Limited Nominal Indexation of Optimal Financial Contracts[1]

Césaire A. Meh
Bank of Canada

Vincenzo Quadrini
University of Southern California

Yaz Terajima
Bank of Canada

January 26, 2010

[1]We would like to thank Craig Burnside, Matthias Doepke and Lutz Weinke for their helpful discussions. We also thank participants at the Annual Conference of the Canadian Economic Association, the Monetary Policy and Financial Frictions conference in Minneapolis, the Asset Markets, Nominal Contracts, and Monetary Policy conference in Munich, the 2009 Society of Economic Dynamics meeting in Turkey, the 2009 Midwest Macroeconomics meeting, the 2009 Society of Computational Economics meeting in Sydney, the 2009 Econometric Society Far Eastern Summer meeting, and seminar attendees at the Bank of Japan, HEC Montreal, Keio University, Osaka University, Queens University, University of Maryland, University of Nagoya, and the University of Tokyo. Quadrini gratefully acknowledges financial support from the National Science Foundation (grant SES 0617937). The views expressed in this paper are those of the authors. No responsibility should be attributed to the Bank of Canada. The paper has previously circulated under the title "Real Effects of Price Stability with Endogenous Nominal Indexation".

Abstract

We study a model with repeated moral hazard where financial contracts are not fully indexed to inflation because nominal prices are observed with delay, as in Jovanovic & Ueda (1997). More constrained firms sign contracts that are less indexed to inflation, and as a result, their investment is more sensitive to nominal price shocks. We also find that the overall degree of nominal indexation increases with uncertainty in the price level. An implication of this is that economies with higher inflation uncertainty are less vulnerable to a price shock of a given magnitude, that is, aggregate investment and output respond to a lesser degree.

1 Introduction

When financial contracts are not indexed to inflation, an unexpected increase in the nominal price redistributes wealth from lenders to borrowers. Doepke & Schneider (2006b, 2006a) and Meh, Ríos-Rull, & Terajima (2008) show empirically that redistribution can be sizeable even for moderate levels of inflation, using U.S. and Canadian data respectively. To the extent that the distribution of wealth is not neutral for investment and production decisions, this could have important macroeconomic effects. Christiano, Motto, & Rostagno (2008) consider nominal debt contracts in a large scale macroeconomic model that incorporates the financial accelerator of Bernanke, Gertler, & Gilchrist (1999) and find that the redistribution of wealth from households to entrepreneurs induced by unexpected inflation contributes significantly to macroeconomic fluctuations.

Although the assumption of 'nominal' debt contracts is clearly supported by the data, it is not obvious why firms and households enter into financial relations that are not fully indexed to inflation. In this paper we propose a mechanism that can rationalize the limited indexation of 'optimal' financial contracts. The mechanism is based on agency problems and lagged observation of 'aggregate' nominal prices.

The model features entrepreneurs who finance investment by entering into contractual relations with financial intermediaries. Because of agency problems induced by information asymmetries, financial contracts are constrained optimal. The key mechanism leading to the limited indexation of these contracts is the assumption that the aggregate nominal price is observed with delay as in Jovanovic & Ueda (1997, 1998). This is motivated by the fact that in reality there is a substantial time lag before the aggregate price level becomes public information.[1] The timing lag creates a time-inconsistency problem that leads to the renegotiation of a contract that is fully indexed to inflation.

We first characterize the optimal long-term contract in which the parties commit not to renegotiate in future periods. The optimal contract with commitment is fully indexed, and therefore inflation is neutral. After show-

[1]This is certainly the case for the GDP deflator. For the consumer price index the time lag is shorter. However, the CPI is an aggregate measure of a representative consumption basket. Because of heterogeneity, what matters is an individual's consumption basket, the price of which could deviate substantially from the nominal price of the representative basket.

ing that this contract is not immune to renegotiation, we characterize the renegotiation-proof contract. In doing so we assume that renegotiation can arise at any time before the observation of the nominal price. Contrary to the environment considered in Martin & Monnet (2006), this assumption eliminates the optimality of mixed strategies.[2]

A key property of the renegotiation-proof contract is the limited indexation to inflation, that is, *real* payments depend on *nominal* quantities. A consequence of this is that unexpected movements in the nominal price have real consequences for an individual firm as well as for the aggregate economy. The central mechanism of transmission is the debt-deflation channel: An unexpected increase in prices reduces the real value of nominal liabilities, improving the net worth of entrepreneurs. The higher net worth then facilitates greater investment and leads to a macroeconomic expansion.

This result can also be obtained in a simpler model in which we impose that financial transactions take place only through nominal debt contracts. However, with this simpler framework we would not be able to study how different monetary regimes or policies affect the degree of indexation, and therefore, how the economy responds to nominal price shocks under different monetary policy regimes. Our model, instead, allows us to study whether an economy with greater nominal price uncertainty features a higher degree of nominal indexation and whether nominal price shocks have different macroeconomic implications given the different degree of 'endogenous' indexation.

Although the theoretical foundation for limited indexation used in this paper has been developed in Jovanovic & Ueda (1997), the structure of our economy and the questions addressed in this paper are different. First, in our environment all agents are risk neutral, but they operate a concave investment technology. Therefore, the role that the concavity of preferences plays in Jovanovic and Ueda is now played by the concavity of the technology. Second, we consider agents that are infinitely lived, and therefore, we solve for a repeated moral hazard problem. This allows us to study how inflation

[2]Building on the results of Fudenberg & Tirole (1990), Martin and Monnet show that the time-consistent policy may also depend on the realization of real output if we allow for mixed strategies. The optimality of the mixed strategies, however, depends on the assumption that, once the agent has revealed his/her type, the contract cannot be renegotiated again. This point is clearly emphasized in the concluding section of Fudenberg & Tirole (1990). In our model we do not impose this restriction, that is, the contract can be renegotiated at any time before the observation of the price level. Consequently, mixed strategies are time-inconsistent in our set up.

shocks impact investment and aggregate output dynamically over time. It also allows us to distinguish the short-term versus long-term effects of different monetary regimes. Third, in our model entrepreneurs/firms are ex-ante identical but ex-post heterogeneous. At each point in time, some firms face tighter constraints and invest less while others firms face weaker constraints and invest more. This allows us to study how nominal price shocks impact investment at different stages of the firm's growth. The paper is also related to a more recent contribution by Jovanovic (2009).

There are several findings. The first is that the optimal contract allows for lower nominal indexation in firms that are more financially constrained (and tend to be smaller in size). As a result, these firms are more vulnerable to inflation shocks. This finding is also relevant for cross-country comparisons, since a country with less developed financial markets is likely to have a larger share of firms with tighter financial constraints. Thus, controlling for the monetary regime, the economies of these countries may be more vulnerable to inflation shocks.

The second finding is that the degree of nominal price indexation increases with the degree of nominal price uncertainty. This implies that the impact of a given inflation shock is bigger in economies with lower price volatility (since contracts are less indexed in these economies). In general, however, economies with greater price uncertainty also face larger inflation shocks on average. Therefore, the overall aggregate volatility induced by these shocks is not necessarily smaller in these economies. In fact, we show in the numerical exercise that the relationship between inflation uncertainty and aggregate volatility is not monotonic: aggregate volatility first increases with inflation uncertainty and then decreases.

The plan of the paper is as follows. In the next section we describe the model. Section 3 characterizes the long-term financial contract with commitment and shows that such a contract is not free from renegotiation. Section 4 characterizes the renegotiation-proof contract and Section 5 discusses the relationship between the monetary regime and the degree of indexation. Section 7 presents additional properties of the model numerically and Section 8 concludes.

2 The model

Consider a continuum of risk-neutral entrepreneurs with utility $E_0 \sum_{t=0}^{\infty} \beta^t c_t$, where β is the discount factor and c_t is consumption. Entrepreneurs have the skills to run an investment technology as specified below. They finance investments by signing optimal contracts with 'competitive' risk-neutral intermediaries. We will also refer to intermediaries as investors. Given the interest rate r, the market discount rate is denoted by $\delta = 1/(1+r)$. We assume that $\beta \leq \delta$, that is, the entrepreneur's discount rate is at least as large as the market interest rate.

The investment technology run by an entrepreneur generates cash revenues $R_t = p_t z_t k_{t-1}^{\theta}$, where p_t is the nominal price level, z_t is an 'unobservable' idiosyncratic productivity shock and k_{t-1} is the publicly observed input of capital chosen in the previous period. We assume that the capital fully depreciates after production, but this assumption is not essential for the results. For notational convenience we denote by $s_t = p_t z_t$ the product of the two stochastic variables, nominal price and productivity. Therefore, the cash revenue can also be written as $R_t = s_t k_{t-1}^{\theta}$.

