Growth, inequality and taxation in uncommitted societies*

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1 Introduction

This paper considers the properties and implementation of optimal allocations in settings with private information, *but without societal commitment*. The environment features a continuum of infinitely-lived agents who exert costly effort that is combined with capital to produce output. These agents receive private shocks to their disutility of effort. They are motivated to truthfully reveal their current disutility from working with future utility rewards and sanctions. These in turn translate into future inequality that a society might have difficulty committing to implement. We model a lack of societal commitment by supposing that allocations are implemented by an uncommitted planner playing a dynamic game with agents. The planner is tempted to undo inequality ex post, but is deterred from doing so by the disruption to future agent incentives that this will cause. In particular, if a planner defection leads agents to believe that future utility rewards and sanctions will not be honoured, then incentives for truthful reporting are undermined. The implied equilibrium restrictions on allocations include conditions that ensure agents are motivated to be truthful and the planner is motivated to continue implementing the allocation. The latter translates into a sequence of *societal credibility constraints* that place lower bounds on continuation

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utilitarian payoffs.[1] We call allocations that satisfy these conditions credible and focus on optimal (according to some societal criterion) credible allocations.

Our analysis begins with several two period examples that illustrate the main themes of the paper. These examples assume, respectively, a committed planner with utilitarian preferences, a committed paternalistic planner who discounts the future less heavily than the agents and a utilitarian planner with a limited ability to commit. Each example features privately informed agents. The environments contained in the first two examples have been considered previously[2] and provide benchmarks for assessing the implications of limited commitment. The third example supplements the constraint set of the other two with a "credibility constraint" that places a lower bound on continuation societal payoffs. In the simple example, this constraint is assumed, in the infinite-horizon model that follows it is derived explicitly as an equilibrium restriction. Intuitively, a society cannot commit to implementing continuation (period 2) allocations with societal payoffs that fall below this bound.

The optimal allocation in each example satisfies a "conditional inverted Euler equation". These equations are reminiscent of traditional Euler equations, but they are expressed in terms of *reciprocals* of marginal utilities of consumption rather than the marginal utilities themselves. They hold for each agent at each date conditional on the agent's past history. With commitment these conditions give rise to intertemporal wedges between an agent's expected intertemporal marginal rate of substitution and the intertemporal shadow price of resources.[3] Intuitively, if an agent transfers resources from one period to the next, she undermines her incentives to exert effort in the succeeding period. This "incentive effect" introduces an additional societal cost of saving that results in the intertemporal wedge. With limited commitment, an additional "credibility effect" is introduced that may reinforce or offset the incentive effect depending on the agent. In particular, under log preference-Cobb Douglas technology assumptions, the rich face an unambiguously positive intertemporal wedge. The planner cannot commit

[1] Sleet and Yeltekin (2008) consider economies that feature probabilistic voting that give rise to similar equilibrium constraints on utilitarian - or mean - continuation payoffs in some cases and, in others, imply that the probability of electoral success is tied to agent payoffs via some other preference aggregator that must be similarly maintained above a lower bound.

[2] By, inter alia, Golsov, Kocherlakota and Tsyvinski (2003), Kocherlakota (2005) and Albanesi and Sleet (2006) in the first case and Farhi and Werning (2007) in the second.

[3] Golosov et al (2003) provide a very general derivation of this result.

to too much inequality ex post. In a decentralisation of the optimal allocation, rich agents must be deterred from saving to the point where their intertemporal marginal rate of substitution is equated with the intertemporal shadow price both because of the incentive effect and because of a "credibility effect": society cannot refrain from redistributing from these agents if they accumulate too much. The incentive and credibility effects are reinforcing. On the other hand, the credibility effect implies that the poor should be encouraged to save, since otherwise society might be unable to refrain from redistributing towards these agents. For the poor the credibility and incentive effects are in opposition and implications for the intertemporal wedge are analytically ambiguous.

In addition to conditional inverted Euler equations, the optimal allocations in each setting satisfy unconditional inverted Euler equations that integrate out individual histories. These equations provide joint restrictions on intertemporal shadow prices and population moments of consumption. Extending our analysis to settings with aggregate public shocks, we obtain a shadow asset pricing kernel for each environment. Kocherlakota and Pistaferri (2008) derive such a pricing kernel in a commitment economy and interpret it as a market price. They argue that it does a better job in reconciling asset prices and consumption data with plausible specifications of preferences than do the pricing kernels from complete or "standard" incomplete market models. Hence, this kernel offers a resolution of the equity premium puzzle. The shadow asset pricing kernel with a paternalistic planner coincides with that from the standard commitment economy up to a multiplicative constant. Hence, it delivers the same restrictions on consumption and risk premia. Under limited commitment, the kernel incorporates an additional "credibility pricing factor". In the special case of log utility and Cobb-Douglas technology this factor is equal to the constant one, and the kernel is identical to that derived by Kocherlakota and Pistaferri.

We provide a tax-market implementation of the optimal allocation in each environment. In the limited commitment environment capital taxation is progressive in the sense that an agent's expected marginal asset tax in the subsequent period is increasing in her current consumption. Thus, consistent with our description of intertemporal wedges, the accumulation of poorer agents is subsidised, while that of richer agents is taxed. In the commitment case, conditional expected marginal asset taxes are zero for all agents and do not exhibit progressivity.[4] Farhi and Werning (2007) have previously shown that capital taxes are progressive in paternalistic settings and this is true in our second example. In addition, all agents in this setting face an expected capital subsidy. Since the paternalistic planner discounts the future less heavily than agents, implementation of its optimal allocation requires the promotion of saving via asset subsidies for all. In contrast with limited commitment, asset subsidies are only available for the poor.

In the remainder of the paper, we extend the model to infinite period horizon settings and formalise credibility as an equilibrium concept in a repeated game played by an uncommitted planner and privately informed agents. This allows us to explicitly derive societal commitment constraints as equilibrium restrictions. The set of credible (i.e. equilibrium) allocations is large, although numerical calculations indicate that the set excludes the (infinite horizon) optimal allocation with commitment. This allocation features all agents ultimately being absorbed by either an immiserating state in which they consume nothing and work a maximal amount or a high utility state in which they do not work at all. Such limiting inequality is not sustainable.

We focus on optimal credible allocations and show that they solve the problem of a committed "pseudo" planner whose preferences are perturbed relative to those of the true planner. With log preferences, we can disentangle the pseudo-planners attitudes towards capital accumulation and inequality. In this case, the pseudo-planner's preferences feature a "split personality" in the sense that she uses one endogenous societal discounting scheme to evaluate the payoff from the dispersion of agent utilities around an "aggregate capital growth path" and another to evaluate the payoff from this path itself. The first discounting scheme weakly exceeds that of the agent and strictly exceeds it when the societal credibility constraints bind. The second is a convolution of this first scheme and the agents' scheme. This pseudo-planner can be interpreted as a hybrid of the committed utilitarian and the committed paternalistic planner.

We obtain a recursive formulation of the pseudo-planner's problem using the techniques of Marcet and Marimon (1999) and implement it numerically. We conclude with a quantitative assessment of optimal taxes in this environment.

[4] See Albanesi-Sleet (2006) and Kocherlakota (2005) for earlier and, in the case of the latter, more general derivations of this result.

2 Three simple economies

To begin with we use three simple two period examples to convey our basic results.[5]

2.1 Basic environment

Preferences Suppose that the economy lasts for two periods. The economy is inhabited by a continuum of initially identical agents and a planner. Agents consume and exert effort in each period of their life. They receive private shocks $\theta_t \in \Theta = \{\widehat{\theta}_1, \widehat{\theta}_2\}$, $\widehat{\theta}_1 < \widehat{\theta}_2$, that affect their disutility from leisure.[6] These shocks are assumed to be i.i.d. with distribution π. Probability distributions over histories of shocks $\theta^t = \{\theta_r\}_{r=1}^t$ are denoted π^t. We appeal to a law of large numbers (see, for e.g. Sun (2006)) and interpret $\pi^t(E)$ as the fraction of agents with shock history $E \subset \Theta^t$. Agent preferences over consumption-effort streams are given by:

$$\sum_{t=1}^{2} \beta^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi(\theta^t),$$

where $u : \mathbb{R}_+ \to \mathbf{D} \subset \mathbb{R} \cup \{-\infty\}$ and $v : [0, T] \to \mathbf{L} \subset \mathbb{R}$ are the agent's current utility from consumption and disutility from effort. $\mathbf{D}$ and $\mathbf{L}$ denote the ranges of u and ν respectively and T is the agent's total per period time endowment. Additionally, u and ν are assumed to be continuously differentiable on the interior of their domains and strictly concave, u is assumed strictly increasing and ν strictly decreasing. The preference shock multiplicatively perturbs disutility from effort. $\beta \in (0, 1)$ is a discount factor.

The planner is utilitarian so that her (continuation) preferences at each date $r = 1$, 2 are given by:

$$W_r = \sum_{t=r}^{2} \beta^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi(\theta^t).$$

Agents prefer to work less if they draw the low shock value $\widehat{\theta}_1$ and, since the planner is benevolent, she would prefer to work

[5] We thank Marco Bassetto for emphasizing the value of such examples to us.

[6] θ may be interpreted as a health shock that makes work more or less costly or, with some slight respecification of the model, a productivity shock.

them less in such circumstances. However, agents must be motivated to reveal their private shocks and this disrupts the provision of taste shock insurance.

Technologies and constraints In each period t, the aggregate labor input $L_t = \sum_{\Theta^t} e_t(\theta^t)\pi^t(\theta^t)$ is combined with aggregate capital K_t to produce output. The initial capital endowment K_1 is given. Output in each period is given by:

$$Y_t = F(K_t, L_t)$$

where $F : \mathbb{R}_+ \times [0, T] \to \mathbb{R}_+$ is smooth, strictly concave and increasing. An allocation $\{\{c_t, e_t\}_{t=1}^{\infty}, K_2\}$ is resource-feasible given K_1 if for $t = 1, 2$,

$$\sum_{\Theta^t} c_t(\theta^t)\pi^t(\theta^t) + K_{t+1} \leq F(K_t, L_t), \tag{1}$$

where $K_3 := 0$. Given that taste shocks are private and agents must be motivated to reveal them, allocations must satisfy the following temporary incentive constraints, for period 1 and each i and $j \neq i$,

$$\begin{aligned} u(c_1(\widehat{\theta}_i)) + \widehat{\theta}_i v(e_1(\widehat{\theta}_i)) + \beta \sum_{k=1}^{2} [u(c_2(\widehat{\theta}_i, \widehat{\theta}_k)) + \widehat{\theta}_k v(e_2(\widehat{\theta}_i, \widehat{\theta}_k))]\pi(\theta_k) \geq \\ u(c_1(\widehat{\theta}_j)) + \widehat{\theta}_i v(e_1(\widehat{\theta}_j)) + \beta \sum_{k=1}^{2} [u(c_2(\widehat{\theta}_j, \widehat{\theta}_k)) + \widehat{\theta}_k v(e_2(\widehat{\theta}_j, \widehat{\theta}_k))]\pi(\theta_k) \end{aligned} \tag{2}$$

and for period 2 and each k, i and $j \neq i$,

$$u(c_2(\widehat{\theta}_k, \widehat{\theta}_i)) + \widehat{\theta}_i v(e_2(\widehat{\theta}_k, \widehat{\theta}_i)) \geq u(c_2(\widehat{\theta}_k, \widehat{\theta}_j)) + \widehat{\theta}_i v(e_2(\widehat{\theta}_k, \widehat{\theta}_j)). \tag{3}$$

The first of these constraints implies that it is optimal for an agent to be truthful in the first period given that it is truthful in the second. The second constraint implies that it is indeed optimal for the agent to be truthful in the second period. We will index these constraints by their dates and histories of shocks so that (2) will be labelled the $(1, \widehat{\theta}_i)$-th constraint and (3) the $(2, \widehat{\theta}_k, \widehat{\theta}_i)$-th constraint.

2.2 A first environment: Commitment

To begin with we assume that the planner can commit to allocations. Given K_1, she selects an allocation $\{\{c_t, e_t\}_{t=1}^{\infty}, K_2\}$ to solve

$$\sup_{\{\{c_t,e_t\}_{t=1}^2,K_2\}} \sum_{t=1}^{2} \beta^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi(\theta^t). \tag{4}$$

subject to (1), (2), (3) and the constraints c_t, $K_2 \geq 0$ and $e_t \in [0, T]$. Denoting the shadow price of resources in period i by q_i, the Lagrange multiplier on the period (t, θ^t)-th incentive constraint by $\eta_t(\theta^t)$ and the optimal allocation with $*$'s, the first order conditions for consumption are given by:

$$q_1 = \left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right] u'(c_1^*(\widehat{\theta}_i)) \tag{5}$$

and

$$q_2 = \left[1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)} + \eta_2(\widehat{\theta}_i, \widehat{\theta}_k) - \eta_2(\widehat{\theta}_i, \widehat{\theta}_l)\frac{\pi(\widehat{\theta}_l)}{\pi(\widehat{\theta}_k)}\right] \beta u'(c_2^*(\widehat{\theta}_i, \widehat{\theta}_k)). \tag{6}$$

Anticipating our later recursive formulations, the planner may be interpreted as assigning all agents the Pareto weight $\gamma_1 = 1$ in the initial period and then updating this to the effective Pareto weight of $\gamma_2 = 1 + \eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}$ in the second period. In both periods the distribution of effective Pareto weights has a cross sectional mean of 1. However, in the second period, they are more dispersed capturing the inequality in continuation Pareto weights, utilities and consumption in period 2 that the optimal provision of incentives in period 1 implies. This dispersion of Pareto weights and utilities is desirable ex ante, but ex post it is costly for a utilitarian planner to provide. An uncommitted planner would be tempted to renege. The first order condition for K_2 is:

$$-q_1 + q_2 F_K(K_2^*, L_2^*) = 0. \tag{7}$$

Combined these conditions imply the well known conditional inverted Euler equation[7]:

$$E\left[\frac{1}{u'(c_2^*)}\middle|\widehat{\theta}_i\right] = \beta F_K(K_2^*, L_2^*)\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right], \tag{8}$$

[7] See, for example, Rogerson (1985) and Golosov et al (2003) for a very general derivation.

where $E\left[\cdot|\widehat{\theta}_i\right]$ denotes an expectation conditional on $\theta_1 = \widehat{\theta}_i$. (8) implies, after an application of Jensen's inequality,

$$u'(c_1^*(\widehat{\theta}_i)) \leq F_K(K_2^*, L_2^*)\beta E\left[u'(c_2^*)|\widehat{\theta}_i\right]$$

with strict inequality if some of the period 2 incentive constraints are binding. Thus there is a wedge between intertemporal marginal rates of substitution $\beta\frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))}$ and the intertemporal marginal rate of transformation $\frac{1}{F_K(K_2^*,L_2^*)}$. Implementation of this allocation in a market economy requires some supplementary institution that prevents agents from equating their intertemporal marginal rates of substitution to $\frac{1}{F_K(K_2^*,L_2^*)}$. The tax code provides one such institution. However, as Albanesi and Sleet (2006) and Kocherlakota (2005) (Collectively: ASK) have shown, there are some subtleties in moving between wedges and taxes. For benchmarking we purposes, we briefly elaborate these.

