Rational Opacity

Cyril Monnet                                Erwan Quintin*
University of Bern and SZ Gerzensee        Wisconsin School of Business

April 19, 2013

Preliminary and incomplete

Abstract

A key motivation for transparency regulations and disclosure rules is the view that corporations are more opaque than they could be. Traditional arguments attribute this tendency to agency problems between managers on one side and stakeholders and/or outsiders on the other. We propose a completely different argument for why corporations limit the flow of information. When secondary markets are shallow, more information can reduce the expected payoff of stakeholders who need to liquidate their positions early. Even given direct and costless control over information design then, the very stakeholders of long-term investment projects have an incentive to curtail information. This holds even if stakeholders have the option to keep information releases to themselves. The existing evidence that financial intermediaries tend to be especially opaque particularly during financial crisis provides direct support that a fundamental trade-off exists between information and liquidity.

Keywords: opacity, liquidity, information

JEL codes:

*We wish to thank Mark Ready, Warren Weber, Ted Temzelides and seminar participants at the University of St-Gallen, Rice University, and the University of Wisconsin for very helpful comments.
Banks may be the black holes of the financial universe; hugely powerful and influential, but to some irreducible extent – unfathomable. Donald P. Morgan (American Economic Review, 2002)

1 Introduction

Corporations are perceived to be opaque in the sense that they have a tendency to withhold news about the quality of the long-term assets they hold. As Morgan (2002) discusses with and emphasis on financial firms, the view that corporations tend to be strategically secretive about what they do motivates regulations that govern foreclosure and transparency requirements as well as the heavy investments most economies make in supervising sectors of activity deemed to be especially opaque and of strategic importance. As Morgan (2002) also points out, traditional arguments attribute this tendency to keep secrets to agency problems between managers and stakeholders or more simply to the inherent fluidity/complexity of corporate holdings.

We propose a different theory. We describe an environment where it is optimal for the very stakeholders of long-term investment projects to incentivize managers to withhold information. The corporations we describe, in other words, are rationally opaque. In fact, information withholding is the constrained efficient outcome for these stakeholders. While more transparency leads to better divesting decisions, it also increases the risk that stakeholders will lose value if they must liquidate their positions early. Since a distinguishing feature of some intermediaries (e.g. banks) is that their stakeholders highly value liquidity, it is only rational – constrained optimal, even – that these should be especially opaque.

If, as we propose, the key benefit of opacity is to mitigate potential liquidation losses, the tendency of corporations to remain opaque should depend on the state of secondary markets for long-term investment projects. In our model, it is only when the depth of secondary markets may affect the liquidation value (a situation Allen and Gale, 2005, describe as “cash-in-the-market” pricing) that stakeholders choose to limit the revelation of new information on asset quality. This key prediction accords well with the existing empirical evidence. Flannery et al. (2004, 2013) propose various proxies for information availability (bid-ask spreads, the effect of trades on prices...), find little evidence that banks and bank-holding companies are especially opaque during tranquil times but find evidence that the opacity of these intermediaries increases disproportionately during financial crises, especially the 2007-09 period.
We develop our argument in a simple model of liquidity needs in the spirit of Diamond and Dybvig (1983) and Jacklin (1987). Agents can invest in a productive long-term project but face the risk that they may need to consume at an interim stage, before the project matures. When they need to liquidate their investment early, they can either scrap the project or, instead, sell it to more patient agents as in Jacklin (1987). We differ in two aspects from the canonical Diamond and Dybvig framework: First the long-term project is risky and its probability of success (its quality) is drawn at the interim stage. Second, when they make the original investment, agents can choose to make that interim information public, or they can opt instead to receive only a coarse signal of the project’s quality.

In this environment, the optimal information design becomes more opaque in a sense we make precise as the risk of early liquidation rises. This occurs because while more information allows investors to scrap early when efficient, it can also reduce the expected payoff when agents are constrained to liquidate their projects. Bad news always lower the project’s fundamental value hence its liquidation value. On the flip side of course, good news do raise the asset’s fundamental value as well but, when secondary markets are shallow, this may not be fully reflected in the liquidation value. This asymmetry – bad news are fully reflected in the liquidation value whereas good news may not be – causes finer information to be costly in expected terms for agents who must liquidate early.

Given this logic, it would seem that the optimal situation for investors would be to observe project quality privately at the interim stage in order to make efficient scrapping decisions without incurring the risk of liquidation losses. That intuition turns out to be correct from an individual point of view, but wrong in equilibrium. As in Milgrom and Stokey (1982), all private information becomes revealed when projects trade in secondary markets. As a result, private information can only hurt investors if they cannot commit to restrict it. It is optimal, therefore, for investors to restrict their access to information in some way. One natural way to implement the desired solution is to delegate the project continuation to a manager with the right incentives. A carefully designed compensation scheme that gives the manager a participation in revenues and a severance payment if they choose to scrap early implements the constrained optimal scrapping policy.

Our paper is related to several strands of the literature. Andolfatto, Berentsen and Waller (2012) show that the threat of undue diligence (that agents decide to acquire private information) can influence the socially optimal disclosure policy. Challes, Mojon and Ragot (2012) show how opacity can lead banks to gamble for resurrection, or in any words, to hold bad
loans on their book for too long. In our environment, such a behaviour is optimal from the point of view of stakeholders. Gorton and Pennachi (1990), Dang, Holmstrom and Gorton (2012), Morgan (2002), Siegert (2012), Sobel and Crawford (198?). Fourth, some papers have tackled the question of opacity and liquidity in different context: Von-Thaden (199?), Diamond (1984), Zetlin-Jones (2013).

Kaplan (2006) also writes a model where banks may choose not to reveal the interim information they obtain about the quality of their assets in an environment with consumption risk in the spirit of Diamond and Dybvig (1983). The mechanism he studies is quite different from ours however: in his model, revealing bad news makes it more costly to induce patient investors to reveal their type.

A different literature argues that deposit insurance is bad because it does not give depositors the incentives to monitor the bank’s activity (see Calomiris and Khan, 1991). Here we give a reason why, even in the absence of deposit insurance, depositors would not monitor their banks. If depositors were to monitor their banks, they would lose the liquidity services that the bank provides. In other words, warning by the monitor could make refinancing difficult. We may see that policy of the Fed to release TALF access only with a long delay as another example for the optimality of opacity when dealing with financial intermediaries.

2 The environment

2.1 Production technologies and preferences

The economy consists of a continuum of islands each inhabited by a mass two of agents. There are three dates \( t = 0, 1, 2 \). On each island, the first half of agents – early investors, henceforth – are endowed with one unit of a consumption good at \( t = 0 \). The other half – the late investors – appear at date \( t = 1 \), and receive an endowment \( A > 0 \) at that point.

