Catching Up and Falling Behind

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Abstract

This paper studies the interaction between technology, which flows in from abroad, and human capital, which is accumulated domestically, as the twin engines of growth in a developing economy. The model displays two types of long run behavior, depending on policies and initial conditions. One is sustained growth, where the economy keeps pace with the technology frontier. The other is stagnation, where the economy converges to a minimal technology level that is independent of the world frontier. Transitions to the balanced growth path display features seen in modern growth miracles: a high savings rate and rapid investment in education.

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This paper develops a model of growth that can accommodate the enormous differences in observed outcomes across countries and over time: periods of rapid growth as less developed countries catch up to the income levels of those at the frontier, long periods of sustained growth in developed countries, and substantial periods of decline in countries that at one time seemed to be catching up. The sources of growth are technology, which flows in from abroad, and human capital, which is accumulated domestically.

The framework shares several features with the one in Parente and Prescott (1994), including a world technology frontier that grows at a constant rate and "barriers" that impede the inflow of new technologies into particular countries. Technology inflows here are modeled as a pure external effect, with the rate of inflow governed by three factors: (i) the domestic technology gap, relative to a world frontier, (ii) the domestic human capital stock, also relative to the world frontier technology, and (iii) the domestic "barrier." The growth rate of the local technology is an increasing function of the technology gap, reflecting the fact that a larger pool of untapped ideas offers more opportunities for the adopting country. It is also increasing in the local human capital stock, reflecting the role of education in enhancing the ability to absorb new ideas. Finally, the "barrier" reflects tariffs, internal taxes, capital controls, currency controls, or any other policy measures that retard the inflow of ideas and technologies.

A key feature of the model is the interaction between local technology and local human capital. There are many channels through which human capital can facilitate the inflow of ideas. Better educated entrepreneurs and managers are better able to identify new products and processes that are suitable for the local market. In addition, a better educated workforce makes a wider range of new products and processes viable for local production, an important consideration for both domestic entrepreneurs interested in producing locally and foreign multinationals seeking attractive destinations for direct investment.

Human capital is modeled here as a private input into production, accumulated with a technology that uses the agent's own time (current human capital) and the local technology as inputs. Human capital also has an external effect, however, through its impact on the rate of technology inflow. Because of this externality, public subsidies to education affect the long run behavior of the economy.

For fixed values of the technology and preference parameters, the model displays two types of long run behavior, depending on the policies in place and the initial conditions. If the technology barrier is low, the subsidy to education is high, and the initial levels for local technology and local human capital are not too far below the frontier, the economy displays sustained growth in the long run. In this region of policy space, and for suitable initial conditions, higher barriers and lower subsidies to education imply slower convergence to the economy's balanced growth path (BGP) and wider steady state gaps between the local technology and the frontier. But inside this region of policy space, changes in the technology barrier or the education subsidy do not affect the long run growth rate. Policies that widen the steady state technology gap also produce lower levels for capital stocks, output and consumption along the BGP, but the long run growth rate is equal to growth rate of the frontier.

Thus, the model predicts that high and middle income countries can, over long periods, grow at the same rate as the world frontier. In these countries the gap between the local technology and the world frontier is constant in the long run, and not too large.

Alternatively, if the technology barrier is sufficiently high, the subsidy to education is sufficiently low, or some combination, balanced growth is not possible: the economy stagnates in the long run. An economy with policies in this region converges to a minimal technology level that is independent of the world frontier, and a human capital level that depends on the local technology and local education subsidy. In addition, even for parameters that permit balanced growth, for sufficiently low initial levels of technology and human capital, the economy converges to the stagnation steady state instead of the balanced growth path.

Thus, low income countries—those with large technology gaps—cannot display modest, sustained growth, as middle and high income countries can. They can adopt policies that trigger a transition to a BGP, or they can stagnate, falling ever farther behind the frontier. Moreover, economies that enjoyed technology inflows in the past can experience technological regress if they raise their barriers: local TFP and per capita income can actually decline during the transition to a stagnation steady state.

Two policy parameters are included in the model, the barrier to technology inflows and a subsidy to human capital accumulation. Although both can be used to speed up transitional growth, the simulations here suggest that stimulating technology inflows is more the potent tool. The intuition for this is twofold. First, human capital accumulation takes resources away from production, reducing consumption in the short run. In addition, human capital accumulation is necessarily slow. Thus, while it eventually leads to higher technology inflows, the process is prolonged. Faster technology inflows increase output immediately, and in addition they increase the returns to human (and physical) capital, thus stimulating further investment and growth.

The rest of the paper is organized as follows. Section 1 discusses evidence on the importance of technology inflows as a potential source of growth. It also documents the fact that many countries are not enjoying these inflows. Indeed, they are falling ever farther behind the world frontier. The model is described in section 2. Section 3 looks at economies that converge to balanced growth paths, and section 4 looks at economies that stagnate. Section 5 describes the implications of the model for growth and development accounting, and also discusses methods for estimating a key technology parameter. In section 6 the model is calibrated, and in section 7 transition paths are simulated for economies that reduce their barriers and/or increase their subsidies to education. Section 8 concludes.

1. EVIDENCE ON THE SOURCES OF GROWTH

Five types of evidence point to the conclusion that differences in technology are critical for explaining differences in income levels over time and across economies, that there is a common (growing) 'frontier' technology that developed economies share, and that poor economies can grow rapidly by tapping into that world technology.[1]

First, growth accounting exercises for individual developed countries, starting with those in Solow (1957) and Denison (1974), invariably attribute a large share of the increase in output per worker to an increase in total factor productivity (TFP). Although measured TFP in these exercises—the Solow residual—surely includes the influence of other (omitted) factors, the search for the missing factors has been extensive, covering a multitude of potential explanatory variables, many countries, many time periods, and many years of effort. It is difficult to avoid the conclusion that technical change is a major ingredient.

Second, development accounting exercises using cross-country data arrive at a similar conclusion, finding that differences in physical and human capital explain only a modest portion of the differences in income levels across countries. For example, Hall and Jones (1999) find that of the 35-fold difference in GDP per worker between the richest and poorest countries, inputs—physical and human capital per worker—account for 4.5-fold, while differences in TFP—the residual—accounts for 7.7-fold. Klenow and Rodriguez-Clare (1997b) arrive at a similar conclusion.

To be sure, the cross-country studies have a number of limitations. Data on hours are not available for many countries, so output is measured per worker rather than

[1]See Prescott (1997) and Klenow and Rodriguez-Clare (2005) for further evidence supporting this conclusion.

per manhour. No adjustment is made for potential differences between education attainment in the workforce and the population as a whole, which might be much larger in countries with lower average attainment. Human capital is measured very imprecisely, consisting of average years of education in the population with at best a rough adjustment for educational quality.[2] Nor is any adjustment made for other aspects of human capital, such as health.

In development accounting exercises, as in growth accounting, the figure for TFP is a residual, so it is surely biased upward. Nevertheless, it is large enough to absorb a substantial amount of downward revision and survive as a key determinant of cross-country differences.

A third piece of evidence is Baumol's (1986) study of the OECD countries. Although criticized on methodological grounds (see DeLong, 1988, and Baumol and Wolff, 1988), the data nevertheless convey an important message: the OECD countries (and a few more) seem to share common technologies. It is hard to explain in any other way the harmony—over many decades—in both their income levels and growth rates. Moreover, as Prescott (2002, 2004) and Ragan (2006) have shown, much of the persistent differences in income levels can be explained by differences in fiscal policy that affect work incentives.

A fourth piece of evidence for the importance of technology comes from data on 'late bloomers.' As first noted by Gerschenkron (1962), economies that develop later have an advantage over the early starters exactly because they can adopt technologies, methods of organization, and so on developed by the leaders. Followers can learn from the successes of their predecessors and avoid their mistakes. Parente and Prescott's (1994, 2000) evidence on doubling times makes this point systematically. Figure 1

[2] Differences in educational quality probably have a modest impact, however. Hendricks (2002) reports that many studies find that immigrants' earnings are within 25% of earnings of native-born workers with the same age, sex, and educational attainment.

reproduces their scatter plot, updated to include data through 2006. Each point in this figure represents one of the 55 countries that had reached a per capita GDP of \$4000 by 2006. On the horizontal axis is the year that the country first reached \$2000, and on the vertical axis is the number of years required to first reach \$4000.

As Figure 1 shows, there is a strong downward trend: countries that arrived at the \$2000 figure later, doubled their incomes more quickly. The later developers seem to have enjoyed the advantage of fishing from a richer pool of ideas, ideas provided by advances in an ever-improving world technology.[3]

A fifth and final piece of evidence supporting the importance of technology is the occurrence, infrequently, of 'growth miracles.' The term is far from precise, and a stringent criterion should be used in classifying countries as such, since growth rates show little persistence from one decade to the next. Indeed, mean reversion in income levels following a financial crisis or similar event implies that an especially bad decade in terms of growth rates is likely to be followed by a good one. But recovery from a disaster is not a miracle.

Nevertheless, over the period 1950-2006 twelve countries (i) enjoyed at least one 20-year episode where average per capita GDP growth exceeded 5%, and (ii) in 2006 had GDP per capita that was at least 45% of the U.S. This group has five members in Europe (Germany, Italy, Greece, Portugal, and Spain), five in east Asia (Japan, Taiwan, Hong Kong, Singapore, and Korea), and two others (Israel and Puerto Rico).

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[3] Figure 1 has a built-in bias, which should be noted. Among countries that have recently reached the \$2000 figure, many have not yet reached \$4000. In particular, the slow growers have not yet reached that goal. The dotted line indicates a region where, by construction, there cannot yet be any observations. Ignoring the pool of countries that in the future will occupy this space biases the impression in favor of the 'advantage of backwardness' hypothesis. An easy way to mitigate the bias is to truncate the last twenty years of data, which eliminates many of the observations for which the \$4000 goal lies in the future. The strong downward trend seems to survive this truncation.

The jury is still out on several others candidates: China, Thailand, Malaysia, and Botswana have met criterion (i) but not (yet) accomplished (ii).[4]

Although each of these five sources of evidence has individual weaknesses, taken together they make a strong case for the importance of international technology spillovers in keeping income levels loosely tied together in the developed countries, and occasionally allowing a less developed country to enjoy a growth spurt during which it catches up to the more developed group.

Not all countries succeed in tapping into the global technology pool, however. Figure 2 shows the world pattern of catching up and falling behind, with the U.S. taken as the benchmark for growth. It plots per capita GDP relative to the U.S. in 2000, against per capita GDP relative to the U.S. in 1960 for 104 countries. Countries that are above the 45^o line have gained ground over that 40-year period, and those below it have lost ground. It is striking how few have gained. The geographic pattern of gains and losses is striking as well. The countries that are catching up are almost exclusively European (plus Israel) and East Asian. With only a few exceptions, countries in Latin America, Africa, and South Asia have fallen behind.

Figure 3 shows 69 of the poorest countries, those in Asia and Africa, in more detail. The six Asian miracles (Israel, Japan, Taiwan, Hong Kong, Singapore, and Korea) are omitted, since they significantly alter the scale.[5] Here the plot shows per capita GDP in 2000 against per capita GDP in 1960, both in levels. The rays from the origin correspond to various average growth rates, and keeping pace with the U.S. over this period requires a growth rate of 2%. The number of countries that have

[4]It is sobering to see how many countries enjoyed 20-year miracles, yet gave up all their (relative) gains or even lost ground over the longer period. Among countries in this groups are Bulgaria, Yugoslavia, Jamaica, North Korea, Iran, Gabon, Libya, and Swaziland.

[5]Also excluded are the oil producers Bahrain, Oman, Saudia Arabia, and countries with population under 1.2 million in 2000, Bahrain, Cape Verde, Comoro Islands, Djibouti, Equatorial Guinea, Mauritius, Reunion, Seychelles Islands.

gained ground relative to the U.S. (points above the 2% growth line) is modest, while the number that have fallen behind (points below the line) is much larger. A shocking number, mostly in Africa, have suffered negative growth over the whole period.

The model developed below focuses on technology inflows as the only source of sustained growth. Other factors are neglected, although some are clearly important.

For example, it is well documented that in most developing economies, TFP in agriculture is substantially lower than it is in the non-agricultural sector.[6] Thus, an important component of growth in almost every fast-growing economy has been the shift of labor out of agriculture and into other occupations. The effect of this shift on aggregate TFP, which is significant, cannot be captured in a one-sector model.

More recently, detailed data for manufacturing has allowed similar estimates for the gains from re-allocation across firms within that single sector. Such misallocation can result from financial market frictions, from frictions that impede labor mobility, or from taxes (or other policies) that distort factor prices.

For example, Hsieh and Klenow (2009) find that in China, improvements in allocative efficiency contributed about 1/3 of the 6.2% TFP growth in manufacturing over the period 1993-2004. Although this gain is substantial, it is a modest part of overall TFP growth in China—in all sectors—over that period.

In addition, evidence from other countries gives reallocation a smaller role. Indeed, Hsieh and Klenow find that in India allocative efficiency declined over the same period, although per capita income grew. In addition, Bartelsman, Haltiwanger, and Scarpetta (2008) find that for the two countries (Slovenia and Hungary) for which they have time series, most of the (substantial) TFP gains both countries enjoyed during the 1990s came from other sources: improvements in allocative efficiency played a minor role. And most importantly, TFP gains from resource reallocation are one-time

[6]See Caselli (2005) for recent evidence that TFP differences across countries are much greater in agriculture than they are in the non-agricultural sector.

gains, not a recipe for sustained growth.[7]

The model here is silent about the ultimate source of advances in the technology frontier, which are taken as exogenous. Thus, it is complementary to the many models that address the sources of technical change more directly, looking at the incentives to invest in R&D, the role of learning by doing, and other factors that affect the pace of innovation.

Nor does the model her have anything to say about the societal factors that lead countries to develop institutions or adopt policies that stimulate or hinder growth, by stimulating technology inflows, encouraging factor accumulation, or any other means. In this sense is complementary to the work of Acemoglu, Johnson, and Simon (2001, 2002, 2005) and others, that looks at the country characteristics associated with economic success, without specifying the more proximate mechanism.

2. THE MODEL

The model is a variant of the technology diffusion model first put forward in Nelson and Phelps (1966) and subsequently developed elsewhere in many specific forms.[8]

[7] In addition, it is not clear what the standard for allocative efficiency should be in a fast-growing economy. Restuccia and Rogerson (2008) develop a model with entry, exit, and fixed costs that produces a non-degenerate distribution of productivity across firms, even in steady state. Their model has the property that the stationary distribution across firms is sensitive to the fixed cost of staying in business and the distribution of productivity draws for potential entrants. There is no direct evidence for either of these important components, although they can be calibrated to any observed distribution. Thus, it is not clear if differences across countries reflect distortions that affect the allocation of factors, or if they represent differences in fundamentals, especially in the 'pool' of technologies that new entrants are drawing from. In particular, one might suppose that the distribution of productivities for new entrants would be quite different in a fast-growing economy (like China) and a mature, slow-growing economy (like the U.S.).

[8] See Benhabib and Spiegel (2005) for an excellent discussion of the long-run dynamics of various versions.

There is a frontier (world) technology $W(t)$ that grows at a constant (exogenously given) rate,

$$\dot{W}(t) = gW(t), \tag{1}$$

where $g > 0$, and in addition each country i has a local technology $A_i(t)$. Growth in $A_i(t)$ has the form

$$\begin{aligned} \frac{\dot{A}_i}{A_i} &= 0, \qquad \text{if} \quad A_i = A_i^{st} \quad \text{and} \quad \frac{\psi_0}{B_i}\frac{\overline{H}_i}{W}\left(1 - \frac{A_i^{st}}{W}\right) < \delta_A, \\ \frac{\dot{A}_i}{A_i} &= \frac{\psi_0}{B_i}\frac{\overline{H}_i}{W}\left(1 - \frac{A_i}{W}\right) - \delta_A, \qquad \text{otherwise,} \end{aligned} \tag{2}$$

where A_i^{st} is a lower bound on the local technology level, B_i is a policy parameter, $\overline{H}_i$ is average human capital, and $\delta_A > 0$ is the depreciation rate for technology.

The technology floor A_i^{st} allows the economy to have a 'stagnation' steady state with a constant technology. Above that floor technology growth is proportional to the ratio $\overline{H}_i/W$ of local human capital to the frontier technology and to the relative gap $1 - A_i/W$ between the current technology and the frontier. The former measures the capacity of the economy to absorb technologies near the frontier, while the latter measures the pool of technologies that have not yet been adopted.

