Explaining Educational Attainment across Countries and over Time†

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Abstract

Consider the following facts. In 1950 rich countries attained an average of 8.1 years of schooling whereas poor countries attained 1.3 years –a 6-fold difference. By 2005, the difference in schooling declined to 2-fold even though per-capita income gaps did not decrease. What explains the educational attainment differences across countries and their evolution over time? We develop a model of human capital accumulation to quantitatively assess the importance of productivity, life expectancy, and growth in explaining educational attainment differences across countries and over time. Calibrating the parameters of the model to reproduce historical data in the United States, we find that the model with cross-country differences in productivity and life expectancy accounts for 89 percent of the difference in schooling levels between rich and poor countries in 1950 and 78 percent of the increase in schooling over time in poor countries. The model generates a faster increase in schooling in poor relative to rich economies even if their income gaps do not decrease.

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1 Introduction

Understanding the sources of income differences across countries and their evolution over time is one of the most important questions in development economics. Human capital accumulation is believed to play a crucial role but a precise assessment of this importance has been hindered by the lack of empirical measures of human capital.[1] We develop a model of human capital accumulation to quantitatively assess the importance of productivity, life expectancy, and growth in explaining educational attainment differences across countries and over time. A calibrated version of the model accounts for 89 percent of the difference in schooling between rich and poor countries in 1950 and 78 percent of the increase in schooling over time in poor countries. Moreover, the model generates a faster increase in schooling in poor relative to rich economies even if their income gaps do not decrease.

Using a panel data of schooling and income per capita for 87 countries from 1950 to 2005, we emphasize the following three facts. First, there are large differences in schooling measures across countries, with average years of schooling of the 25-29 year old population being a factor between 2 and 6-fold in rich relative to poor countries. Second, average years of schooling have increased over time in all countries in our sample. Third, average years of schooling have increased more in poor relative to rich countries. Hence, dispersion in schooling levels have decreased overtime. This occurs despite the fact that dispersion in income levels have not decreased. What can explain the differences and evolution of schooling levels across countries?

The model of human capital accumulation that we develop to account for these facts follows that in Restuccia and Vandenbroucke (2010). It features generations of rational, forward-looking individuals, seeking to maximize lifetime utility. There are two noteworthy elements

[1] Important exceptions include Hendricks (2002) and Schoellman (2010).

of the theory. First, labor supply is endogenous. Restuccia and Vandenbroucke (2010) argue that the endogeneity of labor supply quantitatively matters for the returns to schooling when there are substantial differences/changes in income levels. This was the case in the United States between 1870 and 1970 where income per capita increased by a factor of 7. Thus, it may also matter for the comparison between rich and poor countries since per-capita income differences are no less than a factor of 13 at any point in time between 1950 and 2005. Second, the model features an income effect from non-homothetic preferences. Such preferences are central to the theories of structural transformation in development.[2] There is also renewed interest in this type of preferences for the study of trade patterns, e.g. Markusen (2010) and Fieler (2010); and the study of broader transformations in an economy, e.g., Greenwood and Seshadri (2005). Income effects from non-homothetic preferences generate implications that are consistent with broad empirical patterns. For instance, schooling levels are higher and hours of work lower for people with higher ability and therefore higher income. Across countries, the expenditure share of necessities and hours of work are lower in rich countries relative to poor and rich countries invest more in schooling.

We calibrate a benchmark economy to fit a long time series for schooling and hours in the United States. This calibration puts restrictions on the strength of income effects in the theory. We then perform a series of cross-country quantitative experiments. We allow the levels of productivity and life expectancy as well as their changes over time to vary across countries. We discipline these differences by reproducing the evolution of the cross-country distribution of GDP per capita between 1950 and 2005 as well as two empirical relationships observed across countries. First, the relationship between life expectancy and income and second the relationship between changes in life expectancy and income. Our model, so disciplined, is consistent with the three facts emphasized earlier. First, the model generates

[2] See for instance Laitner (2000), Kongsamut, Rebelo, and Xie (2001), and Gollin, Parente, and Rogerson (2002).

substantial dispersion in schooling. In particular, in 1950, the model accounts for 89 percent of the difference in schooling between rich and poor countries. Second, the model implies that schooling increases in all countries. Third, the model implies a faster increase in schooling for poor countries than for rich countries and, therefore, is consistent with the contraction in the distribution of schooling observed in the cross-country data. This contraction occurs in the model even though, as in the data, there is no reduction in income gaps across countries. For the poorest countries the model accounts for 78 percent of the increase in schooling over time. Considering all countries at once, it accounts for no less than 57 percent of the increase in schooling. We find that when we abstract from cross-country differences in productivity growth and changes in life expectancy, the model still accounts for one third of the increase in schooling across countries.

There is a recent literature in macroeconomics assessing the role of human capital in development by bringing theory to measure human capital, see for instance Manuelli and Seshadri (2006) and Erosa, Koreshkova, and Restuccia (2010), among others. The focus of this literature has been to assess the importance of schooling and human capital differences in explaining income gaps across countries at a point in time. Less emphasis has been placed on understanding the evolution of income and schooling over time.[3] The time dimension of schooling and income data is an additional source of discipline in the channels emphasized by the theory. As a result, we complement the existing literature by emphasizing the time dimension of the data. This is not a trivial task.

The paper is organized as follows. In the next section, we present the facts from a panel of 87 countries from 1950 to 2005 for a measure of educational attainment and income. Section 3 describes the model in detail. In Section 4 we calibrate the model. Section 5 performs a quantitative analysis of cross-country differences in productivity levels, life expectancy, and

[3] One exception is Manuelli and Seshadri (2009).

growth in explaining the patterns in the panel data. We conclude in Section 6.

2 Facts

Data We construct a panel data of schooling and income as follows. We obtain average years of schooling for the population aged 25 to 29 from Barro and Lee (2010). We restrict the sample to the narrow age population to minimize the impact of demographics and other changes on schooling measures across time and space. It is also the definition that is best suited for the model we consider in Section 3. The schooling data is available for a large set of countries from 1950 to 2005 in 5 year intervals. We obtain gross domestic product (GDP) per capita from the Conference Board (2010), Total Economy Database. We restrict the time frame of this data from 1950 to 2005. To abstract from short-run fluctuations in real GDP we filter it using the Hodrick and Prescott (1997) filter with $\lambda = 100$ for yearly observations and keep the trend component of these time series. When we merge these two sets of data, we end up with a sample of 87 countries that have available data for schooling and GDP per capita from 1950 to 2005.

Facts We emphasize three sets of facts that arise from analyzing these data. First, schooling differences across countries are large at any point in time between 1950 to 2005. Second, schooling increases over time in all countries in our sample. Third, schooling differences across countries are smaller in 2005 than they were in 1950. The reduction in schooling differences across countries is systematic and occurs despite the fact that the income gap between rich and poor countries has not decreased. We now document these facts in detail.

