Search, Liquidity and the Dynamics of House Prices and Construction

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Abstract

The dynamics of house prices, sales, construction and population growth in are characterized response to city–specific income shocks for 106 U.S. cities. A dynamic search model of the housing market is then developed in which construction, the entry of buyers, house prices and sales are determined in equilibrium. The theory generates dynamics qualitatively consistent with the observations and a version calibrated to match key features of the U.S. housing market offers a substantial quantitative improvement over models without search. In particular, variation in the time it takes to sell (i.e. the house's *liquidity*) induces transaction prices to exhibit serially correlated growth.

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1 Introduction

We explore the consequences of time-consuming search and matching for the dynamics of house prices, sales, and construction at the city level. First, we characterize the impact of city-specific income shocks on the short-run dynamics of average house prices, home sales, construction and population growth for a panel of U.S. cities. We then develop a model in which the entry of new buyers and the construction of new houses in response to such shocks are endogenously determined. Our theory generates serial correlation in the growth rates of house prices and construction, even if income is strictly mean-reverting following shocks.[1] When calibrated to data on U.S. cities our model accounts for over 80 percent of the variance of house price movements driven by city-specific income shocks and nearly half of the observed autocorrelation of house price growth.

In our empirical analysis, we estimate a structural panel vector auto-regressive (VAR) model using city-level observations on the variables listed above. We focus on conditions *at the city level* because, as a number of authors have noted, a major share of the time-series variation in house prices is local in nature.[2]

We find that housing market dynamics in U.S. cities are characterized by the following: Firstly, house prices are volatile relative to *per capita* incomes. Moreover, house price growth is *much* more volatile than a standard asset pricing model would predict for a simple claim to local *per capita* income. Secondly, house price growth exhibits strong positive serial autocorrelation over the short term, but reverts to its mean over longer periods. Thirdly, sales growth is volatile relative to income and is positively autocorrelated with, but lags, population growth. Fourthly, population growth is more volatile than construction, especially in the short run. Finally, construction is more persistent than population growth, and both exhibit substantially more persistence than fluctuations in income growth.

Some of these observations have been documented previously by other authors using different data sets, but never to our knowledge in a unified study of city-level data.[3] In any case, it has been noted, for example by Capozza, Hendershott and Mack (2004), that a formal theory which accounts for them has proved difficult to construct. In particular, the substantial autocorrelation of house price growth appears to be inconsistent with an asset-pricing approach in which houses are treated as simple claims to local incomes and/or

[1]This behavior has been referred to as "price momentum" in the literature (e.g. Glaeser *et al.* 2011).

[2]For evidence on the U.S. see Abraham and Hendershott (1996), Del Negro and Otrok (2006) and Glaeser et al. (2011). For Canada see Allen et al. (2009).

[3]See, for example, Abraham and Hendershott (1996), Malpezzi (1999) and Meen (2002).

rents. For example, Case and Shiller (1989) argue that the serial correlation in rents cannot explain that of price changes (see also Cutler, Poterba and Summers, 1991). Glaeser *et al.* (2011, p. 29) find that while a dynamic rational expectations model of housing with endogenous construction can generate long-run mean reversion, it "fails utterly at explaining high frequency positive serial correlation of price changes."

Several authors have argued that there are good reasons to expect search and matching to play important roles in housing markets. For example, both the observed positive aggregate co-movement of prices and sales and the fact that both are negatively correlated with average time on the market (Krainer, 2008) are broadly consistent with search theories of housing markets. Moreover, as noted recently by Caplin and Leahy (2011), there is significant negative correlation between vacancies and price growth. Diaz and Jerez (2012) suggest that movements in the division of surplus between buyers and sellers driven by changes in the tightness of housing markets (as predicted by competitive search theory) may be a significant source of fluctuations in house prices.

Here, we construct a framework with search and matching in the housing market in which both the entry of new buyers and the construction of new houses are endogenous. The value of living in a particular city depends jointly on the value of housing and the income that can be earned there relative to those in other locations. Agents enter a city when the expected value of doing so exceeds their next best alternative. They require housing. All rent initially, but many then search for houses to buy. The market for residential housing is characterized by random search with entry of both buyers and sellers — a process motivated by the idea that an agent must find the "right" house to realize utility from home-ownership. New houses are constructed and offered either for sale or for rent by profit-maximizing development firms. Home-owners also put their houses up for sale or rent due to idiosyncratic shocks that either render them dissatisfied with their current house or cause them to exit the city altogether. In this environment, we establish the existence of a unique stationary growth path characterized by constant rates of population growth, migration and construction.

We study the implications of city-specific income shocks by calibrating our model to data on U.S. cities. The theory generates short-term serial correlation in price growth in equilibrium even in the absence of persistent income growth. In the model, an increase in the value of living in a city spurs an immediate increase in house search activity as households enter. It takes time, however, for these buyers to find houses, as well as for construction of new housing to respond. To meet the immediate housing demands of new entrants, some existing vacant houses are shifted to the rental market. As a result, the matching rate for individual house buyers initially declines, while both sales and the rate at which houses sell rise immediately. Therefore, although the value of house search begins to decline after just one period (due to mean reversion in income), the tightness of the housing market (*i.e.* the ratio of buyers to sellers) continues to rise for several more.

As the market tightens houses sell more quickly. This ongoing increase in the *liquidity* of houses causes their expected re-sale value to grow. Because the sales price of a house in part reflects this, it continues to grow for some time in anticipation of easier future re-sale. Over time, as income reverts to its long-run level, the stock of buyers declines as entry slows and many become home-owners. Higher home values induce increased construction so that the decline in vacancies slows and is eventually reversed. The buyer–seller ratio, and hence the liquidity of houses, falls. In anticipation of a less tight market (and slower re-sale) in the future, the house price eventually reverts to its steady-state level.

Although a number of researchers have studied the role of search and matching in housing markets (e.g. Wheaton, 1990; Krainer, 2001; Albrecht at al., 2007; and Head and Lloyd-Ellis, 2012), they have generally treated the aggregate housing stock as fixed, and/or considered only steady-states. Caplin and Leahy (2011) consider the non-steady-state implications of a model with a fixed housing stock. In contrast, we focus on the role of transitional dynamics of prices and construction of new homes in response to shocks. Furthermore, we allow for the turnover of existing homes, which turns out to be crucial for both the qualitative and quantitative nature of price and construction dynamics. Finally, models of housing investment and construction (*e.g.* Davis and Heathcote, 2005) generally abstract from search and matching in the market for houses in order to focus on supply-side factors, whereas in this paper we combine the two in a unified framework.

Our analysis is most closely related to those of Glaeser et al. (2011) and Diaz and Jerez (2012). Glaeser et al. (2011) also study short-term dynamics driven by an estimated process for city-level incomes in a model in which house prices reflect the interaction between local supply conditions and the willingness of households to pay to live in a particular location.[4] They do not, however, consider the role of the ease of re-sale associated with search and matching. Their model has significant success in accounting for short term volatility in prices and construction but fails to generate serial correlation in price growth and cannot account for volatility over a longer time horizon.

Diaz and Jerez (2012) develop and calibrate a search model in which houses are traded because home-owners experience shocks which render them unsatisfied with their current

[4] Their model, like ours, builds on the ideas of Rosen (1979) and Roback (1982).

house (as in Wheaton, 1990). They study the impact of random changes to frequency of these shocks, but do not consider either construction or the entry and exit of buyers, all of which play important roles in our results.[5] In their model, competitive search magnifies fluctuations in house prices due to movements in the shares of surplus accruing to buyers and sellers. When we allow for this in our model (using a matching function similar to theirs) volatility does increase, but only at the expense of a substantial reduction in the serial correlation of price growth.

Because our empirical analysis focuses on *city-specific* income shocks, we develop a model in which the impact of local shocks has negligible effects on the rest of the economy. Several recent papers develop models that allow for cross-city interactions, but limit their analyses in other ways. Van Nieuwerburgh and Weill (2010) study the long-run implications of increasing dispersion of wages across cities for that of housing prices in a model with cross-city migration and endogenous contruction. They do not, however, consider search dynamics (and make no distinction between owning and renting), study short-term price movements or consider the implications of their model for the empirical counterparts of construction, population growth and sales growth at the city level. Karahan and Rhee (2013) study the interaction between illiquid housing markets and labour market outcomes during the recent recession in a model with two representative cities. Again they do not consider short-term movements in the variables we consider at the city level.

Section 2 documents empirical features of the dynamics of housing markets at the city level. Section 3 develops the basic model structure. In Section 4, a search equilibrium is characterized and a deterministic steady-state derived. Section 5 presents both a baseline calibration for our search economy and an alternative economy without search. Section 6 considers the dynamic implications of income shocks in the theory. Section 7 checks the robustness of our main findings by examining four modifications to the baseline search economy. Section 8 concludes. Details regarding the data may be found in Appendix A and all proofs and extended derivations are contained in Appendix B.[6]

[5] They also evaluate their model using aggregate rather than city-level data.

[6] Several additional extensions and calculations appear in a separate appendix, available on-line.

2 Empirical Properties of Housing Markets in U.S. MSA's.

In this section, we characterize the joint dynamics of city-level *per capita* income, house prices, growth in the sales of existing houses, construction rates and population growth for a sample of 106 U.S. metropolitan statistical areas (MSA's). Our data is annual and runs from 1980 through 2008. Details, including sources, are provided in Appendix A. Since we are interested in the dynamics of city-level measures relative to those in other cities, we transform the data by removing common time effects. That is, we estimate a panel regression for each series with time dummies and study the residual components. In what follows we assume these transformed variables are mean stationary.

We assess the assumption of stationarity using several different unit root tests that have been developed for panel data. Most of these tests posit the null hypothesis that all panels contains a unit root.[7] After removing the common time effects, in all cases we strongly reject this hypothesis for all five variables (see the online appendix). An alternative test (Hadri, 2000) posits the null that no panel contains a unit root. Here, we reject for all variables except sales growth. The results are thus inconclusive, suggesting that some panels may be non-stationary. Using income and price growth rates, the results are equally inconclusive.

We address the potential for non-stationarity of *per capita* income and prices to affect our estimates in two ways. First, we use system GMM which has been found to work well when variables are persistent and close to unit root processes. In fact, only mean (rather than covariance) stationarity of the variables is necesary for identification using this method (see Blundell and Bond, 1998 and Binder et al., 2005). In our case, this requires us to assume that long run differences in the *growth rates* of income and price across cities are not permanent. Second, for comparison, we also estimate the panel VAR using the first differences of log income and prices. Although there are differences, the moments upon which we focus and the nature of the impulse responses in the short-run are very similar (see the online appendix).

2.1 A Structural Panel VAR

Movements in house prices, sales, construction and city populations are likely affected by many factors. Here we isolate the dynamics that result from changes to *income* at the city level, which we interpret as a dividend to residence in the city. Our focus is motivated by the

[7] See Breitung (2000), Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003).

theory we present below: When incomes in a particular city rise relative to the average, that city's rate of population growth rises as households move in from outside to take advantage of either the higher income itself or the factor(s) that caused it. Faster entry increases housing demand, raises house prices and, over time, spurs construction. As increased construction pushes costs higher and local income reverts to its trend, entry slows. Eventually, house prices decline and return to their long-run level.

With this in mind, we estimate the following panel VAR model:

$$\mathbf{B}\mathbf{X}_{ct} = \sum_{i=1}^{T} \mathbf{A}_i \mathbf{X}_{ct-i} + \mathbf{F}_c + \boldsymbol{\varepsilon}_{ct}. \tag{1}$$

Here $\mathbf{X}_{ct} = [Y_{ct}, P_{ct}, g_{ct}^S, g_{ct}^H, g_{ct}^N]'$ denotes the vector of the logarithms of the levels of income *per capita* and house prices, sales growth, the growth rate of the stock of houses and population growth in each city at each date. $\mathbf{B}$ and $\mathbf{A}_i$ are matrices of parameters, $\mathbf{F}_c$ is a vector of city fixed–effects and $\boldsymbol{\varepsilon}_{ct} = [\varepsilon_{ct}^Y, \varepsilon_{ct}^P, \varepsilon_{ct}^S, \varepsilon_{ct}^H, \varepsilon_{ct}^N]'$ a vector of structural shocks.

We assume that the shocks are orthogonal and adopt a Cholesky decomposition with the ordering indicated by the definition of $\boldsymbol{\varepsilon}_c$ above. In particular, income does not depend contemporaneously on any of the other variables and house prices depend contemporaneously only on income. This ordering, which is consistent with our theory, emphasizes the importance of shocks to fundamentals that affect current and future income, as well as house prices, sales growth, construction, and population growth contemporaneously. It also allows for shocks to prices that have no *contemporaneous* effect on income, but can affect the other variables, and *future* income.[8] Like our theory, this ordering rules out shocks to current and future income that have no contemporaneous effect on house prices.

We estimate equation (1) for $T = 2$ using the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998).[9] This estimator is asymptotically consistent when the number of panels becomes large for a given time-dimension, thereby avoiding the incidental parameters problem associated with fixed-effects estimators (Nickell, 1981). We use this estimator for several reasons. Firstly, it is generally found to outperform other standard GMM estimators such as that of Arellano and Bond (1991) when the endogenous variables are persistent.[10] Secondly, its asymptotic properties are well understood and it has been extended to the context of panel VARs by Binder, Hsiao and Pesaran

[8]Since g_{ct}^H and g_{ct}^N are growth rates going forward (i.e. $g_{ct}^H = \ln H_{t+1} - \ln H_t$), it seems reasonable that they are able to respond to time t shocks to income and prices. The relative ordering of g_{ct}^H and g_{ct}^N makes little difference for our results.

[9]Estimating the system with more than two lags made little difference for our results.

[10]The system GMM estimator instruments the endogenous variables using lagged differences. We adopt

(2005). Finally, the standard fixed–effect estimator has been found to exhibit a significant finite-sample bias for samples with similar dimensions to ours (*i.e.* moderately large time and panel dimension; see Judson and Owen, 1999). There are, however, some potential pitfalls in using this estimator. These, along with estimates generated using other methods are discussed in the online appendix.

A full set of parameter estimates from our baseline panel VAR is reported in the online appendix. Here, Tables 1 and 2 contains several summary statistics. For each series, the first column contains its average standard deviation relative to that of *per capita* income growth. The second and third columns contain respectively their correlations with the growth of *per capita* income and house prices. The remaining columns contain the first four coefficients of autocorrelation. Our rationale for reporting statistics for income and price *growth* (when the model is estimated using levels) is that we are focussing on short-run fluctuations, and the comovements associated with the *levels* of these variables are dominated by movements in their less precisely estimated long-run responses.

Table 1: Moments from Panel VAR: all shocks

				$\rho(x_t, x_{t-i})$			
Variable (x)	σ_x/σ_y	$\rho(x,y)$	$\rho(x,p)$	$i=1$	$i=2$	$i=3$	$i=4$
Income growth (y)	1.00	1.00	0.43	0.27	0.03	-0.06	-0.09
Price growth (p)	2.75	0.43	1.00	0.56	0.24	0.02	-0.11
Sales growth (s)	2.42	0.30	-0.26	0.63	0.37	0.23	0.15
Construction (h)	0.28	0.26	0.34	0.75	0.49	0.28	0.14
Population Growth (n)	0.49	0.27	0.10	0.39	0.31	0.17	0.10

Note: In all tables, σ_x denotes the standard deviation of x, and $\rho(x,y)$ the correlation of x with y.

Several observations can be made based on Table 1. First, house price growth and sales growth are much more volatile than city-level income growth. Secondly, price changes are more persistent than those of income growth, with a first-order autocorrelation of 0.56 as compared to 0.27. Thirdly, population growth rates are more volatile on average than construction rates. Fourthly, construction rates are more persistent than population growth

an alternative, but asymptotically equivalent approach which has been found to perform better in finite samples. We instrument levels with lagged deviations from the forward mean of the remaining sample (Arellano and Bover, 1995). As such the model is "just identified" (the number of regressors equals the number of instruments), so that system GMM is numerically equivalent to equation-by-equation 2SLS.

rates and both, like price changes, exhibit substantially more persistence than income. Finally, growth in the sales of existing houses at the city level are *negatively* correlated with price growth. This contrasts with others' findings at the aggregate level (*e.g.* Diaz and Jerez, 2012).

