Long-Term Contracts in Unsecured Credit Markets *

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Abstract

This paper studies the effectiveness of longer-term unsecured credit contracts in improving credit access, consumption smoothing, and welfare. I find that longer-term contracts result in higher average borrowing interest rates and hence lower levels of borrowing and fewer borrowers in the equilibrium. In addition, I show that longer-term contracts reduce consumer welfare. I also investigate the welfare implications of interest rate ceilings and show that imposing a modest interest rate cap under long-term contracts improves welfare.

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We have a line of credit with you, but we do have the right at any time to say we're not going to extend that credit to you anymore, ... we have the right to increase the interest rate, because you now have become a riskier customer.

Edward Yingling, President of the American Bankers Association, 2004, Frontline

1 Introduction

In May 2009, the Credit Card Accountability Responsibility and Disclosure (CARD) Act of 2009 was signed into law. This act states that lenders cannot raise interest rates in the first year after a credit card account is opened unless the account is determined to be at least sixty days delinquent. The Act also prohibits lenders from raising credit card interest rates on *existing* balances. A third salient feature of the Act is that it requires lenders to give cardholders a longer notice period for rate increases applicable to any future borrowing on the account. Lastly, the Act proposed interest rate ceilings. The net effect of the first three provisions is to force lenders to commit to longer-term unsecured credit contracts than they do at present, and the effect of the fourth is to limit the risk that borrowers may assume.[1]

The premise of the policy is that restrictions on contracts and on the ability to change contractual terms *ex post* will help consumer welfare. The intuition for limiting repricing is simply that consumers will benefit from the ability to borrow at terms that do *not* fully reflect their *ex post* default risk. In particular, in the presence of rich and frequently updated information on the current "state" of a household, the inability of households to commit to repaying uncollateralized debts will force profit-maximizing lenders to reduce credit access to those whose state shows deterioration. Raising interest rates on both past and any future debts is precisely one such tactic. But such actions by definition undermine the ability of

[1] Usury laws have been commonly practiced in a variety of societies (See Visser and McIntosh (1998), Rockoff (2003), and Persky (2007)). In the US, this regulation was commonly used in the credit market before the Supreme Court case *Marquette National Bank vs. First Omaha Service Corp.* in 1978. The Supreme Court ruled that only the usury ceiling of the state in which a bank is chartered applies to the terms it offers, not the state in which the customer resides; given that some states (Delaware, South Dakota) had very limited restrictions, usury ceilings effectively ceased to exist.

households to use unsecured credit as an effective means of smoothing consumption in the face of shocks. (see Athreya, Tam, and Young 2009*a*) Therefore, by effectively lengthening the duration for which lenders are committed to contractual terms, it does seem possible this policy would enhance the ability of unsecured credit to help households smooth consumption. Naturally, however, such limitations on *ex post* actions will result in changes in *ex ante* lending behavior. In particular, in any long-term commitment to contractual terms, households may need to pay higher premia for borrowing. Nonetheless, households may be willing to pay higher rates in a setting in which large debts do not get repriced at precisely those moments when income is low and is likely to remain low. As a result, total credit use could expand and consumption smoothing, though more expensive *ex ante*, might improve.

As an empirical matter, the frequent repricing of debt is a reality. The available evidence suggests that creditors are now better able to keep track of the evolution of household income and, thereby, household default risk. Moreover, once in possession of this information, it appears difficult if not impossible for creditors to commit to not altering the terms of credit. "Risk-based" pricing is the norm in credit cards and other forms of uncollateralized lending (See Edelberg 2006). The frequent arrival of information on income realizations and default risk, combined with the inability of lenders to credibly commit to refraining from "adverse" repricing, raises the possibility that longer-term contracts with fixed borrowing terms may be desirable for households. Furthermore, from a policy standpoint, given the lack of commitment to not reprice, realizing any welfare gains from long-term contracts may have to come from deliberate policy changes such the ones studied here.

Given the presence of opposing forces at work, the overall effect of the duration of commitment to contractual terms on consumption smoothing and welfare is obviously a quantitative question. The goal of this paper is to develop an environment in which the relative strength of these forces can be gauged, and then to employ the model to understand whether the enacted limitations are likely to improve consumer welfare. The intuition given above makes it clear that defaultable debt, its duration, and its pricing may all matter for consumption smoothing. However, until very recently two common features of the vast literature on consumption and credit over the life cycle have been the presence of implicitly secured borrowing (via the prohibition of default) up to fixed, common, and exogenous credit limits. However, those restrictions prevent the confrontation of two pervasive features of US data on credit use. First, Carroll (2001), among others, finds that approximately up to 20 percent of households have negative financial net worth, while Sullivan, Warren, and Westbrook (2000) show that approximately 15 percent of US households have defaulted on unsecured debt obligations over the past two decades.[2] These findings show that not all household debts are fully collateralized, and that a substantial portion of such debts are not repaid. Default risk, in turn, leads households to face widely varying terms under which they may acquire unsecured credit, as well as the upper bounds (or "credit limits") that are available to them (See Furletti 2002 and Edelberg 2006). Moreover, both unsecured debt accumulation and consumer default do appear to stem primarily from attempts to smooth consumption in the face of income shocks. Gross and Souleles (2002) and Sullivan (2006) document that unsecured debt is frequently accumulated by those who have suffered an adverse shock, while Sullivan, Warren, and Westbrook (2000) finds that 67 percent of bankruptcy filers report income disruption as one of their reasons for declaring bankruptcy. The preceding observations have led to an emerging strand of literature seeking to evaluate the impact of unsecured credit and default on consumption smoothing and risk sharing.[3] The theoretical work by Dubey, Geanakoplos, and Shubik (2005) shows that harsh punishment which leads borrowers to increase commitment to their contracts destroys risk sharing. However, the subsequent quantitative literature has been almost uniformly negative in its assessment of the consumption smoothing role played by unsecured debt and default. For example, Athreya (2002, 2008) and Athreya, Tam, Young (2008) each find that ruling out default altogether would, if feasible, improve welfare *ex ante*.[4]

[2] Wolff (2002) finds that 17.6 percent of U.S. households have negative net worth.

[3] Contributions to this literature include Zame (1993), Livshits, MacGee, and Terlilt (2007), Narajabad (2007), Chartterjee, Corbae, and Ríos-Rull (2008), Nakajima(2008), and Sánchez (2008).

[4] Athreya (2002) models pooling interest rates. Athreya (2008) and Athreya, Tam, and Young (2009a)

Despite the preceding findings, it would likely be premature to conclude that unsecured credit and default cannot play a constructive role in household smoothing, in large part due to a potentially severe modeling restriction imposed by all work to date. Critically, a common feature of all previous work on defaultable consumer debt is that *contracts are repriced at exactly the same frequency as the arrival of income shocks.* This restriction, in turn, is likely to be crucial to driving the normative implications of existing models. The intuition is as follows. All loans must be refinanced every time any information on household income arrives. In a competitive setting, this refinancing will work directly against allowing credit to serve a consumption smoothing role: those realizing bad shocks will, by virtue of being able to default, be forced to pay more to roll over any existing debt, or to acquire new debt, while the reverse will hold for anyone realizing good fortune. In essence, credit "tightens" precisely when households need it most, and "loosens" precisely when they need it least. In contrast, prohibiting debt default (via, say, harsh punishments) would ensure plentiful credit at a risk-free rate. On balance, households prefer the latter, *ex ante*, often by a substantial amount.