Islands are indexed by \( \pi \) which is uniformly distributed over \([0, 1]\). On an island of type \( \pi \), a fraction \( \pi \) of early investors and a fraction \( 1 - \pi \) of late investors have the desire to consume at date 1 while other agents want to consume at date 2. All told then, half of all agents consume at date 1, the other half want to consume at date 2. To simplify the exposition we will treat consumption type realizations as independent across agents but emphasize that this plays no role in any of our results.\(^1\)

\(^1\)We could in fact equally well assume that consumption types are fully correlated across agents without
As of date 0, early investors do not yet know whether they will want to consume early or late hence they seek to maximize:

\[ u(c_1, c_2; \pi) \equiv \pi c_1 + (1 - \pi) c_2, \]

where \( c_1 \) is their expected consumption in period \( t = 1 \) and \( c_2 \) is expected consumption at \( t = 2 \).

All agents have the option to store the consumption good across dates. Agents who receive their endowment at \( t = 0 \) - early investors - also have the option to invest their endowment in a long-term and risky project that yields either \( R > 1 \) or nothing at date 2 per unit of the consumption good invested at date 0. As of date 0, early investors know that the success probability \( q \in [0, 1] \) will be drawn at date 1 from a Borel distribution \( F \) with dense support in \([0,1]\). All projects have the same success probability on a given island. All long-term projects can be scrapped at date 1, and all have a scrap value \( S > 0 \) which is independent of \( q \). Figure 1 shows the important steps of the long-term project.

We make the following assumption on technological parameters:

\[ S < 1 < \int \max(S, \min(qR, A))dF < A < \int qRdF. \]  

(2.1)

In Diamond-Dybvig, scrapping yields the same return as storage. Here, the first inequality guarantees that there is a risk to invest in the long term project and agents would not invest in the risky project if they knew they would scrap it. The last inequality implies that a late investor is willing to spend all his resources on a project even if no new information is provided any effects on our findings.
This inequality also implies cash-in-the-market-pricing, as the price will not be able to be above \( A \), especially if the project is of high quality. By that same logic, the fact that \( \int \max(S, \min(qR, A))dF < A \) will mean that information reduces the sellers’ expected payoff from secondary markets since late investors are already willing to pay \( A \) when no new information is provided.

The fact that \( 1 < \int \max(S, \min(qR, A))dF \) implies that early investors are better off taking their chances with secondary markets than attempting to self-insure by storing a fraction of their date 0 resources. Indeed, that inequality says that even if full information is provided about \( q \), agents expect to get more from secondary markets than from storage.

In appendix 9.2 we show that this assumption implies that the contract we study in this paper – whereby project owners who must consume early sell their projects to secondary markets and consume the proceeds – is the uniquely optimal contract for early investors.

### 2.2 Information

Most of this paper is about precisely what early investors choose to find out about \( q \) once it is drawn in period 1. To learn about \( q \), early investors who invest in the risky project can choose to activate an information technology. This technology sends a message \( m \) once \( q \) is realized at date 1. Early investors are free to choose any message function in the following set:

\[
\{ m : [0, 1] \rightarrow \mathcal{B}([0, 1]) : q \in m(q) \text{ for almost all } q \in [0, 1] \}
\]

where \( \mathcal{B}([0, 1]) \) is the space of Borel subsets of \([0, 1]\).

Restricting the choice of message functions to satisfy \( q \in m(q) \) is without loss of generality\(^2\) and has the advantage that the technology can be thought of as announcing a subset of \([0, 1]\) to which \( q \) belongs. By the same token, we can assume without loss of generality that \( m \) partitions \([0, 1]\) as in Sobel and Crawford (1982). We will assume to start that the message must be public. In section 5.1 we will discuss the option for early agents to keep information to themselves and argue that this does not affect any of our results.

---

\(^2\)He then expects a return \( \frac{qRdF}{A} > 1 \), which dominates storage.

\(^3\)To see why this is without loss of generality take any Borel-measurable mapping \( h \) from \([0, 1]\) to an arbitrary message space. Then the set-valued mapping \( m : [0, 1] \rightarrow \mathcal{B}([0, 1]) \) defined for all \( p \in [0, 1] \) by \( m(p) = h^{-1} \circ h(p) \) has the desired properties and conveys exactly the same information as \( h \). In other words, as long as all agents understand the selected design of the information technology, they can invert any message into a subset of \([0, 1]\).
Denote by $m_\pi$ the message function chosen by agents on island $\pi \in [0, 1]$. Among other information design choices, agents can choose to fully reveal project quality by establishing $m_\pi(q) = \{q\}$ for all $q \in [0, 1]$. One of our main points, however, is that unless $\pi = 0$ (unless there is no risk of early consumption), early investors will opt for much coarser information technology designs. Revealing no information $m_\pi(q) = [0, 1]$ for all $q \in [0, 1]$ is always an option as well but will turn out not to be optimal either unless $\pi = 1$.

2.3 Market for projects

At date 1, all agents on an island $\pi \in [0, 1]$ can buy or sell risky projects in a Walrasian market. The sell side can only comprise early investors since only they had the opportunity to invest in the risky projects. The buy side, as we will discuss below, can only comprise late investors who discover that they want to consume late. All agents take the project price as given and, as we make more precise in the next section, trade for projects when doing so raises their expected utility given the information they have.

In appendix 9.1 we show that our model with Walrasian trade makes the exact same predictions as a model where early investors who wish to consume early are matched with exactly one late investor who wish to consume late and the former get to make the latter a take-it-or-leave it offer.

2.4 Timeline

At date 0, early investors choose the design of the information technology. Since they are all alike, they all agree to choose the design that maximizes their expected payoff as of date 0. Next they decide whether to invest in the long-term project or to store their endowment. At the start of date 1, late investors appear, all consumption types are revealed, and a message $m \in \mathcal{B}([0, 1])$ becomes available to all on a given island $\pi \in [0, 1]$. Agents immediately and correctly translate this message into an expected likelihood

$$E(q|m; \pi) = \frac{\int_m q dF}{\int_m dF}$$

of success for the long-term project. Walrasian markets open, projects trade and/or are scrapped. Early consumers consume while late consumers wait until the returns to their investment is realized and consume the proceeds.
3 Equilibrium given an information structure

In this section we define an equilibrium given a message function \( m^\pi \) for almost all islands \( \pi \in [0, 1] \). The next section will endogenize the choice of information.

