation at the aggregate level. Average years of schooling ($\bar{s} \equiv 30 \times \int_S s d\Psi$) are lower in the poor country. Most of this reduction comes out of a large reduction in schooling for entrepreneurs, highlighting the importance of production for understanding schooling. We also report coefficients from Mincerian regressions of log of earnings on schooling, calculated separately for workers and entrepreneurs, from the model-generated data

$$\log(\text{earnings}) = \alpha + \beta(30 \times s) + \varepsilon.$$

With no frictions the coefficient for entrepreneurs is not defined in our current parameterization of the model because all entrepreneurs make the same schooling decision ($s = 1$).

Model	schooling years		Mincer coefficient	
	workers	entrepreneurs	workers	entrepreneurs
No frictions	11.38	30	0.092	n/a
Frictions	7.014	19.543	0.121	0.082

Table 5: Schooling

[27] Our measure of firm size does not distinguish the number of workers from the quantity of human capital employed. We obtain similar qualitative results when using total revenue as a measure of size.

6 Concluding Remarks

In this paper we investigate the effects of financing frictions in an environment where they can influence both firm-level investment decisions, and household-level schooling decisions. Our preliminary results indicate that there are important interactions between occupational choice, saving, and schooling decisions that were not explored by the existing literature. In particular, understanding how schooling decisions of future entrepreneurs respond to the current and future financial constraints seems to be key for evaluating the effects of financing frictions on economic development.

A Profit Functions

The profit functions are:

$$\Pi^* (h, x) = X_1 (x) \, h^{\frac{\theta}{1-\gamma}},$$
$$\Pi^c (q, h, x) = B_2 (q, x) \, h^{\frac{\theta}{1-(1-\alpha)\gamma}} - B_1 q,$$

where

$$X_1 (x) = \left[X_0 (x) \left(\frac{(1-\alpha)(r+\delta)}{\alpha w} \right)^{1-\alpha} \right]^{\gamma} A (1-\gamma) \, x,$$
$$X_0 (x) = \left[\frac{(1-\alpha)(r+\delta)^{1-\frac{1}{(1-\alpha)\gamma}}}{\alpha w} \right]^{(1-\alpha)\frac{\gamma}{1-\gamma}} (\alpha \gamma A x)^{\frac{1}{1-\gamma}} ,$$
$$B_2 (q, x) = B_0 \left[x (q)^{\alpha\gamma} \right]^{\frac{1}{1-(1-\alpha)\gamma}} ,$$
$$B_0 = \frac{1-(1-\alpha)\gamma}{(1-\alpha)\gamma} w \left[\frac{(1-\alpha)\gamma A \lambda^{\alpha\gamma}}{w} \right]^{\frac{1}{1-(1-\alpha)\gamma}} , \tag{38}$$
$$B_1 = (r+\delta) \lambda. \tag{39}$$

B Constructing Figure 1

To determine the shape of $\bar{q}(h, x)$, notice first that since $\Pi^* (h, x) = \Pi^c (q^* (h, x), h, x) \geq \Pi^c (\bar{q}(h, x), h, x)$, it follows that

$$\bar{q}(h, x) \leq q^* (h, x),$$

with equality if $h = \bar{h}(x)$. Moreover, $q^* (h, x)$ is increasing and convex in h, going through the origin in the (h, q) space. This follows from $k^* (h, x)$ being increasing and convex in h and from the fact that $q^* (0, x) = 0$. Note also that $\bar{q}(h, x)$ is not defined for $h < \bar{h}(x)$, since in this case $wh > \Pi^* (h, x) > \Pi^c (q, h, x)$ for all q.

