Tsunami, *PRELIMINARY*

Yong Kim* University of Southern California

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Abstract

This paper considers an asset market subject to search frictions, where there are adjustment costs to the entry rate of buyers. An implication is that even in asset markets where the search frictions are very small, asset prices respond to changes in liquidity. Another implication is that asset liquidity is a state variable, the dynamics of which are analyzed. I demonstrate that transition paths of liquidity toward its (stable) steady state can exhibit dramatic *divergence* before convergence following small *positive* deviations in the measure of buyers in the market. Thus, adjustment costs to entry are a potential source of volatility by generating large waves of liquidity, or "tsunami", in asset markets. I quantitatively assess the ability of the mechanism to generate asset market booms and busts via the implied price movements.

Keywords liquidity, adjustment cost, asset market, search.

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1 Introduction

This paper considers an asset market subject to search frictions, where there are adjustment costs to the entry rate of new buyers. Adjustment costs to entry is motivated first by the observation that adjustments to the stock of buyers do not occur at an infinite rate, just as adjustments of capital do not occur through changes in investment at an infinite rate. Secondly, changes in adjustment costs allow us to capture how changes in the *flow* of entry can affect outcomes.

An implication of the adjustment cost is that the shadow value of potential buyers searching for assets moves inversely with the relative population of buyers to sellers which determines the *liquidity* of assets.[1] This turns out to mean that even in asset markets where the search friction is very small ("liquid" asset markets), asset sales prices respond to changes in liquidity. This is not the case in models of fixed cost of entry (where the shadow value of potential buyers is constant), and highlights the role of adjustment costs for the relevance of liquidity dynamics in liquid asset markets.

Another implication of the adjustment cost is that the population of buyers becomes a state variable. In the typical framework with perfectly elastic entry of buyers, the population of sellers is a state variable, but the population of buyers is a jump variable which implies their relative size (determining liquidity) is a jump variable. In this paper, asset liquidity is a state variable, the dynamics of which are analyzed.

I demonstrate that transition paths of asset liquidity toward its (globally stable) steady state level can exhibit dramatic divergence away from steady state before convergence following small *positive* deviations in the measure of buyers in the market. This occurs because the implied higher asset liquidity lowers the population of sellers (relative to buyers) which feeds back into higher liquidity. This feedback loop will continue until the population of sellers (relative to buyers) is sufficiently small to halt the increase in asset liquidity. From that point onwards, asset liquidity begins falling and the population of sellers rises in concert toward its steady state level. Thus, in the presence of search frictions, adjustment costs to the entry of buyers are a potential source of volatility by generating large waves of liquidity, or "tsunami", in asset markets.[2]

The associated fluctuation in the relative population of buyers to sellers causes fluctuation in the sales price of assets even when the assessment of asset earnings by buyers and sellers and interest rates remain constant. This is because in the typical search environment I consider, there is a discrete difference in the assessment of earnings by buyers and sellers, such that their relative bargaining power pins down the *effective* earnings used to price assets. Their bargaining power in turn is pinned down by their relative populations.

I assess this mechanism quantitatively in terms of its ability to generate asset market booms and busts via the implied movements of liquidity on the sales price of assets. This is conducted by considering temporary and deterministic

[1] The definition of liquidity followed here is the average speed of resale.

[2] In contrast, negative deviations in the measure of buyers, do not generate these strong feedback effects, so the predicted volatility (relative to steady state levels) is *asymmetric*.

innovations to the entry rate of buyers as inducing deviations in the measure of buyers from steady state levels, and by imposing that asset prices are expected to revert to their steady state levels with certainty. Simulations conducted for a variety of asset markets perform well in the sense that small deviations in the entry of buyers can generate large innovations in prices. This is demonstrated in the context of various asset markets, including liquid markets.

The search-theoretic literature on asset markets includes Duffie, Gârleanu and Pedersen (2005, 2007), Gârleanu (2006), Lagos and Rocheteau (2007), Miao (2006), Rust and Hall (2003), Spulber (1996), Vayanos and Wang (2007), and Weill (2007). I adopt the framework of Kim (2008) who shows that the canonical search model of unemployment can be modified to consider asset markets subject to search friction, and consider adjustment costs of entry as in Vayanos and Wang (2007).[3] Differentiating features of this paper are the analysis of out of steady state dynamics when buyer entry rates are subject to adjustment cost.[4]

The next section presents the model of asset prices with search frictions. Section 3 characterizes the dynamics of buyer and seller populations which determine asset liquidity. Section 4 considers the case of an asset market which is large relative to the pool of potential market entrants. Section 5 considers a reinterpretation of the model in the context of asset rental markets. Section 6 conducts numerical simulations. The last section concludes.

2 Asset market

Consider an asset market in continuous time, populated by three types of risk neutral agents who discount at constant rate r: buyers, owners and sellers. Assets are traded in fixed discrete units, the population of which is fixed and normalized to one. Let $v \geq 0$ denote the measure of buyers, $u \in [0,1]$ denote the measure of sellers, and $\theta \equiv \frac{v}{u}$ denote their ratio. The flow of matches in the economy is $\alpha M(v,u)$ which is a constant returns to scale function of v,u such that $\alpha M(v,u) \equiv \alpha m(\theta) u$. This assumption has the advantage of allowing us to abstract away from scale effects coming from the match technology. $\alpha > 0$ captures the degree of search friction inherent in the market. I assume that $m(0) = 0, m'(\theta) > 0, m''(\theta) \leq 0, \lim_{\theta\to\infty} m'(\infty) = 0$, and that $M(v,u)$ is continuous and differentiable in v,u.

[3] The canonical search unemployment model I refer to is Pissarides (2000), Chapter 1. As will be shown, the counterpart to the entry of buyers is the entry of job vacancies. Adjustments costs to the entry of vacancies have not been extensively considered in that literature. A recent exception is Fujita and Ramey (2007) who adopt such costs to generate hump-shaped response of vacancies to productivity shocks. Blanchard and Diamond (1989) have considered a flow approach to the entry of vacancies in related environments. However, theses analyses do not explore the implications of the transition dynamics of market tightness (the counterpart to asset liquidity) which is the focus of my analysis.

[4] Vayanos and Wang (2007) and the related work of Vayanos and Weill (2008) and Alfonzo (2009) consider outcomes in steady state settings. Duffie, Gârleanu and Pedersen (2007) conduct numerical simulations of out of steady state dynamics, but do not consider innovations to entry rates and associated divergent transition paths analyzed here.

A *buyer* of an asset searches for a seller and assesses the earnings of the asset at $\pi > 0$, but with arrival rate λ reassesses the earnings at πz where $z \in [0, 1)$, forever, and exits the market. The incidence of a match with a seller at rate $\alpha \frac{m(\theta)}{\theta}$, leads to asset purchase at sales price P, and change in status from potential buyer to *owner*.

For an owner, the arrival of the earnings reassessment shock (also at rate λ) leads to a change in status from owner to *seller*, with sales occurring at arrival rate $\alpha m(\theta)$ at sales price P, after which he exits the market. $\alpha m(\theta)$ is a measure of the average speed of resale or liquidity of the asset.[5]

Payment of the price P occurs using a second type of asset for which all agents have a common valuation, and which all agents have access to a common technology for producing. Figure 1 summarizes the movement of agents through the market.

[Figure 1]

Value equations for buyers V, owners J, sellers U and the sales price P are given by

$$\begin{aligned} rV &= \alpha \frac{m(\theta)}{\theta}(J - P - V) - \lambda V + \dot{V}, \qquad (1) \\ rJ &= \pi - \lambda (J - U) + \dot{J}, \\ rU &= \pi z + \alpha m(\theta)(P - U) + \dot{U}. \end{aligned}$$

The flow value of buyers rV, consists of the option value of realizing a match $\alpha \frac{m(\theta)}{\theta}(J - P - V)$, plus the option value of a reassessment shock $-\lambda V$, plus the capital gains $\dot{V}$. The flow value of owners rJ, consists of the assessed earnings flow π, plus the option value of a reassessment shock $-\lambda(J - U)$, plus the capital gains $\dot{J}$. The flow value of sellers rU, consists of the assessed earnings flow πz, the option value of realizing a match $\alpha m(\theta)(P - U)$, plus the capital gains $\dot{U}$.

The match surplus is $S \equiv J - U - V$. Let $\beta \in [0, 1]$ denote the exogenous bargaining power of sellers. Sales prices are determined through Nash Bargaining such that

$$P = U + \beta S. \qquad (2)$$

Given the paths of v, u (determined below), these equations specify the paths of values V, J, U, P and S.

In steady states $\theta = \theta_{ss}$ constant. The sales price in a steady state is given by (all derivations are to be found in the Appendix)

$$P_{ss} = \frac{\pi}{r} \frac{\beta \left(\frac{r}{\alpha m(\theta_{ss})} + 1 \right) + \left(\frac{(1-\beta)r + \lambda}{\alpha m(\theta_{ss})} + \frac{(1-\beta)}{\theta_{ss}} \right) z}{\frac{r+\lambda}{\alpha m(\theta_{ss})} + \beta + \frac{(1-\beta)}{\theta_{ss}}}, \qquad (3)$$

[5] In financial markets, where we observe actual earnings, one can think of potential buyers and owners as being optimists and sellers as pessimists, with the actual (average) earnings stream lying somewhere between π and πz. Alternatively, the gap can be interpreted as a holding cost as in Duffie, Gârleanu and Pedersen (2005).

which is rising in the steady state relative population of buyers θ_{ss}. While the analysis of the paper will focus on out of steady state dynamics, this equation provides some intuition for how the asset price is linked to the asset liquidity through θ (which in turn depends on v, u). Note that as $z \to 1$, the steady state price $P_{ss} \to \frac{\pi}{r}$. Thus, we expect in markets where buyers and sellers assess earnings in a similar way, the relative population of buyers θ_{ss}, does not affect the sales price.

The steady state value of sellers is

$$U_{ss} = \frac{\pi}{r} \frac{\beta\theta_{ss} + \left(\frac{r+\lambda}{\alpha m(\theta_{ss})} + \frac{(1-\beta)}{\theta_{ss}}\right) z}{\frac{r+\lambda}{\alpha m(\theta_{ss})} + \beta + \frac{(1-\beta)}{\theta_{ss}}}. \tag{4}$$

Note, as $z \to 1$, the steady state value of sellers $U_{ss} \to P_{ss} = \frac{\pi}{r}$. Thus, we expect in markets where buyers and sellers assess earnings in a similar way, the spread between selling the asset to a buyer or an agent who has valuation πz for the earnings is small.

In applications to financial markets, we can consider an infinite population of buyers with low earnings assessment πz (and zero cost of entry) who are labelled "dealers". From the seller's viewpoint, such dealers can be matched with instantaneously, and the sales price to such dealers is U. The price at which dealers subsequently sell to "end-user" buyers with assessment π is P. Then the ratio $\frac{P-U}{P}$ has the interpretation of a *bid-ask spread.* This bid-ask spread in steady states is

$$\frac{P_{ss} - U_{ss}}{P_{ss}} = \frac{\beta \frac{r}{\alpha m(\theta_{ss})} (1-z)}{\beta \left(\frac{r}{\alpha m(\theta_{ss})} + 1\right) + \left(\frac{(1-\beta) r + \lambda}{\alpha m(\theta_{ss})} + \frac{(1-\beta)}{\theta_{ss}}\right) z}. \tag{5}$$

2.1 Liquid asset markets

An issue to be addressed is the applicability of liquidity dynamics in markets where asset transactions occur rapidly, or "liquid" markets. As search frictions get smaller $\alpha \to \infty$, the steady state sale price from (3) approaches

$$P_{\alpha\to\infty} = \frac{\pi}{r} \frac{\beta\theta_{ss} + (1-\beta) z}{\beta\theta_{ss} + (1-\beta)}. \tag{6}$$

Note, the dependency of the sales price on the relative population of buyers θ, is maintained. This price is increasing in the bargaining power of sellers β, and the relative valuation of sellers z.