The idiosyncratic productivity shock is *iid* and log-normally distributed, that is $z_t \sim LN(\mu_z, \sigma_z^2)$. The price level is also *iid* and log-normally distributed, that is, $p_t \sim LN(\mu_p, \sigma_p^2)$. For later reference we denote by $\tilde{z}_t$ and $\tilde{p}_t$ the logarithms of these two variables. Thus, $\tilde{s}_t = \tilde{z}_t + \tilde{p}_t$. Given the log-normality assumption, the logarithms of productivity and prices are normally distributed, that is, $\tilde{z}_t \sim N(\mu_z, \sigma_z^2)$ and $\tilde{p}_t \sim N(\mu_p, \sigma_p^2)$.

It is important to emphasize that z_t is not observable directly. It can only be inferred from the observation of the cash revenue R_t and the nominal price p_t. Because k_{t-1} is public information, the observation of the revenue informs us of the value of $s_t = p_t z_t$. Thus, once we observe the nominal price p_t we can infer z_t.

The central feature of the model is the particular timing of information wherein aggregate prices are observed with delay. There are two stages in each period and the price level is observed only in the second stage. In the first stage the cash revenue $R_t = p_t z_t k_{t-1}^{\theta}$ is realized. The entrepreneur is the first to observe R_t and, indirectly, $s_t = p_t z_t$. However, this is not sufficient to infer the value of z_t because the general price p_t is unknown at this stage.

By being the first to observe the cash revenue, the entrepreneur has the ability to divert the revenue for consumption without being detected by the investor (consumption is also not observable). Therefore, there is an information asymmetry between the entrepreneur and the investor which is typical in investment models with moral hazard such as Atkeson (1991), Clementi & Hopenhayn (2006), Gertler (1992) and Quadrini (2004).

In the second stage the general price p_t becomes known. Although the observation of p_t allows the entrepreneur to infer the value of z_t, the investor can infer the true z_t only if the entrepreneur chooses not to divert the revenues in the first stage.

The actual consumption purchased in the second stage with the diverted revenue will depend on the price p_t. Therefore, when the revenue is diverted in the first stage, the entrepreneur is uncertain about the real value of the diverted cash. As we will see, this is the key feature of the model that creates the conditions for the renegotiation of the optimal long-term contract as in Jovanovic & Ueda (1997). Figure 1 summarizes the information timing.

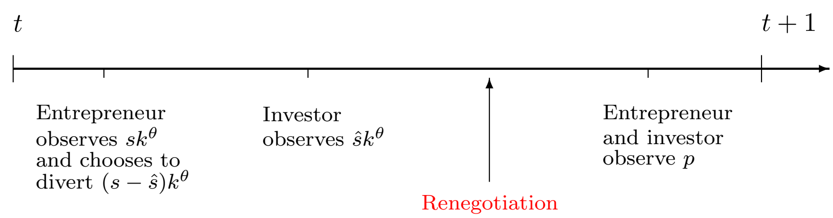

Figure 1: Information timing.

3 The long-term contract

In this section we characterize the optimal long-term contract, that is, the contract signed under the assumption that the parties make a commitment not to renegotiate consensually in later periods. We will then show that this contract is not free from renegotiation given the particular information structure where the nominal aggregate price is observed with delay. The renegotiation-proof contract will be characterized in the next section.

The long-term contract is characterized by maximizing the value for the investor subject to a value promised to the entrepreneur. We write the optimization problem recursively considering the optimization problem solved at the end of the period, after consumption. Assuming that the idiosyncratic productivity is not persistent, the only 'individual' state for the contract at the end of period is the utility q promised to the entrepreneur after consumption.

The contract chooses the new investment, k, the next period consumption, $c' = g(z', p')$, and the next period continuation utility, $q' = h(z', p')$, where z'

and p' are the productivity and the aggregate price for the next period. For the contract to be optimal we have to allow the next period consumption and continuation utility to be contingent on all additional information that becomes available in the next period, that is, z' and p'.

The maximization problem is subject to two constraints. First, the utility promised to the entrepreneur must be delivered (promise-keeping). The contract can choose different combinations of next period consumption $c' = g(z', p')$ and next period continuation utility $q' = h(z', p')$, but the expected value must be equal to the utility promised in the previous period, that is,

$$q = \beta E\Big[g(z', p') + h(z', p')\Big].$$

Second, the entrepreneur can not have an incentive to divert, for any possible realization of $s' = p'z'$ (incentive-compatibility). This requires that the value received when reporting the true s' is not lower than the value of reporting a smaller s' (and diverting the difference in revenue). If the entrepreneur reports $\hat{s}'$, the real value of the diverted revenues is $\phi(s' - \hat{s}')k^\theta/p$, where $\phi \leq 1$ is a parameter that captures the *efficiency* in diverting. Since smaller values of ϕ imply lower gains from diversion, we interpret ϕ as a proxy for the characteristics of financial markets (less developed financial markets have higher ϕ).

At the moment of choosing whether to divert the revenues, which arises in the first stage of the next period, the nominal price p' is unknown. Therefore, what matters is the expected value of the diverted revenue conditional on the observation of s', that is, $E[\phi(s' - \hat{s}')k^\theta/p' \,|\, s']$. Thus, for incentive-compatibility we have to impose the following constraint:

$$E\Big[g(z', p') + h(z', p') \,\Big|\, s'\Big] \;\geq\; E\left[\phi\left(\frac{s' - \hat{s}'}{p'}\right)k^\theta + g\left(\frac{\hat{s}'}{p'}, p'\right) + h\left(\frac{\hat{s}'}{p'}, p'\right) \,\Big|\, s'\right]$$

for all $\hat{s}' < s'$, where s' is the true realization of $p'z'$ and $\hat{s}'$ is the value observed by the investor if the entrepreneur diverts $(s' - \hat{s}')k^\theta$. Notice that the expectation is conditional on the information available to the entrepreneur when he/she chooses to divert. Even if the investor observes $\hat{s}'$, the entrepreneur knows what the true value is s'.

Although the constraint is imposed for all possible values of $\hat{s}' < s'$, we can restrict attention to the lowest value $\hat{s}' = 0$. It can be shown that, if the incentive compatibility constraint is satisfied for $\hat{s}' = 0$, then it will also be satisfied for all $\hat{s}' < s'$. Using this property, the contractual problem can be written as:

$$V(q) = \max_{k,\, g(z',p'),\, h(z',p')} \left\{ -k + \delta E\Big[z'k^{\theta} - g(z',p') + V(h(z',p')) \Big] \right\} \tag{1}$$

subject to

$$E\Big[g(z',p') + h(z',p') \,\Big|\, s' \Big] \geq E\Big[\phi\, z'k^{\theta} + g(0,p') + h(0,p') \,\Big|\, s' \Big] \tag{2}$$

$$q = \beta E\Big[g(z',p') + h(z',p') \Big] \tag{3}$$

$$g(z',p'),\ h(z',p') \geq 0. \tag{4}$$

The problem maximizes the value for the investor subject to the value promised to the entrepreneur. In addition to the incentive-compatibility, which must be satisfied for all possible value of s', and the promise-keeping constraints, we also impose the non-negativity of consumption and continuation utility. These are limited liability constraints.

The following proposition characterizes some properties of the optimal contract.

Proposition 1 *The optimal policies for next period consumption and continuation utility depend only on z', not p'.*

Proof 1 *See Appendix A.*

Therefore, the contract is fully indexed to nominal price fluctuations. The intuition behind this result is simple. What affects the incentive to divert is the 'real' value of the cash revenues. But the real value of revenues depends on z', not p'. Although z' is not observable when the entrepreneur decides whether or not to divert, conditioning the payments on the ex-post inference of z' is sufficient to discipline the entrepreneur. Therefore, we can rewrite the optimal policies as $c' = g(z')$ and $q' = h(z')$.

3.1 Rewriting the optimization problem

It will be convenient to define $u(z') = g(z') + h(z')$ as the next period utility before consumption. Then, imposing the property that the policies of the optimal long-term contract depend only on z', not p', the optimization problem can be split in two sub-programs. The first program optimizes over the input of capital and the total next period utility for the entrepreneur, that is,

$$V(q) = \max_{k,\, u(z')} \left\{ -k + \delta E\Big[z'k^\theta + W(u(z')) \Big] \right\} \tag{5}$$

subject to

$$E\Big[u(z') \,|\, s'\Big] \geq E\Big[\phi\, z'k^\theta + u(0) \,|\, s'\Big]$$

$$q = \beta E u(z')$$

$$u(z') \geq 0$$

The second program determines how the total utility u' will be delivered with immediate or future payments, that is,

$$W(u') = \max_{c',\, q'} \left\{ -c' + V(q') \right\} \tag{6}$$

subject to

$$u' = c' + q'$$

$$c', q' \geq 0$$

This is the problem solved at the end of the next period, after observing p' and, indirectly, z'.