1. There are large differences in educational attainment across countries.

Consider Table 1 which decomposes our sample into ten groups of countries according to the 1950 distribution of GDP per capita –i.e., the countries in each decile are the same in 1950 and in 2005. For each decile, the table reports the average GDP per capita relative to the United States and the average years of schooling. In 1950 there is a 6-fold difference in schooling between the richest and poorest decile of the distribution. In 2005 there remains a noticeable 2-fold difference. These differences are not specific to the top and bottom decile and/or to the initial and end year of our sample. Figure 1 documents them across all countries, for selected years, and shows that there has been a significant dispersion of schooling across all levels of income and at all dates. To put the magnitude of cross-country differences in schooling in perspective, consider that in 1900 in the United States a 35-year old had completed about 7.4 years of schooling. Hence, a 25-29 year old in 2005 in the average poor country still had 2 years less of schooling than a 35-year old in the United Sates in 1900.[4]

2. Educational attainment increased over time in all countries.

   Schooling increased between 1950 and 2005 for every country in our sample. Table 1 conveys an aggregated view of this fact since average years of schooling increase for each decile of the distribution. The increase in schooling between 1950 and 2005 is 37 percent for countries in the top decile and 299 percent for countries in the bottom decile. We note that the increase in educational attainment is positive for all deciles of the income distribution regardless of the initial income level or subsequent income growth relative to the United States. We expand on this fact next.

3. Differences in educational attainment across countries have been reduced substantially over time.

   Poor countries exhibit a tendency to increase their schooling faster than rich countries.

[4]The figure 7.4 years of schooling for the average 35-year old is from Goldin and Katz (2008). Table 1 shows that the years of schooling for the average 25-29 year old in the average poor country is 5.07 in 2005.

In Table 1 this is evidenced by the tendency of the 2005-to-1950 ratio of schooling (last column) to decrease with relative income. This is a remarkable finding given that for some deciles, such as the second and the fourth, relative income did not change between 1950 and 2005. For deciles such as the third or the fifth relative income increased and for the tenth decile relative income decreased. Yet, each group of countries experienced a substantial increase in schooling. A more complete and systematic documentation of the decline in schooling dispersion across countries is to report for each year the cross-sectional elasticity of income levels to schooling. That is, in each year, we regress the log of average years of schooling (for people 25-29 years old) on a constant and the log of GDP per capita, and report the slope coefficient. This cross-sectional elasticity is systematically falling over time as illustrated in Figure 2. The same declining pattern is observed for the time series elasticity, that is the elasticity of income levels to schooling for each country over time, although with only 12 observations per country the pattern has more noise.

3 The Model

Time is continuous. The world comprises a set of closed economies and hence in what follows we focus on a single economy to describe the model. At every moment a generation of homogeneous individuals of size 1 is born and lives for an interval of time of length T_τ. The index τ denotes an individual's generation: the date at which the individual is of age 0. We use t to refer to calendar time.

Individuals are endowed with one unit of productive time at each moment and no assets at birth. There is a worldwide rate of interest r at which borrowing and lending can occur without constraint. The payment to a unit of human capital-hour is denoted by z_t at moment

t. We generically refer to z_t as productivity and assume it grows at a time invariant rate g.

Preferences

Preferences are defined over lifetime sequences of consumption and leisure time as well as over time spent in school. As in Restuccia and Vandenbroucke (2010), we abstract from life-cycle considerations by imposing that consumption and leisure time remain constant throughout the individual's life. Hence, the preferences of an individual of generation τ can be represented by

$$\int_0^{T_\tau} e^{-\rho u}\left[U_\tau(c) + \alpha V(\ell)\right] du + \beta W(s),$$

where ρ is the subjective rate of discount assumed to be equal to the rate of interest, c is consumption, ℓ leisure time, and s time spent in school. The parameters α and β are positive.

We let

$$V(\ell) = \frac{\ell^{1-\sigma} - 1}{1-\sigma},$$

where $\sigma > 0$ and

$$W(s) = \ln(s).$$

We also assume that

$$U_\tau(c) = \ln[c - \bar{c}(z_\tau)],$$

where $\bar{c} : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ and $\lim_{z \rightarrow +\infty} \bar{c}(z) = \mu > 0$. There are two features of preferences that are worth discussing. First, the term $\bar{c}$ introduces a non-homotheticity which has the standard interpretation of a subsistence level above which consumption must remain at every point in time. This feature of preferences plays an important role in both the theoretical and quantitative properties of the model. In particular, when income is sufficiently large (alternatively when $\bar{c} = 0$), preferences display the standard property in modern macroeconomics where income and substitution effects of changes in productivity cancel each other and labor supply –as well as schooling– are constant. But when income is relatively low ($\bar{c} > 0$) both schooling and leisure are increasing in productivity.[5] Second, subsistence consumption is not constant but instead indexed by z_τ. The motivation for this feature is empirical. Allowing $\bar{c}$ to vary is critical to reproducing the historical time series of hours of work in the United States and to analyzing poor economies. We parameterize and discipline the function $\bar{c}$ using the U.S. time series data but we contrast our results with the case where $\bar{c}$ is constant. We note that there is a structural interpretation of time-varying $\bar{c}$ that can be derived from a model with household production where the rate of growth of market and home productivity differ. To the extent that we cannot empirically pin down these growth rates, we opted for a reduced form approach.[6] The assumption that $\bar{c}(z)$ is asymptotically constant allow us to discuss the long-run properties of the model.

Human Capital Technology

Individuals can acquire human capital by spending time in school and purchasing educational services. The human capital technology follows Bils and Klenow (2000) and is described by

$$H(s,x) = x^\gamma h(s) \equiv x^\gamma \exp\left(\frac{\theta}{1-\psi} s^{1-\psi}\right),$$

[5] A formal proof of these statements is in Restuccia and Vandenbroucke (2010).

[6] Consider an individual that can allocate time to market and non-market production and leisure. Market and non-market goods are prefect substitutes in consumption. The individual solves the following optimization problem:

$$\max_{c,h,\ell} \{\ln\left(c - \bar{c} + z_h h^\theta\right) + \ln(\ell) : c + z(\ell + h) = z\},$$

where h stands for home hours and z and z_h are market and home productivity. The solution for home hours is $h = \theta(z_h/z)^{1/(1-\theta)}$. Note that the period utility derived from consumption can be written as $\ln\left(c - \bar{m}(z_h, z)\right)$ where $\bar{m}(z_h, z) = \bar{c} - z_h\theta(z_h/z)^{\theta/(1-\theta)}$ and $\bar{m}$ shits as a function of market and home productivity growth.

where x represents purchases of educational services whose relative price is denoted by q. These services are purchased up front. Hence, x is more appropriately described as the present value of educational services. The parameter $\gamma \in (0,1)$ measures the elasticity of human capital to educational services. At an optimum γ is the share of lifetime income spent by an individual in educational services. The parameters $\theta > 0$ and $\psi > -1$ govern the importance of the time input in the production of human capital.