A higher value for the frontier technology W thus has two effects. It widens the technology gap, which tends to speed up growth, but also reduces the absorption capacity, which tends to retard growth. The first effect dominates if the ratio A_i/W is high and the second if H_i/W is low, resulting in a logistic form.[9]

As in Parente and Prescott (1994), the barrier $B_i \geq 1$ can be interpreted as any policies that impede access to or adoption of new ideas, or reduce the profitability of adoption. For example, it might represent impediments to international trade that reduce contact with new technologies, taxes on capital equipment that is needed to

[9]Benhabib and Spiegel (2005, Table 2) find that cross-country evidence on the rate of TFP growth supports the logistic form: countries with very low TFP also have slower TFP growth. Their evidence also seems to support the inclusion of a depreciation term.

implement new technologies, poor infrastructure for electric power or transportation, or civil conflict that impedes the flow of people and ideas across border. Notice that because of depreciation, technological regress is possible. Thus, an increase in the barrier can lead to the abandonment of once-used technologies.

Growth in A_i is exogenous to the individual household in country i, although it depends on collective household decisions through $\overline{H}_i$. Next consider the decisions of a typical household.

Households accumulate capital, which they rent to firms, and in addition each household is endowed with one unit of time (a flow), which it allocates between human capital accumulation and goods production. Investment in human capital uses the household's own time and human capital as inputs, as well as the technology level. In particular,

$$\dot{H}_i(t) = \phi_0 \left[v_i(t)H_i(t)\right]^{\eta} A_i(t)^{1-\eta} - \delta_H H_i(t), \tag{3}$$

where v_i is time devoted to human capital accumulation, $\delta_H > 0$ is the depreciation rate for human capital, and $0 < \eta < 1$. This technology has CRS jointly in the stocks (A, H) and DRS in the time input. The former fact permits sustained growth. The latter feature rules out the possibility of endogenous growth in the absence of technology diffusion (without increases in A_i), and also insures that time allocated to human capital accumulation is always strictly positive.

Time allocated to production is also augmented by technology and human capital to produce effective labor input. Specifically, the labor offered by a household in country i is augmented by the country technology A_i, which it takes as exogenous, and its own human capital H_i. Thus its effective labor is

$$L_i(t) = \left[1 - v_i(t)\right] A_i^{\beta}(t) H_i^{1-\beta}(t), \tag{4}$$

where $(1 - v_i)$ is time allocated to goods production.

The technology for goods production is Cobb-Douglas, with physical capital K_i

and effective labor L_i as inputs,

$$Y_i = K_i^{\alpha} L_i^{1-\alpha},$$

so factor returns are

$$\begin{aligned} R_i(t) &= \alpha \left(\frac{K_i}{L_i}\right)^{\alpha-1}, \\ \hat{w}_i(t) &\equiv (1-\alpha)\left(\frac{K_i}{L_i}\right)^{\alpha}, \end{aligned} \tag{5}$$

where $\hat{w}$ is the return to a unit of effective labor. Hence the wage for a worker with human capital H is

$$w_i(H,t) = \hat{w}_i A_i^{\beta} H^{1-\beta}. \tag{6}$$

To allow balanced growth, subsidies to education can be incorporated as follows. Time spent investing in human capital is subsidized at the rate $\sigma_i w_i(\overline{H}_i, t)$, where $\overline{H}_i$ is the average human capital level in the economy and $\sigma_i \in [0, 1)$ is a policy variable. Thus, the subsidy is a fraction of the current average wage. The subsidy is financed with a lump sum tax τ_i, and the government's budget is balanced at all dates,

$$\tau_i(t) = v\sigma_i w_i(\overline{H}_i), \qquad t \geq 0. \tag{7}$$

Thus, the budget constraint for the household is

$$\dot{K}_i(t) = (1-v_i)\, w_i(H_i) + v\sigma_i w_i(\overline{H}_i) + (R_i - \delta_K)\, K_i - C_i - \tau_i, \tag{8}$$

where $\delta_K > 0$ is the depreciation rate for physical capital. Households have constant elasticity preferences, with parameters $\rho, \theta > 0$. Hence the household's problem is

$$\max_{\{v_i(t), C_i(t)\}_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} \frac{C_i^{1-\theta}(t)}{1-\theta} \qquad \text{s.t. (3) and (8)}, \tag{9}$$

given σ_i, H_{i0}, K_{i0} and $\left\{A_i(t), \overline{H}(t), R_i(t), w_i(\cdot, t), \tau_i(t), \quad t \geq 0\right\}$.

DEFINITION: Given the policy parameters B_i and σ_i and initial values for the state variables W_0, A_{i0}, H_{i0}, and K_{i0}, a *competitive equilibrium* consists of $\{A_i(t), H_i(t), \overline{H}_i(t), K_i(t), v_i(t), C_i(t), R_i(t), w_i(\cdot, t), \tau_i(t), \quad t \geq 0\}$ with the property that:

i. $\{v_i, C_i, H_i, K_i\}$ solves (9), given σ_i, H_{i0}, K_{i0}, and $\{A_i, \overline{H}_i, R_i, w_i, \tau_i\}$;

ii. $\{R_i, w_i\}$ satisfy (5) and (6);

iii. $\{A_i\}$ satisfies (2) and $\{\overline{H}_i(t) = H_i(t), \quad t \geq 0\}$;

iv. $\{\tau_i\}$ satisfies (7).

The system of equations describing the equilibrium is developed in the Appendix. Two types of behavior are possible in the long run. If the barrier B_i is low enough and/or the subsidy σ_i to education is high enough, balanced growth is possible. Specifically, for some initial conditions the economy converges asymptotically to a BGP, along which A_i, H_i, K_i and C_i all grow at the rate g. Economies of this type are studied in section 3.

Balanced growth is not the only possibility, however. The set of initial conditions that lead to balanced growth shrinks with increases in B_i and reductions in σ_i, at some point disappearing altogether. Thus, if B_i is sufficiently large and/or σ_i is sufficiently small, or if the initial values for A_i and H_i are sufficiently small relative to W, the economy stagnates in the long run. In these economies the technology level converges to its stagnation level A_i^{st}, the stocks of human and physical capital adjust accordingly to H_i^{st}, K_i^{st}, and consumption converges to a constant level C_i^{st}. Economies of this sort are studied in section 4.

3. CATCHING UP: ECONOMIES THAT GROW

For convenience drop the country subscript. In this section BGPs are characterized, where the time allocation $v > 0$ is constant and the state variables A, H, K grow at constant rates. It is easy to show that A, H, K, L, and C grow at the same rate g as the frontier technology, the factor returns R and $\hat{w}$ are constant, the wage $w(H)$ grows at the rate g, and the costate variables Λ_H and Λ_K for the household's problem grow at the rate $-\theta g$.

To study growing economies it is convenient to define the normalized variables

$$\begin{aligned} a(t) &\equiv \frac{A(t)}{W(t)}, \qquad h(t) \equiv \frac{H(t)}{W(t)}, \\ k(t) &\equiv \frac{K(t)}{W(t)}, \qquad c(t) \equiv \frac{C(t)}{W(t)}, \\ \lambda_k(t) &\equiv \frac{\Lambda_K(t)}{W^{-\theta}(t)}, \qquad \lambda_h(t) \equiv \frac{\Lambda_H(t)}{W^{-\theta}(t)}. \end{aligned}$$

The equilibrium conditions can then be written as

$$\begin{aligned} \lambda_h \eta \phi_0 v^{\eta-1} &= (1-\sigma)\,\lambda_k \hat{w} \left(\frac{a}{h}\right)^{\beta+\eta-1}, \qquad (10) \\ c^{-\theta} &= \lambda_k, \\ \frac{\dot{\lambda}_h}{\lambda_h} &= \rho + \theta g + \delta_H - \phi_0 \left(\frac{a}{h}\right)^{1-\eta} v^\eta \eta \left(1 + \frac{1-\beta}{1-\sigma}\frac{1-v}{v}\right), \\ \frac{\dot{\lambda}_k}{\lambda_k} &= \rho + \theta g + \delta_K - R, \\ \frac{\dot{h}}{h} &= \phi_0 v^\eta \left(\frac{a}{h}\right)^{1-\eta} - \delta_H - g, \\ \frac{\dot{k}}{k} &= \kappa^{\alpha-1} - \frac{c}{k} - \delta_K - g, \\ \frac{\dot{a}}{a} &= \frac{\psi_0}{B_i} h\,(1-a) - \delta_A - g, \end{aligned}$$

where

$$\begin{aligned} \kappa &\equiv K/L = k/\,(1-v)\,a^\beta h^{1-\beta}, \qquad (11) \\ R &= \alpha\kappa^{\alpha-1}, \qquad \hat{w} = (1-\alpha)\,\kappa^\alpha, \end{aligned}$$

and the transversality conditions hold if and only if

$$\rho > (1-\theta)\,g, \qquad (12)$$

which insures that the discounted value of lifetime utility is finite.

a. The balanced growth path

Let a^{bg}, h^{bg}, and so on denote the constant values for the normalized variables along the BGP, determined by setting $\dot{\lambda}_h/\lambda_h = \dot{h}/h = ... = 0$ in (10). The interest rate is

$$r^{bg} \equiv R^{bg} - \delta_K = \rho + \theta g,$$

so (12) implies $r^{bg} > g$. The input ratio κ^{bg} then satisfies

$$\alpha \left(\kappa^{bg}\right)^{\alpha-1} = R^{bg} = r^{bg} + \delta_K,$$

which in turn determines the return to effective labor $\hat{w}^{bg}$.

Then use the third and fifth equations in (10) to find that time allocated to human capital accumulation is

$$v^{bg} = \left[1 + \frac{1-\sigma}{1-\beta}\left(\frac{r^{bg}+\delta_H}{g+\delta_H}\frac{1}{\eta} - 1\right)\right]^{-1}. \tag{13}$$

Since $r^{bg} > g$ and $\eta < 1$, the second term in brackets is positive and $v^{bg} \in (0,1)$. Notice that v^{bg} is increasing in the subsidy σ, with $v^{bg} \to 1$ as $\sigma \to 1$. It is also increasing in $1-\beta$, the share for human capital, and in η, the exponent on v, in the production function for human capital. A higher value for $1-\beta$ increases the sensitivity of the wage rate $w(H)$ to private human capital, increasing the incentives to invest, while a higher value for η reduces the force of diminishing returns. Finally, v^{bg} is increasing δ_H and g, reflecting the fact the time allocated to investment in human capital must offset depreciation and allow the stock H to keep pace with W.

The fifth equation in (10) then determines the ratio of technology to human capital,

$$z^{bg} \equiv \frac{a^{bg}}{h^{bg}} = \left(\frac{g+\delta_H}{\phi_0\left(v^{bg}\right)^{\eta}}\right)^{1/(1-\eta)}. \tag{14}$$

Hence this ratio is decreasing in the subsidy rate σ. Note that v^{bg} and z^{bg} do not depend on the barrier B.

The last equation in (10) then implies that a BGP level for the relative technology a^{bg}, if any exists, satisfies the quadratic

$$a^{bg}\left(1-a^{bg}\right)=(g+\delta_A)\frac{B}{\psi_0}z^{bg}. \tag{15}$$

The solutions are as follows.

PROPOSITION 1: If

$$(g+\delta_A)\frac{Bz^{bg}}{\psi_0}<\frac{1}{4}, \tag{16}$$

then (15) has two solutions, and they are symmetric around the value 1/2. Call the higher and lower solutions a_H^{bg} and a_L^{bg}, with

$$0<a_L^{bg}<\frac{1}{2}<a_H^{bg}.$$

If the inequality in (16) is reversed, then no BGP exists.

Figure 4 displays the solutions as functions of B, for two values of σ and fixed values for the other parameters. With σ fixed, a small increase in B moves both solutions toward the value 1/2. For fixed B, a small decrease in σ, which increases z^{bg}, also moves the solutions toward 1/2. For B sufficiently large or σ sufficiently small, the inequality in (16) fails, and no BGP exists.

Notice that the higher solution a_H^{bg} has the expected comparative statics—it increases as the barrier B falls or the subsidy σ rises—while the lower solution a_L^{bg} has the opposite pattern. As we will see later, for reasonable parameter values a_H^{bg} is stable and a_L^{bg} is not. Therefore, since $a_H^{bg}\in[1/2,\ 1]$, this model produce BGP productivity (and income) ratios of no more than two across growing economies.[10] Poor economies cannot grow along BGPs, in parallel with richer ones.

[10] Other factors, like taxes on labor income (which reduce labor supply) can increase the spread in incomes across growing economies. See Prescott 2002, 2004, and Ragan, 2006, for models of this type.

If a BGP exists, changes in B do not affect the growth rate g, the interest rate r^{bg}, the time allocation v^{bg}, or the ratio z^{bg} of technology to human capital. Thus, looking across economies with similar education policies, along BGPs those with higher barriers lag farther behind the frontier, but in all other respects are similar. Stated a little differently, an economy with a higher barrier looks like its neighbor with a lower one, but with a time lag.

b. Transitional dynamics

The normalized system has three state variables, a, h, and k, and two costates, λ_h and λ_k, making the transitional dynamics complicated. The more interesting interactions involve technology and human capital, with physical capital playing a less important role. Thus, for simplicity we will drop physical capital.

To this end, set $\alpha = 0$ and drop the equations for $\dot{k}$ and $\dot{\lambda}_k$. Then $\hat{w} = 1, R = 0$, and all output is consumed, so

$$c = (1 - v)\, a^{\beta} h^{1-\beta},$$

and $c^{-\theta}$ takes the place of λ_k. The first equation in (10) then implies that the time allocation satisfies

$$v^{\eta-1} (1 - v)^{\theta} = \frac{1 - \sigma}{\eta \phi_0} a^{\Delta} h^{-(\Delta+\theta)} \lambda_h^{-1}, \tag{17}$$

where

$$\Delta \equiv \beta (1 - \theta) - (1 - \eta).$$

The transitional dynamics are then described by the remaining three equations in (10), for $\dot{\lambda}_h/\lambda_h$, $\dot{h}/h$, and $\dot{a}/a$.

This system of three equations can be linearized around each of the two steady states, and the characteristic roots determine the long run behavior of the economy. As shown in the Appendix, a sufficient condition for all of the roots to be real is

$\theta \geq 1$. Further conditions that insure exactly two negative roots at a_H^{bg} and exactly one negative root at a_L^{bg} are also discussed. All of the simulations reported here have this pattern for the roots..

Under these conditions the higher steady is locally stable: for any pair of initial conditions a_0, h_0 in the neighborhood of $\left(a_H^{bg}, h_H^{bg}\right)$, there exists a unique initial condition λ_{h0} for the costate with the property that the system converges asymptotically. The lower steady state is not locally stable. Instead, there is a one-dimensional manifold of initial conditions a_0, h_0 in the neighborhood of $\left(a_L^{bg}, h_L^{bg}\right)$ for which the system converges. This manifold defines the boundary of the trapping region for the high (stable) steady state. The dynamics are described in more detail in section 7.

4. FALLING BEHIND: ECONOMIES THAT STAGNATE

In economies where the barrier B is sufficiently high and/or the educational subsidy σ is sufficiently low, condition (16) in Proposition 1 fails and there is no BGP. These economies stagnate in the long run. In addition, even if the policies permit a BGP, for sufficiently low initial conditions the economy converges to the stagnation steady state rather than the BGP. These economies may grow—slowly—during a transition period. But their rate of technology adoption is always less than g, and it declines over time, eventually converging to zero. In this section we will describe the stagnation steady state and transitions to it.

a. The stagnation steady state

At the stagnation steady state the technology level, capital stocks, factor returns, consumption, and time allocation are constant, as are the costates for the household's problem. Let A^{st}, H^{st}, K^{st}, and so on denote these levels. Since consumption is constant, the interest rate is equal to the rate of time preference,

$$r^{st} = R^{st} - \delta_K = \rho,$$

the input ratio κ^{st} satisfies

$$\alpha \left(\kappa^{st}\right)^{\alpha-1} = R^{st} = \rho + \delta_K,$$

and the return to effective labor $\hat{w}^{st}$ depends only on κ^{st}. The steady state time allocation and input ratio, the analogs of (13) and (14), are

$$v^{st} = \left[1 + \frac{1}{\eta}\frac{1-\sigma}{1-\beta}\left(1-\eta+\frac{\rho}{\delta_H}\right)\right]^{-1}, \tag{18}$$

$$z^{st} \equiv \frac{A^{st}}{H^{st}} = \left(\frac{\delta_H}{\phi_0 \left(v^{st}\right)^{\eta}}\right)^{1/(1-\eta)}, \tag{19}$$

so they are like those on a BGP except that the interest rate is $r = \rho$ and there is no growth.

Notice that for economies with the same educational subsidy σ, more time is allocated to human capital accumulation in the growing economy, $v^{bg} > v^{st}$, if and only if

$$\rho > (\theta - 1)\,\delta_H.$$

If θ is sufficiently large, the low willingness to substitute intertemporally discourages investment in the growing economy.