In this paper, I extend quantitative theory in two ways. First, by allowing for long-term contracts and for their coexistence with short-term debt I remove a restriction that may wrongly have implicated defaultable consumer debt in lowering welfare, and use this framework to study a recent policy response that has the potential to improve outcomes. Second, my framework will allow for the possibility of interest rate ceilings, another aspect of recent reforms, to be welfare-enhancing without relying on general equilibrium effects, externalities, or consumer irrationality (see Glaeser and Scheinkman 1996, Andolfatto 2002, and Webb 1999). In contrast, in existing work where debt is repriced contemporaneously with shocks, interest rate limits only restrict choice sets for agents and thereby cannot help account for the existence of usury ceilings.

To understand the effect of long-term contracts on consumer smoothing and welfare, I

model risk-based interest rates.

study two economies. First, I study a benchmark model in which lenders offer pure one-period (henceforth "short-term") contracts. I then calibrate the model to match the key empirical facts of unsecured credit markets in the recent U.S. economy. Next, the benchmark allocation is compared to one in which lenders offer pure two-period (henceforth "long-term") contracts. The difference between these allocations is a quantitative measure of the effect of enforcing long-term contracts.

This paper documents four main implications of lengthening the duration of contracts. First, it shows that longer-term contracts tend to result in higher average interest rates, lower levels of borrowing, and fewer households borrowing. Also, longer-term contracts generally result in lower aggregate default rate and higher discharged debt-to-income ratios. As lenders commit to long-term contracts, on one hand, they expect higher default risks because of a more dispersed distribution of future earnings. Therefore, they charge higher prices. On the other hand, long-term contracts could lower the contractual terms because households could have enough credit not to default under long-term contracts. This could alter households' decisions to default or to defer filing. In equilibrium, the average borrowing interest rate rises as the duration of the contract rises for two reasons. First, the probability of reversing default decisions is low when earnings shocks are persistent.[5] Second, the future level of borrowing does not reflect borrowers' *ex post* default risk, and hence borrowers accumulate more debt. That results in default with higher debt, and hence lenders raises the borrowing interest rate. As the cost of borrowing rises, the level of debt drops, and hence fewer households borrow and default.

Second, longer-term contracts improve intratemporal smoothing (the variance of log consumption at a given age) at the expense of intertemporal consumption smoothing (the variance of mean log consumption over the life cycle). As the duration of debt contracts lengthen, households get worse contractual terms and so they borrow less. As a result, they have less

[5]Earnings shocks are commonly decomposed into transitory shocks and persistent shocks in the labor earnings literature. The literature finds that the persistence of earnings shocks is high. See Lillard and Weiss (1979), MaCurdy (1982), and Abowd and Card (1989). Guvenen (2007) argues for substantial but lower persistence.

ability to smooth the life-cycle earnings. Less debt results in higher levels of accumulated wealth, and then households have greater ability to smooth the intratemporal shock. Also, as noted earlier, defaulters accumulate more debt under long-term contracts. That implies that defaulters have higher consumption prior to defaulting than under short-term contracts.

Third, longer-term contracts lead to welfare reduction. Long-term contracts raise the price of borrowing and hence lower the level of borrowing. As a result, households have less ability to smooth the life-cycle earnings. As in Athreya (2002) and Athreya, Tam, and Young (2008), I also find that eliminating the bankruptcy option leads to the largest welfare gains.[6]

Last, imposing a usury ceiling in an economy can improve welfare. By imposing a usury ceiling into long-term contracts, households who get bad earnings shocks in the future are limited their abilities to borrow and hence default with less debt. Thus, the contractual terms improve *ex ante*. That can improve outcomes. I show that imposing a not too "tight" usury ceiling under long-term contracts can improve the welfare of newborns. However, imposing a usury ceiling weakly reduces the welfare of newborns under short-term contracts.[7]

Pure long-term contracts do allow households to lock in credit terms and prevent these terms from getting bad when they face a bad earnings shocks. But when this is the only type of contract available it forces all borrowers to buy the additional insurance of a long-term contract. Given the absence of asymmetric information in a small open economy, it seems that the coexistence of both short-term and long-term improve outcomes. Such a setting will allow those agents who want protection to obtain it, and also will allow those who do not expected bad shocks in the near future to avoid buying insurance in this way. Therefore, I also study a setting in which households may choose the duration of loan contracts.

My paper is closely related to the work of Mateo-Planas and Ríos-Rull (2007), which models a credit line which prespecifies a credit limit and has a fixed interest rate.[8] Both

[6]If borrowers have full commitment to the debt contract, lenders' commitment is irrelevant.

[7]The model is constructed as an small open economy. A usury ceiling only hurt the high-risk borrowers without benefiting the others.

[8]Narajabad (2007) and Drozd and Nosal(2008) also use credit lines in their models.

their paper and mine assume that households can hold only a single contract in each period. The pricing of contracts highlights the difference between our findings. In their contract, the borrowing limit corresponds to a unique price term.[9] In this paper, the contract is a risk-based pricing function, which maps a sequence of borrowing levels to a unique sequence of price terms, and lenders commit to the terms for certain periods.[10]

The paper is organized as follows. Section 2 presents the main model. Section 3 discusses the strategies to calibrate the model. Section 4 covers the implications for the impact of longer-term contracts and the consequences of imposing usury ceiling in such an environment. Section 5 concludes.

2 Model

The general framework follows Athreya *et al.* (2008). Households live for a maximum of $J < \infty$ periods and are exposed to idiosyncratic risk in labor earnings. Markets for insurance against this risk do not exist.

2.1 Credit Markets

Markets are perfectly competitive and information is symmetric between lenders and borrowers.[11] Households have access to financial intermediaries for the purpose of saving, and the risk-free return on savings is given exogenously. They can also borrow using defaultable debt from a financial intermediary, where the cost of borrowing depends on individual characteristics. Households are allowed to sign only one contract. Lenders commit to the contractual terms for certain period(s). After the contract expires, a new contract is priced based on the household's current characteristics. A household can terminate the existing

[9] The credit line assumption seems at odds with credit card industry practice. Massoud, Saunders, and Scholnick (2006) finds that credit card penalty fees are positively correlate with consumer default risk.

[10] Another related strand of literature concerns long-term sovereign bonds. Notable contributions in this literature are Arellano and Ramanarayanan (2008), Hatchondo and Martinez (2008), and Chatterjee and Eyigungor (2009).

[11] Evans and Schmalensee (2000) suggest that unsecured consumer credit markets are competitive.

contract either by defaulting or by switching to a new contract. It is costly for a household to default, but it is costless to switch to a new contract. In the benchmark, the financial intermediary offers pure short-term contracts.

