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# On the Robustness of Laissez-Faire*

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ABSTRACT

This paper considers a model economy in which preference orderings over observable choices are allowed to be correlated with hidden wealth. However, the planner/government is both uncertain about the nature of this joint distribution and unable to choose among multiple equilibria of any given social mechanism. Thus, we model the planner/government as having a maximin objective in the face of this uncertainty.

Our main theorem is as follows. Once we allow for this kind of uncertainty, the uniquely optimal social contract is laissez-faire, in which agents trade in unfettered markets with no government intervention of any kind.

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*"Primum non nocere"* — medical aphorism

## 1. Introduction

In economies with privately observed effort levels, or privately observed endowments, skills, or preferences, decentralization is problematic. In such economies, to achieve efficiency, every observable aspect of an agent's life must generally be monitored or controlled. That is, one lesson of modern information economics is that the optimal system in the presence of information problems appears as centrally planned as one can imagine.

Such control is desirable since it helps provide insurance. The lucky and unlucky typically have different preferences over observable choices. If a hidden source of wealth is apples, those with high apple endowments will be more willing to trade apples for other goods than those with low apple endowments. If a hidden source of wealth is skill, the highly skilled will be more willing to substitute leisure for consumption. By conditioning insurance payments on these kinds of observable choices, economies can achieve at least partial insurance. For example, Townsend (1982), Green (1987), and Atkeson and Lucas (1992) describe how a social planner can exploit observable choices over current and future consumption to provide social insurance.

In this paper, we re-examine this result. We consider a model economy in which preference orderings over observable choices are allowed to be correlated with hidden wealth. However, the planner/government is uncertain about the nature of this joint distribution. Here, by uncertain, we mean that the planner/government is unable to form a Bayesian prior over what this correlation might be. We also mean that, if there are multiple equilibrium outcomes to a mechanism, the planner/government cannot form a Bayesian prior over these outcomes. We model the planner/government as having a maximin objective in the face of this uncertainty. Thus, the social welfare function is minimized over possible joint distributions and possible equilibrium outcomes. Our main theorem is that once we allow for this kind of uncertainty, the uniquely optimal social contract is laissez-faire, in which agents trade in unfettered markets with no government intervention of any kind.

The logic behind our result is as follows. Consider an environment (Environment 1) such that preference orderings are correlated with hidden wealth. Then, there is an interventionist mechanism that provides some insurance against the wealth risk. Now take another environment (Environment 2), in which the marginal distribution of wealths and the marginal distribution of preferences across agents are both the same as in the original environment, Environment 1. However, in Environment 2, wealths and preferences are independent across agents.

We prove that in Environment 2, the laissez-faire outcome is the best possible, and that it provides as much as social welfare as the laissez-faire outcome in Environment 1. Then, we prove that under the interventionist mechanism, it is an equilibrium in Environment 2 for agents to act just as they would in Environment 1. But since there is no correlation between wealths and preferences, this equilibrium is necessarily worse than the laissez-faire outcome. It follows that the minimax objective of the planner is higher with laissez-faire.

We depict this logic graphically in Figure 1. The interventionist mechanism dominates laissez-faire in Environment 1. Laissez-faire gives the same welfare in both environments. But laissez-faire dominates the interventionist mechanism in Environment 2. It follows that the social planner prefers laissez-faire: its worst performance is better than that of the interventionist mechanism.

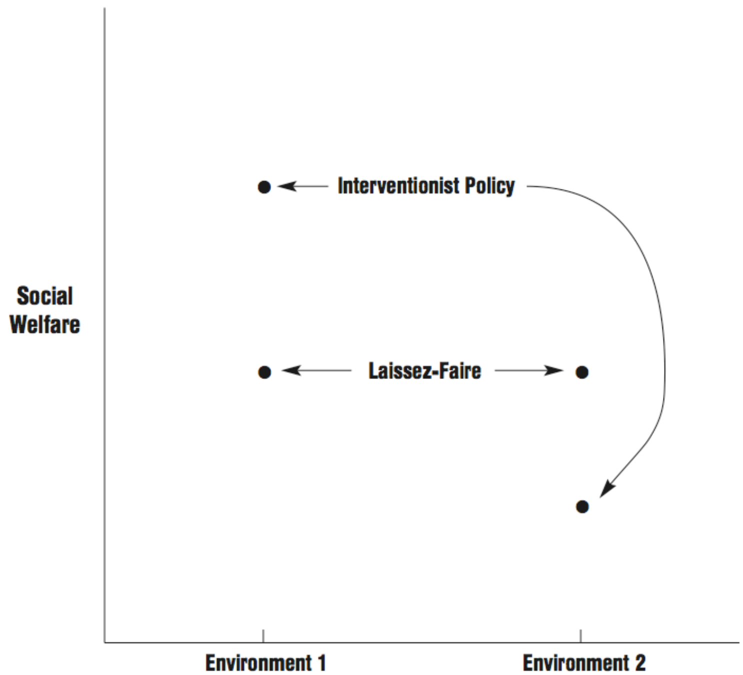


**Figure 1**

Our analysis builds on the recent literature on robust mechanism design. Chung and Ely (2006) use a minimax criterion with respect to environments. We differ from them by extending the minimax criterion to allow for the possibility of multiple equilibrium outcomes (as occurs in Bassetto and Phelan (2006)).