Optimization

The optimization problem for an individual of generation τ is

$$\max_{c,\ell,x,s}\left\{\int_0^{T_\tau} e^{-\rho u}\left(U_\tau(c)+\alpha V(\ell)\right)du+\beta W(s):\right.$$
$$\left.\int_0^{T_\tau} e^{-ru}c\,du+qx=z_\tau\int_s^{T_\tau} e^{(g-r)u}(1-\ell)H(s,x)du\right\}. \quad (1)$$

Our assumptions that consumption and leisure are constant throughout the life cycle and that the rate of interest equals the subjective discount rate imply that the optimization problem can also be written more compactly as:

$$\max_{c,\ell,x,s}\left\{a_\tau\left[U_\tau(c)+\alpha V(\ell)\right]+\beta W(s):a_\tau c+qx=z_\tau(1-\ell)H(s,x)d_\tau(s)\right\},$$

where $a_\tau=\int_0^{T_\tau} e^{-\rho u}du$ and $d_\tau(s)=\int_s^{T_\tau} e^{(g-r)u}du$ are discount terms. Note that the discount term for education includes the foregone labor income of s years of schooling.

We note that our assumption of constant consumption over the lifecycle is not too restrictive since with separable utility in consumption and leisure and $\rho = r$ an individual optimally chooses a constant path of consumption. However, these assumptions do not guarantee a constant path of leisure. We impose a constant leisure profile over the lifecycle for simplicity.[7]

The first order condition for schooling after substituting for x is given by

$$a_\tau U'_\tau(c)c\left[\frac{h'(s)}{h(s)}+\frac{d'_\tau(s)}{d_\tau(s)}\right]+(1-\gamma)\beta W'(s)=0. \tag{2}$$

We make two remarks about this equation. First, when individuals derive no utility from schooling (i.e., $\beta = 0$) the optimal level of schooling s is determined by setting the term in square brackets in (2) to zero. In this case, the optimal level of schooling maximizes lifetime income. To see this, let $i_\tau = z_\tau(1-\ell)x^\gamma h(s)d_\tau(s)$ denote lifetime income and note that the derivative of lifetime income with respect to schooling relative to lifetime income gives exactly the term in square brackets. An increase in s raises lifetime income through human capital accumulation $h'(s)/h(s)$ and reduces lifetime income by the foregone time working $d'_\tau(s)/d_\tau(s)$. At the optimal schooling level, these terms offset each other and lifetime income is maximized. Note that optimal schooling is independent of productivity z. Hence, when $\beta = 0$, schooling is independent of productivity and may differ across generations only through changes in the function d_τ, for instance changes in life expectancy T. Second, when individuals derive utility from schooling ($\beta > 0$) the term in square brackets is negative: the individual chooses more schooling than needed to maximize lifetime income. Third, productivity appears in the equation for optimal schooling only through consumption, that is through the intertemporal budget constraint in (1). The term $U'_\tau(c)c$ is critical, therefore, in driving the effect of productivity on the schooling choice. Given the functional form of U this term is

$$U'_\tau(c)c=\frac{c}{c-\bar{c}(z_\tau)}.$$

[7]We do not have detailed data on the lifecycle behavior of labor supply for generations dating back to the 19th century. In addition, the changes in lifecycle labor supply for recent cohorts are small in comparison with the variation in hours over time. Hence, we think there is little benefit to modeling labor supply over the lifecycle since our focus is on changes across countries and over time.

To illustrate the properties of this function, assume that $\bar{c}$ is a positive constant and consider an increasing path of income and consumption. At low levels of income consumption is close to $\bar{c}$ and the term $U'_\tau(c)c$ is large. Increases in income, reduce the value of $U'_\tau(c)c$ and since the term in square brackets in (2) is negative the left-hand side of (2) increases so that optimal schooling increases. As income rises the term $U'_\tau(c)c$ asymptots 1 and s becomes invariant to changes in income. Hence, qualitatively the model delivers the observed pattern in the data that schooling increases faster for poor than for rich countries. In general $\bar{c}$ is not constant but instead a function of productivity. Since we assume that $\lim_{z\to+\infty} \bar{c}(z) = \mu > 0$, in the long-run s is invariant to productivity. In the quantitative analysis we assume that $\bar{c}(z)$ is an increasing function which affects the rate at which $U'_\tau(c)c$ decreases and, therefore, matters for the change in schooling s. We assess the quantitative importance of this effect in Section 5.

The first order condition for leisure is

$$U'_\tau(c)c - (1-\gamma)\alpha V'(\ell)(1-\ell) = 0. \tag{3}$$

Given the functional form for V, the term $V'(\ell)(1-\ell)$ is decreasing in ℓ. Hence, as income grows and $U'_\tau(c)c$ decreases towards 1, leisure time increases. Asymptotically, leisure time is constant. Note that our choice of $U_\tau(c)$ implies that this asymptotic value of ℓ is in the interior of $(0,1)$.[8]

We define y_τ as the period income of an individual of generation τ at age 35:

$$y_\tau = z_\tau e^{35g}(1-\ell_\tau)H(s_\tau, x_\tau).$$

[8]Functions such as $U_\tau(c) = [(c-\bar{c}(z_\tau))^{1-\eta} - 1]/(1-\eta)$, where $\eta > 1$ imply that $U'_\tau(c)c \to 0$ as $c \to \infty$. In this case the asymptotic long-run value for leisure time is one.

Since we use this measure in our quantitative analysis, we emphasize how increases in productivity affect income. First, an increase in productivity raises income through three channels: a direct effect through z_τ; an indirect effect through increases in schooling s; and another indirect effect through increases in expenditures in education x and therefore human capital. Second, an increase in productivity induces an increase in leisure time and therefore reduce labor income. The increase in leisure hinders the incentive to acquire education.

4 Calibration

We calibrate a benchmark economy to the time-series data for the United States. Although the emphasis of our quantitative exercise is on the cross-country implications from 1950 to 2005, we calibrate the model using the longest possible time series of the variables of interest for the United States. The motivation for this strategy is simple. Since the key channel in the model is the strength of the income effect on schooling and hours of work, the time path of these variables in the United States provides quantitative discipline. The long time series allows for better identification of the parameters of the model. In particular, as is well documented for the United States over time, there is a long-run increase in schooling followed by a slowdown toward the end of the 20th century. Similarly, there is a long-run decline in weekly hours followed by a slowdown. Our calibration procedure exploits these changing trends to discipline the strength of the income effect that is central to the quantitative implications of the model across countries. To summarize, we use the information provided by the historical time series for schooling and hours in the United States to discipline the model's implications across countries and over time.

For the United States, the schooling data that we use is provided by Claudia Goldin and

Larry Katz and serve as the basis of Figures 1.4, 1.5 and 1.6 in their book.[9] The data give years of schooling by birth cohort, completed at age 35 for white people starting in 1876 until 1975. We HP-filtered the time series and used, for calibration purposes, cohorts from 1880 to 1965. These cohorts of people are of age 35 in years 1915 through 2000. The trend in schooling shows that a 35-year-old person in 1915 had completed about 8 years of schooling while the same-age person in 2000 had completed close to 14 years. The weekly hours data that we use are built from various sources: Kendrick (1961, Table A-IV), McGrattan and Rogerson (2004, Table 1), and Whaples (1990, Table 2.1), and are available at census dates from 1830 until 2000. We HP-filtered the data and linearly interpolate between census dates to build a time series of hours from 1830 through 2000. The trend shows a decline from close to 72 hours per week in 1830 to 40 hours in 2000. Importantly, the rate of this decline is non-constant. There is a moderate decline in hours from 1830 to about 1910, followed by a sharp decline until about 1980, and a substantial flattening after 1980. This non-monotonic pattern in the rate of change of hours motivates a key aspect of our calibration strategy that as we discuss below.