In growing and stagnant economies with the same time allocation, the one that is growing has a higher ratio of technology to human capital,

$$\frac{z^{bg}}{z^{st}} = \left(1 + \frac{g}{\delta_H}\right)^{1/(1-\eta)} > 1.$$

For $\rho = \delta_H$ and $\theta = 2$, as will be assumed below, the steady state time allocations are the same, $v^{st} = v^{bg}$.

b. Transitional dynamics

To study dynamics near the stagnation steady state we can proceed as before, setting $\alpha = 0$ to eliminate physical capital, so $\hat{w} = 1$ and all output is consumed. The time allocation during the transition then satisfies the analog of (17),

$$v^{\eta-1}(1-v)^{\theta} = \frac{1-\sigma}{\eta\phi_0} A^{\Delta} H^{-(\Delta+\theta)} \Lambda_H^{-1}, \tag{20}$$

where as before $\Delta \equiv \beta(1-\theta) - (1-\eta)$. The laws of motion for A, H and Λ_H can then be used to study the transition.

5. DIGRESSION ON GROWTH ACCOUNTING

Consider a world with many economies, $i = 1, 2, ..., I$. In each economy i, output per capita at any date t is

$$Y_i(t) = K_i(t)^{\alpha} \left\{ A_i(t)^{\beta} \overline{H}_i(t)^{1-\beta} \left[1 - v_i(t)\right] \right\}^{1-\alpha}.$$

Assume that at each date, any individual is engaged in only one activity, and hours per worker are the same over time and across countries. Then all differences in v—in time allocation between production and human capital accumulation—take the form of differences in labor force participation. Let

$$y_i(t) \equiv \frac{Y_i(t)}{1-v_i(t)} \qquad \text{and} \qquad k_i(t) \equiv \frac{K_i(t)}{1-v_i(t)}$$

denote output and capital per worker, and note that human capital $\overline{H}_i$ is already so measured. Then output per worker can be written three ways,

$$\begin{aligned} y_i(t) &= k_i(t)^{\alpha} \left[\overline{H}_i(t)^{1-\beta} A_i(t)^{\beta}\right]^{1-\alpha}, \qquad (21) \\ y_i(t) &= \left(\frac{k_i(t)}{y_i(t)}\right)^{\alpha/(1-\alpha)} \overline{H}_i(t)^{1-\beta} A_i(t)^{\beta}, \\ y_i(t) &= \left(\frac{k_i(t)}{y_i(t)}\right)^{\alpha/\beta(1-\alpha)} \left(\frac{\overline{H}_i(t)}{y_i(t)}\right)^{(1-\beta)/\beta(1-\alpha)} A_i(t). \end{aligned}$$

Suppose that α and β are known. The income shares for capital and labor, α and $1-\alpha$, could be obtained from NIPA data for any country, and a method for estimating β is described below. Suppose in addition that $\overline{H}_i$ is observable, as well as y_i and k_i. The first equation in (21) is the standard basis for a growth accounting exercise, as in Solow (1957); the second is the version used for the development accounting exercises in Hall and Jones (1999), Hendricks (2002), and elsewhere; and the third is a variation suitable for the model here. In each case the technology level A_i is treated as a residual.

First consider a single economy. A growth accounting exercise based on the first line in (21) in general attributes growth in output per worker to growth in all three inputs, $k_i, \overline{H}_i$ and A_i, and along a BGP the shares are α, $(1-\beta)(1-\alpha)$, and $\beta(1-\alpha)$.

An accounting exercise based on the second line attributes some growth to physical capital only if the ratio k_i/y_i is growing. The rationale for using the ratio is that growth in $\overline{H}_i$ or A_i induces growth in K_i, by raising its return. Here the accounting exercise attributes to growth in capital only increases in excess of those prompted by growth in effective labor. Along a BGP k_i/y_i is constant, and the exercise attributes all growth to $\overline{H}_i$ and A_i, with shares $(1-\beta)$ and β.

The third line applies the same logic to human capital, since growth in A_i induces growth in $\overline{H}_i$ as well as K_i. Since $\overline{H}_i/y_i$ is also constant along a BGP, here the accounting exercise attributes all growth on a BGP to the residual A_i.

Next consider a development accounting exercise involving many countries. Differences across countries in capital taxes, public support to education, and other policies lead to differences in the ratios k_i/y_i and $\overline{H}_i/y_i$, so a development accounting exercise using any of the three versions attributes some differences in labor productivity to differences in physical and human capital. But the second line attributes to physical capital—and the third to both types of capital—only differences in excess of those induced by changes in the supplies of the complementary factor(s), $\overline{H}_i$ and A_i in the second line and A_i in the third.

6. CALIBRATION

The model parameters are the long run growth rate g, the preference parameters (ρ, θ), the share β for technology in producing effective labor, the parameters (ϕ_0, η, δ_H) for human capital accumulation and (ψ_0, δ_A) for technology diffusion, and the policy parameters (σ, B) . Baseline values for these parameters are described below. Experiments with some alternative values are also conducted, to assess the sensitivity of the results.

Growth rate, preferences.—

The growth rate is set at $g = 0.019$, which is the rate of growth of per capita GDP in the U.S. over the period 1870-2003.

The preference parameters are set at $\rho = 0.03$ and $\theta = 2$, which are within the range that is standard in the macro literature.

Calibrating β.—

If the model with many countries is extended to allow some international migration, an estimate of β can be obtained from data on the wages of migrants. The main idea is that any individual's wage rate reflects two components: his own human capital $\hat{H}$ and the technology A_i in the country where he is employed, with weights $1-\beta$ and β. A sample of migrants provides independent variation in H and A, which are highly correlated for native-born workers.[11]

The method discussed here uses cross-sectional data on the wages of immigrants

[11]Heterogeneity in the education subsidy provides some variation, but it would be difficult to get a sharp estimate of β from this kind of data.

from various source countries in a single destination country, but a similar exercise could be carried out with data on emigrants from a single source country working in many destinations, or with combined data. The number of immigrants should be large enough to generate a sample of reasonable size, but small enough relative to the size of the native population in the destination country so that productivity and wage rates there reflect the skills of the native-born. The relevant regression can also accommodate heterogeneity across individual migrants in education and other components of human capital like age and experience.

From (6), the wage of a migrant from country j to country i depends on the return to effective labor $\hat{w}_i$ and the technology A_i in the country i where he is employed, and the human capital $\hat{H}$ that he acquired in his country of origin, where he acquired his education. Thus, the wage in country i of an individual with human capital $\hat{H}$ is

$$W_i(\hat{H}) = \hat{w}_i A_i^{\beta} \hat{H}^{1-\beta},$$

and his wage relative to the wage of the average native-born worker in i is

$$\frac{W_i(\hat{H})}{W_i(\overline{H}_i)} = \left(\frac{\hat{H}}{\overline{H}_i} \right)^{1-\beta}. \tag{22}$$

The right side of (22) involves human capital, which is not directly observable. To relate it to observables, suppose countries i and j are both on their BGPs. In addition, suppose that there is some idiosyncratic variation across dynasties, which produces idiosyncratic variation in their time allocation decisions. In particular, suppose that along the BGP each dynasty in j chooses a constant time allocation, and that their average across dynasties is v_j^{bg}. The human capital level $\hat{H}$ of the immigrant depends on the time allocation decision $\hat{v}$ of his own dynasty and the technology A_j in his country of origin. In particular, along the BGP his human capital is

$$\hat{H} = H_j(\hat{v}) = \left(\frac{\phi_0}{g + \delta_H} \right)^{1/(1-\eta)} \hat{v}^{\eta/(1-\eta)} A_j.$$

A similar relationship holds for $\overline{H}_i$, so

$$\frac{\hat{H}}{\overline{\overline{H}}} = \left(\frac{\hat{v}}{v_i^{bg}}\right)^{\eta/(1-\eta)} \frac{A_j}{A_i}, \tag{23}$$

where v_i^{bg} is the average time allocation (educational attainment) in country i. The first term represents time allocated to human capital accumulation. Here we will identify $\hat{v}$ with years of schooling, which is observable. The second term, which represents the effect of technology in improving the efficacy of time spent in school, is not directly observable. The technology also appears in the production function for goods, however, so output per worker can be used to draw inferences about the technology.

Specifically, for the second term note that output per worker in j relative to i is

$$\begin{aligned}
\frac{y_j(t)}{y_i(t)} &= \frac{1-v_i^{bg}}{1-v_j^{bg}}\left(\frac{K_j}{K_i}\right)^{\alpha}\left(\frac{L_j}{L_i}\right)^{1-\alpha} \\
&= \frac{1-v_i^{bg}}{1-v_j^{bg}}\frac{L_j}{L_i} \\
&= \left(\frac{A_j}{A_i}\right)^{\beta}\left(\frac{\overline{H}_j}{\overline{H}_i}\right)^{1-\beta} \\
&= \frac{A_j}{A_i}\left(\frac{v_j^{bg}}{v_i^{bg}}\right)^{(1-\beta)\eta/(1-\eta)} \qquad ,
\end{aligned} \tag{24}$$

where the second line uses the fact that in steady state—with no taxes on capital, as assumed here—$\kappa^{bg} = K/L$ is the same across countries, the third line uses the definition of L, and the fourth uses the same logic as above to substitute for $\overline{H}_j/\overline{H}_i$. Note that the educational attainment levels in the last line are country averages. Hence the ratio of the technology levels can be backed out from the ratios of output per worker and average educational attainment in the two countries.

Substituting from (24) into (23) and then into (22), we obtain the regression equation

$$\begin{aligned}\ln W(\hat{v}, j) &= \text{constant} + (1-\beta)\frac{\eta}{1-\eta}\ln\hat{v} + (1-\beta)\ln y_j \\ &\quad - (1-\beta)^2\frac{\eta}{1-\eta}\ln v_j^{bg} + \text{error},\end{aligned}$$

for a cross-section of immigrants in i with various educational attainments $\hat{v}$ and from various source countries j. The first term represents the effect of the individual's own educational attainment, while the second and third can be interpreted as adjusting for educational quality.

Hence $1-\beta$ is the coefficient on $\ln y_j$ in a cross section of immigrants in i from various source countries, controlling for the individual's own education and the average educational attainment in his country of origin. Note that this equation could be modified to control for other inputs into human capital—experience, age and so; for taxes on capital that alter the ratio $\kappa = K/L$; and other factors. School quality should not be added, since that is the role of y_j.

Borjas (1987) reports regression results for an equation that is similar to the one above, using data on immigrants in the U.S. from 41 source countries.[12] His regression includes years of schooling, GNP per capita in the immigrant's home country, and some other controls. Thus, home-country output variable is GNP per capita rather than GDP per worker, and there is no control for average educational attainment in the immigrant's home country. In the model here, the omission of v_j^{bg} does not matter if the education subsidy σ_j is the same across countries. In this case v_j^{bg} is the same across countries, and differences in y_j arise only because of differences A_j, which arise from differences in the barriers B_j. But if σ_j varies across countries, and those subsidies are not too large, then σ_j and y_j will be positively correlated. Hence omitting v_j^{bg} will bias the coefficient on y_j downward.

The estimated coefficient in Borjas' equation is $1-\beta \approx 0.12$, which suggests

[12] Jasso and Rosenzweig (1990) and Rosenzweig (2010) for further discussion of immigrant earnings.

that the human capital (individual) component of the wage is small relative to the technology (country) component, $\beta = 0.88$. According to this estimate, an individual in a poor country is wise to focus his time and resources on emigrating rather than acquiring an education.

The baseline simulations here use a more conservative figure, $\beta = 0.60$. Experiments with $\beta = 0.20$, 0.40, and 0.80 are also reported.

Human capital, technology diffusion.—

For human capital accumulation, Heckman (1976) estimates $\eta = 0.52$ and $\delta_H = 0.037$. Here we will set the curvature parameter at $\eta = 0.50$ and the depreciation rate at $\delta_H = 0.03$. The depreciation rate for technology will be set at the same level, $\delta_A = 0.03$. For these parameter values and a common education subsidy, the time allocation is the same along the BGP and in the stagnation steady state.

The constants ϕ_0, ψ_0 involve units for A and H, so one can be fixed arbitrarily. Here ϕ_0 is chosen so that $a_F^{bg}/h_F^{bg} = z_F^{bg} = 1$ in the "frontier" economy, defined as one with no barrier, $B_F = 1$, and with an education subsidy σ_F set so that the steady state time allocation v_F^{bg} is the steady state for a social planner's (welfare maximization) problem. From (13) and (14), this requires

$$\phi_0 = (g + \delta_H) \left(v_F^{bg}\right)^{-\eta}. \tag{25}$$

Here $v_F^{bg} = 0.13486$, $\sigma_F = 0.14464$, and $\phi_0 = 0.1334$. The (normalized) technology level in the frontier country is $a_F = 0.9103$.

The level parameter ψ_0 affects the speed of the transition after a barrier is reduced. It must have the property that (16) holds for the frontier economy, which requires

$$\psi_0 > 4\,(g + \delta_A) = 0.1960. \tag{26}$$

Beyond this, it can be chosen to fit the evidence on catch-up growth. In the simulations here $\psi_0 = 0.60$. As we will see in the next section, this value produces rapid growth after a barrier is reduced, providing a reasonably good fit for the twelve countries identified in section 1 as 'miracles.'

To summarize, the benchmark parameters are

$$\begin{array}{llll} g = 0.019, & \rho = 0.03 & \theta = 2, & \beta = 0.60, \\ \eta = 0.50, & \delta_H = 0.03, & \delta_A = 0.03, & \\ \sigma_F = 0.1446, & \phi_0 = 0.1334, & \psi_0 = 0.60. & \end{array}$$

In all of the simulations the roots are real, two are negative at the high steady state, and two are positive at the low steady state.

7. SIMULATIONS

Figures 5a and 5b display the normalized variables $\left(a_H^{bg}, h_H^{bg}\right)$ at the stable steady state as the education subsidy varies over the range $\sigma \in [0, \sigma_F]$ and the technology barrier over the range $B \in [1, 3]$. Both values are decreasing in B and increasing in σ, and h_H^{bg} is more sensitive to σ. The threshold in (B, σ) $-$space beyond which there is no BGP can also be seen on these figures.

Figures 5c and 5d show (the absolute value of) the negative roots at the stable steady state. Call these $|R_f| > |R_s| > 0$, for fast and slow. A higher barrier slows down transitions: both roots fall in absolute value as B increases. And as the parameters approach the threshold where a BGP ceases to exist, the slow root converges to zero, $|R_s| \to 0$.

Figure 6a shows the (normalized) high and low steady states $ss_{Ji} = \left(a_{Ji}^{bg}, h_{Ji}^{bg}\right)$, $J = H, L$, $i = 1, 2, 3$, for three barriers, $1 = B_F = B_1 < B_2 < B_3$, with the education subsidy fixed at the level for the frontier economy, $\sigma = \sigma_F$. Hence ss_{H1} is the steady state for the frontier economy. Since $\sigma = \sigma_F$ is the same for all three economies, they all have the same steady state ratio $z = a/h = 1$.

The boundary of the basin of attraction around each of the lower steady states is also displayed. This threshold is found by perturbing around the point ss_{Li} using the eigenvector associated with the single negative root at that point, and running the ODE backward. Thus, an economy with policy parameters $\sigma = \sigma_F$ and $B = B_i$, with an initial condition on the curve through the point ss_{Li}, converges to that point. For any initial condition above that curve it converges to the point ss_{Hi}, and for initial conditions below that curve it converges to its stagnation steady state (not shown).

Increasing the barrier B has several effects. First, it moves the stable steady state downward, so the levels for the BGP lag farther behind the frontier. It also moves the lower steady state upward. Thus, a higher barrier reduces the region of initial conditions that produce balanced growth. For $B > B_{crit} \approx 3.1$ the system does not have a balanced growth path.

Figure 6b shows the phase diagram for the policy of the frontier economy, $B_1 = 1$. These transitional paths are computed by perturbing the system away from the point ss_{H1} and running the ODEs backward. Any linear combination (with small weights) of the eigenvectors associated with the two negative roots can be used, giving a 2-dimensional set of allowable perturbations. The adjustment paths in the figure are very flat, indicating that the technology level a adjusts much more rapidly than human capital h.

Figure 7 displays in more detail the adjustment from ss_{H3} to ss_{H1}, the transition for an economy that reduces its barrier from $B_3 = 3$ to $B_1 = 1$, with $\sigma = 0.14$ throughout. In each panel both the exact path (solid) and the linear approximation (broken) are displayed. (The 'exact' path is calculated with the ODE for the first 26 years of the transition, and a linear approximation is used for the continuation after that period.)

Figure 7a shows the transition path in $a, h-$space. Since σ does not change, the steady state ratio $z = a/h = 1$ is the same on both BGPs. Over the first decade of the transition, the technology a grows rapidly while the human capital stock h remains approximately constant. These are normalized stocks, so over the first decade human capital grows at about the same rate, 1.9%, as it would have under the old policy. Over several subsequent decades the technology continues to grow, but more slowly, and the human capital stock grows more quickly. The linear approximation is close to the exact path over the whole transition, although it substantially overstates the speed of adjustment for the first decade.

Figures 7b - 7d display time plots for the 100 years after the policy change. Figure 7b shows the time v allocated to human capital accumulation. Interestingly, it falls rather sharply immediately after the policy change. Two factors are at work. First, consumption smoothing provides a direct incentive to shift the time allocation toward goods production. In addition, since technology is an important input into human capital accumulation, there is an incentive to delay the investment of time in that activity until after the complementary input has increased.