Suppose a two period market economy with taxes in which agents are initially endowed with a quantity of claims b_1 to period 1 consumption, a market for claims to period 2 consumption opens in period 1 and in each period t, agents pay taxes in periods 1 and 2 according to some pair of schedules:

$$T_t(\theta^t, b_t) = \tau_t^0(\theta^t) + \tau_t^1(\theta^t)b_t,$$

where τ_t^0 and τ_t^1 depend on histories of reported shocks.[8] Agents in the market economy solve:

$$\sup_{\{\{c_t,e_t\}_{t=1}^{\infty},b_2\}} \sum_{t=1}^{2} \beta^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi(\theta^t)$$

subject to:

$$\begin{aligned} b_1 &= c_1(\theta_1) + T_1(\theta_1, b_1) + Qb_2(\theta_1) \\ b_2(\theta_1) &= c_2(\theta_1, \theta_2) + T_1(\theta_1, b_2(\theta_1)), \end{aligned}$$

where b_2 gives the agent's period 2 history-contingent saving and Q is the market price for claims. It is natural to consider decentralizations in which $Q = \frac{q_2}{q_1} = \frac{1}{F_K(K_2^*,L_2^*)}$. Given K_1, the tax system $\{T_t\}_{t=1}^2$ and the initial claim endowment b_1 must

[8] Provided histories of shocks are measurable with respect to histories of efforts, the tax system can be respecified in terms of efforts and asset holdings.

be selected so that agents truthfully report their types and choose the desired (market clearing) allocation $\{\{c_t^*, e_t^*\}, K_2^*\}$. As ASK have shown, to rule out joint reporting (effort) and saving deviations, the period 2 marginal asset tax $\tau_2^1(\theta^2)$ must depend on the realized shock in period 2. In particular, implementation can be achieved by setting each $\tau_2^1(\theta^2)$ such that:

$$Qu'(c_1^*(\theta_1)) = \beta(1 - \tau_2^1(\theta^2))u'(c_2^*(\theta^2)). \tag{9}$$

Combining this expression with (8) implies that conditional expectation of period 2 asset taxes is zero:

$$\sum_{\Theta} \tau_2^1(\theta_1, \theta)\pi(\theta) = 0.$$

Before proceeding to alternative environments, we consider the unconditional counterpart of the inverted Euler equation (8)

$$QE\left[\frac{1}{u'(c_2^*)}\right] = \beta E\left[\frac{1}{u'(c_1^*)}\right]. \tag{10}$$

Applying an appropriate law of large numbers, this expression can be interpreted as a joint restriction on the shadow (or, in the implementation, market) price of claims Q and population moments of marginal utilities. Kocherlakota and Pistaferri (2008) extend this basic framework to include publicly observable aggregate shocks $\{Z_t\}_{t=1}^2$. These shocks could, in principle, be to productivity, government spending or they could be common and publicly observable effort taste shocks. A version of (10) conditional on aggregate shocks holds in this environment:

$$Q(Z^2|Z_1)E\left[\frac{1}{u'(c_2^*)}\middle| Z^2\right] = \beta E\left[\frac{1}{u'(c_1^*)}\middle| Z^1\right]\Lambda(Z_2|Z_1), \tag{11}$$

where $\Lambda(Z_2|Z_1)$ is the probability of Z_2 conditional on Z_1 and Q is now the period 1 shadow or market pricing kernel for period 2 goods contingent on the aggregate states now and at the time of delivery. Kocherlakota-Pistaferri (2008) consider the positive content of (11). They ask whether expressions of this sort can reconcile observed asset market prices and consumption data with plausible assumptions about the risk aversion of agents. They argue that they can.

2.3 A second environment: Paternalism

We now assume that the planner uses the discount factor $\widehat{\beta} = \beta(1+\phi)$ rather than β to evaluate future agent payoffs. "The planner knows best" and solves:

$$\sup_{\{\{c_t,e_t\}_{t=1}^{2},K_2\}} \sum_{t=1}^{2} [\beta(1+\phi)]^{t-1} \sum_{\Theta^t} \left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right] \pi(\theta^t)$$

subject to the same constraints as before.[9] The first order conditions from this planning problem are now (5), (7) and

$$\begin{aligned} q_2 \quad = \quad & \left[\frac{\phi}{1+\phi} + \frac{1}{1+\phi}\left[1+\eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right] + \eta_2(\widehat{\theta}_i,\widehat{\theta}_k) - \eta_2(\widehat{\theta}_i,\widehat{\theta}_l)\frac{\pi(\widehat{\theta}_l)}{\pi(\widehat{\theta}_k)}\right] \\ & \times\beta(1+\phi)u'(c_2^*(\widehat{\theta}_i,\widehat{\theta}_k)). \end{aligned} \qquad (12)$$

Only, the first order condition for period 2 consumption has changed. Again anticipating our later recursive formulation, $\frac{\phi}{1+\phi} + \frac{1}{1+\phi}\left[1+\eta_1(\widehat{\theta}_i) - \eta_1(\widehat{\theta}_j)\frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}\right]$ can be interpreted as an updated Pareto weight for period 2. Now the dispersion in period 2 effective Pareto weights induced by the provision of period 1 incentives is moderated by the $\frac{1}{1+\phi}$ term. Intuitively, the paternalistic planner is more patient than the agents and more averse to inequality in period 2. She is willing to trade less insurance in period 1 off against less inequality in period 2.

The conditional inverted Euler equation in this setting is:

$$q_2 E\left[\frac{1}{u'(c_2^*)}\middle|\widehat{\theta}_i\right] = \beta(1+\phi)\left[\frac{\phi}{1+\phi} + \frac{1}{1+\phi}\left(\frac{q_1}{u'(c_1^*(\widehat{\theta}_i))}\right)\right]. \qquad (13)$$

The first order condition for period 1 consumption implies:

$$q_1 E\left[\frac{1}{u'(c_1^*)}\right] = 1$$

which when combined with (7) and (13) implies:

$$E\left[\frac{1}{u'(c_2^*)}\middle|\widehat{\theta}_i\right] = \beta(1+\phi)F_K(K_2^*,L_2^*)\left[\frac{\phi}{1+\phi}E\left[\frac{1}{u'(c_1^*)}\right]\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right]^{-1} + \frac{1}{1+\phi}\right]\frac{1}{u'(c_1^*(\widehat{\theta}_i))} \qquad (14)$$

[9] See Phelan (2006) and Farhi and Werning (2007) for interpretations of these societal preferences in an intergenerational context.

or

$$1 \leq \beta(1+\phi)F_K(K_2^*, L_2^*)H(\widehat{\theta}_i)\frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))}, \tag{15}$$

where $H(\widehat{\theta}_i) = \frac{\phi}{1+\phi}E\left[\frac{1}{u'(c_1^*)}\right]\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right]^{-1} + \frac{1}{1+\phi}$. (15) has indeterminate implications for the sign of the intertemporal wedge $\beta F_K(K_2^*, L_2^*)\frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))} - 1$. We identify three distinct forces. First, the standard period 2 incentive effect present in environment 1 is present here. This works in the direction of a positive wedge and ensures the inequality in (15). Second, the higher planner discount factor introduces the $1+\phi$ term into (15) and works in the opposite direction. Third, when the the period 1 incentive constraints are binding and there is variation in agent consumption in period 1, $H(\widehat{\theta}_1) < 1 < H(\widehat{\theta}_2)$. For agents who receive the low taste shock $\widehat{\theta}_1$ in period 1, work harder and, in consumption terms, are richer, the H term contributes to a positive wedge. For other agents, it contributes to a negative one. Intuitively, if the paternalistic planner was to decentralize her preferred allocation using markets and taxes, her concern with period 2 incentives which are undermined by higher period 2 consumption, would work in the direction of deterring agent saving, while the greater value the planner attaches to period 2 utility would work in the direction of encouraging it. Her desire to avoid too much period 2 inequality works in the other direction of encouraging saving amongst the period 1 poor (the $\widehat{\theta}_2$-shock agents) and discouraging it amongst the rich (the $\widehat{\theta}_1$-shock agents).

Although the model has indeterminate implications for wedges it has sharper ones for marginal asset taxes. In particular, (9), the definition of market prices and (13) imply that:

$$E\left[\tau_2^1\middle|\widehat{\theta}_i\right] = -\phi\frac{u'(c_1^*(\widehat{\theta}_i))}{q_1}. \tag{16}$$

And so conditional expected marginal asset taxes are negative for all agents and are increasing in an agent's period 1 consumption. In this sense they are progressive. This result was first derived by Farhi and Werning (2007).

From (14), it follows that the *unconditional* inverted Euler equation is:

$$QE\left[\frac{1}{u'(c_2^*)}\right] = \beta(1+\phi)E\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right]. \tag{17}$$

In the extended environment with public aggregate shocks considered by Kocherlakota and Pistaferri, the analogue of (17) is:

$$Q(Z^2|Z^1)E\left[\frac{1}{u'(c_2^*)}\middle| Z^2\right] = \beta(1+\phi)E\left[\frac{1}{u'(c_1^*)}\middle| Z^1\right]\Lambda(Z^2|Z^1). \tag{18}$$

Thus, the "stochastic discount factor" in this case equals that from the commitment environment scaled by the constant $1+\phi$, i.e. $SDF = \beta(1+\phi)\frac{E\left[\frac{1}{u'(c_1^*)}\middle| Z^1\right]}{E\left[\frac{1}{u'(c_2^*)}\middle| Z^2\right]}$. Consequently, this model places the same restrictions on (shadow) return premia and consumption as does the commitment model. In particular, if S and B are two different bundles of claims (a stock and a bond, say) with contingent period 2 returns R^S and $R^B(Z^2)$, (18) implies

$$E\left[(R^S - R^B)\frac{E\left[\frac{1}{u'(c_1^*)}\middle| Z^1\right]}{E\left[\frac{1}{u'(c_2^*)}\middle| Z^2\right]}\right] = 0. \tag{19}$$

This is the restriction that Kocherlakota and Pistaferri (2008) test with R^S and R^B interpreted as market returns and $E\left[\frac{1}{u'(c_t^*)}\middle| Z^t\right]$ interpreted as a cross sectional moment.

2.4 A third environment: Limited commitment

Instead of altering the planner's preferences as in the paternalistic case, we now alter her constraints relative to the initial commitment environment. In particular, we supplement the constraint set with the additional "credibility constraint":

$$\sum_{\Theta^t}\left[u(c_t(\theta^t)) + \theta_t v(e_t(\theta^t))\right]\pi(\theta^t) \geq W(K_2), \tag{20}$$

where W is a smooth, increasing function. Later, in an infinite horizon setting, we will explicitly derive the function W as the worst equilibrium payoff function in a dynamic game played by the planner and the agents. Conditions of the form (20) will then emerge as equilibrium incentive constraints for *the planner.* For now, however, we will take the function as given and simply assume that the planner solves (4) subject to additional constraint (20).

If ϕ is interpreted as the multiplier on the credibility constraint, then the first order conditions for consumption from this problem are the same as in the paternalistic case (i.e. (5) and (12)). The only change is to the interpretation of ϕ. In particular, the conditional inverted Euler equation (13) holds. It coincides with the conditional inverted Euler equation from the commitment case only if the credibility constraint does not bind and $\phi = 0$. However, the first order condition for capital is different from both of the preceding cases. It is given by:

$$-q_1 + q_2 F_K\left(K_2^*, L_2^*\right) - \beta\phi W_K(K_2^*) = 0,$$

where W_K denotes the derivative of W with respect to capital. Equivalently,

$$\frac{q_1}{q_2} < \frac{q_1}{q_2}\left[1 + \phi + \phi\left(\frac{\beta W_K(K_2^*)}{q_1} - 1\right)\right] = F_K\left(K_2^*, L_2^*\right). \tag{21}$$

A small ε increase in societal saving in period 1 has the usual period 1 shadow cost of $q_1\varepsilon$ and period 2 shadow benefit of $q_2 F_K\left(K_2^*, L_2^*\right)\varepsilon$. In addition, there is a second shadow cost: higher period 2 capital stocks tighten the credibility constraint. Hence, the marginal cost of capital exceeds the shadow price ratio $\frac{q_1}{q_2}$. Combining (21) with (13) gives:

$$1 \leq \beta\left[\frac{H(\widehat{\theta}_i)}{1 + \frac{\phi}{1+\phi}\left(\frac{\beta W_K(K_2^*)}{q_1} - 1\right)}\right] F_K(K_2^*, L_2^*) H(\widehat{\theta}_i) \frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))}. \tag{22}$$

Hence, the wedge is positive if $\frac{H(\widehat{\theta}_i)}{1+\frac{\phi}{1+\phi}\left(\frac{\beta W_K(K_2^*)}{q_1} - 1\right)} \leq 0$ and indeterminate otherwise. We have previously noted that $H(\widehat{\theta}_1) \leq 1$ with strict inequality if $\phi > 0$. In later sections, we consider an infinitely horizon dynamic game without planner commitment and assume that $F(K, L) = K^\alpha L^{1-\alpha}$ and $u(c) = \ln c$. In this case, we show that the planner's outside option function W is of the form $W(K) = \underline{W} + \alpha \ln K$. Anticipating our later analysis, let us simply assume that F, u *and* W have these functional forms. Then manipulating the first order conditions and resource constraint gives $\frac{\beta W_K(K_2^*)}{q_1} = 1$, so that $F_K\left(K_2^*, L_2^*\right) = \frac{q_1}{q_2}[1 + \phi]$ and (22) reduces to

$$1 \leq \beta F_K(K_2^*, L_2^*) H(\widehat{\theta}_i) \frac{E\left[u'(c_2^*)|\widehat{\theta}_i\right]}{u'(c_1^*(\widehat{\theta}_i))}.$$

Since $H(\widehat{\theta}_1) < 1$, the intertemporal wedge is positive for "rich" agents who receive the low taste shock in period 1.

We may again consider implementing the optimal limited commitment allocation in a market economy. If the asset market price is set to $\frac{1}{F_K(K_2^*,L_2^*)}$ and taxes are set according to (9), then the inverted Euler equation implies:

$$E\left[\tau|\widehat{\theta}_i\right] = \frac{\phi}{1+\phi\left(\frac{\beta W_K(K_2^*)}{q_1}\right)}\left[\frac{\beta W_K(K_2^*)}{q_1} - \frac{u'(c_1^*(\widehat{\theta}_i))}{q_1}\right] \tag{23}$$

As in the paternalistic case (16), we observe that (when the credibility constraint binds and $\phi > 0$) the expected marginal asset tax is increasing in $c_1^*(\widehat{\theta}_i)$ and in this sense is progressive. Now, however, the tax formula includes an additional positive term $\frac{\phi\frac{\beta W_K(K_2^*)}{q_1}}{1+\phi\frac{\beta W_K(K_2^*)}{q_1}}$. Intuitively, like her paternalistic counterpart, the limited commitment planner seeks to moderate ex post inequality since, in this case, it relaxes the credibility constraint. This underpins the dependence of expected marginal asset taxes in period 2 on consumption in period 1. In contrast to the paternalistic case, however, additional capital in the second period confers an additional social cost: it tightens rather than relaxes the credibility constraint. Thus the limited commitment planner does not seek to encourage capital taxation to the degree that the paternalistic planner does. We can nest the expected marginal tax formulas by:

$$E\left[\tau|\widehat{\theta}_i\right] = A_0^i - A_1^i u'(c_1^*(\widehat{\theta}_i)), \tag{24}$$

where $i = \{C, P, LC\}$ for commitment, paternalistic and limited-commitment economies. We observe that in the commitment economy, $A_0^C = A_1^C = 0$, in the paternalistic economy $A_1^P > A_0^P = 0$ and in the limited commitment economy with binding credibility constraint $A_1^{LC} > 0$, $A_0^{LC} > 0$.

The unconditional inverted Euler equation with limited commitment is:

$$E\left[\frac{1}{u'(c_2^*)}\right] = \beta\left[\frac{1}{1+\frac{\phi}{1+\phi}\left(\frac{\beta W_K(K_2^*)}{q_1}-1\right)}\right] F_K(K_2^*, L_2^*) E\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right]. \tag{25}$$

In the special case described above with $\frac{\beta W_K(K_2^*)}{q_1} = 1$ this reduces to

$$QE\left[\frac{1}{u'(c_2^*)}\right] = \beta E\left[\frac{1}{u'(c_1^*(\widehat{\theta}_i))}\right].$$

where $Q = \frac{1}{F_K(K_2^*,L_2^*)}$. Hence, in this case, there is no difference between the commitment and limited-commitment unconditional inverted Euler equations. More generally, after the incorporation of aggregate observable shocks, the asset pricing restriction (19) holds with the stochastic discount factor set to $\beta\mathcal{K}(Z^2)\frac{E\left[\frac{1}{u'(c_1^*)}\middle|Z^1\right]}{E\left[\frac{1}{u'(c_2^*)}\middle|Z^2\right]}$ and $\mathcal{K}(Z^2) = \left[1 + \frac{\phi(Z^2)}{1+\phi(Z^2)}\left(\frac{\beta W_K(K_2^*(Z^2))}{q_1(Z^1)} - 1\right)\right]^{-1}$ interpreted as an additional "credibility factor". This factor equals 1 in the log-Cobb Douglas case.