The hours data are available in calendar time while the model predicts hours by generation. We choose to associate the 1830 hours data with the 1795 generation from the model, i.e. the generation that is 35 years old in 1830. That is, when we compare the model's predictions to the U.S. data we compare the hours chosen by the 1795 generation in the model with the 1830 data on hours. We associate subsequent data points and generations in the same way, until the 1965 generation which corresponds to individuals reaching age 35 in 2000. Thus, our calibration procedure implies that we compute decisions for 171 generations, starting with 1795 and ending with 1965.

We use data on life expectancy and measures of lifetime labor supply to build a sequence of

[9] See Goldin and Katz (2008, Figures 1.4–1.6).

T for 171 U.S. generations. Following the argument in Restuccia and Vandenbroucke (2010), we let $T_{1880} = 44$ for the 1880 generation and let it increase at a rate of 0.3 percent from one generation to the next. This implies that the 1795 generation, the first one we compute with our model, is endowed with $T_{1795} = 34$.

We specify now the function $\bar{c}(z)$. We use a transformed version of the logistic function as follows:

$$\bar{c}(z) = \frac{\mu}{1 + \exp(-z + \omega)},$$

where μ and ω are parameters to be determined. The motivation for this function is the non-monotonic pattern of changes in hours over time. A model with constant $\bar{c}$ implies an income elasticity of hours that falls exponentially as income rises. With constant income growth this implies that hours fall exponentially over time. This pattern of hours in the model can fit the U.S. hours data from 1910 but not the pattern before 1910. Since the U.S. was poorer before 1910 than nowadays, fitting the income elasticity of hours at low levels of income has implications for the model's predictions about poor countries. We assess quantitatively the importance of time-varying $\bar{c}$ in Section 5.3.

We now describe the details of the calibration procedure. We start the calibration of the model by normalizing the productivity parameter $z_{1795} = 1$. We set the discount factor to 4 percent, i.e., $\rho = 0.04$ and, following Bils and Klenow (2000), we choose $\gamma = 0.1$. We pick the rest of the parameters in order to minimize a measure of distance between the model's predictions and relevant U.S. data. Specifically, let λ be the vector of parameters to calibrate:

$$\lambda = [\alpha, \beta, \mu, \omega, \psi, \theta, g]',$$

and let $\hat{s}_t(\lambda)$ and $\hat{n}_t(\lambda)$ represent optimal schooling and work time of generation τ. Let $\mathbf{s}_\tau$ and $\mathbf{n}_\tau$ be their empirical counterpart: $\mathbf{s}_\tau$ is years of schooling for generation τ in the U.S.

data and $\mathbf{n}_\tau$ is the workweek at date $\tau+35$ in the U.S. data. The mapping of hours between the model and the data is done by assuming that there are 112 hours of discretionary time per week.[10] Hence, a 40-hour workweek corresponds to 40/112 units of work time in the model. We find λ by solving the following minimization problem:

$$\min_{\lambda}\left\{\sum_{\tau=1880}^{1965}\left(\frac{\hat{s}_\tau(\lambda)}{\mathbf{s}_\tau}-1\right)^2+\sum_{\tau=1795}^{1965}\left(\frac{\hat{n}_\tau(\lambda)}{\mathbf{n}_\tau/112}-1\right)^2+M'(\lambda)M(\lambda)\right\},$$

where

$$M(\lambda)=\begin{bmatrix}\hat{y}_{1965}(\lambda)/\hat{y}_{1795}(\lambda)/e^{0.02\times 170}-1\\ \bar{c}(z_{1965})/\hat{y}_{1965}(\lambda)/0.02-1\end{bmatrix}.$$

In addition to targeting the time series of hours and years of schooling in the data, our calibration procedure also targets a growth in income per capita of 2 percent per year and a ratio of subsistence consumption to income of 2 percent for the last generation. We obtain this target by interpreting the fact reported by Maddison (2009) that GDP per capita between years 1 and 1500 was between 2 to 4.5 percent of the GDP per capita in the United States in 1970.

Table 2 shows the values of the calibrated parameters. Figures 3 and 4 show the excellent fit of the model to the U.S. data on schooling and hours. Note how the time series of hours implied by the model fits the changing pattern of the rate of change in actual hours. The calibrated function of $\bar{c}(z)$ permits this fit. We show that when $\bar{c}(z)$ is constant, the best fit the model can produce for the time series of hours is a strictly convex pattern that would fail to fit the changing pattern in hours over time.

[10] Assuming that a person needs 8 hours for sleep and other necessary needs, there are $(24-8)\times 7=112$ hours of discretionary time in a week.

5 Cross-Country Experiments

We conduct cross-country experiments using the calibrated model to quantitatively assess the importance of productivity, life expectancy, and growth in explaining educational attainment differences across countries and over time.

5.1 Baseline Experiment

We use the calibration of the benchmark economy and assume that countries are identical to the benchmark economy except on productivity and life expectancy. In particular, we assume that countries differ in the initial level and growth of productivity z and g; and in the initial level and change in life expectancy T and g_T. Broadly speaking, we choose these 4 parameters in each country so that the model reproduces the cross-country data on GDP per capita for 1950 and 2005 as documented in Table 1 and data on life expectancy for 1950 and 2005. A detailed explanation of the discipline imposed on the cross-country parameters for this experiment is in the appendix. Essentially, our approach exploits two empirical links observed across countries: First, the link between income levels and life expectancy and second, the link between income levels and changes in life expectancy over time. Imposing these links in the model implies that we can associate an economy with an arbitrary income level to the level and changes over time in life expectancy. Hence, we search for a combination of productivity level z and growth g that match the relative income gaps in 1950 and 2005. We consider 10 different values of z and g so that the model economies mimic a decile of the distribution of GDP per capita as documented in Table 1. The implied values of z, g, T, and g_T for each economy are reported in the first four columns in Table 3.

There are two sets of results that we emphasize from Table 3: the cross-section implications of the model relative to the data in 1950 and the time-series behavior across countries relative to the data. We start with the cross-section implications in 1950. We find that the model accounts for 89 percent of the difference in schooling between countries in the 1st decile and the United States. To understand how we obtain this statistic, note that for countries in the poorest decile of income in 1950, the model implies 2.3 years of schooling whereas the data is 1.27 years (see Table 1). In 1950 the United States has 10.4 years of schooling which is reproduced by our calibrated benchmark economy. Hence, the model accounts for $(10.40-2.3)/(10.4-1.27) = 89\%$ of the difference. The model accounts for a lower percentage of schooling differences for countries in higher deciles of income: 87% for the 5th decile and 25% for the 10th decile. Therefore, there is a systematic tendency for the model to account for lower fractions of the schooling data as we consider richer countries. This is because the mechanism emphasized in our theory (non-homotheticy in preferences) vanishes at high levels of income to eventually play no role. For rich countries, factors other than income levels have first-order importance in the determination of schooling, e.g., public policy towards education, among others. In poor countries, however, increases in productivity and income allow individuals to move farther away from subsistence consumption having a first-order effect on the allocation of time in schooling.

