Directed Search in the Housing Market

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Abstract

In this paper, we present a directed search model of the housing market. The pricing mechanism we analyze reflects the way houses are bought and sold in the United States. Our model is consistent with the observation that houses are sometimes sold above, sometimes below, and sometimes at the asking price. We consider two versions of our model. In the first version, all sellers have the same reservation value. In the second version, there are two seller types, and type is private information. For both versions, we characterize the equilibrium of the game played by buyers and sellers, and we discuss efficiency. Our model offers a new way to look at the housing market from a search-theoretic perspective. In addition, we contribute to the directed search literature by considering a model in which the asking price (i) entails only limited commitment and (ii) has the potential to signal seller type.

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1 Introduction

In this paper, we present a directed search model of the housing market. We construct our model with the following stylized facts in mind. First, sellers post asking prices, and buyers observe these announcements. Second, there is not a straightforward relationship between the asking price and the final sales price. Sometimes buyers make counteroffers, and houses sell below the asking price. Sometimes houses sell at the asking price. Sometimes – typically when the market is hot – houses are sold by auction above the asking price. Third, a seller who posts a low asking price is more likely to sell his house, albeit at a lower price, than one who posts a higher asking price.[1]

Our model is one of directed search in the sense that sellers use the asking price to attract buyers. However, ours is not a standard directed search model in that we assume only limited commitment to the asking price.[2] The specific form of commitment to the asking price that we assume reflects the institutions of the U.S. housing market. Within a "selling period," buyers who view a house that is listed at a particular price can make offers on that house.[3] A seller is free to reject any offer below her asking price, but she also has the option to accept such an offer. However, if the seller receives one or more *bona fide* offers to buy the house at her asking price (without contingencies), then she is committed to sell.[4] If the seller receives only one such offer at the asking price, then she is committed to transfer the house to the buyer at that price. If the seller receives two or more legitimate offers at her asking price, then she cannot, of course, sell the house to more than one buyer. In this case, the remaining buyers can bid against each other to buy the house. In practice, in some locations, this auction takes the form of bids with escalator clauses. For example, if a house is listed at $1 million, a buyer might submit a bid of that amount together with an offer to beat any other offer the seller might receive by $5,000 up to a maximum of $1.1 million.

Our description of the U.S. housing market is stylized in the sense that there is sometimes ambiguity about what constitutes a *bona fide* offer at the asking price. For example, a seller can often reject a prospective buyer's offer at the asking price if the buyer's ability to secure a mortgage is in question. It is also important to emphasize that the institutional form of limited commitment to the asking price that we are ascribing

[1] Ortalo-Magné and Merlo (2004), using UK data, find that a lower asking price increases the number of visitors and offers that a seller can expect to receive but decreases the expected sales price. Similarly, using Dutch data, De Wit and Van der Klaauw (2010) show that list price *reductions* significantly increase the probability of selling a house.

[2] While we assume limited commitment to the asking price, we do assume full commitment to the selling mechanism, which will be discussed below.

[3] We are thus assuming a many-on-one meeting technology or what Eeckhout and Kircher (2010) call a nonrival meeting technology.

[4] This commitment is typically written into contracts between sellers and their real estate agents.

to the U.S. housing market is not universal. For example, in the Netherlands, the asking price entails no legal commitment whatsoever. Nonetheless, since real estate agents have reputational concerns, asking prices reflect some limited commitment there as well.

Given limited commitment, what determines the asking prices that sellers post and what role do these asking prices play? Of course, the asking price for a spacious house that is located in a desirable neighborhood is typically higher than the asking price for a smaller house in a less desirable neighbourhood. We assume that prospective buyers can observe these and more subtle "vertical" differences among houses, either directly or with the help of real estate agents. Instead, we focus on the role that asking prices play in directing search across houses that buyers view as *ex ante* identical. In particular, we are interested in the question of whether asking prices can direct buyers towards "motivated sellers," that is, those who are particularly eager to sell and are therefore more likely to accept a low counteroffer.

We begin, however, with a basic version of our model in which all sellers are equally motivated, i.e., have the same reservation value. This homogeneous-seller version of our model turns out to be of interest in its own right. After observing all the asking prices in the market, each buyer chooses a seller to visit. Upon visiting the seller, the buyer decides how much he likes her house; that is, he observes the realization of a match-specific random variable. This realization is the buyer's private information. Based on this realization – and without knowing how many other buyers have visited this seller – the buyer chooses among accepting the seller's asking price, making a counteroffer (and, if so, at what level), or simply walking away. The seller then assembles her offers, if any. If no buyer has offered to pay the asking price, the seller decides whether or not to accept her best counteroffer. If she has received one, and only one, offer at the asking price, then she sells the house at that price. If she has received multiple offers at the asking price, she then allows the buyers who made those offers to compete for the house via an ascending bid auction.[5] A payoff-equivalence result holds for this version of the model. All asking prices at or above the seller's reservation value give the seller the same expected payoff; asking prices below the reservation value yield a lower expected payoff. Similarly, buyers are indifferent with respect to any asking price greater than or equal to the common reservation value but strictly prefer any asking price below that level. Any distribution of asking prices greater than or equal to the common seller reservation value constitutes an equilibrium, and there are no equilibria in which any sellers post asking prices below the common reservation value. These equilibria are constrained efficient in

[5] In a tight market, we sometimes observe buyers submitting initial bids above the asking price. We assume that sellers are committed to allowing all buyers who bid at least the asking price to participate in the auction, so it is not in any buyer's interest to make an initial bid above the asking price. Buyers do, however, make bids above the asking price in the subsequent auction.

the sense that, given the level of market tightness, the house always go to the buyer who values it most if that value is above the seller's reservation value or, if not, it is retained by the seller. In addition, when market tightness is endogenous, equilibrium entails the optimal seller entry.

After analyzing the homogeneous-seller case, we consider a version of our model in which sellers have different reservation values and in which these reservation values are private information. Specifically, we examine a model in which there are two seller types – one group with a high reservation value ("relaxed sellers"), the other with a low reservation value ("motivated sellers"). In this heterogeneous-seller version of our model, the asking price can potentially signal a seller's type. Our signaling model is nonstandard in the sense that sellers have both *ex ante* and *ex post* signaling motives. *Ex ante* a seller wants to signal that her reservation value is low. This attracts buyers since buyers prefer to visit a seller who is perceived to be "weak." *Ex post*, however, that is, once any buyers have visited, a seller prefers to have signaled a high reservation value. Buyers will make higher bids when dealing with a seller who is perceived to be "strong."[6] Given the mixed incentives faced by sellers, there are several potential types of equilibria. There are two types of possible pooling equilibria. In a pooling-on-low equilibrium, all sellers post a low asking price – the sellers with the high reservation value mimic those with the low reservation value – while in a pooling-on-high equilibrium, all sellers post a high asking price – the sellers with the low reservation value mimic those with the high reservation value. There is also the possibility of hybrid equilibria. Again, there are potentially two types of these equilibria. In one possible hybrid equilibrium, some but not all of the sellers with the low reservation value mimic those with the high reservation value, while in the other possible hybrid equilibrium, some but not all of the sellers with the high reservation value mimic those with the low reservation value. Under standard restrictions on buyers' out-of-equilibrium beliefs, we show the nonexistence of pooling and hybrid equilibria.

Finally, we consider separating equilibria, in which the two seller types are identified by their posted asking prices. We prove the existence of separating equilibria. Such equilibria are constrained efficient. As in the homogeneous-seller case, the level of entry by sellers is optimal. In addition, the equilibrium allocation of buyer visits across the two seller types is the same as the allocation that a social planner would choose.