Proposition 2 *There exists $\underline{q}$ and $\bar{q}$, with $0 < \underline{q} < \bar{q} < \infty$, such that $V(x)$ and $W(x)$ are continuously differentiable, strictly concave for $x < \bar{q}$, linear* *for $x > \bar{q}$, strictly increasing for $x < \underline{q}$ and strictly decreasing for $x > \underline{q}$. The entrepreneur's consumption takes the form:*

$$c' = \begin{cases} 0 & \text{if} \quad u' < \bar{q} \\ \\ u' - \bar{q} & \text{if} \quad u' > \bar{q} \quad \text{and} \quad \beta < \delta \\ \\ \text{Any value in } [0, u' - \bar{q}] & \text{if} \quad u' > \bar{q} \quad \text{and} \quad \beta = \delta \end{cases}$$

Proof 2 *See Appendix B.*

The typical shape of the value function characterized in the proposition is shown in Figure 2. To understand these properties we should think about q as the entrepreneur's net worth. Because of incentive compatibility, together with the limited liability constraint, the input of capital is constrained by the entrepreneur's net worth. As the net worth increases, the constraints are relaxed and more capital can be invested. This can be seen more clearly by integrating the incentive compatibility constraint over s' and eliminating $Eu(z')$ using the promise-keeping constraint. This gives the condition:

$$\frac{q}{\beta} \geq \phi \bar{z} k^{\theta} + u(0).$$

where $\bar{z} = Ez'$ is the mean value of productivity.

Because $u(0)$ cannot be negative, k must converge to zero as q converges to zero. Then for very low values of q the input of capital is so low and the marginal revenue is so high that marginally increasing the value promised to the entrepreneur leads to an increase in revenues bigger than the increase in q. Therefore, the investor would also benefit from raising q. This is no longer the case once the promised value has reached a certain level ($q \geq \underline{q}$). At this point the value function becomes downward sloping.

The concavity property derives from the concavity of the revenue function. However, once the entrepreneur's value has become sufficiently large ($q > \bar{q}$), the firm is no longer constrained to use a suboptimal input of capital. Thus, further increases in q will not change k, but only involve a redistribution of wealth from the investor to the entrepreneur. The value function will then become linear.

As stated in the proposition, the payments to the entrepreneur (entrepreneur's consumption) is unique only if $\beta < \delta$. In the case of $\beta = \delta$, c and q are not uniquely determined when $u' > \bar{q}$. However, they are determined for $u' \leq \bar{q}$.

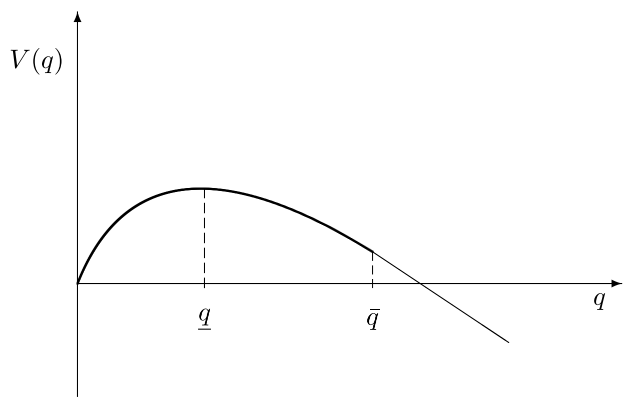

Figure 2: Value of the contract for the investor.

3.2 The long-term contract is not renegotiation-proof

The optimal long-term contract characterized above assumes that the parties commit not to renegotiate in future periods even if changing the terms of the contract could be beneficial, ex-post, for both of them. Obviously, this is a very strong assumption. In this section we show that both parties could benefit from changing the terms of the contracts in later periods or stages. In other words, the optimal long-term contract is not free from (consensual) renegotiation.

Consider the optimal policies for the long-term contract $c' = g(z')$ and $q' = h(z')$. The utility induced by these policies after the observation of s' (and after the choice of diversion) is:

$$\bar{u}' = E\Big[g(z') + h(z') \mid s'\Big] \equiv f(s').$$

Now suppose that, after the realization of s' but before observing p', we consider changing the terms of the contract in a way that improves the investor's value but does not harm the entrepreneur. That is, the value received by the entrepreneur is still $\bar{u}'$. The change is only for one period and then we revert to the long-term contract. In doing so, we solve the following problem:

$$\overline{W}(s', \bar{u}') \quad = \quad \max_{u(z')} E\Big[W(u(z')) \mid s'\Big] \tag{7}$$

subject to

$$\bar{u}' = E\Big[u(z') \mid s'\Big]$$

where $W(.)$ is the value function with commitment defined in (6).

Notice that the optimization problem is now conditional on s' because it is solved after observing the revenues. At this point the agency problem is no longer an issue in the current period since the entrepreneur has already made the decision to divert. Therefore, we do not need the incentive-compatibility constraint. The solution is characterized in the following proposition.

Proposition 3 *If $\bar{u} < \bar{q}$, the solution to problem (7) does not depend on z', that is, $u(z') = \bar{u}'$.*

Proof 3 *Proposition 2 has established that the value function $W(x)$ is strictly concave for $x < \bar{q}$. Therefore, given the promise-keeping constraint $\bar{u} = E[u(z')|s']$, the expected value of $W(u(z'))$ is maximized by choosing a constant value of utility, that is, $u(z') = \bar{u}'$ for all z'.* *Q.E.D.*

This property derives from the concavity of $W(.)$. Because at this stage the incentive problem has already been solved (the entrepreneur has already reported the non-diverted revenues), the expected value of $W(u(z'))$ is maximized by choosing a constant value of utility. Because the optimal $u(z')$ in the long-term contract depends on z', Proposition 3 establishes that this contract is not free from renegotiation. In other words, the the parties would gain from erasing the dependence of the entrepreneur's utility on the true realization of z'.

There is another reason why the optimal long-term contract is not free from renegotiation, even if there is no lag in the observation of the price level. After a sequence of bad shocks, the value of q approaches the lower bound of zero. But low values of q also imply that k approaches zero. Given the structure of the production function, the marginal productivity of capital will approach infinity. Under these conditions, increasing the value of q—that is, renegotiating the contract—will also increase the value for the investor. Essentially, for low values of q the function $V(q)$ is increasing in q, as established in Proposition 2. The proof of this proposition also shows that, if $\beta < \delta$, the increasing segment of the value function will be reached with probability 1 at some future date. When $\beta = \delta$, the renegotiation interval will be reached with a positive probability if the current q is smaller than $\bar{q}$. Therefore, the long-term contract could be renegotiated.

4 The renegotiation-proof contract

Proposition 3 established the important result that any policy that makes the promised utility dependent on z' will be renegotiated. Anticipating this, the contract that is free from renegotiation can only make the promised utility dependent on s', not on z'. This implies that the real payments associated with the renegotiation-proof contract depend on *nominal* quantities. As we will see, this also implies that nominal price fluctuations have real effects.

Consider the following problem:

$$V(q) = \max_{k,u(s')} \left\{ -k + \delta E\Big[z'k^\theta + W(u(s'))\Big]\right\} \tag{8}$$

subject to

$$u(s') \geq \phi E\Big[z'k^\theta \mid s'\Big] + u(0), \qquad \forall\, s'$$

$$q = \beta E u(s')$$

$$u(s') \geq \underline{u}$$

where $W(.)$ is again defined in (6). We have imposed that future utilities can be contingent only on s' since any dependence on z' will be renegotiated after observing s'. Furthermore, we have also imposed that they cannot take a value smaller than $\underline{u}$. As argued in the previous section, the contract may not be free from renegotiation because the value function is strictly increasing for low value of q (see Proposition 2). As shown in Quadrini (2004) and Wang (2000), renegotiation-proof is achieved by imposing a lower bound on the promised utility. This bound, denoted by $\underline{u}$, is endogenously determined. For the moment, however, we take $\underline{u}$ as exogenous and solve Problem (8) as if the parties commit not to renegotiate.

We establish next a property that will be convenient for the analysis that follows.

Lemma 1 *The incentive-compatibility constraint is satisfied with equality.*

Proof 1 *This follows directly from the concavity of the value function. If the incentive compatibility constraint is not satisfied with equality, we can find an alternative policy for $u(s')$ that provides the same expected utility (promise-keeping) but makes next period utility less volatile, and allows for a higher input of capital. The concavity of $W(.)$ implies $EW(u(s'))$ will be higher under the alternative policy.* *Q.E.D.*

Using this result, we can combine the incentive-compatibility constraint with the promise-keeping constraint and rewrite the optimization problem as follows:

$$V(q) = \max_{k} \left\{ -k + \delta E\Big[z'k^{\theta} + W(u')\Big]\right\} \tag{9}$$

subject to

$$u' = \phi\Big[E(z' \mid s') - \bar{z}\Big]k^{\theta} + \frac{q}{\beta} \tag{10}$$

$$\frac{q}{\beta} - \phi\bar{z}k^{\theta} \geq \underline{u} \tag{11}$$

where $\bar{z} = Ez'$ is the mean value of productivity.