We now turn to the time-series implications of schooling across countries. We find that the model accounts for 78 percent of the increase in schooling in poor countries. We compute this statistic as follows. For the economy in the 1st decile, schooling increases from 2.3 years in 1950 to 6.8 in 2005, a $\ln(6.8/2.3)/55 = 2\%$ annual increase. It compares with 2.56% in the data. Thus, for this economy, the model accounts for $2.0/2.56 = 78\%$ of the increase in schooling. Similarly, for countries in the 5th and 10th deciles the model accounts for 79 and 155% of the increase in schooling. We note from Table 3 that schooling increases in all economies in our model, an implication that is consistent with the data. We also note the tendency for poor economies to increase their schooling faster than richer economies even though they are not catching up in relative income. As a consequence of this faster increase in schooling in poor economies, schooling differences in the model are smaller in 2005 than in 1950. This implication of the model is also consistent with the data. In particular, it is consistent with the decreasing income elasticity of schooling across countries displayed in Figure 2. Using Table 3, we compute an approximation of this elasticity with the ratio of relative changes in schooling and income between deciles of the distributions. Comparing the 1st and 10th decile in Table 3, we find an elasticity of 0.53 (i.e., $\ln(9.8/2.3)/\ln(0.83/0.05)$) in 1950 versus 0.32 in 2005. Again, the decrease of this elasticity is evidence of the reduced dispersion in schooling across countries in 2005 and, therefore, of the faster increase in schooling in poorer countries. Comparing the 1st and 5th deciles in Table 3 yields elasticities of 0.55 and 0.24 in 1950 and 2005. Comparing the 5th and 10th deciles yields 0.52 and 0.46.

In terms of hours there is limited data that can be brought to bear on the implications of the model. Nevertheless, we use the available hours data from Conference Board (2010). They report yearly hours per worker and we plot the data against GDP per capita in Figure 5. In the data in 1950, hours of work in poor countries relative to rich is about 1.4, while the same ratio drops to 1.2 in 2005. In the model, the ratio of hours in the poor economy relative to the benchmark is 2.4 in 1950 and drops to 1.7 in 2005. While the comparison here is very crude since the hours data is likely to be more noisy and the better mapping to the model would be hours per capita, the rough comparison suggests that the hours implications of the model are broadly in line with the data in terms of both the magnitude of hours differences in 1950 and the faster decline in hours over time in poor countries.

5.2 Constant Growth Rates across Countries

To illustrate the importance of differences in productivity growth and changes in life expectancy across countries, we conduct two additional experiments. First, we conduct an experiment as in the baseline except that we assume that g is the same in all countries at the value in the benchmark economy. Second, we assume that in addition, changes in life expectancy over time g_T are the same across countries at the value in the benchmark economy. Essentially, we show with these experiments that the implications of the model for the cross-country differences in schooling in 1950 are not substantially affected by the growth components. We also show that even when abstracting from productivity growth and changes in life expectancy, the model still accounts for a substantial portion (more than a third) of the changes in schooling over time across countries.

In the first experiment, we follow the baseline except that we assume constant productivity growth g across countries. Results are reported in Table 4. In the 1950 cross-section the model accounts for 86 percent of the difference in schooling between countries in the 1st decile and the United States. This compares to 89 percent in the baseline. At the 5th and 9th deciles these figures are 85 and 58 percent (87 and 63 in the baseline). Turning to the time-series implications, the model with constant g across economies implies that in the 1st decile, the model accounts for 71 percent of the increase in schooling versus 78% in the baseline experiment. In the 5th decile the figure is 74% (79% in the baseline). Just as in the baseline, the model with constant g predicts a narrowing of the schooling gap relative to income as observed in the data. As discussed previously, we compute an approximation of the cross-country income elasticity of schooling in 1950 and 2005 and show that the implied elasticity is lower in 2005 than in 1950. Comparing the 1st and 10th decile, we find that the income elasticity of schooling falls from 0.50 to 0.29. The corresponding figures, when we compare the 1st and 5th decile are 0.50 and 0.31 and when we compare the 5th and 10th they are 0.49 and 0.27.

In the second experiment, we follow the previous experiment and assume in addition that the change in life expectancy for each country is the same as in the benchmark economy. The results of this experiment are reported in Table 5. We first note that in terms of the cross-section of 1950 the predictions of the model in this experiment are identical to those of the previous experiment.[11] In terms of the time series, abstracting from differences in the growth rate of T reduces the ability of the model to account for the increase in schooling. Nevertheless, even when abstracting from differences in productivity growth and changes in life expectancy, the model accounts 33% of the growth rate of schooling in the 1st decile. At the 5th decile the corresponding figure is 41%.

5.3 Importance of $\bar{c}(z)$

In this section we assess the quantitative importance of our assumption that the subsistence level $\bar{c}$ is time-varying. We conduct an experiment where we assume a constant $\bar{c}$. We calibrate the benchmark economy to U.S. data as in Section 4. The list of parameters to be determined by the minimization program is now $[\alpha, \beta, \bar{c}, \psi, \theta, g]'$. Figures 6 and 7 show the fit of this version of the model against the U.S. data. In particular, Figure 7 shows that the time path of hours exhibits a convex shape that fails to fit the time series behavior of hours, particularly in the earlier period. A consequence of this convex shape is that hours increase "fast" as we consider lower levels of productivity. In our model, where income growth is roughly constant, this behavior reveals an increasing elasticity of hours to income as income decreases. In contrast, the U.S. data exhibits a declining elasticity of hours to income as

[11] This is not a numerical artifact. Instead it is due to the fact that, by design, the experiment implies that the level of life expectancy and income are the same in 1950. The only difference between the two experiments is the level of life expectancy after 1950.

income decreases for the earlier period.[12]

We conduct the same cross-country experiment as described in Section 5.1 using the calibrated parameters of the benchmark economy with a constant $\bar{c}$. Table 6 reports the results. The model generates such large effects on hours and schooling for poor countries that it is not possible to compute economies below 26 percent of the 1950 income per-capita in the benchmark economy. When computing an economy with 26% relative income in 1950, hours reach 100 versus 65.7 for an economy that is 37% of the benchmark. Thus, at this level the labor supply elasticity is very high, as suggested by Figure 7. Finally, we note that the model with a constant $\bar{c}$ exaggerates also on the elasticity of schooling to income. This can be seen by comparing schooling in 1950 for countries of the 8th decile in the baseline experiment, where $\bar{c}$ varies with productivity (Table 3), and the current experiment. In the baseline, schooling is 6.2 years for the 8th decile while in the current experiment it is 5.5 years. To put it differently, we find that at the 8th decile the model with constant $\bar{c}$ accounts for 95% of the schooling gap with the United States in 1950 versus 82% in the baseline experiment.