Figure 7c shows the technology level a, human capital level h, and consumption (output) c relative to the frontier economy. All of these start at about 63% of their levels in the frontier economy. As the phase diagram showed, the technology adjusts rapidly, while human capital adjusts quite slowly. The shift in the time allocation just after the policy change allows consumption to jump immediately from 63% to about 66% of the value in the frontier economy.

Figure 7d shows the growth rate of output (consumption), which jumps from 1.9% to about 5.7% immediately after the policy change, and then falls gradually back to the steady state level.

Figure 8 shows the transitions to ss_{H1} for two economies with different initial conditions, both of which adopt the policies of the frontier economy. One is the transition in Figure 7, which begins at ss_{H3}. The other is for an economy starting at ss_{H4}, with a higher initial stock of human capital but a lower technology level. The point ss_{H4}, which is the steady state for an economy with a higher barrier, $B_4 = 4$, and a higher education subsidy, $\sigma_4 = 0.38$, is constructed so that it has the same productive capacity as ss_{H3} if all time is allocated to goods production.

As Figure 8a shows, the transition from ss_4 is more rapid than the one from ss_{H3}. The economy starting at ss_4 makes a more dramatic shift in its time allocation, away from human capital accumulation and toward production, as shown in Figure 8b. Its technology soon overtakes its neighbor's, and its human capital and consumption exceed its neighbor's over the entire transition path, as shown in Figure 8c. The shift in time allocation following the reform produces a larger immediate jump in consumption growth, as shown in Figure 8d.

The experiment in Figure 8 shows that a poor but well educated country grows rapidly when impediments to technology inflows are reduced. It does not imply that policies to promote human capital accumulation are the most valuable ones, however.

Figure 9 displays the transition paths for two possible policy reforms. Each transition starts at the steady state corresponding to a moderate technology barrier, $B_5 = 1.43$, and no subsidy to education, $\sigma_5 = 0$. One policy involves reducing the barrier to $B_6 = 1$, with no change in educational policy. The other involves implementing a subsidy to education, $\sigma_7 = 0.14$, with no change in the barrier. The policies are constructed so that consumption—and hence welfare—on the BGP is the same after either reform.

As Figure 9a shows, the economy enjoys a much more rapid transition when the technology barrier is reduced: technology increases very quickly along the transition to ss_6. The transition to ss_7 is much slower, with less than half of the gap closed even after 20 years.

Figures 9b and 9c show the time allocations and the transition paths for human capital. Increasing σ raises the steady state time allocated to human capital accumulation. In addition, v lies above the (higher) steady state level during the entire transition, producing a fairly rapid increase in human capital. Reducing the barrier B does not change the steady state time allocation. It does produce a temporary drop in time devoted to human capital accumulation, followed by an overshooting of the steady state level. Human capital nevertheless increases—albeit slowly—along the entire path, a consequence of the technology inflows. The steady state level for human capital is slightly higher, another result of the higher technology level.

Figures 9d and 9e show the transition paths for technology and consumption, and Figure 9f shows the growth rate of consumption. Reducing B produces a rapid increase in the technology level towards its new and higher steady state. Consumption jumps upward at the date of the reform, as time is shifted away from human capital accumulation and toward production. Thereafter consumption continues to grow with the technology towards its new steady state, and its growth rate declines gradually back toward g.

Increasing σ produces a quite different pattern. Technology grows slowly, a result of the higher levels of human capital, and the steady state level for a is slightly higher, for the same reason. Consumption jumps downward at the date of the reform, as time is shifted away from production toward human capital accumulation. Hence consumption growth is negative for one period, and thereafter it is slightly higher than the steady state level. Notice that it takes about a decade for consumption to recover to the level it would have had without the reform.

In terms of consumption—and hence welfare—reducing B clearly dominates raising σ. It also dominates the policy of no reform.

Can the model fit the data from the growth "miracles" discussed in Section 1? Figure 10 shows the transition following a reform from rather low initial levels for a and h. The initial condition, ss_{st}, represents a stagnation steady state, in which there is no subsidy to education, $\sigma = 0$. The pre-reform consumption level in this economy is 20% of the level in the frontier economy. The economy is assumed to adopt the frontier policies, $B = 1$ and $\sigma = 0.14$, so the transition is to ss_{H1}. The exact transition is computed for the first 58 years, and the log-linear approximation is used for the subsequent 42 years.

Figure 10a shows the transition path. Qualitatively, the growth miracle is similar to the transition in Figure 7. There is a long period—here several decades—during which the technology grows rapidly. Human capital declines slightly during this phase of the transition. This initial phase is followed by one where human capital grows more rapidly.

Figure 10b shows the time v allocated to human capital accumulation. Here again v jumps downward just after the policy change. It later overshoots the new (higher) steady state level and declines back toward it asymptotically.

Figure 10c shows the transition paths for the human capital, technology, and consumption. The paths for technology and consumption are qualitatively similar to those in Figure 7, but here there is a decline in human capital that lasts for about 15 years.

Does this transition capture well the "miracle" growth episodes discussed in section 1? The economy in Figure 10 has an initial income level that is about 20% of the frontier economy. The initial levels of GDP per capita relative to the U.S. for the "miracles" were about 40% for Germany and Italy, about 30% for Israel, about 20-23% for Greece, Portugal, Spain, Puerto Rico, Japan, Hong Kong, and Singapore, and about 10% for S. Korea and Taiwan.

Figure 10d shows the growth rate of consumption in the model economy, smoothed growth rates of GDP per capita for the 12 miracles, and an average for the 12 miracles.[13] The fit is not too bad, although the model understates growth during the first half of the transition, and overstates it during the second half.

[13]For most of growth miracles began in 1950, so that is date $t = 0$. In Singapore rapid growth began in 1960, and in South Korea it began in 1963, so those paths are shifted appropriately.

8. CONCLUSIONS

The model developed here has a number of empirical implications. First, as shown in Proposition 1, economies with sufficiently unfavorable policies—a combination of high barriers to technology inflows and low subsidies to human capital accumulation—have no BGP. Thus, the model implies that only middle and upper income economies should grow like the technology frontier over long periods. Low income economies can grow faster (if they have reduced their barriers) or slower (if they have raised them) or stagnate. The empirical evidence is consistent with this prediction. Higher income countries grow at very similar rates over long periods, while low income countries show more heterogeneity (in cross section) and variability (over time).

Second, unfavorable policies reduce the level of income and consumption on the BGP, and also shrink the set of initial conditions for which an economy converges to that path. Thus, the model predicts that a middle-income countries that grow with the frontier should have displayed slower growth during their transitional phases.

Third, as shown in Figure 7, the transition to a (higher income) BGP features a modest period of very rapid TFP growth, accompanied by rapid growth in income and consumption. This initial phase is followed by a (longer) period during which human (and physical) capital are accumulated. Income growth declines toward the rate of frontier growth as the income level approaches the frontier level. Thus, the model has predictions for some features of the transition paths of growth 'miracles.'

Fourth, as shown in Figure 8, low income countries with higher human capital are better candidates to become growth miracles, since TFP and income grow more rapidly in such economies.

The model has one implication for policy, and it is strong. As shown in Figure 9, the model suggests that policies stimulating technology transfer are more effective in accelerating growth than policies stimulating human capital accumulation. The logic behind this conclusion is very simple. Investments in human and physical capital respond to the returns on those assets, and the returns are high when technology is growing rapidly. Thus, the empirical association between high investment rates and rapid growth is not causal: it is technology that drives both. Policies that promote technology inflows increase output immediately and directly, but they also have a second-round effect. They raise the rates of return on private capital, which stimulates private investment. Through this channel, they provide a second impact on output in the future.

Policies that encourage human capital accumulation, on the other hand, reduce output and consumption in the short run and have only a modest and delayed effect on technology inflows. Although they may provide some stimulus to growth in the long run, the model here suggests that the effects are weak and distant.

Human capital accumulation is too slow to deliver a miracle, and physical capital is not important enough as a factor of production. Growth miracles require rapid TFP growth, and rapid TFP growth requires an inflow of technologies from outside.

APPENDIX

A. Equilibrium conditions

The Hamiltonian for the household's problem is

$$\begin{aligned}\mathfrak{H} &= \frac{C^{1-\theta}}{1-\theta} + \Lambda_H \left[\phi_0 \left(vH\right)^{\eta} A^{1-\eta} - \delta_H H\right] \\ &+\Lambda_K \left[\left(1-v\right) w(H) + \left(R-\delta_K\right) K - C + v\sigma w(\overline{H}) - \tau\right],\end{aligned}$$

where to simplify the notation the subscript i's have been dropped. Taking the first order conditions for a maximum, using the fact that in equilibrium $\tau = v\sigma w(\overline{H})$ and $\overline{H} = H$, substituting for $w(H)$ and its derivative, and simplifying gives

$$\begin{aligned}\Lambda_H \eta \phi_0 v^{\eta-1} &= \Lambda_K \left(1-\sigma\right) \hat{w} \left(\frac{A}{H}\right)^{\beta-(1-\eta)}, \\ C^{-\theta} &= \Lambda_K, \\ \frac{\dot{\Lambda}_H}{\Lambda_H} &= \rho + \delta_H - \phi_0 \left(\frac{A}{H}\right)^{1-\eta} \eta v^{\eta} \left[1 + \frac{1-\beta}{1-\sigma}\frac{1-v}{v}\right], \\ \frac{\dot{\Lambda}_K}{\Lambda_K} &= \rho + \delta_K - R, \\ \frac{\dot{H}}{H} &= \phi_0 v^{\eta} \left(\frac{A}{H}\right)^{1-\eta} - \delta_H, \\ \frac{\dot{K}}{K} &= \left(\frac{K}{L}\right)^{\alpha-1} - \frac{C}{K} - \delta_K,\end{aligned} \tag{27}$$

with $\dot{A}/A$ in (2). The transversality conditions are

$$\lim_{t\to\infty} e^{-\rho t}\Lambda_K(t)K(t) = 0 \qquad \text{and} \qquad \lim_{t\to\infty} e^{-\rho t}\Lambda_H(t)H(t) = 0. \tag{28}$$

The system of equations in (1), (2), (27) and (28) completely characterizes the competitive equilibrium, given initial values for the state variables W, A, H, K and the policy parameters B, σ.

The law of motion for A requires $\overline{H}/W$ and A/W to be constant along a BGP, so A and H must also grow at the rate g. Since the production functions for effective labor in (4) and for goods have constant returns to scale, the factor inputs L and K also grows at the rate g, as do output and consumption, and the marginal utility of consumption grows at the rate $-\theta g$. The FOC for consumption then implies that Λ_K grows at the rate $-\theta g$. The factor returns R and $\hat{w}$ are constant along a BGP, and the FOC for time allocation implies that Λ_H grows at the same rate as Λ_K.

The normalized conditions in (10) follow directly from (2) and (27).

B. Linear approximations and stability

Define the constants

$$\Delta = \beta(1-\theta) - (1-\eta), \qquad \chi \equiv \eta \frac{1-\beta}{1-\sigma},$$

$$\Pi_H \equiv \frac{\delta_H + r^{bg}}{\delta_H + g}, \qquad \Pi_H - 1 = \frac{r^{bg} - g}{g + \delta_H},$$

$$\upsilon^{bg} = \left[1 + \frac{1}{\chi}(\Pi_H - \eta)\right]^{-1}, \qquad \chi\left(\frac{1}{\upsilon^{bg}} - 1\right) = \Pi_H - \eta,$$

$$\begin{aligned} \Gamma_2^{bg} &\equiv \eta - \frac{\chi\left(\upsilon^{bg}\right)^{-1}}{\eta - \chi + \chi\left(\upsilon^{bg}\right)^{-1}} \\ &= \eta - \frac{\chi}{\eta\upsilon^{bg} + \chi\left(1 - \upsilon^{bg}\right)} < \eta, \end{aligned}$$

$$\Gamma_3^{bg} = \left[\eta - 1 - \frac{\theta}{1/\upsilon^{bg} - 1}\right]^{-1} < 0,$$

and the log deviations

$$x_1 = \ln\left(a/a^{bg}\right), \qquad x_2 = \ln\left(h/h^{bg}\right), \qquad x_3 = \ln\left(\lambda_h/\lambda_h^{bg}\right).$$

Take a first-order approximation to (17) to get

$$\frac{\upsilon - \upsilon^{bg}}{\upsilon^{bg}} = \Gamma_3\left[\Delta x_1 - (\Delta + \theta)x_2 - x_3\right],$$

Then linearize (10) to find that

$$\begin{aligned}
\dot{x}_1 &\approx (g+\delta_A)\left[-\frac{a_J^{bg}}{1-a_J^{bg}}x_1+x_2\right], \\
\dot{x}_2 &\approx (g+\delta_H)\left[(1-\eta)(x_1-x_2)+\eta\frac{v-v^{bg}}{v^{bg}}\right] \\
&= (g+\delta_H)\left\{(1-\eta)(x_1-x_2)+\eta\Gamma_3^{bg}\left[\Delta x_1-(\Delta+\theta)x_2-x_3\right]\right\}, \\
\dot{x}_3 &\approx -\left(r^{bg}+\delta_H\right)\left[(1-\eta)(x_1-x_2)+\Gamma_2^{bg}\frac{v-v^{bg}}{v^{bg}}\right] \\
&= -\left(r^{bg}+\delta_H\right)\left\{(1-\eta)(x_1-x_2)+\Gamma_2^{bg}\Gamma_3^{bg}\left[\Delta x_1-(\Delta+\theta)x_2-x_3\right]\right\}.
\end{aligned}$$

Hence

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}\approx\begin{pmatrix}-c_{1J} & c_2 & 0\\ c_3 & -c_3-\theta c_4 & -c_4\\ -c_6 & c_6+\theta c_5 & c_5\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix},$$

where

$$\begin{aligned}
c_{1J} &= (g+\delta_A)\,a_J^{bg}/\left(1-a_J^{bg}\right), \\
c_2 &= g+\delta_A, \\
c_3 &= (g+\delta_H)\left(1-\eta+\eta\Gamma_3\Delta\right), \\
c_4 &= (g+\delta_H)\,\eta\Gamma_3, \\
c_5 &= \left(r^{bg}+\delta_H\right)\Gamma_2\Gamma_3, \\
c_6 &= \left(r^{bg}+\delta_H\right)\left(1-\eta+\Gamma_2\Gamma_3\Delta\right)
\end{aligned}$$

The stability of each steady state depends on the roots of the associated characteristic equation,

$$0=\det\begin{pmatrix}-R-c_{1J} & c_2 & 0\\ c_3 & -R-(c_3+\theta c_4) & -c_4\\ -c_6 & c_6+\theta c_5 & -R+c_5\end{pmatrix}$$

$$
\begin{aligned}
&= -R^3 + (c_5 - c_3 - \theta c_4 - c_{1J}) R^2 \\
&\quad - [c_{iJ} (c_3 + \theta c_4) - c_{1J} c_5 - (c_3 + \theta c_4) c_5] R \\
&\quad + c_{1J} (c_3 + \theta c_4) c_5 + c_2 c_4 c_6 \\
&\quad - (R + c_{1J}) c_4 (c_6 + \theta c_5) + c_2 c_3 (R - c_5)
\end{aligned}
$$

$$
\begin{aligned}
&= -R^3 + (m_1 - c_{1J}) R^2 + (m_2 + c_2 c_3 + m_1 c_{1J}) R \\
&\quad + m_2 (c_{1J} - c_2) .
\end{aligned}
$$

where

$$
m_1 \equiv c_5 - c_3 - \theta c_4, \qquad m_2 \equiv c_3 c_5 - c_4 c_6.
$$

Write this equation as

$$
0 = \Psi_J(R) \equiv R^3 - A_{1J} R^2 - A_{2J} R - A_{3J}, \qquad J = H, L,
$$

where

$$
\begin{aligned}
A_{1J} &\equiv m_1 - c_{1J}, \\
A_{2J} &\equiv m_2 + c_2 c_3 + m_1 c_{1J}, \\
A_{3J} &\equiv m_2 (c_{1J} - c_2), \qquad J = H, L.
\end{aligned}
$$

Since $\Gamma_3 < 0$ and $\Gamma_2 - \eta < 0$, it follows that

$$
m_2 = (\delta_H + g) \left(\delta_H + r^{bg}\right) \Gamma_3 (1 - \eta) (\Gamma_2 - \eta) > 0,
$$

and since $a_H^{bg} > 1/2 > a_L^{bg}$, it follows that $c_{1H} > c_2 > c_{1L}$. Hence

$$
\Psi_H(0) = -A_{3H} < 0, \qquad \Psi_L(0) = -A_{3L} > 0,
$$

so Ψ_H has at least one positive real root, and Ψ_L has at least one negative real root.