Table 2 reports statistics associated with the effect of income shocks only. The standard deviation of house price growth generated by income shocks alone is more than half that observed overall. Income shocks also generate somewhat greater persistence of house price growth than it exhibits overall. Similar results hold for sales, construction and population growth rates. In response to a shock to local income, house price growth and sales growth are respectively 60 percent and 32 percent more volatile than income. In contrast, the construction rate exhibits only 11 percent as much volatility as income growth, and population growth 17 percent as much. All four variables, however, exhibit substantially more serial correlation than income growth. Finally, the induced correlation of price and sales growth is small.

Table 2: Moments from Panel VAR: Income shocks only

				$\rho(x_t, x_{t-i})$			
Variable (x)	σ_x/σ_y	$\rho(x,y)$	$\rho(x,p)$	$i=1$	$i=2$	$i=3$	$i=4$
Income growth (y)	1.00	1.00	0.76	0.28	0.03	-0.06	-0.09
Price growth (p)	1.60	0.76	1.00	0.75	0.36	0.05	-0.15
Sales growth (s)	1.32	0.56	0.01	0.59	0.39	0.35	0.35
Construction (h)	0.11	0.55	0.80	0.88	0.61	0.33	0.11
Population Growth (n)	0.17	0.75	0.60	0.67	0.40	0.19	0.09

Figure 1 depicts the implied impulse responses to a shock to relative local income together with the associated 95 percent confidence intervals.[11] Following the shock, local income exhibits positively auto-correlated growth, peaking after one year, and is quite persistent. The resulting rise in the relative house price exhibits considerably more persistence, continuing to rise for three years before starting to revert to its mean. Mean reversion is, however, more rapid overall for house prices than for income. After an initial peak, sales growth slows quickly before rising again in the medium term. Population growth responds immediately to the shock then slows down, whereas the construction rate responds more sluggishly, peaking after two years. A key consequence of the latter is that the ratio of city population to the housing stock rises and remains persistently high following a shock to income.

[11] These are computed using a Monte Carlo simulation provided by Love and Zicchino (2006).

We considered several alternative specifications of the panel VAR, but omit the estimates here for brevity (see the online appendix). Interestingly, when we restrict the VAR so that income follows a univariate AR(2) process, the results are hardly changed. This suggests that feedback effects of the other variables to per capita income are not very large and that it may be reasonable to think of the latter as an exogenous process.

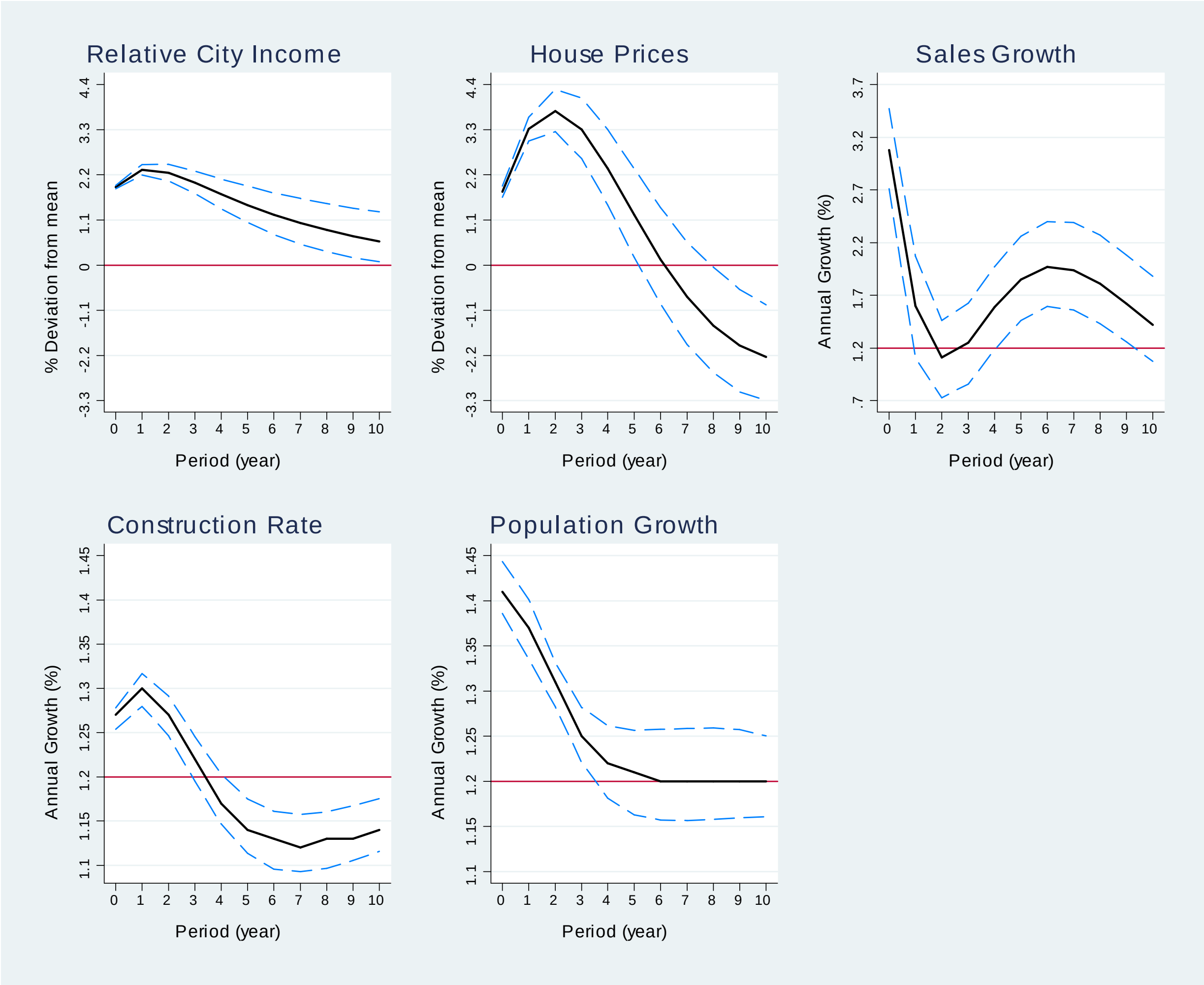

Figure 1: Estimated Impulse Responses to an Income Shock

2.2 Pricing a claim to local income

It is useful to consider the dynamics of the price of a simple claim to local income, as this serves as a benchmark for evaluating the importance of the particular characteristics of *houses* in accounting for the dynamics of their prices. If agents' utility is linear in consumption (as it is in our theory), and if claims to local *per capita* income, y_t, are traded in a frictionless Walrasian market, then their price, P_t^L, will equal the present discounted value of local income:

$$P_t^L = E_t \sum_{i=1}^{\infty} \beta^i y_{t+i}, \tag{2}$$

where β is a discount factor. Imposing the appropriate transversality condition, setting $\beta = 0.96$, and assuming that $\ln(y_t)$ follows the univariate AR(2) process described above, we generate the implied moments for P^L. Table 3 compares these moments with those documented for house prices in U.S. cities in Table 2. The relative volatility of the price of a claim to local income is less than a quarter that of houses in the data. Moreover, despite the fact that income growth exhibits serial correlation, that of P^L does not. The price of these claims immediately capitalizes future income fluctuations, Thus, for it to exhibit persistent growth, extremely high and persistent serial correlation in income growth would be required.

Table 3: Prices of claim to local income: Income shock

			$\rho(x_t, x_{t-i})$			
Variable (x)	σ_x/σ_y	$\rho(x,p)$	$i=1$	$i=2$	$i=3$	$i=4$
Houses in U.S. MSA's	1.60	0.76	0.75	0.36	0.05	-0.15
Claims to MSA Incomes	0.38	0.95	-0.04	-0.04	-0.04	-0.04

3 The model environment

Time is discrete and indexed by t. The economy is populated by a measure Q_t of *ex ante* identical households, growing exogenously at net rate μ and distributed over two locations; the *city* and the "rest of the world". Each period, households enter and exit the city through processes described below. All households living in the city require housing, and they each may either own or rent a house.

Each household is infinitely-lived and discounts the future at rate $\beta \in (0,1)$. In the city, each household is endowed with two types of labour: general and construction. At each date t, a household supplies one unit of general labour inelastically and l_t units of construction labour endogenously, taking the construction wage w_t as given.[12] General labour earns y_t per unit supplied, where y_t follows a stationary stochastic process in log-levels.

[12]Incorporating endogenous supply of general labour is straightforward, but makes no difference for our analysis if agents' preferences are separable in the disutility of the two labour types.

At date t, preferences over consumption c_t, construction labour l_t and housing z_t are given by:

$$U(c_t, l_t, z_t) = c_t - v(l_t) + z_t, \tag{3}$$

where

$$v(l_t) = \zeta^{-\frac{1}{\eta}} \left(\frac{\eta}{1+\eta} \right) l_t^{1+\frac{1}{\eta}} \tag{4}$$

and η and ζ are constants. Here z_t denotes a constant service flow reflecting an owner's personal preference for his/her house. If the owner likes the house he/she owns, $z_t = z^H$. If the owner either does not like the house it owns or is renting, $z_t = 0$. Any depreciation resulting from occupancy is assumed to be offset by maintenance. We denote the cost of maintenance to the owner as m.

In period t, the city has a stock H_t of housing units, each of which is either occupied by a resident owner, rented to a resident, or vacant for sale. The measure of resident home-owners is denoted by N_t, and that of renters by $B_t + F_t$. Here B_t is the measure of renters who would like to own a house (and so are currently searching for one to buy) and F_t is that of renters that are not interested in owning. A measure S_t of houses are for sale, where $S_t = H_t - N_t - B_t - F_t$. Houses for sale include both newly built ones (currently owned by developers) and those put up for sale by resident owners who either do not want them anymore or are moving elsewhere. At the beginning of each period, a house that is not currently owner-occupied can either be rented or listed for sale. Let H_t^R denote the stock of houses available for rent. A rented house earns rent r_t less the maintenance cost m. The rental market is competitive.

In the city, there are a large number of competitive *developers* who operate a technology for the construction of new housing units. Each new house requires one unit of land, which can be purchased in a competitive market at unit price q_t, and $1/\phi$ units of construction labour. Houses constructed at time t become available either for sale or for rent at time $t+1$ and do not depreciate over time. The stock of houses thus evolves according to

$$H_{t+1} - H_t = \phi L_t. \tag{5}$$

Newly built houses are identical to pre-existing ones. Developers can either rent them out or designate them for sale, in which case they remain vacant for at least one period and have exactly the same value as existing vacant houses. Only houses that are occupied require maintenance to offset depreciation.

Land potentially available for residential use, K_t, grows at an exogenous rate equal to that of the aggregate population, μ.[13] To make use of such land in construction, a stochastic conversion cost must be incurred. For simplicity, we assume that the cost of converting each undeveloped and available parcel of land is represented by a random draw c from stationary distribution $\Lambda(\cdot)$ with support $[\underline{c}, \overline{c}]$. This captures the idea that conversion costs across parcels and over time may depend on a variety of factors (*e.g.* topography, current taxes and regulations *etc.*). There is free entry into construction so that only those parcels with $c < q_t$ will be converted in a given period. It follows that the supply of actual residential land for housing, $\hat{H}_t$, changes according to

$$\hat{H}_{t+1} = \hat{H}_t + \Lambda(q_t)A_t, \tag{6}$$

where A_t, the stock of land available for conversion, evolves according to

$$A_{t+1} = (1 - \Lambda(q_t))A_t + \mu K_t. \tag{7}$$

At the beginning of period t, measure μQ_{t-1} of new households arrive in the economy. Each of these households has an alternative value, ε, to entering the city. Here ε is distributed across the new households according to a stationary distribution function $G(\varepsilon)$ with support $[0, \bar{\varepsilon}]$.[14] There exists a critical alternative value ε_t^c, at which a new household is just indifferent to entering the city:

$$\varepsilon_t^c = \bar{W}_t, \tag{8}$$

where $\bar{W}_t$ is the value of being a new entrant to the city. All non-resident households with $\varepsilon \leq \varepsilon_t^c$ enter the city and are immediately separated into two types. A fraction ψ of the new entrants derive utility from owning their own home *per se* and become potential buyers, while the rest do not and become perpetual renters.[15] Let W_t denote the value of being a potential buyer and W_t^f the value of being a perpetual renter. Then,

$$\bar{W}_t = \psi W_t + (1 - \psi)W_t^f. \tag{9}$$

[13] This ensures the existence of a balanced growth path. A similar assumption is made by Davis and Heathcote (2005).

[14] An interpretation of $G(\cdot)$ based on a multi-city model in which agents realize different amenity values from residence in any of the many cities is presented in the online appendix. The key assumption here is that income, y, is truly *city-specific* in that it affects *only* the attractiveness of our representative city to potential entrants.

[15] Given that we focus on versions of the model calibrated to match the long-run ratio of renters to owners this is equivalent to assuming that all entrants become buyers with a given probability in each period.

The choice to move to the city is irreversible in the sense that once a household has entered, they lose access to their alternative opportunities and cannot leave until they recieve another.[16] Searching for a house to own takes at least one period, and potential buyers also rent while searching.

At the end of each period, home-owners are subject to two exogenous shocks. With probability $\pi_n \in (0, 1)$ they leave the city immediately and recieve continuation value Z. In this case, they own a vacant house which they either rent or hold vacant for sale. With probability $\theta \in (0, 1)$ the remaining $(1 - \pi_n)N_t$ of owners find that they no longer derive the utility premium, z^H, from owning their current house. Such "mismatched" owners immediately move out of their current house, put it up for sale, and rent while searching for a new one.[17]

Perpetual renters may also, with probability $\pi_f \in (0, 1)$, experience an exogenous shock that induces them to leave the city. Like home-owners, in this case they move out immediately and receive value Z. Otherwise they remain as renters in the next period. In contrast, we assume that potential buyers are not subject to any shocks while they are searching.[18]

We assume that capital markets are perfect and that the gross interest rate is $1/\beta$. In this case, with free entry into construction, households have no interest in owning houses either as a means of saving or for speculation. As such, it makes no difference whether or not we allow for the trading of vacant houses in a Walrasian market. It is, however, important that in order to receive utility z^H from owning, households must search for the right house through a time-consuming process that depends on the measures of buyers and vacant houses/sellers in the market.

Matching between searching buyers and vacant houses for sale is determined by the function $\mathcal{M}(B_t, S_t)$, which is increasing in both arguments and exhibits constant returns to scale. It follows that a buyer finds a vacant house in the current period with probability

$$\lambda_t = \frac{\mathcal{M}(B_t, S_t)}{B_t} = \lambda(\omega_t). \tag{10}$$

where $\omega_t = B_t/S_t$ is the *tightness* of the housing market. Similarly, each period a seller

[16] This assumption is similar to that in labour market models of on-the-job search in which once a household moves to a new job they cannot return to their previous one

[17] Assuming some or all mis-matched owners remain in their current home while searching yields almost identical results (this is addressed below). In either case, a mis-matched owner who moves out has a vacant house, which in each period may be either rented or held vacant for sale.

[18] This assumption reflects our view that households would not search for a house to buy unless they believed that the probability with which they would leave the city at the end of the quarter were very low — in particular lower than the overall exit rate for either renters or established homeowners. In Section 7, we show that relaxing this assumption makes no difference for our results.

finds a buyer with probability

$$\gamma_t = \frac{\mathcal{M}(B_t, S_t)}{S_t} = \gamma(\omega_t) = \omega_t \; \lambda(\omega_t). \tag{11}$$

Buyers and sellers take the matching probabilities λ_t and γ_t as given.

We associate the rate at which houses sell, γ_t, with their *liquidity*. When this increases (decreases), houses become more (less) liquid, by which we mean they sell more (less) quickly. We parameterize a specific matching function as part of our calibration in Section 5. Within a match, the price is determined according to a simple bargaining scheme in which χ denotes the share of the total match surplus received by the buyer.

Our model has much in common with search and matching models used, for example, in the labour search literature. There are, however, several differences which play a crucial role in our characterization of housing market dynamics. Firstly, every potential home buyer eventually becomes a seller. This implies that expected future market conditions (which determine capital gains) affect a potential buyer's value of owning a home and hence the current transaction price. Secondly, new vacant homes are produced at a unit cost which varies with supply and demand. It is through this avenue that housing price adjustments ultimately occur. Finally, the rental market plays a key role in absorbing new potential buyers into the city while allowing some substitution of non-owned housing between the rental and the vacant-for-sale pools.