Earlier, we mentioned that Townsend (1982) and a large succeeding literature have discussed how interventionist mechanisms are optimal when it is known that agents' wealths are correlated with their intertemporal endowment profiles. However, Allen (1985) shows any mechanism in these settings induces a laissez-faire outcome if agents can engage in hidden borrowing and lending. Cole and Kocherlakota (2001) prove in this same setting that laissez-faire is the best mechanism if agents can engage in hidden lending. In these papers, laissez-faire is optimal because the agents can undo attempts on the part of the planner to improve on laissez-faire. Our argument is quite different. Laissez-faire is optimal in our setting because it rules out the ability of agents to co-ordinate on bad outcomes.

## 2. Setup

In this section, we describe the basic model. In it, individuals face four sorts of risks: shocks to overall wealth, shocks to how that wealth is split across their endowments of various goods, shocks to their general urgency to consume, and shocks to their urgency to consume particular goods. The realization of these shocks is private information. As well, the underlying joint distribution of the shocks is unknown.

More formally, we consider an open economy with a unit measure of agents and $N$ goods. The goods can be traded by the society with an outside world at price vector $p \in \Delta^N$ (the unit simplex in $R_+^N$). Agents have endowments indexed by parameters $y \in Y \subset R_+$ and $z \in Z \subset R^N$ such that an agent's endowment of good $n$ equals $y + z_n/p_n$. Assume $Z$ is such that if $z \in Z$ then $\sum_n z_n = 0$. This formulation ensures that the value of an agent's endowment vector (under prices $p$) equals $y$ while his vector $z$ specifies how this value is distributed across goods.

Agents have preferences indexed by parameters $\theta \in \Theta \subset R_+$ and $\omega \in \Omega \subset R^N$, where if $\omega \in \Omega$ then $\sum_n \omega_n = 0$. Preferences over consumption bundles are given by

$$u(c, \theta, \omega) = -\sum_n \exp(-\alpha c_n) \exp(\frac{\beta_n}{p_n}) \exp(\theta + \frac{\omega_n}{p_n})$$

This specification of preferences, along with the assumption that $\sum_n \omega_n = 0$, ensures that the $\omega_n$ shocks do not incorporate a general urgency to consume, but are instead good specific. (The $\beta_n$ parameters are common across agents. Having them inside the exponentiation function divided by prices is a useful normalization. The restriction to exponential utility over consumption can be dropped if $y$ and $\omega$ are degenerate.)

An agent's *type* is his specification of $t = (y, z, \theta, \omega) \in Y \times Z \times \Theta \times \Omega$. Henceforth, we use the notation $T$ to denote the type space $Y \times Z \times \Theta \times \Omega$. Assume $T$ is finite. Let $\Pi$ be the set of probability measures over $T$. An *environment* is an element $\mu$ in $\Pi$; in an environment $\mu$, agents' types are determined by i.i.d. draws from $\mu$. The agents' realized types are private information.

Let $M$ be a finite message space with elements $m \in M$. Assume $T \subset M$ (so it is always feasible for an agent's message to be "I am of type $t$.") Let $\Phi$ be the space of probability measures over $M$ with elements $\phi \in \Phi$. A *mechanism* is a mapping $\tau : M \times \Phi \to R^N$. The outcome $\tau(m, \phi)$ describes the transfer of goods made to an agent who sends message $m \in M$, when the cross-sectional distribution of messages is $\phi \in \Phi$. A mechanism is *resource-feasible* if for all $\phi$ in $\Phi$:

$$\sum_m \phi(m) \sum_n p_n \tau_n(m, \phi) \leq 0$$

so that the society can afford the transfers being made regardless of the cross-sectional distribution of messages.

A *strategy* $\sigma$ is a mapping from $T$ into $\Phi$ (this allows the agent to mix over elements of $M$ in sending messages). Given a strategy $\sigma$ and an environment $\mu$, define $\phi(\mu, \sigma) \in \Phi$ to be the cross-sectional distribution of messages induced by $\sigma$:

$$\phi(\mu, \sigma)(m) = \sum_t \mu(t)\sigma(t)(m)$$

Next let

$$BR((y, z, \theta, \omega), \tau, \phi) \;\; = \;\; \{m| \; -\sum_n \exp(-\alpha(y + z_n/p_n + \tau_n(m, \phi))) \exp(\beta_n/p_n + \theta + \omega_n/p_n)$$

$$\geq -\sum_n \exp(-\alpha(y + z_n/p_n + \tau_n(m', \phi))) \exp(\beta_n/p_n + \theta + \omega_n/p_n)$$

for all $m' \in M\}$.

That is, $BR(t, \tau, \phi)$ is the set of messages $m$ such that $m$ is a best response given type $t$, mechanism $\tau$, and distribution of messages by others, $\phi$. Given a mechanism $\tau$ and an environment $\mu$, we define a strategy $\sigma$ to be an equilibrium strategy if for all $(t, m) \in T \times M$ such that $\mu(t) > 0$ and $\sigma(t)(m) > 0$, $m \in BR(t, \tau, \phi(\mu, \sigma))$. Note that the agent is assumed to know the environment $\mu$ when choosing his message. Let $E(\mu, \tau)$ denote the set of equilibrium strategies, given the environment $\mu$ and mechanism $\tau$.