We conclude this section with a more detailed analysis of the particular case $u(c) = \ln c$, $v(e) = \ln(T - e)$, $F(K, L) = K^\alpha L^{1-\alpha}$ and $W(K) = \underline{W} + \alpha \ln K$. Again, this anticipates our later analysis in which the functional form W is derived explicitly. Let $\widetilde{c}_t = c_t/K_t^\alpha$ and $\kappa_2 = K_2/K_1^\alpha$, where κ_2 is interpreted as a "capital growth term". We can rewrite the planner's problem as:

$$\alpha \ln K_1 + \sup_{\{\{\widetilde{c}_t, e_t\}_{t=1}^2, \kappa_2\}} \sum_{t=1}^{2} \beta^{t-1} \sum_{\Theta^t} \left[\ln(\widetilde{c}_t(\theta^t)) + \theta_t \ln(T - e_t(\theta^t))\right] \pi(\theta^t) + \beta\alpha \ln \kappa_2 \tag{26}$$

subject to the resource constraints, for $t = 1, 2$,

$$\sum_{\Theta^t} \widetilde{c}_t(\theta^t)\pi^t(\theta^t) + \widetilde{\kappa}_{t+1} \leq L_t^{1-\alpha}, \tag{27}$$

with $\kappa_3 := 0$, the temporary incentive constraints for period 1 and each i and $j \neq i$,

$$\begin{aligned} &\ln(\widetilde{c}_1(\widehat{\theta}_i)) + \widehat{\theta}_i \ln(T - e_1(\widehat{\theta}_i)) + \beta \sum [\ln(\widetilde{c}_2(\widehat{\theta}_i, \widehat{\theta}_k)) + \widehat{\theta}_k \ln(T - e_2(\widehat{\theta}_i, \widehat{\theta}_k))]\pi(\theta_k) \geq \\ &\quad \ln(\widetilde{c}_1(\widehat{\theta}_j)) + \widehat{\theta}_i \ln(T - e_1(\widehat{\theta}_j)) + \beta \sum [\ln(\widetilde{c}_2(\widehat{\theta}_j, \widehat{\theta}_k)) + \widehat{\theta}_k \ln(T - e_2(\widehat{\theta}_j, \widehat{\theta}_k))]\pi(\theta_k), \end{aligned} \tag{28}$$

and for period 2 and each k, i and $j \neq i$,

$$\ln(\widetilde{c}_2(\widehat{\theta}_k, \widehat{\theta}_i)) + \widehat{\theta}_i \ln(T - e_2(\widehat{\theta}_k, \widehat{\theta}_i)) \geq \ln(\widetilde{c}_2(\widehat{\theta}_k, \widehat{\theta}_j)) + \widehat{\theta}_i \ln(T - e_2(\widehat{\theta}_k, \widehat{\theta}_j)), \tag{29}$$

and the credibility constraint:

$$\sum_{\Theta^2} \left[\ln(\widetilde{c}_2(\theta^2)) + \theta_2 \ln(T - e_2(\theta^2))\right] \pi(\theta^2) \geq \underline{W}. \tag{30}$$

We may rewrite problem (26) by first defining a Lagrangian that incorporates the credibility constraint. By rearranging terms in this Lagrangian, the planner's problem can be reformulated as:

$$\sup_{\{\{\widetilde{c}_t, e_t\}_{t=1}^2, \kappa_2\}} \sum_{t=1}^{2} [\beta(1+\phi)]^{t-1} \sum_{\Theta^t} \left[\ln(\widetilde{c}_t(\theta^t)) + \theta_t \ln(T - e_t(\theta^t))\right] \pi(\theta^t) + \beta\alpha \ln \kappa_2 \tag{31}$$

subject to (27)-(29) with ϕ equal to the optimizing multiplier on the credibility constraint. We can interpret this problem as that of a committed "pseudo-planner" with the perturbed discount factor $\beta(1+\phi)$ applied to the utility aggregate $\sum_{\Theta^2}\left[\ln(\widetilde{c}_2(\theta^2)) + \theta_2 \ln(T - e_2(\theta^2))\right]\pi(\theta^2)$ and the standard discount factor β applied to the "capital growth term" $\alpha \ln \kappa_2$. In this sense, the pseudo-planner is a hybrid of a paternalistic planner with discount factor $\beta(1+\phi)$ and a standard planner with discount factor β.

Assuming that the agent's effort choices in period 1 are interior, the first order conditions for $\widetilde{c}_1$, e_1 and κ_2 and the resource constraint imply that the optimal choice of κ_2 in (31) is independent of ϕ. Hence, the optimal choice of capital in period 2 is the same with or without commitment. What differs across the two environments is the amount of insurance offered against taste shocks in period 1 and the amount of consumption and effort inequality in period 2. Both are reduced.

3 An infinite horizon environment

In the remainder of the paper, we consider infinite horizon settings. This allows us to derive credibility constraints explicitly as equilibrium restrictions in a dynamic game and to endogenise the outside option function W. We sharpen our characterisations by focussing on the log case and translate the observations of the preceding section to the infinite horizon setting. Full characterisation of the optimal solution even in this case requires numerical analysis. In particular, it is difficult to analytically determine in which periods credibility constraints bind in the optimal credible allocation problem. Consequently, we seek recursive formulations of our problems[10] that facilitate numerical implementation.

3.1 Preferences and technologies

A continuum of infinitely-lived agents is initially partitioned into a measure space $(\mathbb{R}, \mathcal{B}, \Phi)$ of types w, where $\mathcal{B}$ is the Borel sigma algebra. At this point, we interpret a type simply as a public signal that policy makers can condition allocations upon. Later we will identify types with initial utility promises or Pareto weights that are assigned to agents a priori. As before

[10] Our formulations utilise the methods of Marcet and Marimon (1999).

agents receive random taste shocks $\theta_t \in \Theta := \{\widehat{\theta}_i\}_{i\in\mathbf{I}}$, $\mathbf{I} = 1, 2, \cdots, I$ that affect their disutility from effort and that are i.i.d. across agents and time with distribution π. An allocation is now an infinite sequence of functions $\{c_t, e_t\}_{t=1}^{\infty}$ with for all t, $c_t : \Theta^t \to \mathbb{R}_+$ and $e_t : \Theta^t \to E \subset \mathbb{R}_+$. Assume that u and v are as before and denote their inverses by C and N. The payoff from an allocation is then:

$$\widetilde{U}(\{c_t, e_t\}_{t=1}^{\infty}) = \lim_{S\to\infty}\inf \sum_{t=1}^{S}\sum_{\Theta^t} \beta^{t-1}[u(c_t(\theta^t)) + \theta_t \nu(e_t(\theta^t))]\pi^t(\theta^t).$$

It will be convenient to describe allocations directly in terms of the stream of utility they provide rather than stream of resources they use. To that end, define an *individual allocation* (of utility) to be a sequence $\{\psi_t, \nu_t\}_{t=1}^{\infty}$ where $\psi_t : \Theta^t \to \mathbf{D}$ and $\upsilon_t : \Theta^t \to \mathbf{L}$ give an individual's utility from consumption and disutility from effort at t as functions of past shocks. An agent's payoff from such a sequence $\{\psi_t, \upsilon_t\}_{t=1}^{\infty}$ is then:

$$U(\{\psi_t, \upsilon_t\}_{t=1}^{\infty}) = \lim_{S\to\infty}\inf \sum_{t=1}^{S}\sum_{\Theta^t} \beta^{t-1}[\psi_t(\theta^t) + \theta_t \upsilon_t(\theta^t)]\pi^t(\theta^t).$$

Let **IA** denote the set of bounded individual allocations: $\mathbf{IA} := \{\{\psi_t, \upsilon_t\}_{t=1}^{\infty} | \ \{\psi_t, \upsilon_t\}_{t=1}^{\infty}$ is an individual allocation and $\lim_{S\to\infty} \sum_{t=1}^{S} \sum_{\Theta^t} \beta^{t-1} [|\psi_t(\theta^t)| + \theta_t|\upsilon_t(\theta^t)|]\pi^t(\theta^t) < \infty\}$.We define a *(population) allocation* $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$ to be a sequence of measurable functions $\varphi_t : \mathbb{R}\times\Theta^t \to \mathbb{R}$ and $\upsilon_t : \mathbb{R}\times\Theta^t \to \mathbb{R}$ and a sequence of capital stocks $\{K_{t+1}\}_{t=1}^{\infty}$ such that i) for all $w \in \mathbb{R}$, $\{\varphi_t(w,\cdot), \upsilon_t(w,\cdot)\}_{t=1}^{\infty} \in \mathbf{IA}$ and ii) $f(w) = U(\{\varphi_t(w,\cdot),\ \upsilon_t(w,\cdot)\}_{t=1}^{\infty})$ is Φ-integrable. These technical conditions on population allocations ensure that they have well defined utilitarian payoffs. A population allocation implies a collection of continuation individual allocations $\{\varphi_{t+r}, \upsilon_{t+r}|w, \theta^{t-1}\}_{r=0}^{\infty}$, $t \geq 1$, obtained after each individual history (w, θ^{t-1}). We denote the set of population allocations by **PA**.

As before, effort is combined with capital to produce output. Given an initial capital amount K_1, type distribution Φ and allocation $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$, the amount of output produced in period t is:

$$Y_t = F(K_t, L(\upsilon_t)),$$

where $L(\upsilon_t) = \int_{\mathbb{R}} \sum_{\Theta^t} N(\upsilon_t(w, \theta^t))\pi^t(\theta^t)\Phi(dw)$ is the aggregate labor input.

3.2 Resource and incentive-feasibility

As in our simple economies above, an allocation is feasible if it is resource-feasible and incentive-compatible. Given Φ and K_1, $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$ is *resource-feasible* if for all t,

$$\int_{\mathbb{R}} \sum_{\Theta^t} C(\varphi_t(w, \theta^t))\pi^t(\theta^t)\Phi(dw) + K_{t+1} \leq F(K_t, L(\upsilon_t)) \tag{32}$$

Let $m = \{m_t\}_{t=1}^{\infty}$ denote a reporting strategy, where $m_t : \Theta^t \to \Theta$ gives an agent's t-th period shock report as a function of his t-th period history of current and past shocks. The strategy m induces a sequence of functions $m^t : \Theta^t \to \Theta^t$ that give agent report histories contingent as functions of their true shock histories. An allocation is *incentive-compatible* if for Φ-a.e. w and all m,

$$\sum_{t=1}^{\infty} \sum_{\Theta^t} \beta^{t-1}[\varphi_t(w, \theta^t) + \theta_t \upsilon_t(w, \theta^t)]\pi^t(\theta^t) \geq \sum_{t=1}^{\infty} \sum_{\Theta^t} \beta^{t-1}[\varphi_t(w, m^t(\theta^t)) + \theta_t \upsilon_t(w, m^t(\theta^t))]\pi^t(\theta^t). \tag{33}$$

A population allocation is *fully incentive-feasible* if it satisfies the resource constraints (32) and the incentive constraint (33). To obtain recursive formulations of our problems we replace (33) with a sequence of *temporary incentive-compatibility constraints,* for Φ-a.e. w, all t, θ^{t-1}, i, $j \in \mathbf{I}$,

$$\begin{aligned} &\varphi_t(w, \theta^{t-1}, \widehat{\theta}_i) + \widehat{\theta}_i \upsilon_t(w, \theta^{t-1}, \widehat{\theta}_i) + \beta \sum_{r=1}^{\infty} \sum_{\Theta^r} \beta^{t-1}[\varphi_{t+r}(w, \theta^{t-1}, \widehat{\theta}_i, \theta^r) + \theta_{t+r} \upsilon_{t+r}(w, \theta^{t-1}, \widehat{\theta}_i, \theta^r)]\pi^r(\theta^r) \geq \\ &\varphi_t(w, \theta^{t-1}, \widehat{\theta}_j) + \widehat{\theta}_i \upsilon_t(w, \theta^{t-1}, \widehat{\theta}_j) + \beta \sum_{r=1}^{\infty} \sum_{\Theta^r} \beta^{t-1}[\varphi_{t+r}(w, \theta^{t-1}, \widehat{\theta}_j, \theta^r) + \theta_{t+r} \upsilon_{t+r}(w, \theta^{t-1}, \widehat{\theta}_j, \theta^r)]\pi^r(\theta^r). \end{aligned} \tag{34}$$

Let $\Gamma_0(K_1, \Phi)$ denote the set of *temporary-incentive-feasible* population allocations satisfying (32) and (34). It is well known that (33) implies (34). In addition, if a population allocation satisfies the tail condition:

$$\lim_{S \to \infty} \sup_{m} \beta^S \sum_{t=1}^{\infty} \sum_{\Theta^t} \beta^{t-1}[\varphi_{T-1+t}(w, \theta^{S-1}, m^t(\theta^t)) + \theta_{S-1+t} \upsilon_{T-1+t}(w, \theta^{S-1}, m^t(\theta^t))]\pi^t(\theta^t) = 0 \tag{35}$$

and (34), then it satisfies (33). To verify that an element of $\Gamma_0(K_1, \Phi)$ is fully incentive-feasible, it suffices to verify that it satisfies (35).

4 The infinite-horizon optimal commitment problem

As in the first simple environment, the commitment problem entails maximizing a societal payoff subject to resource and incentive-feasibility conditions. We generalise the earlier objective by allowing supposing that a planner attaches a Pareto weight $\gamma_0(w)$ to members of the w-indexed subpopulation of agents. Assume that the weighting function γ_0 is non-negative-valued and integrable with $\int \gamma_0(w)\Phi(dw) = 1$. Let Ψ_0 denote the distribution over weights implied by Φ and γ_0. These weights may be thought of as primitive elements of the planner's preferences or the result of rewards and penalties accrued in earlier unmodelled periods. To economise on notation, we will rewrite allocations directly as functions of Pareto weights (since the planner would not choose to treat different w-populations of agents with the same Pareto weight differently). The infinite-horizon commitment problem is:

$$\sup_{\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty} \in \Gamma_0(K_1, \Psi_0)} \int_{\mathbb{R}} \gamma_0 U(\{\varphi_t(\gamma_0, \cdot), \upsilon_t(\gamma_0, \cdot)\}_{t=1}^{\infty}) \Psi_0(d\gamma_0). \tag{36}$$

The literature on efficient allocations with private information has tended to focus on dual problems in which a planner minimizes a cost aggregate subject to implementing a given distribution of utilities rather than the primal problem considered here. This is largely because the dual problem can be disaggregated into a family of "component planner problems" that have a natural and well known recursive formulation in terms of utility promises.[11] We will disaggregate the primal problem and use the alternative recursive formulation in terms of Pareto weights developed by Marcet and Marimon (1999).

4.1 Component planner formulations

The next lemma describes how (36) can be disaggregated into a collection of component planner problems. It is the counterpart of the dual component planner disaggregation of Atkeson and Lucas (1992) for our primal setting.

[11] For examples of the dual approach see, inter alia, Atkeson and Lucas (1992) who omit an effort choice and assume a private taste shock to consumption utility and Atkeson and Lucas (1995) who include an effort choice and assume a private productivity shock that is interpreted as a job opportunity.

Lemma 1 *Suppose that* $\{q_t\}_{t=1}^{\infty} \in \prod_{t=1}^{\infty}[\beta^{t-1}\underline{q}, \beta^{t-1}\overline{q}]$, *where* $0 < \underline{q} < \overline{q}$, $\{w_t\}_{t=1}^{\infty} \in \prod_{t=1}^{\infty}[\underline{w}, \overline{w}]$, *where* $0 < \underline{w} < \overline{w}$. *Let* $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}_{t=1}^{\infty} \in \mathbf{PA}$ *and* $\{L_t^*\}_{t=1}^{\infty} \in [0, T]^{\infty}$. *If for* Ψ_0*-a.e.* γ_0, $\varphi_t^*(\gamma_0, \cdot)$ *and* $\upsilon_t^*(\gamma_0, \cdot)$ *solve:*

$$V_1(\gamma_0) = \sup_{\{\psi_t, \nu_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \sum_{\theta^t} \{\beta^{t-1}\gamma_0[\psi_t(\theta^t) + \theta_t \nu_t(\theta^t)] - q_t[C(\psi_t(\theta^t)) + w_t N(\nu_t(\theta^t))]\}\pi^t(\theta^t) \tag{37}$$

subject to $\{\psi_t, \nu_t\}_{t=1}^{\infty} \in \mathbf{IA}$ *and (34), and if, for* $t > 1$, K_t^* *and* L_t^* *solve*

$$\sup_{K_t, L_t} F(K_t, L_t) - \frac{q_{t-1}}{q_t} K_t - w_t L_t \tag{38}$$

while for $t = 1$, L_1^* *solves*

$$\sup_{L_t} F(K_1, L_1) - w_1 L_1 \tag{39}$$

and for all t,

$$\int \sum_{\theta^t} C(\varphi_t^*(\gamma_0, \theta^t))\pi^t(\theta^t)\Psi_0(d\gamma_0) + K_{t+1}^* = F(K_t^*, L_t^*), \tag{40}$$

and

$$\int \sum_{\theta^t} N(\upsilon_t^*(\gamma_0, \theta^t))\pi^t(\theta^t)\Psi_0(d\gamma_0) = L_t^*, \tag{41}$$

where $K_1^* = K_1$, *then* $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}_{t=1}^{\infty}$ *solves (36).*

The Lemma 1 can be interpreted as a first welfare theorem for a particular market economy. In this economy, a representative firm hires capital K_t and L_t on competitive markets in each period at prices $\frac{q_{t-1}}{q_t}$ and w_t. A "component planner" is assigned to each agent. These component planners maximize the Pareto weighted utility net of cost of their client agents subject to the temporary incentive constraints. They do this by trading claims to output and labor with the firm and one another. The Lemma asserts that competitive equilibrium population allocations in this economy solve the planning problem (36).[12]

[12] A second welfare theorem for this environment is complicated by the difficulties in establishing the existence of an appropriate decentralising price sequence $\{q_t\}_{t=1}^{\infty}$.