4 Equilibrium

Because preferences are linear in consumption and capital markets perfect, households are indifferent with regard to the timing of their consumption. The optimal construction labour supply decision is atemporal and yields the supply function

$$l(w_t) = \zeta w_t^{\eta}. \tag{12}$$

The net benefit of supplying labour is

$$x(w_t) = w_t l_t - v(l_t) = \frac{\zeta w_t^{1+\eta}}{1+\eta}. \tag{13}$$

Perpetual renters remain as such until they exit the city. Their value is

$$W_t^f = u_t^R + \pi_f \beta Z + (1-\pi_f)\beta E_t W_{t+1}^f, \tag{14}$$

where $u_t^R = y_t + x(w_t) - r_t$ is the flow utility to a renter. The stock of such renters evolves according to

$$F_t = (1 - \pi_f)F_{t-1} + (1 - \psi)G(\varepsilon_t^c)\mu Q_{t-1}. \tag{15}$$

At the beginning of each period, a house that is not currently occupied by the owner can be either rented or listed for sale. Such houses have value

$$\tilde{V}_t = \max\left[r_t - m + \beta E_t \tilde{V}_{t+1},\ V_t\right], \tag{16}$$

where V_t is the value of a house designated for sale. The value of being a home-owner J_t is then given by

$$\begin{aligned} J_t &= u_t^H + \pi_n \beta\left(Z + E_t \tilde{V}_{t+1}\right) + (1 - \pi_n)\theta\beta\left(E_t W_{t+1} + E_t \tilde{V}_{t+1}\right) \\ &\quad + (1 - \pi_n)(1 - \theta)\beta E_t J_{t+1}, \end{aligned} \tag{17}$$

where $u_t^H = y_t + x(w_t) + z^H - m$ is the flow utility from being a home-owner.

An agent with a vacant house is free to enter the market as a seller at no cost and matches with a prospective buyer with probability γ_t. Within a match, the purchase price, P_t, solves a Generalized Nash bargaining problem to yield

$$P_t = (1 - \chi)\,\beta\left(E_t J_{t+1} - E_t W_{t+1}\right) + \chi\beta E_t V_{t+1}, \tag{18}$$

where we interpret χ as the buyer's bargaining weight. We focus on situations in which the total trade surplus in the housing market is strictly positive:

$$E_t\left[J_{t+1} - W_{t+1} - \tilde{V}_{t+1}\right] > 0. \tag{19}$$

Below we demonstrate that (19) must hold in a steady–state equilibrium.

If a house held vacant for sale is not sold in the current period, the seller keeps it and receives the value of a house that is not currently owner-occupied at the beginning of the next period. The value of a vacant house for sale then satisfies

$$V_t = \gamma_t\ P_t + (1 - \gamma_t)\,\beta E_t \tilde{V}_{t+1}. \tag{20}$$

Given (20), a seller is willing to enter the market if $P_t \geq \beta E_t \tilde{V}_{t+1}$.

Successfully matched buyers pay price P_t and become home-owners in the next period, receiving value J_{t+1}. Unmatched buyers continue to search. Searching buyers rent, and thus receive u_t^R. The value of being a searching buyer, W_t, is then given by

$$W_t = u_t^R + \lambda_t\left(\beta E_t J_{t+1} - P_t\right) + (1 - \lambda_t)\,\beta E_t W_{t+1}. \tag{21}$$

Clearly, a buyer is willing to enter the market if and only if $P_t \leq \beta\left(E_t J_{t+1} - E_t W_{t+1}\right)$.

Given that there is an active market of houses for sale, the stock of buyers at date t is given by:

$$B_t = \theta(1-\pi_n)N_{t-1} + \psi G(\varepsilon_t^c)\mu Q_{t-1} + (1-\lambda_{t-1})B_{t-1}. \tag{22}$$

The stock of home-owners evolves via

$$N_t = (1-\pi_n)(1-\theta)N_{t-1} + \lambda_{t-1}B_{t-1}. \tag{23}$$

Overall, at time t, the *state*, s_t, of the economy is given by level of income in the city, y_t, and the measures of buyers, B_t, home-owners, N_t, permanent renters, F_t, houses, H_t, and land available for construction, K_t. The state evolves via (5), (15), (22), (23), (6) and the stochastic process for non-construction income, y_t.

Definition. A *search equilibrium* is a collection of functions of the state, s_t. These functions are the values of houses vacant for sale, V_t, home-ownership, J_t, new entrants, $\overline{W}_t$, searchers, W_t, permanent renters, W_t^f, the entry value cutoff, ε_t^c, price of houses, P_t, rent, r_t, wage, w_t, the measures H_t^R, B_t, N_t, F_t, H_t, and housing market tightness, ω_t. These functions satisfy:

i. New households enter the market optimally so that (8) and (22) are satisfied;

ii. The values of home-ownership, vacant houses, searching buyers, permanent renters, and new entrants satisfy (9), (14), (17), (20) and (21), respectively.

iii. The owner of a vacant house is indifferent between renting the unit and holding it vacant for sale:

$$\tilde{V}_t = r_t - m + \beta E_t \tilde{V}_{t+1} = V_t; \tag{24}$$

iv. The house price, P_t, satisfies (18).

v. The market for rental housing clears:

$$H_t^R = B_t + F_t; \tag{25}$$

vi. There is free entry into construction:

$$\beta E_t \tilde{V}_{t+1} \leq \frac{w_t}{\phi} + q_t, \qquad H_{t+1} \geq H_t, \tag{26}$$

where the two inequalities hold with complementary slackness;

vii. The construction wage, w_t, clears the market for construction labour:

$$(N_t + B_t + F_t)l_t = L_t. \tag{27}$$

viii. The land price, q_t, clears the market for land, so that $\hat{H}_t = H_t$.

ix. The value of home-ownership is bounded: $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$.

4.1 The equilibrium dynamic system

In an equilibrium with an active housing market (*i.e.* in which (19) holds) the return to renting a house equals the expected gain from holding it vacant for sale. Combining (20) and (24), we have:

$$r_t - m = \gamma_t \left(P_t - \beta E_t V_{t+1}\right). \tag{28}$$

We focus on equilibria in which construction is always positive, *i.e.*, $H_{t+1} > H_t$.[19] It then follows from (5), (24) and (26) that the quantity of new housing constructed in period t is given by

$$H_{t+1} - H_t = \zeta\phi^{1+\eta} \left(N_t + B_t + F_t\right) \left(\beta E_t V_{t+1} - q_t\right)^{\eta}. \tag{29}$$

To obtain a stationary representation of the economy, we normalize the state variables (other than y_t and ω_t) by the total population Q_t, and use lower case letters to represent *per capita* values. Given (15), (22), (23) and (29), the laws of motion for renters, buyers, owners and houses, *per capita,* respectively, are

$$\begin{aligned}
(1+\mu)f_t &= (1-\psi)\,\mu G\left(\bar{W}_t\right) + (1-\pi_f)f_{t-1} && (30)\\
(1+\mu)b_t &= \psi\mu G\left(\bar{W}_t\right) + (1-\lambda_{t-1})\,b_{t-1} + \theta(1-\pi_n)n_{t-1} && (31)\\
(1+\mu)n_t &= (1-\theta)(1-\pi_n)n_{t-1} + \lambda_{t-1}b_{t-1} && (32)\\
(1+\mu)h_{t+1} &= h_t + \zeta\phi^{1+\eta}\left(n_t + b_t + f_t\right)\left(\beta E_t V_{t+1} - q_t\right)^{\eta}. && (33)
\end{aligned}$$

By definition, the tightness of the housing market is

$$\omega_t = \frac{b_t}{h_t - b_t - f_t - n_t}. \tag{34}$$

Moreover, rental market clearing implies

$$h_t^R = b_t + f_t. \tag{35}$$

[19] It is straightforward to show that this requires only sufficient population growth.

Finally, land market clearing together with (7) and (6) implies

$$(1+\mu)h_{t+1} = h_t + \Lambda(q_t)a_t, \tag{36}$$

where $(1+\mu)a_{t+1} = (1-\Lambda(q_t))a_t + \mu k$.

4.2 The deterministic steady-state

We now consider a steady-state in which general income *per capita*, y_t, is constant and normalized to unity. In this setting all normalized quantities and values are constant and their steady-state values are indicated with an asterix. We impose the following assumption:

Assumption 1. *(i)* $\lambda(\omega) \in [0,1]$, $\gamma(\omega) \in [0,1]$, $\lim_{\omega\to\infty} \lambda(\omega) = \lim_{\omega\to 0} \gamma(\omega) = 0$ *and* $\lim_{\omega\to 0} \lambda(\omega) = \lim_{\omega\to\infty} \gamma(\omega) = 1$; *(ii)* $\lambda'(\omega) < 0$, $\gamma'(\omega) > 0$.

Appendix B contains most steady-state calculations, including the proof of the central result:

Proposition 1. *There exists a unique steady-state equilibrium if*

$$\frac{(1-\chi)\beta z^H}{(1-\beta)\left[1-\beta(1-\theta)(1-\pi_n)\right]} > \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(1+\frac{\psi\left(\mu+\pi_f\right)}{A+B}\right)\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta}. \tag{37}$$

Condition (37) ensures that expected utility from homeownership exceeds the cost of producing another house. The proof of Proposition 1 proceeds in three steps. The first establishes that if a steady-state exists, then the surplus from a match in the housing market is positive. This follows from matched buyers strictly preferring owning to renting and there being no cost of selling. It implies that housing transactions always take place as long as the matching rate is positive.

The second and third steps pertain to the relationship between the value of a house for sale, V^*, and market tightness, ω^*. Firstly, as the value of a vacant house rises, more are built and made available for sale, reducing market tightness. Secondly, a less tight market, *lowers* the value of a vacant house. It reduces the rate at which houses sell, γ, lowering their value for a given selling price. It also *raises* the rate at which buyers find houses, reducing the transaction price by lowering surplus from becoming an owner. The interaction of these two forces on the relationship between V^* and ω^*, one positive and one negative, is responsible for the existence of a unique steady-state.

5 Calibration

We linearize the dynamic system for a calibrated version of the economy in a neighborhood of its unique deterministic steady-state. We then solve numerically for the implied local dynamics driven by stochastic movements in y_t using a first-order perturbation method.[20]

5.1 Baseline parameterization

The matching function is Cobb-Douglas,

$$\mathcal{M} = \kappa B_t^{\delta} S_t^{1-\delta}, \tag{38}$$

where $\kappa > 0$ and $\delta \in (0,1)$. Although (38) does not satisfy Assumption 1, in all of the experiments and robustness checks that we conduct the steady-state is interior in that $\gamma(\omega^*) \in (0,1)$ and $\lambda(\omega^*) \in (0,1)$. Since the shocks we consider are small, cases where γ or λ equal unity out of steady-state are sufficiently rare that they do not affect our results.

We set the minimum conversion cost to $\underline{c} = 0$ and let the distribution of conversion costs be

$$\Lambda(c) = \left(\frac{q}{\bar{c}}\right)^{\tau} \quad \text{where} \quad \tau > 0. \tag{39}$$

Table 4: Baseline Calibration Parameters: Steady State

Parameter	Value	Target
β	0.99	Annual real interest rate = 4percent
μ	0.003	Annual population growth rate = 1.2 percent
π_f	0.030	Annual mobility of renters = 12 percent
π_n	0.008	Annual mobility of owners = 3.2 percent
θ	0.012	Fraction of moving owners that stay local = 0.6
ϕ	2.5×10^{-4}	Quarterly permits/construction employment (hours)
$\bar{q}$	3.84	Average land price-income ratio
ξ	1.75	Median price-elasticity of land supply
ψ	0.43	Fraction of households that rent = 32 percent
m	0.0125	Average rent to average income ratio, $r^* = 0.137$
z^H	0.028	Zero net-of-maintenance depreciation
κ	0.76	Vacancy rate = 2 percent
χ	0.0916	Months to sell = months to buy
ζ	0.038	$P^* = 12.8$

[20] In all cases that we consider, the resulting systems of first-order linear difference equations satisfy the conditions for saddle-path stability. We obtained essentially identical results using a second-order perturbation method in Dynare.

Table 4 provides values for those baseline calibration parameters chosen to match steady-state targets. The first eight are set to match the indicated targets directly. The last six are chosen jointly to match the remaining six targets collectively. In Section 6.4, we consider the sensitivity of our results to alternative values. A period equals one quarter, with β set to reflect an annual interest rate of 4 percent, and μ to match annual population growth during the 1990's. Steady-state non-construction income *per capita* is normalized to $\bar{y} = 1$.

The relocation shock probabilities π_f and π_n are set to match the annual fractions of renters and home-owners that move between counties, roughly 12 percent and 3.2 percent respectively according to the Census Bureau. Similarly, θ is set to match the fraction of owners that, conditionally on moving, *do not* change counties (60 percent). Dieleman, Clark and Deurloo (2000) estimate an overall housing turnover rate of eight percent annually (see also Caplin and Leahy, 2008), which is consistent with our quarterly value of $\pi_n + (1 - \pi_n)\theta \simeq 0.02$. The value of exit is set equal to that of being a perpetual renter in steady state: $Z = \bar{u}^R/(1 - \beta)$.

Labor productivity in the construction sector is given by ϕ. The average ratio of permits issued in the U.S. each quarter to the numbers of employees in residential construction is approximately 0.1. If the average work-week is 35 hours (roughly 400 hours per quarter), then the number of permits produced per hour worked is approximately 0.00025 (amounting to 4,000 man-hours per house).

The steady-state unit price of land $\bar{q}$ is set so that the relative share of land in the price of housing is 30 percent (see Davis and Palumbo, 2008, and Saiz, 2010). The average price of a house is approximately 3.2 times annual income or 12.8 times quarterly income. This implies a ratio of the land price to income of 3.84. A related parameter is the price elasticity of land supply. Saiz (2010) studies the relationship between house prices and the stock of housing based on a long difference estimation between 1970 and 2000 for 95 U.S. cities.[21] By instrumenting using new measures of regulatory restrictions and geographical constraints, he is able to infer city level price elasticities that vary due to natural and man-made land constraints. His supply elasticity estimates vary from 0.60 to 5.45 with a population–weighted average of 1.75 (2.5 unweighted). In our steady-state, the price elasticity of land suppy is given by

$$\xi = \frac{\mu\tau}{\mu + \Lambda(\bar{q})}. \tag{40}$$

Given (39) and our value for $\bar{q}$, τ is chosen so that $\xi = 1.75$.

[21] As such we view his estimates as picking up "long-term" dynamics associated with ξ,

We choose the remaining parameters to match jointly six additional steady-state targets. In particular, we match the average fraction of U.S. households that rent (32 percent). In our steady-state this is $(b+f)/(n+b+f)$. We also set rent to 13.7 percent of median income. The income of the average renter in the U.S. is less than half of that of the average owner, reflecting the fact that owners and renters differ systematically. On average, renters in the U.S. allocate 24 percent of their after-tax income to rent (Davis and Ortalo-Magne, 2011). Since in our model households are homogeneous, we target the ratio of rent to the median income of owners and renters (see Head and Lloyd-Ellis, 2012 for details).

Harding et al. (2007) estimate the gross rate of depreciation for a median age house in the U.S. to be about 3 percent annually.[22] Given a depreciation rate, d, and an exactly offsetting cost of maintenance, m, under a simple optimal maintenance program, the implicit steady-state flow utility derived from owning a house is given by[23]

$$z^H = \left(1 + \frac{1-\beta}{\beta d}\right) m. \tag{41}$$

We assume that in the steady-state, it takes the same length of time, on average, to either buy or sell a house, *i.e.* $\omega^* = 1$. Conditional on the other parameters of the model, κ then determines the steady-state vacancy rate. Vacancy rates by MSA are available from the Census Bureau's Housing Vacancy Survey (HVS). In our model, all houses that are vacant are designated for sale. The HVS includes the category "vacant units which are for sale only". In 2000, for example, this category constituted 1 percent of the overall housing stock. Since owner-occupied homes constituted approximately two-thirds of the housing stock, this corresponds to a home-owner vacancy rate of about 1.5 percent.[24] Housing units in the HVS category "vacant units for rent" contains units offered for rent only and units offered *both* for rent and sale. In 2000, these houses constituted a further 2.6 percent of the overall housing stock. In our model, vacant units are technically available for rent in the subsequent period, so it makes sense to include some of the vacant units offered for both rent and sale in our

[22] The resulting actual depreciation rate is less than 1 percent precisely because maintenance is undertaken.