The first constraint defines the law of motion for the next period utility while the second ensures that this is not smaller than the lower bound $\underline{u}$. See Appendix C for the derivation of these two constraints.

Proposition 4 *There exists $\underline{u} > 0$ such that the solution to problem (9) is free from renegotiation.*

Proof 4 *See Appendix D.*

The lower bound $\underline{u}$ ensures that the utility promised to the entrepreneur does not reach the region in which the promised utility would be renegotiated ex-post. This is at the point in which the derivative of the value function is zero, that is, $V_q(q = \underline{u}) = 0$. Therefore, changing the value promised to the entrepreneur does not bring, at the margin, either gains or losses to the investor.

4.1 First order conditions

Denote by $\delta\mu$ the Lagrange multiplier for constraint (11). The first order conditions are:

$$\delta\theta k^{\theta-1}\left[\bar{z}(1-\phi\mu)+\phi E\Big(E(z'|s')-\bar{z}\Big)W_{u'}\right]=1, \tag{12}$$

$$W_{u'}=\max\Big\{V_{q'},-1\Big\}, \tag{13}$$

and the envelope condition is:

$$V_q=\left(\frac{\delta}{\beta}\right)\Big(EW_{u'}+\mu\Big) \tag{14}$$

The investment k is determined by equation (12). If the entrepreneur does not gain from diversion, that is, $\phi=0$, we have the frictionless optimality condition for which the discounted expected marginal productivity of capital is equal to the marginal cost. Notice that with $\phi=0$, constraint (11) will not be binding and $\mu=0$. When $\phi>0$, however, the investment policy will be distorted.

Before continuing, it will be instructive to compare the first order conditions for the renegotiation-proof contract with those for the long-term contract, that is, the optimality conditions derived from Problem (1). In this case we obtain:

$$\delta\theta k^{\theta-1}\left[\bar{z}(1-\phi\mu)+\phi E\Big(z'-\bar{z}\Big)W_{u'}\right]=1 \tag{15}$$

$$W_{u'}=\max\Big\{V_{q'},-1\Big\}, \tag{16}$$

with the envelope condition given in (14).

The comparison of conditions (12) and (15) illustrates how the lack of indexation in the renegotiation-proof contract affects the dynamics of the firm. First notice that the optimality conditions are very similar with the exception of the term z' replacing $E(z'|s')$ for the long-term contract. If there is no price uncertainty, then $E(z'|s')=z'$, and the renegotiation-proof contract is equivalent to the long-term contract, with the exception of the lower bond $\underline{u}$.

Consider first the long-term contract. The term $W_{u'}$ is typically negative and decreasing (due to the concavity of $W(.)$). Thus, $E(z'-\bar{z})W_{u'}$ is negative.

So in general, the input of capital is reduced by a higher volatility of z'. Another way to say this is that capital investment is risky for the investor because a higher k requires a more volatile u' to create the right incentives (see equation (10)). Because the value of the contract for the investor is concave, a higher volatility of u' reduces the contract value.

Now consider nominal price uncertainty. The long-term contract is not affected by nominal price uncertainty since the contract is fully indexed. The renegotiation-proof contract, instead, is not fully indexed. What this implies is that price uncertainty reduces the dependence of the entrepreneur's (expected) value of diversion from the realization of revenues. This is because, with price uncertainty, revenues provide less information about the true value of z' (which ultimately determines the value of diversion). Therefore, the distortions in the choice of capital could be less severe. However, the promised utility will now depend on price fluctuations. Therefore, an unanticipated change in nominal price will impact the promised utility of all firms, with potential aggregate consequences for investment.

4.2 Equilibrium with renegotiation-proof contracts

The equilibrium is defined under the assumptions that there is a unit mass of entrepreneurs or firms, and that investors have unlimited assess to funds (so that the interest rate is constant). The equilibrium is characterized by a distribution of firms over the entrepreneur's value q. The support of the distribution is $[\underline{u}, \bar{q}]$. Because of nominal price fluctuations, the distribution never converges to a steady state distribution. Only in the limiting case of $\sigma_p = 0$ (absence of nominal price uncertainty), the distribution of firms converges to an invariant distribution.

Within the distribution, firms move up and down depending on the realization of the idiosyncratic productivity z and the nominal price level p. A firm moves up in the distribution when it experiences a high value of z (unless it has already reached $q = \bar{q}$), and moves down when the realization of z is low (unless the firm is at $q = \underline{u}$). The idiosyncratic nature of productivity ensures that at any point in time some of the firms move up and others move down. An unexpected nominal price shock, instead, impacts all firms in a monotonic fashion.

5 Monetary policy regimes and indexation

We can use the results established in the previous section to characterize how inflation shocks affect the economy under different monetary regimes. In this framework, monetary regimes are fully characterized by the volatility of the price level, σ_p. Therefore, we will use the terms 'monetary regime' and 'price level uncertainty' interchangeably.

We are interested in asking the following question: suppose that there is a one-time unexpected increase in the price level (inflation shock); how would this shock impact economies with different degrees of aggregate price uncertainty σ_p?

The channel through which the monetary regime affects the financial contract is by changing the expected value of z' given the observation of s', that is $E(z'|s')$. This can be clearly seen from the law of motion of next period utility, equation (10), and from the first order condition (12). As is well known from signaling models, the greater the volatility of the signal, the less information the signal provides. The assumption that $\tilde{p} = \log(p)$ and $\tilde{z} = \log(z)$ are normally distributed allows us to show this point analytically.

Agents start with a prior about the distribution of $\tilde{z}'$, which is the normal distribution $N(\mu_z, \sigma_z^2)$. They also have a prior about $\tilde{s}' = \tilde{z}' + \tilde{p}'$, which is also normal $N(\mu_z + \mu_p, \sigma_z^2 + \sigma_p^2)$. What we want to derive is the posterior distribution of $\tilde{z}'$ after the observation of $\tilde{s}'$. Because the prior distributions for both variables are normal, the posterior distribution of $\tilde{z}'$ is also normal with mean:

$$E(\tilde{z}'|\tilde{s}') = \frac{\sigma_p^2}{\sigma_z^2 + \sigma_p^2}\mu_z + \frac{\sigma_z^2}{\sigma_z^2 + \sigma_p^2}(\tilde{s}' - \mu_p), \tag{17}$$

and variance:

$$Var(\tilde{z}'|\tilde{s}') = \frac{\sigma_z^2 \sigma_p^2}{\sigma_z^2 + \sigma_p^2}. \tag{18}$$

This derives from the fact that the conditional distribution of normally distributed variables is also normal.[3]

Expression (17) makes clear how the volatility of nominal prices, σ_p, affects the expectation of $\tilde{z}'$ given the realization of revenues. In particular, the contribution of $\tilde{s}'$ to the expectation of $\tilde{z}'$ decreases as the volatility of prices increases. In the limiting case in which $\sigma_p = \infty$, $E(\tilde{z}'|\tilde{s}') = \mu_z$. Therefore,

[3] A formal proof can be found in Greene (1990, pp. 78-79). It can also be shown that the covariance between $\tilde{z}$ and $\tilde{p}$ is σ_z^2.

the observation of $\tilde{s}'$ does not provide any information about the value of $\tilde{z}'$. Given this, the law of motion for the next period utility, equation (10), converges to $u' = q/\beta$. Hence, in the limit, the next period utility does not depend on s', that is, the contract becomes fully indexed. Of course, if u' does not depend on s', the contract is not incentive compatible. But this is just a limiting result. With finite values of σ_p, the next period utility does depend on s' but the sensitivity declines with σ_p.

Proposition 5 *Consider a one-time unexpected increase in price Δp. The impact of the shock on the next period promised utility strictly decreases in σ_p and converges to zero as $\sigma_p \to \infty$.*

Proof 5 *See Appendix E.*

The intuition behind this property is simple. When $\sigma_p = 0$, agents interpret an increase in nominal revenues induced by the change in the price level as being derived from a productivity increase, not a price increase. Therefore, the utility promised to the entrepreneur (and therefore, the discounted value of real payments) has to increase in order to prevent diversion. But in doing so, the promised utility increases on average for the whole population. Essentially, the inflation shock redistributes wealth from investors to entrepreneurs. As entrepreneurs become wealthier, the incentive-compatibility constraints are relaxed in the next period and this allows for higher aggregate investment. For positive values of σ_p, however, increases in revenues induced by nominal price shocks are interpreted to a lesser extent as changes in z. As a result, the next period utilities will increase by less on average.