The other roots of Ψ_H are real and both are negative if and only if

$$
\Psi_H(I) > 0, \qquad \text{for some } I < 0.
$$

This condition holds if and only if Ψ_H has (real) inflection points, the lower one occurs at a negative value I_H, and Ψ_H takes a positive value at this point. Similarly, the other roots of Ψ_L are real and both are positive if and only if

$$\Psi_L(I_L) < 0, \qquad \text{for some } I_L > 0.$$

The inflection points of Ψ_J satisfy the quadratic equation

$$0 = \Psi_J'(I) = 3I^2 - 2A_{1J}I - A_{2J}, \qquad J = H, L,$$

so

$$I_J = \frac{1}{3}\left[A_{1J} \pm \sqrt{A_{1J}^2 + 3A_{2J}}\right]. \tag{29}$$

Hence Ψ_H has a negative (real) inflection point if and only if $A_{2H} > 0$, or

$$\begin{aligned} 0 &< m_2 + c_2c_3 + m_1c_{1H} \\ &= m_2 + (c_2 - c_{1H})\, c_3 + (c_5 - \theta c_4)\, c_{1H}. \end{aligned} \tag{30}$$

As shown above, $m_2 > 0$. But since $c_2 < c_{1H}$, the second term is negative if $c_3 > 0$. For the last term note that

$$c_5 - \theta c_4 = \left[\left(r^{bg} + \delta_H\right)\Gamma_2 - \theta\left(g + \delta_H\right)\eta\right]\Gamma_3.$$

Since $\Gamma_3 < 0$, this term is positive if and only if

$$\left(1 - \theta\frac{g + \delta_H}{r^{bg} + \delta_H}\right)\eta - \frac{\chi}{\eta v^{bg} + \chi\left(1 - v^{bg}\right)} < 0.$$

The second term in on the left is clearly negative, so it suffices if

$$\begin{aligned} 0 &\geq r^{bg} + \delta_H - \theta\left(g + \delta_H\right) \\ &= \rho + \delta_H - \theta\delta_H, \end{aligned}$$

or $\rho \leq (\theta - 1)\,\delta_H$.

Similarly, Ψ_L has a positive (real) inflection point if and only if $A_{2L} > 0$, or

$$0 < m_2 + c_2 c_3 + m_1 c_{1L}.$$

The first term on the right is positive, as before. And since $c_2 < c_{1L}$, here the first term is positive if $c_3 > 0$. The condition for the last term is the same as before.

To complete the argument that there are two negative (real) roots at a_H^{bg}, we must show that $\Psi_H(I_H) > 0$, for the inflection point

$$I_H \equiv \frac{1}{3}\left(A_{1H} - \sqrt{A_{1H}^2 + 3A_{2H}}\right) < 0.$$

To complete the argument that there are two positive (real) roots at a_L^{bg}, we must show that $\Psi_L(I_L) < 0$ for the inflection point

$$I_L \equiv \frac{1}{3}\left(A_{1L} + \sqrt{A_{1L}^2 + 3A_{2L}}\right) > 0.$$

Both conditions hold for the benchmark calibration. [Question: are there examples where any of these conditions fail?]

Check sum of roots for $\sigma = 0$, $x_1 \equiv 0$.—

For the case where $x_1 = \dot{x}_1 \equiv 0$ the system is

$$\begin{pmatrix} \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} \approx \begin{pmatrix} -c_3 - \theta c_4 & -c_4 \\ c_6 + \theta c_5 & c_5 \end{pmatrix} \begin{pmatrix} x_2 \\ x_3 \end{pmatrix},$$

so the roots satisfy

$$\begin{aligned} 0 &= [-R - (c_3 + \theta c_4)]\,[-R + c_5] + c_4\,(c_6 + \theta c_5) \\ &= R^2 - m_1 R - m_2. \end{aligned}$$

Since $m_2 > 0$, the roots are real and of opposite sign.

For $\sigma = 0$, the household's optimization problem is undistorted, so the sum of the roots should equal the discount rate,

$$R_1 + R_2 = m_1 = \rho - g\,(1 - \theta) = r^{bg} - g,$$

or

$$\frac{m_1}{(\delta+g)\,\Gamma_3^{bg}} = \frac{r^{bg}-g}{\delta+g}\frac{1}{\Gamma_3^{bg}} = \frac{(\Pi_H-1)}{\Gamma_3^{bg}}.$$

Use the expression above for m_1 to find that this condition holds, since

$$(\Pi_H-1)\,(\eta-1) - \frac{\chi\theta\,(\Pi_H-1)}{\Pi_H-\eta} = \frac{(\Pi_H-1)}{\Gamma_3^{bg}}.$$

REFERENCES

[1] Acemoglu, Daron, and Fabrizio Zilibotti. 2001. Productivity differences, *Quarterly Journal of Economics*, 116(2): 563-606.

[2] Acemoglu, Daron, Simon Johnson, and James A. Robinson. 2001. The colonial origins of comparative development: an empirical investigation, *Am. Ec. Rev.*, 91: 1369-1401.

[3] ______2002. Reversal of fortune: geography and institutions in the making of the modern world income distribution, *Quarterly Journal of Economics*, 117: 1231-1294.

[4] ______2005. The rise of Europe: Atlantic trade, institutional change, and economic growth. American Economic Review, 95: 546-579.

[5] Barro, Robert J. and Jong-Wha Lee. 2000. International data on educational attainment: updates and implications, Center for International Development Working Paper No. 42, Harvard University.

[6] Bartelsman, Eric, John Haltiwanger, and Stefano Scarpetta. 2008. Cross country differences in productivity: the role of allocative efficiency, working paper, University of Maryland.

[7] Baumol, William J, 1986. Productivity growth, convergence, and welfare: what the long-run data show, *American Economic Review*, 76(5): 1072-85.

[8] Baumol, William J and Edward N. Wolff 1988. Productivity growth, convergence, and welfare: reply, *American Economic Review*, 78(5): 1155-59.

[9] Benhabib, Jess and Mark M. Spiegel. 2005. Human capital and technology diffusion, *Handbook of Economic Growth*, Ch. 13, Vol. 1A. P. Aghion and S. Durlauf, eds., North Holland.

[10] Borjas, George J. 1987. Self-selection and the earnings of immigrants, *American Economic Review*, 77(4): 531-553.

[11] Caselli, Francesco. 2005. Accounting for income differences across countries. *Handbook of Economic Growth*, Ch. 9, Vol. 1A. P. Aghion and S. Durlauf, eds., North Holland.

[12] De Long, J Bradford. 1988. Productivity growth, convergence, and welfare: comment, *American Economic Review*, 78(5): 1138-54.

[13] Denison, Edward F.1974. Accounting for United States economic growth, 1929-1969. Washington, DC. Brookings Institution..

[14] Gerschenkron, Alexander. 1965. *Economic Backwardness in Historical Perspective: A Book of Essays*, Books on Demand, 1965.

[15] Hahn, Chin-Hee. 2010. Learning by exporting. Working paper, Korea Development Institute.

[16] Hall, Robert E., and Charles I. Jones. 1999. Why do some countries produce so much more output per worker than others? *Quarterly Journal of Economics* 114: 83-116.

[17] Hendricks, Lutz. 2002. How important is human capital for development? Evidence from immigrant earnings, *American Economic Review* 92: 198-219.

[18] Jasso, Guillermina and Mark R. Rosenzweig. 1995. Do immigrants screened for skills do better than family-reunification immigrants? *International Migration Review*, 29: 85-111.

[19] Jasso, Guillermina and Mark R. Rosenzweig. 2009. Selection criteria and the skill composition of immigrants: a comparative analysis of Australian and U.S. employ-

ment immigration. In Jagdish Bhagwati and Gordon H. Hanson (eds.), *Skilled Migration Today*. New York: Oxford University Press.

[20] Hsieh, Chang-Tai and Peter J. Klenow. 2009. Misallocation and manufacturing TFP in China and India, working paper.

[21] Klenow, Peter J. 1998. Ideas vs. rival human capital: industry evidence on growth models, *Journal of Monetary Economics* 42: 3-24.

[22] Klenow, Peter J. and Andres Rodriguez-Clare. 1997a. Economic growth: A review essay, *Journal of Monetary Economics*, vol. 40(3): 597-617.

[23] Klenow, Peter J. and Andrés Rodríguez-Clare. 1997b. The neoclassical revival in growth economics: has it gone too far? *NBER Macroeconomics Annual* 1997, B. Bernanke and J. Rotemberg ed., Cambridge, MA: MIT Press, 73-102.

[24] Klenow, Peter J. and Andrés Rodríguez-Clare. 2005. Externalities and growth, *Handbook of Economic Growth*, Ch. 11, Vol. 1A. P. Aghion and S. Durlauf, eds., North Holland.

[25] Lucas, Robert E., Jr. 1988. On the mechanics of economic development, *Journal of Monetary Economics* 22:3-42.

[26] Lucas, Robert E., Jr. 2009. Trade and the diffusion of the industrial revolution, *American Economic Journal: Macroeconomics* 1(1): 1-25.

[27] Parente, Stephen L. and Edward C. Prescott. 1994. Barriers to technology adoption and development, *Journal of Political Economy* 102: 298-321.

[28] Parente, Stephen L. and Edward C. Prescott. 2000. *Barriers to Riches*. MIT Press, Cambridge, MA.

[29] Prescott, Edward C. 1997. Needed: a theory of total factor productivity, *International Economic Review* 39(3): 525-551.

[30] Prescott, Edward C. 2002. Prosperity and depression, *AER Papers and Proceedings*, 92: 1-15.

[31] Prescott, Edward C. 2004. Why do Americans work so much more than Europeans? Federal Reserve Bank of Minneapolis *Quarterly Review*, 28(1), July.

[32] Psacharopoulos, George, 1994. Returns to investment in education: a global update, World Development, Elsevier, vol. 22(9): 1325-1343.

[33] Ragan Kelly S. 2006. Taxes, transfers and time use: fiscal policy in a household production model. Ph.D. Thesis, University of Chicago.

[34] Raimondo, Natalia.

[35] Restuccia, Diego, and Richard Rogerson. 2008. Policy distortions and aggregate productivity with heterogeneous establishments, *Review of Economic Dynamics* 11: 707-720.

[36] Rosenzweig, Mark R. 2010. Global wage inequality and the international flow of migrants. Economic Growth Center Discussion Paper No. 983, Yale University.

[37] Solow, Robert M. 1957. Technical change and the aggregate production function. *Review of Economics and Statistics* 39: 312-320.

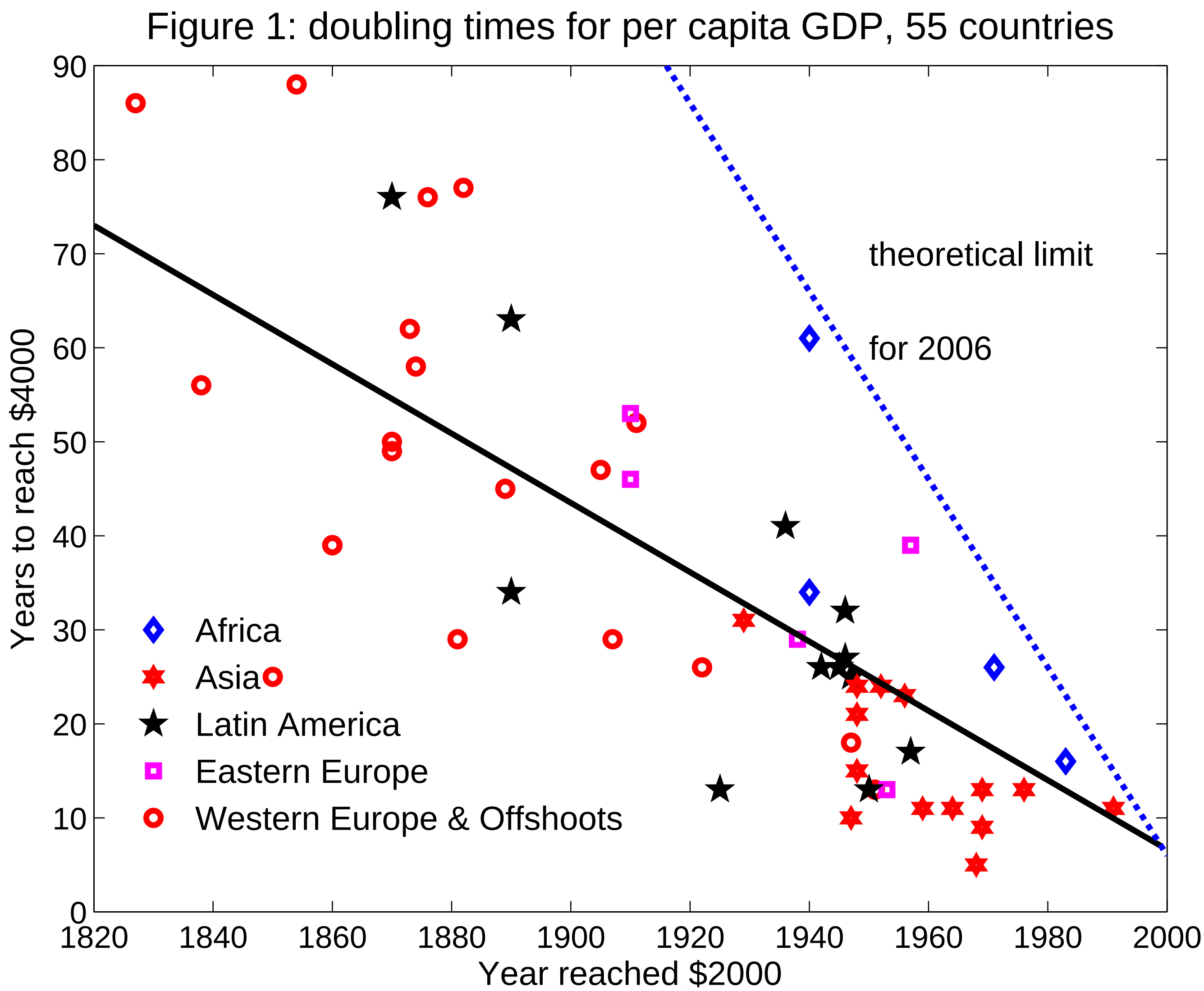
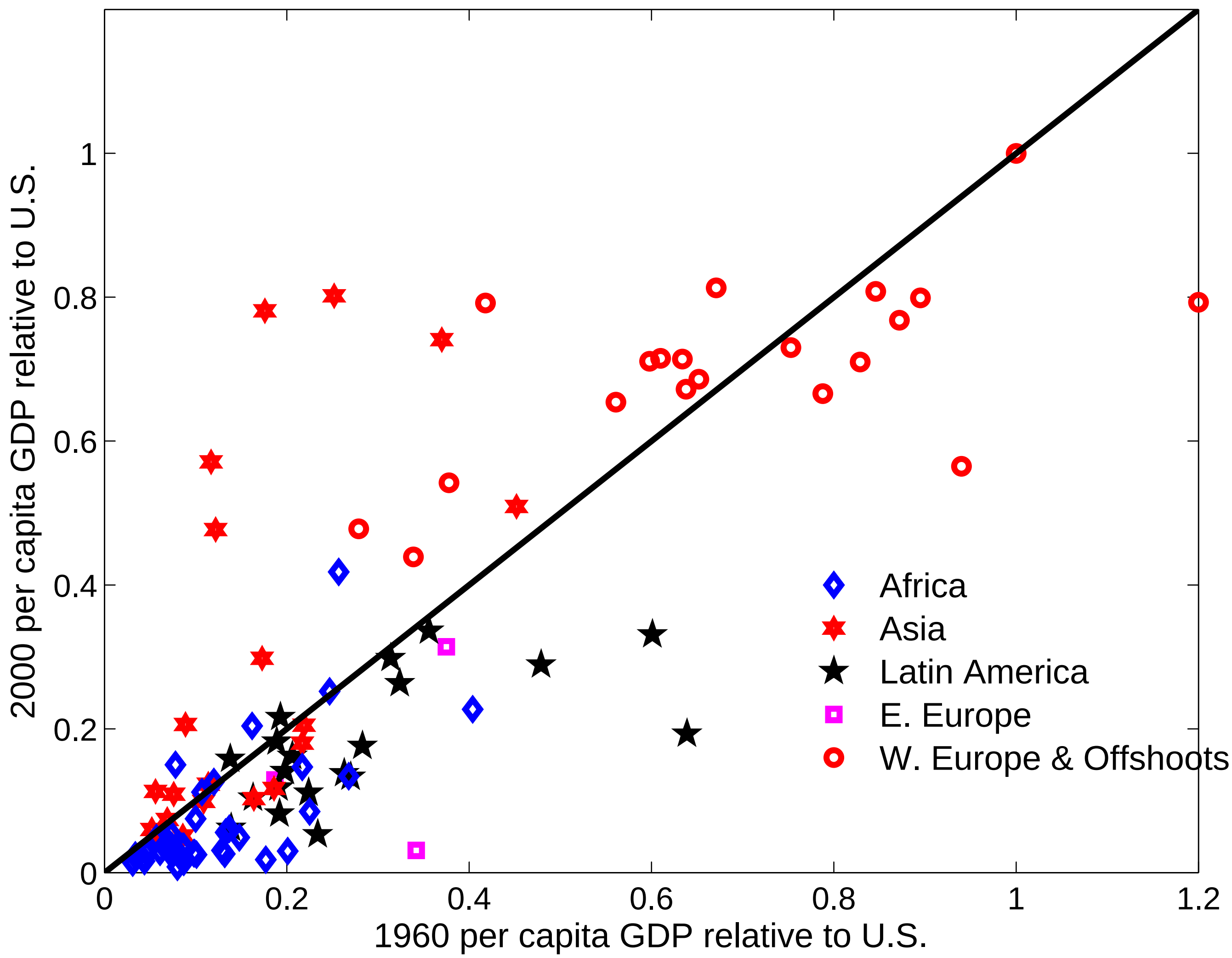