2.2 Households

Households value consumption, discount the future at the rate $\beta < 1$, inelastically supply labor, and attach a negative value λ to all non-pecuniary costs associated with bankruptcy. There is evidence for non-pecuniary costs: Fay, Hurst, and White (1998) and Gross and Souleles (2002) document significant unexplained variation in the probability of default across households even after controlling for many observables. White (1998) finds that at least 15 percent of U.S. households would benefit financially from filing for bankruptcy, but only about 1.5 percent do so. This suggests that there is a large non-pecuniary cost of filing for bankruptcy. These results suggest that unobservable costs associated with bankruptcy are not explicitly pecuniary in nature. The literature often refers to the non-pecuniary cost as "stigma".

In the model presented here, households maximize the expected discounted life-time utility:

$$E \sum_{j=1}^{J} \beta^j \left(\frac{c_j^{1-\sigma}}{1-\sigma} - \lambda \mathbf{1} \left(d_j = 1 \right) \right). \tag{1}$$

$\sigma \geq 0$ is the coefficient of relative risk aversion. A household retires exogenously at age $j^* < J$, and $d_j = 1$ denotes the (indicator) decision rule governing bankruptcy if the household declares bankruptcy and 0 otherwise.

In every period a typical household decides how much to consume (c) and save (b). The household has the option to default on any accumulated debt, as a function of the next period's shock. Three costs are associated with defaulting. As noted earlier, the first is a non-pecuniary cost, λ, borne only in the period in which the default occurs.[12] The second

[12]Athreya *et al.* (2008) assumes λ to be a random variable so that they have more freedom to match the targets in the calibration exercise. However, the assumption of a constant λ does *not* alter the answer to the

is a pecuniary cost, Δ, also paid only in the period of default. The final cost is one-period exclusion.[13]

During the time during which the household is of working age, the household's budget constraint is given by

$$c_j + q(\cdot)b_j + \Delta\mathbf{1}\left(d_j = 1\right) \leq a_j + \omega_{j,y}e\nu, \tag{2}$$

where $q(\cdot)$ is the bond price function which will depend on individual characteristics, the size of bond issuance is b, and the length of the contract is T. Net worth is denoted by a, the pecuniary cost of filing for bankruptcy is denoted by Δ, and current income is denoted by $\omega_{j,y}e\nu$. Labor income is the product of three terms: a deterministic age term $\omega_{j,y}$, where y is interpreted as college, high-school, and non-high-school educated types, as in Hubbard, Skinner, and Zeldes (1994), a persistent shock e that evolves according to the AR(1) process

$$\log\left(e'\right) = \rho\log\left(e\right) + \epsilon', \tag{3}$$

and a purely transitory shock $\log\left(\nu\right)$. The shocks to labor earnings, ϵ and $\log\left(\nu\right)$, are both independent mean zero normal random variables. The budget constraint during retirement is given by

$$c_j + q(\cdot)b_j + \Delta\mathbf{1}\left(d_j = 1\right) \leq a + \theta\omega_{j^*-1}e_{j^*-1}\nu_{j^*-1} + \Theta, \tag{4}$$

where for simplicity in this paper I assume that pension benefits are composed of a fraction $\theta \in (0,1)$ of income in the last period of working life plus a fraction Θ of average income, which is normalized to 1. As seen in the data, bankruptcy is not a retiree phenomenon; thus I deliberately keep the specification of the problem of retirees simple.[14] I assume households

question of this paper, but simply raises the computational burden. The reason is that switching the λ state can also trigger default, and hence borrowing premia rise and credit limits are cut. These hurt households' ability to smooth life-cycle consumption.

[13]Under the current U.S. bankruptcy code, a filing is likely to be judged fraudulent if it is immediately followed by significant asset accumulation. Because filings normally take several months to resolve, it seems that the law imposes a short period of exclusion as a punishment. Longer periods of exclusion that are imposed by the market are not consistent with competitive behavior.

[14]Sullivan *et al.* (2000) find that 7 percent of defaulters are age 65 or older.

have no bequest motive.

2.2.1 Recursive Formulation with a Unique Type of Contracts

Let t denote number of periods that the current contract has been in effect notational. For simplificity, let $x = (e, \nu)$ denote exogenous components of the household's state vector in the current period, and $x^* = (e^*, \nu^*)$ denote the household's state vector at the time when the contract was signed, $t = 1$. Recall, d denotes the decision to file for bankruptcy. Let $s = 1$ denote the (indicator) decision rule governing contract switching if the household opts for a new contract and 0 otherwise. b_{-1} (the size of bond issuance in the previous period),x, q, and t all play a role in default (d) and switching (s) decisions, so they will be part of the household's state vector. In recursive terms, a household of age j, keeping the existing contract ($d = 0$ and $s = 0$), solves

$$v^r\left(b_{-1}, x, j, t, q\right) = \max_b \left\{ \frac{c_j^{1-\sigma}}{1-\sigma} + \beta E_{x'}\left[v(b, x', j+1, \tilde{t}', \tilde{p}')\right] \right\}, \tag{5}$$

switching to a new contract ($d = 0$ and $s = 1$) solves

$$v^s\left(b_{-1}, x, j, 1, q^*\right) = \max_b \left\{ \frac{c_j^{1-\sigma}}{1-\sigma} + \beta E_{x'}\left[v(b, x', j+1, \tilde{t}', \tilde{p}')\right] \right\}, \tag{6}$$

and defaulting on the accumulated debt ($d = 1$) solves

$$v^d\left(0, x, j\right) = \left\{ \frac{c_j^{1-\sigma}}{1-\sigma} - \lambda + \beta E_{x'}\left[v\left(b, x', j+1, 1, \tilde{p}'\right)\right] \right\}, \tag{7}$$

where $\tilde{t}$ denotes the period in which the contract $\tilde{q}$ is obtained, and $\tilde{q}$ denotes the optimal contract in the current period. $x' = (e', \nu')$ denotes the exogenous components of the household's state vector, $\tilde{t}'$ denotes the period in which the contract $\tilde{q}'$ is obtained, and $\tilde{q}'$ denotes the optimal contract in the future period. In addition, q^* denotes the new contract, which depends on the current information set x. The household chooses $d \in \{0, 1\}$ and $s \in \{0, 1\}$

to solve

$$v\left(a, x, j, \tilde{t}, \tilde{q}\right) = \max_{d\in\{0,1\}, s\in\{0,1\}} \left\{v^r\left(b_{-1}, x, j, t, q\right), v^s\left(b_{-1}, x, j, 1, q^*\right), v^d\left(0, x, j\right)\right\}, \tag{8}$$

where

$$a = \begin{cases} 0 & \text{if} \quad d = 1 \\ d_{-1} & \text{otherwise,} \end{cases} \tag{9}$$

and

$$\tilde{t} = \begin{cases} 0 & \text{if} \quad d = 1 \\ 1 & \text{if} \quad s = 1 \\ t & \text{if} \quad s = 0, \end{cases} \tag{10}$$

and

$$\tilde{q} = \begin{cases} 0 & \text{if} \quad d = 1 \\ q^* & \text{if} \quad s = 1 \\ q & \text{if} \quad s = 0. \end{cases} \tag{11}$$