This result suggests that economies with volatile nominal prices are less vulnerable than economies with more stable monetary regimes to the *same* price level shock. However, this does not mean that economies with more volatile prices display lower volatility overall because shocks are larger on average. Ultimately, how different monetary policy regimes affect the business cycle is a quantitative question. But, a-priori we cannot say whether countries with more volatile inflation experience greater or lower macroeconomic instability. This point will be illustrated numerically.

6 Heterogeneous impact of unexpected inflation

As we have already pointed out, the model generates firms's heterogeneity depending on the financial conditions they face. Because all firms have access to the same technology, the financial condition of the firm is identified by the variable q, which can be interpreted as net worth. Lower values of q imply tighter financial conditions which result in lower scales of production because more capital increases the value of repudiation. If q is low (low net worth), the investor is not willing to finance the optimal input of capital. In this section we show that the impact of unexpected inflation is stronger for firms with tighter financial conditions.

The easiest way to show that firms with tighter financial conditions are more vulnerable to surprise inflation is in the case with $\beta = \delta$. In this particular version of the model, firms will eventually reach $q = \bar{q}$ and stay there forever. Therefore, in order to have a non-degenerate steady state distribution of firms we need entry and exit. For example, we could assume that firms exit with some exogenous probability and there is a new mass of firms entering in every period.[4] The new firms are created by entrepreneurs with zero net worth. Therefore, the initial state of the contract will be $\underline{u}$.

With the addition of exogenous exit the optimal contract is essentially the same. However, at any point in time a fraction of firms have $q < \bar{q}$ and the remaining fraction have $q \geq \bar{q}$. The first group of firms face tight financial constraints and operate with a suboptimal input of capital while the second are unconstrained and operates at the optimal scale.

Proposition 6 *Suppose that $\beta = \delta$ and consider a one-time unexpected increase in price Δp. The shock affects only the next period investment of firms with $q < \bar{q}$.*

Proof 6 *The proof is obvious from the discussion above. Once firms have reached the state $q \geq \bar{q}$, their contract value will never fall below $\bar{q}$. Therefore, they will not change the next period input of capital.*

In general, if we think that tight constraints are more likely for young firms (because they have not been around long enough to reach $\bar{q}$) and small firms (because they have been unlucky and pushed back by a sequence of negative shocks), then the model predicts that younger and smaller firms are more vulnerable to unexpected inflation shocks.

[4] This is also the assumption made in Clementi & Hopenhayn (2006), Li (2008) and Quadrini (2004). In these papers there is also endogenous exit. However, the probability of endogenous exiting becomes zero once they reach $\bar{q}$.

Although it cannot be proved analytically, the sensitivity of next period capital (relative to current capital) for firms with $q < \bar{q}$ decreases in q. As q and k increase, the firm gets closer to the unconstrained state. Thus, the benefits from an increase in q are smaller because firms with higher q are more likely to exceed $\bar{q}$ after a positive shock. But after exceeding $\bar{q}$, inflation no longer matters. This result, which will be shown numerically, also applies to the case with $\beta < \delta$. In this case there is always a mass of firms with $q < \bar{q}$ even if there is not exit.

7 Numerical analysis

This section further characterizes the properties of the economy numerically. Although we do not conduct a rigorous calibration exercise, the numerical analysis allows us to illustrate additional properties that cannot be established analytically but are robust to alternative parameter values.

The model period is a year and the discount factor of the entrepreneur is $\beta = 0.95$. The gross real revenue is $z'k^{\theta}$. The idiosyncratic productivity z is log normally distributed with parameters $\mu_z = 0.125$ and $\sigma_z = 0.5$. The decreasing return to scale parameter θ is set to 0.85.

The market discount rate is set to $\delta = 0.96$, which is higher than the entrepreneur discount factor. The parameter ϕ governs the degree of financial frictions (*i.e.*, the return from diversion) and it is set to $\phi = 1$. This means that the entrepreneur is able to keep the whole hidden cash-flow. The general price level is log normally distributed with parameters $\mu_p = 0.01$ and $\sigma_p = 0.02$. We will also report the results for alternative values of σ_p. For the description of the solution technique see Appendix F.

7.1 Some steady state properties

Assuming that the economy experiences a long sequence of prices equal to the mean value $Ep = e^{\mu_p + \sigma_p^2/2} = \bar{p}$, the economy converges to a stationary equilibrium. With some abuse of terminology, we will refer to the stationary equilibrium as 'steady state'. Notice that, even if the realized prices are always the same, agents do not know this in advance, and therefore, they assume that the price level is stochastic and form expectations accordingly.

Panel (a) of Figure 3 reports the decision rule for investment as a function of the entrepreneur's value q in the limiting equilibrium that we termed steady state. Investment k is an increasing function of q. For very high values of q, the capital input is no longer constrained, and therefore, k reaches the optimal scale which is normalized to one.

Panel (b) plots the distribution of firms over their size k in the steady state. As Panel (a) shows, some firms will ultimately reach the highest size. Even if some of them will be pushed back after a negative productivity shock, there is always a significant mass in the largest class.

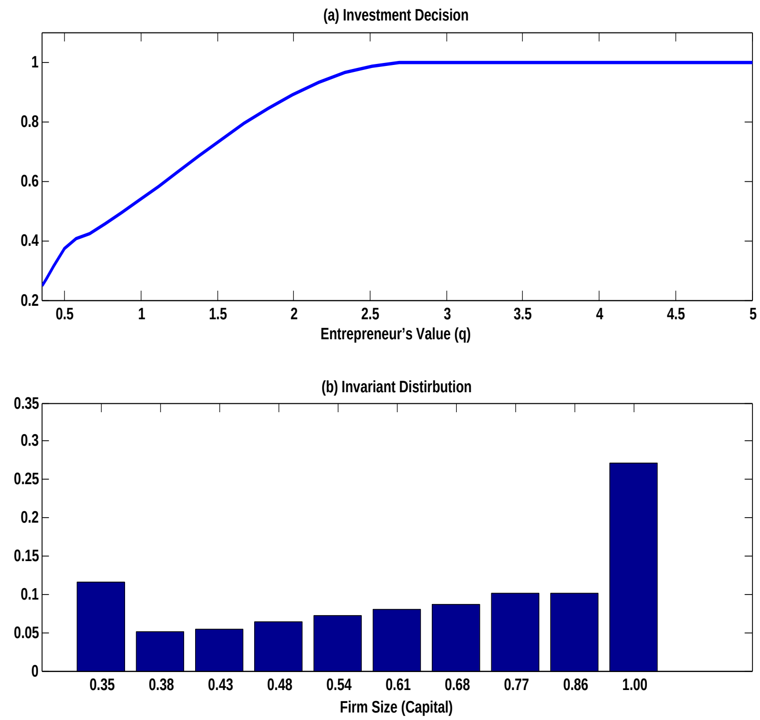

Figure 3: Investment Decision Rule and Firm Invariant Distribution

7.2 Degree of indexation

The central feature of the model is that the degree of indexation depends on nominal price uncertainty. If financial contracts were fully indexed, then a price shock would not affect the values that the entrepreneur and the investor receive from the contract. On the other hand, if contracts were not indexed, a price shock would generate a redistribution of wealth. For example, if entrepreneurs borrow with standard debt contracts that are nominally denominated (instead of using the optimal contracts characterized here), an unexpected increase in the price level redistributes wealth from the investor (lender) to the entrepreneur. Therefore, a natural way to measure the degree of indexation is the elasticity of the next period entrepreneur's value—the promised utility u'—with respect to a nominal price shock.

From equation (10) we can see that the next period promised utility is given by:

$$u' = \phi\Big[E(z' \mid \tilde{z}' + \tilde{p}') - \bar{z}\Big]k^{\theta} + \frac{q}{\beta}$$

We want to determine the change in u' following a deviation Δp in the nominal price from its mean value. For each realization of the idiosyncratic productivity $\tilde{z}'$, this is equal to:

$$\Delta u' = \phi k^{\theta}\Big\{E(z' \mid \tilde{z}' + \mu_p + \Delta\tilde{p}) - E(z' \mid \tilde{z}' + \mu_p)\Big\}$$

Integrating over all possible realizations of $\tilde{z}'$ using the unconditional distribution $N(\mu_z, \sigma_z^2)$, we get the average value $E_{\tilde{z}'}\Delta u'$ for a firm of type q. The elasticity measure is then obtained by dividing this term by $\phi k^{\theta} E_{\tilde{z}'}\{E(z' \mid \tilde{z}' + \mu_p)\} + q/\beta$, that is, the average u' for a firm of type q if $\tilde{p}$ is equal to its mean μ_p.

Interpreting the next period value of the contract for the entrepreneur as the net worth of the firm, the financial contract would be fully indexed when the elasticity is zero. In this case, the net worth is indeed insulated from inflation shocks. If the elasticity is different from zero, the financial contract is imperfectly indexed.