3.1 Optimal trading decisions at the Walrasian stage

Let \( p(\pi, m) \) be the unit price of a long-term project on island \( \pi \in [0, 1] \) when the message \( m \) is issued at the start of period 1. The next section will pin down what this function must be in any equilibrium. Early investors who discover that they have to consume early will scrap their project given \( m \) if and only if \( S \geq p(\pi, m) \). As for early investors who do not experience the consumption shock, they sell their project in the market if \( p(\pi, m) > \max\{E(q|m; \pi)R, S\} \).

Given the message technology in place and as of date 0, early investors thus expect payoff:

\[
\pi \int \max\{S, p(\pi, m)\} dF + (1 - \pi) \int \max(S, E(q|m; \pi)R, p(\pi, m)) dF.
\]

Late investors for their part, consume their endowment if they must consume at date 1. Those who are late consumers spend all their endowment on projects if \( p(\pi, m) \geq \max\{E(q|m; \pi)R, S\} \). Otherwise they store their endowment. In either case, all late consumers consume their realized wealth at date 2.

3.2 Definition

Given a set of message functions \( \{m_\pi : \pi \in [0, 1]\} \), an equilibrium is a set of investment decisions for all early investors, a set of scrapping/selling decisions, and a Walrasian price schedule \( \{p(\pi, m) : \pi \in [0, 1], m \in m_\pi[0, 1]\} \) such that:

1. All decisions are optimal for almost all agents;

2. Demand for projects equals supply at the Walrasian stage on almost all islands.

We establish below that given a set of message functions, a unique equilibrium exists. Since early investors get to select the message function, they select the equilibrium that yields the highest expected payoff to them. That decision is our key concern in this paper.
3.3 Existence

This section establishes that given a message function, there is exactly one equilibrium. To
that end, we show that only one price schedule is compatible with market clearing at the
Walrasian stage.

**Proposition 3.1.** Given a set of message functions \{m_\pi : \pi \in [0,1]\}, an equilibrium exists
and is generically unique. Furthermore, at the Walrasian stage and for all \pi \in [0,1],

\[
p(\pi, m) = \max [S, \min \{E(q|m; \pi)R, A\}]
\]

for all possible message m given \(m_\pi\)\(^4\).

**Proof.** We begin by proving that q must have the form posited above in any equilibrium. Fix
a type \pi \in [0,1] and take \(m \in m_\pi[0,1]\). If \(p(\pi, m) > E(q|m; \pi)R\) then all early investors
would sell their projects at date 1 while there are no buyers. If, on the other hand, \(S < p(\pi, m) < E(q|m; \pi)R\) then only early investors who need to consume early sell their projects.
In that case, all late investors who need to consume at date 2 buy as many projects as they
can afford, namely \(A/p(\pi, m)\) so that demand equals supply if and only if

\[
\pi \frac{A}{p(\pi, m)} = \pi \iff p(\pi, m) = A.
\]

Finally, if \(E(q|m; \pi)R \leq S\) then the optimal strategy for a project holder is to scrap it.
Therefore we must have \(p(\pi, m) = S\) so that all agents are indifferent between buying or
selling the project. This proves the second part of the proposition.

Having so characterized the shape of the Walrasian price schedule, we can now simplify
early investor’s expected payoff given their type \pi and given a message function \(m_\pi\). If project
quality q is drawn at date 1, all agents rightly infer from the message \(m_\pi(q)\) they receive that
the likelihood of success is \(E(q|m_\pi(q); \pi)\). The agent’s payoff, then, is given by:

\[
\pi \int p(\pi, m_\pi(q))dF + (1 - \pi) \int \max (S, E(q|m_\pi(q); \pi)R) dF.
\]

To understand the second integral, recall that agents always have the option to scrap their
project and observe that agents who do not experience a consumption shock are always at
least as well off keeping their projects as selling them (and may in fact be strictly better off

\(^4\)That is \(m \in \{m : \exists p \in [0,1] \text{ such that } m = m_\pi(p)\}\).
doing so when \( p(\pi, m) = A \). Finally, it is easy to see that as long as assumption \( 2.1 \) holds
it is uniquely optimal for agents born at date zero to invest all their endowment in risky projects.

There is an equilibrium only if early investors who consume late hold on to their projects, as otherwise
supply exceeds demand. However, they do so if the price is not too high relative to their expectation
for the project expected return. In addition, late investors will bid for the project as long as its expected return is higher than its price and as long as they have the means to buy it. In particular, if the signal is very good, then late investors might want to buy a lot. However they have limited resources and the maximum they can spend is their endowment. Therefore the price cannot exceed \( A \) and it will equal its expected return otherwise. Finally, if the expected return falls below the scrap value, then holders of the project scrap it and the only possible market price is \( S \) in this case.

The argument is illustrated in figure 3.3. The price must fall between \( S \) and the the expected payoff \( E(q|m_\pi(q); \pi)R \) given the message. As long as the price is the expected payoff, late investors who want to consume late are willing to spend their entire endowment on projects making demand (drawn in blue) the entire interval between 0 and \( \frac{\pi A}{p} = \frac{\pi A}{E(q|m_\pi(q); \pi)R} \). To move beyond that demand level, the price must fall and demand becomes \( \frac{\pi A}{p} \) for \( p \in (0, E(q|m_\pi(q); \pi)R) \). Supply, shown in red, is \([0, \pi] \) when \( p(\pi, m) = S \), is exactly \( \pi \) when \( p(\pi, m) \in (S, E(q|m_\pi(q); \pi)R) \) and becomes \([\pi, 1] \) when \( p(\pi, m) = E(q|m_\pi(q); \pi)R \) since in that case even late consumers with projects are willing to sell.

The figure shows the case where \( \frac{\pi A}{E(q|m_\pi(q); \pi)R} < \pi \) in which case trivial algebra shows that the only equilibrium price is \( p(\pi, m) = A \). In other words, the price is dictated by the resources available in the market rather than the project’s expected payoff. (Under assumption \( 2.1 \) this happens when information is full.) This is a situation Allen and Gale (2004) describe as cash-in-the-market pricing. We will show that in that situation there is a trade-off between information and liquidity. When on the other hand \( \frac{\pi A}{E(q|m_\pi(q); \pi)R} \geq \pi \), then the equilibrium price is the expected payoff and we will find in that case that full information is optimal.