6 Conclusions

We developed a model of human capital accumulation to quantitatively assess the importance of productivity levels, life expectancy, and growth in explaining differences in educational attainment across countries and over time. We calibrated a benchmark economy to reproduce the historical evolution of schooling and hours in the United States. We found that the model

[12] A back-of-the-envelope calculation reveals a consequence of this convex behavior of hours. From Figure 7 note that by 1830 hours are 85 (out of a total of 112) while income per capita is 18% of the 1950 income. It is likely, therefore, that the utility maximization problem is not well-defined for levels of productivity too below 18% since even if individuals worked all their endowment of time, they would not be able to afford $\bar{c}$. In fact, from Table 1, 18% of relative income corresponds to the 5th decile of the 1950 distribution of GDP per capita.

with cross-country differences in productivity and life expectancy accounts for 89 percent of the difference in schooling between rich and poor countries in 1950. The model also accounts for 78 percent of the increase in schooling levels over time in poor countries and the fast catch-up in schooling levels in poor relative to rich countries. Hence, the model explains the convergence in cross-country schooling levels observed in the data even though per-capita income gaps do not decrease. We show that the implications of the model on hours of work are broadly consistent with the available cross-country data.

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A Cross-Country Experiments

In this appendix we describe in more detail our strategy to bring quantitative discipline to our choices of the four parameters varying across countries in the cross-country experiments of Section 5. These parameters are the level and growth rate of productivity z and g and the level of life expectancy T and change over time g_T.

Our procedure is best described in two steps. First, we estimate two empirical relationships observed across countries. The first relationship between the level of life expectancy and income per capita relative to the United States, and the second relationship between the rate of growth of life expectancy and income per capita relative to the United States. Second, we choose the level and growth rate of productivity to reproduce the level of income per capita, relative to the United States, in 1950 and 2005, as documented in Table 1. In our baseline experiment this procedure amounts to solving a system of two equations in two unknowns, z and g. The two equations solved for are the differences between relative income per capita in the model and in the data, in 1950 and 2005. The level and rate of growth of T are pinned downed by applying the two empirical relationship estimated in the first step to the 1950 income of the economy.

We now describe formally our choice of T and g_T for a given economy. We use data on GDP per capita and life expectancy at birth, for a panel of 78 countries between 1960 and 2005.[13] We acknowledge that life expectancy at birth is not the empirical counterpart of T, which would be more appropriately described as the sum of years of schooling and working years. Unfortunately, such data is not available for a large range of countries and years. Let $\mathbf{y}_t^i$ denote the GDP per capita of country i at date t in our data set and let $\mathbf{l}_t^i$ denote life expectancy at birth. Define $\mathbf{g}_\mathbf{l}^i$ as the average annual growth rate of life expectancy at birth

[13] The life expectancy data is from the World Bank and GDP data is from Penn World Tables. The data set is available from the authors upon request.

in country i between 1960 and 2005. First, we estimate the following equation

$$\ln\left(\frac{1+\mathbf{g}_{\mathbf{l}}^{i}}{1+\mathbf{g}_{\mathbf{l}}^{us}}\right) = a_g + b_g \ln\left(\frac{\mathbf{y}_{1960}^{i}}{\mathbf{y}_{1960}^{us}}\right) + u_i.$$

We find $\hat{a}_g = 0.0008$ and $\hat{b}_g = -0.0014$.[14] We use this relationship to calibrate the rate of growth of T for the economy we consider, relative to the rate of growth of T in the benchmark economy. The rate of growth of T in the benchmark economy was calibrated to 0.003 in Section 4. Thus, we use

$$\ln\left(\frac{1+g_T}{1+0.003}\right) = \hat{a}_g + \hat{b}_g \ln\left(\frac{\mathbf{y}_{1950}}{\mathbf{y}_{1950}^{us}}\right)$$

for the rate of growth of T of an economy with relative GDP per capita given by $\mathbf{y}_{1950}/\mathbf{y}_{1950}^{us}$ in 1950. Next, we estimate

$$\ln\left(\frac{\mathbf{l}_{1960}^{i}}{\mathbf{l}_{1960}^{us}}\right) = a_l + b_l \ln\left(\frac{\mathbf{y}_{1960}^{i}}{\mathbf{y}_{1960}^{us}}\right) + v_i,$$

and find $\hat{a}_l = 0.0119$ and $\hat{b}_l = 0.1963$.[15] We proceed similarly as we did for the growth rate. We use this estimation to calibrate the initial level of T relative to the initial level of T in the benchmark economy. The latter was calibrated to 48.87 for the 1915 generation in Section 4. Hence we use

$$\ln\left(\frac{T_{1795}}{48.87}e^{120 g_T}\right) = \hat{a}_l + \hat{b}_l \ln\left(\frac{\mathbf{y}_{1950}}{\mathbf{y}_{1950}^{us}}\right).$$

for the initial level of T of an economy with relative GDP per capita given by $\mathbf{y}_{1950}/\mathbf{y}_{1950}^{us}$ in 1950.

[14] The elasticity b_g is significant at 5%. The r^2 of the regression is 16%.
[15] The elasticity $\hat{b}_l$ is significant at 5%. The r^2 of the regression is 64%.

Table 1: GDP per Capita and Schooling across Countries

	1950		2005		
Decile	y_{50}	s_{50}	y_{05}	s_{05}	s_{05}/s_{50}
1	0.05	1.27	0.06	5.07	3.99
2	0.07	1.65	0.07	6.79	4.12
3	0.10	2.82	0.18	8.47	3.00
4	0.13	1.88	0.13	8.01	4.26
5	0.18	3.55	0.31	10.29	2.90
6	0.21	3.34	0.24	9.16	2.74
7	0.24	4.23	0.33	10.36	2.45
8	0.37	5.30	0.57	10.56	1.99
9	0.59	6.73	0.71	11.74	1.74
10	0.83	8.11	0.78	11.08	1.37
$R_{10/1}$	16.60	6.39	13.00	2.19	-
$R_{9/1}$	11.80	5.30	11.83	2.32	-

Note: y is real GDP per capita relative to that of the United States and s is average years of schooling of the 25-29 year old population. Numbers reported are the average of each decile. The countries in each decile are the same in each year and represent the 1950 distribution of GDP per capita.