Our paper contributes to the growing literature that uses an equilibrium search approach to understand the housing market. Search theory is a natural tool to use to analyze this market since it clearly takes time and effort for buyers to find suitable sellers

[6] The same mixed incentives apply in the labor market. *Ex ante* a job applicant wants to convince a prospective employer that she really wants the job; *ex post*, i.e., once the employer has decided to offer her the job, she wants the employer to believe that she has many other good options.

and vice versa. Most of the papers in this literature assume that search is random. In some of these papers, when a buyer and seller meet, one of the parties (typically the seller) makes a take-it-or-leave-it offer; in others, prices are determined by Nash bargaining. See, for example, Wheaton (1990) and Albrecht et al. (2007). In contrast, in our model, search is directed; that is, sellers post prices to attract buyers. Other models of the housing market that take a directed search approach include Díaz and Jerez (2011), Carillo (2012), and Stacey (2012). Díaz and Jerez (2011) use competitive search theory (Moen 1997) to analyze the problem initially posed in Wheaton (1990), in which shocks lead to mismatch, causing a household to first search to buy a new house and then to look for a buyer for its old house.[7] In equilibrium, all sellers post the same asking price, the asking price and the sales price are the same, and all houses sell with the same probability. In Carillo (2012), buyers also direct their search in response to posted asking prices, but sellers interact with only one buyer at a time. In his model, the asking price is a price ceiling – sometimes the seller gets the asking price, but sometimes the buyer gets the house at the seller's reservation value. Houses never sell above their asking prices because, by assumption, there is never any *ex post* competition among buyers. Finally, Stacey (2012) is the paper that is closest to ours. Using our model as a starting point, albeit with a two-point distribution for the idiosyncratic value that a buyer realizes once he visits a seller, he explores the implications of eliminating any commitment to the asking price. In the heterogeneous-seller version of our model, sellers signal their type by their (limited) commitment to the asking prices they announce, whereas in his model, also with two seller types, sellers signal whether they are motivated or not by the type of real estate contract they sign (high-service/high-fee versus low-service/low-fee).

We also contribute to the directed search literature. In the standard directed search model, there is full commitment in the sense that all transactions must take place at the posted price. In our model, however, there is only limited commitment. The posted price "means something" and is used to attract buyers, but the final selling price need not be the same as the posted price. Camera and Selcuk (2009) also consider a model of directed search with limited commitment. As we do, they assume that sellers post prices and that buyers direct their search in response to those postings. The difference between our approach and theirs comes once buyers choose which sellers to visit. They allow for the possibility of renegotiation, that is, that the final selling price and the posted price may differ, but they are agnostic about the specifics of the renegotiation process. Instead, they deduce some implications of assuming that the selling price is increasing in (i) the asking price and (ii) the number of buyers who visit the seller in

[7] See also Head, Lloyd-Ellis and Sun (2011), who take the competitive search framework used by Díaz and Jerez (2011) and embed it in a model of the housing market with endogenous entry of buyers at the city level that is similar to the one developed in Glaeser, Gyourko, Morales and Nathanson (2010).

question; for example, they prove that all sellers post the same asking price in symmetric equilibrium.[8] Our approach differs from that of Camera and Selcuk (2009) in that we assume a specific price determination mechanism. We take this more specific approach because the price determination mechanism that we analyze is an important one in practice.

We also add to the directed search literature by considering the potential signaling role of the asking price. As in Delacroix and Shi (2008), we consider a model in which the asking price plays the dual role of directing buyer search and signaling seller type. In their model, each seller chooses a pricing mechanism – either price posting or Nash bargaining – and whether to produce a low-quality or a high-quality good. Efficiency has two dimensions, entry and quality, and they ask under what conditions price posting or bargaining is the more efficient mechanism. The answer depends on the bargaining power parameter and on the relative quality of the two goods. In the heterogeneous-seller version of our model, there are also two choice variables in the social planner problem. Seller entry should be at the efficient level, and the probability that a buyer visits a high-type seller should be at the efficient level. The pricing mechanism that we analyze is able to satisfy these two criteria simultaneously. In particular, when buyers are heterogeneous and their type is private information, an auction is an efficient selling mechanism.

Finally, our model is closely related to the papers of Menzio (2007) and Kim and Kircher (2011), which consider the possibility of cheap-talk equilibria in a directed search environment. In the heterogeneous-seller version of our model, it is also natural to ask whether cheap talk might be enough to separate the two seller types. That is, is it enough for sellers to post advertisements announcing their types without commitment of any sort to the asking price? In our setup, the answer is "no" – relaxed sellers would want to mimic their more motivated counterparts.

In constructing our model, we have abstracted from some important features of the housing market. One obvious abstraction is that we ignore real estate agents. We do this to keep our model simple but also because the decision about the asking price, which is the focus of our model, is ultimately the seller's to make. We also abstract from the fact that in the housing market, buyers are often also sellers and their ability to buy may hinge on their ability to sell. Rather than modeling this explicitly as in Wheaton (1990) and Díaz and Jerez (2009), we capture this in the heterogeneous-seller version of

[8] As we show below, in the equilibrium of the homogeneous-seller version of our model, the expected selling price is *not* increasing in the asking price. More fundamentally, our model does not fit the Camera and Selcuk (2009) framework for two reasons. First, in our model, buyers draw idiosyncratic values once they visit a seller; i.e., buyers are *ex post* heterogeneous. Second, we allow for the possibility that sellers may be *ex ante* heterogeneous in the sense of having different reservation values.

our model through the reservation value. A motivated seller, one with a low reservation value, can be thought of as one who has already bought or put a contract on a new house and is thus eager to sell. Finally, houses are, of course, not identical – some are in good condition and located in desirable neighborhoods while others are not – and much of the variation in asking prices across houses reflects this intrinsic heterogeneity. In our model, we assume that buyers can identify these differences, perhaps with the help of real estate agents, perhaps by simply using the web. We are looking at the role that the asking price plays after adjusting for these differences. Sellers set different asking prices, even for houses that are intrinsically identical. These differences reflect the sellers' reservations values. Some are motivated, while others are willing to hold out for a good price.

The remainder of our paper is organized as follows. In the next section, we lay out the structure of the game that we analyze. In Section 3, we analyze the model assuming that all sellers have the same reservation value. In Section 4, we consider the heterogeneous-seller case. We show the nonexistence of pooling and hybrid equilibria and the existence of separating equilibrium. Finally, in Section 5, we conclude.

2 Basic Model

We model the housing market as a one-shot game played by B buyers and S sellers of identical houses. We consider a large market in which both B and S go to infinity but in such a way as to keep $\theta = B/S$, the market tightness, constant. We first analyze the market taking θ as given. Then, once the equilibrium is characterized for any given θ, we allow for free entry of sellers and discuss the efficiency of market equilibrium.

The game has several stages:

1. Each seller posts an asking price a.

2. Each buyer observes all posted prices and chooses one house to bid on. There is no coordination among the buyers.

3. Upon visiting a seller, the buyer draws a match-specific value, x. Match-specific values are private information and are *iid* draws across buyer-seller pairs from a continuous distribution, $F(x)$, with support $[0, 1]$. This distribution is assumed to have an increasing hazard. Buyers do not observe the number of other visitors to the house.

4. The buyer can accept the asking price, a, make a counteroffer, or walk away.

5. If no buyer visits, the seller retains the value of her house.

6. If at least one buyer visits, but no buyer accepts the asking price, then the seller can accept or reject the highest counteroffer. If one or more buyers accept the asking price, then there is an ascending-bid (second-price) auction with reserve price a among those buyers. In this case, the house is transferred to the highest bidder.

A buyer who fails to purchase a house receives a payoff of zero. The payoff for a buyer who draws x and then purchases the house is $x - p$, where p is the price that the buyer pays. If no sale is made, the owner of the house retains its value, while a seller who transfers her house to a buyer at price p receives that price as her payoff.

This is a model of directed search in the sense that buyers observe all asking prices and choose which seller to visit based on these asking prices. It differs from many directed search models in that the sellers make a limited commitment to their asking prices. That is, while in the usual directed search model, sellers (or vacancies) fully commit to their posted prices (wages), here the seller makes only a limited commitment. If only one buyer shows up and accepts the asking price, then the seller agrees to sell at that price, but if more buyers accept the asking price, then the price is bid up. We consider symmetric equilibria in which all buyers use the same strategy. They search optimally given the distribution of posted asking prices and given optimal directed search by other buyers. Buyers bid optimally given the bidding strategy followed by other buyers.

We first consider the case of homogeneous sellers, i.e., the case in which all sellers have the same reservation value s. In setting an asking price, each seller anticipates buyer reaction to her posted price given the distribution of asking prices posted by other sellers. When sellers are homogenous, we show that the only role of the asking price is to ensure that houses do not sell below s. After considering the homogeneous case, we turn to the heterogeneous case in which sellers differ with respect to their reservation values and seller type is private information. In this case, the asking price can potentially signal seller type. We assume that there are two seller types: high types who have reservation value s and low types who have a reservation value that we normalize to zero.

3 Homogeneous Sellers

We begin by considering the case in which all sellers have the same reservation value, s. We first show a payoff equivalence result for asking prices of s or more. Suppose buyers distribute themselves randomly across sellers posting asking prices greater than or equal to s. That is, suppose all sellers posting asking prices in this range face the same expected queue length. Then all asking prices in $[s, 1]$ are revenue equivalent for sellers. This in turn implies payoff equivalence for buyers, confirming the assumption that buyers randomize their visits across sellers posting asking prices in $[s, 1]$.