4.1.1 The firm's problem

The firms problem (38) gives rise to the standard first order conditions:

$$
\begin{aligned}
-q_{t-1} + q_t F_K(K_t^*, L_t^*) &= 0; \\
-w_t + F_L(K_t^*, L_t^*) &= 0.
\end{aligned}
$$

4.1.2 Recursive component planner problem

The component planner problems (37) can be formulated recursively. Following Marcet and Marimon (1999), our recursive formulation relies on a manipulation of the component planning Lagrangian. Details of the method are described in Sleet and Yeltekin (2008). In this subsection, we sketch the formulation.

It can be readily verified that if an allocation solves a relaxed version of (37) with only the local upwards (i.e. the $(i, i+1)$) and the local downwards (i.e. the $(i, i-1)$) temporary incentive constraints, then it satisfies all of the temporary incentive constraints. Thus, we need only impose the local upwards and downwards constraints. Given this, define the date t-continuation component planning problem by:

$$
V_t(\gamma) = \sup_{\{\psi_r, \nu_r\}_{r=1}^{\infty} \in \Omega_1} \sum_{r=1}^{\infty} \sum_{\theta^r} \{\beta^{r-1}\gamma[\psi_r(\theta^r) + \theta_r \nu_r(\theta^r)] - q_{t+r-1}[C(\psi_r(\theta^r)) + w_{t+r-1} N(\nu_r(\theta^r))]\} \pi^r(\theta^r) \tag{42}
$$

where $\Omega_T = \{\{\psi_t, \nu_t\}_{t=1}^{\infty} \in \mathbf{IA} \mid \{\psi_t, \nu_t\}_{t=1}^{\infty}$ satisfies the local upward and downward temporary incentive constraints from period T onwards$\}$. Associated with this problem (42) is a (well defined) Lagrangian $\mathcal{L}_t : \mathbb{R}_+^{K(K-1)} \times \mathbf{IA} \to \mathbb{R}$ that incorporates only the current (i.e. the t-th period) local temporary incentive constraints: assigning them the Lagrangian multiplier $\eta = \{\eta_{i,j}\}_{i,j \in \mathbf{I}} \in \mathbb{R}_+^{K(K-1)}$, $\mathbf{I} = \{(i,j) : 1 \le i, j \le K,\ j = i-1 \text{ or } j = i+1\}$.

$$
\begin{aligned}
\mathcal{L}_t(\eta, \{\psi_r, \nu_r\}_{r=1}^{\infty}; \gamma) \quad = \quad & \sum_{r=1}^{\infty} \sum_{\theta^r} \{\beta^{r-1}\gamma[\psi_r(\theta^r) + \theta_r \nu_r(\theta^r)] - q_{t+r-1}[C(\psi_r(\theta^r)) + w_{t+r-1} N(\nu_r(\theta^r))]\} \pi^r(\theta^r) \\
& + \sum_{i,j \in \mathbf{I}} \eta_{i,j} \Bigg[\psi_1(\widehat{\theta}_i) + \widehat{\theta}_i \nu_1(\widehat{\theta}_i) + \beta \sum_{r=1}^{\infty} \sum_{\Theta^r} \beta^{r-1} [\psi_{r+1}(\widehat{\theta}_i, \theta^r) + \theta_{r+1} \nu_{r+1}(\widehat{\theta}_i, \theta^r)] \pi^r(\theta^r) \\
& -\psi_1(\widehat{\theta}_j) - \widehat{\theta}_i \nu_1(\widehat{\theta}_j) - \beta \sum_{r=1}^{\infty} \sum_{\Theta^r} \beta^{r-1} [\psi_{r+1}(\widehat{\theta}_j, \theta^r) + \theta_{r+1} \nu_{r+1}(\widehat{\theta}_j, \theta^r)] \pi^r(\theta^r) \Bigg],
\end{aligned} \tag{43}
$$

An application of the classical result of Luenberger. Theorem 1, p.217 establishes that $V_t(\gamma)$ is the optimal payoff from a saddle point problem involving $\mathcal{L}_t(\cdot;\gamma)$.and that an optimal multiplier exists for this problem.

Proposition 2 *V_t and $\mathcal{L}_t$ satisfy for each t and $\gamma \in \mathbb{R}_+$,*

$$V_t(\gamma) = \inf_{\eta \in \mathbb{R}_+^{K(K-1)}} \sup_{\{\psi_r, \nu_r\}_{r=1}^{\infty} \in \Omega_2} \mathcal{L}_t(\eta, \{\psi_r, \nu_r\}_{r=1}^{\infty}; \gamma). \tag{44}$$

Additionally, there exists an $\eta^* \in \mathbb{R}_+^{K(K-1)}$ that attains the infimum in (44).

It then follows from (44) at t and $t+1$ and a rearrangement of the terms in (43) that:

$$V_t(\gamma) = \inf_{\Lambda(\gamma)} \sum_{i=1}^{I} \{J_i(\rho_i(\zeta;\eta), \lambda_i(\zeta;\eta); q_t, w_t) + \beta V_{t+1}(\gamma_i'(\gamma;\eta))\} \pi(\widehat{\theta}_i), \tag{45}$$

where the indirect current payoff function is given by $J_i(\rho, \lambda; q_t, w_t) = \sup_{\psi}\{\rho\psi - q_t C(\psi)\} + \sup_v\{\widehat{\theta}_i \lambda v - q_t w_t N(v)\}$, the current utility weights are

$$\begin{aligned} \rho_i(\gamma;\eta) &= \gamma + \sum_{j:\{(i,j)\in\mathbf{I}\}} \eta_{i,j} - \sum_{j:\{(j,i)\in\mathbf{I}\}} \eta_{j,i} \frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)} \\ \lambda_i(\gamma;\eta) &= \gamma + \sum_{j:\{(i,j)\in\mathbf{I}\}} \eta_{i,j} - \sum_{j:\{(j,i)\in\mathbf{I}\}} \eta_{j,i} \frac{\widehat{\theta}_j}{\widehat{\theta}_i} \frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}, \end{aligned}$$

the updated *effective Pareto weights* are:

$$\gamma_i'(\gamma;\eta) = \gamma + \sum_{j:\{(i,j)\in\mathbf{I}\}} \eta_{i,j} - \sum_{j:\{(j,i)\in\mathbf{I}\}} \eta_{j,i} \frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)}. \tag{46}$$

and the constraint correspondence for current multiplier choices is $\Lambda(\zeta) = \{\eta \in \mathbb{R}_+^{K(K-1)} | \ \forall i,\ \gamma_i'(\zeta;\eta) \geq 0\}$ Thus, V_t is a value function in a dynamic programming problem that features the Lagrange multipliers η from the current incentive constraints as choice variables and effective Pareto weights as state variables. Optimal policy functions $\{\eta_t^*\}_{t=1}^{\infty}$ from the problems (45) and the updating functions $\{\gamma_i'\}$ can be used to construct a stochastic process for effective Pareto weights $\{\gamma_t^*\}_{t=1}^{*}$, with $\gamma_{t:}^* : \mathbb{R}_+ \times \Theta^{t-1} \to \mathbb{R}_+$. This, together with $\{\eta_t^*\}_{t=1}^{\infty}$ and the static optimizations used to define the indirect payoff functions J_i can be used to obtain an individual allocation $\{\psi_t^*(\gamma), v_t^*(\gamma)\}_{t=1}^{\infty}$ from the initial Pareto weight γ. We say that $\{\psi_t^*(\gamma), v_t^*(\gamma)\}_{t=1}^{\infty}$ *is induced by* $\{\eta_t^*\}_{t=1}^{\infty}$ *from* γ. .The following result is a corollary of those proven in Sleet and Yeltekin (2008):

Proposition 3 *1) There exists a unique sequence of policy functions* $\{\eta_t^*\}_{t=1}^{\infty}$*, with* $\eta_t^* : \mathbb{R}_+ \to \mathbb{R}_+$*, that attain the infima in the optimization problems (45); 2) each* η_t^* *is continuous; 3) Assume that the non-recursive component planner problem (37) has a solution, then the allocation* $\{u_t^*(\gamma), v_t^*(\gamma)\}_{t=1}^{\infty}$ *induced by* $\{\eta_t^*\}_{t=1}^{\infty}$ *from* γ *solves (37).*

It follows from (46) that the optimal process for effective Pareto weights $\{\gamma_t^*\}_{t=1}^{\infty}$ starting from some $\gamma \in \mathbb{R}_+$ evolves according to:

$$\gamma_{t+1}^*(\gamma, \theta^{t-1}, \widehat{\theta}_k) = \gamma_t^*(\gamma, \theta^{t-1}) + \varepsilon_{t+1}(\gamma, \theta^{t-1}, \widehat{\theta}_k), \tag{47}$$

where the sequence of incentive shocks $\{\varepsilon_{t+1}\}_{t=1}^{\infty}$ is obtained from the optimal incentive multipliers. These incentive shocks incorporate the future rewards and penalties for sending different current reports into the (effective) multiplier process. For example, if $K = 2$,only the upwards local constraint binds. Agents are motivated to report the low $\widehat{\theta}_1$ value (i.e. the low preference for leisure) and to work with a non-negative incentive shock $\varepsilon_{t+1}(\gamma, \theta^{t-1}, \widehat{\theta}_1) := \eta_{t,1,2}^*(\gamma_t^*(\gamma, \theta^{t-1})) \geq 0$ to their continuation effective Pareto weights if they make such a report $\widehat{\theta}_1$ and a non-positive shock $\varepsilon_{t+1}(\gamma, \theta^{t-1}, \widehat{\theta}_2) := -\eta_{t,1,2}^*(\gamma_t^*(\zeta, \theta^{t-1})) \frac{\pi(\widehat{\theta}_2)}{\pi(\widehat{\theta}_1)}$ otherwise. In addition, the incentive shocks satisfy: $E[\varepsilon_{t+1}|\gamma, \theta^{t-1}] = 0$. Consequently, an agent's optimal effective Pareto weight follows a non-negative martingale and, by the martingale convergence theorem almost surely converges. This is a force for continued spreading of agent effective Pareto weights and increasing inequality over time.

4.2 A numerical example

A complete analytical characterisation of the optimal allocation with commitment is quite complicated. This is largely because the economy is in transition and both the shadow resource price and the cross sectional distribution over the effective Pareto weights is evolving over time. In this section, we provide a numerical example.

In our example, $u(c) = \ln c$ and $v(e) = \ln(T - e)$. In this case, in those periods in which the lower bound on effort is not binding on agent, the shadow price of resources and the wage can be explicitly solved for. Denote their values in such periods by $\widehat{q}$ and $\widehat{w}$. If all agents begin period 1 with an identical unit (effective) Pareto weigh and the lower bound on effort is not binding for any agent, then the shadow price and the wage equal these values. Overtime, the distribution of effective Pareto weights becomes more dispersed and eventually some agents become sufficiently wealthy that the lower effort bound is binding. After this, the shadow resource price is lower (but non-negative) and must be solved for numerically. We posit a large upper bound on $\hat{T}$ the time taken to transition to an (approximate) limiting cross sectional distribution of effective Pareto weights from a degenerate initial distribution with mass at 1. We assume that after this date the shadow resource price is constant at a value to be determined. We choose an initial sequence of resource shadow prices $\{q_t^0\}_{t=1}^{\hat{T}}$ for the period $t = 1, \cdots, \hat{T}$ with $q_1^0 = \hat{q}$ and compute the solutions to the component planner problems at these prices. The resulting individual allocations can then be used (together with the initial Pareto distribution) to recover a sequence of cross sectional Pareto weight distributions, net aggregate resource costs and marginal products of labor. In all those periods in which net aggregate resource costs are positive, shadow resource prices are raised and in those in which aggregate resource costs are negative, shadow resource prices are lowered. The wages paid by component planners are adjusted to be closer to marginal products in each period. In this way, a new sequence of shadow prices and wages are obtained $\{q_t^1, w_t\}$. The exercise is then repeated until net resource costs are close to zero in each period and wages equal marginal products.

In all numerical simulations, we observe a spreading in the dispersion of effective Pareto weights until (nearly) all mass is placed on Pareto weights very close to zero or close to an upper bound such that agents do not work regardless of the shock. The following graph plots the (normalized) continuation utilitarian payoff over the first 50,000 periods of a simulation (the blue curve) of an economy in which the initial distribution of Pareto weights is degenerate at one. This continuation payoff falls relentlessly. The blackline is the (normalized) continuation utilitarian payoff in an economy with no insurance. As we show in the next section to be credible, an allocation must maintain normalized continuation utilitarian payoffs above the no insurance payoff. Clearly, the optimal allocation with commitment violates this constraint and is, therefore, not credible.

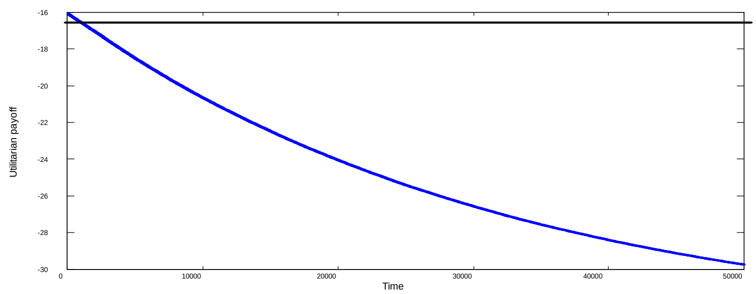

Figure 1: Continuation utilitarian payoffs with commitment

5 A policy game

In this section we describe a policy game. As before agent productivity shocks are not public information and the planner must induce agents to reveal information about them. The optimal allocation of the previous section relied on future allocations to do this. Now, however, we assume that the planner cannot commit to such allocations.

5.1 The stage game

In period t, each agent is publicly identified by a history of public signals $w^{t-1} \in \mathbb{R}^{t-1}$ and a history of past reports $\theta^{t-1} \in \Theta^{t-1}$.[13] The ensuing stage game consists of three sub-periods. In the first, a planner chooses a mechanism $S_t = \{\xi_t, \varphi_t, \upsilon_t, K_{t+1}\}$, where $\xi_t : \mathbb{R}^{t-1} \times \Theta^{t-1} \to \mathbf{P}$ is a measurable function mapping histories of signals and reports to a lottery of current public signals and $\varphi_t : \mathbb{R}^t \times \Theta^{t-1} \to \mathbb{R}_+$ and $\upsilon_t : \mathbb{R}^t \times \Theta^t \to \mathbb{R}$ are a pair of measurable allocation functions that

[13] We assume at the outset that the planner uses direct mechanisms to elicit information. Sleet-Yeltekin (2006) provide a revelation principle for a related environment. Their argument holds here. This revelation principle asserts that from a welfare point of view there is no loss of generality in assuming the planner uses direct mechanisms and induces truthful reporting along the equilibrium path of the game. In addition, restricting the planner to direct mechanisms off the equilibrium path does not alter the planner's incentives or the set of utility distributions that are implementable.

give each agent's utility from consumption and effort as a function of their histories (inclusive of the current public signals).[14] Finally, $K_{t+1} \in \mathbb{R}_{++}$ is an aggregate capital choice. Notice that the lottery ξ_t gives the planner the flexibility to partition the population of agents arbitrarily at any date. In practice the planner is tempted to choose allocation functions that ignore prior partitions of agents into histories, but it seems natural to allow the planner to adopt a finer partition at any date. This formulation also allows us to endogenise the initial distribution over agent types. In the second sub-period, agents receive a private shock, produce and send a message to the planner. Finally, the planner allocates utility according to φ_t and υ_t. The game then proceeds to the next period. As noted, the planner cannot commit in advance to a particular sequence of future mechanisms. However, having selected a mechanism in the first sub-period of the stage game she must execute it in the third; she cannot deviate to some alternative current allocation after hearing the agents' messages. Thus, we assume planner commitment within, but not across periods.