[23] Suppose $z_t^H = z^H(a_t)$ where a denotes house quality. The optimal maintenance program of a home–owner can be written as

$$\begin{aligned} V(a_t) &= \max_{\{m_t, a_t\}} z^H(a_t) - m_t + \beta V(a_{t+1}) \\ \text{s.t. } a_{t+1} &= (1-d)a_t + m_t. \end{aligned}$$

If $z^H(a_t)$ is approximately linear, then the steady-state solution to this program implies (41).

[24] This number is close to the average over the period 1980-2008. More recently, homeowner vacancy rates have exceeded 2.5 percent.

measure of vacancies. For this reason, we assume an additional one percent of the housing stock is vacant and for sale.

Given the values of μ, θ, and π_n from Table 4 and our targets for the overall fraction of renters and vacancy rate, the implied steady-state probability of sale each period is $\gamma^* = 0.76$. This implies an average time for a house to be on the market of just under 4 months.[25] Finally, ζ is chosen so that the price of a house is 3.2 times annual income or 12.8 times quarterly income. Note that the value of χ required to hit the targets implies that most of the surplus from housing transactions goes to the seller.

5.2 The earnings process

Parameterization of the process for general income, y_t, is complicated by the mismatch between the frequency of available city-level income data and the period length assumed in our calibrated model. The income data is available annually, whereas the baseline calibration assumes that each period is a quarter. While the model period length could be increased to one year, this would require, counter-factually that houses for sale remain vacant for at least one year.

Instead, we derive a quarterly process for income that shares key properties at annual frequencies with the process estimated in our panel VAR in Section 2. Specifically, we let the quarterly income process be

$$\ln y_t = a \ln y_{t-1} + b \ln y_{t-4} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon), \tag{42}$$

with $a = 1.0375$, $b = -0.05$ and $\sigma_\varepsilon = 0.011$. The implied annual income process matches the volatility of income growth, σ_y, its first-order autocorrelation, $\rho(y_t, y_{t-1})$, and the sum of the second-, third- and fourth-order autocorrelation coefficients. Table 5 reports these moments for both the estimated and constructed processes.

Table 5: Implied moments for annual income growth process

		$\rho(y_t, y_{t-i})$			
Data	σ_y	$i = 1$	$i = 2$	$i = 3$	$i = 4$
Actual	0.02	0.26	0.02	-0.05	-0.07
Artificial	0.02	0.26	-0.03	-0.04	-0.04

[25] Estimates of the times required to buy and sell vary. Anglin and Arnott (1999) report estimates of up to 4 months. Piazzesi and Schneider (2009) suggest using 6 months. Diaz and Jerez (2012) use 2 months based on a report from the National Association of Realtors. The NAR estimate of "time on the market" is imprecise because houses are sometimes strategically de-listed and quickly re-listed in order to reset the "days on market" field in the MLS listing (see Levitt and Syverson, 2008).

Using the constructed process for earnings in the linearized model, we generate sample paths for the variables of interest and use them to construct "annual" time series.[26]

5.3 Three elasticities

The dynamics of the model depend crucially on three elasticities:

• The elasticity of the distribution of alternative values, $G(\cdot)$, in a neighborhood of the steady-state cut-off, ε^c. This is given by

$$\alpha = \frac{\varepsilon^c G'(\varepsilon^c)}{G(\varepsilon^c)}, \tag{43}$$

and determines the responsiveness of entry to changes in local income and the value of search. In the steady-state, $G(\cdot)$ determines only the measure of searching households *per capita,* b^*. As this cannot be observed, it cannot be used to calibrate α.

• The elasticity of the matching function with respect to the measure of buyers, δ, which determines in part the dynamic relationship between sales and prices. With tightness set to $\omega^* = 1$, δ does not affect the steady-state.

• The wage elasticity of construction labor supply, η, which determines the responsiveness of construction to the value of housing. In a model with no frictions, this elasticity equals that of new housing construction to the sales price. With search frictions the value of a newly constructed house depends on the endogenous absorption rate as well as the transaction price.

In the absence of direct observations, we jointly calibrate α, δ and η so that the relative standard deviations of population growth, construction and growth in the sales of existing houses (σ_n/σ_y, σ_h/σ_y and σ_s/σ_y, respectively) implied by the model exactly match their counterparts in the data (see Table 2).[27]

Note that it is not necessarily possible *a priori* to match all three moments exactly. In a model with no frictions, for example, $\sigma_n = \sigma_h$ by construction and there is no parameter δ. Also, the calibration of these parameters depends on the pricing protocol in the housing market. For this reason any calibration or estimation must be specific to the model posited. That is, independent estimates in the literature inferred from frictionless models

[26] We obtained essentially identical results using an ARMA(1,4) process in which the shock's direct effect is divided over four quarters.

[27] When computing sales of existing houses in our model, we assume that new and existing houses are equally likely to be rented each period.

are inappropriate. Finally, our calibration neither imposes any particular process for house prices, nor determines directly the autocorrelations or comovements of population growth, construction and sales growth. Again, we consider the sensitivity of our results to variations in the calibrated parameters in Section 6.

Table 6: Baseline Calibration Parameters: Non Steady-State

Parameter	Value	Target
α	7.30	Relative volatility of population growth = 0.17
δ	0.86	Relative volatility of sales growth = 1.32
η	1.05	Relative volatility of construction rate = 0.11

5.4 An economy with no search

A version of our economy without search is described in Appendix B. This economy is comparable in many ways to that of Glaeser *et. al.* (2011), although there are some important differences.[28] In this economy an agent who is not a permanent renter realizes the utility gain from home-ownership, z^H, from living in *any* house.

The parameters of this economy are set as in Table 4 except for ψ, m, z^H and ϕ. These are adjusted so that the steady-state again matches the relevant targets. With no search the stock of housing must equal the population. Thus, it is not possible to match the volatilities of both population growth and construction. In this case, we maintain the value of α and set $\eta = 2$, so that the implied relative volatility of population growth, σ_n/σ_y, matches that in the data.[29]

5.5 Steady-state implications

We now consider briefly implications of search frictions for market tightness, house prices and welfare in the steady-state. In particular, consider varying the "productivity" of the matching function, κ, and measure welfare by the steady-state value of entry to the city, $\bar{W}^*$. This measure gives equal weight to the welfare of prospective owners and renters and

[28] In Glaeser et al. (2011) the alternative to living in the city yields a homogeneous payoff so that the elasticity of entry is effectively infinite. In response to a shock, this implies immediate entry of buyers until the price of housing adjusts to keep the value of entering constant.

[29] Fixing η and adjusting α yield similar results.

takes into account the likelihood of buying, selling and exiting the city in the future. Figure 2 depicts house prices and welfare against market tightness as κ varies from 0 and 1.

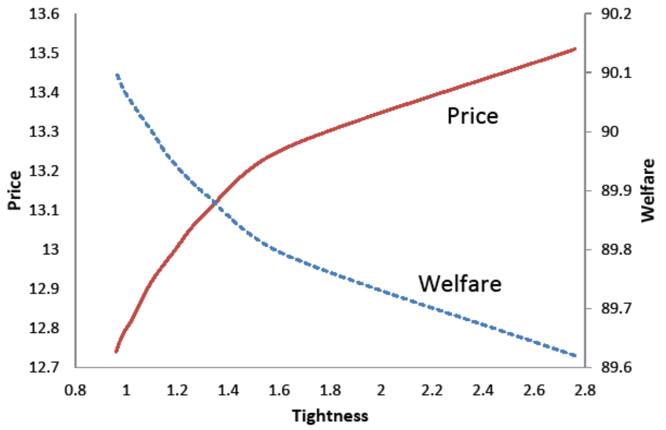

Figure 2: Varying the productivity of matching in steady state

As κ is increased, tightness falls and welfare rises. As they sell faster for a given buyer-seller ratio, $\gamma(\cdot)$, the value of unoccupied homes to their owners rises and the unit value required by developers falls. Both of these factors induce market tightness to decline as the supply of new housing increases relative to demand. The impact on the value of vacant houses and hence on transaction prices is, in general, ambiguous and depends on the relative elasticities of entry and housing supply. In our calibration, steady-state housing prices rise with market tightness. As such, as tightness falls it becomes easier to find a house and the price falls, raising the welfare of new entrants.

6 Equilibrium Dynamics

6.1 Qualitative implications of a shock to local income

We now consider several qualitative implications of the model, noting at the outset that the model's dynamics are not driven by the autocorrelation of income growth inherent in the calibrated income process. A sufficiently persistent AR(1) process for income generates essentially identical dynamics to those reported here.

The implied impulse responses for income, the house price, sales growth (existing houses), construction, and population growth to a local income shock are depicted in Figure 3 for the economies with and without search.[30] In both economies, the shock induces entry and raises population growth. Although the responses of city population growth in the two cases are qualitatively similar, entry is initially much more rapid in the search economy.

In contrast, the responses of house prices, growth in sales of existing houses and the construction rate differ qualitatively across the two economies. The search model exhibits serial correlation in the both price growth and construction, qualitatively in line with the dynamics of the empirical model illustrated in Figure 1. Moreover, as in the empirical model, sales growth spikes quickly and then declines sharply before rising again. The no-search economy generates serial correlation in neither price growth nor construction rates, despite generating substantial serial correlation in the housing stock. Also, without search sales growth rises and then returns monotonically to trend.

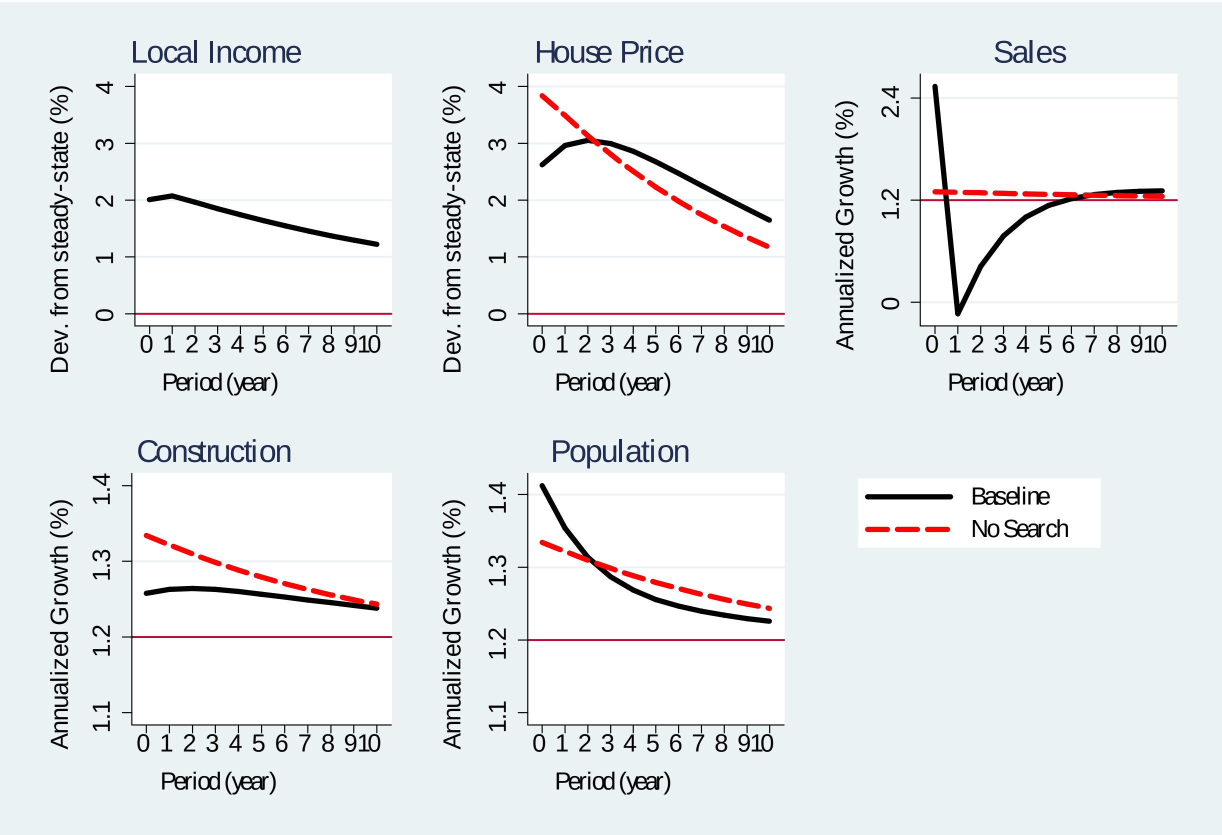

Figure 3: Responses to an income shock with and without search

The force generating serial correlation in both house price growth and the construction

[30] Figure 3 depicts annualized responses to facilitate comparison with Figure 1.

rate and the movements in sales growth in the search economy is changes to the *illiquidity* of housing. To see this, consider Figure 4, which depicts market tightness, ω, together with both its numerator and denominator, buyers and vacancies, respectively. Initially, an increase in the value of living in the city (due here to the income shock) generates an immediate increase in search activity as households enter and some begin searching for houses. Ignoring, for now, any response of the measure of vacant houses for sale, the ratio of buyers to sellers (*i.e.* tightness) increases, reducing the rate at which buyers find homes through the matching process. Because newly-entering buyers are not all immediately matched with sellers, and because entry is persistent (owing to the persistence of the shock), unmatched buyers "build up" in the market over time, generating future increases in both tightness and the rate at which houses sell.

The price of a house reflects in part its future resale value (as home-owners expect to sell the house eventually). Continuing increases in tightness, by lowering future time-to-sell, thus increase the future value of a vacant house, causing the transaction price to grow persistently. That is, as houses become more liquid over time, their value and thus their sales price increases as well.

Both the overall supply of housing and the allocation of houses between the rental market and vacancies for sale respond to the income shock in ways affected by movements in the liquidity of housing. New entrants to the city require housing immediately. This increase in rental demand induces the shifting of vacant houses into the rental market because the stock of housing units cannot respond instantaneously. Irrespective of any change in rent, owners of vacant houses are compensated, to some extent, for supplying their units to the rental market by the return on houses associated with both expected future increases in house sale prices and lower future time on the market. An increased supply of rental housing keeps the rental rate from rising and reinforces the continued entry of buyers which drives the subsequent price growth.

The increased value of vacant houses induces developers to build and thus increases the housing stock. Serial correlation in the construction rate is thus generated through the same mechanism as that in house prices; persistent growth of market tightness and thus reductions in the time required for a resident owner to sell a house. Eventually, as *per capita* income reverts to its steady-state, entry slows and the population growth rate returns to trend. Increased construction lowers market tightness and causes both the value of a vacant house and the transaction price to return to their steady-state values.

In the economy without search, income shocks generate very different dynamics. In-

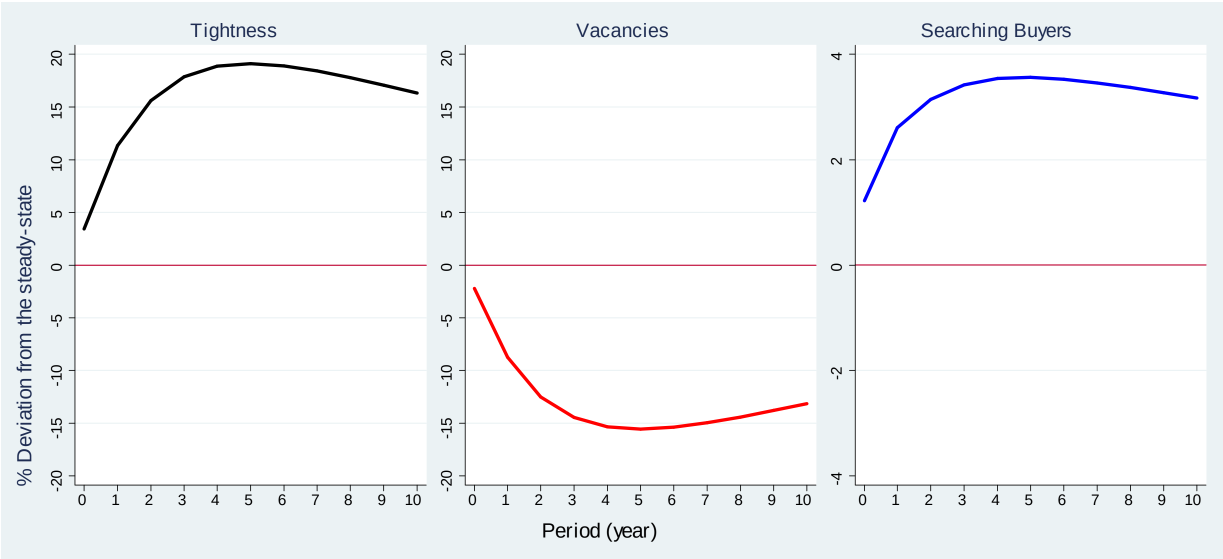

Figure 4: Responses to an income shock: Matching

creased entry leads immediately to a higher house price and increases in both construction and house sales, with all of these variables tracking the population growth rate closely. In addition, rent (which is paid only by permanent renters) behaves very differently. Since there are no vacant houses to be shifted into the rental market in the short-run, new entrant renters bid up the rent immediately as this is necessary to induce developers to build new rental housing (see Figure 5). In contrast, in the search economy rent falls initially because the anticipated growth in prices temporarily induces houses that were previously vacant-for-sale to be supplied to the rental market. Eventually, however, growing demand for both rental and owner occupied housing induces rent to rise.

