Death and Capital
Physical and Human Capital Investment under Missing Annuity Markets

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Abstract

This paper examines the impact of adult mortality on the pattern of investment and economic development. In the presence of high mortality risks and imperfect annuities market, altruistic parents invest more in tangible assets (physical capital, land) that are readily transferable to future generations compared to intangible human capital. This differential effect of mortality can translate into divergent growth paths for economies, differing willingness to adopt modern skill-intensive technologies as well as a late transition from physical to human capital accumulation. Parental altruism can substitute for the absence of annuities reasonably well: investment in tangible assets is typically higher under missing annuities.

KEYWORDS: Life Expectancy, Mortality, Health, Physical Capital, Human Capital, Parental Altruism, Growth

JEL CLASSIFICATION: I10, O10, O40

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1 Introduction

This paper studies the impact of adult mortality on the choice between different income generating assets and its consequence for intergenerational transfer and economic development. We differentiate between physical assets and human capital as alternative sources of future income, one of the key distinction being the latter's "inalienability" (Hart and Moore, 1994). Physical assets such as land and capital are readily transferable across people in a way that human capital is not. This difference becomes important when an investor faces lifetime uncertainty that can cut short his amortization period.

Transferability of physical assets implies that well-functioning annuity markets can deliver a risk-free return on it, but not on human capital. Lifetime uncertainty will hence tilt portfolio allocation in favour of physical assets. In developing countries where mortality risks are high, the predominant form of asset accumulation will consequently be land and physical capital. Patterns of investment and production will shift towards human capital only when adult survival rates improve with the process of development.

This differential impact of mortality on human capital is not predicated on the availability of annuities, however. Markets in such instruments may be absent in developing countries and returns on both physical and human capital may be, it would seem, subject to lifetime uncertainties. This is where the transferability of physical assets becomes salient. Apart from its economic returns, when altruistic parents derive pleasure from bequests, the utility of an asset depends on its transferability to the future generation. The possibility that an investor may die prematurely but leave some of his physical assets for his survivors enhances the internal return on physical assets vis-a-vis human capital. When markets are not fully developed early in development, that constraint can be overcome if parental altruism is strong enough.

That mortality impacts the return on physical or human capital, and thereby overall investment and growth is well known in the literature (see, for example, Blackburn and Cipriani, 1998; Cervellatti and Sunde, 2005; Chakraborty, 2004; Kalemli-Ozcan, 2002 for various mechanisms). These works either focus on the relationship between mortality and the effective rate of time preference or on a single growth-promoting asset, usually human capital. By identifying more clearly the differential impact of mortality on the choice between different assets, we add to this literature and highlight their relative importance in various stages of development.

Related to our work, Galor and Moav (2004) differentiate between physical and human capital in terms of their technological characteristics. According to these authors, the fundamental asymmetry between human and physical capital accumulation is that the former is subject to quicker diminishing returns due to physiological constraints. Our focus on another asymmetry, in transferability, complements Galor and Moav. If longevity is positively associated with modern economic development, physical capital will be the prime engine of prosperity in the early stages of development and gradually displaced by human capital as living conditions improve. This predicted switch from physical to human capital echoes the central thesis of Galor and Moav's work without appealing to technological differences.

It has been argued that mortality has a significant (negative) impact on growth in the initial stages of development only because early development is characterized by imperfect annuity markets. We show that parental altruism, if strong enough, can substitute for such market imperfections, even if perfect diversification of mortality risks may not be possible. Thus at another level our work is an extension of Kotlikoff and Spivak (1981) who show that resource sharing between household members with independent mortality risks can substantially compensate for missing annuities.

The following two sections present the overall structure of the economy and analyze the extent to which intergenerational altruism can compensate for missing annuity markets when individuals invest in a single physical asset. Human capital is introduced in section 4 which demonstrates how mortality differentially impacts the optimal allocation across the two assets. A general equilibrium version in section 5 generalizes the result by incorporating pecuniary externalities.

2 Structure of the Model

In a discrete-time overlapping-generations economy individuals potentially live for two periods. For convenience, we will refer to these periods as "youth" and "middle-age". Individuals live in youth for sure but they may or may not survive into middle-age, the probability of surviving being a constant $p \in [0, 1]$.

At the end of their youth, individuals give birth to a single offspring, before they realize their mortality shock. They are perfectly altruistic toward their children and care for the child's lifetime utility (Becker, 1981). Individuals are endowed with a share of the family income in youth, which constitutes their first period income. They also inherit the tangible asset stock of the family (e.g. physical capital and/or land) upon the death of the parent. First period income is used for consumption purposes, for investment in improving the productivity of physical assets, and/or for acquiring human capital. The latter two activities determine second period income. If an agent survives into the middle-age he consumes a part of his second period income and transfers the remainder to his offspring as "intentional bequest". When he does not survive, his share of second period income either goes to the annuity issuer (in the case of perfect annuities) or to her offspring as "unintentional bequests" (when annuities markets are absent). Parents derive pleasure from both these bequests.

All agents in a generation are identical *ex ante*. The expected lifetime utility V_t of a young adult at t with income endowment y_t received either as intended or unintended bequest is

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}. \tag{1}$$

Here β is the subjective discount rate, γ represents the intensity of parental altruism and utility from death has been normalized to zero. Even though altruism is pure in that parents care about their offsprings' lifetime welfare, they do not necessarily discount their offsprings' lifetime utility at the same rate as they discount their own future consumption. In fact it may be biologically "natural" to assume altruism is limited, $\gamma \leq \beta$.

3 Altruism Substituting for Missing Annuities: A Single Tangible Asset

We begin by assuming there is a single tangible asset that we refer to as land for convenience. A land stock of T generates an income $f(T)$ where the production function satisfies the usual conditions $f(0) = 0$, $f' > 0$ and $f'' < 0$. Let $1 - \theta$ denote the share of output in any period that a parent intends to share with his offspring. If the parent is alive in middle-age, he consumes $\theta f(T)$ leaving the rest for his offspring. If he does not survive, that $1 - \theta$ share goes either to the annuity issuer (under perfect annuities) or to the offspring (when annuities are unavailable). Note that in the latter case the offspring ends up with a higher income endowment. We assume θ is exogenous and non-zero, determined by social customs and convention.

3.1 Optimization under Perfect Annuities

A young adult's decision at t is

$$Max\ u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} + x_t &= (1-\theta) f(T_t), \\ c_{2t+1} &= \theta f(T_{t+1})/p, \\ T_{t+1} &= (1-\delta) T_t + x_t, \end{aligned}$$

where $\delta \in [0, 1]$ is the depreciation rate of land quality. The middle-age constraint incorporates our annuity market assumption. Since the parent is committed to sharing $1-\theta$ fraction of the family income that period, she can pledge only $\theta f(T_{t+1})$ to the annuity issuer. Zero expected profits in the annuity market imply annuities are actuarially fair, that is, the annuity pays $\theta f(T_{t+1})/p$ in the event of survival while the annuity issuer keeps $\theta f(T_{t+1})$ in the event of death. Expected investment returns are independent of mortality risk.

As mentioned above, parental premature death has no effect on the offspring's budget constraints (given T_t). Rewrite the optimization problem as the dynamic programming problem (DPP)

$$V(T_t) = \max_{\{T_{t+1}\}} \{u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma V(T_{t+1})\}$$

subject to

$$\begin{aligned} c_{1t} &= (1-\theta) f(T_t) + (1-\delta) T_t - T_{t+1}, \\ c_{2t+1} &= \theta f(T_{t+1})/p. \end{aligned}$$

The necessary and sufficient first order condition for T_{t+1} is

$$u'(c_{1t}) = \beta\theta u'(c_{2t+1}) f'(T_{t+1}) + \gamma V'(T_{t+1})$$

which when combined with the envelope condition

$$V'(T_t) = [1-\delta + (1-\theta) f'(T_t)] u'(c_{1t})$$

yields the Euler equation

$$u'(c_{1t}) = \beta\theta u'(c_{2t+1}) f'(T_{t+1}) + \gamma[1-\delta + (1-\theta) f'(T_{t+1})] u'(c_{1t+1}) \tag{2}$$

Closed form solutions for the investment rate can be obtained only under two specific utility functions, logarithmic and linear, in addition to full depreciation of land and constant output elasticity of land.