Notice that in the filing period, $\tilde{t} = 0$ means that the contract is terminated and $q = 0$ for $\forall b < 0$. The defaulter can obtain a new contract in the future period. When $t > T$, then $s = 1$; that is, a new contract must be obtained. In this case, the household's problem simplifies to

$$v\left(a, x, j, \tilde{t}, \tilde{q}\right) = \max_{b, d\in\{0,1\}, s=1} \left\{v^s\left(b_{-1}, x, j, 1, q^*\right), v^d\left(0, x, j\right)\right\}, \tag{12}$$

where

$$\tilde{t} = \begin{cases} 0 & \text{if} \quad d = 1 \\ 1 & \text{if} \quad s = 1, \end{cases} \tag{13}$$

and

$$\tilde{q} = \begin{cases} 0 & \text{if} \ d = 1 \\ q^* & \text{if} \quad s = 1. \end{cases} \tag{14}$$

2.2.2 Price Function

The main innovation in this paper is to extend short-term contracts into long-term contracts. A perfectly-informed, diversified, and competitive financial intermediary offers households individually-priced credit contracts with duration T. Let i^* denote the current information set for a lender, $\widehat{\pi}^{b_j}$ denote the function that assigns a probability of default to a loan of size b, and let $\widehat{\pi}^{s_j}$ denote the function that assigns a probability of switching a contract to a loan of size b under information set i^* at age j. Let r denote the risk-free return rate on savings, and ϕ represents transactions costs for lending, so that $r + \phi$ is the risk-free rate on borrowing. When the length of contract is restricted, as in all preceding works, to short-term, the lender's zero-profit condition is

$$q\left(b_j, i^*, 1\right) b_j = \begin{cases} \frac{b_j}{1+r} & \text{if } b_j \geq 0 \\ \frac{(1-\widehat{\pi}^{b_j})b_j}{1+r+\phi} & \text{if } b_j < 0. \end{cases} \tag{15}$$

They are the same price functions as in Athreya, Tam, and Young (2008), and completely standard in the unsecured debt literature (see Livshits, MacGee, and Tertilt 2006, Chatterjee and Satyajit, P. Dean Corbae, Makoto Nakajima, and José-Víctor Ríos-Rull 2007).[15]

Allowing for more flexible durations of defaultable debt contracts, now consider the case $T > 1$ I demonstrate that perfect competition implies the following two propositions.

If $b_j \geq 0$, the lender's zero-profit condition must satisfy

$$q\left(b_j, i^*, T\right) b_j = \frac{b_j}{1+r}. \tag{16}$$

b_j is the face value of bond that the lender promises to pay the household who is currently age j in the next period (at age $j+1$). Lenders cannot default, so the present value of the household's receivable equals $\frac{b_j}{1+r}$. Recall, q is the bond price function, and it depends on

[15] See Athreya, Tam, and Young (2008) for more details on the short-term contract.

the bond issuance b, information set i^*, and the duration of contracts T. The lender sells its bond at price $q(b_j, i^*, T)$, so $q(b_j, i^*, T)\, b_j$ is the present value of the lender's receivable. Imposing the zero-profit condition, Equation (16) has to hold. It can be further simplified to $q(b_j, i^*, T) = \frac{1}{1+r}$.

If $b_j < 0$, the lender's zero-profit condition must satisfy

$$q(b_j, i^*, T)\, b_j + \sum_{t=j+1}^{j+T-1} E\left[\left(\prod_{k=j}^{t-1} \frac{\left(1-\widehat{\pi}^{s_k}-\widehat{\pi}^{d_k}\right)}{1+r+\phi}\right)\left(q(b_t, i^*, T)\, b_t\right)\right]$$

$$= \frac{\left(1-\widehat{\pi}^{d_j}\right) b_j}{1+r+\phi} + \sum_{t=j+2}^{j+T} E\left[\left(\prod_{k=j}^{t-1} \frac{\left(1-\widehat{\pi}^{s_k}-\widehat{\pi}^{d_k}\right)}{1+r+\phi}\right)\left(\frac{\left(1-\widehat{\pi}^{d_t}\right) b_t}{1+r+\phi}\right)\right]. \quad (17)$$

Recall, given a household who is at age j, $\widehat{\pi}^s$ is the probability of switching to new contract, and $\widehat{\pi}^d$ is the household's default probability at age $j+1$. So the conditional survival probability of the contract is $\left(1-\widehat{\pi}^s-\widehat{\pi}^d\right)$, and the conditional repayment probability equals $\left(1-\widehat{\pi}^d\right)$.

The first term on the left hand side of Equation (17) is the payment that the lender lends to the household today. The second term is the present value of expected net payment(s) that the lender lends to the household until the contract expires. Therefore, the left-hand-size of Equation (17) is the present values of the (expected) total payment(s) that the lenders lends to the household under this contract.

The first term on the right hand side of Equation (17) is the present value of expected repayment that the lender receives from the household tomorrow. The second term is the present value of expected net repayment(s) that the lender receives from the household until the contract expires. Therefore, the right hand side of Equation (17) is the present value of the (expected) repayment(s) that the lenders receive from the household under this contract. Imposing a break-even condition, Equation (17) has to hold.

The compactness of the range for q implies that the household problem has a compact opportunity set. To show this, I first show $\widehat{\pi}^d \colon b_{|i^*} \rightarrow [0,1]$ and $\widehat{\pi}^s \colon b_{|i^*} \rightarrow [0,1)$.

$\widehat{\pi}^d$ and $\widehat{\pi}^s$ are mutually exclusive.

Equation (7) shows that event $s = 1$ and event $d = 1$ are disjoint. Therefore, $\widehat{\pi}^d$ and $\widehat{\pi}^s$ are mutually exclusive.

Rational expectations for the financial intermediary requires that the probability of default used to price debt must be consistent with that observed in equilibrium, implying that

$$\widehat{\pi}^b = \sum\nolimits_{e',\nu'} \pi_e\left(e'|e\right)\pi_\nu\left(\nu'\right) d\left(\hat{b}, q, e', \nu'\right), \tag{18}$$

where $\hat{b}$ is the current optimal decision rule based on the household's characteristics and the price function q, and $d(\cdot)$ is the default decision that is governed by the debt level $\hat{b}$, the (old) contract q, and the income states e' and ν'. The probability is given that an agent will default in state (e', ν') tomorrow at debt level b with contract q, and integrating over all such events tomorrow is the relevant default risk. Let $i^* = (y, e, \nu, j)$.

$\widehat{\pi}^d \colon b_{|i^*} \rightarrow [0,1]$

Recall, households have no bequest motive. When $j = J$, it implies $v^{s=0} = v^{s=1}$. And given Equation (4), Equation (8) is simplified to

$$v\left(\tilde{a}, x, J, \tilde{t}, \tilde{q}\right) = \max_{d\in\{0,1\}}\left\{\frac{\left(a + \theta\omega_{j^*-1}e_{j^*-1}\nu_{j^*-1} + \Theta\right)^{1-\sigma}}{1-\sigma}, \frac{\left(\theta\omega_{j^*-1}e_{j^*-1}\nu_{j^*-1} + \Theta - \Delta\right)^{1-\sigma}}{1-\sigma} - \lambda\right\}. \tag{19}$$

So $\exists$ an a^* such $(a + \theta\omega_{j^*-1}e_{j^*-1}\nu_{j^*-1} + \Theta)^{1-\sigma} = (\theta\omega_{j^*-1}e_{j^*-1}\nu_{j^*-1} + \Theta - \Delta)^{1-\sigma} - \lambda(1-\sigma)$. $\forall a < a^*$, then $d = 1$, so $\widehat{\pi}^d = 1$. $\forall a \geq a^*$, then $d = 0$, so $\widehat{\pi}^d = 0$. Use the backward induction, $\widehat{\pi}^d$ is a closed set and bounded in $[0,1]$ for $\forall j$.