In what follows, we consider the problem of a planner who wants to choose a mechanism $\tau$ that maximizes his objective. For a given policy $\tau$, environment $\mu$, and equilibrium $\sigma$, we formulate this objective as a social welfare function,

$$V_{SP}(\tau, \mu, \sigma) = -\sum_{(y,z,\theta,\omega)} \lambda(y, z, \theta, \omega) \sum_{m \in M} \sigma(y, z, \theta, \omega)(m) \sum_{n=1}^{N} \exp(-\alpha(y + \frac{z_n}{p_n} + \tau_n(m, \phi(\mu, \sigma)))) \exp(\frac{\beta_n}{p_n} + \theta + \frac{\omega_n}{p_n})$$

where $\lambda(y, z, \theta, \omega) = \mu(y, z, \theta, \omega)\kappa(y, \theta)$ such that $\kappa(y, \theta) > 0$ for all $(y, \theta)$. If $\kappa(y, \theta) = 1$, then the social welfare function $V_{SP}(\tau, \mu, \sigma)$ is simply the average utility in the population, and thus $V_{SP}$ is the standard utilitarian or ex-ante welfare criterion. By allowing $\kappa(y, \theta) \neq 1$, we allow for redistributive motives by the social planner. By not allowing $\kappa$ to depend on $z$ or $\omega$, we restrict the social welfare function to be symmetric across goods. We don't allow the social planner to put high weight on those who enjoy a particular good (say, art), or those who have high endowments of one good or another, holding constant the overall value of their endowment bundle.

Across environments and equilibria, we assume that the planner has two forms of *ambiguity.* First, the planner is unable to form a subjective prior over the set of possible environments, other than he is certain it falls in some exogenous set of possible environments $X$. Second, given a mechanism and an environment, there may be multiple equilibria, and the planner cannot formulate a subjective prior over which of these equilibria will occur. Given these two forms of ambiguity, the planner wants to design a mechanism that works well regardless of the actual environment or the equilibrium that gets played. Hence, we define the payoff to the social planner from a given mechanism $\tau$ as $V_{SP}(\tau)$, as:

$$V_{SP}(\tau) = \inf_{\mu \in X} \inf_{\sigma \in E(\mu,\tau)} V_{SP}(\tau, \mu, \sigma)$$

## 3. Examples

Here are two examples that fit into the above framework.

*Example 1.* (Atkeson-Lucas) Let the number of goods, $N$, equal two, representing date 1 and date 2 consumption. Prices $(p_1, p_2)$ equal $(\frac{1}{2}, \frac{1}{2})$ and $\beta_1 = \beta_2 = 0$ (a zero interest rate and no discounting). Let the set of possible environments, $X$, be a singleton, $\mu$, such that each agent has a 50% chance of being type 1 or type 2. Let both types have an endowment of 1 unit in each period, or $y = 1$ and $z = (0, 0)$. For type 1 agents, let $\theta = \frac{5}{2}$ and $\omega = (\frac{1}{4}, -\frac{1}{4})$. For type 2 agents, let $\theta = \frac{3}{2}$ and $\omega = (-\frac{1}{4}, \frac{1}{4})$. That is, those with a general urgency to consume (the high $\theta$ type 1 agents) prefer first to second period consumption, and those without a general urgency to consume (the low $\theta$ type 2 agents) prefer second to first period consumption.

*Example 2.* (Mirrleesian) Set $N = 2$, and think of the goods as being consumption and effective time, where a negative transfer of effective time is providing labor to the market and a positive transfer is akin to receiving a service. This is like a (richer than usual) Mirrleesian setup. (As in the previous example, let prices $(p_1, p_2)$ equal $(\frac{1}{2}, \frac{1}{2})$ and $\beta_1 = \beta_2 = 0$.) Again, assume $X$ is a singleton, $\mu$, such that each agent has a 50% chance of being type 1 or type 2. Let $\theta = 0$ and $\omega = (0, 0)$ for both types. For type 1 agents, let $y = \frac{1}{2}$ and $z = (\frac{1}{4}, -\frac{1}{4})$. For type 2 agents, let $y = 1$ and $z = (0, 0)$. That is, a type 1 agent has an endowment of good 1 (the consumption good) of $\frac{1}{2} + \frac{1}{4}/p_1 = 1$ and an endowment of good 2 (effective labor) of $\frac{1}{2} - \frac{1}{4}/p_2 = 0$. That is, type 1's are disabled. Type 2's have an endowment of one unit of each good.

## 4. The Laissez-Faire Mechanism

In what follows, we will be especially interested in the properties of the *laissez-faire mechanism.* Define $c_{LF}(t)$ to be the solution to:

$$\begin{aligned} c_{LF}(y, z, \theta, \omega) &= \arg\max_c \sum_n -\exp(-\alpha c_n)\exp(\frac{\beta_n}{p_n})\exp(\theta + \frac{\omega_n}{p_n}) \\ & \quad s.t. \sum_n p_n c_n \leq y \end{aligned}$$

This is the optimal choice for an individual agent with type $t$, when confronted with the budget set defined by the price vector $p$. One can analytically solve (albeit with a bit of tedious algebra) for $c_{j,LF}(t)$ as:

$$c_{j,LF}(y, z, \theta, \omega) = y + \frac{(\beta_j + \omega_j)p_j^{-1} - \log p_j - \sum_n(\beta_n - p_n \log p_n)}{\alpha}$$

Note that $c_{LF}(t)$ does not depend on $\theta$ since $\theta$ acts as a multiplicative shock on the entire utility function.