3.1.1 Example 1: Logarithmic Preferences

Suppose

$$u(c) = \ln c, f(T) = AT^{\alpha}, \delta = 1$$

Then (2) becomes

$$\frac{1}{(1-\theta)AT_t^{\alpha} - T_{t+1}} = \frac{\alpha\beta p A T_{t+1}^{\alpha-1}}{AT_{t+1}^{\alpha}} + \frac{\alpha\gamma(1-\theta)AT_{t+1}^{\alpha-1}}{(1-\theta)AT_{t+1}^{\alpha} - T_{t+2}} \tag{3}$$

We use a guess-and-verify approach to get the policy function for investment. Suppose we conjecture that

$$T_{t+1} = \mu f(T_t) = \mu A T_t^{\alpha}.$$

Substituting this in the LHS of the Euler equation, we get

$$\frac{1}{1-\theta-\mu}\frac{1}{AT_t^{\alpha}}.$$

Leading our candidate policy function by one period and substituting it on the RHS gives us

$$\frac{\alpha\beta p A}{AT_{t+1}} + \frac{\alpha\gamma(1-\theta)A}{1-\theta-\mu}\frac{1}{AT_{t+1}}.$$

Now equate the LHS and RHS expressions to get

$$\begin{aligned}
\frac{1}{1-\theta-\mu}\frac{1}{AT_t^{\alpha}} &= \left[\alpha\beta p + \frac{\alpha\gamma(1-\theta)}{1-\theta-\mu}\right]\frac{A}{AT_{t+1}} \\
&\Rightarrow \quad T_{t+1} = \left[\alpha\beta p(1-\theta-\mu) + \alpha\gamma(1-\theta)\right] AT_t^{\alpha}
\end{aligned}$$

which takes the same functional form as our candidate policy function. Equating coefficients and solving for μ leads to

$$\mu = \frac{\alpha(\beta p + \gamma)(1-\theta)}{1+\alpha\beta p}.$$

Concavity of the policy function ensures that, even if families differed in their initial land holdings $\{T_0\}$, all families eventually converge to the same asset holding T^* as long as they face the same p. T^* is the fixed point of

$$T_{t+1} = \frac{\alpha(\beta p + \gamma)}{1+\alpha\beta p}(1-\theta)AT_t^{\alpha}. \tag{4}$$

Since first period income is $(1-\theta)AT_t^{\alpha}$, the investment rate is $\alpha(\beta p + \gamma)/(1+\alpha\beta p)$ which is increasing in the survival probability as long as $\alpha\gamma < 1$. If families differed in their survival rates as well, those facing longer lives would converge to a higher steady-state wealth. Even though actuarially fair annuity markets ensure consumption smoothing does not depend on lifetime uncertainty (Barro and Friedman, 1977), an increase in p affects the saving rate since annuities now offer a lower return on saving for the same level of investment.

3.1.2 Example 2: Linear Preferences

Under linear preferences, equation (2) becomes, assuming $\delta = 1$ and $f(T) = AT^{\alpha}$ as before,

$$\begin{aligned} 1 &= [\beta\theta + \gamma(1-\theta)]\, f'(T_{t+1}) = [\beta\theta + \gamma(1-\theta)]\, \alpha A T_{t+1}^{\alpha-1} \\ &\Rightarrow \; T_{t+1} = [\alpha A\, \{\beta\theta + \gamma(1-\theta)\}]^{1/(1-\alpha)} \end{aligned}$$

This solution is valid as long as

$$T_t \geq \left[\frac{[\alpha A\, \{\beta\theta + \gamma(1-\theta)\}]^{1/(1-\alpha)}}{A(1-\theta)} \right]^{1/\alpha} \equiv \hat{T},$$

otherwise the individual is at a constrained optimum where he invests his entire first period income in land

$$T_{t+1} = (1-\theta) A T_t^{\alpha}$$

and consumes only in middle-age.

3.2 Optimization under Missing Markets

When annuities are unavailable, the offspring's initial income depends on parental survival whose realization we denote by $z_t \in \{a, d\}$ corresponding to "alive" and "deceased" respectively. We can denote the offspring's initial endowment as

$$y_t = y(T_t, z_t) = \begin{cases} (1-\delta)T_t + (1-\theta) f(T_t), & \text{if } z_t = a \\ (1-\delta)T_t + f(T_t), & \text{if } z_t = d \end{cases}$$

The DPP in this case is

$$V(T_t, z_t) = \max \left\{ u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V(T_{t+1}, z_{t+1}) \right\}$$

subject to

$$\begin{aligned} c_{1t} &= y(T_t, z_t) - T_{t+1} \\ c_{2t+1} &= \theta f(T_{t+1}) \\ z_{t+1} &\sim iid \end{aligned}$$

where expectations are taken with respect to z_{t+1}.

FOC for land investment is now

$$u'(c_{1t}) = \beta p \theta u'(c_{2t+1}) f'(T_{t+1}) + \gamma E_t V_1(T_{t+1}, z_{t+1})$$

and the envelope condition

$$V_1(T_t, z_t) = u'(c_{1t}) y_1(T_t, z_t).$$

Leading the envelope condition by one period and using it in the FOC we get

$$u'(c_{1t}) = \beta p \theta u'(c_{2t+1}) f'(T_{t+1}) + \gamma E_t \left[u'(c_{1t+1}) y_1(T_{t+1}, z_{t+1}) \right] \quad (5)$$

3.2.1 Example 1: Logarithmic Preferences

Now make the same functional assumptions as before: $u(c) = \ln c$, $f(T) = AT^{\alpha}$ and $\delta = 1$ which implies

$$y_1(T, z) = \begin{cases} (1-\theta)\alpha A T_t^{\alpha-1}, & \text{if } z_t = a \\ \alpha A T_t^{\alpha-1}, & \text{if } z_t = d \end{cases}$$

We will have two versions of (5) depending on the realization of z_t. Optimal land investment at t will obviously depend on z_t so that $T_{t+1} = T(T_t, z_t)$. But since z takes discrete values and y depends on z only through a scaling constant, lets conjecture that the effect of z on T is through a scaling constant (that is the functional form of T itself does not depend on z).

Given T_t, suppose we denote future assets as $T_{a,t+1}$ and $T_{d,t+1}$ for the two realizations of parental survival. For $z_t = a$, we have

$$\underbrace{\frac{1}{(1-\theta)AT_t^{\alpha} - T_{a,t+1}}}_{u'(c_{1t}^{a})} = \beta p \theta \underbrace{\frac{\alpha A T_{a,t+1}^{\alpha-1}}{\theta A T_{a,t+1}^{\alpha}}}_{u'(c_{2t+1}^{a}) f'(T_{t+1})} + \gamma \left[p \underbrace{\frac{(1-\theta)\alpha A T_{a,t+1}^{\alpha-1}}{(1-\theta)AT_{a,t+1}^{\alpha} - T_{a,t+2}}}_{u'(c_{1t+1}^{a}) y_1(T_{t+1}, a)} + (1-p) \underbrace{\frac{\alpha A T_{a,t+1}^{\alpha-1}}{AT_{a,t+1}^{\alpha} - T_{d,t+2}}}_{u'(c_{1t+1}^{d}) y_1(T_{t+1}, d)} \right] \quad (6)$$

and for $z_t = d$

$$\underbrace{\frac{1}{AT_t^{\alpha} - T_{d,t+1}}}_{u'(c_{1t}^d)} = \beta p \theta \underbrace{\frac{\alpha A T_{d,t+1}^{\alpha-1}}{\theta A T_{d,t+1}^{\alpha}}}_{u'(c_{2t+1}^d) f'(T_{t+1})} + \gamma \left[p \underbrace{\frac{(1-\theta)\alpha A T_{d,t+1}^{\alpha}}{(1-\theta) A T_{d,t+1}^{\alpha} - T_{a,t+2}}}_{u'(c_{1t+1}^a) y_1(T_{t+1}, a)} + (1-p) \underbrace{\frac{\alpha A T_{d,t+1}^{\alpha-1}}{A T_{d,t+1}^{\alpha} - T_{d,t+2}}}_{u'(c_{1t+1}^d) y_1(T_{t+1}, d)} \right] \tag{7}$$

As before we will use a guess-and-verify approach: $T_{a,t+1} = \mu A T_t^{\alpha}$ and $T_{d,t+1} = \nu A T_t^{\alpha}$.

Equation (6) then becomes

$$\frac{1}{1-\theta-\mu} \frac{1}{AT_t^{\alpha}} = \frac{\alpha}{T_{a,t+1}} \left[\beta p + \frac{\gamma p (1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu} \right]$$

so that

$$T_{a,t+1} = \alpha(1-\theta-\mu) \left[\beta p + \frac{\gamma p (1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu} \right]$$

or

$$\mu = \alpha(1-\theta-\mu) \left[\beta p + \frac{\gamma p (1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu} \right]. \tag{8}$$

Similarly, using our policy function guesses, (7) becomes

$$\frac{1}{1-\nu} \frac{1}{AT_t^{\alpha}} = \frac{\alpha}{T_{d,t+1}} \left[\beta p + \frac{\gamma p (1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu} \right]$$

from which it follows that

$$T_{d,t+1} = \alpha(1-\nu) \left[\beta p + \frac{\gamma p (1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu} \right] AT_t^{\alpha}.$$

Comparing coefficients, we must have

$$\nu = \alpha(1-\nu) \left[\beta p + \frac{\gamma p (1-\theta)}{1-\theta-\mu} + \frac{\gamma(1-p)}{1-\nu} \right]. \tag{9}$$

Dividing (8) by (9)

$$\frac{\mu}{\nu} = \frac{1-\theta-\mu}{1-\nu} \Rightarrow \mu = (1-\theta)\nu \tag{10}$$

which also verifies our conjecture that the effect of z on T is only through a scaling constant. Make use of this relationship between μ and ν in (9) to obtain

$$\nu = \frac{\alpha(\beta p + \gamma)}{1 + \alpha \beta p} \tag{11}$$

and

$$\mu = \frac{\alpha(\beta p + \gamma)}{1 + \alpha \beta p}(1-\theta). \tag{12}$$

Note the investment propensity does not depend on parental survival and, in particular, is exactly as before, $\alpha(\beta p + \gamma)/(1 + \alpha \beta p)$.

3.2.2 Example 2: Linear Preference

Once again, equation (5) becomes

$$1 = \beta p\theta f'(T_{t+1}) + \gamma\{p(1-\theta) + (1-p)\}f'(T_{t+1})$$

which gives, assuming the same functional form for f,

$$T_{t+1} = [\alpha A\{p\beta\theta + \gamma\,(1-p\theta)\}]^{1/(1-\alpha)}$$

if

$$T_t \geq \left[\frac{[\alpha A\,\{p\beta\theta + \gamma(1-p\theta)\}]^{1/(1-\alpha)}}{A(1-\theta)}\right]^{1/\alpha} \equiv \bar{T},$$

and

$$T_{t+1} = (1-\theta)AT_t^{\alpha}$$

otherwise. Since marginal utility is independent of consumption level, bequest/income uncertainty has no effect on land investment. Note also that investment in the unconstrained case is lower under missing markets as long as $\beta > \gamma$. It is only when $\beta = \gamma$ (which also ensures that $\bar{T} = \hat{T}$) that the altruistic motive completely eliminates utility loss due to lifetime uncertainty.