We next show that any distribution of asking prices on $[s, 1]$ is an equilibrium. We do this by showing that no seller wants to post an asking price below s. If a seller were to post an asking price $a < s$, the expected arrival rate of buyers would be greater than it would have been had she posted $a \geq s$, but the expected payoff would be lower. Since buyers understand that sellers will not accept counteroffers below $a \leq s$, a seller who posts an asking price $a \leq s$ is essentially offering a second-price auction with reserve price a. Using a result from Albrecht, Gautier and Vroman (2012), we show that this seller would prefer to post $a = s$.[9] Since asking prices $a \geq s$ are payoff equivalent and since sellers do not want to post asking prices below s, any distribution of asking prices greater than or equal to s constitutes an equilibrium.

3.1 Payoff Equivalence

Consider a seller posting $a \geq s$. If $a = s$, the seller is posting a second-price auction with reserve price s. If $a > s$, some buyers may choose to make counteroffers between s and a, while buyers who draw higher valuations may accept the asking price. The buyers who choose to make counteroffers are essentially engaging in a sealed-bid first-price auction (relevant only if no buyers accept a) while any who accept a are participating in a second-price auction.

Our payoff equivalence result follows from standard auction theory, although we need to account for the fact that the number of buyers is random. First, consider the case in which the number of buyers visiting a particular seller is given. A statement of revenue equivalence is given in Proposition 3.1 of Krishna (2010):

> Suppose that values are independently and identically distributed and all buyers are risk neutral. Then any symmetric and increasing equilibrium of any standard auction, such that the expected payment of a bidder with value zero is zero, yields the same expected revenue to the seller.

The selling mechanism that we consider is a standard auction since the mechanism dictates that the buyer who makes the highest bid of s or more (the highest counteroffer if no buyer accepts a; the highest bid in the second-price auction if one or more buyers accepts a) gets the house. The equilibrium is increasing since buyer bids are increasing

[9] Albrecht, Gautier, and Vroman (2012) shows that in a large market, when sellers have reservation value 0 and compete by advertising reserve prices for second-price auctions, each seller posts an asking price of 0.

in x. Finally, a buyer who draws $x \leq s$ gets value zero from this selling mechanism and pays nothing.[10]

We also have payoff equivalence for buyers across all asking prices of s or more. The surplus associated with a particular house is the maximum of the seller's reservation value and the highest value drawn by a buyer. Any surplus that doesn't go to the seller necessarily goes to the winning bidder (and any losing bidders get zero). Revenue equivalence for sellers thus implies payoff equivalence for buyers, so buyers are equally willing to visit any seller posting an asking price in $[s, 1]$.

Payoff equivalence for buyers and sellers continues to hold when the number of buyers is a random draw from a finite number of potential bidders (McAfee and McMillan 1987, Harstad, Kagel and Levin 1990). In our directed search setting, the expected queue length across all sellers offering the same expected payoff, i.e., across all sellers posting $a \geq s$, must be equal. In a large market, this means that the number of buyers visiting a particular seller is a Poisson random variable, a random draw from a distribution with unbounded support. The same argument used to show revenue and payoff equivalence when the set of potential buyers is finite can be used to show that it holds in this case as well.[11]

3.2 Equilibrium

We have shown that buyers and sellers are indifferent across all asking prices $a \geq s$. To show that any distribution of asking prices over $[s, 1]$ constitutes an equilibrium, we must show that no seller would choose to post an asking price below s. Note that if a seller posts $a < s$, she is in effect offering a second-price auction with reserve price a. The reason is that buyers know that there is never any point to making a counteroffer since counteroffers would always be rejected. The seller's problem of choosing an optimal reserve price in a second-price auction can be posed as a constrained maximization problem. The seller advertises a reserve price, a, to maximize her expected payoff subject to the constraint that a buyer who visits the seller can expect to receive at least as high a payoff as is available elsewhere in the market. The rate at which buyers visit this seller, ξ, adjusts so that the value of visiting this particular seller is the same as that of visiting any of the other sellers. The constrained maximization problem can thus be

[10] We assume that buyers who draw $x < s$ simply walk away without submitting a counteroffer.

[11] With a random number of buyers, an individual buyer's optimal bid is a weighted average of his optimal bids conditional on competing with $n = 0, 1, 2, ...$ other buyers; specifically, $b(x) = \frac{\sum p_n F(x)^n b(x;n)}{\sum p_n F(x)^n}$, where p_n is the probability the buyer is competing with n other buyers and $b(x;n)$ is the optimal bid at x when facing n other buyers. The only issue with a potentially infinite number of bidders is the convergence of the weighted average. With Poisson weights, convergence is assured.

written as[12]

$$\max\ \Pi(a,\xi) \text{ subject to } V(a,\xi)=\overline{V}, \tag{1}$$

where a is the reserve price, ξ is the Poisson arrival rate of buyers, and $\overline{V}$ is the market level of buyer utility. The seller and buyer payoffs are $\Pi(\cdot)$ and $V(\cdot)$, respectively, with

$$\begin{aligned}
\Pi(a,\xi) &= s+\xi\int_a^1 (v(x)-s)e^{-\xi(1-F(x))}f(x)dx \\
&= s+(1-e^{-\xi})\int_a^1 (v(x)-s)g(x)dx \\
V(a,\xi) &= \int_a^1 (x-v(x))e^{-\xi(1-F(x))}f(x)dx \\
&= \int_a^1 (1-F(x))e^{-\xi(1-F(x))}dx,
\end{aligned}$$

where

$$v(x)=x-\frac{1-F(x)}{f(x)}$$

is the "virtual valuation function," which can be interpreted as the marginal revenue associated with a buyer of type x (Bulow and Roberts 1989), and

$$g(x)=\frac{\xi e^{-\xi(1-F(x))}f(x)}{1-e^{-\xi}}$$

is the density of the highest valuation drawn by the buyers visiting a particular seller conditional on the seller having at least one visitor.[13] As is standard, we assume that $v(x)$ is increasing in x. The expected seller payoff is s plus the probability that at least one buyer visits the seller times the integral of $v(x)-s$ against the density of the highest valuation. Finally, the buyer payoff, $V(a,\xi)$, is $x-v(x)$ times the probability that no other buyer draws a value greater than x integrated against the density of x.

[12] This formulation of the problem follows Peters and Severinov (1997).

[13] The derivation of $g(x)$ is as follows. Let H denote the event that a particular buyer draws the highest valuation. Using Bayes Law,

$$f(x|H)\equiv g(x)=\frac{P(H|x)f(x)}{P(H)}.$$

The probability that a buyer who has drawn x has the highest valuation is

$$P[H|x]=e^{-\xi(1-F(x))}.$$

The unconditional probability that any one buyer has the highest valuation is

$$P[H]=\int_0^1 e^{-\xi(1-F(x))}f(x)dx=\frac{1-e^{-\xi}}{\xi}.$$

Lemma 1 *The asking price that solves the constrained maximization problem (1) is* $a^* = s$. *The corresponding Poisson arrival rate is the solution to* $V(s, \xi^*) = \overline{V}$.

The proof is given In the Appendix and parallels one given in Albrecht, Gautier, and Vroman (2012), which deals with the case of $s = 0$ [14] This means that no sellers post asking prices below s, and since all asking prices $a \geq s$ are payoff equivalent, any distribution of asking prices on $[s, 1]$ is an equilibrium.

Summarizing,

Proposition 1 *Any configuration of asking prices over* $[s, 1]$ *constitutes an equilibrium of the homogeneous-seller model. All such equilibria are payoff equivalent. Further, there are no equilibria in which any sellers post asking prices below* s.

Proposition 1 states that there is an infinity of equilibria in the homogeneous-seller model, but we have shown that all of these equilibria are payoff equivalent. We can thus choose one of these equilibria, for example, the one in which all sellers post $a = s$, to demonstrate some of the properties of equilibrium. In particular, we now show that the probability of sale and the average selling price vary with θ and s, the exogenous parameters of the model, in the expected way.

Consider first the probability that any particular house is sold. This is

$$P[\text{Sale}] = 1 - e^{-\theta(1-F(s))}.$$

As expected, as the market gets tighter, i.e., as θ increases, the probability that a house sells increases. In addition, also as expected, as sellers become "less motivated," i.e., as s increases, the probability of a sale decreases.