In contrast, in a model with a perfectly elastic supply of buyers at some fixed cost of entry $\bar{V}$ $\left(\text{where } 0 < \bar{V} < \frac{\pi}{r+\lambda}\right)$, the steady state sales price is $\tilde{P}_{ss} = \frac{\pi}{r} \left(\frac{\beta\left(\frac{r}{\alpha m(\theta_{ss})} + 1\right)\left[1 - (r+\lambda)\frac{\bar{V}}{\pi}\right] + \frac{(1-\beta) r + \lambda}{\alpha m(\theta_{ss})} z}{\frac{r+\lambda}{\alpha m(\theta_{ss})} + \beta} \right)$. In this case, as search frictions get smaller $\alpha \to \infty$,

$$\tilde{P}_{\alpha\to\infty} = \frac{\pi - (r+\lambda)\bar{V}}{r},$$

which is *independent* of the relative population of buyers θ. Thus, the dynamics of the relative population of buyers modelled in this paper are applicable to markets where search frictions are very small, *in a way that is not true without adjustment costs of entry.* The discussion here shows that the difference lies in the endogenous response of the value of buyers V downwards when θ rises (see discussion relating to V_{ss} below).

For the value of sellers, from (4), as search frictions get smaller $\alpha \to \infty$, then $U_{\alpha\to\infty} = P_{\alpha\to\infty}$. Thus, from (5), we expect that in markets where asset transactions occur rapidly, the spread between selling the asset to a buyer or an agent who has valuation πz for the earnings is small.[6]

Proposition 1: *Asset prices in liquid markets* ($\alpha \to \infty$) *respond to the relative population of buyers* θ. *In such markets, the spread between sales prices and the value of sellers is small,* $\alpha \to \infty$, $\Rightarrow U_{\alpha\to\infty} = P_{\alpha\to\infty}$.

Finally, the steady state value of vacancies is $V_{ss} = \frac{\pi}{r}\left(\frac{r}{r+\lambda}\right)\frac{\frac{(1-\beta)}{\theta}}{\frac{r+\lambda}{\alpha m(\theta_{ss})}+\beta+\frac{(1-\beta)}{\theta_{ss}}}$. Thus, as $\alpha \to \infty$, then

$$V_{\alpha\to\infty} = \frac{\pi}{r}\left(\frac{r}{r+\lambda}\right)\frac{(1-\beta)}{\beta\theta_{ss}+(1-\beta)}. \tag{7}$$

This moves inversely with $P_{\alpha\to\infty}$. Note the $\frac{r}{r+\lambda}$ term here implies that increases in $P_{\alpha\to\infty}$ are not matched by decreases in $V_{\alpha\to\infty}$ one for one when $z = 0$. In particular, when $\frac{r}{r+\lambda}$ is small, large movements in the sale price are associated with small movements in the value of being a buyer.

3 Dynamics of liquidity

To complete the characterization of outcomes, we need to determine the paths of buyers v, and sellers u. Let $x > 0$ denote the entry rate of new buyers. From the structure of the market detailed above, the dynamics of v, u are given by

$$\begin{aligned} \dot{v} &= x - \lambda v - \alpha\frac{m(\theta)}{\theta}v \\ \dot{u} &= \lambda(1-u) - \alpha m(\theta)u, \end{aligned} \tag{8}$$

where the initial v_0, u_0 are given. The net increase in the stock of buyers consists of entry minus the exit from reassessment shocks and matches. The net increase in the stock of sellers consists of the flow of owners experiencing reassessment shocks minus the exit from matches.

Assumption 1: *The entry rate of buyers* x, *is exogenous.*

This assumption captures adjustment costs in the entry rate of new buyers in a simple way. Given $V \geq 0$, the adjustment cost is characterized by a zero cost of entry up to entry rate x, and an infinite cost of entry thereafter. In

[6] While adjustment costs to entry and perfectly elastic entry have both been considered in the literature before, I am not aware of previous work highlighting this observation between these two approaches.

the Appendix, I consider a more general variant of this specification where the adjustment cost function is smooth and differentiable, such that in equilibrium, the entry rate of new buyers responds (positively) to the value of buyers V. However, the substantive insights of such a model are not different from those of this simple specification.

Proposition 2: *There exists a unique steady state pair* $\{v_{ss}, u_{ss}\}$*, which is globally stable.* v_{ss} *is rising in* x, u_{ss} *is falling in* x*, and their ratio* θ_{ss} *is rising in* x*.*

All proofs are to be found in the Appendix. Figure 2 summarizes the dynamics of v, u. Initial v_0, u_0 lie in one of fours possible regions. If they are in regions b, d in the Figure, both v, u converge monotonically to the steady state. If they are in regions a, c in the Figure, v converges monotonically, but this is not necessarily the case for u.

[Figure 2]

From (8), the law of motion for θ is given by

$$\begin{aligned} \dot{\theta} &= \left[\frac{x}{v} - \lambda - \alpha\frac{m(\theta)}{\theta} - \lambda\frac{(1-u)}{u} + \alpha m(\theta)\right]\theta \qquad (9) \\ &= \frac{x}{u} - \alpha m(\theta) - \frac{\lambda}{u}\theta + \alpha m(\theta)\theta. \end{aligned}$$

The locus of points $\theta_\theta(u)$ where $\dot{\theta} = 0$ is given by

$$\frac{x}{u} + \alpha m(\theta_\theta(u))\theta_\theta(u) = \alpha m(\theta_\theta(u)) + \frac{\lambda}{u}\theta_\theta(u). \qquad (10)$$

The right hand side is a weakly *concave* function of θ, and the second term on the left hand side is a weakly *convex* function of θ (with one strictly so).

Given the positive intercept term $\frac{x}{u} > 0$ this implies for a given u, *if* a solution to (10) exists, there are in general *two* loci of solutions which are denoted $\{\theta_{\theta 1}(u), \theta_{\theta 2}(u)\}$. Moreover, the concavity/convexity of the terms in (*10*) imply from (9) that for $\theta(u)$ lying between the two solutions, $\dot{\theta} < 0$, and for $\theta(u)$ lying above and below these solutions, $\dot{\theta} > 0$.

Note that since $m(0) = 0$, $\dot{\theta} > 0$ when $\theta = 0$. Then a sufficient condition for the existence of a solution to (10) is to show that $\dot{\theta} \le 0$ for some $\theta(u)$. Note also that $\min\{\theta_{\theta 1}(0), \theta_{\theta 2}(0)\} = \frac{x}{\lambda}$, and $\min\{\theta_{\theta 1}(1), \theta_{\theta 2}(1)\} > 0$ if it exists.

Lemma 1: *Solutions for (10)* $\{\theta_{\theta 1}(u), \theta_{\theta 2}(u)\}$
(i) exist for $u \le u_{ss}$*,*
(ii) exist for the entire support of $u \in [0,1]$ *if* $x \le \lambda$*,*
(iii) exist for the entire support of $u \in [0,1]$ *if* $x \le \hat{x}$ *where*

$$\begin{aligned} \hat{x} &\equiv \min_{1 \ge u > u_{ss}} \Lambda(u), \\ \textit{where } \Lambda(u) &\equiv \max_{\theta}\{u\alpha m(\theta)(1-\theta) + \lambda\theta\}. \end{aligned}$$

Combining Lemma 1 (ii) and (iii), the upper bound on x which ensures these solutions exist for the entire support of $u \in [0,1]$, satisfies $\hat{x} \ge \lambda$.

Meanwhile, from (8) the locus of points $\theta_u(u)$ where $\dot{u} = 0$ is given by

$$\alpha m(\theta_u(u)) = \frac{\lambda}{u}(1-u). \tag{11}$$

From this and (8), we can state the following:

Lemma 2: *$\theta_u(u)$ is falling in u, with $\theta_u(0) = \infty, \theta_u(1) = 0$. $\theta > \theta_u(u) \Rightarrow \dot{u} < 0$ and $\theta < \theta_u(u) \Rightarrow \dot{u} > 0$.*

From Proposition 1, we know there is a unique steady state θ_{ss} which implies that the $\theta_u(u)$ locus crosses either $\theta_{\theta 1}(u)$ or $\theta_{\theta 2}(u)$, but not both. Given that (i) $\theta_u(u)$ covers the entire support of $\theta \geq 0$, (ii) $\min\{\theta_{\theta 1}(0), \theta_{\theta 2}(0)\} = \frac{x}{\lambda}$ and $\min\{\theta_{\theta 1}(1), \theta_{\theta 2}(1)\} > 0$, and (iii) uniqueness of steady state, we can then observe the following:

Proposition 3: *If $x \leq \hat{x}$ then*

$$\theta_{ss} = \min \ \{\theta_{\theta 1}(u_{ss}), \theta_{\theta 2}(u_{ss})\}.$$

This implies that under the condition of the Proposition, the locus of solutions $\max\{\theta_{\theta 1}(u), \theta_{\theta 2}(u)\}$ must lie to the right of $\theta_u(u)$ in the (θ, u) space and not cross (since the steady state is unique). The results also imply that in the positive quadrant of (θ, u) there are at least 5 regions bounded by the three loci $\{\theta_{\theta 1}(u), \theta_{\theta 2}(u), \theta_u(u)\}$. To clarify the dynamics further using a graph, we can consider the following special case. The reader will be able to verify however, that much of the argument is not particular to this special case from the discussions above.

3.1 Special case

Remark 1: *If $x = \lambda$, from (10) the loci of points $\theta_\theta(u)$ where $\dot{\theta} = 0$ is given by*

$$(\theta_\theta(u) - 1)\left(\alpha m(\theta_\theta(u)) - \frac{\lambda}{u}\right) = 0. \tag{12}$$

This implies when $x = \lambda$, the set of solutions to $\dot{\theta} = 0$ are $\theta_{\theta 1}(u) = 1$, $\alpha m(\theta_{\theta 2}(u)) = \frac{\lambda}{u}$. Thus, $\theta_{\theta 2}(u)$ is falling and convex in u (since $\alpha m(\theta)$ is increasing and concave in θ). Furthermore, from (11) the distance

$$\alpha m(\theta_{\theta 2}(u)) - \alpha m(\theta_u(u)) = \lambda, \tag{13}$$

is a constant. In steady states: $\theta_{ss} = 1$, and $v_{ss} = u_{ss} = \frac{\lambda}{\lambda + \alpha m(1)}$. Figure 3 summarizes the dynamics of θ, u when $x = \lambda$.

[Figure 3]

Initial θ_0, u_0 lie in one of six possible regions. If they are in regions C, E, both θ, u converge monotonically to the steady state. If they are in regions B, D, θ converges monotonically to the steady state, but this is not necessarily the case for u. If they are in region F, u converges monotonically, but θ diverges before converging. Note that region F will exist or not exist depending on whether $\theta_{\theta 1}(\tilde{u}) = \theta_{\theta 2}(\tilde{u})$ for some $\tilde{u} < 1$, i.e. depending on whether these two loci cross. If they do cross, they must do so at a point to the right of the steady state given Proposition 3, i.e. $\tilde{u} > u_{ss}$.

The region of interest is A. Here θ diverges before converging, and the potential degree of divergence is *unbounded*. Meanwhile, u can also diverge before converging. Consider a *positive* deviation in the measure of buyers in the market, relative to the steady state, such that the economy finds itself in region A. From (8), the implied higher θ lowers the population of sellers u, which feeds back into higher θ. This feedback loop will continue until the population of sellers u, is sufficiently small to halt the increase in θ. From that point onwards, we are in region B, where θ begins falling while u continues falling, until we are in region C. Then u rises in concert with falling θ, monotonically toward its steady state level. Thus, a small positive perturbation to the population of buyers is a potential source of volatility by generating a large wave in the relative population of buyers θ, which determines the liquidity of the asset. I label this a "tsunami" effect.

3.2 Outcomes in liquid markets

I now turn to the discussion of the limiting relative population of buyers θ as search frictions go to zero. Recall from (3) the steady state sales price in this case is given by $P_{ss} = \frac{\pi}{r}\frac{\beta\theta_{ss}+(1-\beta)z}{\beta\theta_{ss}+(1-\beta)}$. (The following is shown in the Appendix proof of Proposition 2.)