Average investment under missing annuity markets is higher than under annuities when preferences are logarithmic even though the investment *rate* itself is invariant to the availability of markets. For linear utility, however, missing markets does depress investment unless parents value an extra unit of their offspring's consumption exactly as they would their own. In the first case, the degree of parental "selfishness" has no bearing on the investment rate. In the second case, it does.

These parametric examples identify two effects at work: the role of consumption smoothing and the relative valuation parents place on their own consumption vis-a-vis their children's. To understand how investment responds more generally to these two effects, we choose a more general parametric example – CES preferences – and using numerical methods examine how missing markets alter parental incentives and optimal choices.

3.3 A More General Case

3.3.1 The Net Marginal Benefit of Missing Markets

Closed form solutions are hard to obtain except for the polar cases of log and linear preference. But to gain intuition on the "general" case, it is instructive to consider the CES utility function $u(c) = c^{1-\sigma}/(1-\sigma)$ where we restrict $\sigma \in (0,1)$.[1] Whether or not investment suffers due to missing annuity markets ultimately relates to whether altruism can compensate for the utility loss that a parent suffers due to missing markets.

Based on the Euler equations in the two cases above, consider optimal choices in a stationary state. The expected marginal utility loss an individual suffers if annuity markets were to suddenly disappear, discounted appropriately, is

$$\begin{aligned} \Gamma &\equiv \beta\theta \left[u'\left(\frac{\theta f(T)}{p}\right) - pu'\left(\theta f(T)\right)\right] \\ &= \beta\theta[\theta f(T)]^{-\sigma} p^{\sigma}(1-p^{1-\sigma}) \end{aligned}$$

The marginal benefit, on the other hand, comes from the offspring enjoying higher income under parental death which the parent does take into consideration. Weighted by the degree of parental altruism, and denoting by T' the offspring's land investment under parental death, this benefit is

$$\begin{aligned} \Psi &\equiv \gamma \left[\begin{array}{c} p(1-\theta)u'\left((1-\theta)f(T)-T\right) + (1-p)u'\left(f(T)-T'\right) \\ -(1-\theta)u'\left((1-\theta)f(T)-T\right) \end{array} \right] \\ &= \gamma(1-p)\left[\{f(T)-T'\}^{-\sigma} - (1-\theta)\{(1-\theta)f(T)-T\}^{-\sigma}\right] \end{aligned}$$

Denote by ϕ the investment propensity out of first period income. Under missing annuity markets this income is different (higher) for an offspring whose parent dies prematurely. But suppose the individual maintains his savings propensity when annuity markets "disappear". By exclusively identifying the effect of missing annuities on the incentive to invest, we foreshadow which way optimal investment would respond to missing annuities and hence, whether missing markets are costly for investment.

Given our assumptions above, the net marginal benefit from missing markets is (ignoring the common term $f(T)^{-\sigma}$)

$$\Delta(p) \equiv \Psi - \Gamma = \gamma(1-p)(1-\phi)^{-\sigma}[1-(1-\theta)^{1-\sigma}] - \beta\theta^{1-\sigma}p^{\sigma}(1-p^{1-\sigma})$$

[1] With von Neumann Morgenstern preferences, the normalization of utility from death matters. Since we have normalized this value to zero, an increase in the survival rate would *decrease* expected future utility if $\sigma > 1$ (since $u < 0$ for all positive consumption levels).

For linear utility ($\sigma = 0$), this simplifies to $\Delta = -(\beta - \gamma)(1-p)\theta$. As long as $\beta > \gamma$, the consumption loss from missing markets cannot compensate for the lifetime utility gain the offspring enjoys. The individual would lower his investment in this case as section 3.2.2 shows. When $\beta = \gamma$, that is when parents value an extra unit of their offspring's consumption exactly as they would their own, altruism fully compensates for missing markets and investment is unaffected.

For logarithmic utility ($\sigma = 1$), on the other hand, $\Delta = 0$ irrespective of p, β and γ. This happens because both the marginal benefit and loss are zero due to the logarithmic function. In fact, we can interpret it as the familiar balancing of income and substitution effects.[2] Suppose the individual is consuming an endowment of future consumption goods ω, which is priced at unity, which gives a marginal utility $u'(\omega)$. If now he were given a higher endowment, say $a\omega$, but at the same time its price were to increase by the same proportion to a, then optimality requires that the individual would compare the marginal utilities with price ratio: he would prefer this new endowment if $u'(a\omega)/u(\omega) > a$. The endowment effect is measured by a pure income effect while the price change is purely the substitution effect which cancel out for log preferences: $u'(a\omega)/u(\omega) = a$.

The response of Δ to changes in p for CES preferences is presented in Figures 1 and 2. The response of net marginal benefit to p is as expected: the net marginal benefit is positive for all values of p and decreasing in p (Figure 1). That is, in these cases, missing markets leaves the individual no worse off and usually strictly better off (at the same investment rate as under annuities). It follows then that the lack of annuities would actually encourage investment relative to actuarially fair annuity markets. Figures 1(a) and (b) show that this result does not depend sensitively on the value of σ: lower σ has the effect of raising the relative return of accidental bequests since the marginal utility of the offspring is less sensitive to windfall gains.

As one would expect from our intuition from the linear case above, a higher subjective discount rate (β) relative to altruism intensity (γ) tends to reduce the net marginal benefit of missing markets. An increase in parental "selfishness" would then raise the cost of missing annuity markets. Figure 1(c) uses the same set of numerical values as Figure 1(a) but now posits $\gamma < \beta$.

The monotonicity of the net marginal benefit function does depend sensitively on θ. For instance, under $\sigma = 0.9$, when θ becomes smaller, from 3/4 (Figure 2(a)) to 1/2 (Fig-

[2] Recall from above that the value function is logarithmic in this case.

ure 2(c)), p has a non monotonic effect on Δ. The net benefit at first decreases with p and then increases. The offspring gets a relatively large share of output now, which decreases by a lot the marginal utility of the offspring's consumption under parental death. This reduces the attractiveness of the accidental bequest motive to the parent, unless p is relatively small as well in which case the individual's expected marginal utility from self-consumption is also small.

3.3.2 Optimal Investment with and without Annuity Markets

Following up on the intuition we get from considering the net marginal benefit, we explicitly consider how optimal investment compares in the two cases under CES preferences. First, we make another simplification. We assume $\alpha = 1$ so that marginal returns to land are independent of the investment level. We can make considerable progress under this assumption without having to solve the dynamic path of investment.

The assumption is also not unreasonable in that results do not depend on it qualitatively. In fact, the log case from above (for $\alpha = 1$) will still be nested. But the linear case will not be due to corner solutions. For linear utility and $f(T) = AT$, land investment is independent of p under annuity markets as long as $[\beta\theta + \gamma(1-\theta)]A \geq 1$. When annuities are missing, land investment is positive and independent of p iff $p \geq [1-\gamma(1-p\theta)]/(\beta\theta A)$, zero otherwise; investment is now a weakly increasing function of the survival probability.

Denote by ϕ the investment rate under annuity markets. Under actuarially fair annuities, consumption levels for a given T are

$$\begin{aligned} c_1 &= y - T', \\ c_2 &= \theta A T'/p, \end{aligned}$$

where $y = (1-\theta)AT$, T' denotes the land stock one period ahead and T'' two periods ahead. The Euler equation

$$[(1-\theta)AT - T']^{-\sigma} = \theta\beta A[\theta AT'/p]^{-\sigma} + \gamma(1-\theta)A[(1-\theta)AT' - T'']^{-\sigma}$$

under CES preferences implicitly defines the investment rate $\phi(p)$ as a function of the survival probability

$$\left(\frac{1-\phi}{\phi}\right)^{-\sigma} = A^{1-\sigma}\left[\beta p^{\sigma}\theta^{1-\sigma} + \gamma(1-\theta)^{1-\sigma}(1-\phi)^{-\sigma}\right].$$

When annuity markets are missing, consumption levels and investment choices depend on parental survival. It can be shown though that optimal investment *rates* under parental survival and death are identical for a linear production function. Let ψ denote the investment rates in this case and denote by T'_a and T'_d investments under parental survival and death respectively. The Euler equation, for an income endowment y in the first period, is now

$$\begin{aligned}[y - T'_a]^{-\sigma} &= \theta\beta p A[\theta A T'_a]^{-\sigma} + \gamma A[p(1-\theta)\{(1-\theta)AT'_a - T''_a\}^{-\sigma} \\ &\quad + (1-p)\{AT'_a - T''_d\}^{-\sigma}]\end{aligned}$$

where without loss of generality we have specified the problem for an adult who parent survives in middle-age. Simplifying, the investment rate $\psi(p)$ solves[3]

$$\left(\frac{1-\psi}{\psi}\right)^{-\sigma} = A^{1-\sigma}\left[\beta p \theta^{1-\sigma} + \gamma p(1-\theta)^{1-\sigma}(1-\psi)^{-\sigma} + \gamma(1-p)(1-\psi)^{-\sigma}\right].$$

Figure 3 compares ϕ to ψ for various values of p. At $p = 0$, the two investment rates are

$$\begin{aligned}\phi &= [\gamma A^{1-\sigma}(1-\theta)^{1-\sigma}]^{1/\sigma}, \\ \gamma &= [\gamma A^{1-\sigma}]^{1/\sigma}.\end{aligned}$$

Clearly $\psi(0) > \phi(0)$: when future survival is impossible, parents know for sure their offsprings would benefit from land investment and enjoy a higher endowment under missing annuities than under annuities.[4] At $p = 1$, the two rates are equal since parental bequests are same in both cases. As Figure 3 implies, investment is thus higher under missing annuities and this result is not qualitatively affected by σ or γ.