Next, conditional on a sale, the expected price is

$$E[P] = \frac{\int_s^1 v(x)g(x)dx}{\int_s^1 g(x)dx}.$$

Since neither $v(x)$ nor $g(x)$ depend on s,

$$\begin{aligned} \frac{\partial E[P]}{\partial s} &= \frac{-v(s)g(s)\int_s^1 g(x)dx + g(s)\int_s^1 v(x)g(x)dx}{\left(\int_s^1 g(x)dx\right)^2} \\ &= \frac{g(s)\int_s^1 (v(x) - v(s))g(x)dx}{\left(\int_s^1 g(x)dx\right)^2} > 0, \end{aligned}$$

[14]The lemma generalizes results in Julien, Kennes, and King (2000) and in Eeckhour and Kircher (2010). In different contexts, they show that if all buyers have the same valuation (not less than the common seller reservation value), then the equilibrium reserve price in a competing auctions game is the seller reservation value. That is, if the distribution of buyer valuations is degenerate at $x \geq s$, then the equilibrium reserve price is s.

where the inequality follows from our assumption that $v(x)$ is increasing. As sellers become more motivated, fewer houses are sold, but those that do sell are sold at a higher price on average.

Finally, to examine how the expected price varies with θ, write

$$E[P] = \int_s^1 v(x)h(x;\theta)dx,$$

where

$$h(x;\theta) = \frac{g(x;\theta)}{\int_s^1 g(x;\theta)dx}.$$

Note that $H(x;\theta) = \int_s^x h(t;\theta)dt$ satisfies first-order stochastic dominance with respect to θ; that is, $\theta' > \theta$ implies $H(x;\theta') < H(x;\theta)$ for all $x \in (s,1)$.[15] Now, write

$$v(x) = v(s) + \int_s^x v'(t)dt,$$

so

$$\begin{aligned} E[P] &= v(s) + \int_s^1 h(x;\theta) \int_s^x v'(t)dtdx \\ &= v(s) + \int_s^1 v'(t) \int_t^1 h(x;\theta)dxdt \\ &= v(s) + \int_s^1 v'(t)(1 - H(t;\theta))dt. \end{aligned}$$

Finally, since $v'(t) > 0$ (by assumption) and $\dfrac{\partial(1 - H(t;\theta))}{\partial\theta} > 0$ (by first-order stochastic dominance with respect to θ), we have $\dfrac{\partial E[P]}{\partial\theta} > 0$.

3.3 Endogenizing θ - Efficiency

It is clear that in the model with homogeneous sellers, the mechanism that we analyze is efficient in the sense that once buyers match with sellers, no mutually profitable transactions are left unconsummated. Further, if more than one buyer draws a valuation above the seller's reservation value, the house is necessarily sold to the buyer with the highest valuation. The only remaining efficiency question is whether, once we allow for free entry on the seller side of the market, the buyer/seller ratio is constrained efficient.

[15] We thank Xiaoming Cai for suggesting this argument. The intuition is that as the number of visitors a seller can expect increases, the distribution of the highest valuation drawn among those visitors becomes more "favorable" from the seller's point of view. The algebra required to verify this formally is a bit tedious but is available on request.

In Albrecht, Gautier, and Vroman (2013), we prove that in a large market in which sellers compete by posting reserve prices for second-price auctions, the free-entry equilibrium level of seller entry is constrained efficient. The mechanism that we consider in our housing model is equivalent to a second-price auction in the sense that, given any level of market tightness, expected buyer and seller payoffs are the same in the infinity of payoff-equivalent equilibria of our model as they would be if sellers were to compete by posting reserve prices for second-price auctions. In particular, sellers have the same incentive to enter as they would if houses were sold by second-price auctions. We can therefore apply the efficiency result from Albrecht, Gautier, and Vroman (2013) to conclude:

Proposition 2 *Free-entry equilibrium is efficient in the homogeneous-seller case.*

4 Heterogeneous Sellers

When all sellers have the same reservation value, the only role that asking prices play is to ensure that houses never sell below that common value. Why then do buyers care about asking prices? The reason, in our view, is that the asking price signals a seller's type, that is, how eager she is to sell her house. We now develop this idea in the heterogeneous-seller version of our model.

We suppose that sellers are heterogeneous with respect to their reservation values. For simplicity, we consider two seller types. A fraction q of the sellers, the high (H) types ("relaxed sellers"), have reservation value s, as in the homogeneous case. The remaining sellers, the low (L) types ("motivated sellers"), have a lower reservation value, which we normalize to 0. Seller type is private information, but q is common knowledge. The model with heterogeneous sellers is a signaling game, so we consider *Perfect Bayesian Equilibria*. In a Perfect Bayesian Equilbrium, (i) buyer and seller strategies must constitute a subgame perfect equilibrium given buyer beliefs and (ii) buyers must update their beliefs from their priors using Bayes Law at every node of the game that is reached with positive probability using the equilibrium strategies. A seller's strategy is a choice of an asking price and, in case no buyer accepts her asking price, a reaction (accept or reject) to the highest counteroffer received, if any. A buyer's strategy is a choice of which seller to visit – or a distribution of probability across all sellers – together with a choice of whether to make a counteroffer (and, if so, at what level) or to accept the seller's asking price once x is observed. Buyers form beliefs about sellers' types based on their asking prices. As in the homogeneous-seller case, we only consider symmetric equilibria in which all buyers use the same strategy.

There are three types of Perfect Bayesian Equilibria to consider in which sellers follow pure strategies. In a separating equilibrium, each seller posts an asking price that is type-revealing. There are also two types of pooling equilibria to consider – one in which type-H sellers mimic type-L sellers by posting asking prices below s ("pooling-on-low") and one in which type-L sellers mimic type-H sellers by posting asking prices of s or more ("pooling-on-high"). Finally, hybrid equilibria, in which one seller type randomizes between a high price and a low price, also need to be considered. In a "mixing-by-lows" equilibrium, type-L sellers randomize between posting a low price and a high price, while type-H sellers all post a high price, and in a "mixing-by-highs" equilibrium, type-H sellers randomize between posting a low versus a high price, while all type-L sellers post a low price.

The asking price has the potential to signal seller type, but the incentives for one type to mimic the other are not straightforward in our model. *Ex ante* each seller wants buyers to believe that she is type L because this increases her expected queue length, but *ex post*, once buyers have allocated themselves across sellers, every seller wants buyers to believe that she is type H because this belief leads to higher bids on average. Sellers, however, have only one signal and must trade off the benefit of longer queues in the first stage against higher bids in the second stage. This is why the two types of pooling equilibria and the two types of hybrid equilibria are conceivable in our setting.

As it turns out, despite the incentives to mimic, neither pooling equilibria nor hybrid equilibria exist in our model. If all sellers were to post high asking prices, then a type-L seller would want to deviate to an asking price of zero. Indeed, the type-L seller would want to deviate to zero independent of buyer beliefs about what that deviation might signal. This argument rules out the existence of pooling-on-high and mixing-by-lows equilibria. On the other hand, if all sellers were to post a low price and a type-H seller were to deviate to an asking price of s or more, then the reaction that buyers would have to that deviation would depend on their beliefs about the deviant's type. However, using a standard restriction on out-of-equilibrium beliefs, we argue that buyers should view the deviation as a probability-one signal that the seller is type H. Given this belief, it is then in the interest of the type-H seller to deviate to the higher price. This rules out the existence of pooling-on-low and mixing-by-highs equilibria. In short, there are no equilibria in which any type-H seller pretends to be a type L nor are there equilibria in which any type-L seller pretends to be type H. The *ex ante* incentive to lower the asking price to attract buyers dominates for type L; the *ex post* incentive to set a price that elicits higher bids (and ensures that the house will not be sold at a price below s) dominates for type H.

The equilibria that do exist separate the two seller types. Type-L sellers post low prices, and type-H sellers post high prices of s or more, and there is a separating equilibrium for each parameter combination, $\{q, s, \theta\}$. More precisely, similar to the homogeneous-seller case, there is an infinity of payoff-equivalent equilibria for each parameter configuration. Other separating equilibria in which type-H sellers post higher asking prices than type-L sellers do but in which the higher asking prices are less than s are also conceivable, but we rule out this possibility by invoking the same restriction on out-of-equilibrium beliefs that we use to show the nonexistence of pooling-on-low and mixing-by-highs equilibria.

Separating equilibria in which the high types post asking prices of s or more – the only equilibria that exist, given our equilibrium refinement – are efficient. The first sense in which these equilibria are efficient is that once buyers have allocated themselves across sellers, sales are consummated if and only if the net surplus from doing so is positive, and when a house is sold it always goes to the buyer with the highest valuation. Second, a social planner would prefer that type-L sellers have longer queues on average than do type-H sellers. Separating equilibrium gets these queue lengths just right. That is, the equilibrium rates at which buyers visit the two seller types equal the rates that a social planner would choose. Finally, we discuss entry in the heterogeneous-seller model. Since there are two seller types and since the type-H sellers have the least to gain from entering the market, we endogenize their entry. As in the homogeneous case, the equilibrium level of entry is efficient.