In appendix 9.1 we prove that this is the exact payoff that would obtain in a version of our environment where early investors who experience liquidity shocks are matched with one late investor who wishes to consume late and make a take-it or leave-it offer to these agents. All the results we establish below regarding the rational choice of information technology hold in that simple search environment as well. Furthermore, we show that proportional bargaining does not change the qualitative nature of our results either.
4 Rational Opacity

We are now in a position to characterize the information design decisions of early investors. To that end, it will be instructive to consider a sequence of increasingly intricate cases.

4.1 Full information

Consider first the case where the signal is public information. This means that, in effect and for all $\pi$ and $q$, the message truly reveals the project’s quality, or $m_{\pi}(q) = q$. At date 1, if it turns out that $qR \leq S$, all projects are scrapped. Early investors who must consume early consume $S$ at date 1 while their counterparts who want to consume at date 2 store the scrapping proceeds and consume $S$ at date 2. Late investors consume immediately or store their endowment to consume it at date 2. In this case, the price is the face value of the project, $p(\pi, q) = S$ for all $q \leq S/R$.

If, on the other hand, $qR > S$ then all projects are continued, early consumers sell their projects to late investors who want to consume at date 2 and consume $p(\pi, q) = \min\{qR, A\}$. Late investors consume immediately or store their endowment to consume it at date 2.

4.2 No information

Consider next the case where the signal cannot be observed so that for almost all Walrasian $\pi$ and all $q$, the message reveals is completely uninformative, so that $m_{\pi}(q) = [0, 1]$.

5 Or more generally, $[0, 1] \subset m_{\pi}(q)$. 

Figure 2: Walrasian market for projects under cash-in-the-market Pricing
all projects are continued at date 1 since, absent any new information, their expected return continues to exceed 1. All early investors who experience a consumption shock sell their project at date 1 for $A$ and they immediately consume that amount. At this stage, the face value of the project is $\int qRdF > A$ so that early investors with no consumption shock prefer to keep their project to maturity. Late investors who must consume early consume their endowment. All other agents consume the project’s payoff at date 2.

### 4.3 Either full or no information

Assume now that agents only have a choice between revealing everything and revealing nothing. That is, they can choose to be in either of the two situations we considered above. If early investors on islands of type $\pi \in [0,1]$ choose to reveal nothing, then we can write their payoff as

$$
\int_0^1 qRdF - \pi \int_0^1 (qR - A) dF.
$$

Therefore the expected payoff of investors who chose to remain uninformed is formed of the expected value of the project minus the expected loss from selling the project in case they turn impatient. Notice that impatient investors make a loss only when there is cash-in-the-market pricing. If they choose to provide full information, then we can write the investors’ payoff as

$$
\int_0^S SdF + \int_{\frac{S}{R}}^1 qRdF - \pi \int_{\frac{A}{R}}^1 (qR - A) dF.
$$

Figure 3: Price under full information (black) and no information (red).
Again, the expected payoff of investors who chose to be fully informed is formed of the expected value of the project (now accounting for the efficient scrapping decision) minus the expected loss from selling the project in case they turn impatient. Notice that the loss from selling the project is higher for informed agents. They however gain from knowing when to scrap the project if it turns out to be bad. Therefore, these two expressions make the information trade-off clear. If early investors knew for a fact that they will want to consume late then information is useful because it enables them to make the optimal scrapping decision. On the other hand, if they knew for a fact that they will sell, the information can only reduce their payoff because $\int qRdF > A > S$. Formally,

**Proposition 4.1.** Assume date-0 agents can only choose whether to reveal all information at date 1 or no information. Then there exists a threshold $\pi^F \in (0, 1)$ such that agents of type $\pi \in [0, 1]$ choose to send a perfectly informative message only provided $\pi \leq \pi^F$.

**Proof.** Simply note that

$$\int_0^{\pi^s} SdF + \int_{\pi^s}^A qRdF + \int_A^1 AdF < A$$

while

$$\int_0^{\pi^s} SdF + \int_{\pi^s}^1 qRdF > \int qRdF.$$  

The result immediately follows. \hfill \square

In other words, agents who face a greater refinancing risk rationally choose to withhold information about the quality of the project. As we will point out below in the more general case, this immediately implies that the equilibrium allocation when information design is endogenous is Pareto inefficient. Indeed, in expected terms, projects that agents should scrap are not scrapped when no information is revealed. This is contrary to the corporate finance literature (see for example Holmstrom and Tirole, 1998) where usually agents scrap the project in too many instances.

### 4.4 Rational information design

Assume now that agents can design the message function in any way they wish. We will show that the rational message is one that partitions the space of quality into two non-overlapping
intervals: scrap or hold. We will also show that investors with higher $\pi$ will choose larger “hold”-intervals. In this sense, these investors prefer more opacity.

We have already established that revealing information cannot improve early investors’ payoff in any way if they must consume early since their payoff in that case is at its maximum when no information whatsoever is received. The only point of establishing a more revealing information design, then, is to induce a better scrapping decision when early investors are not constrained to sell. It follows directly that there is no need for the message function to partition $[0, 1]$ in more than two subsets: scrap or hold.

While early investors who are not constrained to sell would like to be informed when $qR < S$, there is no value in terms of expected payoff as of date 0 in finding out early in what part of $[\frac{S}{R}, 1]$ the project’s quality $q$ actually falls. Put another way, the value of marginal information in $[\frac{S}{R}, 1]$ is non-positive. On the other hand, revealing that information could reduce their payoff when they are constrained to consume early and rely on secondary markets to do so.

These simple observations give us the first source of opacity in this environment. It is not rational for early investors to reveal any information beyond what is strictly necessary to induce efficient scrapping decisions. Late investors, for their part, would greatly value finer information, but they have no means to induce the original investors to provide it. It should then intuitive that the two subsets, scrap and hold, are non-overlapping intervals, but we show this formally in the appendix.