6.2 Quantitative implications

The quantitative implications of income shocks are assessed by considering a series of moments. These, along with the corresponding moments for the U.S. economy, are presented in Tables 7 and 8.

Table 7 contains the standard deviations of the growth rates of house prices, sales, the housing stock, and population relative to that of local *per capita* income, the correlations of these variables with local income growth, and the correlation of sales and price growth. The first column reports the corresponding moments from our empirical analysis in Section 2. The second column reports moments for the search model with the baseline calibration, and the third reports moments for the economy with no search. Table 8 contains the first four autocorrelation coefficients for the same variables in both the data and the search model. The model with no search generates no endogenous persistence for any of these variables.

Table 7: Volatilities and co-movements

Moment	US Cities	Baseline	No search
σ_p/σ_y	1.60	1.45	1.82
σ_s/σ_y	1.32	1.32*	0.14
σ_h/σ_y	0.11	0.11*	0.17
σ_n/σ_y	0.17	0.17*	0.17*
$\rho(p,y)$	0.76	0.98	0.98
$\rho(s,y)$	0.56	0.47	0.13
$\rho(h,y)$	0.54	0.51	0.37
$\rho(n,y)$	0.76	0.58	0.37
$\rho(s,p)$	0.02	0.23	0.00

Note: A * indicates a calibrated target.

Table 8: Autocorrelations (106 cities, 1981-2008)

$\rho(x_t, x_{t-i})$	$i=1$	$i=2$	$i=3$	$i=4$	$i=1$	$i=2$	$i=3$	$i=4$
	Price Growth				Sales Growth			
US Cities	0.75	0.37	0.05	-0.15	0.59	0.39	0.35	0.35
Baseline	0.36	0.09	0.03	-0.01	0.12	-0.13	-0.09	-0.06
	Population Growth				Construction Rate			
US Cities	0.67	0.40	0.19	0.09	0.88	0.61	0.33	0.11
Baseline	0.83	0.65	0.52	0.43	0.97	0.91	0.85	0.79

The search economy generates a standard deviation of price growth that is more than 80 percent of that observed in the data. In contrast, the volatility generated by the model with no search is much higher. The search model is also able to account for almost half of the first-order autocorrelation of price growth, and much of that in sales growth. The model with search also does relatively well in terms of the rankings of volatility, correlation with income growth and serial correlation for the four variables. That is, it is consistent with the observation that price growth is the most volatile and most correlated with income growth, followed by population growth and then by construction and sales growth, whereas for persistence they are ranked in the opposite order. It does, however, substantially understate the correlations of sale growth, construction and population growth with income

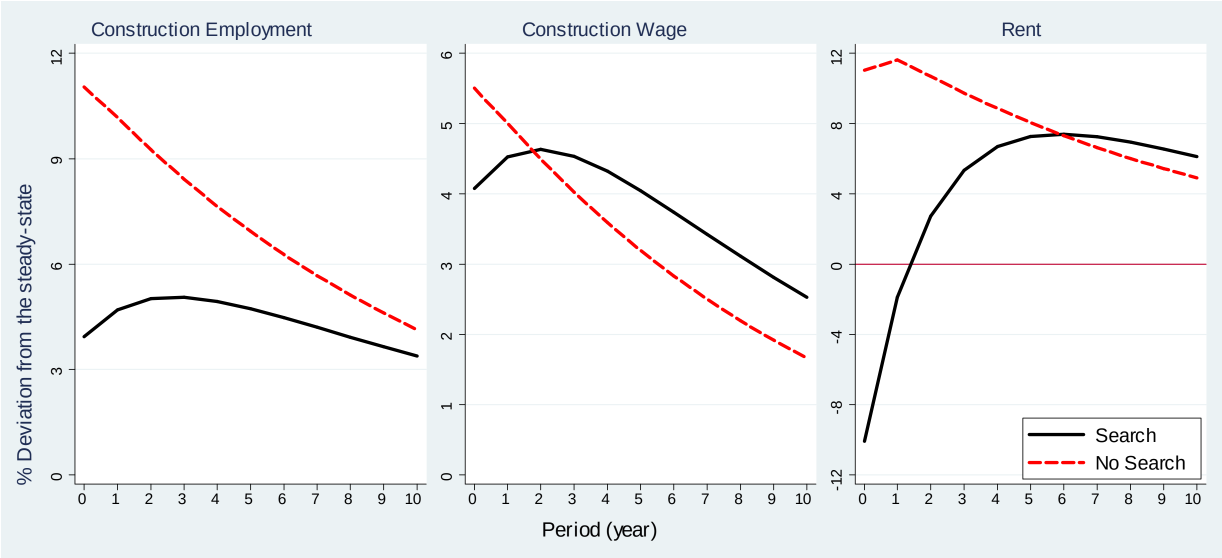

Figure 5: Construction employment, wages, and rent with and without search

growth and overstate the persistence of both population growth and construction. Finally, while the models with and without search both predict a low correlation between sales and price growth and the search model predicts sales growth that is rather less persistent than in the data.

6.3 The dynamics of rent and construction labor

The variables studied in our empirical analysis was limited largely because of data availability. The model, of course, makes predictions for several variables for which we have more limited data (fewer cities and/or a shorter time span). Here we compare the predictions of the model for city-level wages and employment in the construction sector and for rents with their counterparts in what data we have. Impulse responses for these three variables in the models with and without search are depicted in Figure 5.

6.3.1 Construction Wages and Employment

Construction labour data is available on a consistent basis only for some of the cities in our sample. For this reason drop the cities for which these are not available and re-estimate the panel VAR with the construction variables. The implications for co-movements among the other five variables remain largely unchanged, so here we focus here only on the construction variables. The first two panels of Figure 6 depict the impulse responses of construction

Figure 6: Impulse responses for construction employment, wages, and rent

employment and wages resulting from a shock to income, with the associated confidence intervals. Interestingly and in accordance with the theory, construction wages follow a very similar pattern to house prices in Figure 1 while construction labour behaves very much like the construction rate.

Table 9: Construction wages and employment (98 cities, 1981-2008)

	Construction Wage					
			$\rho(w_t, w_{t-i})$			
Moment	σ_w/σ_y	$\rho(w, y)$	$i=1$	$i=2$	$i=3$	$i=4$
US Cities	0.58	0.96	0.41	0.04	-0.19	-0.23
Baseline	0.50	0.99	0.29	0.03	-0.03	-0.03
	Construction Employment					
			$\rho(l_t, l_{t-i})$			
Moment	σ_l/σ_y	$\rho(l, y)$	$i=1$	$i=2$	$i=3$	$i=4$
US Cities	1.41	0.79	0.60	0.18	-0.15	-0.34
Baseline	0.58	0.97	0.39	0.13	0.04	-0.01

Table 9 contains moments for growth of both the construction wage and employment in the data and the model. Clearly, the model overstates the volatility of employment. Both in the data and the model, however, the volatility of employment exceeds that of wages.

This is consistent with the fact that the labour supply elasticity exceeds unity. The model understates the persistence in both variables, but captures their high correlation with income.

6.3.2 Rents

Panel data for MSA averages of (quality-controlled) rents over a reasonable time period appear to be unavailable. There is, however, data on "fair market rents" by MSA which is available on an annual basis going back to 1985 for the 106 cities in our sample. Here we use the adjusted data constructed by van Nieuwerbugh and Weill (2010) (see Appendix A). We re-estimate the panel VAR over the shorter time period with the inclusion of rents. One issue that must be dealt with is the fact that rents are commonly set for a year and may be difficult to adjust immediately in response to shocks. If we were to order rents before income in the VAR, however, this would effectively "force" there to be no initial response in rents to the income shock. Instead, we include rents at time $t+1$ ordered after income in the panel VAR and document the implications. Once again the results for the other variables are robust to these changes, so we discuss only the response of rent.

The right-most panel of Figure 6 illustrates the impulse response for rent together with the implied confidence interval. As predicted by the model, rents initially decline following the shock and then subsequently rise. The initial decline in rents in the data is, however, much smaller than that predicted by the model and is not statistically significant. Rents subsequently rise quite slowly for four years following the shock before mean-reverting, but again the confidence interval is very wide and includes zero.

Quantitatively the predictions of the model for rent do not accore well with the data. Although the persistence documented in Table 10 is similar, both the volatility of rent growth and its correlation with income growth are an order of magnitude lower in the data than in the model. Of course, the correspondence is even worse for the model without search: volatility is much higher and the correlation with income is perfect. Moreover, the persistence is much lower.

Table 10: Growth in Fair Market Rents (106 cities, 1985-2008)

			$\rho(r_t, r_{t-i})$			
Moment	σ_r/σ_y	$\rho(r, y)$	$i=1$	$i=2$	$i=3$	$i=4$
US Cities	0.09	0.01	0.75	0.28	-0.15	-0.39
Baseline	5.51	0.73	0.71	0.38	0.18	0.07

There are several potential reasons for the poor match between the behavior of rent in the the model and the data. Fair market rent may not provide an accurate measure of actual rents in a given city. Moreover, the model assumes that all rents are re-set every quarter, whereas actual rents are almost certainly adjusted less frequently — say annually or when a new lease is signed. In this sense the average rent (measured in the data) probably moves much more sluggishly than the marginal rent paid by a tenant, which is measured in the model.

The insensitivity of rents to city-level income movements in the short run *is* consistent with the empirical findings of Saiz (2007). Using an instrumental variables approach, he finds that MSA level incomes have no significant impact on rents, whereas their impact on house prices is much larger and significant. In the longer run he finds a stronger relationship between rents and incomes suggesting that rents adjust very slowly in response to income shocks.

6.3.3 Demographic shocks

It is straightforward to derive the implications of direct shocks to city populations (*i.e.* movements that are not driven by income) from our panel VAR. The interpretation of such a shock, however, depends on exactly how it is modeled. For example, we could think of shocks to the distribution, $G(\cdot)$, as driving entry. Alternatively, we could think of an unobserved shock to the utility associated with a particular city that induces entry endogenously (e.g. amenities). For the sake of brevity, it is useful to observe that the impulse responses to population growth shocks in our panel VAR are qualitatively similar to those generated by income shocks. If we were to introduce additive utility shocks to the model with features similar to the current income process, we could generate reasonably similar impulse responses. We have focussed on the role of income shocks here precisely because in the absence of direct observations on amenties (and the utility they generate) we have no way to discipline such an exercise.

6.4 Alternative calibrations

We now depart from the baseline calibration to examine the sensitivity of our results to changes in the values of several parameters. Table 11 reports the implications of alternative choices of the specified parameters for the relative volatility and the first-order autocorrelation of price growth. In each case the remaining parameters are adjusted so as to continue to hit the targets listed in Table 4.

Perhaps not surprisingly, increasing either the elasticity of new construction supply or the elasticity of land supply results in a decrease in the volatility of price growth and an increase in its serial correlation. We have chosen the alternative values of these parameters to be at the extremes of the range of typical estimates. As may be seen within this range, these moments are much more sensitive to new construction supply elasticity than land supply elasticity.[31] Indeed for low values of η we obtain price volatilities that exceed those observed in the data. This, however, comes at the expense of a reduction in serial correlation, although it remains positive.

A trade-off between volatility and serial correlation may be seen for all the parameter changes considered in Table 11. Directly increasing the elasticity of entry, for example, implies a greater responsiveness of new entrants to current market conditions in the city and correspondingly less of a lag in entry. Consequently, price growth becomes more volatile and less serially correlated. Of course, when we adjust this parameter we no longer match the relative volatility of population growth.

The impact of raising the elasticity of the matching function with respect to buyers, δ (or, equivalently, reducing its elasticity with respect to vacancies), depends on whether the increase in tightness is more the result of the rise in the measure of buyers or the fall in the measure of vacancies. In our baseline calibration the increase in tightness is due proportionately more to the decrease in vacancies, so that price volatility falls with δ.

Table 11: Volatility and Persistence of Price growth: Sensitivity Results

Moment	Baseline Calibration		New housing supply elasticity (η)		Entry elasticity (α)		Matching Elasticity (δ)	
			.10	10	3.6	15	.50	.95
σ_p/σ_y	1.45		2.77	0.25	0.55	2.38	1.75	1.39
ρ_1^p	0.36		0.28	0.61	0.74	0.27	0.31	0.37
	Land supply elasticity (ξ)		Vacancy Rate (v)		Homeowner exit probability (π_n)		Housing Utility (z^H)	
	0.01	100	.01	.03	.004	.012	.01	.04
σ_p/σ_y	1.79	1.45	1.80	1.20	1.50	1.35	0.86	1.58
ρ_1^p	0.32	0.36	0.31	0.40	0.30	0.44	0.70	0.32

Targeting a higher steady-state vacancy rate of 3 percent implies a less effective matching process in the housing market. As a result, it takes longer for households to find houses and

[31] This is true even though the range of labour supply elasticities considered is proportionately larger than that of land supply elasticities.

market tightness grows to a higher level as buyers exit the pool of searchers more slowly. This tends to generate more serial correlation and less volatility in price growth. When homeowners are more likely to exit the city (*i.e.* when π_n increases), they put more weight on future market conditions. If these conditions are expected to improve due to gradual entry, the persistence of current price growth increases in anticipation.

When we divorce housing utility, z^H, from the maintenance cost, (41) no longer holds, and decreasing z^H raises the serial correlation of price growth and lowers its volatility. The reason for this is that, in order to maintain the targets for the steady-state house price and vacancy rate, a reduction in z^H necessitates a reduction in the buyers' share of the surplus in house transactions. The house price thus becomes increasingly sensitive to future market conditions and the slow increase in the likelihood of sale translates into more gradual price growth.

7 Robustness

Here we summarize four alternative environments to assess further the robustness of our findings (see the on-line appendix for details). Overall, we find our results to be robust in the sense that these alternatives behave similar qualitatively to our basic model. Quantitatively, our baseline results with respect to both volatility and co-movements are also very robust. With regard to the degree of serial correlation in house price growth, however, they are to some extent sensitive to changes in the model of price determination in housing transactions.

7.1 Exiting Buyers

In our baseline model we assume that unsuccessful potential buyers do not exit the city. This assumption reflects our view that this rate is likely to be very low. Since we have no data on the rate at which searchers re-locate, however, we cannot verify this assertion. Instead we check the robustness of our results by extending the model to allow for exiting buyers and assume that unsuccessful buyers exit at the same rate, π_n, as homeowners. Once we re-calibrate the model's parameters to match the same targets as before, our results are almost identical to those of our baseline calibration.

7.2 Mismatched owners remain in their houses

We have assumed that mismatched owners put their houses up for sale immediately and become renters. Mismatched owners are, however, indifferent between doing this and remaining in their current house while searching and putting their vacant house up for sale only once they have purchased a new one. If we model them as choosing the latter alternative, the only difference that arises is an additional state variable: the measure of mismatched owners remaining in their homes. In particular mismatched owners values are the same in both cases.

Proceeding in this way, if the parameters of the model were kept at their baseline values, a somewhat tighter housing market would result. In this case, however, the implied steady-state fraction of renters would be too low to match the calibration target. Recalibrating, (by lowering ψ) market tightness (and everything else) remains much as in our baseline case.