4 Mortality, Altruism and the Pattern of Investment

We turn next to the effect of lifetime uncertainty on portfolio choice. Specifically we now assume that people have access to a second investment vehicle, human capital. All individuals are born with the same level of human capital (normalized to zero) but have the ability to invest in it.

[3]Substituting $\sigma = 1$ gives us the investment rates $\phi = \psi = (\gamma + \beta p)/(1 + \beta p)$, same as in (4), (11) and (12) under $\alpha = 1$.

[4]There is a discontinuity in ϕ at $p = 0$. For arbitrarily small p, annuity purchases are positive but zero at $p = 0$.

As above, we continue to assume that the first asset is land. This specific interpretation is important now since we assume returns to the tangible asset or human capital do not depend on the other asset. This is more likely true of traditional activities involving land – farming and small-scale business enterprise – than modern technologies involving physical capital.

The family shares its income from both land and human capital. Specifically a middle-aged parent shares $1 - \theta_1$ fraction of the family's land income and $1 - \theta_2$ fraction of his labor income with the child. If all that is being shared is family income, it is natural to assume $\theta_1 = \theta_2$. Human capital opportunities (urban areas), though, can be geographically removed from traditional farming (rural areas) requiring skilled workers to migrate elsewhere. If sharing of labor earnings is relatively more difficult, $\theta_2 < \theta_1$. It is also conceivable that land ownership in developing countries is not as well defined as human capital ownership (which is embodied in a person in any case). Consequently land is a family property with every member having some right over its produce: $\theta_1 > 0$, $\theta_2 = 0$. This, of course, requires us to assume that the young can contribute to farm activities without seriously hampering their learning process.

Denote by e_t parental investment in human capital at time t and labor earnings in the second period of life as $h_{t+1} = g(e_t)$ where g is an increasing concave function satisfying $g(0) = 0$. We normalize the return to human capital at 1.

We will first establish results under linear utility, which abstracting from consumption smoothing, starkly brings out the role of asset returns and their dependence on annuities and altruism. We show below that, the non-transferability of human capital across generations tilts investment in favor of tangible assets even in the absence of annuities.

4.1 Optimization under Perfect Annuities

Given his income y_t, an adult in period t maximizes his expected lifetime utility

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} + x_t + e_t &= y_t, \\ c_{t+1} &= \theta_1 f(T_{t+1})/p + \theta_2 g(e_t), \\ T_{t+1} &= (1-\delta)T_t + x_t. \end{aligned}$$

Unlike before the offspring's first period income/bequest is uncertain under annuities as long as $\theta_2 > 0$:

$$y_{t+1} = \begin{cases} (1-\theta_1)f(T_{t+1}) + (1-\theta_2)g(e_t), & \text{with prob. } p \\ (1-\theta_1)f(T_{t+1}), & \text{with prob.} 1-p \end{cases}$$

Note specifically the non-transferability aspect – the offspring still enjoys income from land in the event of parental death but not his labor earnings. Assume $\delta = 1$. The FOCs are

$$\begin{aligned} e_t &: u'(c_{1t}) = \theta_2 \beta p u'(c_{2t+1}) g'(e_t) + \gamma E_t \frac{\partial V_{t+1}}{\partial e_t} \\ T_{t+1} &: u'(c_{1t}) = \theta_1 \beta u'(c_{2t+1}) f'(T_{t+1}) + \gamma E_t \frac{\partial V_{t+1}}{\partial T_{t+1}} \end{aligned}$$

and the Envelope conditions

$$\begin{aligned} \frac{\partial V_{t+1}}{\partial e_t} &= \begin{cases} (1-\theta_2)u'(c_{1t}^a)g'(e_t), & \text{with prob. } p \\ 0, & \text{with prob. } 1-p \end{cases} \\ \frac{\partial V_{t+1}}{\partial T_{t+1}} &= \begin{cases} (1-\theta_1)u'(c_{1t+1}^a)f'(T_{t+1}), & \text{with prob. } p \\ (1-\theta_1)u'(c_{1t}^d)f'(T_{t+1}), & \text{with prob. } 1-p \end{cases} \end{aligned}$$

Together these imply the Euler equations are

$$u'(c_{1t}) = p\left[\theta_2 \beta u'(c_{2t+1}) + \gamma(1-\theta_2)u'(c_{1t+1}^a)\right] g'(e_t)$$

for human capital investment and

$$u'(c_{1t}) = \left[\theta_1 \beta u'(c_{2t+1}) + \gamma(1-\theta_1)\left[p u'(c_{1t}^a) + (1-p)u'(c_{1t}^d)\right]\right] f'(T_{t+1})$$

for land investment.

4.1.1 Example: Linear Preferences

Assuming linear utility requires us to explicitly recognize non-negativity constraints on consumption levels. We reformulate the problem as

$$\begin{aligned} V(T_t, e_{t-1}, z_t) &= \max_{\{T_{t+1}, e_t\}} \{y_t - T_{t+1} - e_t + \beta p\left[\theta_1 f(T_{t+1})/p + \theta_2 g(e_t)\right] + \gamma E_t V(T_{t+1}, e_t, z_{t+1}\} \\ \text{subject to} &: T_{t+1} + e_t \le y_t. \end{aligned}$$

The Lagrangian

$$\mathcal{L} = y_t - T_{t+1} - e_{t+1} + \beta p\left[\theta_1 f(T_{t+1})/p + \theta_2 g(e_t)\right] + \gamma E_t V_{t+1} + \lambda\left[y_t - T_{t+1} - e_t\right]$$

leads to the FOCs

$$
\begin{aligned}
e_t &: -1 + \theta_2 \beta p g'(e_t) + \gamma E_t \frac{\partial V_{t+1}}{\partial e_t} = \lambda \\
T_{t+1} &: -1 + \theta_1 \beta f'(T_{t+1}) + \gamma E_t \frac{\partial V_{t+1}}{\partial T_{t+1}} = \lambda
\end{aligned}
$$

and the Envelope conditions

$$
\begin{aligned}
\frac{\partial V_{t+1}}{\partial e_t} &= \begin{cases} (1-\theta_2) g'(e_t), & \text{with prob. } p \\ 0, & \text{with prob. } 1-p \end{cases} \\
\frac{\partial V_{t+1}}{\partial T_{t+1}} &= (1-\theta_1) f'(T_{t+1})
\end{aligned}
$$

Together these imply

$$
-1 + p\left[\theta_2 \beta + \gamma(1-\theta_2)\right] g'(e_t) = \lambda
$$

for e_t and

$$
-1 + \left[\theta_1 \beta + \gamma(1-\theta_1)\right] f'(T_{t+1}) = \lambda
$$

for T_{t+1}.

The exact solution depends on the Lagrange multiplier. If $\lambda > 0$, implying the non-negativity constraint holds with equality and first period consumption is zero. The Euler equations lead to

$$
-1 + p\left[\theta_2 \beta + \gamma(1-\theta_2)\right] g'(e_t) = -1 + \left[\theta_1 \beta + \gamma(1-\theta_1)\right] f'(T_{t+1})
$$

or

$$
\frac{g'(e_t)}{f'(T_{t+1})} = \frac{\theta_1 \beta + \gamma(1-\theta_1)}{p\left[\theta_2 \beta + \gamma(1-\theta_2)\right]}.
$$

If we assume now that $f(T) = AT^\alpha$ and $g(e) = Be^\alpha$, then this equation gives us the optimal ratio of investment in human capital vis-a-vis land as

$$
\rho \equiv \frac{e_t}{T_{t+1}} = \left[\frac{pB\left[\theta_2 \beta + \gamma(1-\theta_2)\right]}{A\left[\theta_1 \beta + \gamma(1-\theta_1)\right]}\right]^{1/(1-\alpha)}.
$$

Higher longevity evidently tilts investment in favour human capital. Moreover, if the assets are treated symmetrically in terms of intergenerational income sharing ($\theta_1 = \theta_2$), the optimal ratio depends only on relative expected returns.

If, on the other hand, first period consumption is positive, $\lambda = 0$, and the two Euler equations can be solved independently as

$$
p\left[\theta_2 \beta + \gamma(1-\theta_2)\right] g'(e_t) = 1,
$$

and

$$[\theta_1\beta + (1-\theta_1)\gamma]\, f'(T_{t+1}) = 1.$$

Optimal solutions for land and human capital investment are now

$$e_t = [Bp\,[\theta_2\beta + \gamma(1-\theta_2)]]^{1/(1-\alpha)}$$

and

$$T_{t+1} = [A\,[\theta_1\beta + (1-\theta_1)\gamma]]^{1/(1-\alpha)}$$

which differs from the previous situation in that land investment is insensitive to mortality. Note, however, that the investment ratio ρ is still the same and an increase in p tilts investment in favor of human capital.