Figure 1: doubling times for per capita GDP, 55 countries
theoretical limit
for 2006
Africa
Asia
Latin America
Eastern Europe
Western Europe & Offshoots
Years to reach $4000
Year reached $2000
0
10
20
30
40
50
60
70
80
90
1820
1840
1860
1880
1900
1920
1940
1960
1980
2000

Figure 2: catching up and falling behind, 1960-2000



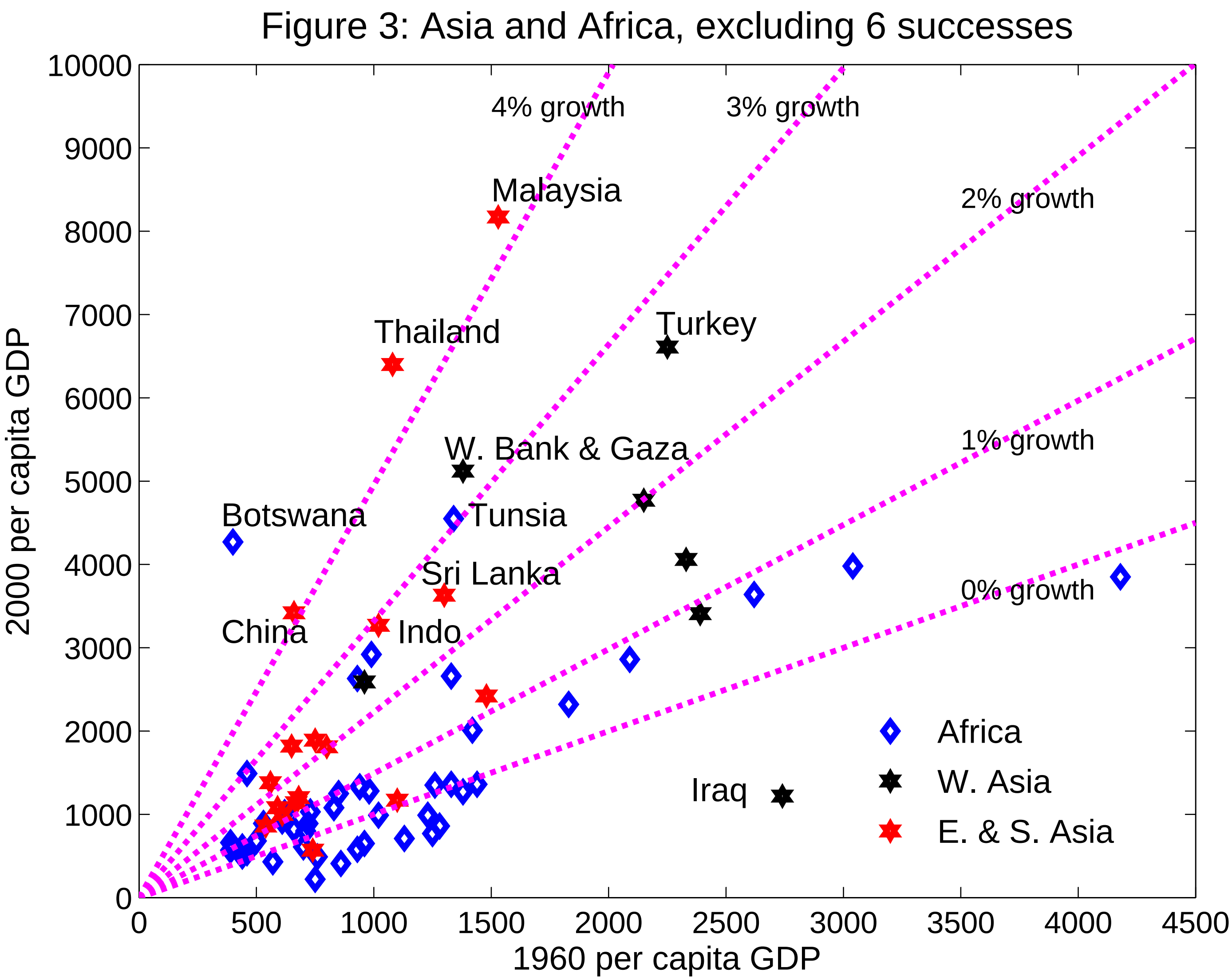


Figure 3: Asia and Africa, excluding 6 successes
4% growth
3% growth
2% growth
1% growth
0% growth
Malaysia
Thailand
Turkey
W. Bank & Gaza
Botswana
Tunsia
Sri Lanka
China
Indo
Iraq
Africa
W. Asia
E. & S. Asia
2000 per capita GDP
1960 per capita GDP
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
500
1500
2500
3500
4500