7.3 Endogenous exit

In the baseline model, the exit rate of home-owners is exogenous. Suppose with probablity π_n a home-owner receives an opportunity to exit the city, the value of which is a random draw from a distribution of continuation values. Only those who receive draws that exceed their value of staying, J_t, will choose to exit. As an example let these continuation values be uniformly distributed on an interval $[0, \bar{Z}]$. To match the mobility targets, $\bar{Z}$ must be chosen so that the expected exit value is the same as in the baseline calibration. Because, out of the steady-state J_t varies relative to $\bar{Z}$,the relevant elasticities must also be re-calibrated to match the volatilities of population, construction and sales growth.

In this case, the resulting dynamics remain qualitatively unchanged. Quantitatively, price growth volatility is slightly higher and its serial correlation slightly lower than in baseline case. These effects are very small and stem from the fact that following a positive income shock, the exit rate initially declines slightly, and then gradually returns to its steady-state.

7.4 Competitive search

To investigate the importance of particular assumptions regarding the determination of prices within matches in the housing market, we also consider a version in which house sellers post prices and search is competitive in the sense of Moen (1997). The search process we have in mind is similar to that considered by Diaz and Jerez (2012). For brevity, we describe the changes required to the environment in the online appendix. Here, we discuss only the implications of modeling search in this way for our overall results.

With competitive search the respective shares of the trade surplus accruing to buyers and sellers in a transaction depend on the tightness of the market. In the case of a Cobb-Douglas matching function, this amounts to imposing the restriction that $\delta = \chi$. In our calibration scheme, this imples that δ is reduced to 0.1 and it is no longer possible to match the volatility of sales growth. Moreover, we are unable to to find a combination of α and η such that we match both the relative volatilities of construction and population growth. We therefore set $\eta = 1.05$ as in our baseline calibration and choose $\alpha = 5.6$ to match σ_h/σ_y. Calibrated in this way, the competitive search model generates qualitatively similar price growth dynamics to the baseline case. Quantitatively, however, house price growth is more volatile and less serially autocorrelated.

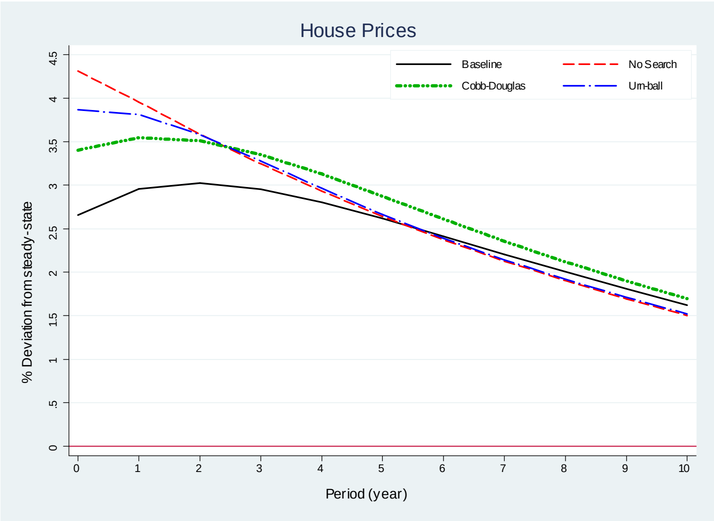

Figure 7: Price growth under competitive search with alternative matching functions

In contrast to our baseline random search model, the nature of the matching function now matters for dynamics because the bargaining weight depends on market tightness. To explore this issue, we consider an alternative matching function for which the equilibrium shares of the surplus received by the buyers and sellers are not constant. Specifically, consider

$$\mathcal{M}(B,S) = S\upsilon(1 - e^{-\varrho\frac{B}{S}}). \tag{44}$$

If $\varrho = 1$, the matching probabilities are equivalent to the "urn-ball" matching process assumed by Diaz and Jerez (2012). Here we consider a somewhat more general form in order to calibrate the model to the same targets as for our baseline calibration above. This generalization could be motivated along the lines of Albrecht, Gauthier, and Vroman (2003), where ϱ denotes the average number of applications to purchase per period and υ indexes the effort required to process each application. Given the other parameters of our baseline calibration, the matching function parameter values needed to achieve the baseline targets are $\upsilon = 0.78$ and $\varrho = 3.73$.

Figure 7 illustrates the effect of a shock to city income on house prices for the competitive search economy with matching functions that are Cobb-Douglas and specified as in (44) ("Urn-ball") as well as for both the baseline and no-search economies of Section 6. Clearly, the nature of price determination affects the serial correlation of price growth to an extent which depends on the matching function. This effect depends on the initial response of prices to an income shock and the resulting extent of entry. The more the share of the surplus received by the buyer falls as tightness rises, the greater the initial price increase. The greater the initial price increase, the less entry and as a result, the smaller the response of tightness. Since tightness moves slowly, a smaller response leads to less autocorrelation of house price growth.

8 Concluding Remarks

This paper makes two main contributions. First, it provides a parsimonious characterization of the impact of relative income shocks across U.S. cities on the short-run dynamics of average house prices, sales of existing homes, construction and population growth. Second, it presents a model that helps to understand these joint dynamics. To do this, time-consuming search and matching are introduced into a dynamic model of housing markets with endogenous entry and construction. Three key features of the model are (1) that it takes time for potential buyers to match with a house they want, and the length of this time depends on market conditions; (2) that home buyers foresee that they will eventually sell; and (3) that unoccupied housing can be rented temporarily to new entrants who are searching for a home to own.

Equilibrium dynamics depend on three key elasticities which are indeterminate in the steady-state and so are calibrated to match the volatilities of population growth, construction, and sales growth relative to income growth. The calibrated model captures well qualitatively the observed dynamics. In particular, the model generates serial correlation in price growth, construction, and sales of existing houses. Quantitatively, the model accounts for more than 80 percent of the variance of house prices associated with local income shocks, and nearly half of the first-order autocorrelation of price growth. Moreover, it improves both qualitatively and quantitatively upon the implications of an economy without search on several dimensions.

Appendix A: The Data

This appendix provides details on data sources, definitions and calculations. Our unit of observation is a metropolitan statistical area (MSA). We use the 2006 MSA definitions. Our sample consists of 106 MSAs from 1980 to 2008.

Local incomes: We define local incomes as the total income from all sources. Our MSA level data are from the Regional Economic Accounts compiled by the Bureau of Economic Analysis (BEA, Table CA01).

House prices: Following van Niewenburgh and Weil (2010), we form a time series of home prices for each city by combining level information from the 2000 Census with time series information from the Federal Housing Finance Agency (FHFA). From the 2000 Census, we use nominal home values for the median single-family home. From the FHFA we use the Home Price Index (HPI) from 1980 to 2008. The HPI is a repeat-sales index for single family properties purchased or refinanced with a mortgage below the conforming loan limit. In contrast to Van Niewenburgh and Weil (2010), we combine prices for MSA divisions into those for MSAs by using population–weighted averages of the division level prices. We need to do this because the housing stock data (described below) can only be constructed using permits at the MSA level.

Sales of existing houses: We obtained quarterly estimates of the sales of existing houses for each city from Moody's Analytics (www.economy.com). Annual sales were computed as the sum of quarterly sales over the year.

Populations: City populations are taken from the BEAs Regional Economic Accounts (Table C02). Throughout we assume that city populations are proportional to the number of households. Although there has been a general decline in people per household in the U.S., this is an economy–wide trend that is removed after controlling for time–fixed effects.

Housing Stocks: We form a time series for housing stocks for each city by combining information from the 2000 Census with times series information from the U.S. Department of Housing and Urban Development (HUD). From the 2000 Census, we use the estimated number of housing units. This data was only available at the county level, so we summed across the counties within the relevant MSAs. From HUD we used annual permits issued for each city from 1980 to 2008. According to the U.S. Census Bureau, approximately

97.5 percent of permits issued each year translate into housing starts, 96 percent of which are completed. We therefore constructed housing stocks H_t according to $H_{t+1} = H_t + 0.936 \times \text{Permits}_t$.

Construction Employment: Construction employment is taken from the BEAs Regional Economic Accounts (Table C25).

Construction Wages: These are computed as construction earnings divided by construction employment. The earnings data is taken from the BEAs Regional Economic Accounts (Table C06).

Rents: Rent data is taken from the data set constructed by van Niewerburgh and Weill (2010) using the Fair Market Rents database (FMR), published annually by HUD. The FMR reports the 40th, 45th or 50th percentile of the gross rent distribution of all units occupied households who moved to their present residence within the past 15 months. Van Niewerburgh and Weill (2010) aggregate to the MSA level using population weighted averages and also adjust for the fact that the reported rent percentile changes over time (see their appendix D.3 for details).

Appendix B: Extended Calculations and Proofs

1. The household's optimization problem: This can be expressed as

$$\max_{c_t, l_t} E_t \sum_{t=0}^{\infty} \beta^t U_t(c_t, l_t, z_t) \qquad \text{s.t.} \quad E_t \sum_{t=0}^{\infty} \beta^t c_t \leq E_t \sum_{t=0}^{\infty} \beta^t \left[y_t + w_t l_t - \Omega_t \right] \tag{A1}$$

where Ω_t denotes the net value of all housing transactions. It follows that the dynamic optimization problem is equivalent to

$$\max_{l_t} E_t \sum_{t=0}^{\infty} \beta^t \left[y_t + w_t l_t - v(l_t) + z_t - \Omega_t \right] \tag{A2}$$

The solution to the (static) construction labour supply problem yields (12).

2. The deterministic steady-state: In a steady-state, equations (30), (31), (32) and

(33) respectively imply

$$
\begin{aligned}
f^* &= \frac{(1-\psi)\mu G(\bar{W}^*)}{\mu+\pi_f} && \text{(A3)}\\
b^* &= \frac{\psi\mu G(\bar{W}^*)}{\mu+\lambda(\omega^*)-\frac{\theta(1-\pi_n)\lambda(\omega^*)}{\mu+\pi_n+\theta(1-\pi_n)}} && \text{(A4)}\\
n^* &= \frac{\lambda(\omega^*)}{\mu+\pi_n+\theta(1-\pi_n)}b^* && \text{(A5)}\\
h^* &= \frac{\phi^\eta\left(n^*+b^*+f^*\right)}{\mu}\left(\beta V^*-\bar{q}\right)^\eta, && \text{(A6)}
\end{aligned}
$$

where (36) implies that $\bar{q}$ satisfies $\Lambda(\bar{q})=\mu h^*/(k-h^*)$. Note that the value of k, the ratio of potential residential land to the total population, is arbitrary. It follows that for any given distribution function $\Lambda(\cdot)$ we can choose k to obtain any target value for $\bar{q}$. Thus the steady-state value of $\bar{q}$ is effectively exogenous. In the steady-state, the values of owners, buyers and vacant houses, the house price and rent, must satisfy:

$$
\begin{aligned}
J^* &= \bar{u}^H+\pi_n\beta Z+\pi_n\beta V^* && \text{(A7)}\\
&\quad +(1-\pi_n)\theta\beta\left(W^*+V^*\right)+(1-\pi_n)(1-\theta)\beta J^* \\
W^* &= \bar{u}^R+\lambda(\omega^*)(\beta J^*-P^*)+(1-\lambda(\omega^*))\beta W^* && \text{(A8)}\\
V^* &= \gamma(\omega^*)P^*+(1-\gamma(\omega^*))\beta V^* && \text{(A9)}\\
P^* &= (1-\chi)\beta\left(J^*-W^*\right)+\chi\beta V^* && \text{(A10)}\\
r^* &= m+\gamma(\omega^*)\left(P^*-\beta V^*\right) && \text{(A11)}\\
W_f^* &= \bar{u}^R+\pi_f\beta Z+(1-\pi_f)\beta W_f^* && \text{(A12)}\\
\bar{W}^* &= \psi W^*+(1-\psi)W_f^*, && \text{(A13)}
\end{aligned}
$$

where $\bar{u}^H=\bar{y}+x(w^*)+z^H-m$ and $\bar{u}^R=\bar{y}+x(w^*)-r^*$ and $Z=\bar{u}_R/(1-\beta)$. The first five equations of this system can be solved for J^*, W^*, V^*, P^* and r^*. The last two equations can be used to determine $\bar{W}^*$ and W_f^*.

3. Proof of Proposition 1. As discussed in the text, we first prove the following three results:

R1. *In the steady-state, the surplus from a match in the owned housing market must be positive and is given by*

$$
J^*-W^*-V^*=\frac{z^H}{1-\beta(1-\pi_n)(1-\theta)+\left(\frac{1-\beta+\pi_n\beta}{1-\beta}\right)\beta\lambda(\omega^*)\chi}. \tag{A14}
$$

R2. *In the steady-state, there exists a negative "supply-side" relationship between the value of a house for sale and market tightness:*

$$V^* = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(1+\frac{\psi\left(\mu+\pi_f\right)}{A\gamma\left(\omega^*\right)+B\omega^*}\right)\right]^{\frac{1}{\eta}}+\frac{\bar{q}}{\beta}, \tag{A15}$$

where $A=\frac{\mu+\psi\pi_f+(1-\psi)\pi_n}{\mu+\pi_n+\theta(1-\pi_n)}$ and $B=\mu+\psi\pi_f$.

R3. *In a steady-state, there exists a positive "demand–side" relationship between the value of a house for sale and market tightness:*

$$V^* = \frac{\gamma\left(\omega^*\right)(1-\chi)\beta z^H}{(1-\beta)\left[1-\beta(1-\theta)(1-\pi_n)\right]+\left(1-\beta+\pi_n\beta\right)\beta\lambda\left(\omega^*\right)\chi}. \tag{A16}$$

To show R1, substituting (A10) into (A7), (A8), (A9) and (A11) yield

$$V^* = \gamma\left(\omega^*\right)(1-\chi)\beta\left(J^*-W^*-V^*\right)+\beta V^* \tag{A17}$$

$$W^* = Z+\frac{\lambda(\omega^*)\chi}{1-\beta}\beta\left(J^*-W^*-V^*\right) \tag{A18}$$

$$r^* = m+\gamma\left(\omega^*\right)(1-\chi)\beta\left(J^*-W^*-V^*\right) \tag{A19}$$

and

$$\begin{aligned} J^*-W^*-V^* = {} & \bar{u}^H+\pi_n\beta Z+\pi_n\beta V^*+(1-\pi_n)\theta\beta\left(W^*+V^*\right)+(1-\pi_n)(1-\theta)\beta J^* \\ & -\bar{u}^R-\left[\lambda(\omega^*)\chi+\gamma\left(\omega^*\right)(1-\chi)\right]\beta\left(J^*-W^*-V^*\right)-\beta W-\beta V^* \end{aligned} \tag{A20}$$

Given the definitions of u^R and u^H and (A19), we have

$$\bar{u}^H-\bar{u}^R = z^H+\gamma\left(\omega^*\right)(1-\chi)\beta\left(J^*-W^*-V^*\right). \tag{A21}$$

Then (A14) is obtained using (A18), (A20) and (A21).

For R2, use (34), (35) and (A6) to derive

$$\frac{b^*}{\omega^*}+b^*+n^*+f^* = \frac{\phi^\eta\left(n^*+b^*+f\right)}{\mu}\left(\beta V^*-\bar{q}\right)^\eta. \tag{A22}$$

It follows that (A15) can be obtained by substituting (A3), (A4) and (A5) into the above. Then, $V^S(\omega^*)$ is strictly decreasing in ω^* because $\gamma'(\omega)>0$ from Assumption 1. Note that (A15) implies that houses will always be built in steady state so long as $\mu>0$ since this ensures that $\beta V>\bar{q}$.

For R3, (A14) and (A17) yield (A16). Recall from Assumption 1 that $\lambda'(\omega) < 0$ and $\gamma'(\omega) > 0$. Thus the right-hand side of (A16) is increasing in ω^*.

Finally, note now that $V^S(\omega)$ is decreasing in ω (R2) and $V^D(\omega)$ is increasing in ω^* (R3), a SSE must be unique if it exists. Existence requires that $V^D(\omega)$ and $V^S(\omega)$ intersect at a positive value of ω. That is if $V^D(0) < V^S(0)$ and $V^D(\infty) > V^S(\infty)$. Recall from Assumption 1 that $\lambda(\infty) = 0$, $\gamma(0) = 0$, $\lambda(0) = \gamma(\infty) = 1$, $\lambda'(\omega) < 0$ and $\gamma'(\omega) > 0$. It follows that necessary and sufficient conditions on the parameters are given by (37).