4.2 Optimization under Missing Markets

As before when annuities are not available, we assume the offspring enjoys the entire land income. Given his income y_t, an adult in period t maximizes his expected lifetime utility

$$V_t = u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} + x_t + e_t &= y_t \\ c_{t+1} &= \theta_1 f(T_{t+1}) + \theta_2 g(e_t) \\ T_{t+1} &= (1-\delta)T_t + x_t \end{aligned}$$

and recognizing that the child's first period income will be stochastic

$$y_{t+1} = \begin{cases} (1-\theta_1)f(T_{t+1}) + (1-\theta_2)g(e_t), & \text{w.p. } p \\ f(T_{t+1}), & \text{w.p.} 1-p \end{cases}$$

FOCs are

$$\begin{aligned} e_{t+1} &: \;\; u'(c_{1t}) = \theta_2\beta p u'(c_{2t+1})g'(e_t) + \gamma E_t \frac{\partial V_{t+1}}{\partial e_t} \\ T_{t+1} &: \;\; u'(c_{1t}) = \theta_1\beta p u'(c_{2t+1})f'(T_{t+1}) + \gamma E_t \frac{\partial V_{t+1}}{\partial T_{t+1}} \end{aligned}$$

and Envelope conditions

$$\begin{aligned} \frac{\partial V_{t+1}}{\partial e_t} &= \begin{cases} (1-\theta_2)u'(c^a_{1t+1})g'(e_t), & \text{with prob. } p \\ 0, & \text{with prob. } 1-p \end{cases}, \\ \frac{\partial V_{t+1}}{\partial T_{t+1}} &= \begin{cases} (1-\theta_1)u'(c^a_{1t+1})f'(T_{t+1}), & \text{with prob. } p \\ u'(c^d_{1t+1})f'(T_{t+1}), & \text{with prob. } 1-p \end{cases}. \end{aligned}$$

Hence optimality conditions for the two assets become

$$u'(c_{1t}) = p\left[\theta_2\beta u'(c_{2t+1}) + \gamma(1-\theta_2)u'(c^a_{1t+1})\right] g'(e_{t+1})$$

for e_t and

$$u'(c_{1t}) = \left[\theta_1\beta p u'(c_{2t+1}) + \gamma\left[p(1-\theta_1)u'(c^a_{1t}) + (1-p)u'(c^d_{1t})\right]\right] f'(T_{t+1})$$

for T_{t+1}.

4.2.1 Example: Linear Preferences

Incorporating the non-negativity constraint on first period consumption $T_{t+1} + e_t \leq y_t$, the Lagrangian is

$$\mathcal{L} = y_t - T_{t+1} - e_t + \beta p\left[\theta_1 f(T_{t+1})/p + \theta_2 g(e_t)\right] + \gamma E_t V_{t+1} + \lambda\left[y_t - T_{t+1} - e_t\right],$$

the associated FOCs

$$\begin{aligned} e_t &: \quad -1 + \theta_2\beta p g'(e_t) + \gamma E_t \frac{\partial V_{t+1}}{\partial e_t} = \lambda \\ T_{t+1} &: \quad -1 + \theta_1\beta f'(T_{t+1}) + \gamma E_t \frac{\partial V_{t+1}}{\partial T_{t+1}} = \lambda \end{aligned}$$

and Envelope conditions

$$\begin{aligned} \frac{\partial V_{t+1}}{\partial e_t} &= \begin{cases} (1-\theta_2)g'(e_t), & \text{with prob. } p \\ 0, & \text{with prob. } 1-p \end{cases}, \\ \frac{\partial V_{t+1}}{\partial T_{t+1}} &= \begin{cases} (1-\theta_1)f'(T_{t+1}), & \text{with prob. } p \\ f'(T_t), & \text{with prob. } 1-p \end{cases}. \end{aligned}$$

Together these imply

$$-1 + p\left[\theta_2\beta + \gamma(1-\theta_2)\right] g'(e_t) = \lambda$$

for e_t and

$$-1 + \left[\theta_1\beta + \gamma\left[p(1-\theta_1) + (1-p)\right]\right] f'(T_{t+1}) = \lambda$$

for T_{t+1}.

If $\lambda > 0$, the associated Euler equations lead to

$$-1 + p\left[\theta_2\beta + \gamma(1-\theta_2)\right] g'(e_t) = -1 + \left[\theta_1\beta + \gamma p(1-\theta_1) + (1-p)\right] f'(T_{t+1})$$

or

$$\frac{g'(e_t)}{f'(T_{t+1})} = \frac{\theta_1\beta + \gamma(1-p\theta_1)}{p\left[\theta_2\beta + \gamma(1-\theta_2)\right]}.$$

For the same technologies as above the optimal ratio of land vis-a-vis human capital investment is now

$$\rho = \left[\frac{B}{A}\frac{p\left[\theta_2\beta + \gamma(1-\theta_2)\right]}{\left[\theta_1\beta + \gamma(1-p\theta_1)\right]}\right]^{1/(1-\alpha)}$$

which is lower than under perfect annuities. This is specifically due to land investment being higher.

If $\lambda = 0$, the Euler equations can be solved independently and the solutions would be the same as in the previous section. For land, the condition is

$$\left[\theta_1\beta p + \gamma\{p(1-\theta_1) + (1-p)\}\right] f'(T_{t+1}) = 1$$

and for human capital the same as before

$$p\left[\theta_2\beta + \gamma(1-\theta_2)\right] g'(e_t) = 1.$$

Hence

$$\rho = \left[\frac{B}{A}\frac{p\{\theta_2\beta + \gamma(1-\theta_2)\}}{\gamma + \theta_1 p(\beta-\gamma)}\right]^{1/(1-\alpha)}.$$

4.3 A More General Case

To generalize this result on the differential effect of mortality we appeal to CES preferences, full depreciation of land and linear production technologies for the two assets, $f(T) = AT$ and $g(e) = Be$, with $B \geq A$. Without loss of generality we impose $\theta_1 \equiv \theta$ and $\theta_2 = 0$.[5]

We start with the annuity markets case. Let ϕ and η denote the investment propensities in land and human capital. The Euler equations for the two assets are

$$[y - T' - e']^{-\sigma} = \theta\beta A[\theta AT'/p + Be']^{-\sigma} + \gamma(1-\theta)A[(1-\theta)AT' - T'' - e'']^{-\sigma}$$

for an income endowment of y. The investment rates (ϕ, η) solve the pair of equations

$$(1-\phi-\eta)^{-\sigma} = \theta\beta pB\left(\frac{\theta A\phi}{p} + B\eta\right)^{-\sigma} \quad (13)$$

$$\left(1 - \frac{A}{pB}\right) = \gamma A^{1-\sigma}(1-\theta)^{1-\sigma}\phi^{-\sigma} \quad (14)$$

[5] $\theta_2 > 0$ would only accentuate the effect of p on human capital investment since premature parental death would eliminate the ability to enjoy a share of parental labor income.

The role of p in portfolio allocation can be examined by considering the relative investment rates η/ϕ in human-to-physical capital.

When annuities are missing let ψ and ν denote the investment propensities in land and human capital. The Euler equations for an individual whose parent has survived (and bequeaths y) are

$$\begin{aligned}[y - T_a' - e_a']^{-\sigma} &= \theta\beta p A[\theta A T_a' + B e_a']^{-\sigma} + \gamma p(1-\theta)A[(1-\theta)AT' - T_a'' - e_a'']^{-\sigma} \\ &\quad + \gamma(1-p)A[AT' - T_d'' - e_d'']^{-\sigma}.\end{aligned}$$

The investment rates now solve the pair of equations[6]

$$(1-\psi-\nu)^{-\sigma} = \theta\beta p B(\theta A\psi + B\nu)^{-\sigma} \tag{15}$$

$$\left(1 - \frac{A}{B}\right) = \gamma A^{1-\sigma}\psi^{-\sigma}[p(1-\theta)^{1-\sigma} + (1-p)]. \tag{16}$$

Here our object of interest is the response of relative investments ν/ψ to changes in p.

Figures 4 and 5 illustrate how the relative responsive of investments to p in the presence and absence of annuities. The blue solid lines correspond to η/ϕ and the red dashed lines to ν/ψ.

In Figure 4(a), both relative investment rates are increasing in survival. While individuals may or may not diversify away mortality risks on physical assets via altruism, p still has a differentially higher impact on human capital investment. The relative investment rates rise faster in Figure 4(b) compared to Figure 4(a) where γ is higher. Since human capital investment is immune to the degree of parental altruism, a lower value of γ does not affect it as much as it dampens physical capital investment. Note also the curvature of the two relative investment rates. Under missing annuities, the switch from physical assets to human capital occurs at a faster rate. Indeed, as Figures 1–3 foreshadowed, land investment is higher for the parameter values used in Figure 4 so that human capital investment is lower relative to the annuities case.

Finally Figure 5 establishes that our results are not sensitive to the curvature of the utility function: investment in human capital rises faster for a smoother function (lower σ) under both cases because the parent does not have to "compensate" for strongly diminishing marginal utility of the offspring by investing more in land.

[6] Suppose $B = \omega A$ where $\omega > 1$. For very low values of p, the LHS of equation (14) can turn negative as returns to human capital are not high enough to compensate for mortality risk. To avoid that we restrict to $p \in [1/\omega, 1]$. $\omega > 1$ also ensures that $A < B$ for equation (16).

5 Pecuniary Externalities

The interpretation of the tangible asset is more general than we have insisted on so far. Specifically, any form of physical asset that is transferable ought to face the same kind of incentives vis-a-vis inalienable human capital. But there is a subtle difference between modern forms of physical capital – equipment, machinery, business enterprise, workshop and cottage industry – and land. While it is relatively easy to imagine traditional activities involving land do not involve skills or other forms of human capital, that assumption is harder to justify with physical capital. In other words, physical and human capital may be complementary production inputs and if so, the accumulation of physical assets can depend on the survival probability through pecuniary externalities that we have ruled out so far.