Figure 4: technology ratios a^{bg}, for $\sigma_2 < \sigma_1$

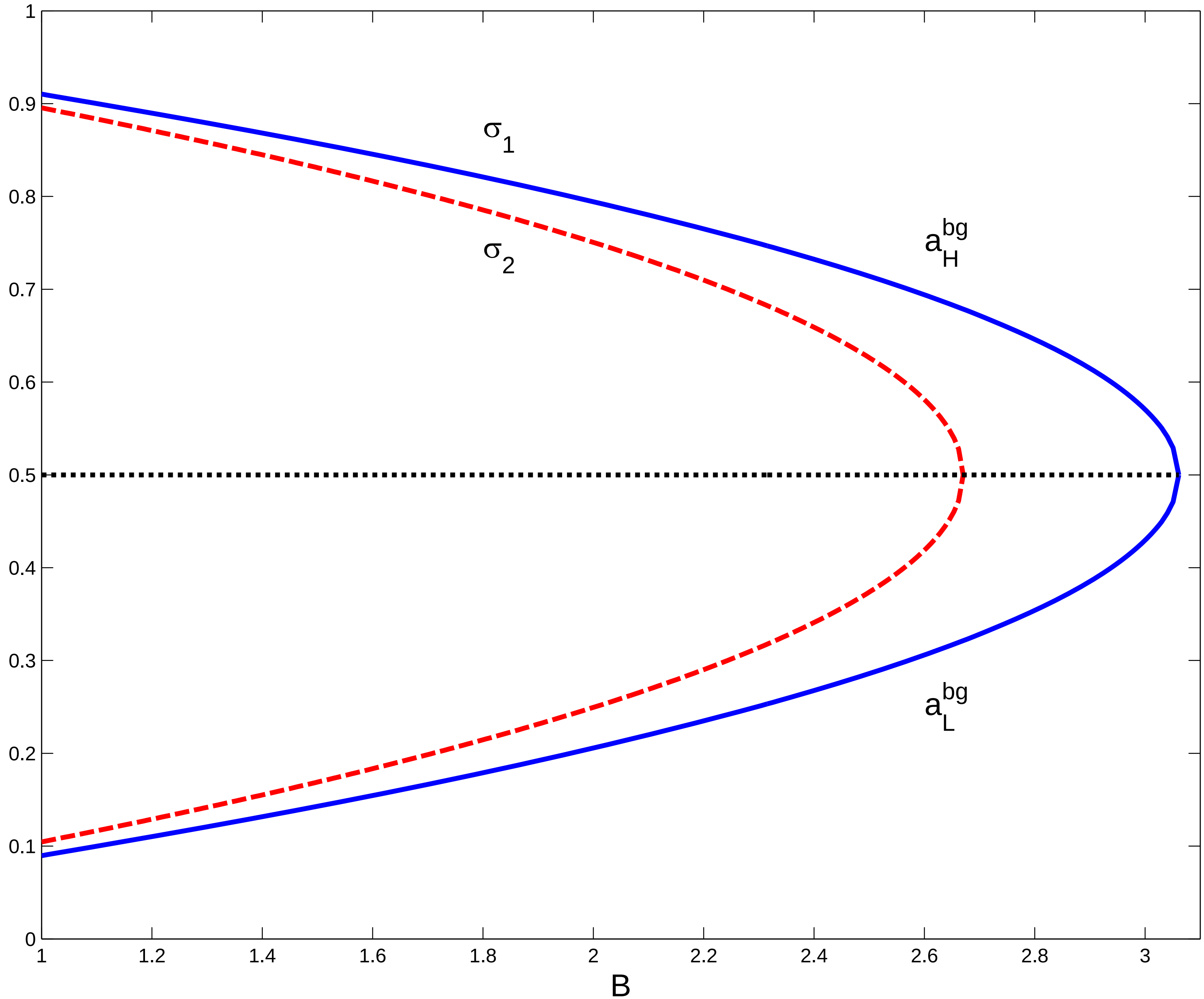


Figure 5a: technology a^{bg}

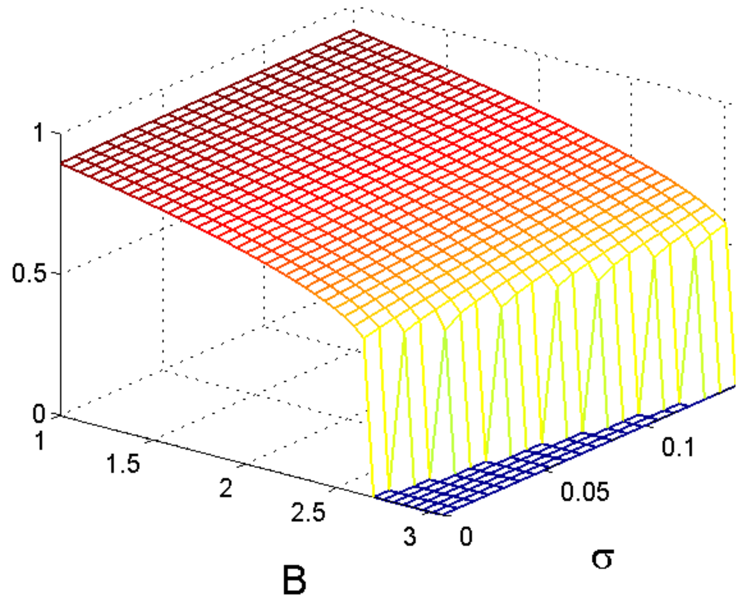


Figure 5b: human capital h^{bg}

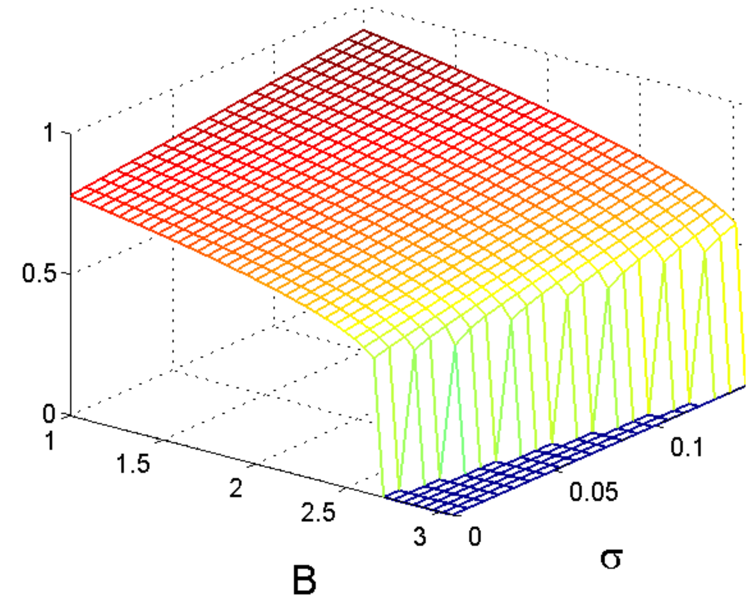


Figure 5c: "fast" root, $-R_f$

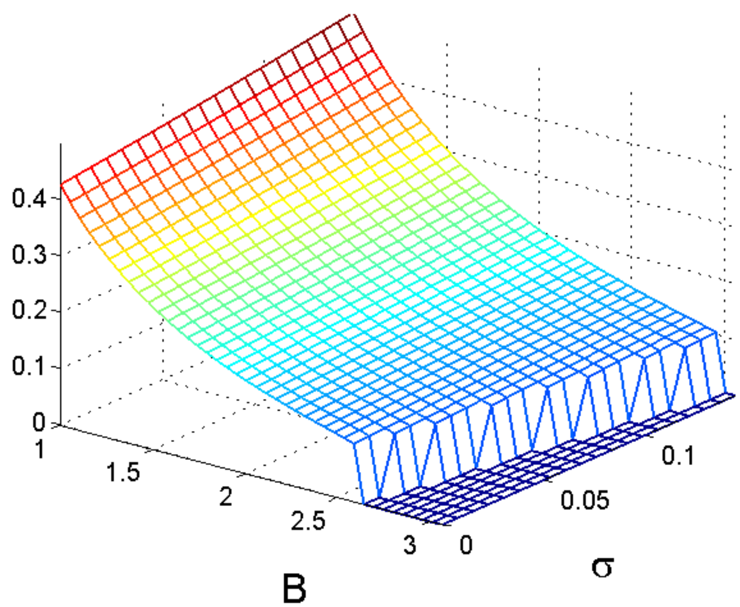


Figure 5d: "slow" root, $-R_s$

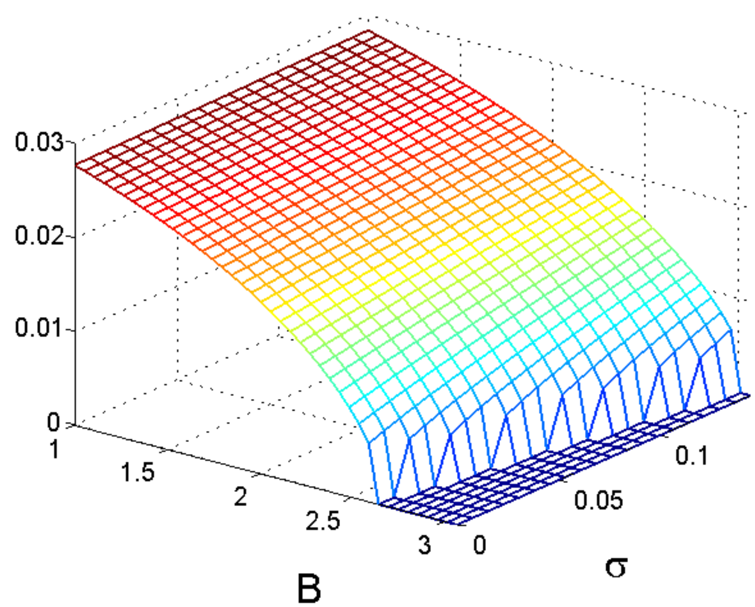


Figure 6a: basins of attraction for BGPs

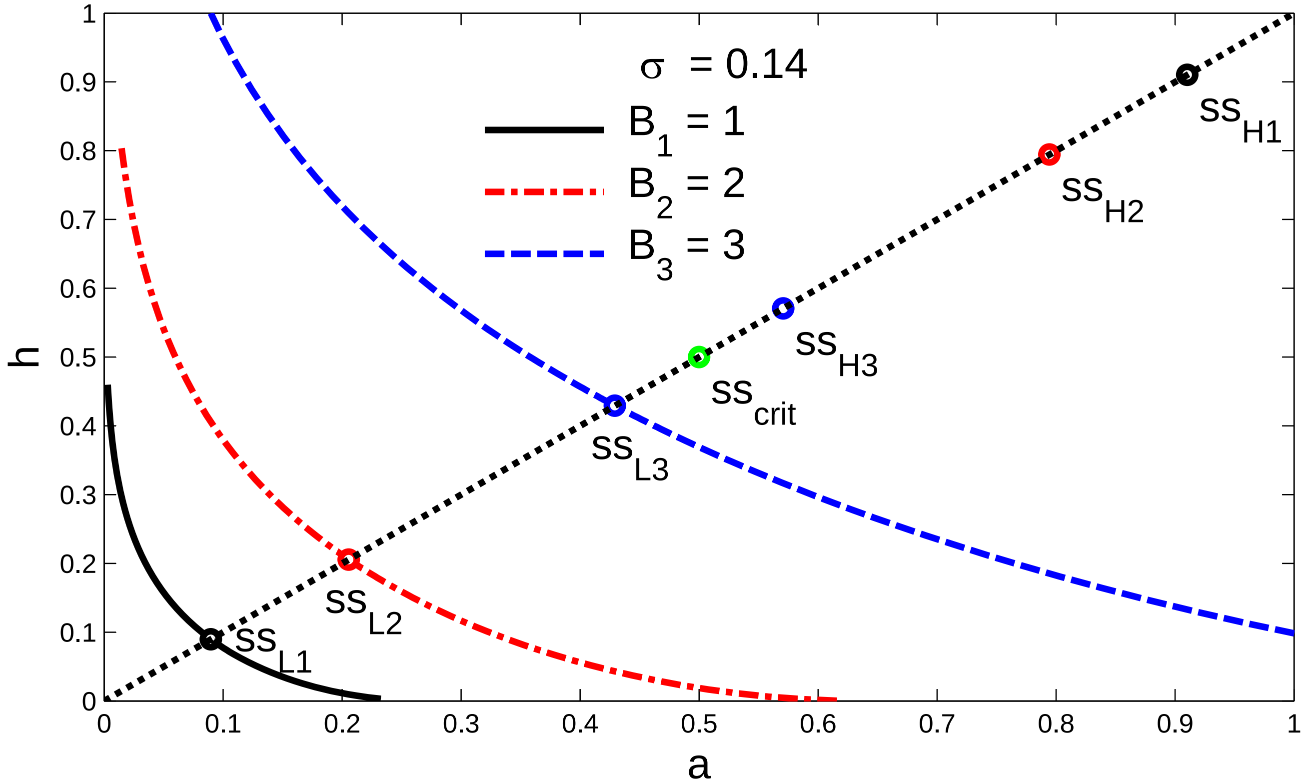
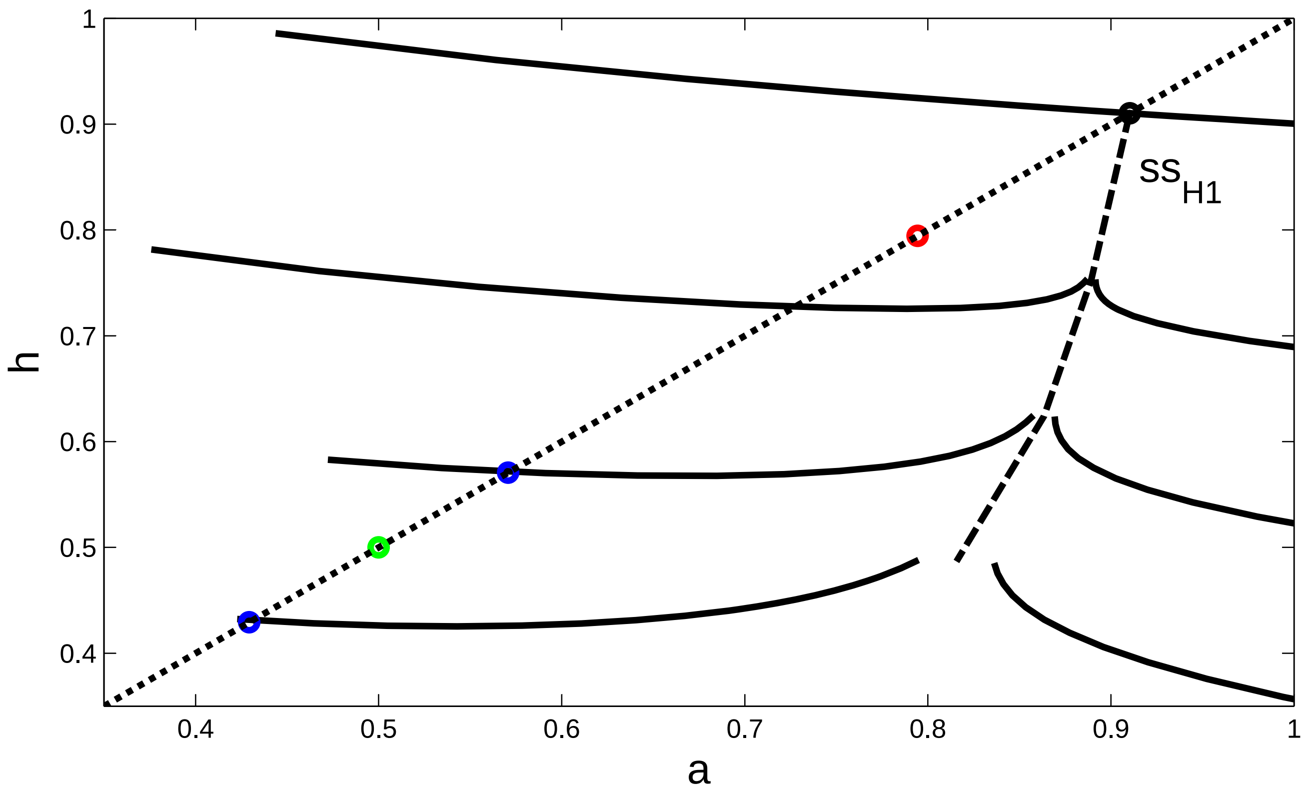


Figure 6b: phase diagram for B_1



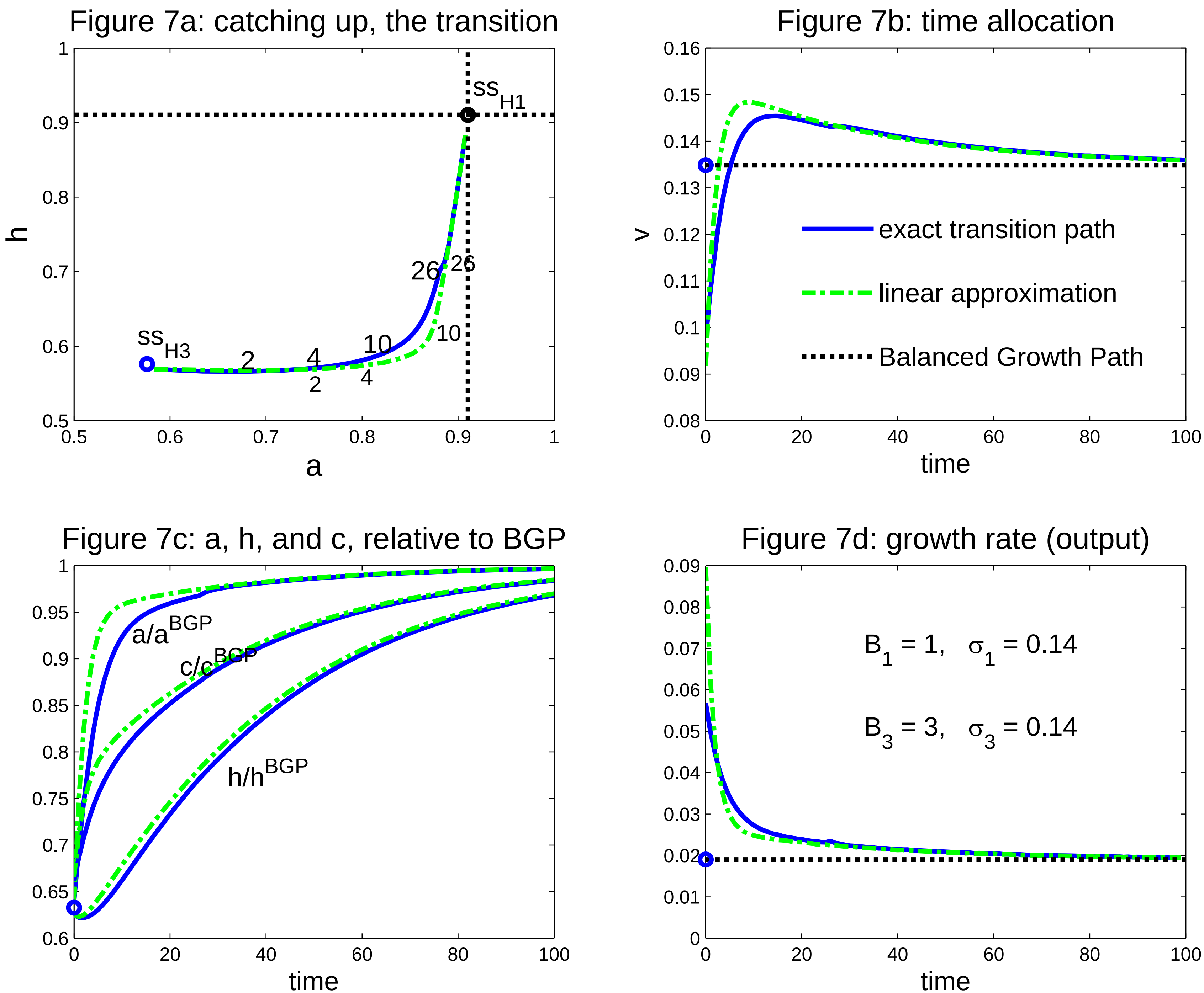


Figure 7a: catching up, the transition
h
a
ss H1
ss H3
2
4
10
26
Figure 7b: time allocation
v
time
exact transition path
linear approximation
Balanced Growth Path
Figure 7c: a, h, and c, relative to BGP
a/a BGP
c/c BGP
h/h BGP
time
Figure 7d: growth rate (output)
B1 = 1, σ1 = 0.14
B3 = 3, σ3 = 0.14
time

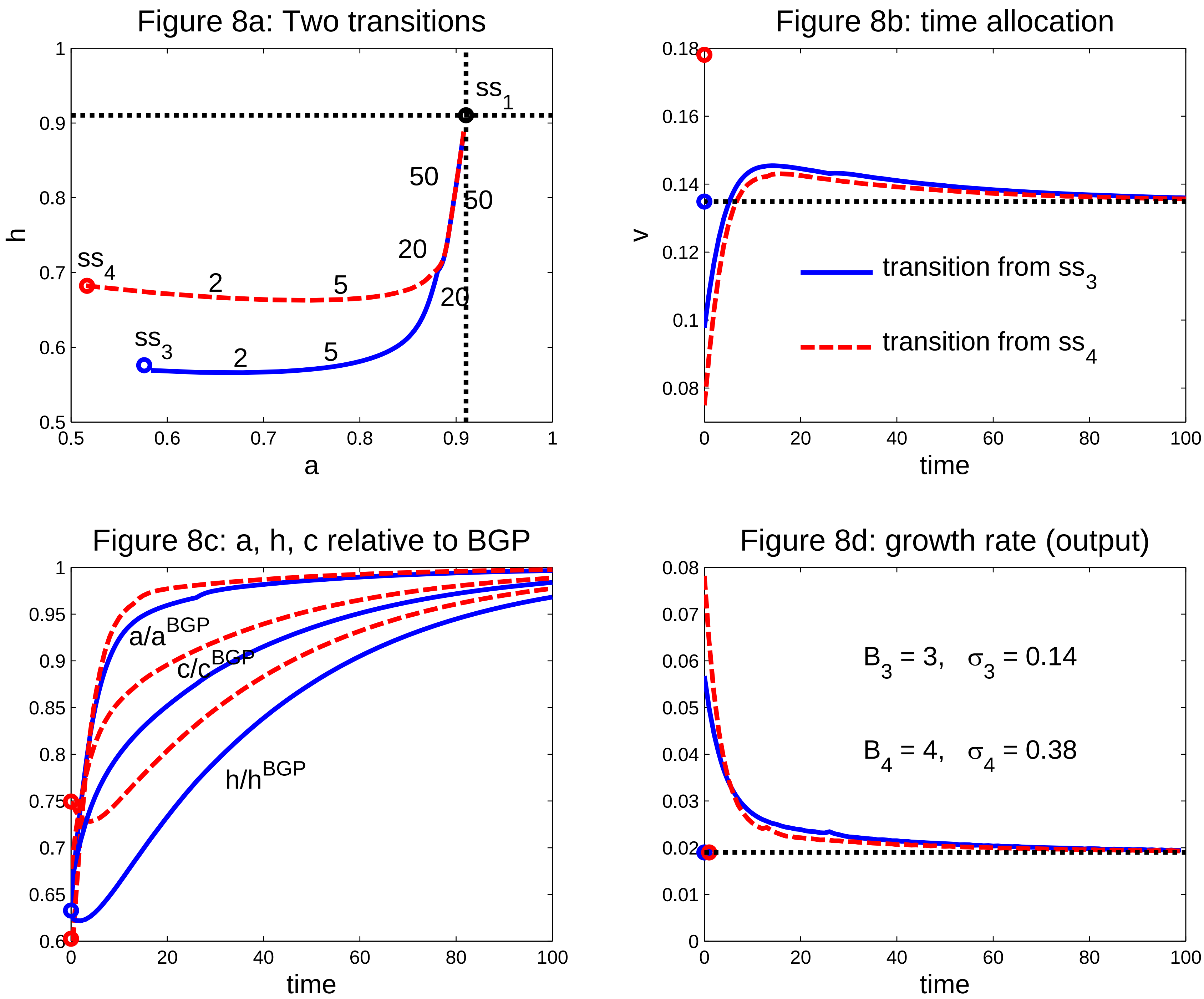


Figure 8a: Two transitions
ss1
ss4
ss3
50
50
20
20
2
5
2
5
h
a
0.5
0.6
0.7
0.8
0.9
1
Figure 8b: time allocation
v
time
transition from ss3
transition from ss4
0.18
0.16
0.14
0.12
0.1
0.08
0
20
40
60
80
100
Figure 8c: a, h, c relative to BGP
a/aBGP
c/cBGP
h/hBGP
time
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
Figure 8d: growth rate (output)
B3 = 3, σ3 = 0.14
B4 = 4, σ4 = 0.38
time
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0