The probability of switching to another contract must also be consistent with that observed in the stationary equilibrium, implying that

$$\widehat{\pi}^s = \sum\nolimits_{e',\nu'} \pi_e\left(e'|e\right)\pi_\nu\left(\nu'\right) s\left(\hat{b}, q, e', \nu', d = 0\right), \tag{20}$$

where $s(\cdot)$ is the probability that the agent will switch to a new contract in state (e', ν') tomorrow at debt level b under contract q and $d = 0$, and integrating over all such future events is the relevant contract switching probability.

$\widehat{\pi}^s$: $b_{|i^*} \rightarrow [0, 1)$

The transition matrix is bounded on $[0, 1)$. Therefore so is $\widehat{\pi}^s$.

2.2.3 Recursive Formulation with Multiple Types of Contracts

The single contract framework can be easily extended to model multiple contracts with different lengths. Lenders offer N contracts to agents and the lengths of contracts are given by $\Omega = \{T_1, T_2, ..., T_N\}$. Notice that the length of the contract T is now a state variable. From Equation (8), the household's problem is modified to

$$v\left(a, x, j, \tilde{t}, \tilde{q}, \tilde{T}\right) = \max_{b, d \in \{0,1\}, s \in \{0,1\}, T \in \Omega} \left\{ \begin{array}{l} v^r\left(b_{-1}, x, j, t, q, T^*\right), v^d\left(0, x, j\right), \\ v^s\left(b_{-1}, x, j, 1, q^*, T \in \Omega\right) \end{array} \right\}. \tag{21}$$

The household chooses b, $d \in \{0, 1\}$, $s \in \{0, 1\}$, and $T \in \Omega$ to maximize indirect utility. $v^r\left(d_{-1}, x, j, t, q, T^*\right)$ is the value of committing to the existing contract with the duration T^*. $v^d\left(0, x, j\right)$ is the value of exercising the default option. $v^s\left(d_{-1}, x, j, 1, q^*, T\right)$ is the value of switching to a new contract with T (N of these contracts). Now,

$$a = \begin{cases} 0 & \text{if} \quad d = 1 \\ d_{-1} & \text{otherwise,} \end{cases} \tag{22}$$

and

$$\tilde{t} = \begin{cases} 0 & \text{if} \quad d = 1 \\ 1 & \text{if} \quad s = 1 \\ t & \text{if} \quad s = 0, \end{cases} \tag{23}$$

and

$$\tilde{q} = \begin{cases} 0 & \text{if} \quad d = 1 \\ q^{*} & \text{if} \quad s = 1 \\ q & \text{if} \quad s = 1, \end{cases} \tag{24}$$

and

$$\tilde{T} = \begin{cases} T^{*} & \text{if} \quad s = 0 \\ T^{o} & \text{if} \quad s = 1, \end{cases} \tag{25}$$

where $\tilde{T}$ is the duration of the contract that provides the household's highest utility level, and the duration of a new contract q^{*} is T^{o}. In the benchmark, both short-term contracts and long-term contracts are offered. That is, $\Omega = \{1, 2\}$. The equilibrium contract price functions are the same as in the previous subsection.

2.2.4 Computational Strategy

This subsection lays out the algorithm used to solve this model. To compute this model, I follow Athreya *et al.* (2008) closely. First, I use finely spaced grids for a, and use linear interpolation to evaluate points off of the grid. Similarly, I linearly interpolate q at the points off of the grid for b. The Bellman equation is solved using backward induction. To compute the equilibrium price function, I use a simulated iterative approach as described in the following:

1. Guess an initial function $q^{0}(b, y, e, \nu, j, T)$;

2. For each $k \subset 0, 1, ..., T-1$ solve the household problem to obtain

   (a) the optimal savings rule for b, $g(a, y, e, \nu, j+k, q^{0})$,

   (b) the default decision, $d(e', \nu' | a, y, e, \nu, j+k, q^{0})$,

   (c) and the contract switching decision, $s(e', \nu' | a, y, e, \nu, j+k, q^{0})$,

(d) then compute

$$\widehat{\pi}^{d_{j+k}}\left(b, y, e, \nu, j+k\right) = \sum_{e'} \sum_{\nu'} \pi_e\left(e'|e\right) \pi_\nu\left(\nu'\right) d\left(e', \nu'\right), \tag{26}$$

(e) and

$$\widehat{\pi}^{s_{j+k}}\left(b, y, e, \nu, j+k\right) = \sum_{e'} \sum_{\nu'} \pi_e\left(e'|e\right) \pi_\nu\left(\nu'\right) s\left(e', \nu'\right); \tag{27}$$

3. Given (y, e, ν, j), simulate the model forward for T periods, then for each b_j

   (a) use $g\left(a, y, e, \nu, t, q^0\right)$ to compute $E(b_t)$ and $E(q^0(b, y, e, \nu, j, T)b_t)$ for $t \subset j+1, j+2, ..., j+T$

   (b) then compute $q^*\left(b_j, y, e, \nu, j\right)$ by solving

$$q^*\left(b_j, y, e, \nu, j, T\right) b_j = \left\{ \begin{array}{c} \frac{\left(1-\widehat{\pi}^{b_j}\right)b_j}{1+r+\phi} + \sum_{t=j+2}^{T+j} \left(\prod_{k=j}^{t-1} \frac{\left(1-\widehat{\pi}^{s_k}-\widehat{\pi}^{d_k}\right)}{1+r+\phi} \right) \times \\ \left(\frac{\left(1-\widehat{\pi}^{b_t}\right)E(b_t)}{1+r+\phi} - E(q^0\left(b_t, y, e, \nu, j, T\right) b_t) \right) \end{array} \right\}; \tag{28}$$

4. Set

$$q^1\left(b, y, e, \nu, j\right) = \Xi q^0\left(b, y, e, \nu, j, T\right) + \left(1-\Xi\right) q^*\left(b, y, e, \nu, j, T\right)$$

   and repeat until the pricing function converges, where $\Xi \in (0, 1]$.

Because the household's value function is continuous but not differentiable or concave, I use golden section search (see Press *et al.* 1993 for details of the golden section algorithm) to find the optimal savings behavior after bracketing with a coarse grid search.