We establish in this section, by means of a Cobb-Douglas production function that utilizes physical capital and skilled labor to produce a final consumption good, that our intuition from the previous sections generalize. An increase in p has a relatively more pronounced effect on human capital investment that increases its aggregate supply. This raises the equilibrium return on the complementary input, physical capital, encouraging its accumulation. The net effect of a higher p is similar to section 4 except that it now tilts investment *and* production towards human capital.

The aggregate technology is CRS in aggregate physical (K) and human capital (H) stocks

$$Y_t = AK_t^{\alpha}H_t^{1-\alpha}$$

where $\alpha \in (0, 1)$. In perfectly competitive factor and goods markets, wage per efficiency unit of labor and rental on capital (assume $\delta = 1$) are

$$\begin{aligned} w_t &= (1-\alpha)A(K_t/H_t)^{\alpha}, && (17) \\ r_t &= \alpha A(H_t/K_t)^{1-\alpha}. \end{aligned}$$

We assume a unit measure of people are born at each date, p fraction of whom survive into middle age. Since agents are *ex ante* identical in their preferences and survival and make the same optimizing choices (see below), we denote individual holdings of the two assets by k and h. The aggregate stocks are then

$$\begin{aligned} K_t &= k_t, \\ H_t &= ph_t, \end{aligned}$$

where h is the human capital of each middle-aged person before experiencing their survival shock. As with the CES case of the previous section, individuals do not possess any human capital endowment in youth and depend on their families to consume out of the returns on physical capital. Labor income in youth as well as sharing of labor income across generations are easy to incorporate and do not change our results qualitatively.

In their youth individuals invest x_t in physical capital and e_t in their human capital that yields asset levels, the following period,

$$\begin{aligned} k_{t+1} &= f(x_t) \\ h_{t+1} &= g(e_t) \end{aligned}$$

where the production functions f and g are concave and satisfy $f(0) = 0 = g(0)$. Unlike the standard model output is not converted into physical capital one-for-one. This assumption is necessary to allow for relative price effects on k since marginal utility will be constant (see below).

We focus exclusively on the case of missing annuities. The decision problem is to maximize expected lifetime utility

$$V_t \equiv u(c_{1t}) + \beta p u(c_{2t+1}) + \gamma E_t V_{t+1}$$

subject to

$$\begin{aligned} c_{1t} &= y_t - x_t - e_t, \\ c_{2t+1} &= \theta r_{t+1} k_{t+1} + w_{t+1} h_{t+1}, \end{aligned}$$

given the income endowment y_t, stochastic first period income for the offspring

$$y_{t+1} = \begin{cases} (1-\theta) r_{t+1} k_{t+1}, & \text{w.p. } p \\ r_{t+1} k_{t+1}, & \text{w.p. } 1-p \end{cases}$$

and

$$k_{t+1} = (1-\delta) k_t + x_t.$$

The middle-age budget constraint embodies our assumption that returns to physical capital are shared with the offspring and ownership of that asset is costlessly passed on to him if the parent dies prematurely.

The first order conditions for optimal investment are

$$x_t : u'(c_{1t}) = \theta \beta p u'(c_{2t+1}) r_{t+1} + \gamma \left\{ p(1-\theta) u'(c^a_{1t+1}) + (1-p) u'(c^d_{1t+1}) \right\} r_{t+1} f'(x_t) \tag{18}$$

and

$$e_t : u'(c_{1t}) = \beta p u'(c_{2t+1}) w_{t+1} g'(e_t) \tag{19}$$

To proceed further, we will simplify by assuming linear utility and

$$\begin{aligned} f(x) &= a x^{\chi}, \\ g(e) &= b e^{\chi}, \end{aligned}$$

where $\chi \in (0,1)$. Suppose also that all families start with a relatively high initial endowment of physical capital k_0 so that the individual is at an unconstrained optima and $c_1 > 0$. Equations (18) and (19) then lead to optimal investment decisions of

$$\begin{aligned} x_t &= \left[a\chi\left\{\gamma + p\theta(\beta - \gamma)\right\} r_{t+1}\right]^{1/(1-\chi)} \\ e_t &= \left[b\beta p \chi w_{t+1}\right]^{1/(1-\chi)} \end{aligned}$$

Individual stocks of the two assets are

$$\begin{aligned} k_{t+1} &= a^{1/(1-\chi)} \left[\chi\{\gamma + p\theta(\beta-\gamma)\}\right]^{\chi/(1-\chi)} r_{t+1}^{\chi/(1-\chi)} \\ h_{t+1} &= b^{1/(1-\chi)} (\chi\beta p)^{\chi/(1-\chi)} w_{t+1}^{\chi/(1-\chi)} \end{aligned}$$

and the ratio of aggregate capital stocks

$$\frac{K_t}{H_t} = \frac{k_t}{p h_t} = \left(\frac{a}{b}\right)^{1/(1-\chi)} \left[\frac{\gamma + p\theta(\beta-\gamma)}{\beta p}\right]^{\chi/(1-\chi)} \frac{1}{p} \left(\frac{r_t}{w_t}\right)^{\chi/(1-\chi)}. \tag{20}$$

From (17), on the other hand,

$$\frac{r_t}{w_t} = \frac{\alpha}{1-\alpha}\frac{H_t}{K_t} \tag{21}$$

Using (20) and (21), we can solve for the equilibrium factor ratio

$$\frac{K_t}{H_t} = \frac{a}{b}\left(\frac{\alpha}{1-\alpha}\right)^{\chi} \frac{1}{p^{1-\chi}} \left[\frac{\gamma + p\theta(\beta-\gamma)}{\beta p}\right]^{\chi} \tag{22}$$

which is a decreasing function of p for $\beta \geq \gamma$.

Investment in physical capital depends positively on its return, r. Since K and H are complementary inputs, an increase in the supply of human capital induced by p, would raise returns to physical capital and encourage its investment. Equilibrium supply of physical capital now depends positively on p. But as equation (22) shows, this second round effect is not enough to bias the equilibrium response away from human capital.

It is also easy to show that output per capita

$$Y = \Gamma p^{\frac{1-\alpha}{1-\chi}} \left[\frac{1}{\beta}\left\{\gamma + \theta p(\beta - \gamma)\right\}\right]^{\frac{\alpha\chi}{1-\chi}}$$

in this economy depends positively on longevity (as long as $\beta \geq \gamma$). Since both physical and human capital depreciate fully, the economy will jump straight to this steady-state output level assuming a high enough k_0. If capital did not fully depreciate, however, the transition path would also depend on p. Not only would low-p countries converge to a lower steady-state, their transition would be slower too. These high mortality economies would rely more intensively on physical capital, the switch from physical to human capitals as engines of development occurring later and remaining incomplete.

Obviously such a transition can be triggered by health and mortality improvements. The widespread mortality reductions (not limited to child and infant survival) in late nineteenth century Western Europe may have spurred accumulation and innovation towards newer generations of technologies that were biased towards human capital.[7] If newer technologies in the twentieth century have been skill oriented, as a body of work now argues, it has implications for developing countries. For instance an increase in the return to human capital B in a low-p country would see a more muted response in skill acquisition compared to a high-p country. High mortality, in other words, biases the response away from newer technologies. The lack of catch-up in parts of the developing world plagued by epidemics and health challenges may be as much to do with the low return from adopting modern technologies as with institutional constraints that prevent such adoption.[8]

6 Conclusion

Two themes underlie our study of the effects of mortality on economic development. When people face lifetime uncertainties, they are more inclined to invest in tangible assets that can be passed on to their survivors. Mortality dissuades human capital accumulation relative to physical capital and land. This has implications for long-run growth,

[7]See Cutler *et al* (2006) on mortality reduction. On technological change, Abramovitch (1993) writes, as quoted in Galor and Moav (2004): "In the nineteenth century, technological progress was heavily biased in a physical capital-using direction. ... In the twentieth century, however, the physical capital-using bias weakened; it may have disappeared altogether. The bias shifted in an intangible (human and knowledge) capital-using direction and produced the substantial contribution of education and other intangible capital accumulation to this century's productivity growth..."

[8]Similar distributional implications are possible if households differed in their survival rates: low-p households would exhibit a preference towards tangible assets and benefit less from skill-biased technological change.

convergence, and technology adoption. From an accounting point of view, moreover, high mortality societies would rely on physical capital accumulation more intensively relative to low mortality ones.

The second contribution of this paper has been to establish these results without appealing to the standard assumption of perfect annuities. Annuity markets are more likely to be underdeveloped in poorer societies. We have demonstrated that the parental altruism motive can substitute for missing annuity markets reasonably well and in particular, for investment in tangible assets. Future work will delineate the implications of this for lifetime utility.

In conclusion, a caveat is in order. The appeal of life insurance policies, which we rule out in our analysis, is to precisely get away from the problem of non-transferability of human capital (Fischer, 1973). We conjecture that our results are robust to the availability of such policies, even though this remains to be established rigorously. Since the demand for a life insurance policy would be positive only for human capital investment, such a policy would simply raise the effective cost of education and training vis-a-vis investment in tangible assets. The very fact that the effective cost of human capital is now higher because of the need to purchase life insurance suggests that the differential effect of mortality on human capital may remain even when such a policy guarantees a risk-free return.