We now give the details of these arguments.

4.1 Nonexistence of Pooling or Hybrid Equilibria

We begin by showing the nonexistence of pooling and hybrid equilibria. In a pooling equilibrium, all sellers post the same asking price. There are two cases to consider. First, all sellers could post a high asking price, e.g., $a = s$. Second, they could all post a low asking price, e.g., $a = 0$. We refer to the two cases as "pooling on high" and "pooling on low" and analyze them in turn.

Consider first a candidate pooling-on-high equilibrium; e.g., suppose all sellers post $a = s$. Buyers know that a seller posting the candidate equilibrium asking price is type H with probability q and type L with probability $1-q$. A buyer who draws a low enough value of x makes a counteroffer below s, which only type-L sellers accept, while a buyer who draws a higher value of x may prefer to accept s.[16] If one or more buyers accepts s, then an English auction, limited to those buyers who accepted s, follows. From the perspective of a type-L seller, this mechanism defines a standard auction. The house is

[16] The reason for the conditional language ("may prefer") is that if s is sufficiently close to one and/or q is sufficiently close to zero, buyers prefer making counteroffers to accepting s for all $x \in [0, 1]$.

sold to the buyer who makes the highest bid, buyer bids are increasing in their valuations, and a buyer who draws $x = 0$ pays (and receives) nothing. Fixing the expected arrival rate of buyers, Proposition 3.1 in Krishna (2010) implies that, for a type-L seller, this auction is revenue equivalent to one in which she posts $a = 0$. However, if the type-L seller deviates to $a = 0$, her expected arrival rate increases, so her expected payoff increases. Type-L sellers thus want to deviate from $a = s$ to $a = 0$. This simple argument shows the nonexistence of pooling-on-high equilibria, and the same reasoning rules out the existence of a hybrid equilibrium in which all type-H sellers post $a = s$ while type-L sellers mix between $a = s$ and $a = 0$. It is worth noting that buyer beliefs about what a deviation to an asking price of zero might signal about the deviant's type play no role in the argument. A buyer who visits a seller posting $a = 0$ doesn't care about the seller's type. The buyer's optimal bid, namely, accept $a = 0$, is the same regardless of the seller's type, as is his expected payoff. Pooling on any other $a > s$ can also be ruled out as an equilibrium by the same argument. Type-L sellers face a standard auction and can increase their payoff by deviating to $a = 0$. Similarly, hybrid equilibria in which all type-H sellers post an asking price above s and the type-L sellers mix between that asking price and zero are also ruled out.

We do, however, use a restriction on out-of-equilibrium beliefs to prove the nonexistence of pooling-on-low and mixing-by-highs equilibria. To understand the equilibrium refinement that we use, suppose all sellers post $a = 0$. According to the Intuitive Criterion (Cho and Kreps 1987), buyers should believe that a deviation to $a = s$ signals type H with probability one if (i) the deviation is strictly profitable for a type-H seller conditional on buyers believing that the deviation signals type H with probability one and (ii) the deviation is strictly unprofitable for a type-L seller for any beliefs that buyers might hold about the deviant's type. Let μ be the probability that buyers attach to type H given a deviation to $a = s$. By Lemma 1, a deviation from $a = 0$ to $a = s$ is strictly profitable for type H if $\mu = 1$, and by the argument that we used to rule out pooling-on-high or mixing-by-lows equilibria, the same deviation is strictly unprofitable for a type L so long as $\mu > 0$. If $\mu = 0$, however, the type-L seller would neither gain nor lose by deviating to $a = s$. All asking prices are revenue equivalent for this type so long as buyers continue to believe that her reservation value is zero with probability one. The Intuitive Criterion is thus not strong enough to rule out the candidate pooling-on-low equilibrium. Instead, we appeal to the "D1 refinement" (Banks and Sobel 1987; see also Fudenberg and Tirole 1992, p. 452). Refinement D1 requires buyers to set $\mu = 1$ if the set of beliefs that make type L willing to deviate to $a = s$ is a strict subset of the set of beliefs that make type H willing to deviate to $a = s$. We have already argued that type L is willing to deviate to $a = s$ only if $\mu = 0$ and that type H is willing to deviate to $a = s$

if $\mu = 1$. To show that the condition required for D1 holds, it thus suffices to show that type H is also willing to deviate to $a = s$ when $\mu = 0$. Suppose then that a type-H seller deviates from $a = 0$ to $a = s$ but that buyers view the deviation as a probability-one signal that the deviant is type L; i.e., buyers set $\mu = 0$. The arrival rate of buyers to the deviant is the same that she would have realized had she continued to post $a = 0$. Some buyers who visit the deviant will make counteroffers below s; others may accept s. If the type-H deviant always accepted the highest bid received, i.e., if she were to accept the highest counteroffer when no buyer accepts s, then, by revenue equivalence, her expected payoff would be the same as it would have been had she continued to post $a = 0$. However, the deviant has the option to reject counteroffers below s, and it is in her interest to do so. In short, if $\mu = 0$, a type-H seller benefits by deviating to $a = s$ because (i) the expected number of buyers visiting does not change and (ii) she can reject bids below her reservation value.

Finally, if all sellers post $a = 0$, and if buyers believe that a deviation to $a = s$ signals type H with probability one, then – again, by Lemma 1, – it is in the interest of the type-H seller to deviate. Pooling-on-low equilibria are ruled out by this reasoning, and a similar argument gives the nonexistence of a hybrid equilibrium in which type-H sellers mix between a low price and a high price. We have used $a = 0$ as a convenient example of a candidate pooling-on-low equilibrium, but we can also rule out pooling on other asking prices below s. In particular, if all sellers post $a \in (0, s)$, then, by the same argument that we used to rule out pooling-on-high equilibria, type-L sellers want to deviate to an asking price of zero.

Summarizing, we have shown:

Proposition 3 *Neither pooling-on-high nor mixing-by-lows equilibria exist in the heterogeneous-seller version of the model. In addition, if buyers view an asking price of $a \geq s$ as a probability-one signal that the seller posting that price is type H, then neither pooling-on-low nor mixing-by-highs equilibria exist.*

4.2 Separating Equilibria

A natural separating equilibrium to consider is one in which all type-L sellers post $a = 0$, all type-H sellers post $a = s$, and in which buyers believe that $a = 0$ signals type L with probability 1 while $a = s$ signals type H with probability 1. This configuration satisfies an obvious efficiency criterion, namely, that a house is sold if and only if the highest buyer valuation is greater than or equal to the seller's reservation value.

To prove the existence of this type of equilibrium, note first that, given buyer beliefs, a type-H seller strictly prefers posting the high asking price to the low one. This follows directly from Lemma 1 since, given the hypothesized buyer beliefs, the choice between $a = 0$ versus $a = s$ is one of choosing between reserve prices for a second-price auction. Similarly, Lemma 1 implies that a type-L seller strictly prefers posting $a = 0$ to $a = s$. There is, however, the question of whether either seller type would want to deviate to some asking price other than 0 or s. As discussed above when we ruled out the existence of a pooling-on-low equilibrium, a type-L seller strictly prefers $a = 0$ to any $a' > 0$ if posting a' would lead buyers to place any positive probability on the possibility that the deviant might be type H. Further, even if buyers believe that a deviation to a' signals L with probability one, type-L sellers are no better off posting a' than posting zero. In short, type-L sellers have no incentive to deviate from the conjectured equilibrium configuration. Next, consider a type-H seller. If a type H deviates to $a' > s$ and buyers view $a' > s$ as a probability-one signal that the deviant is type H, then the type-H seller neither gains nor loses by the deviation. Finally, it cannot be in the interest of a type-H seller to deviate to $a' \in (0, s)$. The argument is by contradiction. Specifically, since it is not in the interest of type L to deviate to a', buyers should believe the deviant is type H with probability 1. But if $\mu = 1$, then Lemma 1 shows that it is not in the interest of a type-H seller to post a'. In short, given our restriction on out-of-equilibrium buyer beliefs, type-H sellers have no incentive to deviate downward from $a = s$. Finally, of course, if all type-L sellers post $a = 0$ and all type-H sellers post $a = s$, then buyer beliefs are consistent, and the equilibrium exists.