3 Calibration

The calibration is conducted using the benchmark framework. I follow the literature by setting the relative risk aversion parameter, σ, to 2, and the exogenous risk-free return on saving, r, to 0.01.[16] I then set θ, the fraction of income in the last period of working life, to 0.35. j^*, the exogenous retirement (model) age, is set to 45, and Θ is set to 0.2, yielding an overall replacement rate of about 55 percent.[17] In addition, I set $\phi = 0.03$ to generate a 3 percent spread between risk-free return rate received by households and the risk-free return rate on borrowing.[18] The median income in the United States is roughly $\$40,000$, and the median filing cost is equal to \$1200. I therefore set $\Delta = 0.03$.[19]

The income process is taken from Hubbard, Skinner, and Zeldes (1994), which estimates separate processes for non-high-school, high-school, and college-educated workers for the period 1982-1986. Figure (1) displays the path for $\omega_{j,y}$ for each type; the large hump that is present in the profile for college-educated workers implies that they will want to borrow early in life to a greater degree than the other types will.[20] For each period the process is discretized with 11 grid points for e and 3 points for ν. The resulting processes are

$$\begin{aligned}
\log(e') &= 0.95\log(e) + \epsilon' \\
\epsilon &\sim N(0, 0.033) \\
\log(\nu) &\sim N(0, 0.04)
\end{aligned}$$

for non-high school educated agents,

$$\begin{aligned}
\log(e') &= 0.95\log(e) + \epsilon' \\
\epsilon &\sim N(0, 0.025) \\
\log(\nu) &\sim N(0, 0.021)
\end{aligned}$$

[16]The risk-free rate is consistent with Mehra and Prescott (1985).

[17]See Livshits *et al.* (2007)

[18]The spread between returns on savings and capital returns is consistent with the transactions costs measured by Evans and Schmalensee (1999).

[19]This cost is an estimate which is inclusive of filing fees, lawyer costs, and the value of time. Sullivan *et al.* (2000) finds that lawyer costs are between \$750 and \$1500. So $\Delta = \$1200$ is a ballpark measure.

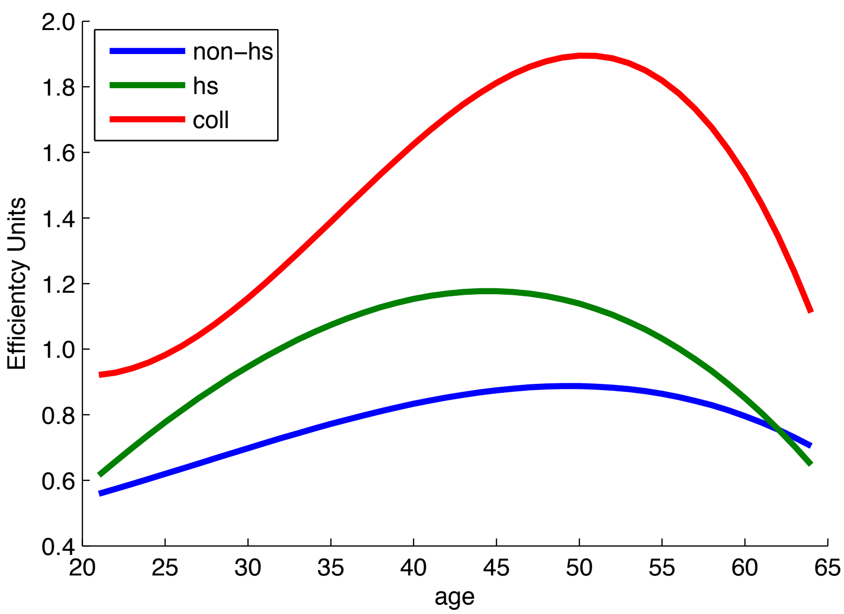
[20]The deterministic life-cycle earnings profile is approximated by a cubic polynomial function in age.

for high school educated agents, and

$$\log(e') = 0.95\log(e) + \epsilon'$$
$$\epsilon \sim N(0, 0.016)$$
$$\log(\nu) \sim N(0, 0.014)$$

for college educated agents; the measures of the three groups are 16, 59, and 25 percent, respectively.

Figure 1: ω over the Life Cycle



Note: Figure (1) plots the deterministic life-cycle earnings ω for each type y.

I calibrate the other parameters (β, λ) to match four targets: the 20.0 percent of households with negative financial net worth, an aggregate negative net worth to GDP ratio of 4.5 percent, a bankruptcy filing rate of 1.2 percent, and a median discharge to median income ratio of 50 percent.[21] The target for bankruptcy is consistent with a model in which

[21]The value of the measure of the negative net worth of households is taken from Carroll (2001). For the measure of the negative net worth to GDP ratio, I roughly average the numbers from Livshits *et al.* (2007) at 8.7 percent and Sánchez (2008) at 0.7 percent to get a ballpark measure of 4.7 percent.

only income is uncertain; that is, there are no shocks to expenses. Expense shocks create involuntary creditors who allow households to suddenly accumulate very large debts with no corresponding changes in measured consumption. The difficulties in measuring rare "catastrophic" shocks, their true "uninsurability" (for example, Medicaid and emergency rooms are always available to deal with medical shocks), and their persistence can lead to a serious mismeasurement of the roles of credit use and bankruptcy in managing income risk. I therefore calibrate the model's bankruptcy target to be the net of this measure. Sullivan, Warren, and Westbrook (2000) show that one-fifth of filers report that health played some role in their decision to file. Using an overall filing rate of 1.5 percent, the target becomes 1.2 percent. The calibrated parameter values are summarized in Table 1, along with all other parameters.

Table 1: Parameter Value

Pension	Punishment	Preference	Intermediary	Demography
$\theta = 0.350$	$\Delta = 0.030$	$\sigma = 2.000$	$r = 0.020$	$j^* = 45$
$\Theta = 0.200$	$\lambda = 0.259$	$\beta = 0.955$	$\phi = 0.030$	$J = 64$

I compare the model's simulated moments with the data targets, and then discuss the role played by the model parameters in generating the simulated moments. Table 2 presents the aggregate statistics for the unsecured credit market under the benchmark economy. As seen from Table 2, this calibration does a reasonable job matching the aggregate default probability, the fraction of negative net worth households, and the aggregate negative net worth to GDP ratio by choosing β and λ. However, it underestimates the median discharge to median income ratio. Sullivan *et al.* (2000) finds that both medical shocks and small-business failures contribute to large discharges in bankruptcy filings. These shocks have modest effects on filings rates, and the target default rate is net of such shocks.

β plays an important role in generating enough negative-net-worth households to match the empirical finding. With a lower β, agents discount future utility to a greater degree. As a result, agents borrow more and then default with higher levels of debt. As both defaults and discharges rise, lenders then raise the default premium. In equilibrium, the fraction of

Table 2: Calibration

	Benchmark	Data
Default Rate	0.93%	1.20%
Fraction of Borrowers	0.218	0.200
Discharge/Income Ratio	0.220	0.500
Debt/GDP Ratio	0.041	0.045

negative-net-worth households rises.

λ plays an essential role in generating the debt to discharged income ratio. Increasing λ raises the cost of defaulting, so agents default less often for the same level of debt. In a perfectly competitive market, lenders lower the borrowing premium. Then agents borrow more. As a net effect, the aggregate default rate goes down and the discharge ratio goes up.[22] To better match the discharge ratio, the calibration calculation has to sacrifice the aggregate default rate.

4 Results

This section is organized into four subsections. First, I study equilibria in the benchmark model. Second, I study the role played by long-term contracts in consumption variation over time and across states as well as the welfare consequences. I begin with comparing the facts regarding debt, discharged debt, and bankruptcy between long-term-contract economy and the benchmark. I then evaluate the effect of long-term contracts in consumption smoothing and welfare. Third, I examine the welfare implications of imposing interest rate ceilings. Last, I study equilibria in an economy in which one-period and two-period contracts are offered.