5.2 Histories and strategies

This subsection introduces notation and definitions for the infinitely repeated game. Define an *aggregate history* H_t, $t \geq 1$, to be a sequence $\{K_1, \{S_r\}_{r=1}^t\}$ of current and past stage mechanisms chosen by the planner and the initial capital stock. Set the history H_0 to be just the initial capital stock. Let $\mathbf{H}^t$ denote the set of t-period aggregate histories and let $\mathbf{S}_t$ denote the set of t-period stage mechanisms. A planner strategy $\sigma = \{\sigma_t\}_{t=1}^{\infty}$ is a collection of functions that map from aggregate histories to a current mechanism, $\sigma_t : \mathbf{H}^{t-1} \to \mathbf{S}_t$.[15] Any given σ induces a sequence of aggregate histories recursively from an initial $H_0 = K_1$ according to: $H_t = (H_{t-1}, \sigma_t(H_{t-1}))$. Let $\sigma|H_r$ denote the continuation of σ after aggregate history H_r and, for $t \geq r$, let $H_t(\sigma|H_r)$ denote the period t aggregate history and $S_t(\sigma|H_r)$ the period t stage mechanism induced by

[14] Here we slightly abuse notation by using the symbols φ_t and υ_t to denote allocation functions that condition utilities on an entire history of public signals rather than simply an initial period signal.

[15] As in Chari and Kehoe (1990), we consider an economy inhabited by a large strategic player, the planner, and a population of atomistic agents. This structure motivates our formulation of strategies; the planner's strategy is conditioned on past histories of capital stocks and planner actions only. The planner's strategy does not allow its treatment of an agent to depend on the past actions of other agents.

$\sigma|H_r$ along its outcome path. Let $\varphi_t(\sigma|H_r)$ and $\upsilon_t(\sigma|H_r)$ denote the corresponding t-th period allocation functions.

The period t *individual public history* of an agent $\left(H_t, w^t, \theta^{t-1}\right) \in \mathbf{H}^t \times \mathbb{R}^t \times \Theta^{t-1}$ augments H_t with the agent's signal-report history. The behavior of an agent is described by a *reporting strategy*, $m = \{m_t\}_{t=1}^{\infty}$, where $m_t : \mathbf{H}^t \times \mathbb{R}^t \times \Theta^{t-1} \times \Theta \to \Theta$ gives the t-th period report of an agent contingent on his individual public history and current shock. Let $m^t : \mathbf{H}^t \times \mathbb{R}^t \times \Theta^t \to \Theta^t$ be the induced period t report history function that gives period t report histories as functions of the aggregate history and the true sequence of shocks experienced by the agent. Whenever they are well defined, let $W_t(\sigma, m|H_{t-1})$ be the continuation payoff for a *utilitarian* planner induced by the strategy pair (σ, m) after the aggregate history H_{t-1} and let $U_t(\sigma, m|H_t, w^{t-1}, \theta^{t-1})$ be the continuation payoff for the agent induced by (σ, m) after $(H_t, w^{t-1}, \theta^{t-1})$. Finally, let $Q^t(m, H^t)$ denote the distribution over individual histories (w^t, θ^t) induced by m and H_t. We define a planner strategy σ to be *resource-feasible* given report strategy m if after each aggregate history the allocations induced by (σ, m) satisfy our earlier resource constraints. More formally, we have:

Definition 4 *Given a report strategy m, a mechanism $S = (\xi, \varphi, \upsilon, K)$ is resource-feasible at H_{t-1} if:*

$$\int_{\mathbb{R}^t} \sum_{\Theta^t} C(\varphi(w^t, \theta^t)) Q^t(m, H^{t-1}, S) + K \leq F\left(K_t, \int_{\mathbb{R}^t} \sum_{\Theta^t} N(\upsilon(w^t, \theta^t)) Q^t(m, H^{t-1}, S)\right).$$

Similarly, an aggregate history $H_t = \{K_1, \{S_r\}_{r=1}^{t}\}$ is resource-feasible given m if S_t is resource-feasible at $H_{t-1} = \{K_1, \{S_r\}_{r=1}^{t-1}\}$ given m and, if $t > 1$, H_{t-1} is resource-feasible given m. Let $\mathbf{H}^t(m)$ denote the set of t-period resource-feasible aggregate histories given m. Finally, a planner strategy σ is resource-feasible given m if after each $H_{t-1} \in \mathbf{H}^{t-1}(m)$, $\sigma_t(H_{t-1})$ is resource-feasible at H_{t-1}.

We will also impose the following bound as a primitive constraint on planner strategies.

Definition 5 *A planner strategy σ has well defined payoffs given a report strategy m if for all t and H_{t-1}, $W_t(\sigma, m|H_{t-1})$ is well defined and for all t, H_t and almost all (w^{t-1}, θ^{t-1}), $U_t(\sigma, m|H_t, w^{t-1}, \theta^{t-1})$ is well defined.*

Let $\mathbf{S}(m)$ denote the set of resource-feasible planner strategies with well defined payoffs given m.

5.3 Credible equilibria

A *credible equilibrium* is defined as follows.

Definition 6 *(σ, m) is a* ***credible equilibrium*** *if $\sigma \in \mathbf{S}(m)$ and:*

1. *(Agent optimality)* $\forall t, H_t, w^{t-1}, \theta^{t-1}, \widehat{m}$,

$$U_t(\sigma, m|H_t, w^{t-1}, \theta^{t-1}) \geq U_t(\sigma, \widehat{m}|H_t, w^{t-1}, \theta^{t-1});$$

2. *(Planner optimality)* $\forall t, H_{t-1} \in \mathbf{H}^{t-1}(m), \widehat{\sigma} \in \mathbf{S}(m)$,

$$W_t(\sigma, m|H_{t-1}) \geq W_t(\widehat{\sigma}, m|H_{t-1}).$$

The first of these conditions requires that the continuation of the agent's message strategy is optimal after all public individual histories given that the planner plays according to σ in the future. The second condition requires that after all resource-feasible aggregate histories the planner is better off adhering to the strategy σ than deviating to some alternative resource-feasible strategy $\widehat{\sigma}$.

Worst credible equilibria We next define uninformed planner and uninformative message strategies and show that they constitute a worst utilitarian credible equilibrium for any initial capital stock K_1. Under the uninformed planner strategy σ^{UI}, the planner commits not to make use of any information that the agents reveal and provides incentives for agents to babble in the aftermath of a planner defection. We first fix $K_1 \geq 0$ and construct the continuation strategy $\sigma^{UI}|K_1$. This construction makes use of two collections of stage game mechanisms. The first collection $\{S_t^{UI}\}_{t=1}^{\infty}$ describes the on-equilibrium path behavior of the planner. Each $S_t^{UI} = (\xi_t^{UI},\ \varphi_t^{UI},\ \upsilon_t^{UI}, K_{t+1}^{UI})$ is obtained by setting $\forall t,\ w,\ \theta^t$,

$\varphi_t^{UI}(w^t,\theta^t)=u_t^{UI}$, $v_t^{UI}(w^t,\theta^t)=v_t^{UI}$, where $\{u_t^{UI},v_t^{UI},K_{t+1}^{UI}\}_{t=1}^{\infty}$ solves:

$$\underline{W}(K_1) = \sup_{\{u_t,v_t,K_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty}\left[\beta^{t-1}(u_t+E[\theta]v_t)\right] \tag{48}$$

$$\text{subject to } \forall t, \qquad C(u_t)+K_{t+1}\le F(K_t,N(v_t)).$$

$\{\xi_t^{UI}\}$ is an arbitrary sequence of probability distributions on $\mathbb{R}$. (48) implies that the mechanisms $\{S_t^{UI}\}_{t=1}^{\infty}$ induce the optimal allocation of an uninformed planner. To describe behavior after a prior defection, suppose that the planner plays $S_{t-1}=(\xi_{t-1},\varphi_{t-1},\upsilon_{t-1},K_t)$ in period $t-1$ and construct the successor mechanism $\widehat{S}_t(S_{t-1})=\{\widehat{\xi}_t^{UI},\widehat{\varphi}_t^{UI},\widehat{\upsilon}_t^{UI},\widehat{K}_{t+1}^{UI}\}$ as follows. Let $\widehat{\theta}_i$ be some element of Θ and define $\Delta(w,\theta^{t-2})=\max_{\theta,\theta'}[\varphi_{t-1}(w,\theta^{t-2},\theta')+\theta\upsilon_{t-1}(w,\theta^{t-2},\theta')-\varphi_{t-1}(w,\theta^{t-2},\widehat{\theta}_i)$ $-\theta\upsilon_{t-1}(w,\theta^{t-2},\widehat{\theta}_i)]$. Then set $\widehat{\xi}_t^{UI}=\xi_t^{UI}$, $\widehat{\upsilon}_t^{UI}=\upsilon_t^{UI}$, $\widehat{K}_{t+1}^{UI}=K_{t+1}^{UI}$, for $(w^t,\theta^t)=(w,\theta^{t-2},\widehat{\theta}_i,\theta)$, set $\widehat{\varphi}_t^{UI}(w,\theta^t)=u_t^{UI}$; otherwise set $\widehat{\varphi}_t^{UI}(w,\theta^t)=u_t^{UI}-\frac{\Delta(w,\theta^{t-2})}{\beta}$. Hence, the successor mechanism $\widehat{S}_t(S_{t-1})$ punishes any agent who did not "babble" and send the common message $\widehat{\theta}_i$ in period $t-1$. $\sigma^{UI}|K_1$ is then obtained recursively by setting $\sigma_1^{UI}(K_1)=S_1^{UI}$ and, for $t>1$ and all aggregate histories H_{t-1} that follow K_1, by setting $\sigma_t^{UI}(H_{t-1})=S_t^{UI}$ if $S_{t-1}=\sigma_{t-1}^{UI}(S^{t-2})$ and $\sigma_t^{UI}(S^{t-1})=\widehat{S}_t(S_{t-1})$ otherwise. The strategy is uninformed in the sense that the planner makes no use of any information it has to provide insurance. Repeating this argument for all initial K_1 values determines σ^{UI}. The uninformative message strategy m^{UI} is defined as for all H_t, w^t, θ^{t-1}, θ_t,

$$m_t^{UI}(H_t,w^t,\theta^{t-1},\theta)=\begin{cases}\theta \text{ if } H_t=(H_{t-1},\sigma_t^{UI}(H_{t-1})) \\ \hat{\theta}_i \text{ otherwise.}\end{cases}.$$

In other words, m^{UI} is truthful if the planner chooses $\sigma_t^{UI}(H_{t-1})$ and commits not to use the truthful signals that it receives. Otherwise, m^{UI} requires that the agent babbles by sending out a common message $\hat{\theta}_i$.

Lemma 7 (σ^{UI},m^{UI}) *constitutes a worst utilitarian credible equilibrium*

Proof: To show that (σ^{UI},m^{UI}) is a credible equilibrium first note that if the planner adheres to σ^{UI} in the current and future periods, then agents receive the same payoff regardless of their current message. They may as well report their true shock. If the planner has defected in the current period, but is anticipated to adhere to σ^{UI} in the future, then agents anticipate a next period penalty for not sending $\widehat{\theta}_i$ large enough that it is in their interests to send this report. Hence, m^{UI} is optimal for agents after all histories. m^{UI} requires that agents report truthfully if $S_t = \sigma_t^{UI}(H_{t-1})$, but send a common signal otherwise. Thus, the planner cannot induce agents to reveal information unless it commits not to use that information by playing a mechanism of the form S_t^{UI} or $\widehat{S}(S_{t-1})$. Given this the planner might as well adhere to σ^{UI} and treat all agents equally except for the measure zero set of agents who fail to send the message $\widehat{\theta}_i$ following a planner defection. Thus, σ^{UI} is optimal for the planner and (σ^{UI}, m^{UI}) is a credible equilibrium. Clearly, the planner can guarantee herself the utilitarian payoff W^{UI} by choosing S_t^{UI} in each period independently of the agents behavior. Hence, W^{UI} is a lower bound on equilibrium utilitarian payoffs and since (σ^{UI}, m^{UI}) attains this payoff, it is a worst utilitarian equilibrium. ■

It follows from the definition of (σ^{UI}, m^{UI}) that after any history H_{t-1} culminating in capital K_t, the continuation payoff induced by (σ^{UI}, m^{UI}) is $\underline{W}(K_t)$.

5.4 Credible allocations

Our formulation of planner strategies incorporates a conditional lottery ξ_t over agent public signals in every period ξ_t. It is convenient to focus upon credible equilibria (σ, m) such that equilibrium allocation functions condition only upon an agent's period 1 lottery outcome and the agent's (reported) shock history only, i.e. such that for all t, w_1, $w^{t-1}, w^{t-1\prime}$ and θ^t, $\varphi_t(\sigma)(w_1, w^{t-1}, \theta^t) = \varphi_t(\sigma)(w_1, w^{t-1\prime}, \theta^t)$. From a welfare point of view, this property is without loss of generality: any credible equilibrium that fails to satisfy it can be replaced with one that does by integrating out lotteries after period 1. Any credible equilibrium subject to this restriction induces a population allocation along its equilibrium path. These allocations must satisfy the resource and incentive constraints (32) and (34) and a sequence of credibility constraints:

$$\forall t, \qquad \int \sum_{s=0}^{\infty} \beta^s \sum_{\Theta^{t+s}} \left[\varphi_{t+s}(w^{t+s}, \theta^{t+s}) + \theta_{t+s} \upsilon_{t+s}(w^{t+s}, \theta^{t+s})\right] \pi(\theta^{t+s}) \Phi(dw) \geq \underline{W}(K_t), \tag{49}$$

where $\Phi = \xi_1$. Conversely, any allocation satisfying these conditions is credible. Formally, we have the following result.

Proposition 8 *An initial distribution over signals Φ and a population allocation $\{\varphi_t, \upsilon_t, K_{t+1}\}_{t=1}^{\infty}$ is credible if and only if it satisfies (32), (34) and (49).*

We sketch the proof (see Sleet-Yeltekin (2006) for a formal argument). If an allocation satisfies the conditions in the proposition then a supporting credible equilibrium can be constructed as follows. The planner implements the sequence of allocation functions $\{\varphi_t, \upsilon_t\}_{t=1}^{\infty}$ and agents are truthful provided there has been no prior defection. If there has been a current or prior defection agents switch to the play of m^{UI} and the planner switches to the play of σ^{UI}. Given the play of agents, it follows that the best continuation payoff the planner can get by defecting from the allocation at t is $\underline{W}(K_t)$ and since the allocation's continuation payoffs exceed this, the planner would never wish to defect. Planner optimality along the equilibrium path is thus ensured, (32) and (34) ensure resource-feasibility and agent optimality along this path. Following a planner defection, (σ^{UI}, m^{UI}) is played. Since this is a credible equilibrium, agent and planner optimality and resource-feasibility hold in all periods subsequent to the defection periods. Hence, the conditions in the proposition are sufficient for credibility. Conversely, the planner can always guarantee herself a continuation payoff of $\underline{W}(K_t)$ after any history by playing σ^{UI}. Hence, no credible allocation can give a normalised continuation utilitarian payoff below this - the planner would simply defect away. This implies that (49) is necessary. Necessity of the other conditions follows from the requirement that agent behavior is optimal and strategies are resource-feasible in a credible equilibrium.