In addition to the separating equilibrium just described, there are other payoff-equivalent separating equilibria. In particular, a situation in which all type L sellers post $a = 0$, while type H sellers post any distribution of asking prices over $[s, 1]$, and buyers believe $a = 0$ signals type L and that asking prices of s or more signal type H, is also a Perfect Bayesian Equilibrium. There are also payoff-equivalent equilibria in which type-L sellers post a distribution of low asking prices while type-H sellers post a distribution of high asking prices.

Without restrictions on out-of-equilibrium beliefs, there may also exist payoff-inferior separating equilibria. Suppose, for example, that all type-L sellers post $a = 0$ while all type-H sellers post $a = s - \varepsilon$. Suppose further that if a seller were to post $a = s$, buyers would believe this signaled type L with probability one. Then it would not be in the interest of a type-H seller to deviate to $a = s$. This configuration entails a loss of surplus relative to the equilibrium in which all type-L sellers post $a = 0$ and all type-H sellers post $a = s$ since when type-H sellers post an asking price below s, houses are sometimes transferred from a type-H seller to a buyer even though the buyer values the house less than the seller does.

We rule out these “unnatural” separating equilibrium by appealing to the same refinement that we used to show the nonexistence of pooling-on-low equilibria. Specifically, given that buyers believe that a deviation to $a = s$ signals type H with probability one, it is in the interest of type H to make that deviation. This argument rules out any candidate equilibrium in which type-H sellers are assumed to post asking prices below their reservation value.

Summarizing, we have the following results on separating equilibrium:

Proposition 4 *Suppose buyers view a deviation to an asking price above the highest asking price posted by type-L sellers as a probability-one signal that the deviant is type H. Then there exists a separating equilibrium in which all type-L sellers post $a = 0$, all type-H sellers post $a = s$, and in which buyers believe that $a = 0$ signals type L with probability 1 while $a = s$ signals type H with probability 1. There also exist payoff-equivalent separating equilibria in which type-L sellers post a distribution of low asking prices, type-H sellers post a distribution of asking prices over $[s, 1]$, and in which buyers believe that low (high) asking prices signal type L (H) with probability one. Finally, there exist no separating equilibria in which type-H sellers post asking prices below s.*

4.2.1 Buyer Optimality Condition

A continuum of payoff-equivalent separating equilibria exist for each parameter configuration, $\{q, s, \theta\}$. Relative to the homogeneous-seller version of the model, an additional issue to consider is the question of how buyers allocate themselves across the two seller types. Suppose buyers visit type H with probability r and type L with probability $1-r$. For given q and θ, this implies an arrival rate of $\theta_L = \dfrac{(1-r)\theta}{1-q}$ to type-L sellers and of $\theta_H = \dfrac{r\theta}{q}$ to type-H sellers. Given r, the expected payoff for a buyer who visits a type-L seller is

$$V_L(r) = \int_0^1 (1 - F(x))e^{-\theta_L(1-F(x))}dx,$$

while the expected payoff for a buyer who visits a type-H seller is

$$V_H(r) = \int_s^1 (1 - F(x))e^{-\theta_H(1-F(x))}dx.$$

This gives the following *Buyer Optimality Condition*

$$V_L(r) \geq V_H(r) \text{ with equality if } r > 0. \tag{2}$$

Since (i) $V_L(r)$ is increasing in r, (ii) $V_H(r)$ is decreasing in r, (iii) $V_H(0) \geq V_L(0)$, and (iv) $V_L(q) \geq V_H(q)$, there is a unique $r \in [0, q]$ that satisfies the Buyer Optimality Condition.

To get a sense for the Buyer Optimality Condition, we consider a simple example. Suppose X follows a standard uniform distribution, so $F(x) = x$ for $0 \leq x \leq 1$. Then

$$
\begin{aligned}
V_L(r) &= \frac{1 - e^{-\theta_L} - \theta_L e^{-\theta_L}}{\theta_L^2} \\
V_H(r) &= \frac{1 - e^{-\theta_H(1-s)} - \theta_H(1-s)e^{-\theta_H(1-s)}}{\theta_H^2}.
\end{aligned}
$$

The shaded areas of Figures 1 and 2 show the set of (s, θ) combinations for which $r > 0$ for two different values of q. The pattern shown in these figures is intuitive. When s is not too high, buyers do not lose much by visiting a type-H seller, and when θ is not too low, the market is relatively tight so buyers have an incentive to visit the type-H sellers. As q increases, there are relatively fewer type-L sellers to visit so buyers have more incentive to visit the type-H sellers. In the non-shaded areas in Figures 1 and 2, where s is relatively high and/or θ is relatively low, separating equilibria exist with $r = 0$, i.e., buyers do not visit the type-H sellers.

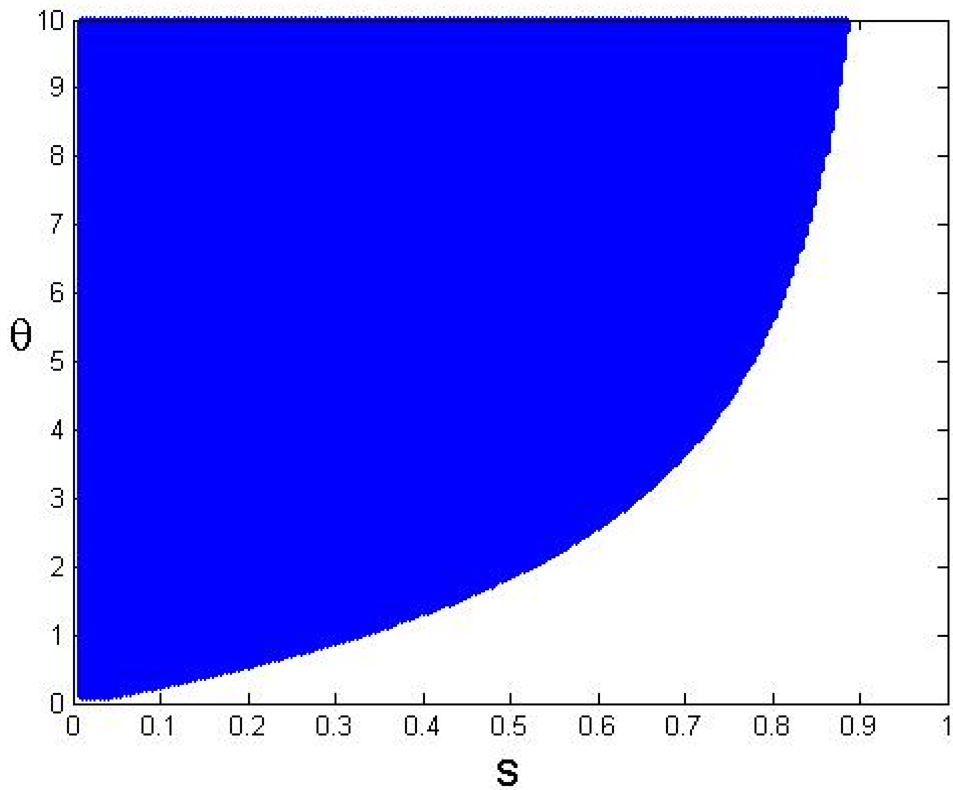

Figure 1: (s, θ) combinations for which $r > 0$ for $q = 0.2$

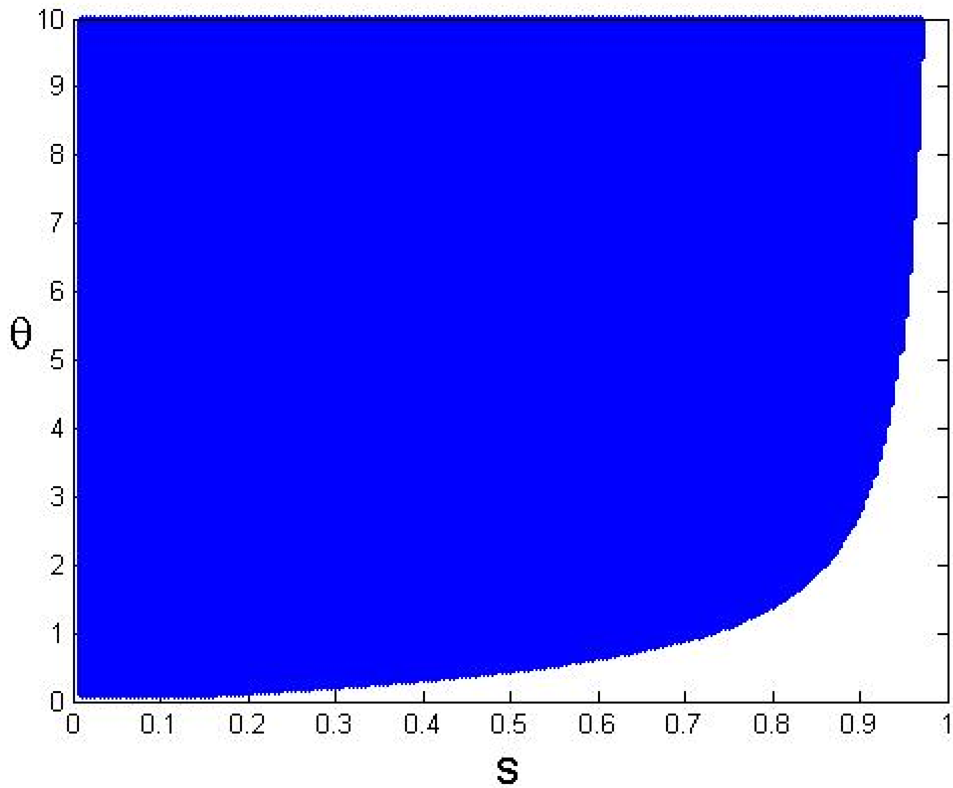

Figure 2: (s, θ) combinations for which $r > 0$ for $q = 0.8$

4.2.2 Equilibrium with Free Entry

We next consider the equilibrium entry of sellers. Type-L sellers enter so long as