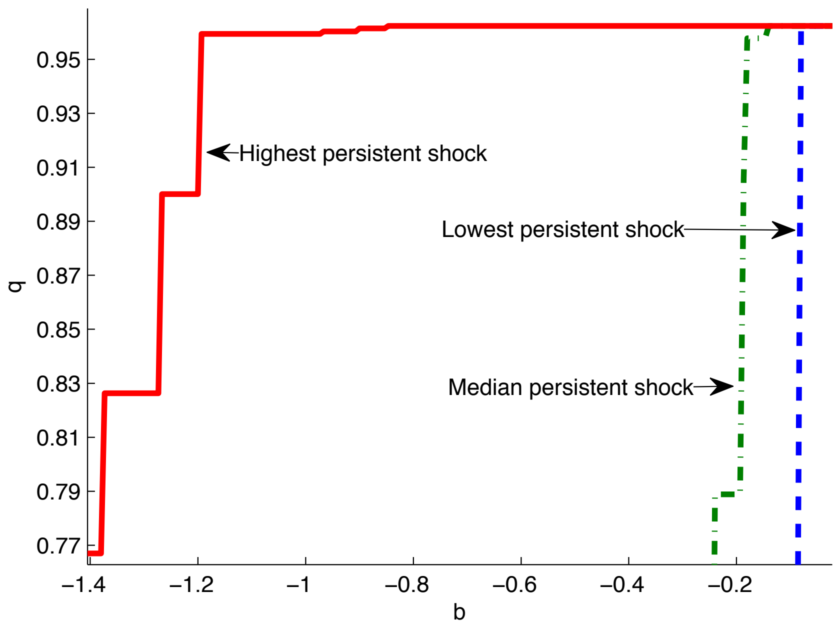
[22] Athreya (2004) and Livshits MacGee, Tertilt (2006) for a more complete discussion.

4.1 The Benchmark Economy

In this subsection, I first discuss the pricing functions. Then, debt and default over the life cycle is evaluated. Lastly, I explain what triggers a default.

4.1.1 Price Functions

Figure 2: Equilibrium Price Functions, Coll, Age 29



Note: q is the bond price, b is the bond issuance, and e is the persistent state.

The equilibrium pricing function is the key to generating the model's aggregate statistics. The household's future transitory state does not depend on its current state. Therefore, borrowers are all of the same type in equilibrium, and lenders cannot offer different contracts to households. But low ν means low b, so the transitory shock does affect default.

With respect to persistent shocks, lenders now can offer different contracts to different types of borrowers. Figure 2 plots pricing functions for the college type at age 29 under short-term contracts. As expected, the higher the realization of e the more credit is available at any given q. For a low realization of e, borrowing can occur at the risk-free rate up to some specified level of debt, after which the price of the bond price goes to zero (interest rate goes to ∞). For higher realizations of e, the decrease in q is more gradual, meaning that some risky borrowing will occur in equilibrium. The pricing functions for non-college types look similar at any given level of debt. Similar pictures arise for older households— weakly increasing for better realizations. Because they have higher incomes, middle-aged households can borrow substantially more than younger households.

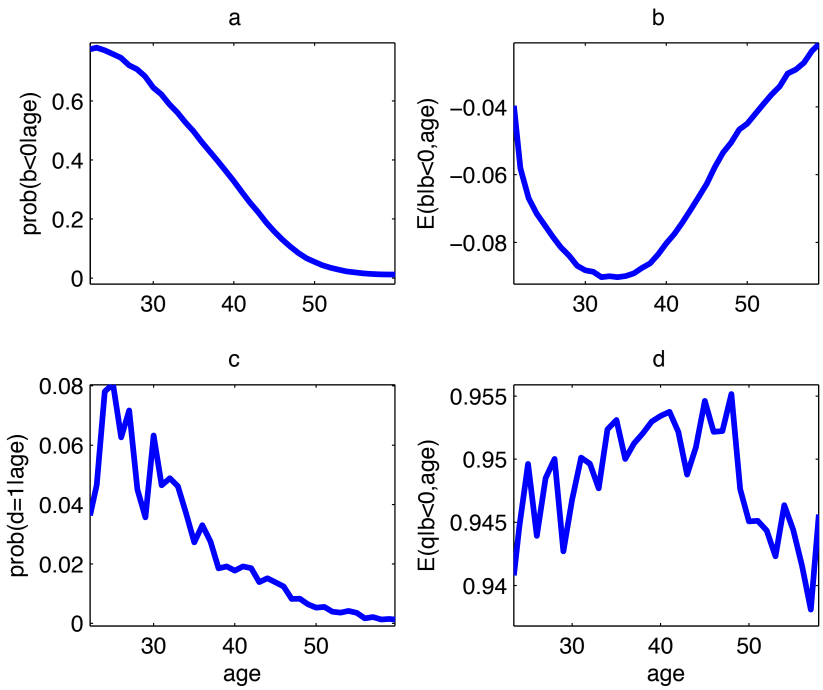
4.1.2 Debt and Default over the Life Cycle

Here I investigate debt and default over the life cycle. All newborns are born with no capital, and they borrow to smooth the life-cycle earnings hump and shocks. Therefore, the young are the most likely population to use the unsecured credit market for the purpose of smoothing consumption. Figure 3a plots the fraction of households which have negative net worth over the life cycle and shows that these households are mostly young. The young have two motives for borrowing: intratemporal and intertemporal consumption smoothing. As seen in Figure 1, the young face the deterministic life-cycle earnings hump, and they want to access the credit market in order to smooth their consumption. The middle-aged are close to the peak of their life-cycle earnings, and soon they will face a downward sloping earnings profile, so they need to accumulate resources to smooth future consumption. To do that, they start unloading their debts, either by defaulting on it or paying it off. Figure 3b plots the mean level of debt over the life cycle and shows the young accumulate debt until the age of 35. At that point, households begin unloading their debt. As a result, the fraction of borrowers declines over the life cycle.

Figure 3c plots the life cycle default distribution and shows that 85 percent of defaulters are 45 years old or younger, which is consistent with the observation of Sullivan *et al.* (2000).[23] As shown in Figure 3c, defaults occur primarily during the early stages of the life cycle. The middle-aged will default if they have accumulated enough debt to do so. However, most are accumulating wealth in preparation for retirement. The elderly face no uncertainty,

[23]Sullivan *et al.* (2000) observes that most defaulters are young. More than 75 percent of defaulters are 45 years old or younger.

Figure 3: Credit Distributions over the Life Cycle



Note: Figure 3*a* plots the fraction of households who are net borrowers, Figure 3*b* plots the mean size of debt, Figure 3*c* plots the fraction of defaulters, and Figure 3*d* plots the average bond rate over the life cycle.