We define the laissez-faire mechanism $\tau_{LF}$ as follows: for messages $m \in T$, the $j$th component of $\tau_{LF}(m)$ is given by:

$$\begin{aligned}
\tau_{j,LF}(y,z,\theta,\omega) &= c_{j,LF}(y,z,\theta,\omega) - y - z_j/p_j \\
&= \frac{(\beta_j+\omega_j)p_j^{-1} - \log p_j - \sum_n(\beta_n - p_n \log p_n)}{\alpha} - \frac{z_j}{p_j}
\end{aligned}$$

(For messages $m \notin T$, $\tau_{j,LF}(m) = 0$ for all goods $j$.) Note that $\tau_{LF}$ is a resource-feasible mechanism. More subtly, because of the properties of exponential utility, $\tau_{LF}$ is independent of $y$.

Under the laissez-faire mechanism, the decision of what type to report is equivalent to deciding which zero-value transfer vector to purchase. Because $\tau_{LF}$ is independent of $y$ and $\theta$, reports regarding $y$ and $\theta$ are irrelevant. Hence, it is straightforward to see that given the laissez-faire mechanism, there are many equilibria. In particular, for any $\mu$, any reporting strategy $\sigma$ such that $\sigma(y,z,\theta,\omega)(y',z',\theta',\omega') = 0$ if $(z',\omega') \neq (z,\omega)$ is an equilibrium under $\tau_{LF}$. However, these equilibria are all consumption-equivalent, in the sense that in any of them, an agent with type $t$ ends up consuming $c_{LF}(t)$.

Given $\mu$, the planner's welfare from the laissez-faire mechanism is equal to:

$$\begin{aligned}
V_{LF}(\mu) &= -\sum_{(y,z,\theta,\omega)} \lambda(y,z,\theta,\omega) \sum_j \exp(-\alpha\, c_{j,LF}(y,z,\theta,\omega)) \exp(\beta_j/p_j + \theta + \omega_j/p_j) \\
&= -\exp(\sum_n \beta_n - p_n \ln p_n) \sum_{(y,z,\theta,\omega)} \mu(y,z,\theta,\omega)\kappa(y,\theta)\exp(-\alpha y + \theta)
\end{aligned}$$

Examination of $V_{LF}(\mu)$ shows the laissez-faire mechanism provides the same expected utility for any two $\mu$'s with the same marginal distribution of $(y, \theta)$.

## 5. The Social Planner's Problem

In this section, we show that the laissez-faire mechanism is the uniquely optimal mechanism for the social planner. In the first subsection, we discuss the apparent shortcomings of the laissez-faire mechanism when we assume that the environment is known. In the second subsection, we show that this apparent problem disappears once we allow for sufficient uncertainty about the environment.

### A. The Problem with the Laissez-Faire Mechanism

In the previous section, we saw that the planner's payoff from the laissez-faire mechanism in an environment $\mu$ equals:

$$-\exp(\sum_n \beta_n - p_n \ln p_n) \sum_{(y,z,\theta,\omega)} \mu(y, z, \theta, \omega)\kappa(y, \theta) \exp(-\alpha y + \theta)$$

Under the laissez-faire mechanism, because the agent can trade the various goods in the market at price vector $p$, the agent is left unaffected by the risk associated with shocks to the composition of his endowment, $z$, and good specific shocks to his preferences, $\omega$. However, the variation in his overall wealth $y$ and overall urgency to consume $\theta$ impacts the agent's utility adversely; not surprisingly, the laissez-faire mechanism leaves the agent uninsured against these risks and thus makes the planner (who cares about a weighted average of the ex-ante utility of the agents) worse off.

Agents' wealths and urgencies to consume are private information. Is it still possible to design a mechanism that improves upon the laissez-faire mechanism? If $\mu$ is known to the planner, it is well-understood that the answer to this question is yes. The construction works as follows. Suppose first that the planner knows the environment $\mu$, and $\mu$ is such that $(z, \omega)$ is not independent of $(y, \theta)$. Consider the Atkeson-Lucas example from Section 3. In that example, those with a general urgency to consume (the high $\theta$ type 1 agents) prefer first to second period consumption, and those without a general urgency to consume (the low $\theta$ type 2 agents) prefer second to first period consumption. Since $X$ is a singleton, there is no uncertainty regarding the environment. Thus the optimal plan solves a standard direct mechanism Bayesian implementation problem as in Atkeson and Lucas.

For $\alpha = 1$ and $\kappa(y, \theta) = 1$ for all $(y, \theta)$, the optimal direct mechanism for this example can be numerically solved for $\tau_1(t = 1) = 0.93$, $\tau_2(t = 1) = -0.37$,$\tau_1(t = 2) = -.78$ and $\tau_2(t = 2) = 0.22$, resulting in type 1 (urgent, or high $\theta$) agents consuming $(1.93, 0.63)$,with market value 1.28 (under prices $(\frac{1}{2}, \frac{1}{2})$) and type 2 agents consuming $(0.22, 1.22)$ with market value 0.72. Note that since the value of the each agent's endowment is 1, this plan transfers wealth from the less urgent (low $\theta$) agents to the more urgent (high $\theta$) agents.[1]

This plan provides insurance against an urgency to consume (high $\theta$) by transferring wealth from the low $\theta$ to high $\theta$ agents. This makes all agents better off from an ex-ante point of view. The plan is incentive compatible since it "tilts" consumption of the high $\theta$ agents toward good 1. This is a prospect enjoyed by truthful type 1 agents, but sufficiently disliked by type 2's to dissuade them from falsely claiming to be type 1's in order to receive the wealth transfer. This mechanism does suffer from a loss in allocative efficiency; however, the first-order insurance gain offsets the second-order loss in allocative efficiency. More generally,

[1]This plan assumes the profile of reported types equals the true profile of types $\mu$. To make this plan the unique equilibrium outcome, let $\tau = \tau_{LF}$ if the profile of reported types is not equal to $\mu$. This plan is always feasible and has truth-telling as a dominant strategy.

if the planner knows $\mu$, the planner can design a mechanism that improves ex-ante welfare for all $\mu$'s but the (non-generic) ones in which tastes/endowments are independent of wealths.