Thus, the credibility constraint on allocations *assumed* in the two period examples of Section 2 emerges endogenously as an equilibrium restriction in the infinitely repeated game. Pareto optimal credible allocations can then be obtained by supplementing (36) with the sequence of constraints (49), i.e by solving:[16]

$$\sup \int_{\mathbb{R}} \gamma_0 U(\{\varphi_t(\gamma_0, \cdot), \upsilon_t(\gamma_0, \cdot)\}_{t=1}^{\infty}) \Psi_0(d\gamma_0) \tag{50}$$

[16]The interpretation here is that the planner generates some cross sectional distirbution over public signals in the initial period and then agents are assigned a Pareto weight that is conditional on their signal.

subject to (32), (34) and (49). One particular case of interest is that in which $u(c) = \ln c$, $v(e) = \ln(T - e)$ and $F(K, L) = DK^{\alpha}L^{1-\alpha}$. In this case, $\underline{W}$ can be solved for analytically as:

$$\underline{W}(K) = \underline{W}_0 + \frac{\alpha}{1 - \alpha\beta} \ln K.$$

6 Optimal infinite-horizon credible allocations

For all $\beta \in (0, 1)$, Pareto optimal allocations with commitment are usually not credible, since they imply ever increasing inequality (see the calculated example in Section 4.2. We focus then on Pareto optimal *credible* allocations. Again, full analytical characterization is complicated by the task of determining the periods in which the credibility constraints bind. We seek a recursive, numerically implementable formulation.

6.1 Pseudo planner problems

Let $\mathcal{L}$ denote a Lagrangian that incorporates the credibility constraints:

$$\mathcal{L}(\{\varphi_t, \upsilon_t, K_{t+1}\}; \{\phi_t\}) = \int \gamma_0 \sum_{t=1}^{\infty} \beta^{t-1} \sum_{\theta^t} \left[\varphi_t(\gamma_0, \theta^t) + \theta_t \upsilon_t(\gamma_0, \theta^t)\right] \pi^t(\theta^t) \Psi_0(d\gamma_0)$$
$$+ \sum_{t=1}^{\infty} \phi_t \beta^{t-1} \prod\nolimits_{r=1}^{t-1} (1 + \phi_r) \left[\int \sum_{r=0}^{\infty} \beta^r \sum_{\theta^{t+r}} \left[\varphi_{t+r}(\gamma_0, \theta^{t+r}) + \theta_{t+r} \upsilon_{t+r}(\gamma_0, \theta^{t+r})\right] \pi^{t+r}(\theta^{t+r}) \Psi_0(d\gamma_0) - \underline{W}(K_t)\right].$$

Here $\phi\beta^{t-1}(1 + \phi)^{t-1}$ is the Lagrange multiplier on the t-th credibility constraint. We have the following result.

Lemma 9 *Let $L = \{\{\phi_t\}_{t=1}^{\infty} \in \mathbb{R}_+^{\infty} : \sum_{t=1}^{\infty} \phi_t \beta^{t-1} \prod_{r=1}^{t-1} (1 + \phi_r) < \infty\}$ and*

$$\inf_{\{\phi_t\} \in L} \sup_{\{\varphi_t, \upsilon_t, K_{t+1}\} \in \Gamma_0(K_1, \Psi_0)} \mathcal{L}(\{\varphi_t, \upsilon_t, K_{t+1}\}; \{\phi_t\}) \tag{51}$$

then $\{\varphi_t, \upsilon_t, K_{t+1}\}$ solves (50).

Now, for $\{\varphi_t, \upsilon_t, K_{t+1}\} \in \mathbf{PA}$ and $\{\phi_t\}_{t=1}^{\infty} \in L$ we can apply Abel's summation by parts lemma to obtain:

$$\mathcal{L}(\{\varphi_t, \upsilon_t, \kappa_{t+1}\}; \{\phi_t\}) = \int \sum_{t=1}^{\infty} B_1^t \gamma_1^t(\gamma_0) \sum_{\theta^t} \left[\varphi_t(\gamma_0, \theta^t) + \theta_t \upsilon_t(\gamma_0, \theta^t)\right] \pi^t(\theta^t) \Psi_0(d\gamma_0) - \sum_{t=1}^{\infty} (B_1^t - \beta B_1^{t-1}) \underline{W}(K_t).$$

where $B_j^0 = 1$ and for $t \geq 1$, $B_j^t = \beta^{t-1} \prod_{r=0}^{t-1}(1+\phi_{j+r})$, $\gamma_j^1(x) = \frac{x}{1+\phi_j}$, $\gamma_j^{t+1}(x) = \frac{\phi_{t+j}}{1+\phi_{t+j}} + \frac{\gamma_j^t(x)}{1+\phi_{t+j}}$. Thus, if B_1^{*t} and $\gamma_1^{*t}(\gamma_0)$ are obtained from the minimising multipliers in (51), then any solution to (52) below is an optimal credible allocation.

$$\sup_{\{\varphi_t, \upsilon_t, K_{t+1}\} \in \Gamma(K_1, \Psi_0)} \int \sum_{t=1}^{\infty} B_1^{*t} \gamma_1^{*t}(\gamma_0) \sum_{\theta^t} \left[\varphi_t(\gamma_1, \theta^t) + \theta_t \upsilon_t(\gamma_1, \theta^t)\right] \pi^t(\theta^t) \Psi_0(d\gamma_0) - \sum_{t=1}^{\infty} (B_1^{*t} - \beta B_1^{*t-1}) \underline{W}(K_t). \tag{52}$$

We call (52) a *pseudo-planner's problem.* The pseudo-planner is committed and faces only resource and incentive constraints. However, her objective is altered relative to both the uncommitted planner (who is utilitarian) and the societal criterion used in (50). In particular, the pseudo-planner uses a perturbed discounting scheme $\{B_1^{*t}\}_{t=1}^{\infty}$ to evaluate future utilities from consumption and effort. It is a convolution of the agents' discounting scheme $\{\beta^t\}$ and the optimal multipliers from the credibility constraints $\{\phi_t^*\}$. Notice that for all t, $B_1^{*t+1}/B_1^{*t} \geq \beta$ with equality if and only if the optimal credibility multiplier ϕ_{t+1}^* equals 0. The pseudo planner also uses a perturbed Pareto weight sequence $\gamma_1^{*t}(\gamma_0)$ that converges towards 1 in all periods in which the credibility constraints bind. In these periods, the endogenous Pareto weights $\gamma_1^{*t}(\gamma_0)$ revert to the cross sectional mean and the initial rewards and penalties captured by the period 1 cross sectional distribution of Pareto weights Ψ_0 are washed out. This convergence in periods in which the credibility constraints bind stems from the need to maintain the uncommitted planner's *equally weighted* utilitarian payoff above a lower bound.

The first component of the pseudo-planner's objective (52) coincides exactly with that of a committed paternalistic planner who has discounting scheme $\{B_1^{*t}\}_{t=1}^{\infty}$ and who must deliver a particular particular distribution of initial rewards or sanctions to agents that are captured by Pareto weights γ_0 or, equivalently, by some initial distribution of utility promises.[17] The second component of the (52) incorporates an additional shadow cost of accumulating capital over and above that paid by the paternalistic planner. As in our third simple economy, this stems from the fact, other things equal, additional capital accumulation tightens the credibility constraint.

The pseudo-planner interpretation is sharpened by consideration of the log-Cobb Douglas case with $u(c) = \ln c$, $v(e) = \ln(T-e)$ and $F(K, L) = K^{\alpha} L^{1-\alpha}$. In this case, consider redefining variables according to $u_t(w, \theta^t) = \varphi_t(w, \theta^t) - \alpha \ln K_t$ and

[17] In the latter case, the multipliers γ_0 can be interpreted as Lagrange mulipliers on a set of utility promise-keeping constraints. See Farhi and Werning (2007) for a paternalistic problem formulated directly in terms of these constraints.

$\kappa_{t+1} = K_{t+1}/K_t^{\alpha}$. Respecifiying the Pareto optimal credible allocation problem and, hence, the pseudo-planner's problem in terms of the "normalised population allocation" gives:

$$\sup_{\{u_t,\upsilon_t,\kappa_{t+1}\}\in\widetilde{\Gamma}(\Psi_0)} \int \sum_{t=1}^{\infty} B_1^{*t}\gamma_1^{*t}(\gamma_0)\sum_{\theta^t}\left[u_t(\gamma_0,\theta^t)+\theta_t\upsilon_t(\gamma_0,\theta^t)\right]\pi^t(\theta^t)\Psi_0(d\gamma_0)+\frac{\beta\alpha}{1-\beta\alpha}\sum_{t=1}^{\infty}B_1^{*t}\ln\kappa_{t+1},$$

where $\widetilde{\Gamma}(\Psi_0)$ is the set of incentive and resource-feasible allocations and we omit unimportant constants. For the purposes of exposition, suppose that $B_1^{*t} = b^{*t}$ for some constant $b^* \in [\beta, 1)$. If a paternalistic planner was systematically applying this discount, then her objective would be

$$\sup_{\{u_t,\upsilon_t,\kappa_{t+1}\}\in\widetilde{\Gamma}(\Psi_0)} \int \sum_{t=1}^{\infty} b^{*t}\gamma_1^{*t}(\gamma_0)\sum_{\theta^t}\left[u_t(\gamma_0,\theta^t)+\theta_t\upsilon_t(\gamma_0,\theta^t)\right]\pi^t(\theta^t)\Psi_0(d\gamma_0)+\frac{b^*\alpha}{1-b^*\alpha}\sum_{t=1}^{\infty}b^{*t}\ln\kappa_{t+1},$$

and the coefficient on the second "capital growth" term is raised from $\frac{\beta\alpha}{1-\beta\alpha}$ to $\frac{b^*\alpha}{1-b^*\alpha}$. Thus the pseudo-planner in this case is a hybrid of a the paternalistic planner with discount b^* and the standard, committed planner with discount β. Both the Pareto-weighted societal and the utilitarian continuation payoff at $t+1$ incorporate the term: $\frac{\alpha}{1-\beta\alpha}\ln\kappa_{t+1}$. This describes how these continuation payoffs are affected by "capital growth" at t, $\kappa_{t+1} = K_{t+1}/K_t^{\alpha}$, *holding the other terms in the normalised population allocation constant*. By raising the continuation societal payoff at $t+1$ and κ_{t+1} relaxes the credibility constraints in periods 1 through to t. Summing up these contributions to the pseudo-planner's shadow payoff delivers the term $\frac{\beta\alpha}{1-\beta\alpha}b^{*t}\ln\kappa_{t+1}$. Permanent increases in the capital stock at $t+1$ raise the continuation utilitarian payoff and the payoff to planner available from defection and play of the worst equilibrium by equal amounts. These contributions net out and do not appear. In contrast, the paternalistic planner's continuation payoff at $t+1$ incorporates the term $\frac{\alpha}{1-b^*\alpha}\ln\kappa_{t+1}$. This is discounted back to period 1 to give the contribution $\frac{\alpha}{1-b^*\alpha}b^{*t}\ln\kappa_{t+1}$.

6.2 A component planner formulation for optimal credible allocations

As in the commitment case, we can use a Lagrangian argument to decentralise (52) into a family of component planning problems and a representative firm's problem.

Lemma 10 *Suppose that* $\{q_t\}_{t=1}^{\infty} \in \prod_{t=1}^{\infty}[\beta^{t-1}\underline{q}, \beta^{t-1}\overline{q}]$*, where* $0 < \underline{q} < \overline{q}$*,* $\{w_t\}_{t=1}^{\infty} \in \prod_{t=1}^{\infty}[\underline{w}, \overline{w}]$*, where* $0 < \underline{w} < \overline{w}$ *and* $\{B_1^{t-1}\}_{t=1}^{\infty}$ *is such that each* $B_1^{t-1} \geq 0$ *with* $B_1^0 = 1$ *and* $\sum_{t=1}^{\infty} B_1^{t-1} < \infty$*. Let* $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}_{t=1}^{\infty} \in \mathbf{PA}$ *and* $\{L_t^*\}_{t=1}^{\infty} \in [0, T]^{\infty}$*. If*

1. *(Component planner optimality) for* Ψ_0*-a.e.* γ_0*,* $\varphi_t^*(\gamma_0, \cdot)$ *and* $\upsilon_t^*(\gamma_0, \cdot)$ *solve:*

$$\widetilde{V}_1(\gamma_0) = \sup_{\{\psi_t, \nu_t\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \sum_{\theta^t} \{B_1^{*t}\gamma_1^{*t}(\gamma_0)[\psi_t(\theta^t) + \theta_t\nu_t(\theta^t)] - q_t[C(\psi_t(\theta^t)) + w_t N(\nu_t(\theta^t))]\}\pi^t(\theta^t) \tag{53}$$

   subject to $\{\psi_t, \nu_t\}_{t=1}^{\infty} \in \mathbf{IA}$ *and (34),*

2. *(Firm optimality) for* $t > 1$*,* K_t^* *and* L_t^* *solve*

$$\sup_{K_t, L_t} F(K_t, L_t) - \frac{q_{t-1}}{q_t} K_t - w_t L_t \tag{54}$$

   while for $t = 1$*,* L_1^* *solves*

$$\sup_{L_1} F(K_1, L_1) - w_1 L_1 \tag{55}$$

3. *(Market clearing) for all* t*,*

$$\int \sum_{\theta^t} C(\varphi_t^*(\gamma_0, \theta^t))\pi^t(\theta^t)\Psi_0(d\gamma_0) + K_{t+1}^* = F(K_t^*, L_t^*), \tag{56}$$

   and

$$\int \sum_{\theta^t} N(\upsilon_t^*(\gamma_0, \theta^t))\pi^t(\theta^t)\Psi_0(d\gamma_0) = L_t^*, \tag{57}$$

   where $K_1^* = K_1$*,*

4. *(Preference determination)*

$$\left(B_1^{*t} - \beta B_1^{*t-1}\right)\left(\int \sum_{r=0}^{\infty} \beta^r \sum_{\Theta^{t+r}} [\varphi_{t+r}^*(\gamma_1, \theta^{t+r}) + \theta_t \upsilon_{t+r}^*(\gamma_1, \theta^{t+r})]\pi^{t+r}(\theta^{t+r})\Psi_1(d\gamma_1) - \underline{W}(K_t)\right) = 0, \tag{58}$$

   with $B_1^0 = 1$*.*

   then $\{\varphi_t^*, \upsilon_t^*, K_{t+1}^*\}_{t=1}^{\infty}$ *solves (52).*

We omit the proof which is similar to Lemma 1.

6.2.1 The recursive component planner problem without commitment

As before the component planner problems admit a recursive formulation that uses the multipliers from the component planner Lagrangian. Define the continuation component planner problems for $t = 1, 2, \ldots$, by

$$\widetilde{V}_t(\gamma) = \sup_{\{\psi_s, \nu_s\}_{s=1}^{\infty} \in \Omega_1} \sum_{s=1}^{\infty} \sum_{\theta^s} \{B_t^{*s} \gamma_t^s(\gamma) [\psi_s(\theta^s) + \theta_s \nu_s(\theta_s)] - q_{t+s-1}[C(\psi_s(\theta^s)) + w_{t+s-1} N(\nu_s(\theta^s))]\} \pi^s(\theta^s). \tag{59}$$

These problems resemble those for the commitment case with the exception that they incorporate the modified discounting B_t^{*s} and Pareto weighting schemes $\gamma_t^s(\gamma)$. As in the commitment case, rearranging the Lagrangian for (59) that incorporates the period t incentive constraint, defining V_t by:

$$\widetilde{V}_t(\gamma) = B_t^{*1} V_t \left(\frac{\gamma + \phi_t^*}{1 + \phi_t^*} \right)$$

and using $q_{t+s-1}^* = \frac{q_{t+s-1}}{B_t^{*1}}$ and $b_{t+1}^* = \frac{B_{t+1}^{*2}}{B_t^{*1}}$ delivers the dynamic programming problem:

$$V_t(\gamma) = \inf_{\Lambda(\gamma)} \sum_{i=1}^{I} \left\{ J_i \left(\rho_i(\zeta; \eta), \lambda_i(\zeta; \eta); q_t^*, w_t \right) + b_{t+1}^* V_{t+1} \left(\gamma'_{i,t+1}(\gamma; \eta) \right) \right\} \pi(\widehat{\theta}_i), \tag{60}$$

where

$$\gamma'_{i,t+1}(\gamma; \eta) = \frac{\phi_{t+1}^*}{1 + \phi_{t+1}^*} + \frac{1}{1 + \phi_{t+1}^*} \left(\gamma + \sum_{j:\{(i,j) \in \mathbf{I}\}} \eta_{i,j} - \sum_{j:\{(j,i) \in \mathbf{I}\}} \eta_{j,i} \frac{\pi(\widehat{\theta}_j)}{\pi(\widehat{\theta}_i)} \right). \tag{61}$$

(61) gives a law of motion for effective Pareto weights for this setting. Notice that when the credibility constraint in period t does not bind, $\phi_t = 0$ and the law of motion reduces to that in the commitment case. When the credibility constraint in t does bind, $\phi_t^* > 0$ and the law of motion incorporates a force for mean reversion of the agent's effective Pareto weight. As in the commitment case, the agent's effective Pareto weights are augmented with "incentive shocks" derived from the multipliers on the incentive constraints. We then have the following analogue of Proposition 3.