4. Economy without search: The dynamic system is given by (14), (30), (36) and the following

$$\begin{aligned}
(1+\mu)n_t &= (1-\pi_n)n_{t-1} + \psi\mu G\left(\bar{W}_t\right) && \text{(A23)}\\
(1+\mu)h_{t+1} &= h_t + \zeta\phi^{1+\eta}\left(n_t + f_t\right)\left(\beta E_t P_{t+1} - q_t\right)^{\eta} && \text{(A24)}\\
J_t &= u_t^H + \beta\pi_n\left(Z + E_t P_{t+1}\right) + \beta(1-\pi_n)E_t J_{t+1} && \text{(A25)}\\
\bar{W}_t &= \psi\left(J_t - P_t\right) + (1-\psi)W_t^f && \text{(A26)}\\
r_t &= m + P_t - \beta E_t P_{t+1}. && \text{(A27)}
\end{aligned}$$

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Appendix C: Empirical Results

Panel Unit Root Tests

Table C1 reports the test statistics and P-values for four different panel unit root tests for each of the variables used in the panel VAR (other standard tests produce similar results). Recall that cross-sectional means at each date have been removed. As noted in the main text, the null hypothesis in each case is rejected at the 95% confidence level, with one exception (the Hadri test for sales growth, g^S).

Table C1: Panel Unit Root Test Statistics

	Y		P		g^S		g^H		g^N	
Breitung	-1.82	[0.03]	-12.45	[0.00]	-12.16	[0.00]	-12.52	[0.00]	-6.84	[0.00]
IPS	-4.68	[0.00]	-12.97	[0.00]	-32.88	[0.00]	-11.49	[0.00]	-13.07	[0.00]
- ave. lag	1.07		1.92		0.87		1.19		0.94	
Fisher - P	338.4	[0.00]	785.9	[0.00]	1196.7	[0.00]	445.8	[0.00]	573.21	[0.00]
- Z	-4.34	[0.00]	-16.56	[0.00]	-26.03	[0.00]	-10.30	[0.00]	-12.37	[0.00]
Hadri	80.75	[0.00]	72.95	[0.00]	-1.26	[0.90]	36.12	[0.00]	45.21	[0.00]

P-values in square brackets.

The Breitung (2000) test assumes that all panels have a common autoregressive parameter. The null hypothesis is that all series contain a unit root. The alternative hypothesis is that the series are stationary. Breitung's (2000) Monte Carlo simulations suggest that his test is substantially more powerful than other panel unit-root tests for the modest-size dataset he considered (N=20, T=30).

The Im-Pesaran-Shin (IPS) (2003) test relaxes the assumption of a common autoregressive parameter and instead allows each panel to have its own. The null hypothesis is that all panels have a unit root. The alternative hypothesis is that the fraction of panels that are stationary is nonzero. Specifically, if we let N_0 denote the number of stationary panels, then the fraction N_0/N tends to a nonzero fraction as N tends to infinity. This allows some (but not all) of the panels to possess unit roots under the alternative hypothesis. We allow the number of lags to be chosen optimally and the average is reported.

The Fisher-type tests conduct unit-root tests for each panel individually, and then combine the p-values from these tests to produce an overall test. We have reported two versions of the test (other versions yield similar results). In both cases augmented Dickey-Fuller unit-root tests are used. The P-test combines p-values using the inverse chi-squared transformation and the Z test uses the normal transformation.

The Hadri (2000) LM test differs from the other three in that it has as the null hypothesis that all the panels are stationary. The alternative hypothesis is that at least some of the panels contain a unit root. Hadri (2000) states that his tests are appropriate for panel datasets in which T is large and N is moderate, such as the Penn World Tables frequently used for cross-country comparisons.

Full Panel VAR Results

Table C2 documents the parameter estimates for the baseline estimation of the Panel VAR discussed in Section 2. Estimating a panel VAR raises a number of econometric issues. A basic problem in dynamic panel data models with fixed effects is that the lagged dependent variables are, by construction, correlated with the individual effects. This renders the least squares estimator biased and inconsistent. Consistent estimation requires some transformation to eliminate fixed effects. A within transformation wipes out the individual effects by taking deviations from sample means, but the resulting within-group estimator is inconsistent when the number of panels becomes large for a given time-dimension (Nickell, 1981).

Table C2: System GMM (2SLS) estimates

	Y		P		g^S		g^H		g^N	
$Y(-1)$	1.23	(0.05)	0.45	(0.09)	0.38	(0.23)	0.01	(0.01)	0.06	(0.02)
$P(-1)$	-0.01	(0.01)	1.37	(0.04)	-0.53	(0.08)	0.01	(0.00)	-0.02	(0.00)
$g^S(-1)$	0.00	(0.00)	-0.06	(0.01)	0.07	(0.03)	0.00	(0.00)	0.01	(0.00)
$g^H(-1)$	0.74	(0.12)	1.28	(0.26)	0.06	(0.83)	0.74	(0.04)	0.39	(0.09)
$g^N(-1)$	-0.23	(0.17)	0.15	(0.28)	2.23	(1.23)	0.08	(0.04)	0.24	(0.19)
$Y(-2)$	-0.33	(0.05)	-0.61	(0.08)	-0.47	(0.22)	-0.02	(0.01)	-0.06	(0.03)
$P(-2)$	0.02	(0.01)	-0.46	(0.05)	0.63	(0.08)	-0.01	(0.00)	0.02	(0.00)
$g^S(-2)$	-0.00	(0.00)	-0.01	(0.01)	0.04	(0.03)	-0.00	(0.00)	0.00	(0.00)
$g^H(-2)$	-0.67	(0.09)	-1.30	(0.19)	-2.77	(0.60)	-0.14	(0.03)	-0.21	(0.04)
$g^N(-2)$	0.16	(0.06)	0.53	(0.12)	1.02	(0.38)	0.03	(0.01)	0.16	(0.05)

Standard errors in parenthesis. No. of observations = 2438

Given this inconsistency, the literature focuses mainly on a first-difference transformation to eliminate the individual effect while handling the remaining correlation with the (transformed) error term using instrumental variables and GMM estimators (e.g. Arellano and Bond, 1991). However, the Arellano-Bond estimator is known to suffer from a weak instruments problem when the relevant time series are highly persistent, as they are in our case. As Blundell and Bond (1998) demonstrate this can result in large finite-sample biases. In our baseline estimation we use the system GMM estimator proposed by Arellano and Bover (1995) and Blundell and Bond (1998). This estimator is consistent when the number of panels becomes large for a given time-dimension and is less likely to suffer from the weak instruments problem. Another reason for focussing on this estimator is that its properties are fairly well understood and it has been studied in the context of panel VARs by Binder, Hsiao and Pesaren (2005).

Analysis of Regional Sub-samples

Table C3: Moments from system GMM estimation for regional sub-samples – income shock

	Region	Relative Std. Dev.	Corr. with Inc. Growth	Corr. with Price App.	Autocorrelation year 1	year 2	year 3	year 4
Income Growth	Coastal	1.00	1.00	0.76	0.27	0.01	-0.07	-0.06
	Interior	1.00	1.00	0.68	0.14	-0.05	-0.10	-0.10
	Sunbelt	1.00	1.00	0.51	0.34	0.12	0.02	-0.04
Price Appreciation	Coastal	1.71	0.76	1.00	0.69	0.20	-0.12	-0.23
	Interior	1.10	0.68	1.00	0.69	0.31	-0.02	-0.25
	Sunbelt	0.89	0.51	1.00	0.89	0.66	0.42	0.20
Sales Growth (existing)	Coastal	3.11	0.19	-0.33	0.90	0.83	0.74	0.65
	Interior	1.11	0.30	-0.01	0.69	0.54	0.47	0.42
	Sunbelt	2.21	0.66	-0.09	0.81	0.58	0.45	0.38
Construction Rate	Coastal	0.05	0.10	0.24	0.92	0.79	0.68	0.61
	Interior	0.15	0.52	0.66	0.88	0.66	0.45	0.27
	Sunbelt	0.15	0.63	0.65	0.86	0.57	0.27	0.02
Population Growth	Coastal	0.15	0.04	-0.30	0.93	0.88	0.83	0.77
	Interior	0.11	0.70	0.39	0.60	0.32	0.16	0.13
	Sunbelt	0.35	0.79	0.26	0.73	0.48	0.26	0.11

We now consider the results of estimating the panel VAR model on various sub-samples of both cities and time periods. Table C2 provides key moments for local earnings, house prices, construction rates, and ratios of housing stocks to city population based on shocks to local income in the panel VAR for each of the three sub-samples. Several, key observations are apparent. The standard deviation of house prices is roughly equal to that of local earnings in the full sample. Both construction rates and housing stock-population ratios are much less volatile than local earnings. House prices, construction rates and housing stock-population ratios are all strongly positively correlated with local earnings, although for inland cities these correlations are somewhat weaker. The higher and more persistent autocorrelation in both house price growth and population growth relative to earnings growth can also be observed in all the sub-samples.

Certain features of these moments and impulse response functions in Figure A27 conform to *a priori* expectations regarding population and price movements. In particular, coastal cities typically have more inelastic land supply than sunbelt cities. Accordingly, in response to demand shocks, price volatility tends to be higher and population and construction volatility tend to be lower in the coastal cities.

Alternative estimators

There are several potential problems with using the system GMM estimator for a sample with the dimensions considered here. While it is usually thought to be suitable for typical microeconometric panels, with only a few waves but a large number of individuals, here we have moderately large number of cities and a moderately long time series. Moreover, GMM estimators tend to have a larger standard error compared to the within-group estimator and may suffer from a finite sample bias due to weak instruments. Here we address these issues by comparing our estimates with those of two alternative estimators: OLS with no fixed effects and a standard within-groups estimator (WGE). Although the WGE is inconsistent as the number of panels becomes large, this should be less of a problem given the dimensions of our sample.

For the sake of brevity we do not report here all of the estimation results for each estimator. Instead Table C4 reports only the sum of the coefficients on the lagged dependent variables for each equation under each estimator, as suggested by Blundell and Bond (1998). As may be seem, the OLS estimates yields the most persistent processes for each variable. This reflects the upward bias due to the fact any fixed effect is attributed to persistent effects of the shocks. The WGE etimates yield the least persistent processes, which reflects

the downward bias. The system GMM (2SLS) estimator implies persistence that lie between these two extremes.

Table C4: Implied persistence: sum of coefficients on lagged dependent variable

Equation	WGE	$2SLS$	OLS
Y	0.90	0.90	0.99
P	0.87	0.92	0.98
g^S	-0.12	-0.11	-0.03
g^H	0.60	0.60	0.72
g^N	0.06	0.40	0.45

Table C5 documents the same set of moments as we have previously considered, for each of the estimators. While there are clearly some differences across estimators, the same broad pattern emerges as that depicted in Table 2. The biggest outliers come from those based on OLS estimation. This is because the omission of city level fixed effects forces any permament differences to show up as high persistence. The system GMM (2SLS) estimator implies a price growth response that is the most volatile and the least persistent.

Table C5: Moments from estimation using alternative estimators – income shocks

	Estimator	Relative Std. Dev.	Corr. with Inc. Growth	Corr. with Price App.	Autocorrelation year 1	year 2	year 3	year 4
Income Growth	WGE	1.00	1.00	0.67	0.24	0.04	-0.04	-0.09
	2SLS	1.00	1.00	0.76	0.28	0.03	-0.06	-0.09
	OLS	1.00	1.00	0.47	0.21	0.08	0.02	-0.00
Price Growth	WGE	1.90	0.67	1.00	0.81	0.48	0.12	-0.19
	2SLS	1.60	0.76	1.00	0.75	0.36	0.05	-0.15
	OLS	1.22	0.47	1.00	0.88	0.60	0.31	0.09
Sales Growth	WGE	1.75	0.60	0.02	0.57	0.11	-0.15	0.27
	2SLS	1.32	0.56	0.01	0.59	0.39	0.35	0.35
	OLS	1.98	0.59	0.12	0.71	0.37	0.25	0.25
Cons. Rate	WGE	0.12	0.41	0.90	0.84	0.47	0.09	-0.20
	2SLS	0.11	0.55	0.80	0.88	0.61	0.33	0.11
	OLS	0.40	0.19	0.44	0.98	0.95	0.91	0.88
Pop. Growth	WGE	0.16	0.89	0.84	0.57	0.20	-0.07	-0.25
	2SLS	0.17	0.75	0.60	0.67	0.40	0.19	0.09
	OLS	0.38	0.47	0.55	0.91	0.84	0.77	0.73

Alternative Specifications

Table C6 documents the relevant moments due to income shocks from the panel VAR for two alternative specifications.[32] The first specification, labelled "AR(2) Income", restricts the equation for income so that income depends only on its own lagged values. The specification labelled "All growth" uses growth rates of per capita incomes and prices in the VAR rather than levels. As may be seen by comparing to Table 2, restricting the income process to be univariate has negligible effects. This suggest thats lagged feedback effects of prices and population on per capita income are of second order importance. Specifying the VAR so that incomes and prices are in growth rates rather than in log levels has somewhat larger

[32] We have considered others including alternative definitions of the construction rate and other definitions of income. Similar patterns emerge in all cases.

effects on our results, but does not change the broad conclusions. Note that, by construction, the level of relative income under this specification is permanently high following a shock. However, this has little impact on the moments that we consider here.

Table C6: Moments from system GMM estimation for alternative specifications – income shocks

	Specification	Relative Std. Dev.	Corr. with Inc. Growth	Corr. with Price Growth	Autocorrelation year 1	year 2	year 3	year 4
Income Growth	Baseline	1.00	1.00	0.76	0.28	0.03	-0.06	-0.09
	AR(2) Income	1.00	1.00	0.75	0.26	0.02	-0.05	-0.07
	All growth	1.00	1.00	0.47	0.23	0.07	0.03	0.01
Price Growth	Baseline	1.60	0.76	1.00	0.75	0.36	0.05	-0.15
	AR(2) Income	1.62	0.74	1.00	0.76	0.38	0.07	-0.12
	All growth	1.10	0.47	1.00	0.87	0.58	0.30	0.10
Sales Growth	Baseline	1.32	0.56	0.01	0.59	0.39	0.35	0.35
	AR(2) Income	1.40	0.58	-0.02	0.62	0.44	0.40	0.41
	All growth	1.34	0.83	0.16	0.44	-0.08	-0.23	-0.26
Cons. Rate	Baseline	0.11	0.55	0.80	0.88	0.61	0.33	0.11
	AR(2) Income	0.13	0.52	0.74	0.90	0.68	0.45	0.26
	All growth	0.13	0.44	0.98	0.90	0.67	0.42	0.23
Pop. Growth	Baseline	0.17	0.75	0.60	0.67	0.40	0.19	0.09
	AR(2) Income	0.17	0.77	0.57	0.68	0.42	0.22	0.12
	All growth	0.18	0.85	0.85	0.69	0.40	0.20	0.07

Appendix D: A Multi-City Environment

There are M symmetric cities, indexed by $i = 1, \ldots, M$, where M is finite, but *large* in the sense that no individual city has a significant effect on aggregate quantities. The cities can be of identical or different sizes; what is important is that they all be small in this sense. We will focus on City 1, which will correspond to the *representative* or *average* city that was considered in the text.

Each city can be described as in Section II, except that here city-specific quantities are indexed by i. In particular, at each point in time, income in City i is denoted $\widetilde{y}_i$. Define average income across all cities by $\overline{y} = \sum_{i=1}^{M} \widetilde{y}_i$, and let $y_i \equiv \frac{\widetilde{y}_i}{\overline{y}}$. We will assume that in the steady-state, $y_i = 1$ for all i. We will think of the deviation of City 1 income from the average, y_1, as following a stochastic process just as in the text. This is straightforward under the assumption that City 1 is small so that fluctuations in $\widetilde{y}_1$ have no effect on $\overline{y}$. Alternatively, we can dispense with y_i and consider fluctuations in the *level* of City i income, $\widetilde{y}_i$. What is important in what follows is that the shocks considered be truly *city-specific.* That is, that fluctuations in either y_1 or $\widetilde{y}_1$, have negligible effects on income and/or housing market conditions in all other cities.