Does the assumption that $\mu$ is known matter? The usual answer would be no. The above mechanism is incentive-compatible. Hence, by the revelation principle, there is an equilibrium in which agents truthfully reveal their types. The planner can then condition transfers upon the distribution of announced types. Under this view, societal welfare is the same, whether or not the planner can observe $\mu$. By conditioning transfers on the joint distribution of announcements, it is possible to design a better mechanism than the laissez-faire mechanism.

But this analysis leaves out the possibility of multiple equilibria. The revelation principle guarantees only that there is *an* equilibrium in which agents truthfully report their types. There may be another equilibrium in which agents do not do so - and this equilibrium could be worse. In the next subsection, we discuss what happens once we allow for this possibility.

### B. The Main Theorem

In this subsection, we state and prove the main theorem of the paper: If the environment space $X$ is *complete*, then $\tau_{LF}$ is the uniquely optimal mechanism. Our definition of a complete environment space is as follows:

Definition 1. *An environment space $X$ is complete if for all $\mu \in X$, there exists $\mu' \in X$ such that*

1. $V_{LF}(\mu') \leq V_{LF}(\hat{\mu})$ *for all* $\hat{\mu} \in X$,
2. *for all* $(y, z, \theta, \omega)$, $\mu'(y, z, \theta, \omega) = (\sum_{y',\theta'} \mu'(y', z, \theta', \omega))(\sum_{z',\omega'} \mu'(y, z', \theta, \omega'))$, *and*
3. *for all* $(z, \omega)$, $\sum_{y',\theta'} \mu'(y', z, \theta', \omega) = \sum_{y',\theta'} \mu(y', z, \theta', \omega)$.

Completeness requires that for every $\mu \in X$, there exists a $\mu'$ which attains the minimum of $V_{LF}$, where $(y, \theta)$ is independent of $(z, \omega)$, and where $\mu'$ has the same marginal distribution of $(z, \omega)$ as $\mu$.

*Example 1.* $X = \Pi$, the set of all possible measures on $Y \times Z \times \Theta \times \Omega$. In this case, since $Y$ and $\Theta$ are finite sets, the minimum of $V_{LF}$ is attained by all $\mu$ which put all mass on the lowest point of $Y$ and highest point of $\Theta$.

*Example 2.* All points in the environment set $X$ have the same marginal distribution of $(y, \theta)$ (thus laissez-faire gives the same value of the social welfare function for all $\mu \in X$), the same marginal distribution of $(z, \omega)$, and $X$ includes a $\mu'$ such that $(y, \theta)$ and $(z, \omega)$ are independent.

*Example 3.* In the earlier Atkeson and Lucas example, $X$ was a singleton, $\mu$, with two types (each 50% of the population). Type 1 had $(y, z, \theta, \omega) = (1, (0, 0), \frac{5}{2}, (\frac{1}{4}, -\frac{1}{4}))$ and type 2 had $(y, z, \theta, \omega) = (1, (0, 0), \frac{3}{2}, (-\frac{1}{4}, \frac{1}{4}))$. Let $X$ contain this $\mu$ as well as $\mu'$ with four types (each 25% of the population).

1. $(y, z, \theta, \omega) = (1, (0, 0), \frac{5}{2}, (\frac{1}{4}, -\frac{1}{4}))$,
2. $(y, z, \theta, \omega) = (1, (0, 0), \frac{5}{2}, (-\frac{1}{4}, \frac{1}{4}))$,
3. $(y, z, \theta, \omega) = (1, (0, 0), \frac{3}{2}, (\frac{1}{4}, -\frac{1}{4}))$,
4. $(y, z, \theta, \omega) = (1, (0, 0), \frac{3}{2}, (-\frac{1}{4}, \frac{1}{4}))$.

Before moving to our main result, we prove an important supporting lemma: If $\mu$ is known and $(y, \theta)$ is independent of $(z, \omega)$ then laissez-faire is the solution to the standard Bayesian mechanism design problem. In such a problem, the message space equals the type space (or $M = T$) and the profile of messages $\phi$ is assumed to equal the true profile of types $\mu$. Given these restrictions, let $BMDP$ be defined as

$$
\begin{aligned}
(BMDP) \qquad & \max_{\delta} - \sum_{(y,z,\theta,\omega)} \mu(y,z,\theta,\omega)\kappa(y,\theta) \\
& \sum_{n} \exp(-\alpha(y + \frac{z_n}{p_n} + \delta_n(y,z,\theta,\omega))) \exp(\frac{\beta_n}{p_n} + \theta + \frac{\omega_n}{p_n}) \\
& \text{subject to} \\
& \sum_{t} \mu(t) \sum_{n} p_n \delta_n(t) \leq 0 \\
& \text{and for all } t \text{ such that } \mu(t) > 0, t \in BR(t, \delta, \mu).
\end{aligned}
$$

LEMMA 1. Consider $\mu$ such that for all $(y, z, \theta, \omega)$

$$
\mu(y,z,\theta,\omega) = (\sum_{y',\theta'} \mu(y',z,\theta',\omega))(\sum_{z',\omega'} \mu(y,z',\theta,\omega')).
$$

Then $\delta$ solves BMDP if and only if $\delta(t) = \tau_{LF}(t)$ for all $t$ such that $\mu(t) > 0$.