Remark 2: *As search frictions disappear, $\alpha \to \infty$, the steady state relative population (i) $\theta_{ss} \to \infty$ if $\frac{x}{\lambda} > 1$, (ii) $\theta_{ss} \to 0$ if $\frac{x}{\lambda} < 1$, and (iii) $\theta_{ss} \to 1$ if $\frac{x}{\lambda} = 1$.*

The intuition for this result is when $\frac{x}{\lambda} \neq 1$, the absence of search friction implies that either the measure of sellers or the measure of buyers goes to zero, whereas the measure of their respective counterparties (buyers or sellers) does not. In these cases, the sales price converges to either $\frac{\pi}{r}$ or $\frac{\pi z}{r}$. In the case where $\frac{x}{\lambda} = 1$, the evolution of buyers and sellers from (8) are identical. As search frictions disappear, both the measure of buyers and sellers converge to zero, and their ratio does not diverge to infinity or zero.

In the special case where $x = \lambda$ discussed above using Figure 3, as search frictions disappear such that $\alpha \to \infty$, the $\theta_u(u)$ locus is vertical at the origin, and $\theta_u(1) = 0$, with the implication of squeezing out areas C, D in Figure 3. Meanwhile, from (13) the distance between $\theta_{\theta 2}(u), \theta_u(u)$ goes to zero, squeezing out areas B, E in Figure 3. These imply that as we let the search frictions go to zero, the only two regions which remain are A, F in Figure 3. Appendix Figure A1 shows how the loci in Figure 3 are altered when considering a very liquid market, where search frictions are present but small.

Rubinstein and Wolinsky (1985) demonstrate that in a stationary environment of buyers and sellers, the steady state sales price may not converge to $\frac{\pi}{r}$ (the value to an agent with high assessment π forever) or $\frac{\pi z}{r}$ (the value to an agent with low assessment πz forever), as the duration between matches goes to zero. Instead, prices are a function of the steady state relative population of buyers θ_{ss}. In the setting of Figure 1, the model characterizes an overlapping generations structure of the asset market which mimics the result of Rubinstein and Wolinsky under the assumption $x = \lambda$. The substantive implication of this is when the relative population of buyers θ is out of steady state, and transitioning to that level, sales prices can respond even in liquid markets.

Duffie, Gârleanu and Pedersen (2005) construct a stationary environment of buyers and sellers subject to search frictions in a fixed population. They assume that agents can hold at most one asset at a given point in time. Thus, in their model, sellers cannot enter the pool of potential entrants until they have sold their asset. This differs from the market structure in Figure 1, where I do not restrict the population of entry and exit in this way. Thus, their results relating to pricing as frictions approach zero do not apply to my environment.

3.3 Relating values to shadow values of social planner

Consider a social planner facing the problem

$$\max_{v,u,q} \int_0^\infty e^{-rt} \left[\pi(q-u) + \pi z u\right] dt \text{ s.t.} \tag{14}$$

$$\begin{aligned}
\dot{v} &= x - \lambda v - \alpha \frac{m(\theta)}{\theta} v, \\
\dot{u} &= \lambda (q-u) - \alpha m(\theta) u, \\
\dot{q} &= 0.
\end{aligned}$$

where the initial v_0, u_0 are given. Then we can observe the following.

Proposition 4: *When the match function M, (i) has a constant elasticity $\eta \equiv \frac{uM_u}{M}$, and (ii) the Hosios (1990) condition is satisfied $\eta = \beta$, decentralized values equations (1) coincide with shadow value equations for the planner.*

In the numerical analysis, I specify the match function to meet these restrictions (using a Cobb-Douglas specification) so we can interchangeably talk about decentralized values and shadow values relating to the planner problem (14).

4 Large asset market

The benchmark model above implicitly assumed that entry and exit occurred from a large pool of potential entrants, the size of which was not affected by what is occurring in the asset market. This section discusses outcomes when the latter is not the case for a large asset market. Specifically, I now assume the reassessment shock λ makes buyers or owners transition into a pool of *potential entrants*. I further assume each agent can only have high earnings assessment π for one asset at a given point in time.

Let $n \geq 1$ denote the exogenous sum of the population of buyers, owners and potential entrants. I refer to this as the size of the market which is bounded below by the population of assets. Transition from the pool of potential entrants $\frac{x}{\delta}$, to the pool of buyers v, occurs at exogenous rate δ following an asset assessment shock. Figure 4 summarizes the movement of agents through the market.

[Figure 4]

The feasible set of v, u is given by the condition that states that the population of the potential pool of entrants $\frac{x}{\delta}$, is non-negative

$$\begin{aligned} \frac{x}{\delta} &\equiv n - v - (1-u) \geq 0, \\ &\Rightarrow n - 1 \geq v - u, \\ &\Rightarrow \frac{n-1}{u} + 1 \geq \theta. \end{aligned} \tag{15}$$

Given $n \geq 1$, this specifies an upper bound for $\theta(u)$. The dynamics of v are now given by

$$\dot{v} = \delta(n - 1 + u - v) - \lambda v - \alpha \frac{m(\theta)}{\theta} v, \tag{16}$$

and the dynamics of u are unchanged from (8).

From (16) and (8), the locus of points where $\dot{v} = 0, \dot{u} = 0$ are respectively given by

$$\begin{aligned} \frac{\lambda\delta}{\lambda+\delta} n - \alpha M(v_v(u), u) &= \lambda v_v(u), \\ \lambda - \alpha M(v_u(u), u) &= \lambda u, \end{aligned} \tag{17}$$

In steady states v_{ss}, u_{ss} satisfy both equations. Taking the difference in these equations implies in steady states

$$v_{ss} - u_{ss} = \frac{\delta}{\lambda+\delta} n - 1.$$

This implies the feasibility condition (15) is always satisfied in steady state since $n - 1 > \frac{\delta}{\lambda+\delta} n - 1$.

The dynamics of the relative population of buyers is given by

$$\begin{aligned} \dot{\theta} &= \delta\left(\frac{n-1}{u} + 1 - \theta\right) - \lambda\theta - \alpha m(\theta) - \lambda\frac{(1-u)}{u}\theta + \alpha m(\theta)\theta \\ &= \delta\left(\frac{n-1}{u} + 1 - \theta\right) - \alpha m(\theta) - \frac{\lambda}{u}\theta + \alpha m(\theta)\theta. \end{aligned} \tag{18}$$

Then the locus of points where $\dot{\theta} = 0$ is given by solutions which are denoted $\{\theta_{\theta 1}(u), \theta_{\theta 2}(u)\}$, as before. Recall, the set of feasible θ, u is given by (15). Suppose $\theta(u) = \frac{n-1}{u} + 1$, its maximum feasible value, then

$$\dot{\theta} > 0 \Leftrightarrow (n-1)\frac{\alpha m\left(\frac{n-1}{u} + 1\right)}{\frac{n-1}{u} + 1} > \lambda,$$

which is a lower bound on n, where this bound is falling in u. If n is sufficiently small, this condition is not satisfied for any $u \in [0, 1]$.[7]

Proposition 5: *The locus of* $\max\{\theta_{\theta 1}(u), \theta_{\theta 2}(u)\}$ *lies outside of the feasible set of* θ, u *for* n *sufficiently low.*

A substantive implication of this Proposition is that tsunami effects can be avoided by limiting the size of the market. In the context of the model there is no reason for doing so, but in other contexts there may be. However, this entails a cost since lower n will lower the $v_v(u)$ locus from (17) (and not change the $v_u(u)$ locus), and thereby increase the steady state level of sellers u_{ss}, who assess assets earnings less.

5 Rental market

A related but different market structure that can be readily considered is the rental market for assets. In such a market there are two distinct populations of agents: *renters* and *landlords*. Assets are owned by landlords all the time.

Buyers v characterized in the benchmark model above are relabelled unmatched renters with high earnings assessment π, and sellers u are relabelled unmatched landlords with low earnings assessment πz. A match forms a bilateral union between renters and landlords causing renters to transition from being unmatched to matched and landlords to transition from being unmatched to matched. π is the asset earnings flowing to the renter during the match. When the match is terminated following a downward reassessment of earnings by the renter at rate λ, the asset reverts to the landlord who searches for another renter, and derives a flow of earnings πz during the search.

The rental rate w of the asset is assumed to be determined by a continuous process of Nash bargaining between these parties during the match, where β is the bargaining power of the landlord. P now has the interpretation as the price of an asset which is currently rented out, which is equal to the value of a matched landlord. U is the price of an asset which is not currently rented out, which is equal to the value of an unmatched landlord. The rental rate w is determined by the value equation for a matched landlord which is

$$rP = w - \lambda(P - U) + \dot{P}, \tag{19}$$

given $P, U, \dot{P}$. The flow value of a rented asset rP, consists of the rental income flow w, plus the option value of a reassessment shock $-\lambda(P - U)$, plus the capital gains $\dot{P}$. Given this reinterpretation, all other value equations remain the same as above in equations (1). In particular, V now denotes the value of an unmatched renter, J now denotes the sum of the value of a matched renter-landlord pair, and $J - P$ denotes the value of a matched renter. Then we can note the following:

Proposition 6: *Outcomes for market tightness* θ*, and market prices* P*, are identical in rental markets.*

[7] This would be the case if $n = 1$, for example.

Thus, whether assets are transacted through transfer of ownership or through rent is irrelevant for the market dynamics and price setting outcomes.[8] This allows us to discuss outcomes in both types of markets interchangeably.

Rearranging (19) in terms of the rent to price ratio implies

$$\frac{w}{P}=r+\lambda\left(\frac{P-U}{P}\right)-\frac{\dot{P}}{P}. \tag{20}$$

We can interpret $\lambda\left(\frac{P-U}{P}\right)$ as the liquidity premium of the market. Prices rise relative to rents when this premium falls. This equation suggests an alternative interpretation of the rental rate w as the effective earnings level used to price assets under the original (ownership) market structure discussed above. Although the assessment of earnings by buyers and sellers respectively remain constant, the effective earnings used to price assets is time varying depending on the relative bargaining power of buyers and sellers, which in turn is determined by the relative population parameter θ.

Combining (19), with equations (1) and (2), the rental rate (effective earnings) can be expressed as (derivation in Appendix)

$$\begin{aligned} w &= \pi z+\beta\pi\left(1-z\right)\frac{r+\lambda+m\left(\theta\right)}{r+\lambda+\beta\alpha m\left(\theta\right)+\left(1-\beta\right)\alpha\frac{m(\theta)}{\theta}} \\ &\quad +\dot{S}\frac{\left(1-\beta\right)\beta\left(\alpha m\left(\theta\right)-\alpha\frac{m(\theta)}{\theta}\right)}{r+\lambda+\beta\alpha m\left(\theta\right)+\left(1-\beta\right)\alpha\frac{m(\theta)}{\theta}}. \end{aligned} \tag{21}$$

This equation shows that the rental rate w, is increasing in the relative population of unmatched renters to unmatched landlords θ. In particular, in a steady state where $\theta=\theta_{ss}$, as we let $\alpha\rightarrow\infty$, $w_{\alpha\rightarrow\infty}=\pi\frac{\beta\theta_{ss}+(1-\beta)z}{\beta\theta_{ss}+(1-\beta)}=rP_{\alpha\rightarrow\infty}$ from equation (6). Thus, even in liquid markets the rental rate responds to θ. Interpreting the rental rate as the effective level of earnings used to price assets as implied by equation (20), this is essentially why asset prices respond to θ even in liquid markets where the liquidity premium $\lambda\left(\frac{P-U}{P}\right)$ is very small.[9]

6 Numerical simulations

We now turn to a quantitative assessment of the mechanisms discussed. I do this in the context of specific asset markets and cover several different cases. The parameterization is as follows. The match function is specified as Cobb-Douglas

$$\alpha M\left(u,v\right)=\alpha v^{1-\eta}u^{\eta}, \tag{22}$$

where recall $\alpha>0$ captures the degree of search friction inherent in the market, and η captures the constant elasticity of the match function. The Hosios

[8]This corresponds to the insight in the canonical search unemployment model (Pissarides, 2000) that wages (a rental rate for labor) are irrelevent for determining market tightness.

[9]In contrast, in a setting where the value of unmatched renters is fixed $V=\bar{V}$, as $\alpha\rightarrow\infty$, $\tilde{w}_{\alpha\rightarrow\infty}=\pi-(r+\lambda)\bar{V}$, which is independent of θ_{ss}.

condition, $\eta = \beta$, is imposed so we can equate decentralized values to social planner's shadow values. I set $\eta = 0.5$ in the benchmark, and report results for different $\eta \in [0,1)$.