So we can restrict our search for optimal message functions to the following class, indexed by $\bar{q} \in [0, 1]$. For $q \in [0, 1]$,

$$m(q) = \begin{cases} [0, \bar{q}] & \text{if } q < \bar{q} \\ (\bar{q}, 1] & \text{otherwise}. \end{cases}$$

At date zero then, early investors on island $\pi \in [0, 1]$ need only choose $\bar{q}$. We refer to $\bar{q}$ as the scrapping threshold. An obvious possibility is to set $\bar{q} = \frac{S}{R}$ which would enable late consumers to always make the efficient scrapping choice. In this case, the message is designed to convey the most information subject to the constraints we have outlined above. That possibility, however, turns out to be optimal only in the extreme case where early investors know they will consume late, i.e. $\pi = 0$. To establish this as well as characterize the relationship between
\(\pi\) and \(\bar{q}\), note that early investors’ payoff given \(\pi\) and \(\bar{p}\) is:

\[
V(\bar{q}; \pi) \equiv \int_0^{\bar{q}} SdF + \int_{\bar{q}}^1 qRdF - \pi \int_{\bar{q}}^1 (qR - A) dF.
\] (4.3)

Since \(V\) is continuous on a compact set, an optimal \(\bar{q}\) exists for all \(\pi \in [0, 1]\). We cannot guarantee uniqueness, however, as the payoff is not necessarily concave in the scrapping threshold for arbitrary density functions. But this has no bearing on our ability to fully characterize comparative statics because \(V\) is submodular: the early investor’s marginal payoff is decreasing in \(\pi\). Therefore, the higher the risk of early consumption, the greater the cost of increasing the scrapping threshold. Intuitively, there is a tradeoff between the desire to scrap when it is efficient to do so and the fact that better information can lower the project’s resale value. Naturally, this implies that investors who face a relatively low risk of early consumption will choose a higher scrapping threshold. And inversely, investors facing a high early consumption risk will prefer a lower scrapping threshold and possibly no information at all. We summarize this discussion in the following proposition,

**Proposition 4.2.** Let \(q(\pi)\) be any maximizer of (4.3). There exists \(\pi^* \in (0, 1)\) such that \(q(\pi)\) is zero on \((\pi^*, 1]\) and decreases on \([0, \pi^*]\). Furthermore, \(q(\pi) = \frac{S}{R}\) if and only if \(\pi = 0\).

**Proof.** That \(q(0) = \frac{S}{R}\) is clear. On the other hand,

\[
V_1 \left( \frac{S}{R}, \pi \right) = \pi f \left( \frac{S}{R} \right) (S - A)
\]

which is strictly negative as long as \(\pi > 0\). This means that \(q(\pi) < \frac{S}{R}\) for all \(\pi > 0\).

\(V\) is strictly submodular on \((0, \frac{S}{R}) \times (0, 1)\) as

\[
V_{21}(\bar{q}; \pi) = -f(\bar{q}) (A - \bar{q}R) \leq -f(\bar{q}) (A - S) < 0,
\]

since \(\bar{q} < \frac{S}{R}\). Since \(V\) is strictly submodular on \((0, \frac{S}{R}) \times (0, 1)\), the Monotone Selection Theorem implies that \(q\) must decrease monotonically until it reaches zero. That it reaches zero strictly before \(\pi = 1\) follows from the fact that \(V_1(0, 1) = f(0) (S - A) < 0\).

**Corollary 4.3.** The equilibrium allocation under rational information design is Pareto inefficient.
Agents on almost all islands $\pi$ choose a scrapping threshold that induces them to keep the project in some states of the world although they should not. Therefore, the total expected output is strictly below what would prevail under full information as is, therefore, aggregate expected consumption. The inefficiency is arising from the fact that ignorance is bliss for those agents who have to sell their project. This is not a trivial result: while ignorance allows agents to escape the downside of scrapping their project if it turns out to be bad, it also prevent them to cash it in when it is good. Hence, to choose ignorance agents have to be quite sure that they will have to sell.

5 Private information

So far we have assumed that if information is made available to some agents, then it is public information. However it would seem that the optimal situation for early investors would be to observe project quality privately at date 1 in order to make efficient scrapping decision without incurring the risk of refinancing losses. This section shows that this intuition is wrong: While it is true that each investor has an incentive to be better informed than any other, general equilibrium arguments imply that private information can only hurt them. Since investors are unable to commit to not act on their private information, aggregate variables will make public any private information. Therefore private information can only hurt investors if they cannot commit to restrict it.

5.1 Investors always observe the signal

Assume that early investors always observe the interim signal perfectly or, equivalently, that they have the option to privately establish a technology that reveals the project quality to themselves and themselves only. In that case and as long as late investors observe the supply of projects, the Walrasian trading stage reveals all private information which means that the equilibrium allocation is the same as in the full information case.

Proposition 5.1. If early investors observe project quality privately then the only equilibrium allocation is the full information equilibrium allocation.

Proof. Consider a candidate price $p(\pi)$ for projects on island $\pi \in (0, 1)$ at date 1. We know that $p(\pi) \geq S$ in any equilibrium. If $S < p(\pi) < qR$ then only early consumers supply their projects and, upon observing that demand, buyers infer that $q$ is distributed with strictly
positive continuous density over \([p(\pi), 1]\). It follows that demand for projects is \(\pi \frac{A}{p(\pi)}\). The only case is which this is an equilibrium, therefore, has \(p(\pi) = 1\) and \(qR > 1\).

If \(p(\pi) > qR\) then all potential sellers sell, from which buyers infer that \(q\) is distributed with strictly positive continuous density over \([0, p(\pi)]\) so that demand is zero, which can’t be in equilibrium.

The only equilibrium, then, has \(p(\pi) = \min(S, qR)\) if \(qR < A\) and \(p(\pi) = 1\) if \(qR \geq A\) exactly as in the full information case.

This result should not come as a surprise: market prices reveal all private information in this environment like they do in Milgrom and Stokey (1982). Since unbridled access to private information lead to an inferior allocation (the full information allocation) from the point of view of early investors, early investors have an incentive to observably commit to remain ignorant. The next section explores this possibility.

5.2 Observable private information design

Assume that agents can make the design of private information they selected observable to late investors, or, equivalently, that they can somehow commit to it. The same argument as above implies that all private information is revealed at the Walrasian stage so that there is no effective distinction between private and public information. This implies:

**Proposition 5.2.** If the design of the information technology is observable, the rational information design choice is the same for almost all islands regardless of whether the message is private or public.

**Proof.** The argument is the same as in the proof of proposition 5.1 with \(E(q, m_{\pi}(q))R\) playing the role of \(qR\).

Put another way, all the results we established in the previous section go through unaffected when information is private rather than public. In addition, this section says that investors who must confront refinancing risks have incentives to observably commit to reveal any information they have (say, via delegated monitoring) or to not trade on the basis of that information (say via regulations that ban trading on the basis of undisclosed information.)
5.3 Implementation by delegation

The analysis above suggests that agents have an incentive to commit in some fashion to ignore or, at least, not act on the private information they observe. In this section we show that a natural way to implement the desired solution is to delegate the project continuation to a risk-neutral representative agent (a manager of sorts) with the right incentives. To see this, assume that the coalition of early investors hire an agent with no holdings in the project and give her the authority to scrap the project at date 1. Assume further that the agent in question, and only she, is given full access to the signal at date $t = 1$.