*Proof.* In this proof, we first show the resource constraint holds as an equality in any solution to BMDP. Next we show that BMDP has a unique solution. From there, we show that this solution does not depend on the agent's announcement of $y$ or $\theta$. Finally, we use these results, along with the independence assumption in the statement of the lemma to show that $\tau_{LF}$ solves BMDP.

First note that if $\delta$ solves BMDP, the resource constraint holds with equality. To see this, suppose $\sum_t \mu(t) \sum_n p_n \delta_n(t) + \epsilon = 0$ for $\epsilon > 0$. Define $\overline{\delta}$ such that $\overline{\delta}_n(t) = \epsilon + \delta_n(t)$. Mechanism $\overline{\delta}$ has a strictly higher objective function value than $\delta$, satisfies the resource constraint with equality, and satisfies the incentive constraints since (from the assumption of exponential utility) $\epsilon$ cancels from each side of the incentive constraints.

Next we show that if $\delta$ and $\delta'$ each solve $BMDP$, then for all $t$ such that $\mu(t) > 0$, $\delta(t) = \delta'(t)$. (That is, in terms of outcomes, the solution to $BMDP$ is unique.) To this end, let $u_n(t) = \exp(-\alpha\delta_n(t))$ and consider choosing $u : T \to R^N$ instead of $\delta$. Then the objective function and incentive constraints become linear in the choice variables and the resource constraint becomes

$$\sum_t \mu(t) \sum_n p_n(-\log(u_n(t))/\alpha) \leq 0.$$

Suppose $u_0$ and $u_1$ each solve BMDP and $u_0(t) \neq u_1(t)$ for some $t$ such that $\mu(t) > 0$. Let $\overline{u} = \gamma u_0 + (1-\gamma)u_1$ for some $\gamma \in (0,1)$. From the linearity of the objective function and the incentive constraints, $\overline{u}$ also solves BMDP if it satisfies the resource constraint. Since $-\log(u)$ is a strictly convex function of $u$, the resource constraint is satisfied as a strict inequality, contradicting the optimality of $u_0$ and $u_1$.

Next we establish that if $\delta^*$ solves $BMDP$, then for all $(y^0, \theta^0)$, ,$(y^1, \theta^1)$ and $(z, \omega)$ such that $\mu(y^0, z, \theta^0, \omega) > 0$ and $\mu(y^1, z, \theta^1, \omega) > 0$, $\delta^*(y^0, z, \theta^0, \omega) = \delta^*(y^1, z, \theta^1, \omega)$. That is, in terms of outcomes, $\delta^*$ does not depend on the announcement of $(y, \theta)$. Again let the choice variable be $u$ instead of $\delta$ and suppose $u$ solves $BMDP$ with $u(y^0, \overline{z}, \theta^0, \overline{\omega}) \neq u(y^1, \overline{z}, \theta^1, \overline{\omega})$ for some $(y^0, \theta^0)$, $(y^1, \theta^1)$ and $(\overline{z}, \overline{\omega})$ such that $\mu(y^0, \overline{z}, \theta^0, \overline{\omega}) > 0$ and $\mu(y^1, \overline{z}, \theta^1, \overline{\omega}) > 0$. Assume without loss of generality that

$$(1) \qquad \sum_n p_n(-\log(u_n(y^1, \overline{z}, \theta^1, \overline{\omega}))/\alpha) \geq \sum_n p_n(-\log(u_n(y^0, \overline{z}, \theta^0, \overline{\omega}))/\alpha)$$

Next note that if one divides each side of the incentive condition in the definition of $BR(t, \delta, \mu)$

by $\exp(-\alpha y+\theta)$, $(y,\theta)$ falls from the incentive condition. Thus if $(y^1,\overline{z},\theta^1,\overline{\omega}) \in BR((y^1,\overline{z},\theta^1,\overline{\omega}),\delta,\mu)$ then $(y^0,\overline{z},\theta^0,\overline{\omega}) \in BR((y^1,\overline{z},\theta^1,\overline{\omega}),\delta,\mu)$ as well. This implies

$$
\begin{aligned}
& -\sum_n \exp(-\alpha(y^1+\overline{z}_n/p_n))u_n(y^1,\overline{z},\theta^1,\overline{\omega})\exp(\beta_n/p_n+\theta^1+\overline{\omega}_n/p_n) \\
(2) \qquad = \; & -\sum_n \exp(-\alpha(y^1+\overline{z}_n/p_n))u_n(y^0,\overline{z},\theta^0,\overline{\omega})\exp(\beta_n/p_n+\theta^1+\overline{\omega}_n/p_n).
\end{aligned}
$$

Define $\hat{u}(y^1,\overline{z},\theta^1,\overline{\omega}) = u(y^0,\overline{z},\theta^0,\overline{\omega})$ and $\hat{u}(y,z,\theta,\omega) = u(y,z,\theta,\omega)$ for all other $(y,z,\theta,\omega)$. From (2) and the definition of $\hat{u}$,

$$
\begin{aligned}
& -\sum_n \exp(-\alpha(y^1+\overline{z}_n/p_n))u_n(y^1,\overline{z},\theta^1,\overline{\omega})\exp(\beta_n/p_n+\theta^1+\overline{\omega}_n/p_n) \\
= \; & -\sum_n \exp(-\alpha(y^1+\overline{z}_n/p_n))\hat{u}_n(y^1,\overline{z},\theta^1,\overline{\omega})\exp(\beta_n/p_n+\theta^1+\overline{\omega}_n/p_n).
\end{aligned}
$$

and thus the objective function of BMDP is equal under $u$ and $\hat{u}$. Likewise, (2) implies that $\hat{u}$ is incentive compatible. Finally, (1) implies that

$$
\sum_n p_n(-\log(u_n(y^1,\overline{z},\theta^1,\overline{\omega}))/\alpha) \geq \sum_n p_n(-\log(\hat{u}_n(y^1,\overline{z},\theta^1,\overline{\omega}))/\alpha)
$$

which implies $\hat{u}$ is resource feasible. This contradicts that $u$ uniquely solves BMDP.