As in the text, the population of the economy is given by Q_t, and grows at gross rate $1+\mu$. Every period, each new household that enters the economy draws M potential amenity values, $a_i \in [0, \overline{a}]$ (one for each city), from distributions $F_i(a)$, for $i = 1, \ldots M$. Here for simplicity we will assume $F_i(\cdot) = F(\cdot)$ for all i., and that $\overline{a}$ is sufficiently large that a positive measure of households chooses to enter all cities in each time period. Amenity values are in utils, and like both consumption and housing services enter households' utility linearly. Utility from amenities, also like that from income, is realized only when the household chooses a particular city in which to live, and locates there.

For each new household j, let $\overline{W}_{ij}$ denote the value of being a new entrant to City i, defined just as in (9). Since new entrants to any city are identical, variation in $\overline{W}_{ij}$ across households is induced solely by variation in the amenity value, a_{ij}; in particular, $F_i(\overline{W}(a)) = F_i(a) = F(a)$ where a is the amenity value that generates $\overline{W}(a)$ given all other city attributes (income, house prices, the housing market tightness, etc.). Let $\varepsilon_j \equiv \max[\overline{W}_{2j}, \ldots, \overline{W}_{Mj}]$. That is, ε_j denotes the highest alternative value to entering City 1 for each new household j. Since M is finite, ε exists for all new households and identifies a single best alternative with probability 1. Similarly, the probability that $\varepsilon_j = \overline{W}_{1j}$, that is that a household is indifferent between entering City 1 and some other city, approaches zero as M becomes large.

Let $G(\varepsilon)$ denote the distribution of the highest alternative value, ε, across households. In a situation in which all cities other than City 1 are identical, $\varepsilon_j$is the value of entering that city for which household j has the highest amenity value. Thus, $G(\cdot)$ satisfies:

$$G(\varepsilon) = [F(a^*)]^{M-1} \tag{A28}$$

where $a^* \in [0, \overline{a}]$ is the amenity value which generates the maximum value ε. Note, however, that for $G(\varepsilon)$ to be well-defined, it is not required for all cities other than City 1 to be identical. Finally, note that the entry cutoff, ε_t^c, in this case satisfies $\varepsilon_t^c = \overline{W}_{1t}$, just as in (8). That is, any household with a maximum alternative value below $\overline{W}_{1t}$ enters City 1.

When a household leaves their city of residence due to the realization of a relocation shock (which happens with probability π for both home-owners and permanent renters), we assume that they are effectively in the same situation as a new household who has just arrived in the economy. That is, they re-draw, *in the current period*, from the amenity distribution for each city, and choose the city which yields the highest value. The expected continuation value following a relocation shock for any household currently resident in *any* city is thus given by

$$Z = \overline{\varepsilon} \equiv \int G(\varepsilon)d\varepsilon. \tag{A29}$$

From (A29) it is clear that Z depends only on the distribution of amenity values, $F(\cdot)$. Also, note that since City 1 is small, the probability that a household which exits it due to a relocation shock returns immediately is negligible.

Let POP_t denote the population of City 1 in period t. The population evolves via

$$POP_{t+1} = POP_t + \underbrace{\mu G(\overline{W}_{1t})Q_t + \pi \mathbb{Z}_t G(\overline{W}_{1t})}_{\text{new entrants}} - \underbrace{\pi[N_t + F_t]}_{\text{exiters}}, \tag{A30}$$

where $\mathbb{Z}_t$ denotes the measure of agents that exit *all other* cities in period t, and is assumed to be unaffected by conditions in City 1. On the balanced growth path, we assume first that all cities are symmetric, so that $G(\overline{W}_{1t}) = 1/M$ for all t. Similarly, $\mathbb{Z}_t = M(N_t + F_t)$. Thus, from (A30) $POP_{t+1} = (1 + \mu)POP_t$.

Finally, note that it is not important that we model the shock to City 1's income as being relative to the average. Any stable distribution of income across cities will give rise to a well-defined distribution of alternative values for City 1 (although (A28) will no longer apply). A direct increase in City 1 income, $\widetilde{y}_1$, will thus lead to entry for the same reasons as before. Again, the magnitude of the response will be determined by the properties of $G(\cdot)$ in a neighborhold of $\varepsilon = \widetilde{y}_1$ along the balanced growth path.

Suppose now that the economy is subject to aggregate income shocks which affect all cities symmetrically. Because utility is linear, adding a common component, y_{ct}, to city-level income of the form,

$$\widetilde{Y}_{it} = \widetilde{y}_{it} + y_{ct} \qquad i = 1, \ldots, N \tag{A31}$$

will have no effect whatsoever, as it affects neither the ranking of cities by new entrants or relocaters nor the demand for housing. Common shocks to construction costs and/or population growth will affect housing markets within each city, but will have no effect on mobility as they will not change the ranking of cities across prospective entrants as this is determined only by the amenity distribution, $F(\cdot)$.

Appendix E: Details of Robustness Exercises

Exiting Buyers

We assume that unsuccessful buyers exit at the end of the period with the same probability, π_n, as homeowners. This implies that the value of being a searching buyer is now

$$W_t = u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1 - \lambda_t)\,\beta\left[\pi_n Z + (1 - \pi_n) E_t W_{t+1}\right]. \tag{A32}$$

and the stock of buyers at date t is given by:

$$B_t = \theta(1 - \pi_n) N_{t-1} + \psi G(\varepsilon_t^c)\mu Q_{t-1} + (1 - \lambda_{t-1})(1 - \pi_n) B_{t-1}. \tag{A33}$$

The rest of the model remains unchanged.

Mismatched Owners remain in their houses

Proposition 2. *In equilibrium, mismatched owners are indifferent between the following two arrangements:*

(1) putting up their house for sale or rent immediately and renting while searching;

(2) remaining in their current house while searching, then putting their vacant house up for sale once they are matched with a new one.

Proof: The value of being a mis-matched owner who remains in their house while they search for a new one is given by

$$\tilde{J}_t = y_t + x_t - m + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) + (1 - \lambda_t)\beta E_t \tilde{J}_{t+1}. \tag{A34}$$

The value of becoming a renter immediately and putting the vacant house up for sale is given by

$$\begin{aligned}
W_t + V_t &= u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1 - \lambda_t)\beta E_t W_{t+1} + \gamma_t P_t + (1 - \gamma_t)\beta E_t V_{t+1} \\
&= u_t^R + \lambda_t \left(\beta E_t J_{t+1} - P_t\right) + (1 - \lambda_t)\beta E_t W_{t+1} + \gamma_t (1 - \chi)\beta E_t \left(J_{t+1} - W_{t+1}\right) \\
&\quad + \gamma_t \chi \beta E_t V_{t+1} + (1 - \gamma_t)\beta E_t V_{t+1} \\
&= y_t + x_t - r_t + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1}\right) \\
&\quad + \gamma_t (1 - \chi))\beta E_t \left(J_{t+1} - W_{t+1} - V_{t+1}\right) + (1 - \lambda_t)\beta \left(E_t W_{t+1} + E_t V_{t+1}\right). \qquad \text{(A35)}
\end{aligned}$$

Given (24), the above implies that

$$\begin{aligned} W_t + V_t &= y_t + x_t - m + \lambda_t \left(\beta E_t J_{t+1} - P_t + \beta E_t V_{t+1} \right) \\ &\quad + (1-\lambda_t) \beta E_t \left[W_{t+1} + V_{t+1} \right]. \end{aligned} \tag{A36}$$

Since $\lim_{T\to\infty} \beta^T E_t \tilde{J}_{t+T} = \lim_{T\to\infty} \beta^T E_t \left[W_{T+1} + V_{T+1} \right] = 0$, solving forwards implies that

$$\tilde{J}_t = W_t + V_t. \tag{A37}$$

QED

Let $\tilde{n}_t$ denote mismatched owners who remain in their (owned) homes. Then the flows of households between states is now described by (30) and

$$(1+\mu)\tilde{n}_t = \theta(1-\pi_n)n_{t-1} + [1-\lambda_{t-1}]\,\tilde{n}_{t-1} \tag{A38}$$

$$(1+\mu)b_t = \psi\mu G\left(\bar{W}_t\right) + [1-\lambda_{t-1}]\,b_{t-1} \tag{A39}$$

$$(1+\mu)n_t = (1-\theta)(1-\pi_n)n_{t-1} + \lambda_{t-1}\left(b_{t-1} + \tilde{n}_{t-1}\right) \tag{A40}$$

Market tightness is given by

$$\omega_t = \frac{b_t + \tilde{n}_t}{h_t - b_t - \tilde{n}_t - f_t - n_t} \tag{45}$$

and the housing stock evolves according to

$$(1+\mu)h_{t+1} = h_t + \phi^\eta \left(n_t + \tilde{n}_t + b_t + f_t\right)\left(\beta E_t V_{t+1} - \bar{q}\right)^\eta. \tag{A42}$$

Endogenous Exit

The implied value of being a homeowner is now given by

$$\begin{aligned} J_t &= u_t^H + \beta\pi_n E_t \left(Z_{t+1}^* + e_{t+1} V_{t+1} \right) + \theta\beta E_t (1-\pi_n e_{t+1}) \left(W_{t+1} + V_{t+1} \right) \\ &\quad + \beta(1-\theta) E_t (1-\pi_n e_{t+1}) J_{t+1}, \end{aligned} \tag{A43}$$

where $Z_t^* = \left(\bar{Z}^2 - J_t^2\right)/2\bar{Z}$ and $e_t = 1 - J_t/\bar{Z}$ is the exit probability conditional on having an opportunity.

Competitive Search

By entering sub-market (ω_t, P_t), a seller sells a house at P_t with probability $\gamma(\omega_t)$. The seller chooses to enter a sub-market that maximizes his/her expected return. It follows that the value of a vacant house for sale satisfies

$$V_t = \max_{(\omega_t, P_t)} \left\{ \gamma(\omega_t) P_t + [1 - \gamma(\omega_t)] \beta E_t \tilde{V}_{t+1} \right\}. \tag{A44}$$

Free entry of sellers implies that all active sub-markets (*i.e.* sub-markets with $\lambda, \gamma \in (0,1)$) in equilibrium must offer the sellers the same payoff V_t, although (ω_t, P_t) varies across sub-markets. It follows that the relationship between the listed price and market tightness that must be satisfied by all active sub-markets:

$$\gamma(\omega_t(P_t)) = \frac{V_t - \beta E_t \tilde{V}_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}}. \tag{A45}$$

Thus, it is sufficient to index sub-markets by the posted price P_t alone.

Buyers also decide in each period which sub-market to enter. The value of being a buyer W_t is therefore given by

$$W_t = u_t^R + \max_{P_t} \left\{ \lambda(\omega_t(P_t)) (\beta E_t J_{t+1} - P_t) + [1 - \lambda(\omega_t(P_t))] \beta E_t W_{t+1} \right\}. \tag{A46}$$

In equilibrium, the set of active sub-markets is *complete* in the sense that there is no other sub-market that could improve the welfare of any buyer or seller.

Let $\epsilon(\omega_t)$ denote the elasticity of the measure of matches with respect to the measure of buyers. We then have the following proposition:

Proposition 3. *In a competitive search equilibrium, there is only one active sub-market. In this market, the share of the surplus from house transactions that accrues to the buyer is equal to the elasticity of the measure of matches with respect to the measure of buyers:*[33]

$$\chi(\omega_t) = \epsilon(\omega_t). \tag{A47}$$

Proof: The first-order condition to the optimization problem in (A46) yields

$$\lambda'(\omega_t)\omega_t'(P_t) (\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}) - \lambda(\omega_t(P_t)) = 0, \tag{A48}$$

[33] This result is a special case of that derived in Moen (1997).

where $\omega_t(P_t)$ and $\omega_t'(P_t)$ are implicitly determined by (A45). This implies

$$\frac{\beta E_t J_{t+1} - P_t - \beta E_t W_{t+1}}{P_t - \beta E_t \tilde{V}_{t+1}} = -\frac{\lambda(\omega_t(P_t))/\lambda'(\omega_t(P_t))}{\gamma(\omega_t(P_t))/\gamma'(\omega_t(P_t))}, \tag{A49}$$

which can be used together with (A45) to solve for P_t, given the values $E_t J_{t+1}$, $E_t W_{t+1}$ and $E_t \tilde{V}_{t+1}$. Then one can solve for ω_t from (A45). Note that (A45) implies that $\omega_t'(P_t) < 0$ given $\gamma'(\omega) > 0$ from Assumption 1.

The trade surplus in the housing market is strictly positive. Given the boundary condition $\lim_{T\to\infty} \beta^T E_t J_{t+T} = 0$, it is clear that the household's equilibrium values are bounded, which implies that the trade surplus is also bounded. Thus $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} \in (0, \infty)$, where we have incorporated that $V = \tilde{V}$ in the equilibrium. Recall condition (ix) of the equilibrium definition that $\gamma(\omega_t),\ \lambda(\omega_t) \in (0,1)$ for all active sub-markets. Also recall from part (ii) of Assumption 1 that $\lambda'(\omega) < 0$, $\gamma'(\omega) > 0$. These conditions imply that $\epsilon(\omega) \in (0,1)$ where

$$\epsilon(\omega_t) = \frac{B_t}{\mathcal{M}} \cdot \frac{\partial \mathcal{M}}{\partial B_t} = \frac{1}{1 - \frac{\gamma(\omega_t)/\gamma'(\omega_t)}{\lambda(\omega_t)/\lambda'(\omega_t)}}. \tag{A50}$$

Define $LHS(P_t)$ as the left-hand side of (A49) and $RHS(P_t)$ the right-hand side. Given (A50), it is clear that

$$RHS(P_t) = \frac{\epsilon(\omega_t(P_t))}{1 - \epsilon(\omega_t(P_t))}. \tag{A51}$$

Because $\epsilon(\omega) \in (0,1)$, we have $RHS(P_t) \in (0,\infty)$ for all P_t. Moreover, recall $\omega_t'(P_t) < 0$ from (A45) and $\epsilon'(\omega) \leq 0$ from Assumption 1. Thus $RHS'(P_t) \geq 0$.

For any given V_t, J_t, W_t, one can verify that $LHS'(P_t) < 0$ because $\beta E_t J_{t+1} - \beta E_t W_{t+1} - \beta E_t V_{t+1} > 0$. Recall from (20) and (21) that the price in an active sub-market satisfies

$$\beta E_t V_{t+1} \leq P_t \leq \beta E_t J_{t+1} - \beta E_t W_{t+1}. \tag{A52}$$

It follows that

$$\begin{aligned} LHS(P_t = \beta E_t V_{t+1}) &= \infty > RHS(P_t = \beta E_t V_{t+1}) && \text{(A53)} \\ LHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}) &= 0 < RHS(P_t = \beta E_t J_{t+1} - \beta E_t W_{t+1}), && \text{(A54)} \end{aligned}$$

where the two inequalities are because $RHS(P_t) \in (0,\infty)$ for all P_t. The above results imply a unique $P_t^* \in (\beta E_t V_{t+1},\ \beta E_t J_{t+1} - \beta E_t W_{t+1})$ that satisfies

$$\frac{\beta E_t J_{t+1} - P_t^* - \beta E_t W_{t+1}}{P_t^* - \beta E_t V_{t+1}} = -\frac{\lambda(\omega_t^*(P_t^*))/\lambda'(\omega_t^*(P_t^*))}{\gamma(\omega_t^*(P_t^*))/\gamma'(\omega_t^*(P_t^*))}, \tag{A55}$$

and a unique $\omega_t^*(P_t^*)$ that satisfies

$$\omega_t^*(P_t^*) = \gamma^{-1}\left(\frac{V_t - \beta E_t V_{t+1}}{P_t^* - \beta E_t V_{t+1}}\right). \tag{A56}$$

Thus, there is a single active sub-market in the directed search equilibrium.

Equation (A55) may be written as

$$\frac{\chi(\omega)}{1-\chi(\omega)} = \frac{\epsilon(\omega)}{1-\epsilon(\omega)}, \tag{A57}$$

where $\chi(\omega)$ denotes the buyer's share of the surplus in a sub-market with tightness ω. The right-hand side of the above is the ratio of the elasticities of the number of matches with respect to the numbers of buyers and sellers. It follows that $\chi(\omega) = \epsilon(\omega)$. **QED**

The share of the surplus accruing to the buyer for the urn ball matching function is given by

$$\chi(\omega) = \epsilon(\omega) = \frac{\varrho\omega}{e^{\varrho\omega} - 1}, \tag{A58}$$

which is decreasing in market tightness ω.