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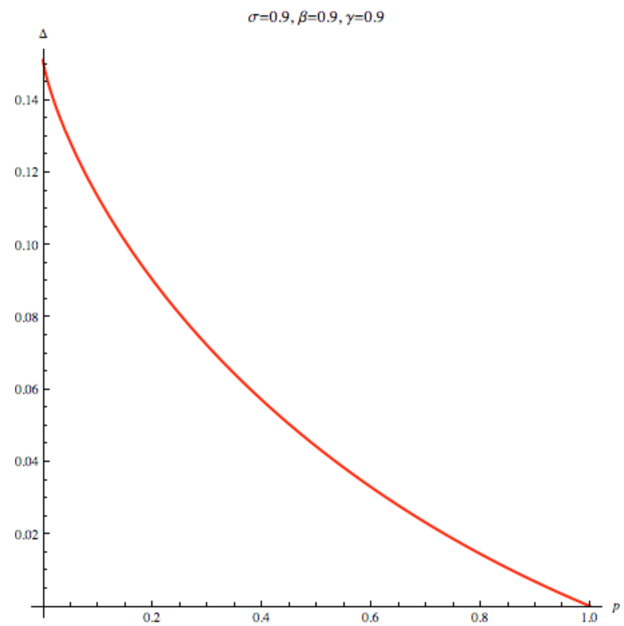
(a)



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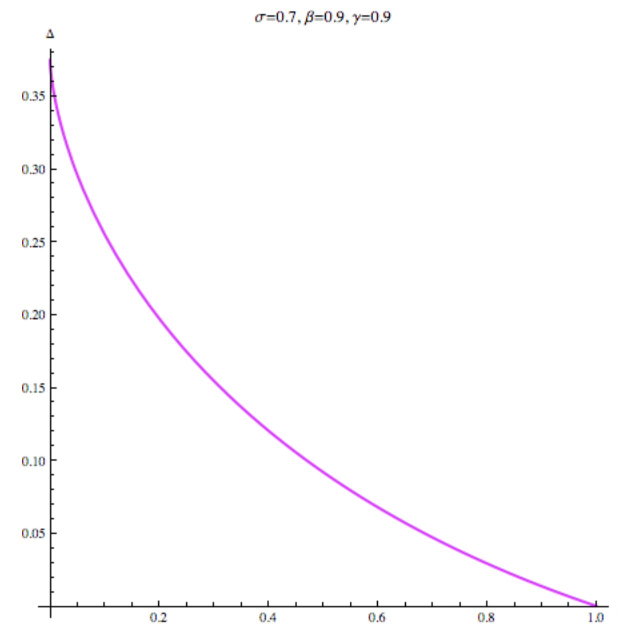

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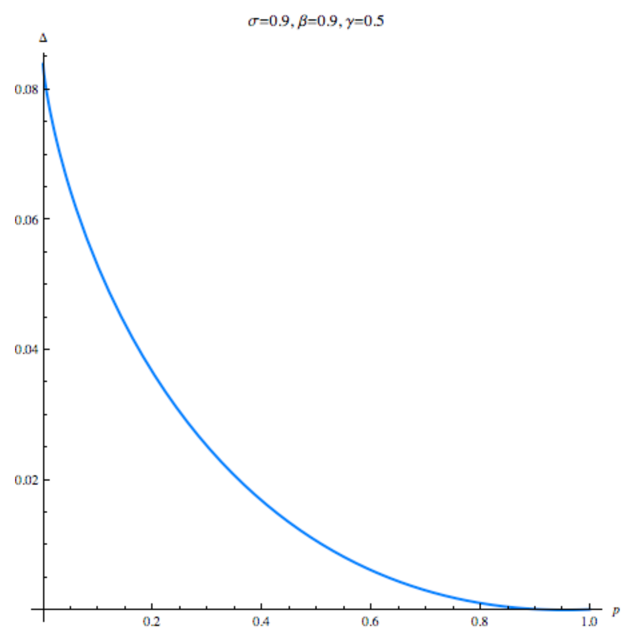

Figure 1

The Net Marginal Benefit of Investing in Physical Assets under Missing Markets relative to Annuity Markets