Next consider the full information planning problem, imposing independence and that $\delta(y,z,\theta,\omega) = \delta(y',z,\theta',\omega)$, or,

$$
(FIP) \qquad \max_{\delta} -\sum_{y,\theta}\mu(y,\theta)\kappa(y,\theta)\sum_{z,\omega}\mu(z,\omega)\sum_n \exp(-\alpha(y+\frac{z_n}{p_n}+\delta_n(z,\omega)))\exp(\frac{\beta_n}{p_n}+\theta+\frac{\omega_n}{p_n})
$$

subject to

$$
\sum_{y,\theta}\mu(y,\theta)\sum_{z,\omega}\mu(z,\omega)\sum_n p_n\delta_n(z,\omega) \leq 0,
$$

where (with some abuse of notation) $\mu(y,\theta) = \sum_{z',\omega'} \mu(y,z',\theta,\omega')$, $\mu(z,\omega) = \sum_{y',\theta'} \mu(y',z,\theta',\omega)$ and $\delta(z,\omega) = \delta(y,z,\theta,\omega)$ for any $(y,\theta)$. This can be rewritten as

$$\Big[\sum_{y,\theta} \mu(y,\theta)\kappa(y,\theta)\exp(-\alpha y + \theta)\Big] *$$
$$\Big[\max_{\delta} - \sum_{z,\omega} \mu(z,\omega) \sum_n \exp(-\alpha(z_n/p_n + \delta_n(z,\omega)))\exp(\beta_n/p_n + \omega_n/p_n)$$
$$\text{subject to}$$
$$\sum_{z,\omega} \mu(z,\omega) \sum_n p_n \delta_n(z,\omega) \leq 0\Big].$$

This inner maximization problem can be analytically solved for $\delta = \tau_{LF}$. Since $\tau_{LF}$ solves the full information problem subject to a restriction that $\delta$ not depend on $(y,\theta)$, it solves $BMDP$, since the incentive conditions of $BMDP$ incorporate this restriction. QED

We now move to our main result.

Theorem 1. *Suppose the environment space $X$ is complete. Suppose $\tau$ is a mechanism such that there exists an environment $\mu \in X$, strategy $\sigma \in E(\tau,\mu)$, type $t$ and message $m$, such that $\mu(t) > 0$, $\sigma(t)(m) > 0$, and $\tau(m,\phi(\mu,\sigma)) \neq \tau_{LF}(t)$. Then $V_{SP}(\tau_{LF}) > V_{SP}(\tau)$.*

In words, the theorem says that if $\tau$ is a mechanism that delivers different equilibrium allocations from the laissez-faire mechanism in some environment, then $\tau$ must be strictly worse for the social planner.

It is useful to begin by verbally sketching the proof of this theorem. Consider a $\tau$ such that there exists a $\mu$ and an equilibrium strategy $\sigma \in E(\tau,\mu)$ such that a non-laissez-faire policy is implemented. The idea of the proof is to consider another environment, $\mu'$, which attains the minimum of $V_{LF}$, where $(y,\theta)$ and $(z,\omega)$ are independent of each other, and where the marginal distribution of $(z, \omega)$ is the same as under $\mu$. Next, one constructs an equilibrium strategy, $\sigma' \in E(\tau, \mu')$ which produces the same profile of messages as $\sigma$ does under $\mu$, and thus the non-laissez-faire outcome occurs when the environment is $\mu'$. (Exponential utility and multiplicative taste shocks play a key role in this demonstration; the incentive constraints are unaffected by the level of $y$ and $\theta$, and so incentive constraints are the same in $\mu'$ as in $\mu$.) From Lemma 1, if the environment is $\mu'$, laissez-faire gives a strictly better payoff to the social planner than $\tau$. Finally, since $\mu'$ attains the minimum payoff to the social planner under laissez-faire, that $\tau$ gives a lower payoff in this environment implies $V_{SP}(\tau_{LF}) > V_{SP}(\tau)$. We now formalize the above logic.

*Proof.* Let $\tau$ be such that there exists an environment $\mu \in X$, strategy $\sigma \in E(\tau, \mu)$, type $t$ and message $m$, such that $\mu(t) > 0$, $\sigma(t)(m) > 0$, and $\tau(m, \phi(\mu, \sigma)) \neq \tau_{LF}(t)$. Since $X$ is complete, there exists $\mu'$ such that $V_{LF}(\mu') \leq V_{LF}(\hat{\mu})$ for all $\hat{\mu} \in X$,

$$\mu'(y, z, \theta, \omega) = (\sum_{y', \theta'} \mu'(y', z, \theta', \omega))(\sum_{z', \omega'} \mu'(y, z', \theta, \omega')),$$

and

$$\sum_{y', \theta'} \mu'(y', z, \theta', \omega) = \sum_{y', \theta'} \mu(y', z, \theta', \omega)$$

for all $(y, z, \theta, \omega) \in T$.