Proposition 11 *1) There exists a unique sequence of policy functions $\{\eta_t^*\}_{t=1}^{\infty}$, with $\eta_t^* : \mathbb{R}_{++} \to \mathbb{R}_+$, that attain the infima in the optimization problems (60); 2) each η_t^* is continuous; 3) Assume that the non-recursive problem (53) has a solution, then the allocation $\{u_t^*(\gamma), v_t^*(\gamma)\}_{t=1}^{\infty}$ induced by $\{\eta_t^*\}_{t=1}^{\infty}$ from γ solves (53).*

It follows from (61) that the optimal process for effective Pareto weights $\{\gamma_t^*\}_{t=1}^{\infty}$ starting from some $\gamma \in \mathbb{R}_+$ evolves according to:

$$\gamma_{t+1}^*(\gamma, \theta^{t-1}, \widehat{\theta}_k) = (1 - \omega_{t+1}) + \omega_{t+1}\gamma_t^*(\gamma, \theta^{t-1}) + \varepsilon_{t+1}(\gamma, \theta^{t-1}, \widehat{\theta}_k), \tag{62}$$

where the sequence of incentive shocks $\{\varepsilon_{t+1}\}_{t=1}^{\infty}$ obtained from the optimal incentive multipliers and again has a conditional expectation of 0 and $\omega_{t+1} = \frac{1}{1+\phi_{t+1}^*}$. As before the incentive shocks are a source of dispersion in individual effective Pareto weights, and, hence, consumption and utility. Now, binding credibility constraints are a source of mean reversion for these weights, reflecting the fact that the planner cannot credibly implement allocations that feature too much ex post inequality.

7 Wedges and taxes without commitment

The wedge and tax results from the two period examples with limited commitment extend with only small modification to the infinite horizon credibility setting.

7.0.2 An intertemporal wedge

The first order conditions for $\psi_t^*(\gamma_1, \theta^t)$ from the component planner problem expressed in terms of optimal consumption $c_t^*(\gamma_1, \theta^t)$ imply:

$$B_1^{*t}\rho_t^*(\gamma_1, \theta^t) = \frac{q_t}{u'(c_t^*(\gamma_1, \theta^t))} \tag{63}$$

where $\rho_t^*(\gamma_1, \theta^{t-1}, \hat{\theta}_i) = \gamma_t^*(\gamma_1, \theta^{t-1}) + \sum_{j:(i,j)\in\mathbf{I}} \eta_{i,j,t}^*(\gamma_t^*(\gamma_1, \theta^{t-1})) - \sum_{j:(j,i)\in\mathbf{I}} \eta_{j,i,t}^*(\gamma_t^*(\gamma_1, \theta^{t-1}))\frac{\pi(\hat{\theta}_j)}{\pi(\hat{\theta}_i)}$ is the optimal current utility weight. Together (63) and the definitions of ρ_{t+1}^* and γ_{t+1}^* give the *conditional inverted Euler equation*:

$$\sum_{\theta} \frac{1}{\beta R_{t+1} u'(c_{t+1}^*(\gamma_1, \theta^t, \theta))}\pi(\theta) = \frac{(1-\omega_{t+1})}{q_t/B_1^{*t}} + \frac{\omega_{t+1}}{u'(c_t^*(\gamma_1, \theta^t))}. \tag{64}$$

where $R_{t+1} := \frac{1}{\beta}\left(\frac{q_t}{B_1^{*t-1}}\right)\left(\frac{q_{t+1}}{B_1^{*t}}\right)^{-1}$. Applying Jensen's inequality, (64) can be restated as:

$$u'(c_t^*(\gamma_1, \theta^t)) \leq \left[\frac{(1-\omega_{t+1})}{q_t/B_1^{*t}} u'(c_t^*(\gamma_1, \theta^t)) + \omega_{t+1}\right] \beta R_{t+1} \sum_{\theta} u'(c_{t+1}^*(\gamma_1, \theta^t, \theta))\pi(\theta). \tag{65}$$

From (63) and the definition of ρ_t^*,

$$\frac{(1-\omega_{t+1})}{q_t/B_1^{*t}} = \int_{\mathbb{R}} \sum_{\Theta^{t+1}} \frac{1}{u'(c_t^*(\gamma_1,\theta^t))} \pi^{t+1}(\theta^{t+1}) \Psi_1(d\gamma_1). \tag{66}$$

It then follows that "rich" agents for whom:

$$\frac{1}{u'(c_t^*(\gamma_1,\theta^t))} \geq \int_{\mathbb{R}} \sum_{\Theta^{t+1}} \frac{1}{u'(c_t^*(\gamma_1,\theta^t))} \pi^{t+1}(\theta^{t+1}) \Psi_1(d\gamma_1) \tag{67}$$

face an "intertemporal wedge" between their expected intertemporal marginal rate of substitution and the intertemporal shadow price R_{t+1}

$$u'(c_t^*(\gamma_1,\theta^t)) \leq \beta R_{t+1} \sum_{\theta} u'(c_{t+1}^*(\gamma_1,\theta^t,\theta))\pi(\theta).$$

For "poorer" agents, i.e. those for whom the inequality in (67) is reversed, the sign of the intertemporal wedge is ambiguous. The sign of this wedge is determined by "incentive" and "credibility" effects that are reinforcing for the rich and offsetting for the poor. Suppose that we consider decentralizing the optimal allocation in a market economy with taxes, something we will do explicitly in the next section. On the one hand, the "incentive effect" identified by Golosov et al (2003) in commitment models implies that it is optimal in such a decentralisation to deter all agents who face binding incentive-compatibility constraints from saving enough to equate their individual intertemporal marginal rate of substitution to the shadow price R_{t+1}. Such saving will raise the cost of providing incentives in subsequent periods, a cost that absent taxes the agent does not fully internalise. On the other hand, there is a "credibility effect" that stems from the planner's inability to commit to high levels of inequality ex post. When the credibility constraints bind, this effect implies that rich and poor agents must be deterred from accumulating too much and too little respectively. With respect to the rich, this reinforces the need to deter savings and results in an unambiguously positive wedge. With respect to the poor, it introduces a social motive for encouraging saving that offsets the incentive effect. The resulting implications for the intertemporal wedge are ambiguous.

The first order condition for capital implies that

$$F_K(K_t^*,L_t^*) = \frac{q_{t-1}}{q_t}\left[\frac{B_1^{*t}}{\beta B_1^{*t-1}} + \frac{B_1^{*t}-\beta B_1^{*t-1}}{\beta B_1^{*t-1}}\left(\frac{W_K(K_t^*)}{q_{t-1}}-1\right)\right].$$

In the log-Cobb Douglas case, manipulation of the first order conditions and the resource constraint gives $\frac{W_K(K_t^*)}{q_{t-1}} = 1$, so that $F_K(K_t^*, L_t^*) = R_t^{-1}$ and R_t is very naturally interpreted as a societal discount rate.

From (63) and the definition of ρ_t^*,

$$\frac{1}{q_t/B_1^{*t}} = \int_{\mathbb{R}} \sum_{\Theta^{t+1}} \frac{1}{u'(c_t^*(\gamma_1, \theta^t))} \pi^{t+1}(\theta^{t+1}) \Psi_1(d\gamma_1). \tag{68}$$

Integrating (64) over agent histories and substituting for $\frac{1}{q_t/B_1^{*t-1}}$ using (68) gives the unconditional inverted Euler equation:

$$\int_{\mathbb{R}} \sum_{\Theta^{t+1}} \frac{1}{\beta R_{t+1} u'(c_{t+1}^*(\gamma_1, \theta^{t+1}))} \pi^{t+1}(\theta^{t+1}) \Psi_1(d\gamma_1) = \int_{\mathbb{R}} \sum_{\Theta^t} \frac{1}{u'(c_t^*(\gamma_1, \theta^t))} \pi^t(\theta^t) \Psi_1(d\gamma_1). \tag{69}$$

This equation does not include the credibility multiplier terms ω_{t+1}. An equivalent restriction is obtained in the case with commitment, except that there $R_{t+1} = \frac{q_t}{q_{t+1}} = F_K(K_{t+1}^*, L_{t+1}^*)$ (as opposed to $R_{t+1} = \frac{1}{\beta}\left(\frac{q_t}{B_1^{*t}}\right)\left(\frac{q_{t+1}}{B_1^{*t+1}}\right)^{-1}$ with R_{t+1} equalling $F_K(K_{t+1}^*, L_{t+1}^*)$ in the log-Cobb Douglas case). As noted in Section 2 of the paper, unconditional inverted Euler equations have underpinned recent asset pricing tests; we return to these equations later in the section.

7.0.3 A tax-market implementation

Suppose that agents face a tax system $\{T_t(\gamma_1, y^t, b_t)\}_{t=1}^{\infty}$ and can trade claims to (pre-tax riskless) capital in a bond market. Following Kocherlakota (2005), we assume that the tax system is linear in an agents wealth:

$$T_t(\gamma_1, y^t, b_t) = T_t^0(\gamma_1, y^t) + \tau_t(\gamma_1, y^t) b_t.$$

As shown in Albanesi-Sleet (2006) and Kocherlakota (2005), tax-market implementations typically enlarge the set of allocations available to agents and some care must be taken in choosing the optimal tax functions so as to rule out double deviations in which agents save too much in the present and work too little in the future. Given the form of the tax function assumed above such deviations are ruled out only if the tax function satisfies a collection of state-by-state Euler equations:

$$Q_{t+1} u'(c_t^*(\gamma_1, \theta^t)) = \beta(1 - \tau_{t+1}(\gamma_1, \theta^{t+1})) u'(c_{t+1}^*(\gamma_1, \theta^{t+1})).$$

where $Q_{t+1} = \left[F_K(K^*_{t+1}, L^*_{t+1})\right]^{-1}$. Clearly, this ensures that the agents' Euler equation:

$$Q_{t+1} u'(c^*_t(\gamma_1, \theta^t)) = \beta \sum_\theta (1 - \tau_{t+1}(\gamma_1, \theta^t, \theta)) u'(c^*_{t+1}(\gamma_1, \theta^t, \theta))\pi(\theta).$$

holds at the optimal allocation. With an appropriate choice of T_0 and initial wealths we may show that agents choose the optimal allocations. The state-by-state equations and the inverted Euler equation then implies, after some manipulation:

$$\begin{aligned}\sum_\theta \tau_{t+1}(\gamma_1, \theta^t, \theta)\pi(\theta) &= \frac{\frac{B_1^{*t+1} - \beta B_1^{*t}}{\beta B_1^{*t}} \frac{W_K(K^*_{t+1})}{q_t}}{1 + \frac{B_1^{*t+1} - \beta B_1^{*t}}{\beta B_1^{*t}} \frac{W_K(K^*_{t+1})}{q_t}} \left(\frac{W_K(K^*_{t+1})}{q_t} - \frac{u'(c^*_t(\gamma_1, \theta^t))}{q_t / B_1^{*t}} \right) \\ &= A_0 - A_1 u'(c^*_t(\gamma_1, \theta^t)),\end{aligned}$$

where $A_0 \geq 0$, $A_1 \geq 0$ and these inequalities are strict if the period t credibility constraint binds. This expression reproduces the expected marginal asset tax formula from the two period example. As noted there, the decentralisation of committed and paternalistic planning problems gives rise to similar tax functions. In the former, $A_0 = A_1 = 0$ (see Albanesi-Sleet (2006), Kocherlakota (2005)); in the latter $A_1 > A_0 = 0$ (see Farhi-Werning (2007)).

As noted, in the log-Cobb Douglas case, $R_{t+1} = \frac{1}{\beta}\left(\frac{q_t}{B_1^{*t}}\right)\left(\frac{q_{t+1}}{B_1^{*t+1}}\right)^{-1} = [F_K(K^*_{t+1}, L^*_{t+1})]^{-1} = Q^{-1}_{t+1}$. Using the state-by-state Euler equation and (63) integrated over agent histories and Jensen's inequality, we obtain the sharper characterisation:

$$\begin{aligned}\int \sum_{\theta^{t+1}} \tau_{t+1}(\gamma_1, \theta^{t+1})\pi^{t+1}(\theta^{t+1})\Psi_1(d\gamma_1) &= (1 - \omega_{t+1}) \left[1 - \frac{\int u'(c^*_t(\gamma_1, \theta^t))\pi^t(\theta^t)\Psi_1(d\gamma_1)}{q_t / B_1^{t-1}} \right] \\ &\leq 0.\end{aligned}$$

and the cross sectional average of marginal asset taxes is negative.

7.0.4 Asset pricing

We extend the environment of the earlier sections slightly. Now, suppose that in addition to the private idiosyncratic shocks θ_t assumed previously, there are also public, aggregate shocks. For our purposes, it will not matter very much what these shocks are to. They could be aggregate preference, government spending or productivity shocks. For concreteness, we will assume that they are to the latter and that the productivity parameter in the production function Z now follows a stochastic process $\{Z_t\}_{t=1}^{\infty}$ with probability law P and each Z_t taking its values in a finite set $\mathbf{Z} \subset \mathbb{R}_{++}$. Much of our previous analysis goes through with little alteration. In our planning problems, allocations and multipliers are now functions of past histories of productivity shocks. Similarly, in our tax-market decentralization, optimal taxes now condition on past productivity histories. Additionally, this decentralization now features a set of asset markets in which claims on aggregate states are traded. To make our basic point, we restrict attention to the log-Cobb Douglas case.

The analogue of the conditional inverted Euler equation (64) in this setting is:

$$\beta \frac{q_{t+1}(Z^{t+1})}{q_t(Z^t)} \frac{B_1^{*t-1}(Z^{t+1})}{B_1^{*t}(Z^t)} \left(\frac{K_{t+1}^*(Z^t)}{K_t^*(Z^{t-1})} \right)^{-\alpha} \sum_{\theta} \frac{1}{\beta u'(c_{t+1}^*(\gamma_1, \theta^t, \theta, Z^{t+1}))} \pi(\theta) = \frac{(1-\omega_{t+1}(Z^{t+1}))}{\frac{q_t(Z^t)}{B_1^{*t-1}(Z^t)} K_t^{*-\alpha}(Z^{t-1})} + \frac{\omega_{t+1}(Z^{t+1})}{u'(c_t^*(\gamma_1, \theta^t, Z^t))}. \tag{70}$$

Now define $Q_{t+1}(Z^{t+1}) = \beta \frac{q_{t+1}(Z^{t+1})}{q_t(Z^t)} \frac{B_1^{*t-1}(Z^{t+1})}{B_1^{*t}(Z^t)} \left(\frac{K_{t+1}^*(Z^t)}{K_t^*(Z^{t-1})} \right)^{-\alpha}$. This can be interpreted as the societal shadow price of a unit of period resources at $t+1$ in state Z_{t+1} in terms of period t resources after history Z^t. Integrating over agents, we obtain:

$$Q_{t+1}(Z^{t+1}) = \left[\int_{\mathbb{R}} \sum_{\theta^{t+1}} \frac{1}{\beta u'(c_{t+1}^*(\gamma_1, \theta^{t+1}, Z^{t+1}))} \pi^{t+1}(\theta^{t+1}) \Psi_1(d\gamma_1) \right]^{-1}$$
$$\times \left[\frac{(1-\omega_{t+1}(Z^{t+1}))}{\frac{q_t(D^t)}{B_1^{*t-1}(D^t)} K_t^{*-\alpha}(Z^{t-1})} + \int_{\mathbb{R}} \sum_{\theta^t} \frac{\omega_{t+1}(Z^{t+1})}{u'(c_t^*(\gamma_1, \theta^t, Z^t))} \pi^t(\theta^t) \Psi_1(d\gamma_1) \right].$$