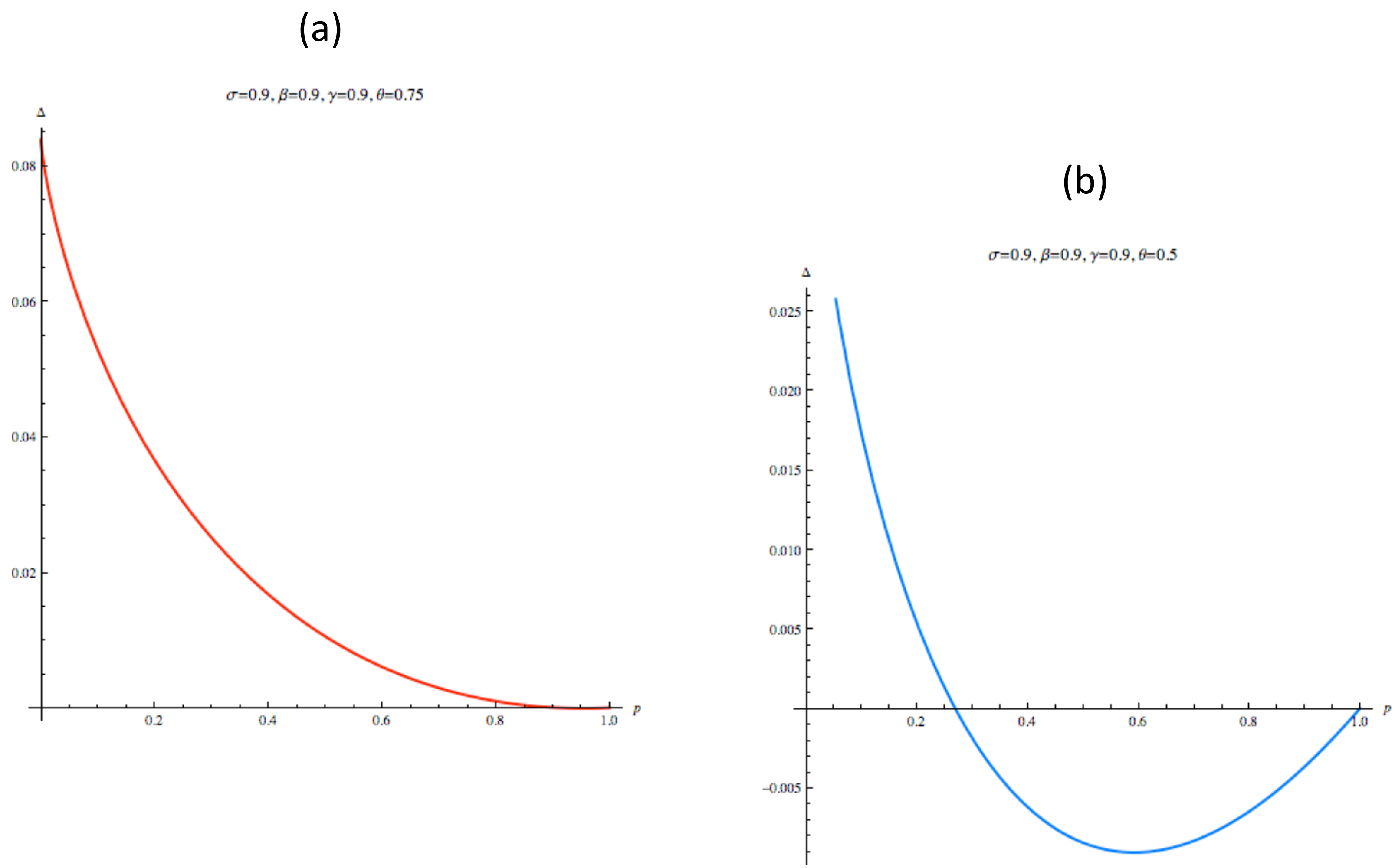

Figure 2 Non-monotonicity of Net Marginal Benefits

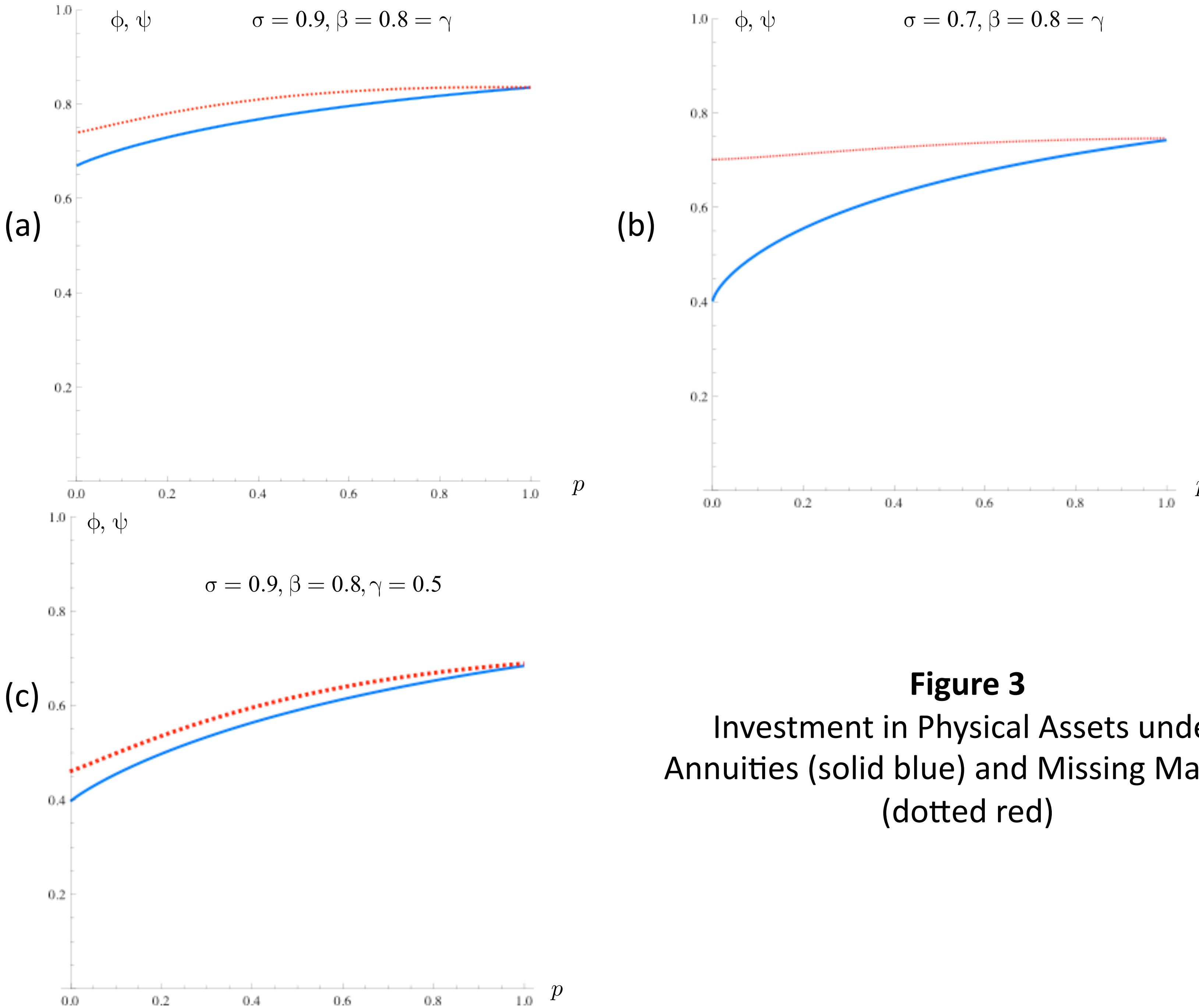

Figure 3

Investment in Physical Assets under Annuities (solid blue) and Missing Markets (dotted red)

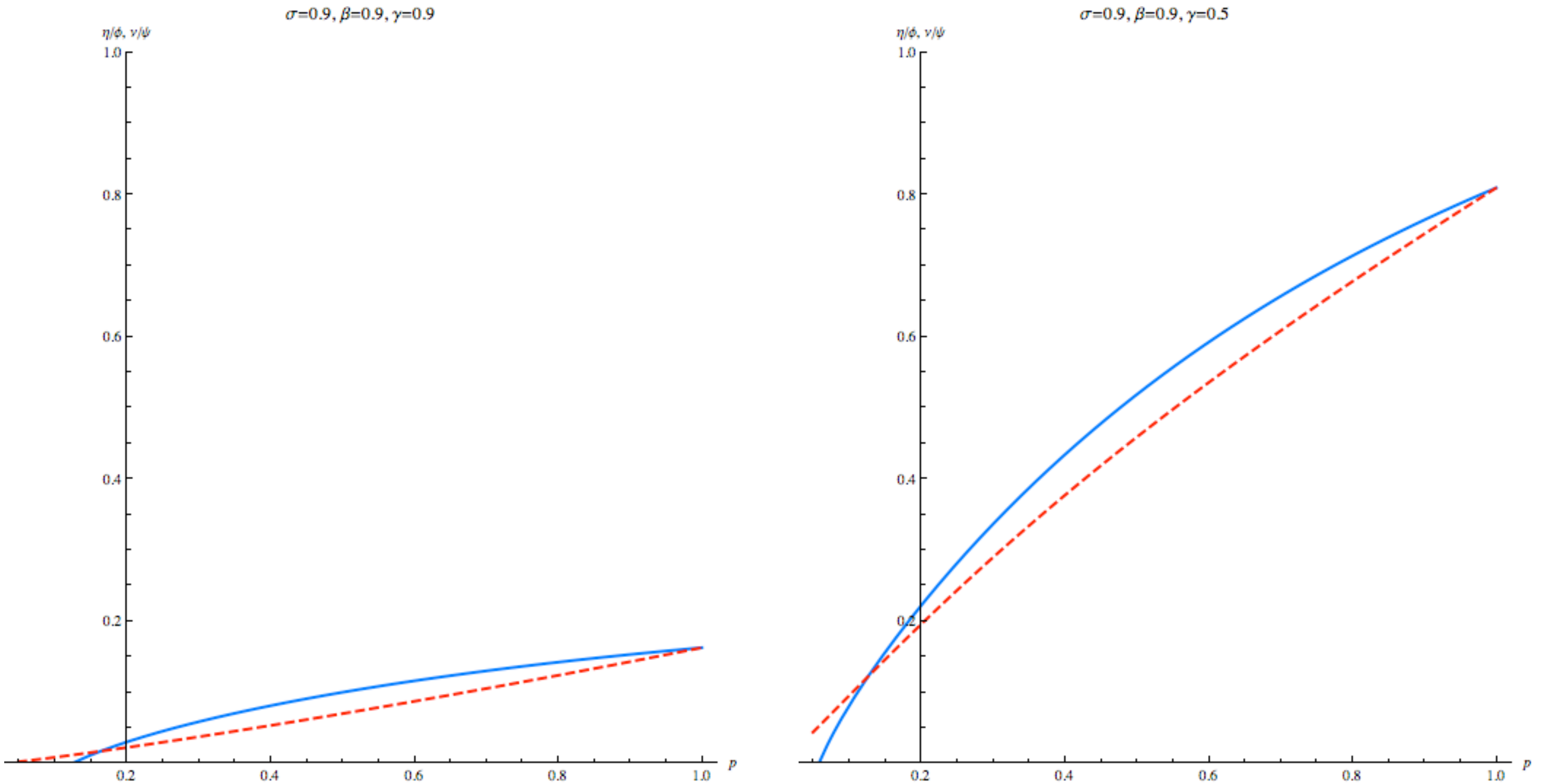

Figure 4

Relative Investment in Human-to-Physical Capital under Annuities (solid blue) and Missing Markets (dotted red)

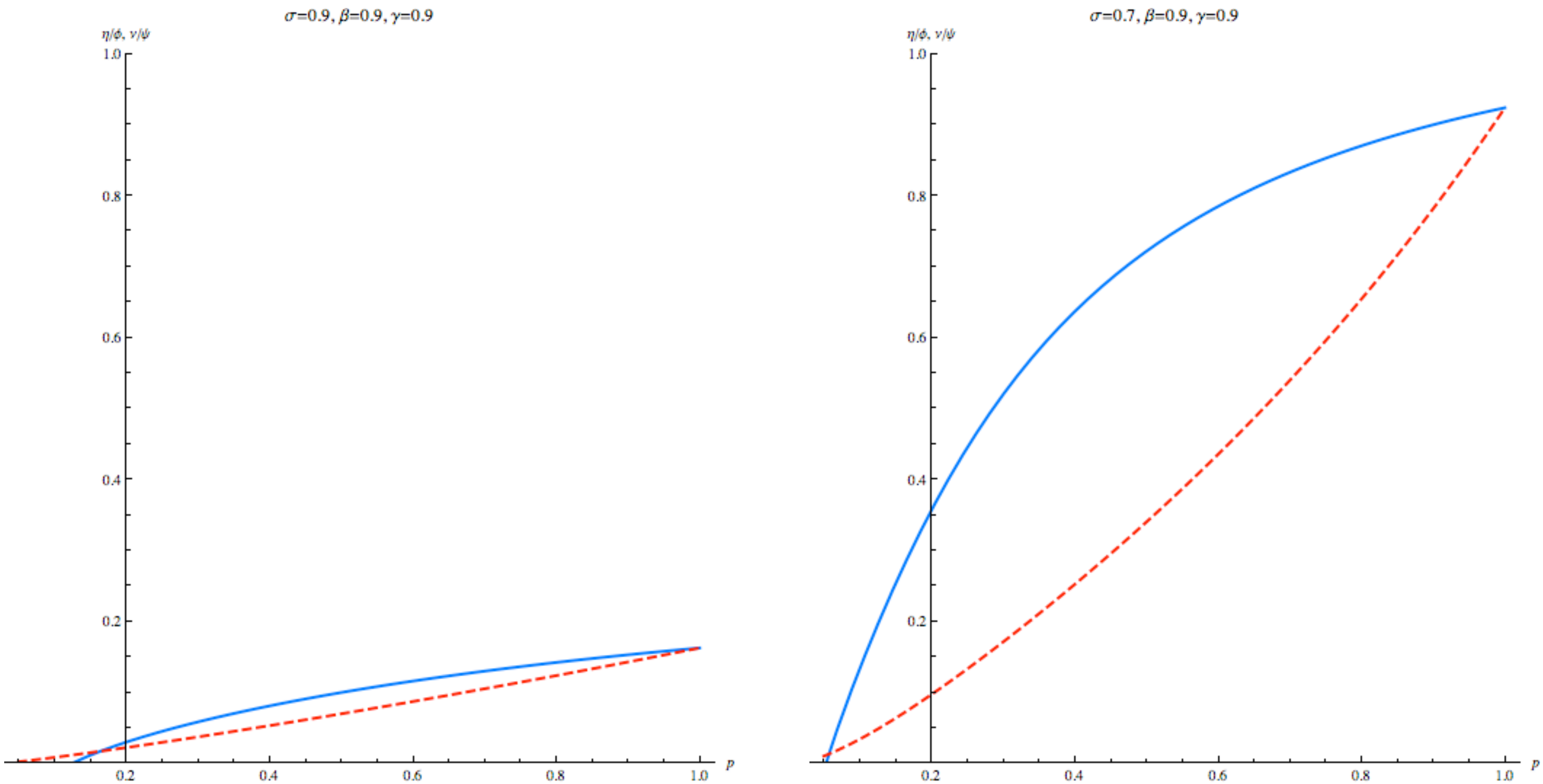

Figure 5

Response of Relative Investment in Human-to-Physical Capital to σ