Given the reporting strategy $\sigma$, define $\sigma'$ to be any reporting strategy that satisfies:

$$\sigma'(y, z, \theta, \omega)(m) = \frac{\sum_{(y', \theta')} \mu(y', z, \theta', \omega)\sigma(y', z, \theta', \omega)(m)}{\mu'(y, z, \theta, \omega)}$$

for all $m$ and all $t = (y, z, \theta, \omega)$ such that $\mu'(t) > 0$. Note that for all $m$:

$$\begin{aligned}
\phi(\mu', \sigma')(m) &= \sum_{t \in T} \mu'(t)\sigma'(t)(m) \\
&= \sum_{(y,z,\theta,\omega)} \mu'(y, z, \theta, \omega) \frac{\sum_{(y',\theta')} \mu(y', z, \theta', \omega)\sigma(y', z, \theta', \omega)(m)}{\mu'(y, z, \theta, \omega)} \\
&= \phi(\mu, \sigma)(m)
\end{aligned}$$

This means that if agents use strategy $\sigma'$ and the environment is $\mu'$, the cross-sectional distribution of messages is the same as if they use strategy $\sigma$ and the environment is $\mu$.

We now claim that $\sigma' \in E(\tau, \mu')$. For this it is sufficient to show $m \in BR(t, \tau, \phi(\mu', \sigma'))$ for all $(t, m)$ such that $\mu'(t) > 0$ and $\sigma'(t)(m) > 0$.

Take such a $(t, m)$ as given, and let $t = (y, z, \theta, \omega)$. By the construction of $\mu'$ and $\sigma'$, that $\mu'(t) > 0$ and $\sigma'(t)(m) > 0$ implies there exists $(\overline{y}, \overline{\theta})$ such that $\mu(\overline{y}, z, \overline{\theta}, \omega) > 0$ and $\sigma(\overline{y}, z, \overline{\theta}, \omega)(m) > 0$. Since $\sigma \in E(\tau, \mu)$,

$$\begin{aligned}
& -\sum_n \exp(-\alpha(\overline{y} + z_n/p_n + \tau_n(m, \phi(\mu, \sigma)))) \exp(\beta_n/p_n + \overline{\theta} + \omega_n/p_n) \\
\geq \; & -\sum_n \exp(-\alpha(\overline{y} + z_n/p_n + \tau_n(m', \phi(\mu, \sigma)))) \exp(\beta_n/p_n + \overline{\theta} + \omega_n/p_n)
\end{aligned}$$

for all $m'$. Next, replace $\phi(\mu, \sigma)$ with $\phi(\mu', \sigma')$ (since they are equal), and multiply each side by $\exp(-\alpha\overline{y} + \overline{\theta}) / \exp(-\alpha y + \theta)$, giving

$$\begin{aligned}
& -\sum_n \exp(-\alpha(y + z_n/p_n + \tau_n(m, \phi(\mu', \sigma')))) \exp(\beta_n/p_n + \theta + \omega_n/p_n) \\
\geq \; & -\sum_n \exp(-\alpha(y + z_n/p_n + \tau_n(m', \phi(\mu', \sigma')))) \exp(\beta_n/p_n + \theta + \omega_n/p_n)
\end{aligned}$$

for all $m'$. Thus, $m \in BR(t, \tau, \phi(\mu', \sigma'))$, implying $\sigma' \in E(\tau, \mu')$.

Note that Lemma 1 and $\sigma' \in E(\tau, \mu')$ implies, under environment $\mu'$, the payoff to the social planner is strictly less under mechanism $\tau$ than mechanism $\tau_{LF}$. That $\mu'$, by assumption, minimizes $V_{LF}(\mu)$ then implies $V_{SP}(\tau) < V_{SP}(\tau_{LF})$. QED.

## 6. Conclusion

The main result of this paper — that laissez-faire delivers the uniquely optimal minmax policy — depends on two key characteristics of our model.

1. There is sufficient uncertainty about the correlation structure of shocks such that the actual environment might be one where laissez-faire is strictly optimal.
2. If a government implements a non-laissez-faire policy and the actual environment turns out to be the one where laissez-faire is optimal, a non-laissez-faire outcome is implemented. That is, the government cannot implement a policy that delivers the laissez-faire outcome when laissez-faire is optimal and a non-laissez-faire outcome when a non-laissez-faire outcome is optimal.

The assumptions of exponential utility and multiplicative taste shocks are used to deliver the second of these characteristics. In our opinion, these assumptions capture the real constraints which disallow governments from finely tuning mechanisms.

It is the first characteristic — that laissez-faire might be optimal — which appears to us as the more strict, depending on the application. Recall that laissez-faire is optimal when hidden wealth is independent of preferences over observable choices. If the hidden source of wealth is skill, such as in a Mirrlees model, it may be difficult to argue for the possibility of no correlation between total wealth and an agent's willingness to trade time for money. On the other hand, if the hidden source of wealth is a shock to endowments, such as in the model of Green (1987), then whether the lucky are more or less willing to save than the unlucky will depend on the timing of the shocks. If high endowment agents learn of their high endowments when they receive them, they will be more willing to save than low endowment agents. On the other hand, if agents learn of their endowments in advance, the lucky (those who know they will have a high endowment next period) will be less willing to save than the unlucky. That there may be no correlation between luck and desire to save or borrow appears, at least to us, entirely possible.

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