Consider then the class of compensation scheme whereby the manager receives a fixed payment $M > 0$ if the problem is scrapped – think of it as a severance payment – and, if the project is continued, receives a payment $\alpha R$ if the projects succeeds– think of this part of her compensation as a participation in revenues. For simplicity, we assume that the delegate has no mass so that, in particular, the payment she receives does not affect the expected surplus generated by the project. We now have:

**Proposition 5.3.** Let $\bar{q}(\pi)$ be an optimal scrapping threshold on island $\pi \in [0,1]$. Let the delegate’s compensation scheme $(M^\pi, \alpha^\pi)$ be such that

$$M^\pi = \alpha^\pi \bar{q}(\pi) R.$$

Then the delegate implements the optimal scrapping policy and, correspondingly, early investors expect the constrained-efficient payoff.

**Proof.** Assume that the (fully but privately informed) delegate observes that $q < \bar{q}(\pi)$. Then, since $\alpha^\pi q R < M^\pi$, the delegate chooses to scrap, as desired. The converse holds by the exact same logic and the compensation scheme, therefore, leads to exactly the desired policy. \(\square\)

Note that the proposition does not pin down the level of the compensation scheme so that in principle, the entire one-dimensional space of schemes that satisfies the desired property implement the optimal policy. Since the delegate has no mass, investors are indifferent across such schemes as long as they involve finite payments.\(^6\)

\(^6\)This feature, of course, is neither here nor there given our purposes. But if one insisted on pinning down the level of the scheme as well from fundamental considerations, this could be accomplished via a straightforward extension. Assume that there are many potential delegates that compete for the job and have a reservation income $w > 0$. Assume further that instead of having literally zero mass, the delegates have small mass. The compensation scheme must be such that, ex ante $E(\max M, \alpha q R) \geq w$. Since investors will
6 The depth of secondary markets and opacity

A dominant message of the empirical literature prompted by the findings of Morgan (2002) is that opacity as proxied by the variance in the rating of banks seems to increase during periods where secondary markets are under stress or illiquid. Our model contains a possible explanation for these findings. When secondary markets are deeper – as proxied in our model by, say, a higher $A$ – the potential losses associated with secondary markets become smaller and the need to withhold information diminishes. For example, if $A > \int_{1/\pi}^1 qRdF$ then the early investor’s payoff is the project’s expected value regardless of the information structure and, therefore, full information is optimal since it maximizes expected output, for any $\pi \in (0, 1)$. Put another way, liquidity-minded stakeholders will choose to withhold bad news when and only when they expect secondary market prices to be affected by the depth of those markets.

This situation – depth affecting expected value of investment projects in secondary markets – is a situation Allen and Gale (2012) describe as cash-in-the-market pricing. Our model thus contains a possible rationale for why information seems to flow more effectively when secondary markets function normally than when they are distressed. The following result characterizes the relationship between $A$ and the degree of opacity more completely. It says that the effect of secondary market depth on the optimal information policy is U-shaped.

**Proposition 6.1.** There exists $\bar{A}(\pi) \leq \int_{1/\pi}^1 qRdF$ such that $\bar{q}(\pi) = \frac{S}{R}$ – information revelation is at its first best level – if $A \leq S$ or $A > \bar{A}(\pi)$. Furthermore, any selection $\bar{q}(\pi)$ from the set of optimal scrapping thresholds first decreases and then increases on $[S, \bar{A}(\pi)]$.

**Proof.** Define

$$V(\bar{q}; A) \equiv \pi \left\{ \int_0^\bar{q} SdF + \int_{\bar{q}}^1 \max \left( \min(E(q|q \geq \bar{q}), R, A), S \right) dF \right\} \left(1-\pi\right) \left\{ \int_0^\bar{q} SdF + \int_{\bar{q}}^1 qRdF \right\}.$$

If $A \leq S$, secondary markets are of no use and first best info is optimal. When $A$ begins exceeding $S$, the argument behind proposition 4.2 implies directly that $\bar{q}(\pi) < \frac{S}{R}$. When on the other hand $A$ is sufficiently large, $\int_{\bar{q}}^1 \max \left( \min(E(q|q \geq \bar{q}), R, A), S \right) dF = \int_q^1 \max \left( E(q|q \geq \bar{q}), R, A, S \right) dF$ for any $\bar{q} \in [0, \frac{S}{R}]$ so that $V(\bar{q}; A) \equiv \left\{ \int_0^\bar{q} SdF + \int_{\bar{q}}^1 qRdF \right\}$ and full information is once again optimal. This gives us a right threshold above which $\bar{q}(\pi) = \left\{ \frac{S}{R} \right\}$ is optimal.

\[\text{now seek to economize on the delegate compensation and given free entry, the inequality must in fact hold as an equality. Interpreting as is standard the zero-mass case as the limit of the sequence of schemes that would prevail as the size of the delegate converges to zero, the limit compensation scheme must also have expected value } w \text{ which now selects a unique point in the manifold described in the proposition.}\]
In the middle section, we must have at any optimal \( \bar{q} \) that \( \min(E(q|q \geq \bar{q})R, A) = A \) or \( \bar{q} = \frac{S}{R} \). Indeed, if \( \min(E(q|q \geq \bar{q})R, A) < A \) then secondary markets are not used and withholding information, therefore, can only destroy expected value. Let \( q(A) \) be an optimal information choice under the assumption that agents can sell their project for \( A \) no matter what. (This is the case we considered in the rational information design section since assumption 2.1 guarantees that agents can sell their projects for \( A \). In the context of this proof however, we allow \( A \) to be outside the bounds implied by that assumption.) As long as \( \min(E(q|q \geq q(A))R, A) = A \), \( q(A) \) is in fact an optimal choice, so that \( q(A) \in \bar{q}(\pi) \). Furthermore, \( V \) is easily shown to be submodular in that section so that any selection from \( \bar{q} \) falls in that region as \( A \) rises. Eventually however as \( A \) rises, \( E(q|q \geq q(A))R < A \). At that point the solution is to set \( \bar{q} \) so that \( E(q|q \geq \bar{q})R = A \) so that \( \bar{q} \) begins to rise with \( A \) until \( \bar{q}(\pi) = \{ \frac{S}{R} \} \). This completes the proof.

\[ \square \]

7 The impact of transparency regulations

What would be the consequences in our model of imposing transparency say via regulatory intervention? Assumption 2.1 guarantees that the risky project is undertaken even if full information is imposed and, given full information, the project is always scrapped when efficient so that aggregate consumption is maximized in aggregate terms. Our analysis shows that the solution is not a Pareto improvement over the full information case however since early investors are worse off in expected terms. Getting early investors to support such a regulation would require imposing taxes on (extracting transfers from) late investors. Be that as it may, the model appears to provide a potential rationale for transparency-imposing regulations.