Now, by the implicit function theorem,

$$\frac{d\bar{q}(h, x)}{dh} = \frac{w - \Pi_2^c (\bar{q}(h, x), h, x)}{\Pi_1^c (\bar{q}(h, x) \, h, x)}.$$

The denominator is always positive since $\bar{q}(h,x) \leq q^*(h,x)$, so that increasing q relaxes the capital constraint and increases profits. To study the sign of the numerator, notice that $\Pi^c(q,h,x)$ is a polynomial in h and q

$$\Pi^c(q,h,x) = B_0\left(xh^\theta\right)^{\frac{1}{1-(1-\alpha)\gamma}}(q)^{\frac{\alpha\gamma}{1-(1-\alpha)\gamma}} - B_1 q,$$

where $B_0, B_1 > 0$ are defined in (38) and (39). It follows that: (i) $\Pi^c(q,0,x) = -B_1 q < 0$ for $q > 0$, and (ii) $\Pi^c(q,h,x)$ is concave in h. As a result, $\Pi^c(q,h,x)$ as a function of h must cross wh twice, say at values h_1 and $h_2 > h_1$. It then follows that $w - \Pi^c_2(\bar{q}(h,x),h,x) < 0$ for $h < h_1$, $w - \Pi^c_2(\bar{q}(h,x),h,x) > 0$ for $h > h_2$, with the derivative $d\bar{q}(h,x)/dh$ changing sign somewhere in (h_1,h_2). So, starting from $\bar{h}(x)$, $\bar{q}(h,x)$ first decreases, and then increases with h.

C Aggregation

The individual input demands from problem (Pf′) can be written as

$$l = \frac{\left[x\left(h^*\right)^\theta \frac{1-\tau^y}{(1+\tau_k)^{\alpha\gamma}}\right]^{\frac{1}{1-\gamma}}}{\int_S \left[x\left(h^*\right)^\theta \frac{1-\tau^y}{(1+\tau_k)^{\alpha\gamma}}\right]^{\frac{1}{1-\gamma}} d\Psi} L \equiv \omega_l L$$

$$k = \frac{\left[x\left(h^*\right)^\theta \frac{1-\tau^y}{\left(1+\tau^k\right)^{1-\gamma(1-\alpha)}}\right]^{\frac{1}{1-\gamma}}}{\int_S \left[x\left(h^*\right)^\theta \frac{1-\tau^y}{\left(1+\tau^k\right)^{1-\gamma(1-\alpha)}}\right]^{\frac{1}{1-\gamma}} d\Psi} K \equiv \omega_k K.$$

Aggregate production is then

$$\begin{aligned} Y &= \int_S y d\Psi \\ &= \int_S \left(1-\tau_y\right) Ax\left(h^*\right)^\theta \left(k^\alpha l^{1-\alpha}\right)^\gamma d\Psi \\ &= TFP\left(K^\alpha L^{1-\alpha}\right)^\gamma \end{aligned}$$

where

$$TFP \equiv A \int_S (1-\tau_y)\, x\, (h^*)^\theta\, \omega_k^{\alpha\gamma} \omega_l^{(1-\alpha)\gamma} d\Psi.$$

D Numerical Algorithm

We solve the model using value function iteration.

1. **Discretization:** Discretize ω into $\{\omega_0, \ldots, \omega_{N_\omega}\}$. Note that $\omega' \equiv \max\{\Pi, wh\} + q$ and $q \geq -\phi\omega$ implies $\omega' \geq \max\{\Pi, wh\} - \phi\omega$. We may then choose $\omega_0 = \max\{\Pi, wh\} / (1+\phi) > 0$. The upper bound ω_{N_ω} is sufficiently high that increasing it further has a negligible effect on the solution.

   The VAR(1) process for abilities is discretized into a Markov chain using the procedure described in Tauchen and Hussey (1991).

2. **Occupational choice and production:** Solve for $\omega'(x, h, q)$ given the current guess for prices w and r.

   (i) Compute the threshold level of saving $q^*(x, h)$.

   (ii) Compute profits $\Pi(x, h, q)$.

   (iii) Compute next generation's wealth $\omega'(x, h, q)$.

3. **Saving and education:** Solve for the decision rules $e(\omega, z, x)$, $s(\omega, z, x)$, and $q'(\omega, z, x)$, given $\omega'(x, h, q')$ from step 2, and given the current guess for prices.

   (i) Guess value function $V^j(\omega, z, w)$ at gridpoints.