I normalize $\pi = 1$, and set the relative assessment of sellers to zero $\pi z = 0$. To pin down the entry rate, I assume the steady state ratio of buyers to sellers is one, $\theta_{ss} = 1$. This implies $x_{ss} = \lambda$ in steady states, and implies a convenient characterization for the asset liquidity which is independent of η in steady states, $\alpha m(\theta_{ss}) = \alpha\theta_{ss}^{1-\eta} = \alpha$.

Then in each market, I calibrate two transition variables α, λ to match two data points in steady state (i) turnover rates, and (ii) sales duration or unemployment rates (one implies the other given the turnover rate). These data restrict the parameters as follows

$$
\begin{aligned}
\textit{turnover rate} &= \alpha\frac{\lambda}{\lambda+\alpha}, \qquad (23)\\
\textit{sales duration} &= \frac{1}{\alpha}, \ \textit{or unemployment rate } u_{ss} = \frac{\lambda}{\lambda+\alpha}.
\end{aligned}
$$

Finally, the interest rates is specified as the sum of the average riskless rate, risk premia and depreciation rate.[10]

The numerical experiments are conducted as follows. I assume that at a determinate date $t = 0$, the entry of buyers $x \neq \lambda$ for a finite period of time, before and after which $x = x_{ss} = \lambda$ forever. I evaluate the transition path of $\{\theta\}_0^T$ at discrete points in time using the equation for the law of motion for θ (9) above. I choose T sufficiently large such that θ at time T is observationally equivalent to its steady state value.[11]

Then I take the sequence $\{\theta\}_0^T$ and solve for value equations (1) and (2), imposing that values are equal to their steady state levels in period T. The sequences $\{P,U,V,J\}_{-\tilde{T}}^T$ are solved backwards up to period $-\tilde{T}$ sufficiently large such that at time $-\tilde{T}$, values are observationally equivalent to their steady state levels. In practice, at monthly frequencies, I set $T = 1000, -T = -500$.

This is for the case where the innovations to the entry rate are assumed to be perfectly foreseen. To consider the other extreme where they are not foreseen at all until period $t = 0$ (and perfectly foreseen thereafter), I set values as equal to their steady state levels up to just before period $t = 0$.

6.1 Owner occupied housing market

The benchmark analysis is conducted for the market for owner occupied housing. The average annual turnover of owner occupied homes reported by the Census is 6.2%. The average duration until sale reported by the National Association of Realtors is 6.0 months. From (23), these imply an average tenure of 189

[10]The model did not consider depreciation of assets arising from asset exit. It would be straightforward to consider exogenous entry and exit of assets which keep the stock of assets constant, and motivate such asset depreciations.

[11]Two other interesting dimensions which generate dynamics responses in θ are λ, α. To keep the analysis parsimonious I focus on the implications of innovations to x.

months, and an average seller inventory (unemployment rate) of 3.0%. These numbers coincide with those reported by Piazzesi and Schneider (2009) from the American Housing Survey. The interest rate is set at an annual rate of 9%, which corresponds to the sum of a 4% riskless rate and 5% depreciation rate. Table 1 summarizes the choice of parameters at monthly frequencies for the housing market.

Table 1: Monthly transition variables for owner occupied housing market.

$Parameter$	α	$\lambda = x_{ss}$	r
$Value$	0.17	0.0053	0.0075

Figure 5 shows how the population of buyers v, sellers u, and relative population θ, behave following a 50% increase in the entry rate sustained over 12 months from period $t = 0$ onwards. A short period of high vacancies is followed by a long period of unemployment which is below steady state. The tsunami effect is evident in the movement of the relative population of buyers θ which determines the asset liquidity $\alpha m(\theta)$.

Figure 6 plots the associated values of surplus S, seller U, buyer V, and sales price P. Prices P rise in anticipation of the tsunami effect rising to a peak of 46% above the steady state level, and fall gradually down to the steady state. The percentage difference between P and value of sellers U, $\frac{P-U}{P}$ is smaller as the price rises, implying a greater share of the sales price embodies the resale option of the asset. The value of buyers moves inversely with the sales price as the greater entry of buyers entails greater competition among existing buyers.

Note to consider unexpected innovations to entry rates, we simply consider the values, such as the sales price, as equal to their steady state levels up to period $t = 0$. In this case, the anticipation effects on transaction prices will not be present.

[Figure 5] & [Figure 6]

The sensitivity of prices to the increase in the entry rate from steady state levels is not affected much by the concentration of this increase. An alternative experiment where the entry rate is increased by 5% for a duration of 120 months, results in a peak price which is 34% above the steady state level.

Next I highlight the asymmetric nature of deviations. Figure 7 plots the peak or trough of the relative population θ as percentage changes relative to steady state levels, following positive or negative displacement of entry rates x, in the range of $\pm 50\%$ of steady state levels sustained over 12 months from period $t = 0$. The effects on the relative buyer population θ are clearly asymmetric as shown by the convex relationship between x and peak/trough θ. This asymmetry is attributed to the tsunami effect.

[Figure 7] & [Figure 8]

Figure 8 plots the associated peak or trough in sales prices P as percentage changes relative to steady state levels. The effects on prices are again asymmetric as shown by the convex relationship between x and peak/trough P. This implies that relative to a distribution of innovations in entry rates x, the resulting distribution of prices P, will be skewed positively. Figure 8 also displays the results of comparative statics involving changing the match elasticity η and interest rate r. The effect of lower η (set at 0.25) is to magnify the percentage change in P following a given innovation to the entry rate of buyers. Higher r (set at a 0.015 monthly rate) increases the weight on near term changes in θ on the asset price so also magnifies the percentage change in P following a given innovation to the entry rate of buyers. Thus, with lower η and higher r, it is possible to generate much larger percentage changes in the asset price following a given path of x.

6.2 Comparison with other markets

To see how the dynamics of the model vary further in parameters, I consider related experiments for three other markets (New York Stock Exchange (NYSE), market for capital reallocation, and labor market) and compare results. To facilitate comparisons I assume the effective interest rate is common across markets, and note the results are not substantively affected by this restriction. The data used to restrict parameters α, λ in each market are chosen as follows.

NYSE

Annual turnover rates for the NYSE are targeted at 124% as reported by Piazzesi and Schneider (2008). Of these, assuming half of transactions are to dealers (rather than end buyers), I target an annual turnover rate of 62%. This is assuming half of trades occur at the bid price (from sellers to dealers) and half of trades occur at the ask price (from dealers to buyers).[12] In financial markets, the average duration until sale from a low valuation seller to a high valuation buyer is not well defined. Neither is the measure of sellers of low valuation. Instead, I target the average bid-ask spread in the market as follows. From (5), the bid-ask spread in steady states is

$$\frac{P_{ss} - U_{ss}}{P_{ss}} = \frac{\frac{r}{\alpha m(\theta_{ss})}}{\frac{r}{\alpha m(\theta_{ss})} + 1} = \frac{r}{r+\alpha}. \tag{24}$$

I target an average bid-ask spread of 40 basis points (0.4%) for the NYSE.[13] From (23) and (24), these imply average search durations for sellers (who assess earnings as πz) to end buyers (who assess earnings as π) of 0.52 months, and average ownership tenure of 18 months.[14][15]

[12] Other trades between these prices are viewed as convex combinations of trades to dealers and end buyers.

[13] Spreads are higher for small cap stocks and higher in other exchanges such as the NASDAQ.

[14] As mentioned above, the interest rate is set at the same rate as the housing market, a 9% annual rate, to facilitate comparisons. However, from (24), using a (more realistic) lower interest rate would imply that the bid-ask spread is lower.

[15] The implied search duration of buyers is also 0.52 months. I interpret this as the time

Capital

Eisfelt and Rampini (2006) report annual turnover rates of 5.5% of the existing capital stock. Capacity utilization rates from surveys by the Federal Reserve average 81%. These are taken to imply that the capital unemployment rate is 19% on average. From (23), these imply average search durations for unemployed capital of 41 months, and average capital tenure of 179 months.

Labor

Labor, just like any other asset, provides a stream of earnings the claims to which are potentially tradeable. The discussion on rental markets in Section 5 showed how after relabelling variables, the asset market model above can be reinterpreted as a rental market model, a particular example of which is the canonical search model of unemployment (Pissarides, 2000) with adjustment costs to the entry of vacancies.[16] Recall that the determination of liquidity was not affected by this reinterpretation.

Parameters for search frictions have been extensively studied in the context of labor, and our benchmark elasticity of the match function $\eta = 0.5$, falls within the range of typical estimates. Shimer (2005) reports average annual labor turnover rates of 35%, and average unemployment rates of 6.8%. From (23), these imply average search durations for unemployed workers of 2.2 months and average job tenure of 30 months.

Table 2 summarizes the choice of parameters at monthly frequencies for these markets and compares them with those for the housing market. Housing and capital have similar arrival rates of assessment shocks, but the liquidity of capital is much lower. The NYSE and the labor market have higher arrival rates of assessment shocks and higher asset liquidity than the housing market. Between the NYSE and the labor market, both have similar arrival rates of assessment shocks, and the NYSE has much higher asset liquidity.

Table 2: Monthly transition variables for four markets.

$Market$	α	$\lambda = x_{ss}$	r
housing	0.17	0.0053	0.0075
$NYSE$	1.9	0.055	0.0075
capital	0.024	0.0056	0.0075
labor	0.45	0.033	0.0075

6.2.1 Results

We begin the analysis with a comparison for the housing market versus the NYSE. Here, the common experiment across markets I consider is the impact of a 5% increase in entry rates, relative to the steady state level, which is sustained for 12 months from period $t = 0$ onwards.

taken to shop around and research into stocks purchased on the NYSE.

[16]See Shimer (2005) for a need for a model of unemployment along these lines motivated by the persistence of movements in vacancies. Fujita and Ramey (2007) adopt adjustment costs in the entry of vacancies to generate hump-shaped response of vacancies to productivity shocks.

Figure 9 plots the relative population θ, following this increase in entry rates for the housing market versus the NYSE. Figure 10 plots the associated movements in sales prices P. Note that the higher liquidity of assets in the NYSE does not imply the forces analyzed are dampened. Actually, the implied movement of the relative buyer population θ, and sales prices P is much greater in this example. In particular, the peak of the sales price is 9.2% higher than steady state for the NYSE versus only a 2.9% increase for the housing market.

[Figure 9] & [Figure 10]

These Figures also suggest that in the more liquid market, the adjustment back to steady state θ, P following the innovation to entry rates is substantially faster. For the sales price P, half the adjustment back to steady state from the peak occurs after 128 months for the housing market, and 17 months for the NYSE.[17] Thus, for a given frequency of data, one is more likely to detect serial correlation in price movements for the housing market than the NYSE following innovations in the entry rate. On the other hand, the anticipation effects (of foreseen changes in entry rates) on the sales prices are common across these two markets, which suggests that the liquidity of the market does not effect this dimension as much.

Next consider outcomes in these markets together with the other two markets, capital and labor. Here, the common experiment across markets I consider is the impact of a 50% decrease in entry rates, relative to the steady state level, which is sustained for 12 months from period $t = 0$ onwards. Focusing on innovations which decrease entry rates allows us to abstract away from tsunami effects to study relative speeds of transition across markets. Figure 11 plots the relative population θ, where here we compare outcomes for the housing market and NYSE with outcomes for the capital market and labor market. This Figure again confirms that the adjustment of θ occurs faster in markets associated with higher asset liquidity: the NYSE and the labor market.

Figure 12 plots the seller population u, following the 50% decrease in entry rates sustained over 12 months from period $t = 0$, across all four markets. This shows that the impact of the innovations to entry rates on seller populations (which are unemployment rates) is strongest in liquid markets where there is a greater degree of asset turnover.

[Figure 11] & [Figure 12]

7 Conclusion

In an asset market characterized by search frictions and adjustment costs to the entry rate of buyers, this paper has highlighted the following. First, asset prices are sensitive to the relative population of buyers to sellers, even in liquid markets. Second, the transition dynamics of the relative population of buyers

[17] Recall the innovation to entry rates lasts for 12 months from period $t = 0$. The trough in sales price is reached in period $t = 5$ for the housing market and period $t = 0$ for the NYSE.

can display strong propagation and divergence along its path to a stable steady state. This occurs because of a feedback loop between the relative population of buyers to sellers, which determines asset liquidity, and the population of sellers, which is decreasing in liquidity.