That prediction is fragile, however. Suppose that we alter assumption 2.1 to become:

\[ S < \int \max(S, \min(qR, A))dF < 1 < A < \int qRdF. \]  

(7.1)

In other words, the expected payoff from fully informed secondary markets is now dominated by the storage payoff, but it continues to be the case that selling to uninformed secondary markets dominates storage. This complicates the optimal information analysis by forcing us to consider different subcases but does not change the nature of any of our results.

On the other hand, the consequences of imposing transparency via policy intervention
become drastically different. Indeed, consider early investors with a high risk of early consumption. If π is high enough, constraining early investors to provide full information will cause them to opt for storage, thus causing a decline in investing activity since those same agents would choose to invest if they could opt for no (or, more generally, less) information. As in Andolfato et. al. (2012) therefore, more transparency can imply less investment and hence destroys surplus.

8 Conclusion

9 Appendix

9.1 Bargaining

Here we analyze the case where early consumers and newborn on each island π are bilateraly matched in period 1 and they bargain over the project after the realization of the public signal m. For simplicity, we assume that agents split the surplus from trade using proportional bargaining. We denote by θ the share of the surplus of early consumers. Given m is public, all agents expected return for the long-term project is \( \max\{E (q|m, \pi) R; S\} \). Let \( p(m) \leq 1 \) be the agreed transfer of resources between the early consumers and the newborn. Since early consumers will scrap the project if they do not trade, their surplus from trade is simply

\[
p(m) - S
\]

The surplus from trade of newborn is

\[
\max\{E (q|m, \pi) R; S\} - p(m)
\]

With proportional bargaining, \( p(m) \) has to satisfy

\[
(1 - \theta)(p(m) - S) = \theta \{\max\{E (q|m, \pi) R; S\} - p(m)\}
\]

and arranging, together with the resource constraint \( p(m) \leq 1 \), we have

\[
p(m) = \min \{\theta \max\{E (q|m, \pi) R; S\} + (1 - \theta)S; 1\}
\]

21
As is usual in the context of proportional bargaining, newborns extract more of the surplus as their bargaining power $1 - \theta$ increases, in which case the transfer decreases to $S$. Also, if $\theta = 1$ then early consumers extract all the surplus from newborns and the solution is equivalent to a take-it or leave-it offer.

The agents expected payoff on island $\pi$ then is as before

$$\pi \int p(m_\pi(q))dF + (1 - \pi) \int \max[S, E(q|m_\pi(q); \pi)R]dF.$$

### 9.1.1 Full information

With full information $m_\pi(q) = \{q\}$, and at date 1, if it turns out that $qR \leq S$, all projects are scrapped. Date 0 agents who must consume early consume $S$ at date 1 while their counterparts who want to consume at date 2 store the scrapping proceeds and consume $S$ at date 2. Newborns at date 1 store their endowment to consume it at date 2.

If, on the other hand, $qR > S$ then all projects are continued, early consumers sell their projects to newborns and consume $\min\{\theta qR + (1 - \theta)S, 1\}$. All other agents consume the project’s payoff at date 2.

### 9.1.2 Either full or no information

Assume now that agents only have a choice between revealing everything and revealing nothing. If agents on island $\pi$ choose to reveal nothing, then their payoff is:

$$\pi \min\left\{\theta \int qRdF + (1 - \theta)S, 1\right\} + (1 - \pi) \int qRdF. \quad (9.1)$$

If they choose to provide full information, then their payoff is:

$$\pi \left\{ \int_0^S SdF + \int_0^{\frac{1 - (1 - \theta)S}{\theta R}} [\theta qR + (1 - \theta)S]dF + \int_0^{\frac{1 - (1 - \theta)S}{\theta R}} dF \right\} + (1 - \pi) \left\{ \int_0^S SdF + \int_0^1 qRdF \right\}. \quad (9.2)$$

With take-it or leave-it offers from early consumers (i.e. $\theta = 1$) we obtain the same expressions as in the text. However, it should be obvious that one of our main result also holds for other values of $\theta \in (0, 1)$.

**Proposition 9.1.** Assume date-0 agents can only choose whether to reveal all information at date 1 or no information. Then there exists a threshold $\pi^*(\theta) \in (0, 1)$ such that agents of
type $\pi \in [0,1]$ choose to send a perfectly informative message only provided $\pi \leq \pi^*(\theta)$. The threshold $\pi^*(\theta)$ is decreasing in $\theta$ and $\pi^*(0) = 1$.

Proof. Simply note that

$$\int_0^{\frac{S}{\pi}} SdF + \int_{\frac{S}{\pi}}^{\frac{1-(1-\theta)S}{\theta R}} \theta q R + (1-\theta)SdF + \int_{\frac{1-(1-\theta)S}{\theta R}}^{0} dF < 1$$

while

$$\int_0^{\frac{\hat{q}}{\pi}} SdF + \int_{\frac{\hat{q}}{\pi}}^{1} q R dF > \int q R dF.$$ 

The first part of the result immediately follows. That the threshold is decreasing in $\theta$ follows from the fact that the surplus of early consumers from being ignorant increases with their bargaining power $\theta$. Finally, at $\theta = 0$ early consumers always obtain $S$ independently of the signal $m$. So they do not care whether they are informed or not. However late consumers prefer to know when they should liquidate. Hence, with $\theta = 0$ all agents on all islands prefer full information. \qed

9.1.3 Rational information design

Again, as before, we can restrict our search for optimal message functions to the following class, indexed by $\bar{q} \in [0,1]$. For $q \in [0,1]$,

$$m(q) = \begin{cases} [0, \bar{q}] & \text{if } q < \bar{q} \\ (\bar{q}, 1] & \text{otherwise.} \end{cases}$$

At date zero then, agents of type $\pi \in [0,1]$ choose $\bar{q}$.

**Proposition 9.2.** The optimal information level $\bar{q}(\pi, \theta)$ for agents of type $\pi \in [0,1]$ decreases strictly with $\pi$ and $\theta$. Furthermore, $\bar{q}(0, \theta) = \bar{q}(\pi, 0) = S/R$ and $\bar{q}(1, \theta) = 0$ for all $\theta > 0$.