$$\theta_L \int_0^1 v(x) e^{-\theta_L(1-F(x))} f(x) dx \geq A.$$

Assume this free-entry condition holds as a strict inequality, so that all motivated sellers enter the market. If there were some type-L sellers who chose not to enter, then it could not be in the interest of any type-H sellers to enter, and we would be back in the homogeneous-seller case. We let B be the measure of buyers in the market and L be the measure of type-L sellers, and we define $\phi = B/L$, the exogenous ratio of buyers to type-L sellers. The interesting entry question therefore has to do with type-H sellers, and in equilibrium, the free-entry condition for this type is

$$s + \theta_H \int_s^1 (v(x) - s) e^{-\theta_H(1-F(x))} f(x) dx \leq A + s \text{ with equality if } \theta_H > 0. \tag{3}$$

Once we impose this free-entry condition for type-H sellers, the model is described by two parameters, ϕ and s. Again, it is useful to consider the standard uniform example. Figure 3 shows the set of (ϕ, s) combinations that are consistent with entry by type-H sellers. All else equal, the lower is s, i.e., the smaller is the difference in motivation between type-L and type-H sellers, the more incentive there is for type-H sellers to incur the advertising cost and enter the market. Similarly, the higher is ϕ, the more incentive there is for entry by relaxed sellers since as ϕ rises, the number of visiting buyers that a seller can expect increases.

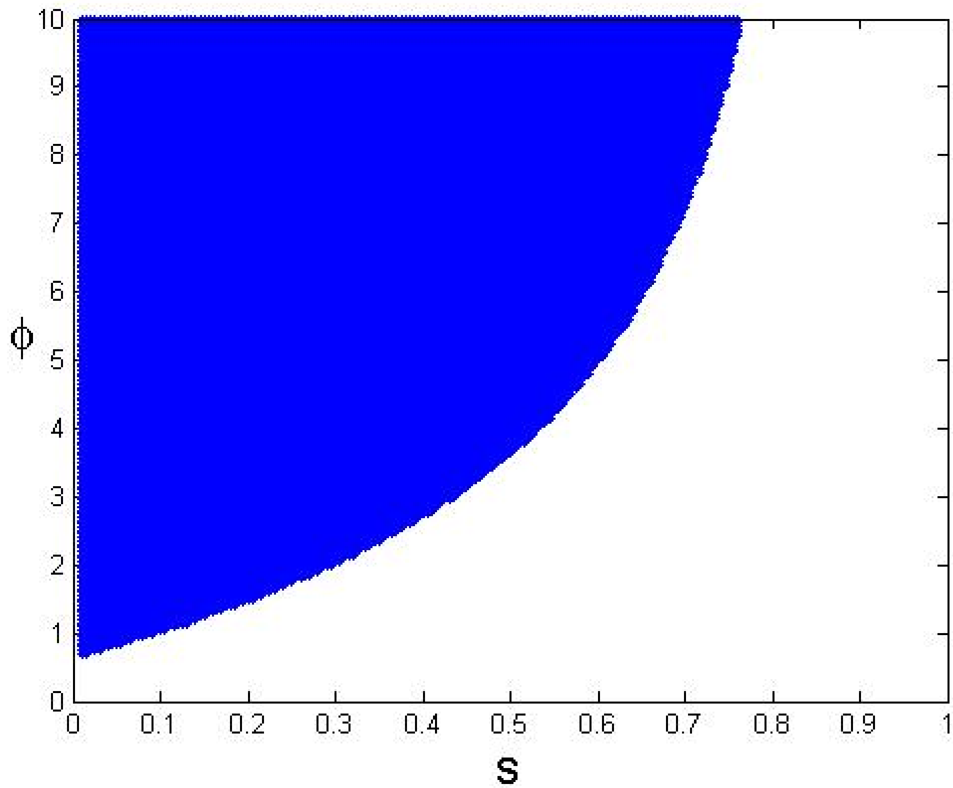

Figure 3: (s, ϕ) combinations with $H > 0$

4.3 Efficiency

In the heterogeneous-seller version of the model, there are three dimensions of efficiency to consider. First, as in the homogeneous seller version of the model, once buyers are matched to sellers, the selling mechanism that we consider is efficient. In separating equilibrium, the house is sold if and only if the highest buyer valuation exceeds the seller's reservation value, and if the house is sold, it goes to the buyer with the highest valuation. Second, again as in the homogeneous-seller case, there is the question of whether the equilibrium levels of seller entry are the same as the levels that a social planner would choose. Finally – and this is specific to the heterogeneous-seller version of the model – there is the question of whether buyers allocate themselves efficiently across the two seller types. To address these questions about queue lengths and seller entry, we formulate the social planner's objective and argue that our selling mechanism implements the social optimum.

The social planner wants to choose the level of entry by type-H sellers, H, and the arrival rates, θ_L and θ_H, to the two seller types to maximize total surplus in the market. We can derive an expression for total surplus as follows. Consider a type-L seller, and suppose n buyers visit this seller. Then the expected surplus associated with this seller is $E[Y_n]$, where $Y_n = \max[X_1, ..., X_n]$. That is, the surplus associated with this seller (again, conditional on being visited by n buyers) is the maximum valuation drawn by the n visitors. Similarly, the expected surplus associated with a type-H seller who is visited by n buyers is $E\max[s, Y_n]$ since this seller retains her house if no buyer draws a valuation above s. These can be found using standard results on order statistics.

The social planner's problem can then be expressed on a per-buyer basis as one of choosing H, and the arrival rates, θ_L and θ_H, to the two seller types to maximize the net surplus

$$\frac{L}{B}\left(1 - \int_0^1 e^{-\theta_L(1-F(x))}dx\right) + \frac{H}{B}\left(1 - \int_s^1 e^{-\theta_H(1-F(x))}dx - (A+s)\right).$$

Equivalently, we can express the planner's problem in terms of q and r. That is, the choice variables in the planner's problem, namely, the measure of type-H sellers in the market and the Poisson rates at which the two seller types receive visitors can be written as functions of q and r. the social planner's problem is one of choosing q and r to maximize

$$\frac{1}{\phi}\left(1 - \int_0^1 e^{-(1-r)\phi(1-F(x))}dx\right) + \frac{1}{\phi}\left(\frac{q}{1-q}\right)\left(1 - \int_s^1 e^{-r\left(\frac{1-q}{q}\right)\phi(1-F(x))}dx - (A+s)\right).$$

We have shown in Albrecht, Gautier, and Vroman (2013) for the case in which all sellers post their reservation values that the equilibrium and social planner values of q and r are the same. Since all equilibria in our housing model are payoff equivalent to the one in which the sellers post their reservation values, all other equilibria will also be efficient in the sense that entry of sellers is at the level the social planner would want as are the queue lengths at the two types of sellers. That is,.

Proposition 5 *Free-entry equilibrium is efficient in the heterogeneous-seller model.*

5 Conclusions

In this paper, we construct a directed search model of the housing market. The mechanism that we analyze captures important aspects of the way houses are bought and sold in the United States. Sellers post asking prices, and buyers direct their search based on these prices. When a buyer visits a house, he can walk away, make a counteroffer, or offer to pay the asking price. If no buyers offer to pay the asking price, the seller can accept or reject the best counteroffer (if any) that she receives. If at least one buyer offers to pay the asking price, the seller is committed to sell her house at a price equal to the highest bid that follows from the competition among those buyers.