8 Appendix

Derivation of equations (3)-(5). From (1) and (2), the derivation for P imposing steady state is given by

$$\begin{aligned}
rS &= \pi(1-z) - \lambda S - \alpha m(\theta)\beta S - \alpha\frac{m(\theta)}{\theta}(1-\beta)S \\
&\Rightarrow \; S = \frac{\pi(1-z)}{r+\lambda+\alpha m(\theta)\beta+\alpha\frac{m(\theta)}{\theta}(1-\beta)}, \\
P &= \frac{\pi z}{r} + \frac{\alpha m(\theta)}{r}\beta S + \beta S \\
&= \frac{\pi}{r}\left(z + \frac{\beta(r+\alpha m(\theta))(1-z)}{r+\lambda+\beta\alpha m(\theta)+(1-\beta)\alpha\frac{m(\theta)}{\theta}}\right) \\
&= \frac{\pi}{r}\left(\frac{\beta(r+\alpha m(\theta)) + \left((1-\beta)r+\lambda+(1-\beta)\alpha\frac{m(\theta)}{\theta}\right)z}{r+\lambda+\beta\alpha m(\theta)+(1-\beta)\alpha\frac{m(\theta)}{\theta}}\right).
\end{aligned}$$

From (1) and (2) we get the steady state value of buyers by

$$\begin{aligned}
rU &= \pi z + \frac{\beta\alpha m(\theta)\pi(1-z)}{r+\lambda+\beta\alpha m(\theta)+(1-\beta)\alpha\frac{m(\theta)}{\theta}} \\
&= \frac{\beta\alpha m(\theta)\pi + \left(r+\lambda+(1-\beta)\alpha\frac{m(\theta)}{\theta}\right)\pi z}{r+\lambda+\beta\alpha m(\theta)+(1-\beta)\alpha\frac{m(\theta)}{\theta}}.
\end{aligned}$$

In the case when $V = \bar{V}$, the steady state price is given by

$$\begin{aligned}
rS &= \pi(1-z) - (r+\lambda)\bar{V} - \lambda S - \alpha m(\theta)\beta S \\
&\Rightarrow \; S = \frac{\pi(1-z)-(r+\lambda)\bar{V}}{r+\lambda+\alpha m(\theta)\beta}, \\
P &= \frac{\pi z}{r} + \frac{\alpha m(\theta)}{r}\beta S + \beta S \\
&= \frac{\pi}{r}\left(z + \frac{\beta(r+\alpha m(\theta))\left[(1-z)-(r+\lambda)\frac{\bar{V}}{\pi}\right]}{r+\lambda+\beta\alpha m(\theta)}\right) \\
&= \frac{\pi}{r}\left(\frac{\beta(r+\alpha m(\theta))\left[1-(r+\lambda)\frac{\bar{V}}{\pi}\right] + ((1-\beta)r+\lambda)z}{r+\lambda+\beta\alpha m(\theta)}\right).
\end{aligned}$$

The value of vacancies

$$\begin{aligned}
(r+\lambda)\bar{V} &= (1-\beta)\frac{\pi(1-z)-(r+\lambda)\bar{V}}{\frac{r+\lambda}{\alpha\frac{m(\theta)}{\theta}}+\beta\theta} \\
\frac{r+\lambda}{\alpha\frac{m(\theta)}{\theta}}+\beta\theta &= (1-\beta)\left(\frac{\pi(1-z)}{(r+\lambda)\bar{V}}-1\right)
\end{aligned}$$

Proof of Proposition 2. First show existence and uniqueness. Imposing that $\dot{v} = 0, \dot{u} = 0$ respectively implies in steady states

$$\begin{aligned} v_{ss} &= \frac{x}{\lambda + \alpha \frac{m(\theta_{ss})}{\theta_{ss}}}, \\ u_{ss} &= \frac{\lambda}{\lambda + \alpha m(\theta_{ss})}. \end{aligned}$$

Taking their ratio and rearranging

$$\begin{aligned} \lambda \theta_{ss} + \alpha m(\theta_{ss}) &= \frac{x}{\lambda}(\lambda + \alpha m(\theta_{ss})), \\ \lambda \theta_{ss} - x &= \alpha m(\theta_{ss}) \left(\frac{x}{\lambda} - 1 \right). \end{aligned}$$

The left hand side is a linear increasing function of θ_{ss}. Since $m(\theta_{ss})$ is concave and $\lim_{\theta \to \infty} m'(\theta) = 0$, there exists a unique solution. Whether the right hand side is increasing or decreasing in θ_{ss} depends on whether $\frac{x}{\lambda} \lesseqgtr 1$. If $\frac{x}{\lambda} > 1$, then $\theta_{ss} > \frac{x}{\lambda}$, if $\frac{x}{\lambda} < 1$, then $\theta_{ss} < \frac{x}{\lambda}$. If $\frac{x}{\lambda} = 1$, then $\theta_{ss} = 1$ regardless of α. As $\alpha \to \infty$, $\theta_{ss} \to \infty$ if $\frac{x}{\lambda} > 1$, or $\theta_{ss} \to 0$ if $\frac{x}{\lambda} < 1$.

Let $v_v(u), v_u(u)$ denote the loci of solutions to $\dot{v} = 0, \dot{u} = 0$ respectively. The proof of stability requires showing that when the two loci cross $\left(\frac{dv_v}{du} - \frac{dv_u}{du}\right) > 0$, which implies the absolute slope of $\frac{dv_v}{du}$ is lower than $\frac{dv_u}{du}$, as shown in Figure 2. At a point where they cross the following conditions are satisfied

$$\begin{aligned} \frac{dv_v}{du} &= -\frac{M_v}{\lambda}\frac{dv_v}{du} - \frac{M_u}{\lambda}, \\ 1 &= -\frac{M_v}{\lambda}\frac{dv_u}{du} - \frac{M_u}{\lambda}. \end{aligned}$$

Rearranging these imply

$$\begin{aligned} \frac{dv_v}{du} &= -\frac{M_u}{\lambda + M_v} \leq 0, \\ \frac{dv_u}{du} &= -\frac{\lambda + M_u}{M_v} < 0. \end{aligned}$$

since $M_v = (1-\eta)\alpha \frac{m(\theta_{ss})}{\theta_{ss}} > 0, M_u = \eta \alpha m(\theta_{ss}) \geq 0$ where $(1-\eta) = \frac{\theta_{ss} m'(\theta_{ss})}{m(\theta_{ss})} \in (0,1)$. Then, after further rearranging

$$\begin{aligned} -M_v \frac{dv_u}{du} &= \lambda - (\lambda + M_v)\frac{dv_v}{du} \\ &\Rightarrow M_v \left(\frac{dv_v}{du} - \frac{dv_u}{du} \right) = \lambda - \lambda \frac{dv_v}{du} > 0. \end{aligned}$$

Given $M_v = (1-\eta)\frac{m(\theta_{ss})}{\theta_{ss}} > 0$ and $\frac{dv_v}{du} \leq 0$, this implies local stability of steady state equilibrium. Then, given uniqueness, global stability of the steady state is readily confirmed using the phase diagram of Figure 2 and noting $\frac{dv_v}{du} \leq 0$, $\frac{dv_u}{du} < 0$.

Proof of Lemma 1. From (10), $\dot{\theta} > 0$ for $\theta = 0$ regardless of u. Next I establish that $\dot{\theta} < 0$ for some feasible $\theta(u)$ when $u < u_{ss}$. Suppose $\theta(u) = \frac{v_v(u)}{u} \Rightarrow \dot{v} = 0$, since $v_v(u)$ is defined as the locus of buyers as a function of sellers where this is true. When $u < u_{ss}, \dot{v} = 0$ implies $\dot{u} > 0$ (from Figure 2) which implies $\dot{\theta} < 0$. This establishes the claim.

Note that $\dot{\theta} > 0$ for θ arbitrarily large. This implies there are two solutions to (10) when $u < u_{ss}$. Finally for $u = u_{ss}$ at least one solution must exist given the existence of steady state. Note if a solution to (10) does not exist for some $u > u_{ss}$, $\dot{\theta} > 0$ throughout the range of θ.

To show the second part of the Lemma we need to show there exists a θ such that $\dot{\theta} \leq 0$ for any $u \in [0, 1]$ if $x \leq \lambda$. Suppose $\theta = 1$, then from (9)

$$\dot{\theta} = \frac{x}{u} - \frac{\lambda}{u} \leq 0.$$

This verifies the claim.

For the third part, note from (9) that a solution will not exist given u iff

$$\begin{aligned} \frac{x}{u} &> \alpha m(\theta) + \frac{\lambda}{u}\theta - \alpha m(\theta)\theta, \ \forall\theta \\ &\Leftrightarrow \\ x &> u\alpha m(\theta)(1-\theta) + \lambda\theta, \ \forall\theta. \end{aligned}$$

Then under the condition of the Lemma a solution always exists.

Proof of Proposition 4. Let μ_V denote the co-state variable associated with v, μ_{J-U} denote the co-state variable associated with u, μ_J denote the co-state variable associated with q. The Euler conditions imply

$$\begin{aligned} -(r + \lambda + M_v)\mu_V + M_v\mu_{J-U} &= -\dot{\mu}_V \\ (r + \lambda + M_u)\mu_{J-U} - M_u\mu_V &= \pi + \dot{\mu}_{J-U}, \\ r\mu_J + \lambda\mu_{J-U} &= \pi + \dot{\mu}_J. \end{aligned}$$

The difference $(\mu_J - \mu_{J-U})$ is

$$r(\mu_J - \mu_{J-U}) = M_u(\mu_{J-U} - \mu_V) + (\dot{\mu}_J - \dot{\mu}_{J-U}).$$

Meanwhile, the sum $(\mu_{J-U} - \mu_V)$ is

$$(r + \lambda + M_u + M_v)(\mu_{J-U} - \mu_V) = \pi + (\dot{\mu}_{J-U} - \dot{\mu}_V).$$

Note $M_u = \eta\alpha m(\theta)$, $M_v = (1-\eta)\alpha\frac{m(\theta)}{\theta}$. Under the conditions of the Proposition, and comparison with equations (1), μ_V is equal to the value of being a buyer V, and μ_{J-U} is equal to the surplus from being an owner versus seller

$J-U$. Then $\left(\mu_{J-U}-\mu_V\right)$ is equal to the match surplus S. μ_J is equal to the asset value of an owner J, and $\left(\mu_J-\mu_{J-U}\right)$ is equal to the asset value of a seller U.