Proof. Fix $\pi \in [0,1]$. Given $\bar{q} \leq \frac{S}{R}$ the agent’s payoff is:

$$\pi \left\{ \int_0^{\bar{q}} SdF + \int_{\bar{q}}^{1} \min \{\theta E[q R | q \geq \bar{q}] + (1-\theta)S, 1 \} dF \right\} + (1-\pi) \left\{ \int_0^{\bar{q}} SdF + \int_{\bar{q}}^{1} q R dF \right\}.$$
Differentiating this expression with respect to $\bar{q}$ yields

$$
S f(\bar{q}) - \pi \min \{\theta E[qR|q \geq \bar{q}] + (1 - \theta)S; 1\} f(\bar{q})
$$

(9.3)

$$
-\mathbb{I}_{\{\theta E[qR|q \geq \bar{q}] + (1 - \theta)S < 1\}} \theta \pi \int_{\bar{q}} f(q) dF - (1 - \pi)qRf(\bar{q}).
$$

where $\mathbb{I}$ is an indicator function. Since $\min \{\theta E[qR|q \geq \bar{q}] + (1 - \theta)S; 1\} > S$, this expression is strictly negative when $\bar{q} = S/R$ unless $\pi = 0$. Therefore only when $\pi = 0$ do agents choose to reveal the efficient level of information. If $\pi = 1$, at the other extreme, (9.3) is strictly negative even if $\bar{q} = 0$ so that no information is revealed. Also, since $\bar{q} < S/R$ for all interior $\pi$ the derivative is uniformly decreasing as $\pi$ rises through $(0, 1)$ which implies that $\bar{q}$ decreases strictly, as claimed. Turning to the effect of $\theta$, when $\theta = 0$ the derivative is strictly positive if $\bar{q} < S/R$ and strictly negative if $\bar{q} > S/R$. Therefore, the maximum is attained at $\bar{q} = S/R$, and all agents prefer more information when they have no bargaining power. The case with $\theta = 1$ is as in the text. Finally, the derivative is uniformly decreasing as $\theta$ rises through $(0, 1)$ which implies that $\bar{p}$ decreases strictly.

The result on efficiency is the same as before. We leave aside the case with bargaining under private information as it is substantially more difficult.

### 9.2 Constrained optimality

Consider a social planner who seeks to maximize the ex-ante welfare of early investors. The planner faces several constraints. First, as a matter or resource feasibility, consumption by early investors who must consume early must come from the endowment of late investors or from scrapping. Late investors can store or consume their endowment however so that any transfer from them must be remunerated at least at a net return of zero. Furthermore, early investors can enter into trades with late investors themselves so that, in particular, the proposed plan must give early consumers at least what they could get from selling their project to late investors.

It follows that for a given information set-up $m^\pi$ the social planner solves:

$$
\max_\pi \int c_1(m^\pi(q)) dF + (1 - \pi) \int c_2(m^\pi(q)) dF
$$

subject to
\[ \pi c_1(m^\pi(q)) \leq x \max \left( S, \min(E(q|m^\pi(q)R), \frac{\pi A}{x}) \right) \]
\[ (1 - \pi)c_2(m^\pi(q)) \leq (1 - x) \max (S, E(q|m^\pi(q)R)) \]
\[ c_1(m^\pi(q)) \geq \max (S, \min(E(q|m^\pi(q)R, A)) \]

where \( x \) is the fraction of projects that are scrapped or liquidated at date 1, while \( c_2(m^\pi(q)) \) is the consumption late consumers expect to get when message \( m^\pi(q) \) is emitted. (Late investors realize this expected payoff by consuming the proceeds of all surviving projects at date 2.) The first two constraints are resource constraints, the last constraint is a participation constraint for early consumers. Inspection of this program shows that the objective is maximized when \( x = \pi \) (that’s because \( \max (S, E(q|m^\pi(p)R)) \geq \max (S, \min(E(q|m^\pi(p)R, \frac{\pi A}{x})) \)) but this means that the planner solves exactly the problem we study in the body of the paper.

This analysis suggests that a covenant that bars early consumers from selling their project to late investors could improve welfare. But that is not the case. As long as early investors can enter into debt contracts with late investors, they can say that they are late consumers and sell \( c_2(m^\pi(q)) \) to late investors. Incentive compatibility (truth-telling) for early investors who want to consume at \( t = 1 \) thus requires:

\[ c_1(m^\pi(q)) \geq \max (c_2(m^\pi(q)), A). \]

After replacing the participation constraint with this new constraint, it remains true that the objective is maximized when \( x \) is a low as possible, which happens yet again when \( x = \pi \) so that, once again, the planner is solving the problem we study in the body of the paper.

### 9.3 Non-overlapping intervals

**Lemma 9.3.** The scrapping message is of the form \([0, \bar{q}] - Q\) where \( \bar{q} \leq \frac{S}{R} \) and \( F(Q) = 0 \).

**Proof.** If \( \pi = 0 \) the result is obvious so assume that \( \pi > 0 \). We have already argued that revealing any information within \([\frac{S}{R}, 1]\) has no value and can only reduce the early investors payoffs. It follows that the scrapping message is F-essentially contained in \([0, \frac{S}{R}]\). Now take any set \( \bar{Q} \) of positive F-mass in the scrapping message and assume by way of contradiction that there is a set of positive F-mass \( Q \) that is not in the scrapping set and such that \( \bar{Q} \cap Q = \emptyset \) and \( \bar{Q} \leq \bar{Q} \). Let \( q_{low} = ess \sup \bar{Q} \) and \( q_{high} = ess \inf \bar{Q} \). Then \( q_{low} \leq q_{high} \). But then the
gain from adding \( Q \) to the scrapping message is bounded below by 
\[
F(Q) (S - (1 - \pi)q_{low}) \geq F(Q) (S - (1 - \pi)q_{high}) \geq 0
\]
where the second inequality follows from the premise that 
\( q_{low} \leq q_{high} \) while the final equality must hold if it is profitable to scrap in \( \overline{Q} \). If \( q_{low} < q_{high} \) the inequality is in fact strict and gains from adding \( Q \) to the scrapping message are strictly positive. If \( q_{low} = q_{high} \) then \( Q \cap \overline{Q} = \emptyset \) implies that either \( q < q_{low} \) for all \( q \in Q \) or \( q > q_{high} \) for all \( q \in \overline{Q} \). In either case, gains from adding \( Q \) to the scrapping message are once again strictly positive. The scrapping message, therefore, cannot have a hole of positive mass, and if it contains some \( \overline{q} \), it also contains almost all \( q < \overline{q} \). F-essentially therefore, the scrapping message is an interval that starts at the origin.

10 References


Siegert, Caspar (2012) “Optimal Opacity and Market Support,” mimeo, University of Munich