In the homogeneous-seller version of this model, that is, when we assume that all sellers have the same reservation value, s, we show that any distribution of asking prices over $[s, 1]$ is consistent with equilibrium. Furthermore, our model implies that houses sometimes sell below, sometimes at, and sometimes above the asking price. Thus, our model is consistent with equilibrium price dispersion for identical houses sold by identical sellers in terms of both asking prices and final sales prices. This free-entry equilibrium is also constrained efficient. That is, when sellers have to pay an advertising cost to enter the market, the free-entry and the social planner levels of market tightness coincide.

In the heterogeneous-seller version of the model, only separating equilibria exist. In separating equilibrium, the sellers with the low (high) reservation value identify themselves by posting a low (high) asking price. That is, the asking price also plays a signaling role by allocating buyers across the two seller types. Equilibrium is again constrained efficient. The fraction of buyers who visit high-type sellers and the level of market tightness equal the values that a social planner would choose. Of course, we are not arguing that there are no inefficiencies in the housing market, but rather that the pricing mechanism and the fact that buyers do not directly observe seller types is not a source of inefficiency.

Our paper contributes both to the growing literature that uses equilibrium search theory to model the housing market and to the directed search literature. Our contribution to the housing literature is to build a directed search model that captures the main features of the house-selling process in the United States. We explain the role of the asking price and its relationship to the sales price, and we show that the mechanism we analyze is constrained efficient. Finally, our contribution to the directed search literature is to analyze a model in which there is only limited commitment and the posted price also plays a signaling role.

References

[1] Albrecht, J., A. Anderson, E. Smith and S. Vroman (2007), Opportunistic Matching in the Housing Market, *International Economic Review,* 48, 641-64.

[2] Albrecht, J., P. Gautier and S. Vroman (2012), A Note on Peters and Severinov, 'Competition Among Sellers Who Offer Auctions Instead of Prices', *Journal of Economic Theory*, 147, 389-92.

[3] Albrecht, J., P. Gautier and S. Vroman (2013), Efficient Entry in Competing Auctions, Georgetown University Working Paper 13-05..

[4] Banks, J. and J. Sobel (1987), Equilibrium Selection in Signaling Games, *Econometrica*, 55, 647-61.

[5] Bulow, J. and J. Roberts (1989), The Simple Economics of Optimal Auctions, *Journal of Political Economy*, 97, 1060-90.

[6] Camera, G. and C. Selcuk (2009), Price Dispersion with Directed Search, *Journal of the European Economic Association*, 7, 1193-1224.

[7] Carillo, P. (2012), An Empirical Stationary Equilibrium Search Model of the Housing Market, *International Economic Review*, 53, 203-34.

[8] Cho, I-K. and D. Kreps (1987), Signaling Games and Stable Equilibria, *Quarterly Journal of Economics*, 102, 179-221.

[9] Delacroix A. and S. Shi (2008), Pricing and Signalling with Frictions, mimeo.

[10] De Wit, E. and B. van der Klaauw (2010), Asymmetric Information and List-Price Reductions in the Housing Market, mimeo.

[11] Díaz, A. and B. Jerez (2011), House Prices, Sales and Time on the Market: A Search-Theoretic Approach, mimeo.

[12] Eeckhout J. and P. Kircher (2010), Sorting versus Screening: Search Frictions and Competing Mechanisms, *Journal of Economic Theory*, 145, 1354-1384.

[13] Fudenberg, D. and J. Tirole (1992), *Game Theory*, MIT Press.

[14] Glaeser, E., J. Gyourko, E. Morales and C. Nathanson (2010), Housing Dynamics, mimeo.

[15] Harstad, R., J. Kagel and D. Levin (1990), Equilibrium Bid Functions for Auctions with an Uncertain Number of Buyers, *Economic Letters*, 33, 35-40.

[16] Head, A., H. Lloyd-Ellis and A. Sun (2011), Search, Liquidity and the Dynamics of House Prices and Construction, mimeo.

[17] Julien, B., J. Kennes, and I. King (2000), Bidding for Labor, *Review of Economic Dynamics, 3,* 619-49.

[18] Kim, K. and P. Kircher (2011), Efficient Cheap Talk in Directed Search: On the Non-essentiality of Commitment in Market Games, mimeo.

[19] Krishna, V. (2010), *Auction Theory*, Elsevier/Academic Press, 2nd ed.

[20] McAfee, P. and J. McMillan (1987), Auctions with a Stochastic Number of Bidders, *Journal of Economic Theory*, 43, 1-19.

[21] Merlo, A. and F. Ortalo-Magné (2004), Bargaining over Residential Real Estate: Evidence from England, *Journal of Urban Economics*, 56, 192-216.

[22] Menzio G. (2007), A Theory of Partially Directed Search, *Journal of Political Economy*, 115, 748-769.

[23] Moen, E. (1997), Competitive Search Equilibrium, *Journal of Political Economy*, 105, 385-411.

[24] Peters, M. and S. Severinov (1997), Competition among Sellers who Offer Auctions Instead of Prices, *Journal of Economic Theory*, 75, 141-179.

[25] Stacey, D. (2012), Information, Commitment, and Separation in Illiquid Housing Markets, mimeo.

[26] Wheaton, W. (1990), Vacancy, Search, and Prices in a Housing Market Model, *Journal of Political Economy,* 98, 1270-92.

6 Appendix: Proof of Lemma 1 (Deriving the optimal reserve price in a second-price auction when the seller reservation value is $s \geq 0$)

Proof. This proof parallels our proof in Albrecht, Gautier, and Vroman (2012), which considered only the case in which the seller's reservation value is zero. The Lagrangean for (1) is

$$L(a, \xi, \lambda) = \Pi(a, \xi) + \lambda \left(V(a, \xi) - \overline{V} \right)$$

with first-order conditions

$$\begin{aligned}
\frac{\partial L(\cdot)}{\partial a} &= \Pi_a(a^*, \xi^*) + \lambda^* V_a(a^*, \xi^*) = 0 \\
\frac{\partial L(\cdot)}{\partial \xi} &= \Pi_\xi(a^*, \xi^*) + \lambda^* V_\xi(a^*, \xi^*) = 0 \\
\frac{\partial L(\cdot)}{\partial \lambda} &= V(a^*, \xi^*) - \overline{V} = 0.
\end{aligned}$$

To show that these conditions hold when $a^* = s$, note first that

$$\Pi_a(s, \xi^*) + \lambda^* V_a(s, \xi^*) = 0$$

implies

$$\lambda^* = \xi^*.$$

This follows from

$$\Pi_a(s, \xi^*) = (1 - F(s))\xi^* e^{-\xi^*(1-F(s))} \quad \text{and} \quad V_a(s, \xi^*) = -(1 - F(s))e^{-\xi^*(1-F(s))}.$$

We thus need to verify that

$$\Pi_\xi(s, \xi^*) + \xi^* V_\xi(s, \xi^*) = 0, \tag{4}$$

where ξ^* is the solution to $V(s, \xi^*) = \overline{V}$. Since $\overline{V}$, as a parameter of the problem, can take on any positive value, so too can ξ^*. We thus need to show that (4) holds for any positive value of ξ^*. This is done by direct computation. Note first that

$$V_\xi(s, \xi^*) = -\int_s^1 (1 - F(x))^2 \, e^{-\xi^*(1-F(x))} dx.$$

We then have

$$
\begin{aligned}
\Pi_\xi(s,\xi^*) &= \int_s^1 \left(x - \frac{1-F(x)}{f(x)} - s\right) e^{-\xi^*(1-F(x))} f(x)dx \\
&\quad -\xi^* \int_s^1 \left(x - \frac{1-F(x)}{f(x)} - s\right)(1-F(x))e^{-\xi^*(1-F(x))} f(x)dx \\
&= \int_s^1 \left(x - \frac{1-F(x)}{f(x)} - s\right) e^{-\xi^*(1-F(x))} f(x)dx \\
&\quad -\xi^* \int_s^1 (x-s)(1-F(x))e^{-\xi^*(1-F(x))} f(x)dx - \xi^* V_\xi(s,\xi^*).
\end{aligned}
$$

To prove (4) we thus need to show

$$
\int_s^1 \left(x - \frac{1-F(x)}{f(x)} - s\right) e^{-\xi^*(1-F(x))} f(x)dx = \xi^* \int_s^1 (x-s)(1-F(x))e^{-\xi^*(1-F(x))} f(x)dx. \tag{5}
$$

This final equality is verified by integrating the right-hand side of (5) by parts with $u = (x-s)(1-F(x))$ and $dv = \xi^* f(x)e^{-\xi^*(1-F(x))}dx$. ■