Derivation of equation (21). From (1) and (2)

$$\begin{aligned} rS &= \pi(1-z)-\left(\lambda+\beta\alpha m(\theta)+(1-\beta)\alpha\frac{m(\theta)}{\theta}\right)S+\dot{S}, \\ \beta\left(rS+\lambda S-\dot{S}\right) &= \beta\pi(1-z)-\beta\beta\alpha m(\theta)S-\beta(1-\beta)\alpha\frac{m(\theta)}{\theta}S, \\ rU+\dot{U} &= \pi z+\alpha m(\theta)S. \end{aligned}$$

From (19)

$$\begin{aligned} w &= rP+\lambda(P-U)-\dot{P} \\ &= r(U+\beta S)+\lambda\beta S-\dot{U}-\beta\dot{S} \\ &= rU+\dot{U}+\beta\left(rS+\lambda S-\dot{S}\right) \\ &= \pi z+\alpha m(\theta)S+\beta\pi(1-z)-\beta\beta\alpha m(\theta)S-\beta(1-\beta)\alpha\frac{m(\theta)}{\theta}S \\ &= \beta\pi(1-z)+\pi z+(1-\beta)\beta S\left(\alpha m(\theta)-\alpha\frac{m(\theta)}{\theta}\right) \\ &= \beta\pi(1-z)+\pi z \\ &\quad+(1-\beta)\beta\left(\alpha m(\theta)-\alpha\frac{m(\theta)}{\theta}\right)\frac{\pi(1-z)+\dot{S}}{r+\lambda+\beta\alpha m(\theta)+(1-\beta)\alpha\frac{m(\theta)}{\theta}} \\ &= \pi z+\beta\pi(1-z)\frac{r+\lambda+\alpha m(\theta)}{r+\lambda+\beta\alpha m(\theta)+(1-\beta)\alpha\frac{m(\theta)}{\theta}} \\ &\quad+\dot{S}\frac{(1-\beta)\beta\left(\alpha m(\theta)-\alpha\frac{m(\theta)}{\theta}\right)}{r+\lambda+\beta\alpha m(\theta)+(1-\beta)\alpha\frac{m(\theta)}{\theta}}. \end{aligned}$$

In steady states where $\theta=\theta_{ss}$, as $\alpha\to\infty$

$$\begin{aligned} w_{\alpha\to\infty} &= \pi z+\pi(1-z)\frac{\beta\theta_{ss}}{\beta\theta_{ss}+(1-\beta)} \\ &= \pi\frac{\beta\theta_{ss}+(1-\beta)z}{\beta\theta_{ss}+(1-\beta)}. \end{aligned}$$

8.1 Model with smooth adjustment costs

Consider the following modified social planner problem

$$\max_{x,v,u,q} \int_0^\infty e^{-rt} \left[\pi(q-u) + \pi z u - Ac(x)\right] dt \text{ s.t.}$$

$$\begin{aligned}
\dot{v} &= x - \lambda v - \alpha \frac{m(\theta)}{\theta} v, \\
\dot{u} &= \lambda (q-u) - \alpha m(\theta) u, \\
\dot{q} &= 0.
\end{aligned}$$

where the initial v_0, u_0 are given. There are adjustment costs to the entry of new buyers such that $c'(x) > 0, c''(x) > 0, c(0) = 0$. The Euler conditions imply

$$\begin{aligned}
Ac'(x) &= \mu_V > 0, \\
-(r + \lambda + M_v)\mu_V + M_v \mu_{J-U} &= -\dot{\mu}_V, \\
(r + \lambda + M_u)\mu_{J-U} - M_u \mu_V &= \pi + \dot{\mu}_{J-U}, \\
r\mu_J + \lambda \mu_{J-U} &= \pi + \dot{\mu}_J.
\end{aligned}$$

Inverting the first expression implies $x = x\left(\frac{\mu_V}{A}\right)$, where $x' > 0$, from the properties of the adjustment cost function $c(x)$. Note $M_u = \eta m(\theta)$, $M_v = (1-\eta)\frac{m(\theta)}{\theta}$. Such a model with smooth adjustment costs permits endogenous changes in entry rates arising from innovations to the interest rate r, and earnings assessment π which affect the value of buyers μ_V.

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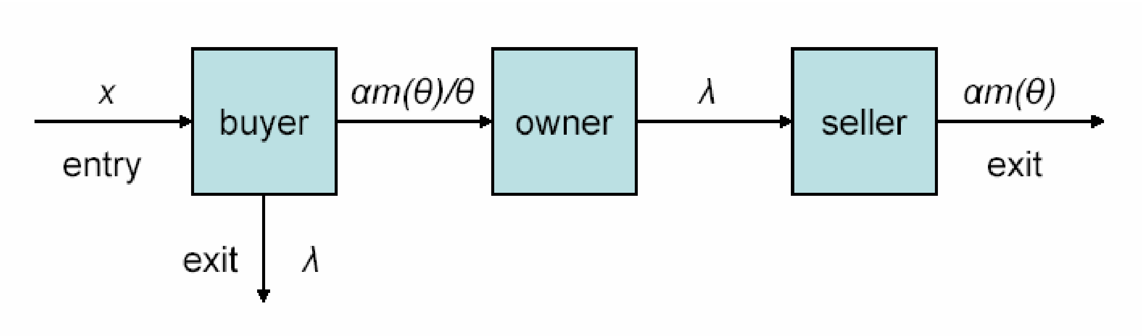

Figure 1: Movement of agents through the market.

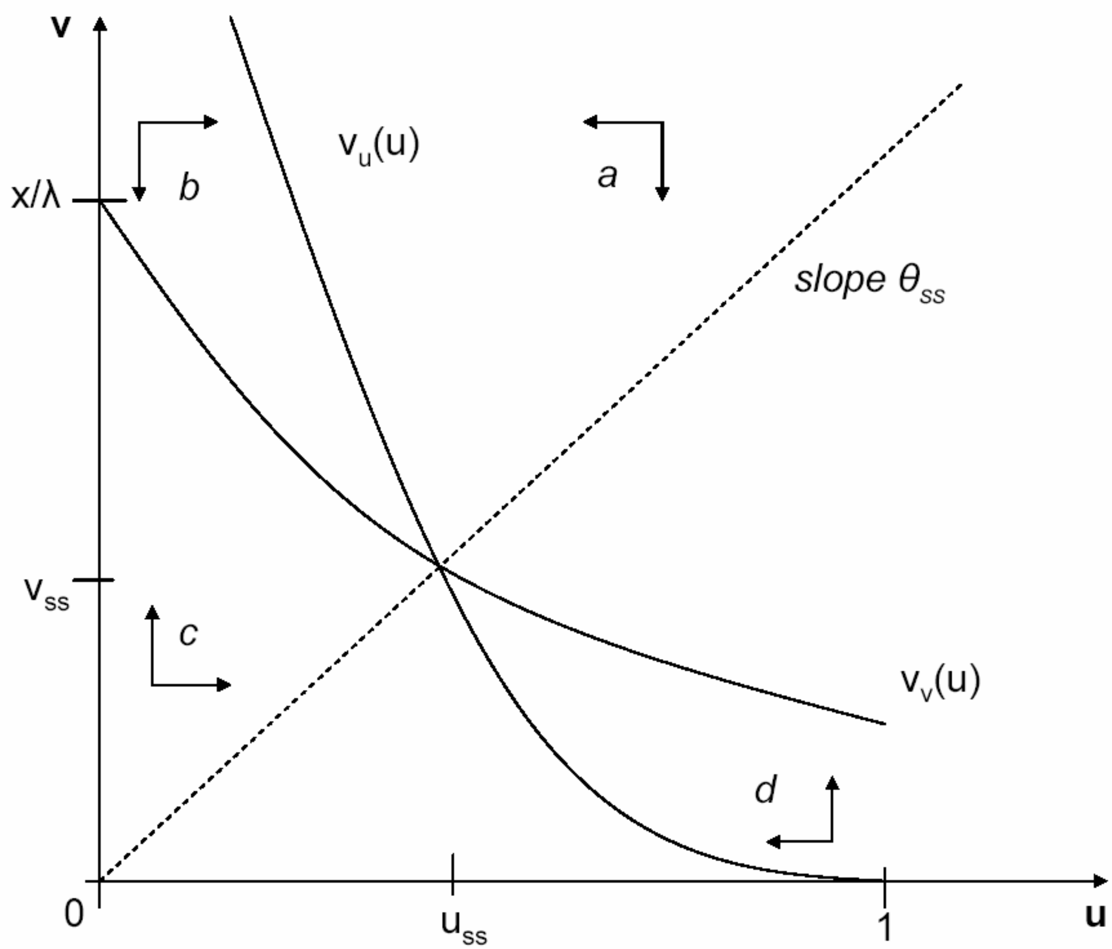

Figure 2: Dynamics of buyers *v*, and sellers *u*.

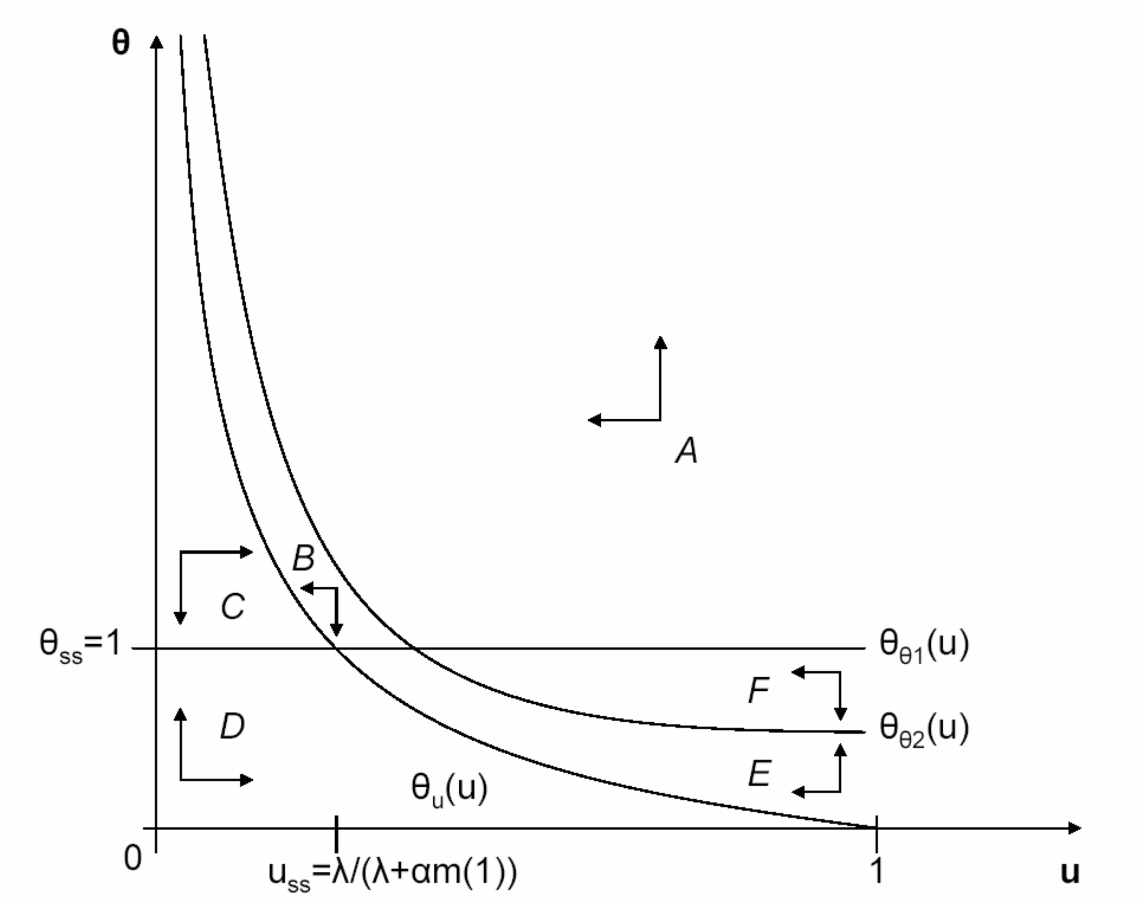

Figure 3: Dynamics of relative population buyers $\boldsymbol{\theta}$, and sellers u, when x=$\boldsymbol{\lambda}$.

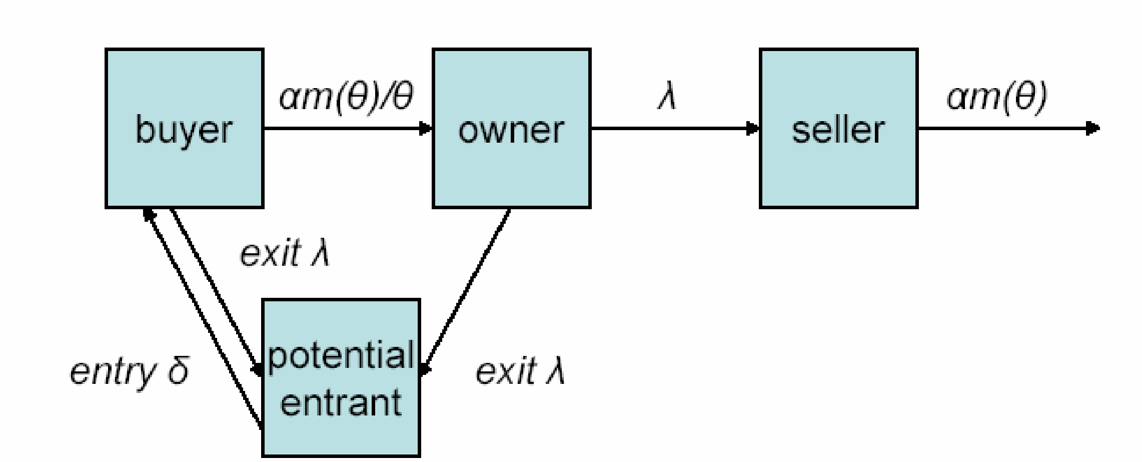

Figure 4: Movement of agents through a large market.