Figure 4 plots the elasticity as a function of the current value of the firm (current promised utility q). The elasticity is computed for a 25 percent increase in the nominal price.

The first feature shown by the figure is that the optimal contract is not fully indexed: for any q, a positive inflation shock redistributes wealth to the firm.

The second feature is that the degree of indexation increases with the entrepreneur's value, and therefore, with the size of the firm. Because the next period entrepreneur's value affects next period investment in a monotonic relation that is close to linear (see Figure 3), this implies that the investment of constrained firms is more vulnerable to inflation shocks.

Table 1 presents the overall degree of indexation in an economy with low nominal price uncertainty ($\sigma_p = 0.02$) and with high nominal price uncer-

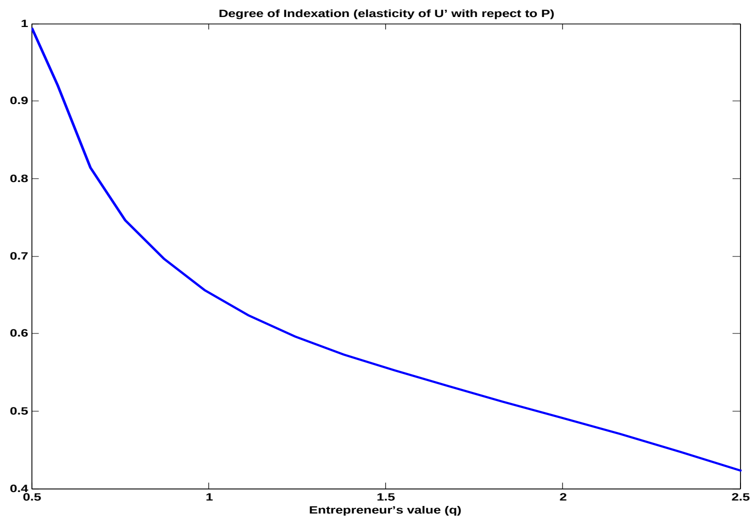

Figure 4: Degree of Indexation as a Function of Entrepreneur's Value (q)

tainty ($\sigma_p = 1.5$). The aggregate degree of indexation is computed by adding the elasticity of each firm of type q weighted by the steady state distribution and for a 25 percent increase in the nominal price.

Table 1: Degree of Indexation for Different Price Level Uncertainty

	Elasticity
Low price uncertainty ($\sigma_p = 0.02$)	0.992
High price uncertainty ($\sigma_p = 1.5$)	0.115

As can be seen from the table, the degree of indexation increases with price uncertainty. For example, when $\sigma_p = 0.02$, the elasticity is almost 1 while it is only in the order of 0.1 when $\sigma_p = 1.5$. What this implies is that, when prices are very stable, an unexpected increase in the nominal price of 1 percent leads to almost a 1 percent increase in the net worth of the firm. Conversely, when there is high price uncertainty, a 1 percent increase in the nominal price leads only to a 0.1 percent increase in the firm's net worth. The result that the degree of indexation is higher in economies with high nominal price uncertainty is consistent with the experience of several countries such as Brazil and Argentina where price uncertainty has been high and indexation widely diffuse.

7.3 Aggregate investment, output and price level uncertainty

Table 2 presents aggregate capital and output for economies with low and high price level uncertainty. The table highlights that the stock of capital is bigger when price level uncertainty is higher.

Table 2: Aggregate Capital and Output for Different Price Level Uncertainty

	Capital	*Output*
Low price uncertainty ($\sigma_p = 0.02$)	0.644	0.835
High price uncertainty ($\sigma_p = 1.5$)	0.963	1.187

This finding arises from the characteristics of the contractual frictions. When the price level is very volatile, the observation of the nominal revenues by the firm in the first stage of the period does not provide enough information about the actual value of productivity z'. The signal becomes noisier and the information content of the signal is smaller. This implies that the incentive to divert is not affected significantly by the observation of revenues. Because of this, the value of the contract for the entrepreneur is less volatile and the distribution of firms over k is more concentrated around the optimal investment.

This finding may appear to conflict with the fact that countries with monetary policy regimes that feature greater nominal price uncertainty are also countries with lower output per-capita. However, it is also plausible to assume that in these countries the contractual frictions, captured by the parameter ϕ, are higher than in rich countries. As we will see later, more severe contractual frictions may offset the impact of greater price level uncertainty on capital accumulation.

7.4 Heterogeneous response to inflation shocks

The impulse responses to a nominal price shock are computed assuming that the economy is in the steady state when the shock hits. As before, we define a steady state as the limiting equilibrium to which the economy converges after the realization of a long sequence of prices equal to the mean value $Ep = e^{\mu_p + \sigma_p^2/2} = \bar{p}$ (although agents do not know this sequence in advance and they take into account price uncertainty in their decisions).

Starting from this equilibrium, we assume that the economy is hit by a one-time price level shock. After the shock, future realizations of p revert to the mean value $\bar{p}$ and the economy converges back to the same steady state.

We start examining the response of different size classes of firms concentrating on two groups: (i) firms that are currently at $q = \overline{q}$; and (ii) firms that are at $q < \overline{q}$. We label the first group 'large firms' and the second group 'small firms'. Figure 5 plots the average capital of firms with $q < \overline{q}$ (small firms) and $q = \overline{q}$ (large firms) in response to an unexpected one-time increase in the nominal price level.

The top panels of Figure 5 show that the average (per-firm) capital of large firms does not change in response to the same since these firms are able to implement the optimal investment. However, the shock has a positive effect on the average (per-firm) size of small firms, that is, constrained firms become larger on average. This effect is much bigger when there is low price uncertainty.

The bottom panels of Figure 5 plot the response of the fraction of large (unconstrained) and small (constrained) firms. The relative mass of large firms increases after the shock. As for the average firm size, the effect is much bigger when price uncertainty is low.

In summary, an unexpected rise in the nominal price increases the average size of constrained firms and the mass of unconstrained firms. Both effects contribute to increasing aggregate investment and capital.

7.5 Aggregate response to inflation shocks

Figure 6 presents the dynamics of aggregate capital after a one-time increase in the nominal price level when the price uncertainty is low ($\sigma_p = 0.02$) and high ($\sigma_p = 1.5$). The aggregate capital increases at impact and slowly converges to the initial level. Although the shock is temporary, the effect is persistent. As discussed above, this follows from the fact that a larger

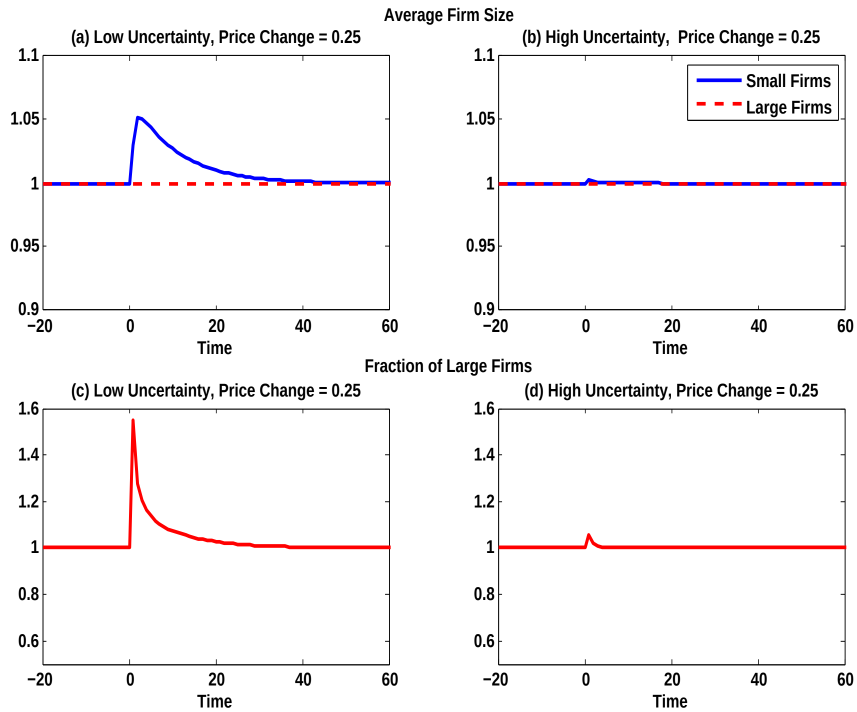

Figure 5: Responses of Average Firm Size and the Relative Number of Small and Large Firms to a Positive Price Level Shock in Regimes with Different Price Level Uncertainty.

number of firms become unconstrained and the average size of constrained firms increases. The aggregate impact of the shock, however, becomes quite small when price uncertainty is very high. This follows from the fact that, with high nominal price uncertainty, contracts are characterized by a high degree of nominal indexation. Thus, the nominal price shock has a small redistributive effect.