But the agent's period t first order condition is:

$$\frac{\rho_t^*(\gamma_1, \theta^t, Z^t)}{\frac{q_t(Z^t)}{B_1^{*t-1}(Z^t)} K_t^{*-\alpha}(Z^{t-1})} = \frac{1}{u'(c_t^*(\gamma_1, \theta^t, Z^t))},$$

with $\int_{\mathbb{R}} \sum_{\Theta^t} \rho_t^*(\gamma_1, \theta^t, Z^t) = 1$. Consequently, the analogue of the unconditional inverted Euler equation (69) for this case is:

$$Q_{t+1}(Z^{t+1}) = \left[\int_{\mathbb{R}} \sum_{\theta^{t+1}} \frac{1}{\beta u'(c_{t+1}^*(\gamma_1, \theta^{t+1}, Z^{t+1}))} \pi^{t+1}(\theta^{t+1}) \Psi_1(d\gamma_1) \right]^{-1} \int_{\mathbb{R}} \sum_{\theta^t} \frac{1}{u'(c_t^*(\gamma_1, \theta^t, Z^t))} \pi^t(\theta^t) \Psi_1(d\gamma_1). \tag{71}$$

In the decentralisation, Q_{t+1} serves as the (pre-tax) pricing kernel for claims purchased after each history Z^t and contingent on Z_{t+1}. The formula (71) does not depend on the credibility multipliers and is identical to the one that emerges in an environment without such constraints. Consider two bundles of contingent claims i and j at t that pay out the returns R_{t+1}^i

and R^j_{t+1} and that each cost one unit of Z^t-consumption. We then have that

$$0 = E[M_{t+1}(R^i_{t+1} - R^j_{t+1})]$$

where $M_{t+1}(Z^{t+1}) = \frac{\int_{\mathbb{R}} \sum_{\theta^t} \frac{1}{u'(c^*_t(\gamma_1,\theta^t,Z^t))} \pi^t(\theta^t)\Psi_1(d\gamma_1)}{\int_{\mathbb{R}} \sum_{\theta^{t+1}} \frac{1}{u'(c^*_{t+1}(\gamma_1,\theta^{t+1},Z^{t+1}))} \pi^{t+1}(\theta^{t+1})\Psi_1(d\gamma_1)}$. This is exactly the condition that Kocherlakota-Pistaferri derive and empirically evaluate in a setting with societal commitment. They investigate whether it can reconcile consumption and asset pricing data with plausibly parameterized agent utility functions. They argue that it can and, in particular, suggest that it can resolve the equity premium puzzle. However, it is less certain whether the implications of a dynamic private information model with commitment are consistent with observed *consumption allocations* since many models of this sort imply increasing inequality and immiseration of some fraction of the population. In contrast, optimal credible allocations can imply ergodic distributions for consumption that are more empirically reasonable and, in addition, retain the pricing kernels derived by Kocherlakota-Pistaferri (2008) that better reconcile consumption and asset pricing data than those obtained in conventional market settings.

8 Numerical analysis of credible steady states

8.1 Steady state: Definition

In this section, we restrict attention to the log-Cobb Douglas and focus on optimal credible allocations that attain a steady state. Such steady states are characterised by time invariant cross sectional distributions over utility, consumption, effort and effective Pareto weights and a constant aggregate capital level. To that end, we consider allocations that maximize Lagrangians in which the multipliers satisfy, for all t, $\phi_t = \phi$ and $q_t = q\beta^{t-1}(1+\phi)^{t-1}$. Hence, in the analogous virtual planning problem, the endogenous discount factors are of the form $B^{t-1} = \beta^{t-1}(1+\phi)^{t-1}$ and the effective Pareto weights (prior to being augmented with incentive shocks) follow $\gamma^t(x) = (1-\omega) + \omega\gamma^{t-1}(x)$. Then our "steady state" analogue of pseudo-planner problem is:

$$\sup_{\{u_t,\upsilon_t,\kappa_{t+1}\}_{t=1}^{\infty}\in\Gamma(\Psi_1)} \int \sum_{t=1}^{\infty} B^{t-1}\gamma^t(\gamma_1)\sum_{\theta^t}[u_t(\gamma_1,\theta^t)+\theta_t\upsilon_t(\gamma_1,\theta^t)]\pi^t(\theta^t)\Psi_1(d\gamma_1)+\frac{\beta\alpha}{1-\beta\alpha}\sum_{t=1}^{\infty}B^t\ln\kappa_{t+1}.$$

This problem can be decomposed into an aggregate capital growth problem:

$$\max_{\{\kappa_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} B^{t-1}\left[\frac{\beta\alpha}{1-\beta\alpha}\ln\kappa_{t+1}-q\kappa_{t+1}\right]$$

with solution:

$$\kappa_{t+1}=\kappa^*(q):=\frac{\beta\alpha}{1-\beta\alpha}\frac{1}{q},$$

and a family of component planner problems:

$$V(\gamma_1)=\sup_{\Omega_1}\sum_{t=1}^{\infty}B^{t-1}\sum_{\theta^t}[\gamma_t(\gamma_1)\{u_t(\gamma_1,\theta^t)+\theta_t\upsilon_t(\gamma_1,\theta^t)\}-q\{\exp(u_t(\gamma_1,\theta^t))+w\upsilon_t(\gamma_1,\theta^t)^{1-\alpha}\}]\pi^t(\theta^t), \tag{72}$$

where w is a steady state wage. The value function V satisfies the Bellman equation:

$$V(\gamma)=\inf_{\eta}\sup_{\{u,\upsilon\}}\sum_{\Theta}\left[\rho(\gamma,\theta,\eta)u(\gamma,\theta)+\theta\lambda(\gamma,\theta,\eta)\upsilon(\gamma,\theta)-q\{\exp(u(\gamma,\theta))+w\upsilon(\gamma,\theta)\}+BV(\gamma'(\gamma,\theta,\eta))\right]\pi(\theta), \tag{73}$$

where the current utility weights ρ and λ are as before and the law of motion for effective Pareto weights (augmented with incentive shocks) satisfies:

$$\gamma'(\gamma,\widehat{\theta}_k,\eta)=(1-\omega)+\omega\gamma+\omega\left[\sum_{j=k+1,k-1}\eta_{k,j}-\sum_{j=k+1,k-1}\eta_{j,k}\frac{\pi(\widehat{\theta}_{k+j})}{\pi(\widehat{\theta}_k)}\right]. \tag{74}$$

For a pair of "aggregate multipliers" (q,ϕ) and a wage w, the dynamic programming problem (73) yields a triple of time invariant optimal policy functions $\{\eta^*(q,\phi,w),u^*(q,\phi,w),\upsilon^*(q,\phi,w)\}$. These functions map the current effective Pareto weight γ to an optimal collection of incentive multipliers $\eta^*(\gamma;q,\phi)$ and an optimal current allocation $\{u^*(\gamma;q,\phi),\upsilon^*(\gamma;q,\phi)\}$. Note that the first order conditions for utilities imply:

$$u_k^*(\gamma,q,\phi) = \ln\left(\frac{\rho(\gamma,\widehat{\theta}_k,\eta^*(\gamma))}{q}\right), \tag{75}$$

$$\upsilon_k^*(\gamma,q,\phi) = \ln\left(\min\left[\frac{\widehat{\theta}_k\lambda(\gamma,\widehat{\theta}_k,\eta^*(\gamma))}{qw},T\right]\right). \tag{76}$$

Together $\eta^*(q,\phi)$ and $\gamma'(q,\phi)$ yield an optimal law of motion for effective Pareto weights $\gamma^*(q,\phi)$:

$$\gamma_k^*(\gamma;q,\phi) = (1-\omega) + \omega\gamma + \omega\left[\sum_{j=k+1,k-1}\eta_{k,j}^*(\gamma;q,\phi) - \sum_{j=k+1,k-1}\eta_{j,k}^*(\gamma;q,\phi)\frac{\pi(\widehat{\theta}_{k+j})}{\pi(\widehat{\theta}_k)}\right]. \tag{77}$$

Given an initial Pareto weight γ_0, $\{\eta^*(q,\phi), u^*(q,\phi), \upsilon^*(q,\phi)\}$ induces an individual allocation. Moreover, $\{\eta^*(q,\phi), u^*(q,\phi), \upsilon^*(q,\phi)\}$ and $\kappa^*(q)$ induce a normalized population allocation in the obvious way. Finally, $\gamma^*(q,\phi)$ implies a law of motion $T_{q,\phi}$ for the cross sectional distribution of effective Pareto weights $\{\Psi_t\}_{t=1}^{\infty}$, where

$$\Psi_{t+1}(B) = T_{q,\phi}(\Psi_t)(B) = \int\sum_{k=1}^{K} 1(\gamma_k^*(\gamma;q,\phi)) \in B)\pi(\widehat{\theta}_k)\Psi_t(d\gamma),\ B\in\mathcal{B}$$

and $\Psi_1(B) = T_{0,\phi}(\Psi)(B) = \int 1(z(\gamma;\phi)\in B)\Psi_0(d\gamma),\ \ B\in\mathcal{B}$. The following Proposition is a corollary of results proven in Sleet and Yeltekin (2007).

Proposition 12 *$T_{q,\phi}$ admits a unique fixed point $\Psi_{q,\phi}^*$.*

Now, substituting the first order conditions (8.1) and (75) and the expression for the wage into aggregate resource constraint implies that:

$$\frac{1}{1-\beta\alpha}\frac{1}{q} = D^{\frac{1}{\alpha}}\frac{w}{(1-\alpha)}^{-\frac{1-\alpha}{\alpha}}. \tag{78}$$

If for all γ in the support of $\Psi_{q,\phi}^*$ and all $\widehat{\theta}_k$, $\frac{\widehat{\theta}_k\lambda(\gamma,\widehat{\theta}_k,\eta^*(\gamma))}{qw} \le T$, then the definition of the wage w reduces to:

$$w^{1-\alpha} = (1-\alpha)D\left(\frac{E[\theta]}{q}\right)^{-\alpha}. \tag{79}$$

Together (78) and (79) determine q and w independently of $\Psi_{q,\phi}^*$ (subject to the condition that for all (γ,θ) in the support of $\Psi_{q,\phi}^*$ the upper bound on leisure is non-binding for any agent). Collecting results we have the following.

Lemma 13 *Let q and w solve (78) and (79) and let $\phi\ge 0$ and $B=\beta(1+\phi)$. Let $\{\eta^*(q,\phi,w), u^*(q,\phi,w), \upsilon^*(q,\phi,w)\}$ denote the optimal policy functions that solve (73) at (q,ϕ,w) and let $\Psi_{q,\phi}^*$ denote the induced invariant distribution for effective Pareto weights. If $\Psi_1 = \Psi_{q,\phi}^*$, for all γ in the support of $\Psi_{q,\phi}^*$ and all $\widehat{\theta}_k$, $\frac{\widehat{\theta}_k\lambda(\gamma,\widehat{\theta}_k,\eta^*(\gamma))}{qw} \le T$ and*

$$\frac{1}{1-\beta}\int_{\mathbb{R}}\sum_{\theta}[u^*(q,\phi,w)(\gamma,\theta) + \theta\upsilon^*(q,\phi,w)(\gamma,\theta)]\pi(\theta)\Psi_{q,\phi}^*(d\gamma) + \frac{1}{1-\beta}\ln\kappa^*(q) = W^{UI},$$

where $\kappa^*(q) = \frac{\beta\alpha}{1-\beta\alpha}\frac{1}{q}$, *then the normalized population allocation implied by* $\{\eta^*(q,\phi,w), u^*(q,\phi,w), \upsilon^*(q,\phi,w)\}$ *and* κ *is stationary and solves* (**??**).

8.2 Steady State: Numerical example

We now provide an illustrative numerical example. Suppose that the parameters $\{\beta, \alpha, T\}$ are given by $\{0.9, 0.5, 1\}$. Assume that there are 4 shock values equally spaced between 0.9 and 1.1. Finally, assume that the probability of any shock realisation is 0.25.

We obtain an (approximate) optimal steady state allocation satisfying the conditions of Lemma 13. The optimal multiplier ϕ is 0.02 and B the effective societal discount factor for the distributional component of the virtual planning problem is 0.92 (compared to the agent discount factor of 0.9). The following graphs describe key aspects of the allocation and its tax-market implementation. The first graph illustrates the optimal policy functions for effective Pareto weights. Specifically, it shows the adjustments in the optimal effective Pareto weight $\gamma_k^{\prime *}(\gamma) - \gamma$ contingent on the shock $\hat{\theta}_k$. The support of the stationary distribution $\Psi^*_{q,\psi}$ corresponds to the region of γ values between the two arrows. All γ values to the left of the first downwards arrow are such that the optimal adjustment in the Pareto weight $\gamma_k^{\prime *}(\gamma) - \gamma$ is positive for all shocks; all γ values to the right of the first downwards arrow are such that the optimal adjustment in the Pareto weight $\gamma_k^{\prime *}(\gamma) - \gamma$ is negative for all shocks. Hence, the γ-values below the first arrow and above the second are transient. The next figure shows the optimal effort levels

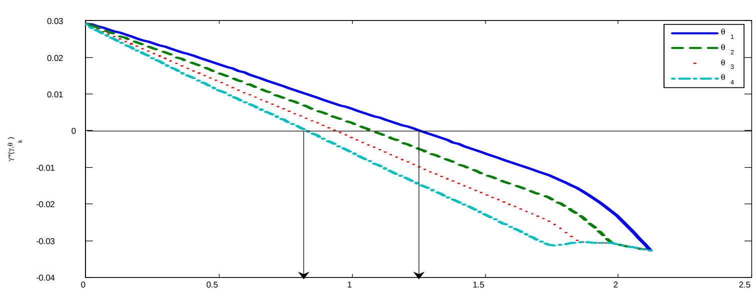

Figure 2: Optimal effective Pareto policies

at each γ value. Unsurprisingly, these are falling in γ and the shock $\hat{\theta}_k$. The figure confirms that the upper bound on leisure (or lower bound of 0 on effort) does not bind for any γ in the support of $\Psi^*_{q,\phi}$.

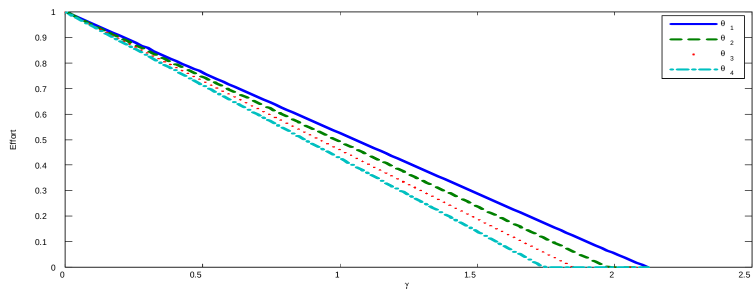

Figure 3: Optimal effort choices

Our next figure illustrates expected marginal asset taxes at $t+1$ conditional on an agent's current effective Pareto weight and shock at t. Again, the vertical lines indicate the support of the steady state distribution (although taxes are defined for all Pareto weights). We express these expected marginal asset taxes as taxes on asset income rather than wealth. i.e. they show $E_t[\hat{\tau}(\gamma, \theta_t, \theta_{t+1})]$, where $\widehat{\tau}(\gamma, \theta_t, \theta_{t+1})$ satisfies $1 + (R-1)\widehat{\tau}(\gamma^*_t(\gamma_1, \theta^{t-1}), \theta_t, \theta_{t+1}) = R(1 - \tau_t(\gamma_1, \theta^{t+1}))$. Over much of the Pareto weight state space, and over the support $\Psi^*_{q,\psi}$ in particular, these tax rates are fairly small, between -2% and 2%. However, they are neither all zero (as in Kocherlakota (2005)) or all negative (as in Farhi-Werning (2007)). They become large and negative at low γ values. Poor agents are given large subsidies to accumulate more wealth; rich agents are lightly taxed to deter too much accumulation.

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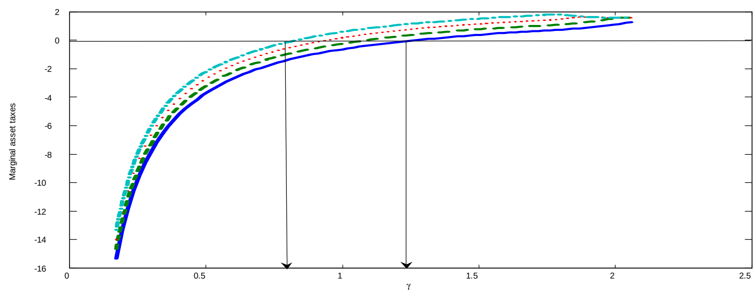

Figure 4: Expected marginal asset taxes

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