Figure 9a: Transitions for two policies

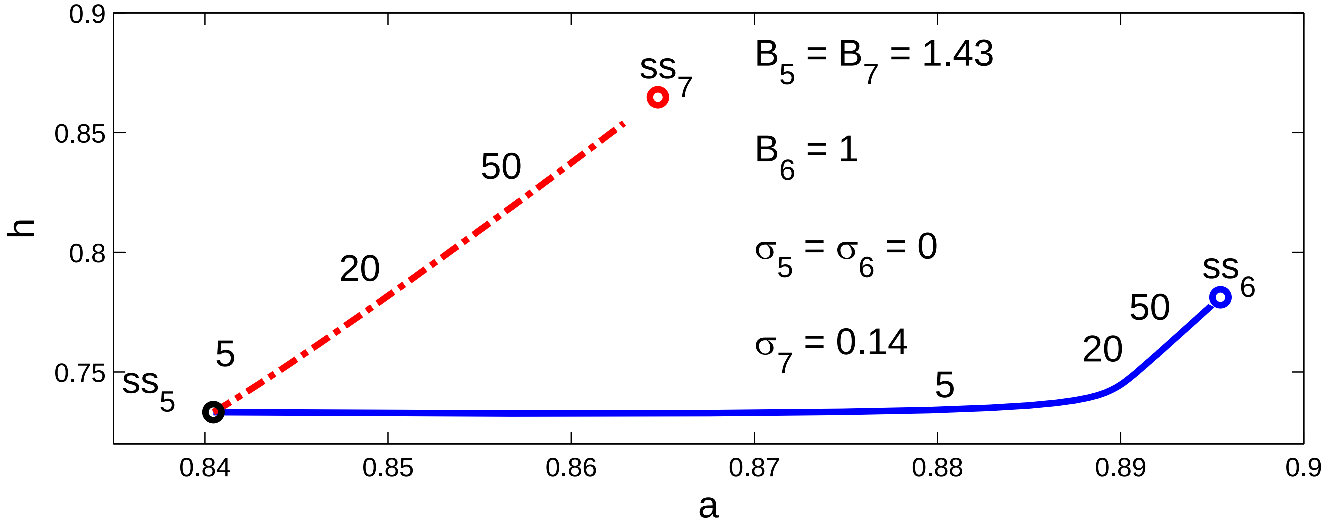


Figure 9b: time allocations

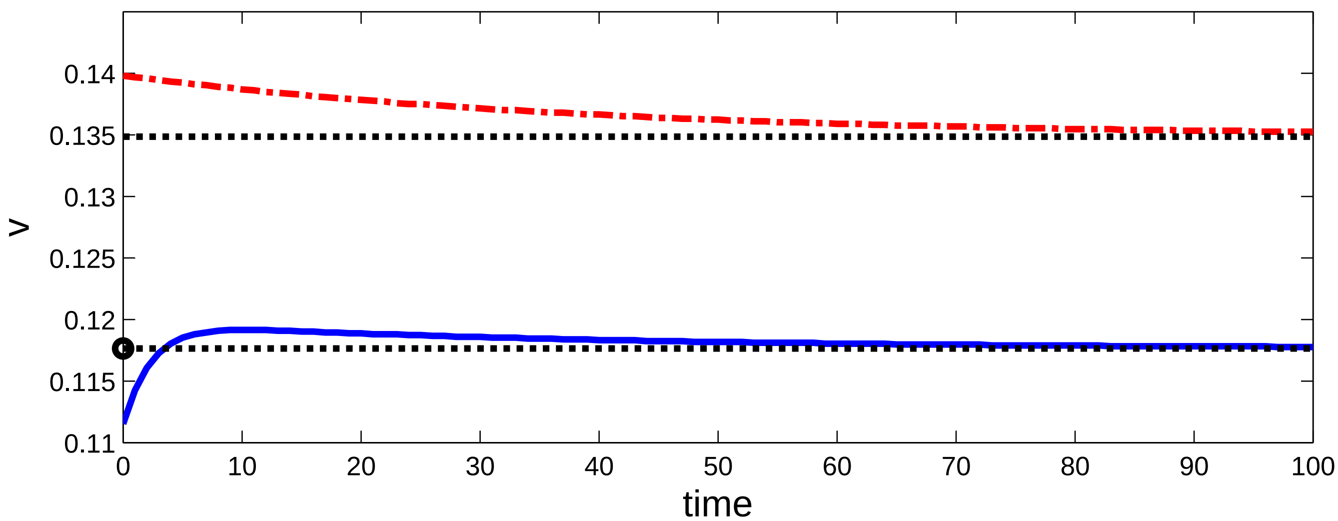


Figure 9c: human capital, h

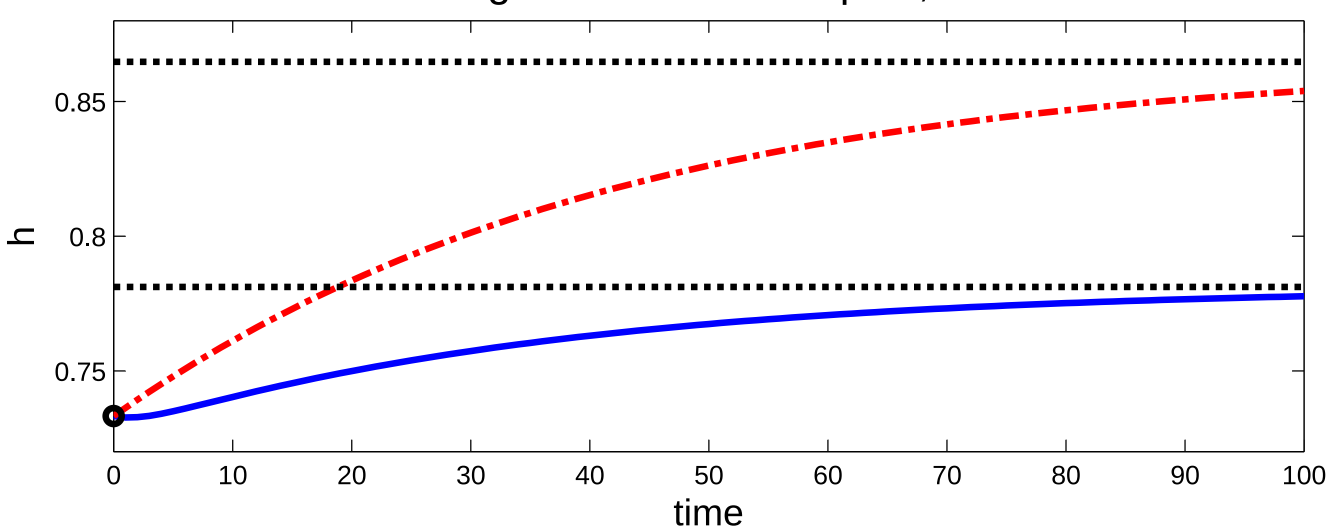


Figure 9d: technology levels, a

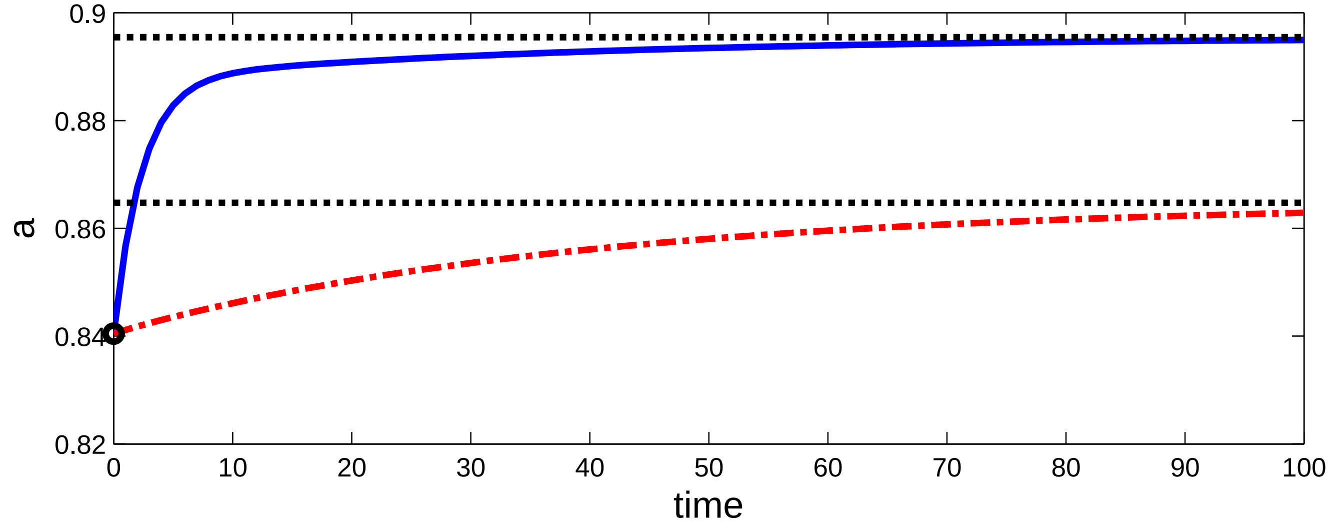


Figure 9e: consumption, c

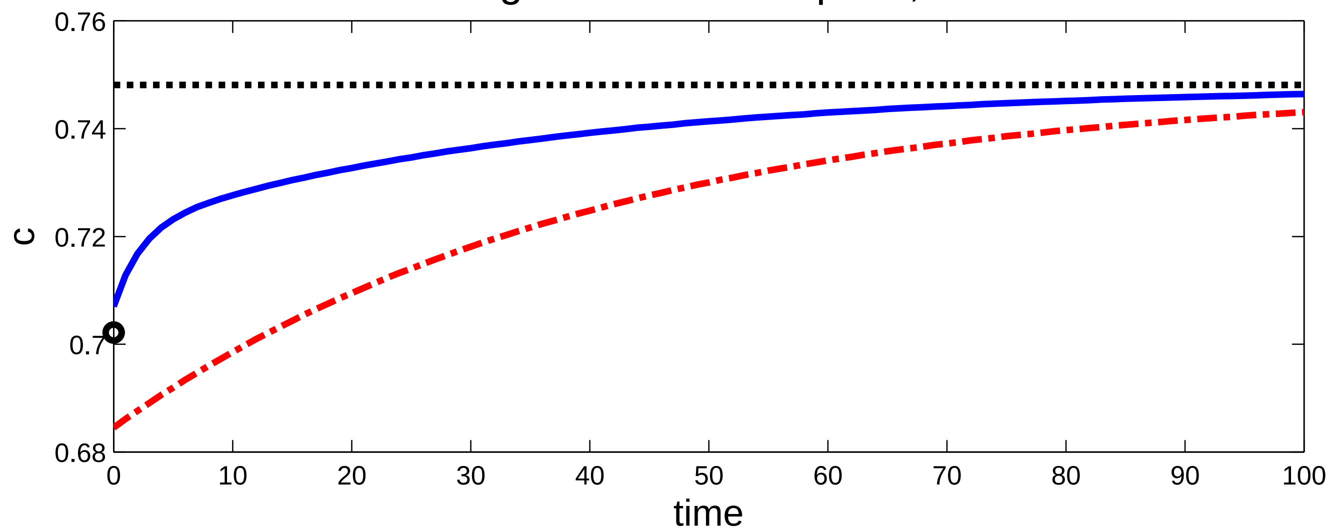


Figure 9f: growth rates (output)

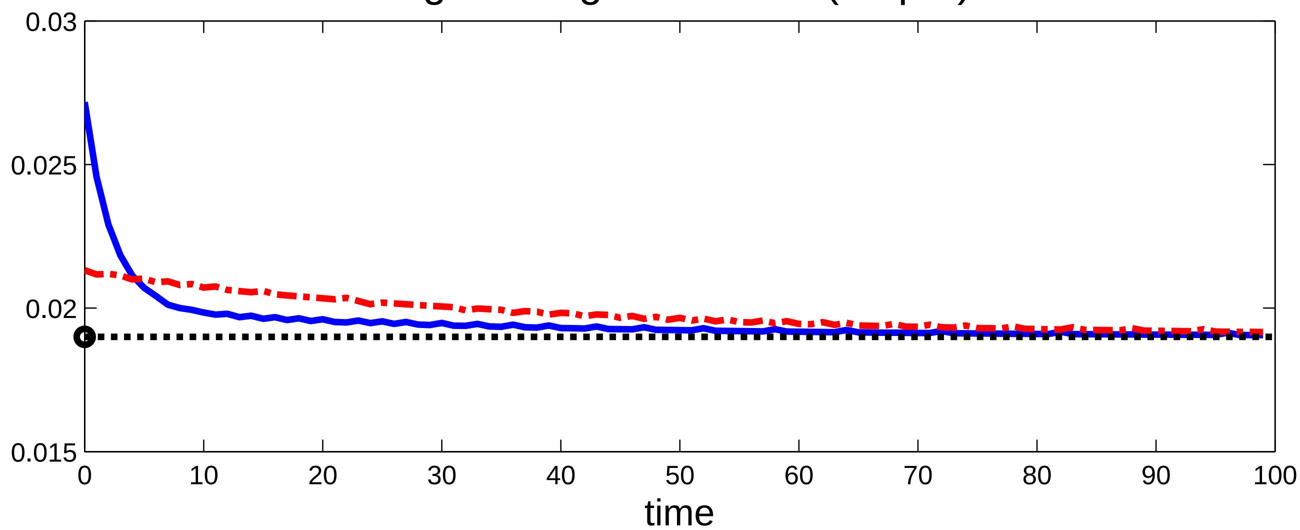


Figure 10a: transition for a miracle

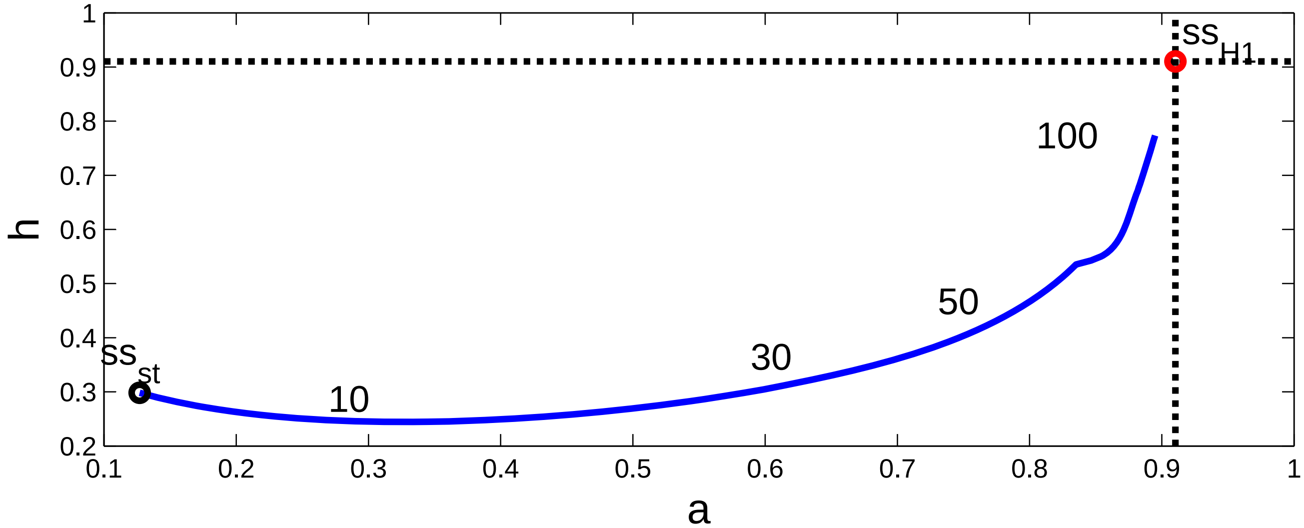


Figure 10b: time allocation

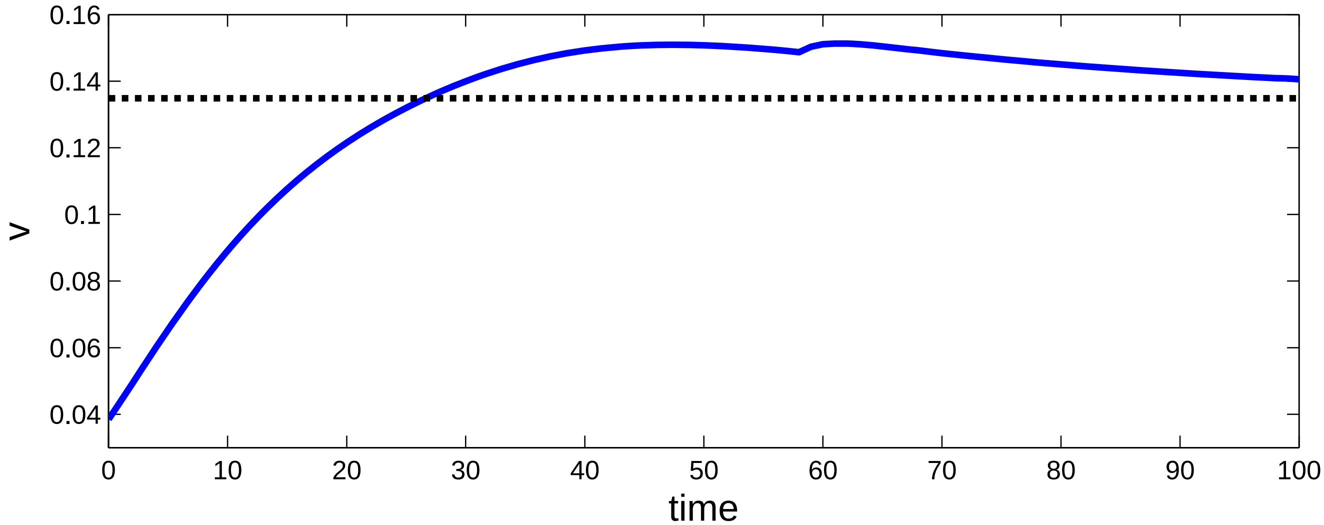


Figure 10c: a, h, and c, relative to BGP

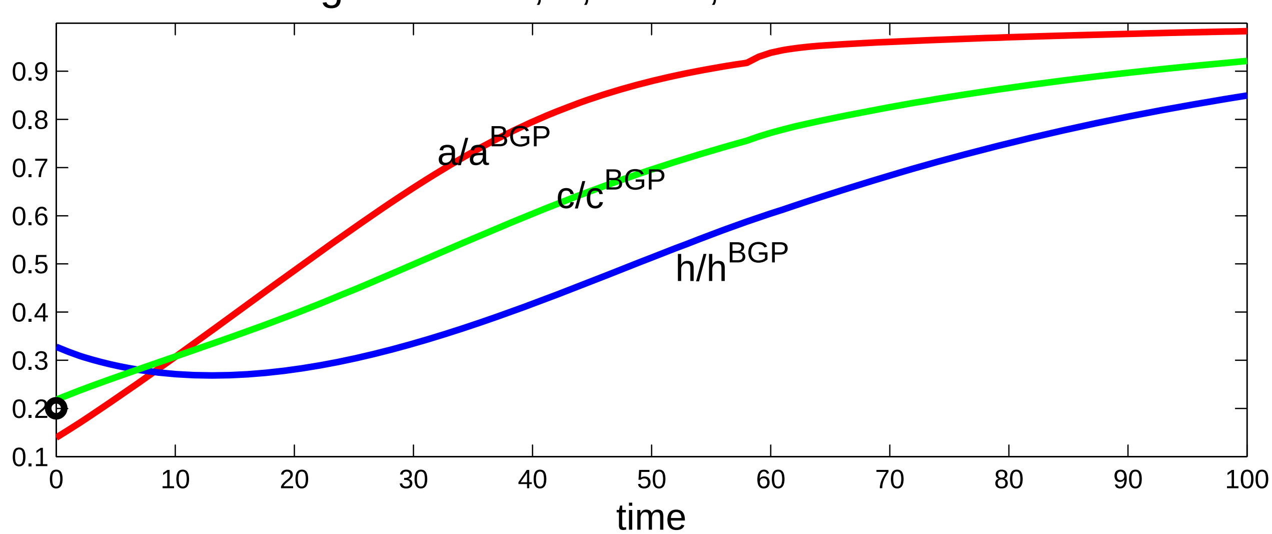


Figure 10d: Growth rates