   (ii) Solve for the right-hand-side of the Bellman equation:

   $$V^{j+1}(\omega, z, x) = \max_{c,e,s,q} \left\{ u(c) + \beta \sum_{z',x'} \pi(z', x' | z, x)\, V^j(\omega'(x, h, q), z', x') \right\}$$

   subject to (4)-(6).

   First try an interior solution for q. If $q \geq -\phi\omega$ then the solution has been found. Otherwise set $q = -\phi\omega$ and find s and e subject to this constraint. V^j

is approximated by a piecewise linear function for future wealth levels outside of the grid.

(iii) Iterate until $V^{j}(\omega, z, x) \approx V^{j+1}(\omega, z, x)$.

4. **Invariant distribution:** Approximate by simulating a large cross-section of $N + 1$ agents over a sufficiently large number of T periods. Decision rules are linearly interpolated over a very fine grid.

5. **Market clearing:** Check whether the labor and capital markets clear. Compute excess demand for labor and capital from the invariant distribution as:

$$
\begin{aligned}
EDL(w,r) &= \frac{1}{N}\sum_{n=2}^{N+1}\left[I_{n-1}l_{n-1} + s_n\bar{l} - (1 - I_{n-1})\,h_{n-1} - (1 - s_n)\,\bar{h}_0\right] \\
EDK(w,r) &= \frac{1}{N}\sum_{n=2}^{N+1}\left[I_{n-1}k_{n-1} - q_n\right],
\end{aligned}
$$

where I_n is an indicator which takes the value of 1 if household n chooses entrepreneurship and 0 otherwise. Iterate on market prices until $EDL(w,r) \approx 0$ and $EDK(w,r) \approx 0$.

References

Amaral, P. S. and Quintin, E. (2010). Limited enforcement, financial intermediation, and economic development: A quantitative assessment. *International Economic Review*, **51** (3), 785–811.

Atkeson, A. and Kehoe, P. (2005). Modeling and measuring organizational capital. *Journal of Political Economy*, **113**, 1026–1053.

Banerjee, A. V. and Duflo, E. (2005). Growth theory through the lens of development economics. In S. Durlauf and P. Aghion (eds.), *Handbook of Economic Growth*, vol. 1A, Elsevier - North Holland, pp. 473–552.

Beck, T., Demirgüç-Kunt, A. and Levine, R. (2000). A new database on financial development and structure. *World Bank Economic Review*, **14**, 597–605.

Bhattacharya, D., Guner, N. and Ventura, G. (2013). Distortions, endogenous managerial skills and productivity differences. *Review of Economic Dynamics*, **16** (1), 11–25.

Bowles, S. and Gintis, H. (2002). The Inheritance of Inequality. *Journal of Economic Perspectives*, **16** (3), 3–30.

Buera, F. J., Kaboski, J. P. and Shin, Y. (2011). Finance and development: A tale of two sectors. *American Economic Review*, **101** (5), 1964–2002.

— and Shin, Y. (2013). Financial frictions and the persistence of history: A quantitative exploration. *Journal of Political Economy*, **121** (2), 221 – 272.

Castro, R., Clementi, G. L. and MacDonald, G. (2004). Investor protection, optimal incentives, and economic growth. *Quarterly Journal of Economics*, **119** (3), 1131–1175.

—, — and — (2009). Legal institutions, sectoral heterogeneity, and economic development. *Review of Economic Studies*, **76** (2), 529–561.

Clausen, A. and Strub, C. (2013). A general and intuitive envelope theorem, University of Pensylvania.

Córdoba, J. C. and Ripoll, M. (2013). What explains schooling differences across countries? *Journal of Monetary Economics*, **60** (2), 184–202.

Dunn, T. and Holtz-Eakin, D. (2000). Financial Capital, Human Capital, and the Transition to Self-Employment: Evidence from Intergenerational Links. *Journal of Labor Economics*, **18** (2), 282–305.

Erosa, A. and Hidalgo-Cabrillana, A. (2008). On finance as a theory of TFP, cross-industry productivity differences, and economic rents. *International Economic Review*, **49** (2), 437–473.