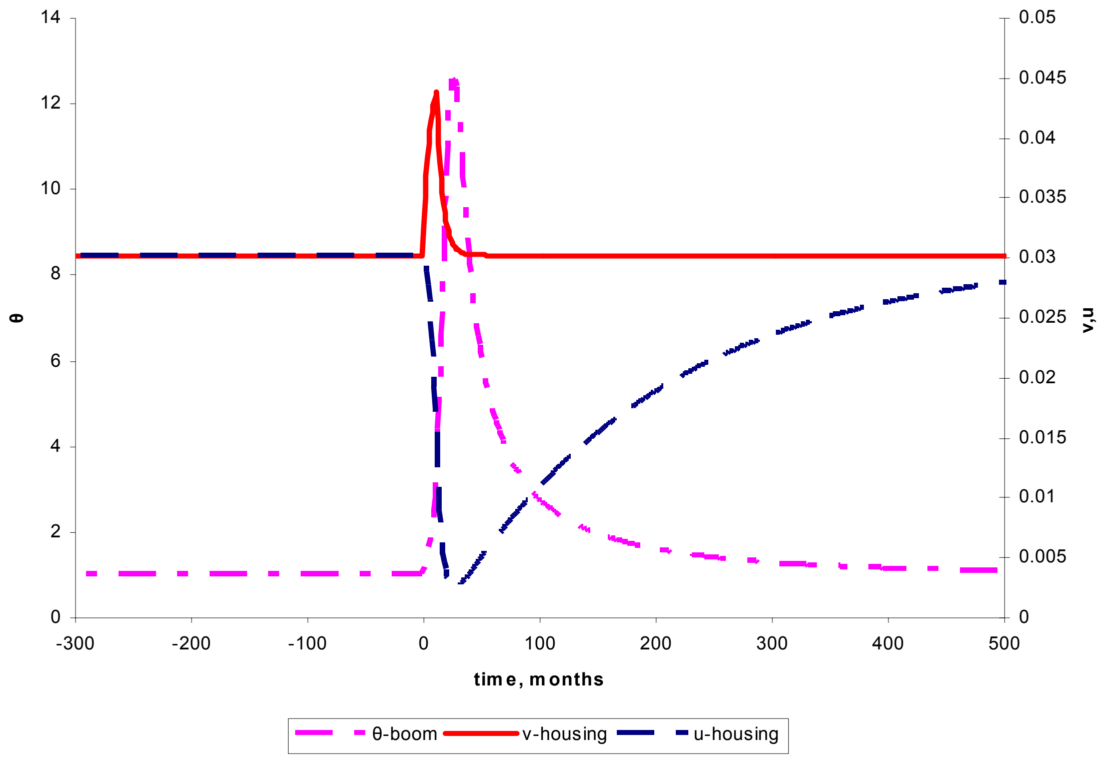

Figure 5: Population of buyers *v*, sellers *u*, and relative population *θ*, following 50% increase in entry rate sustained over 12 months from period t=0.

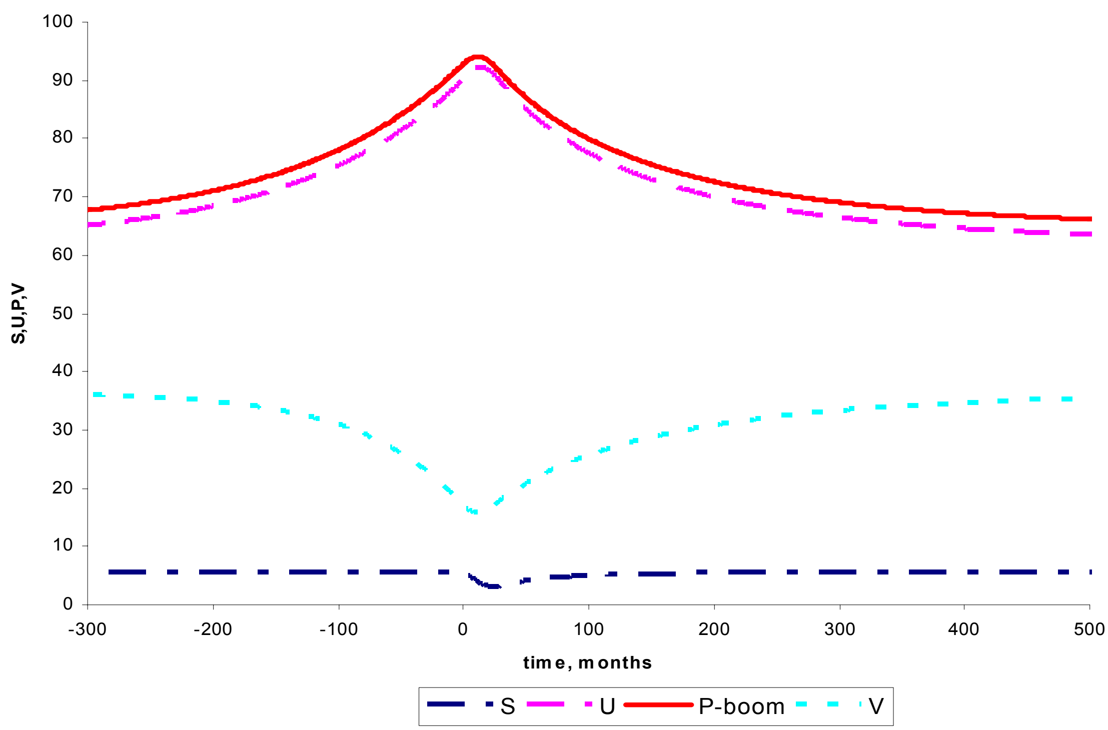

Figure 6: Values of surplus *S*, seller *U*, buyer *V*, and sales price *P*, following 50% increase in entry rate sustained over 12 months from period t=0.

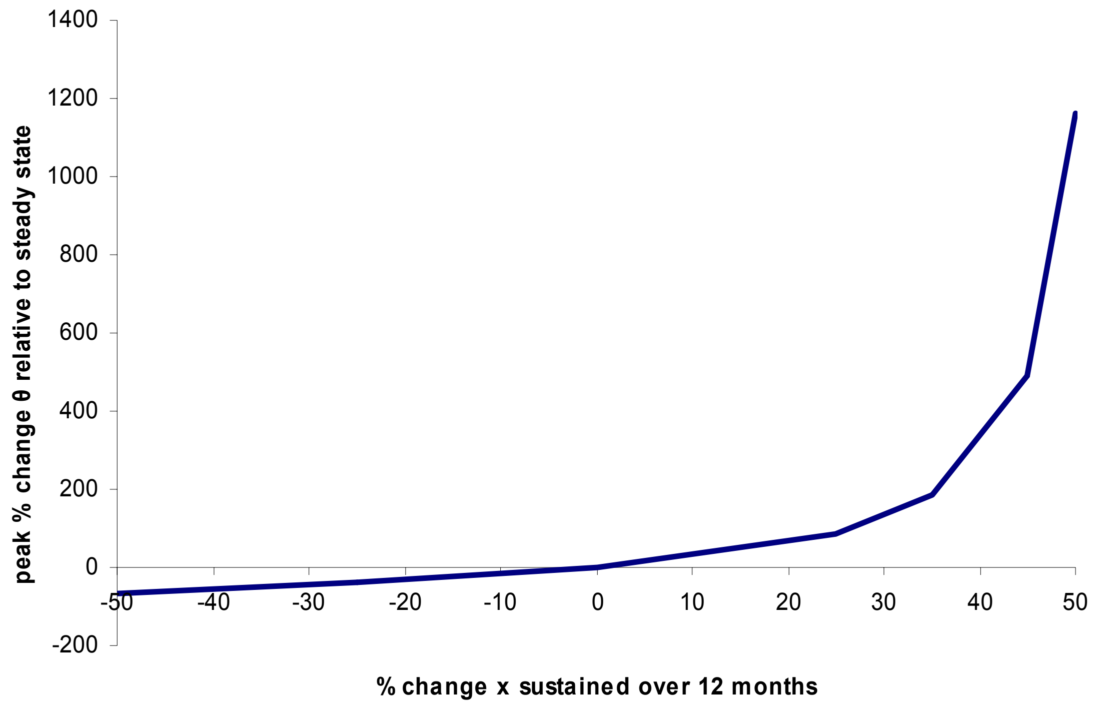

Figure 7: Peak/trough relative population *θ*, following positive/negative displacement of entry rate x, relative to steady state level sustained over 12 months from period t=0.

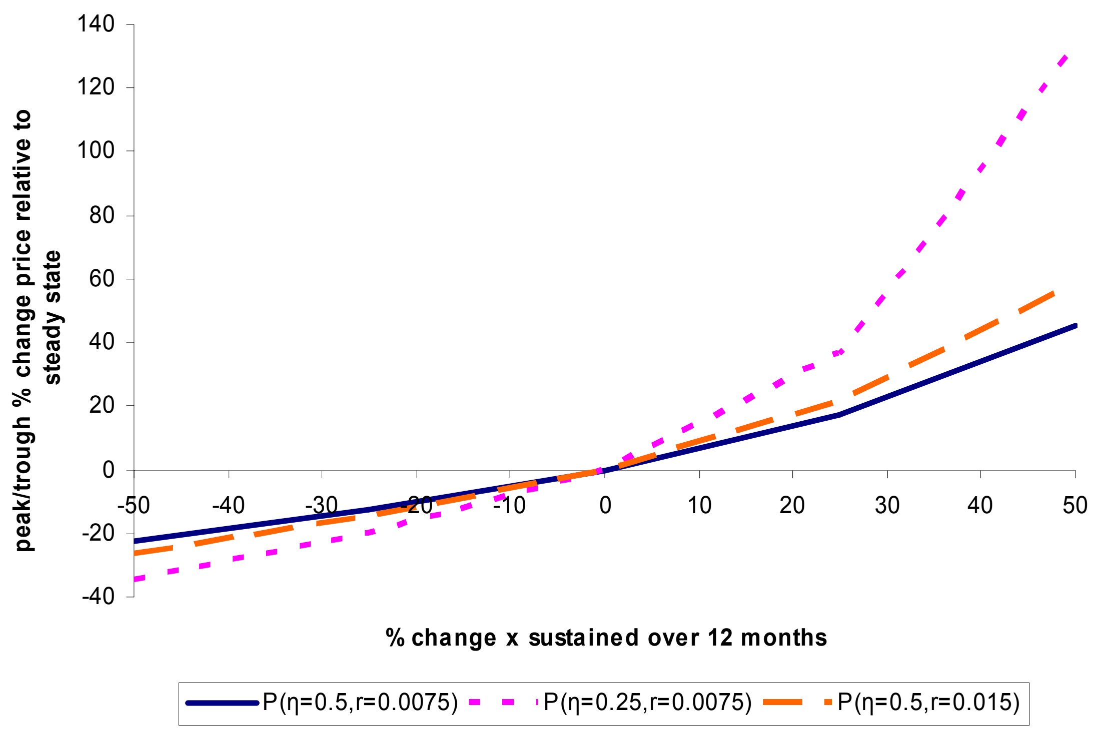

Figure 8: Peak/trough price *P*, following positive/negative displacement of entry rate x, relative to steady state level sustained over 12 months from period t=0.