Figure 6 suggests that countries with a monetary policy regime that is characterized by low nominal price uncertainty are more vulnerable than countries with greater price uncertainty to the *same* nominal price shock. However, countries with greater price uncertainty experience on average larger shocks. This leads to the following question: Are economies with low price uncertainty more unstable than economies with high price uncertainty? To answer this question, we conduct a simulation exercise for several

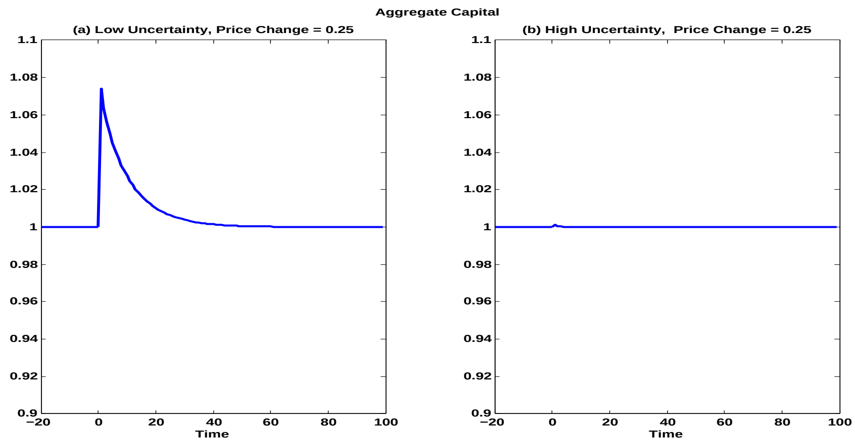

Figure 6: Response of Aggregate Capital to a Positive Nominal Price Shock in Regimes with Different Price Uncertainty.

economies that differ only in the volatility of the price level, σ_p. Each economy is simulated for 20,000 periods. We report the standard deviation of investment and output in Table 3.

Table 3: Volatility of Investment and Output for Different Nominal Price Uncertainty

Price-Level Uncertainty	*Standard Deviation of Capital*	*Standard Deviation of Output*
$\sigma_p = 0.02$	0.008	0.009
$\sigma_p = 0.20$	0.073	0.082
$\sigma_p = 1.50$	0.134	0.147
$\sigma_p = 1.70$	0.120	0.130

Before discussing the results, it is useful to describe intuitively how the volatility of investment and output changes when σ_p increases. There are two opposing effects. On the one hand, a high σ_p reduces the volatility of investment since the contracts are more indexed. On the other, a higher σ_p implies that on average the economy experiences larger price shocks.

Table 3 shows that these two opposing forces lead to a non-monotonic relation between nominal price uncertainty and the volatility of investment and output. For low or moderate values of σ_p, the volatility of investment increases with σ_p. This means that the fact that the economy experiences larger shocks dominates the lower elasticity to each shock (greater indexation). However, for high values of σ_p, the volatility of investment decreases with σ_p, implying that the higher degree of indexation more than offsets the increase in the magnitude of the price shocks. Recall from the previous analysis that the economy converges to full indexation as σ_p converges to ∞.

7.6 Price-level uncertainty and financial development

In this section we discuss how the interaction between nominal price uncertainty and the degree of financial development affects the level and volatility of the real economy. In our model the degree of financial development is captured by the parameter ϕ. A high value of ϕ corresponds to a less developed financial system since firms can gain more from the diversion of resources.

In the previous experiments, ϕ was set to 1. In this section we will compare the previous results with an alternative economy where $\phi = 0.5$. We think of the economy with $\phi = 0.5$ as an economy with a 'more developed financial system'. The standard deviations of aggregate capital and output are reported in Table 4.

As expected, investment is lower when financial markets are less developed. This is because when ϕ is high, financial constraints are tighter and, as result, investment is lower on average. We can also see that investment, for a given level of price uncertainty, is more volatile in the economy with a less developed financial system.

How can we interpret these results? We know that some of the low income countries experience high volatility of inflation. As we have seen in Table 2, our model predicts that these countries should have a higher stock of capital (after controlling for the technology level of these countries). At the same time, they are also likely to face more severe contractual frictions which, according to our model, induce a lower stock of capital. If the impact of financial development dominates the impact of greater price uncertainty, the model still predicts that poorer countries have less capital as in the data.

The finding of this section can also be interpreted along normative point of view: nominal price uncertainty could be welfare improving in countries with lower financial development since it offsets the negative impact of limited

Table 4: Standard deviation of investment and aggregate investment for different degree of financial development and price-level uncertainty.

	More developed financial system ($\phi = 0.50$)	*Less developed financial system* ($\phi = 1.00$)
Low Price Level Uncertainty ($\sigma_p = 0.02$)		
Aggregate Capital	0.803	0.644
Standard Deviation of Capital	0.006	0.008
Moderate Price-Level Uncertainty ($\sigma_p = 0.20$)		
Aggregate Capital	0.812	0.658
Standard Deviation of Capital	0.050	0.073
High Price-Level Uncertainty ($\sigma_p = 1.5$)		
Aggregate Capital	0.984	0.963
Standard Deviation of Capital	0.092	0.134
Extreme Price-Level Uncertainty ($\sigma_p = 1.70$)		
Aggregate Capital	0.986	0.955
Standard Deviation of Capital	0.085	0.130

contract enforcement on capital accumulation.

8 Conclusion

In this paper we have studied a model with repeated moral hazard where financial contracts are not fully indexed to inflation because, as in Jovanovic & Ueda (1997), the nominal price level is observed with delay.

Nominal indexation is endogenously determined in the model and heterogeneous across firms. In particular, we find that more constrained firms operate under contracts with a lower degree of nominal indexation and they are more vulnerable to inflation shocks. As a result, the impact of inflation shocks on aggregate investment and output depends on the extent of financial markets frictions.

Another finding is that the overall degree of nominal indexation increases with price uncertainty. An implication of this is that economies with higher price uncertainty are less vulnerable to a given inflation shock, that is, investment and output respond less. However, these economies experiences larger shocks on average. Therefore, they may still face higher macroeconomic volatility.

Appendix

A Proof of Proposition 1

To simplify the proof we make a change of variables in Problem (1). Define $y = k^{\theta}$. After substituting $k = y^{\frac{1}{\theta}}$, the optimization problem becomes:

$$V(q) = \max_{y,\, g(z',p'),\, h(z',p')} \left\{ - y^{\frac{1}{\theta}} + \delta E\Big[z'y - g(z',p') + V(h(z',p')) \Big] \right\} \quad (19)$$

subject to

$$E\Big[g(z',p') + h(z',p') \mid s' \Big] \geq E\Big[\phi\, z'y + g(0,p') + h(0,p') \mid s' \Big] \quad (20)$$

$$q = \beta E\Big[g(z',p') + h(z',p') \Big] \quad (21)$$

$$g(z',p'),\; h(z',p') \geq 0. \quad (22)$$

The change of variables makes the incentive-compatibility constraint linear in all the decision variables. It is then easy to show that this is a well defined concave problem and (19) satisfies the Blackwell conditions for a contraction mapping. Therefore, there is a unique fixed point V^*. The mapping preserves concavity. This implies that the fixed point for V^* is concave, although not necessarily strictly concave.

Consider a particular solution $S_1 \equiv \{y_1, g_1(z',p'), h_1(z',p')\}$, where the next period consumption and continuation utility are dependent on both z' and p'. Now consider the alternative solution $S_2 \equiv \{y_2, g_2(z'), h_1(z')\}$, where $y_2 = y_1,\; g_2(z') = \int_{p'} g_1(z',p')dF(p'),\; h_2(z') = \int_{p'} h_1(z',p')dF(p')$. In the alternative solution, the next period consumption and continuation utility are contingent only on z', not p'.

We can verify that, if S_1 satisfies all the constraints to problem (19), then the constraints are also satisfied by S_2. Therefore, S_2 is a feasible solution. The next step is to show that S_2 provides higher value than S_1. This follows directly from the concavity of the value function. Essentially, by choosing S_2 we make the next period utility less volatile and increase $EV(h(z',p'))$. *Q.E.D.*

B Proof of Proposition 2

In the proof of Proposition 1, we established that the value function is concave (although not strictly). By verifying the condition of Theorem 9.10 in Stokey, Lucas, & Prescott (1989), we can also establish that the value function is differentiable.