—, Koreshkova, T. and Restuccia, D. (2010). How important is human capital? a quantitative theory assessment of world income inequality. *Review of Economic Studies*, **77** (4), 1421–1449.

Evans, D. and Jovanovic, B. (1989). An estimated model of entrepreneurial choice under liquidity constraints. *Journal of Political Economy*, **97**, 808–27.

Fairlie, R. W. and Robb, A. M. (2008). *Race and Entrepreneurial Success: Black-, Asian-, and White-Owned Businesses in the United States*, *MIT Press Books*, vol. 1. The MIT Press.

Foster, L., Haltiwanger, J. and Syverson, C. (2008). Reallocation, firm turnover, and efficiency: Selection on productivity or profitability? *American Economic Review*, **98** (1), 394–425.

Galor, O. and Zeira, J. (1993). Income distribution and macroeconomics. *Review of Economic Studies*, **60** (1), 35–52.

Gollin, D. (2002). Getting income shares right. *Journal of Political Economy*, **110** (2), 458–474.

Greenwood, J., Sanchez, J. and Wang, C. (2013). Quantifying the impact of financial development on economic development. *Review of Economic Dynamics*, **16** (1), 194–215.

Guner, N., Ventura, G. and Yi, X. (2008). Macroeconomic implications of size-dependent policies. *Review of Economic Dynamics*, **11** (4), 721–744.

Henly, S. E. and Sanchez, J. M. (2009). The U.S. establishment-size distribution: secular changes and sectoral decomposition. *Economic Quarterly*, **95** (4), 419–454.

Hertz, T., Jayasundera, T., Piraino, P., Selcuk, S., Smith, N. and Verashchagina, A. (2008). The Inheritance of Educational Inequality: International Comparisons and Fifty-Year Trends. *The B.E. Journal of Economic Analysis & Policy*, **7** (2), 1–48.

Hsieh, C.-T. and Klenow, P. J. (2009). Misallocation and manufacturing TFP in China and India. *The Quarterly Journal of Economics*, **124** (4), 1403–1448.

— and — (2010). Development accounting. *American Economic Journal: Macroeconomics*, **2** (1), 207–223.

Hurst, E. and Lusardi, A. (2004). Liquidity Constraints, Household Wealth, and Entrepreneurship. *Journal of Political Economy*, **112** (2), 319–347.

— and — (2008). Liquidity constraints and entrepreneurship. household wealth, parental wealth, and the transition in and out of entrepreneurship. In D. Furchtgott-Roth (ed.), *Overcoming Barriers to Entrepreneurship in the United States*, Lexington Books, pp. 47–68.

Kambourov, G. and Manovskii, I. (2009). Accounting for the changing life-cycle profile of earnings, University of Toronto.

Kehoe, T. J. and Levine, D. K. (1993). Debt-constrained asset markets. *Review of Economic Studies*, **60** (4), 865–888.

Lochner, L. and Monge-Naranjo, A. (2012). Credit constraints in education. *American Economic Review*, forthcoming.

Lochner, L. J. and Monge-Naranjo, A. (2011). The nature of credit constraints and human capital. *American Economic Review*, **101** (6), 2487–2529.

Manuelli, R. E. and Seshadri, A. (2014). Human capital and the wealth of nations. *American Economic Review*, forthcoming.

Midrigan, V. and Xu, D. Y. (2013). Finance and misallocation: Evidence from plant-level data. *American Economic Review*, forthcoming.

Moll, B. (2014). Productivity losses from financial frictions: Can self-financing undo capital misallocation? *American Economic Review*, forthcoming.

—, Townsend, R. M. and Zhorin, V. (2013). Financial obstacles and inter-regional flow of funds, MIT.

Restuccia, D. and Rogerson, R. (2008). Policy distortions and aggregate productivity with heterogeneous plants. *Review of Economic Dynamics*, **11** (4), 707–720.

Tauchen, G. and Hussey, R. (1991). Quadrature-Based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models. *Econometrica*, **59** (2), 371–96.