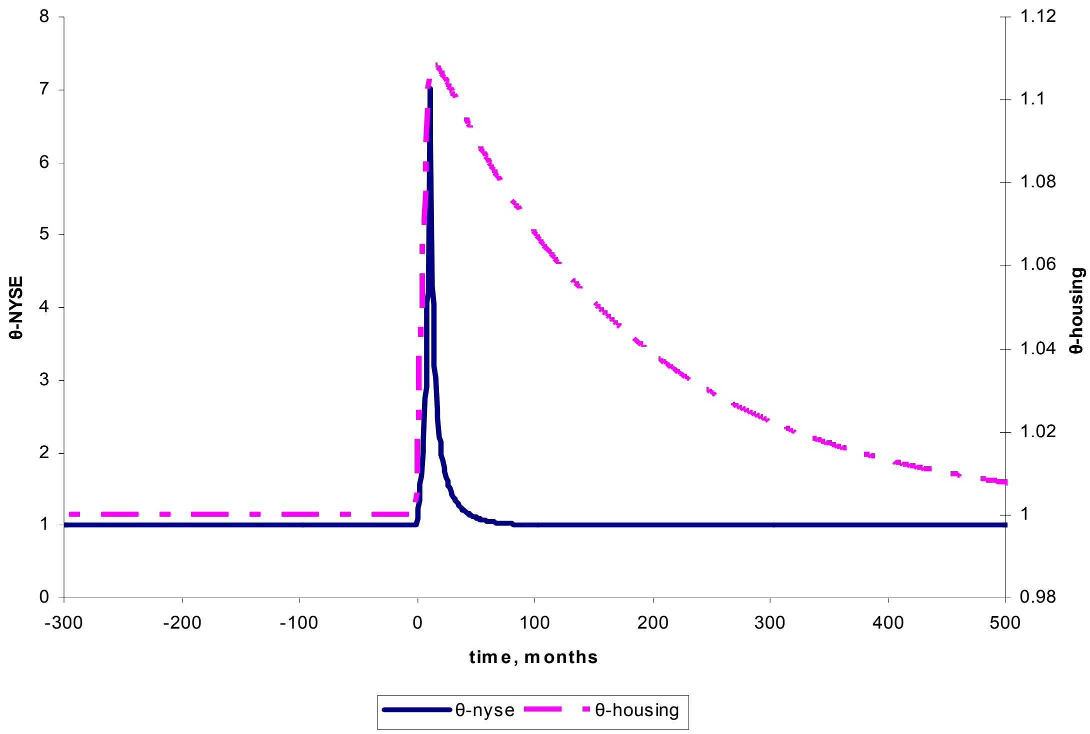

Figure 9: Relative population *θ*, following 5% increase in entry rate sustained over 12 months from period t=0, housing market versus NYSE.

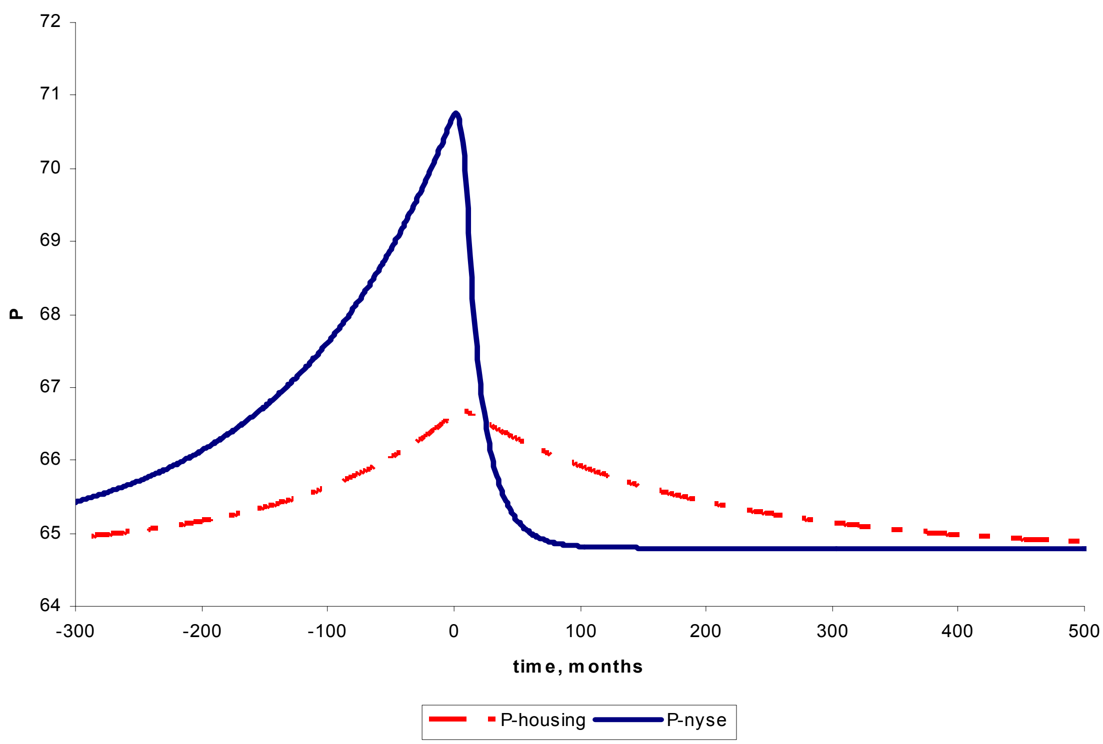

Figure 10: Sales price *P*, following 5% increase in entry rate sustained over 12 months from period t=0, housing market versus NYSE.

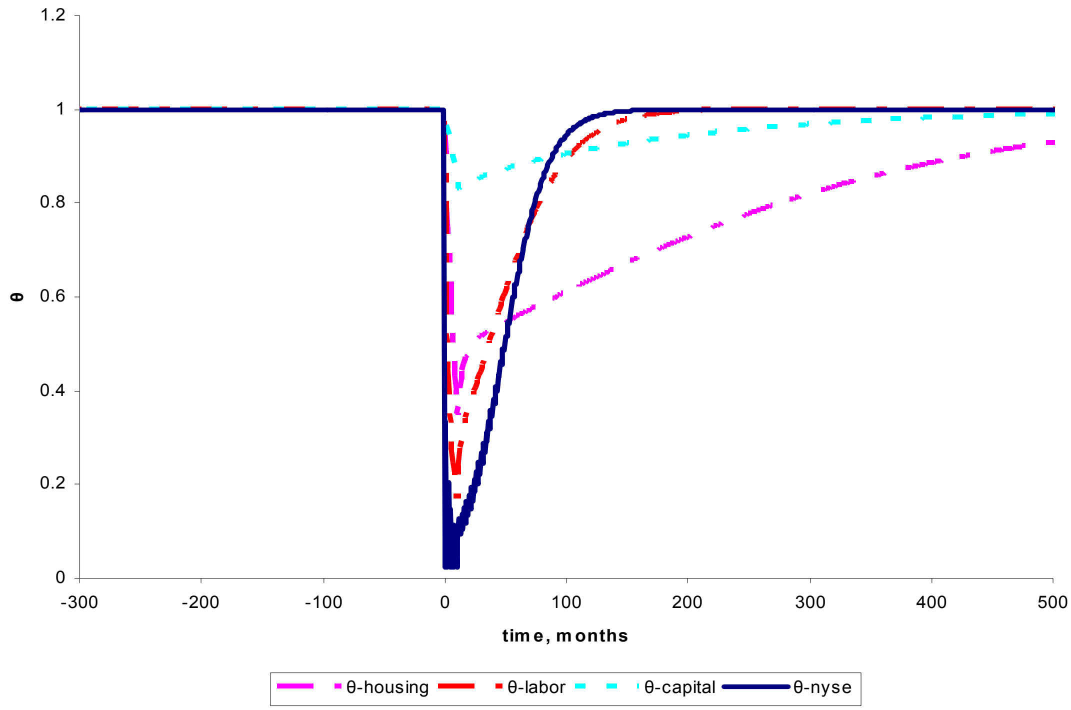

Figure 11: Relative population *θ*, following 50% decrease in entry rate sustained over 12 months from period t=0, housing market, NYSE, capital market and labor market.

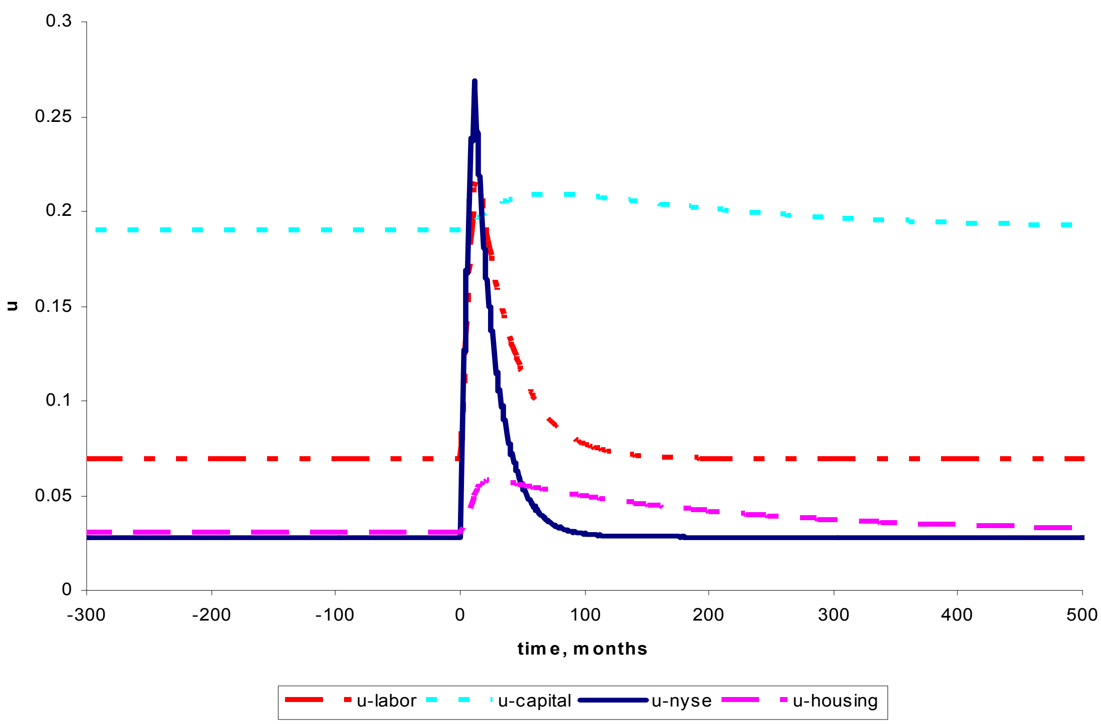

Figure 12: Unemployment rate *u*, following 50% decrease in entry rate sustained over 12 months from period t=0, housing market, NYSE, capital market and labor market.

Appendix Figures

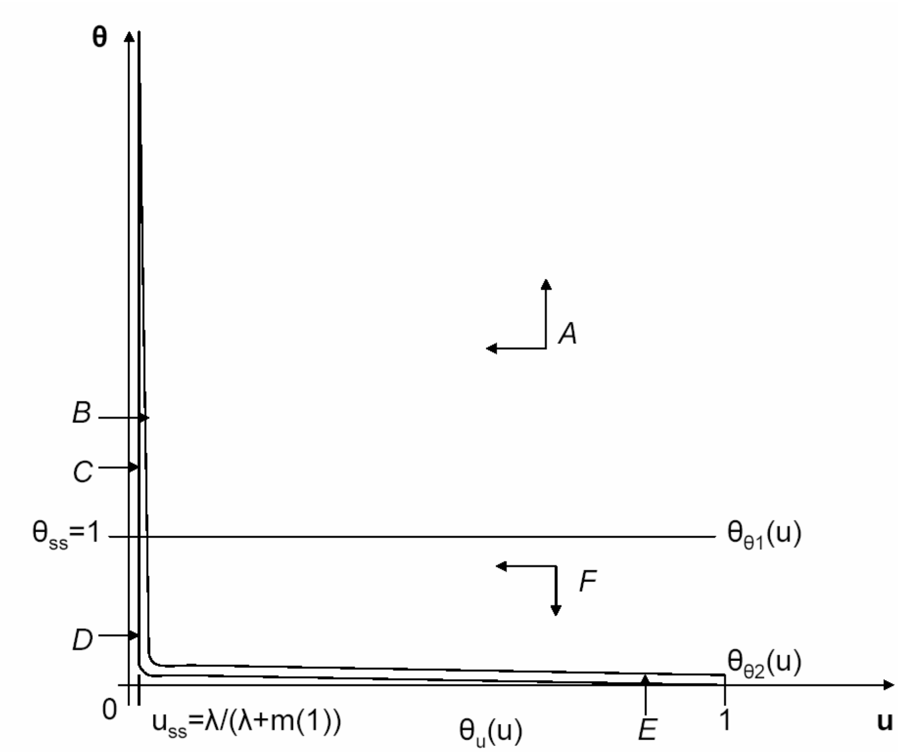

Figure A1: Dynamics of relative population buyers *θ*, and sellers *u*, when *x*=*λ* for liquid market.