Table 2: Calibration

Preferences	$\rho = 0.04$, $\alpha = 3.39$, $\beta = 1.75$
	$\sigma = 0$, $\mu = 1.24$, $\omega = 1.73$
Technology	$\gamma = 0.1$
	$\theta = 0.038$, $\psi = 0.09$
Productivity	$g = 0.020$, $z_{1795} = 1$
Demography	$g_T = 0.003$, $T_{1795} = 34$

Table 3: Model's Implications – Cross-Country Differences in the Levels of Productivity and Life Expectancy, z and T, and their Rates of Growth, g and g_T

Calibrated Parameters				1950 Results			2005 Results		
z_{1795}	g (%)	T_{1915}	T_{1970}	Rel. y	s	$1-\ell$	Rel. y	s	$1-\ell$
0.02	2.61	28.0	43.1	0.05	2.3	104.3	0.06	6.8	65.3
0.04	2.30	29.3	44.4	0.07	2.9	84.2	0.07	7.4	62.8
0.01	3.42	31.5	46.3	0.10	3.2	85.0	0.18	8.4	56.6
0.10	2.20	33.1	47.8	0.13	4.0	69.9	0.13	9.1	54.5
0.02	3.30	35.3	49.7	0.18	4.5	70.0	0.31	10.4	47.3
0.09	2.55	36.4	50.6	0.21	4.9	66.2	0.24	10.8	47.1
0.06	2.91	37.4	51.4	0.24	5.1	66.0	0.33	11.3	45.1
0.06	3.12	40.7	54.2	0.37	6.2	62.7	0.57	12.9	41.7
0.28	2.49	44.6	57.3	0.59	8.1	51.9	0.71	14.6	39.8
1.08	1.88	47.7	59.7	0.83	9.8	44.5	0.78	15.9	39.0
1.00	2.04	48.9	57.6	1.00	10.4	43.5	1.00	14.9	38.5

Note: y is output per capita, s is average years of schooling, and $1-\ell$ is hours worked.

Table 4: Model's Implications – Cross-Country Differences in the Levels of Productivity and Life Expectancy, z and T, and the Rate of Growth of Life Expectancy, g_T

Calibrated Parameters				1950 Results			2005 Results		
z_{1795}	g (%)	T_{1915}	T_{1970}	Rel. y	s	$1-\ell$	Rel. y	s	$1-\ell$
0.05	2.04	28.0	43.1	0.05	2.5	91.3	0.04	6.8	65.3
0.06	2.04	29.3	44.4	0.07	3.0	81.2	0.06	7.4	63.0
0.09	2.04	31.5	46.3	0.10	3.6	72.9	0.09	8.3	58.8
0.12	2.04	33.1	47.8	0.13	4.0	69.4	0.12	9.1	54.9
0.17	2.04	35.3	49.7	0.18	4.6	66.3	0.16	10.2	50.2
0.20	2.04	36.4	50.6	0.21	4.9	65.0	0.18	10.7	48.2
0.22	2.04	37.4	51.4	0.24	5.2	63.7	0.21	11.2	46.6
0.35	2.04	40.7	54.2	0.37	6.5	57.5	0.33	12.8	42.7
0.59	2.04	44.6	57.3	0.59	8.3	49.2	0.57	14.6	40.0
0.83	2.04	47.7	59.7	0.83	9.8	45.1	0.85	15.9	38.9
1.00	2.04	48.9	57.6	1.00	10.4	43.5	1.00	14.9	38.5

Note: y is output per capita, s is average years of schooling, and $1-\ell$ is hours worked.

Table 5: Model's Implications – Cross-Country Differences in the Levels of Productivity and Life Expectancy, z and T

Calibrated Parameters				1950 Results			2005 Results		
z_{1795}	g (%)	T_{1915}	T_{1970}	Rel. y	s	$1-\ell$	Rel. y	s	$1-\ell$
0.05	2.04	28.0	33.0	0.05	2.5	91.3	0.04	4.0	68.4
0.06	2.04	29.3	34.6	0.07	3.0	81.2	0.06	4.5	65.6
0.09	2.04	31.5	37.1	0.10	3.6	72.9	0.09	5.3	60.9
0.12	2.04	33.1	39.1	0.13	4.0	69.4	0.11	6.0	56.6
0.17	2.04	35.3	41.7	0.18	4.6	66.3	0.15	7.1	51.3
0.20	2.04	36.4	42.9	0.21	4.9	65.0	0.17	7.7	49.1
0.22	2.04	37.4	44.1	0.24	5.2	63.7	0.19	8.2	47.4
0.35	2.04	40.7	48.0	0.37	6.5	57.5	0.30	10.1	43.0
0.59	2.04	44.6	52.6	0.59	8.3	49.2	0.53	12.3	40.1
0.83	2.04	47.7	56.2	0.83	9.8	45.1	0.80	14.2	39.0
1.00	2.04	48.9	57.6	1.00	10.4	43.5	1.00	14.9	38.5

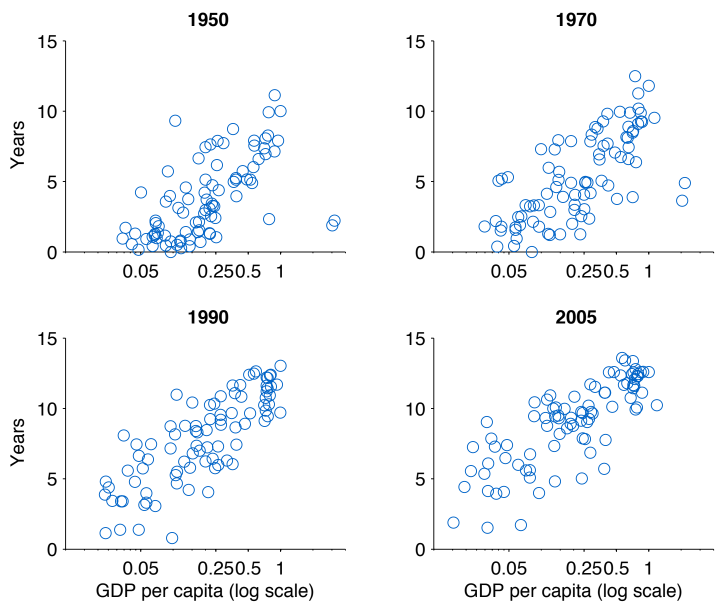
Note: y is output per capita, s is average years of schooling, and $1-\ell$ is hours worked.

Table 6: Model's Implications with $\bar{c}$ Constant – Cross-Country Differences in the Levels of Productivity and Life Expectancy, z and T, and their Rates of Growth, g and g_T

Calibrated Parameters				1950 Results			2005 Results		
z_{1795}	g (%)	T_{1915}	T_{1970}	Rel. y	s	$1-\ell$	Rel. y	s	$1-\ell$
-	-	-	-	0.05	-	-	0.06	-	-
-	-	-	-	0.07	-	-	0.07	-	-
-	-	-	-	0.10	-	-	0.18	-	-
-	-	-	-	0.13	-	-	0.13	-	-
-	-	-	-	0.18	-	-	0.31	-	-
-	-	-	-	0.21	-	-	0.24	-	-
-	-	-	-	0.24	-	-	0.33	-	-
0.06	2.74	40.7	54.2	0.37	5.5	65.7	0.57	13.0	44.1
0.36	1.94	44.6	57.3	0.59	8.0	49.4	0.71	15.0	42.2
1.15	1.42	47.7	59.7	0.83	9.7	44.9	0.78	16.5	41.5
1.00	1.06	48.9	57.6	1.00	10.3	44.3	1.00	15.3	41.1