Consider the incentive-compatibility constraint $E[u(z')|s'] \geq \phi E(z'|s')y + u(0)$ and the promise-keeping constraint $q = \beta Eu(z')$. The IC constraint can be integrated over p' to get $Eu(z') \geq \phi \bar{z} y + u(0)$. Remember that we have made the change of variable $y = k^{\theta}$. Using this condition with the promise-keeping constraint we can write:

$$q = \beta Eu(z') \geq \beta \phi \bar{z} y \tag{23}$$

This says that, as q converges to zero, y (and therefore $k = y^{\frac{1}{\theta}}$) also converges to zero. This also implies that the marginal cost of y converges to zero (or equivalently, the marginal productivity of capital converges to infinity). Therefore, starting from a value of q close to zero, by marginally increasing q we can increase the marginal revenue by a large margin, which makes the value of the contract for the investor higher. Therefore the function $V(q)$ is increasing for very low values of q.

Define $\bar{k}$ as the input of capital for which the expected marginal revenue is equal to the interest rate, that is, $\theta k^{\theta-1} = 1/\delta$. Obviously, the input of capital chosen by the contract will never exceed $\bar{k}$.

Now consider a very large q, above the level that makes $\bar{k}$ feasible, that is, condition (23) is satisfied. Because the contract will never choose a value of $k > \bar{k}$, further increases in q will not change the input of capital. This implies that $V(q)$ (the value for the investor) decreases proportionally to the increase in q. Therefore, for q above a certain threshold $\bar{q}$, the value function is linear. Given that the value function is linear for $q > \overline{q}$, it is easy to see from Problem (6) that $c' = u' - \bar{q}$ if $\beta < \delta$. However, if $\beta = \delta$, there are multiple solutions for c'.

Below the threshold $\bar{q}$, however, q does constrain k. The strict concavity of the value function derives from the fact that the revenue function is strictly concave. The optimal policy for c' then becomes obvious. *Q.E.D.*

C Derivation of equations (10) and (11)

Consider the incentive-compatibility constraint

$$u(s') = \phi E(z'|s')k^\theta + u(0). \tag{24}$$

Integrating over s' we get $Eu(s') = \phi E\{E(z'|s')\}k^\theta + u(0)$. Because $E\{E(z'|s')\} = \bar{z}$, this can also be written as:

$$Eu(s') = \phi \bar{z} k^\theta + u(0). \tag{25}$$

Consider now the promise-keeping constraint $q = \beta Eu(s')$. Using equation (25), this can be written as:

$$\frac{q}{\beta} = \phi \bar{z} k^\theta + u(0). \tag{26}$$

Using this to eliminate $u(0)$ in (24) we get:

$$u(s') = \phi\Big[E(z' \mid s') - \bar{z}\Big]k^\theta + \frac{q}{\beta}, \tag{27}$$

which is equation (10).

The lower bound on total utility, $u(s') \geq \underline{u}$, requires $u(0) \geq \underline{u}$. This is because $u(s')$ is increasing in s'. From equation (26) we have that $u(0) = q/\beta - \phi\bar{z}k^\theta$. Therefore, the condition $u(0) \geq \underline{u}$ can be written as:

$$\frac{q}{\beta} - \phi \bar{z} k^\theta \geq \underline{u}, \tag{28}$$

which is equation (11).

D Proof of Proposition 4

See Quadrini (2004).

E Proof of Proposition 5

Consider the law of motion for the next period utility (10) which for convenience we rewrite here:

$$u' = \phi\Big[E(z'|s') - \bar{z}\Big]k^\theta + \frac{q}{\beta} \tag{29}$$

The effect of the shock is to increase $E(z'|s')$ for each realization of z'. For convenience we can focus on the conditional expectation where the variables are expressed in log form, that is, $E(z'|s') = E(e^{\tilde{z}'}|\tilde{s}')$.

Given the distributional assumptions about $\tilde{z}'$ and $\tilde{p}'$, the conditional expectation is equal to:

$$E(e^{\tilde{z}'}|\tilde{s}') = e^{\frac{\sigma_p^2}{\sigma_z^2+\sigma_p^2}\mu_z + \frac{\sigma_z^2}{\sigma_z^2+\sigma_p^2}(\tilde{s}'-\mu_p) + \frac{\sigma_z^2\sigma_p^2}{2(\sigma_z^2+\sigma_p^2)}}$$

Given a realization of the aggregate log-price $\tilde{p}'$ and the idiosyncratic log-productivity $\tilde{z}'$, the firm observes $\tilde{s}' = \tilde{z}' + \tilde{p}'$. We want to compute how a deviation of the log-price from its mean μ_p affects the conditional expectation of firms. More specifically, we want to compare the case in which the observed revenue is $\tilde{s}_1 = \tilde{z} + \mu_p$ with the case in which the revenue is $\tilde{s}_2 = \tilde{z} + \mu_p + \Delta$. This is done by computing the ratio of conditional expectations $E(z|\tilde{s}_2)/E(z|\tilde{s}_1)$. Using the formula for the conditional expectation written above we get:

$$\frac{E(z|\tilde{s}_2)}{E(z|\tilde{s}_1)} = e^{\frac{\sigma_z^2}{\sigma_z^2+\sigma_p^2}\Delta}$$

Therefore, the change in the conditional expectation decreases with σ_p. From the law of motion (29) we can then observe that, for each z, the change in next period utility decreases with σ_p. *Q.E.D.*

F Solution method

The solution is based on the iteration of the unknown function $V_q = \psi(q)$. We create a grid of points for q and guess the value of the function $\psi(q)$ at each grid point. The values outside the grid are joined with step-wide linear functions. The detailed steps are as follows:

1. Create a grid for $q \in \{q_1, ..., q_N\}$.
2. Guess $V_q^i = \psi(q_i)$, for $i = 1, ..., N$.
3. Solve for k and μ at each grid point of q:

    (a) Check first for the binding solution:

    - Solve for k using (11).

- Solve for μ using (12).

(b) If the μ from the binding solution is smaller than zero, the solution is interior. The interior solution is found as follows:

- Set $\mu = 0$.
- Solve for k using (12).

4. Given the solutions for k and μ, find $W_{u'}$ using (13). Then update the guess for the function $\psi(q)$ at each grid point using the envelope condition (14).

5. Restart from step 3 until convergence in the function $\psi(q)$.

References

Atkeson, A. (1991). International lending with moral hazard and risk of reputation. *Econometrica*, *59*(4), 1069–89.

Bernanke, B., Gertler, M., & Gilchrist, S. (1999). The financial accelerator in a quantitative business cycle framework. In Taylor, J. B. & Woodford, M. (Eds.), *Handbook of Macroenomics*, Vol. 1C, chap. 21, pp. 1341–96. North Holland, Amsterdam: Elsevier Science.

Christiano, L. J., Motto, R., & Rostagno, M. (2008). Financial factors in economic fluctuations. Unpublished manuscript, Northwestern University and European Central Bank.

Clementi, G. & Hopenhayn, H. A. (2006). A theory of financing constraints and firm dynamics. *Quarterly Journal of Economics*, *121*(1), 229–65.

Doepke, M. & Schneider, M. (2006a). Aggregate implications of wealth redistribution: the case of inflation. *Journal of the European Economic Association*, *4*(2-3), 493–502.

Doepke, M. & Schneider, M. (2006b). Inflation and the redistribution of nominal wealth. *Journal of Political Economy*, *114*(6), 1069–97.

Fudenberg, D. & Tirole, J. (1990). Moral hazard and renegotiation in agency contracts. *Econometrica*, *58*(6), 1279–1319.

Gertler, M. (1992). Financial capacity and output fluctuations in an economy with multiperiod financial relationships. *Review of Economic Studies*, *59*(2), 455–72.

Greene, W. H. (1990). *Econometric Analysis*. MacMillan, New York.

Jovanovic, B. & Ueda, M. (1997). Contracts and money. *Journal of Political Economy*, *105*(4), 700–708.

Jovanovic, B. & Ueda, M. (1998). Stock-returns and inflation in a principal-agent economy. *Journal of Economic Theory*, *82*(1), 223–47.

Jovanovic, B. (2009). Nominal shocks and long-term contracts. Unpublished Manuscript, Department of Economics, New York University.

Li, S. M. (2008). Employment flows with endogenous financiang constraints. Unpublished manuscript, University of Melbourne.

Martin, A. & Monnet, C. (2006). Contracts and money revisited. *Berkeley Electronic Press, Topics in Macroeconomics*, *6*(1).

Meh, C., Ríos-Rull, J. V., & Terajima, Y. (2008). Aggregate and welfare effects of redistribution of wealth under inflation and price-level targeting. Bank of Canada, Working Paper No. 2008-31.

Quadrini, V. (2004). Investment and liquidation in renegotiation-proof contracts with moral hazard. *Journal of Monetary Economics*, *51*(4), 713–751.

Stokey, N. L., Lucas, R. E., & Prescott, E. C. (1989). *Recursive Methods in Economic Dynamics.* Harvard University Press, Cambridge, Massachusetts.

Wang, C. (2000). Renegotiation-proof dynamic contracts with private information. *Review of Economic Dynamics*, *3*(3), 396–422.