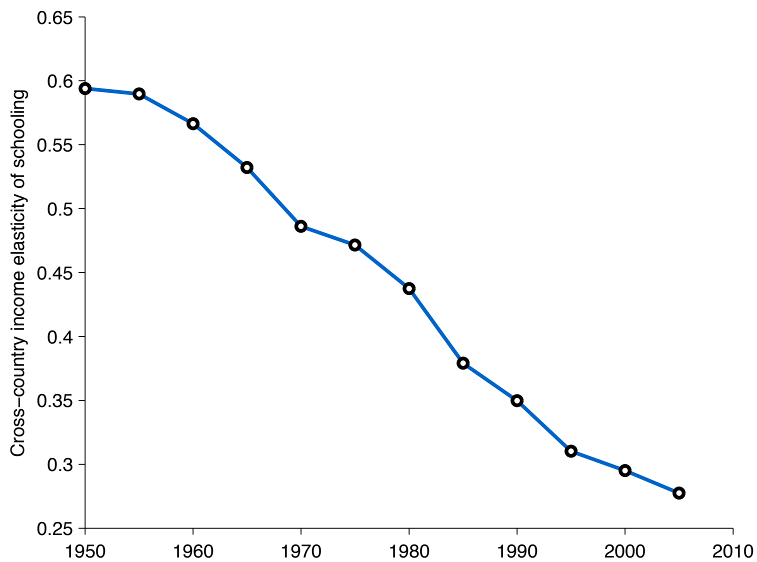
Note: y is output per capita, s is average years of schooling, and $1-\ell$ is hours worked.

Figure 1: Average Years of Schooling Population 25 to 29 – Selected Years



Note: The source of data is Barro and Lee (2010) for schooling and Conference Board (2010), Total Economy Database for GDP per capita. The horizontal axis measures GDP per capita relative to the United States. The vertical axis measures average years of schooling for the 25-29 population.

Figure 2: Income Elasticity of Schooling across Countries



Note: For each year, we regress log average years of schooling on a constant and log real GDP per capita across countries in our sample. The slope coefficient is plotted for each year.

Figure 3: Years of School Completed at age 35 – model and U.S. data

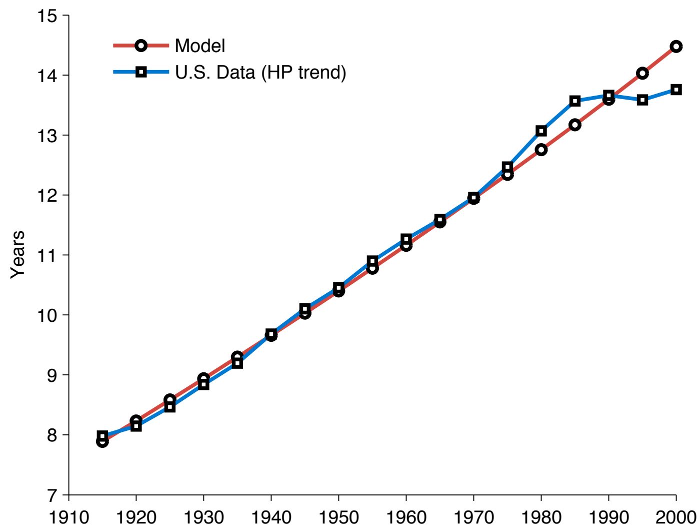


Figure 4: Hours of Work – model and U.S. data

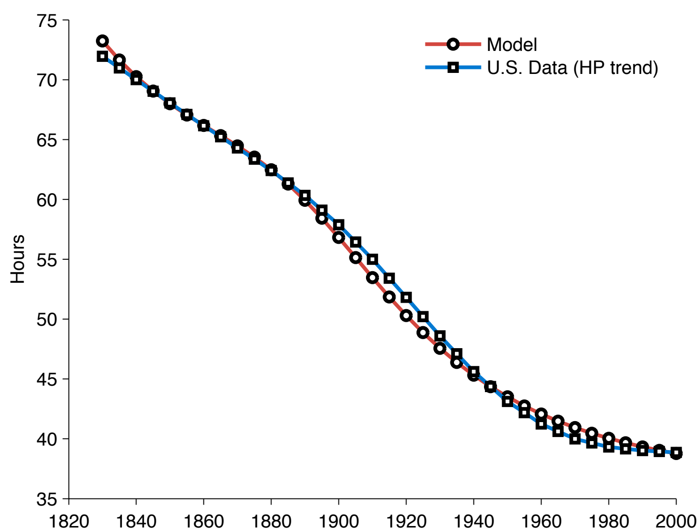
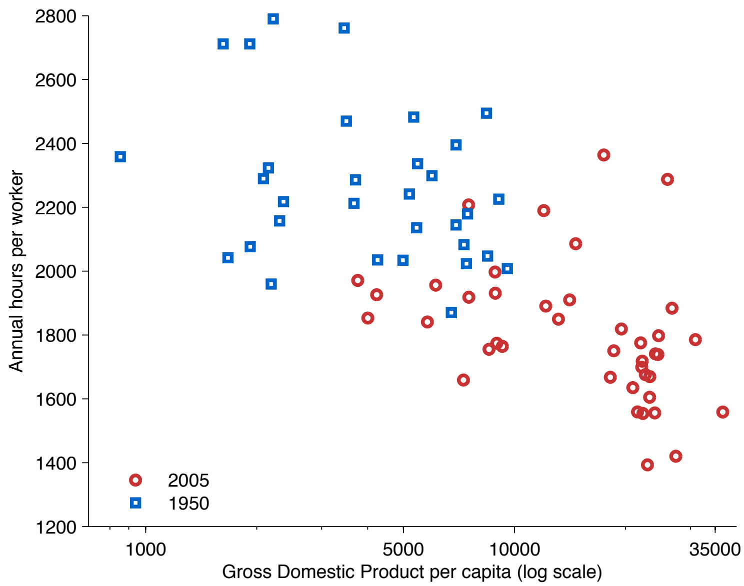


Figure 5: Work Hours across Countries



Note: Average annual hours per worker from Conference Board (2010), Total Economy Database.

Figure 6: Years of School Completed at age 35 – model with constant $\bar{c}$ and U.S. data

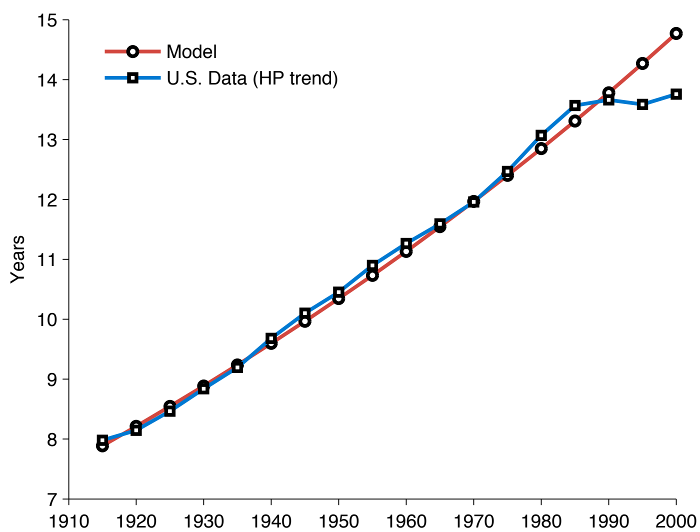


Figure 7: Hours of Work – model with constant $\bar{c}$ and U.S. data