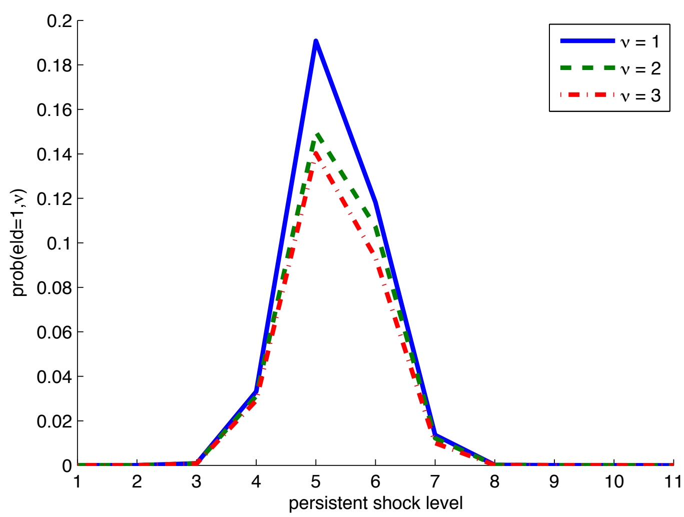
and therefore lenders price out default.

Figure 3*d* plots the distribution of mean q over the life cycle and shows that, as expected, the young pay higher average interest rates relative to the middle-aged. However, borrowers who are 45 years old or elder pay higher rates than the young. These borrowers close or have passed the peak of their life-cycle earnings, so the reason they borrow is to smooth the transitory shock. As noted earlier, current transitory shocks have no predictive power for default. Therefore, these borrowers pay higher interest rates on average.

4.1.3 Default over Earnings States

Here I study the role that earnings risk plays in triggering defaults. Figure 4 plots the default density over persistent states e for a given transitory state ν. It shows that default rarely happens when e is very high or very low; as in the data, the medium values of e trigger the most defaults.[24] Households with a very high e do not default because they expect high future income, and therefore that they will be able to pay off their debts. In addition, high-income households have to receive a large consumption compensation to make up for the losses incurred because of non-pecuniary costs λ, all else equal. Therefore, such households have no reason to default. Households with a very low e would default if they have enough debt to do so. As shown in Figure 2, low e households usually face tight borrowing limits, and hence do not have much risky debt. Moderate e households could have a lot of debt because they have relatively loose credit limits. When they experience income disruption, they have enough debt that they would benefit from default.

Figure 4: The Income Shocks of Defaulters



Note: e is persistent shock, and ν is transitory shock.

Transitory shocks, ν, matter for default decisions as well. As shown in Figure 4, households default more when they have the lowest ν for any given realization of e. Transitory shock ν has no predictive power for default risk, so lenders *cannot* distinguish the borrower's type. In equilibrium, lenders offer the same contract to all households for a given e, j, and y, and hence the post-default benefits are the same across households. However, low e households have a higher marginal utility of consumption than the high e households because high

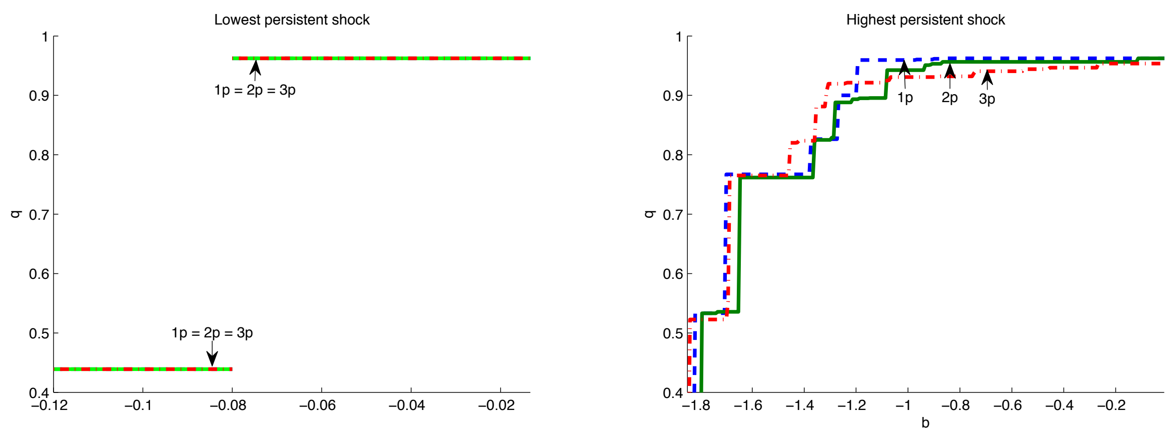
[24]See Sullivan, Warren, and Westbrook (2000).

e households have higher income, all else equal. As a result, low e households are more likely to default than are high e households.

4.2 The Consequences of Lengthening Contracts

In this subsection, I study pricing functions across contract lengths.

Figure 5: Price Functions Across T



(a) (b)

Note: Equilibrium Price Functions for college-educated households of age 29.

4.2.1 Price Functions

Here I study pricing functions across different contractual lengths. Figure 5 plots the equilibrium pricing function for one-period, two-period, and three-period contracts. It shows that as would be expected for any given (j, e, b), q is smaller under long-term contracts than under a short-term contract up to some specified level of debt in the equilibrium. The borrower's expected income more disperses under long-term than under short-term contracts. Therefore, for a given price function, there is more default as the contract lengthens. In equilibrium, the contractual term is worsen as the contract lengthens. Furthermore, short-term contracts can price out intentional default but long-term contracts cannot. Such behaviors cause borrowing premia to rise further. Figure 5 also shows that q is larger under long-term contracts than under a short-term contract for some specified level of debt in the equilibrium. Under what circumstances do long-term contracts offer better terms than short-term contracts? The answer needs to satisfy two conditions: first, long-term contracts have to provide enough credit for the future-period borrowing; and second, the household's future borrowing b' must be not too much larger than current borrowing b. Under the first condition, households do not file for bankruptcy in a future period, so default for the first-period debt is small and the interest rate is "close" to the risk-free rate. Under the second condition, the level of debt that is expected to be discharged by filing bankruptcy under long-term contracts in the third period is similar to that under short-term contracts in the second period. However, the present value of expected losses under long-term contracts is smaller than under short-term contracts, so the borrowing cost in equilibrium is lower under long term contracts. Furthermore, borrowers with a bad shock in the second period will not default if they draw a good shock on the following period. That improves the contractual term *ex ante*. Finally, pricing functions of low realizations of e look more similar across contract lengths than for high realizations of e. The intuition is that low realizations of e cannot get much worse than today in the future, and the *ex post* lucky borrowers switch to new contracts. Therefore, they look similar across different lengths of contracts.

4.2.2 Aggregate Statistics

Here I evaluate how the duration of contracts alters the aggregate statistics. Table 3 presents the aggregate statistics for the benchmark economy, the two-period-contract economy, and the three-period-contract economy. The table shows that the aggregate default rate, the fraction of households with negative net worth, and the aggregate negative net worth to GDP ratio fall as the duration of the contract lengthens, while the median discharge to median income ratio rises. As shown in Figure 5, the long-term q functions lie below the short-term pricing functions. That is, borrowers pay higher interest rate on average under longer-term contracts. As the cost of financing current consumption rises, agents borrow less and defer their consumption. Thus, the debt-to-GDP ratio falls. A lower level of debt implies less benefits from default, holding all else equal, so the aggregate default probability declines as T increases.

Table 3: Unsecured Credit Market Aggregates Across T

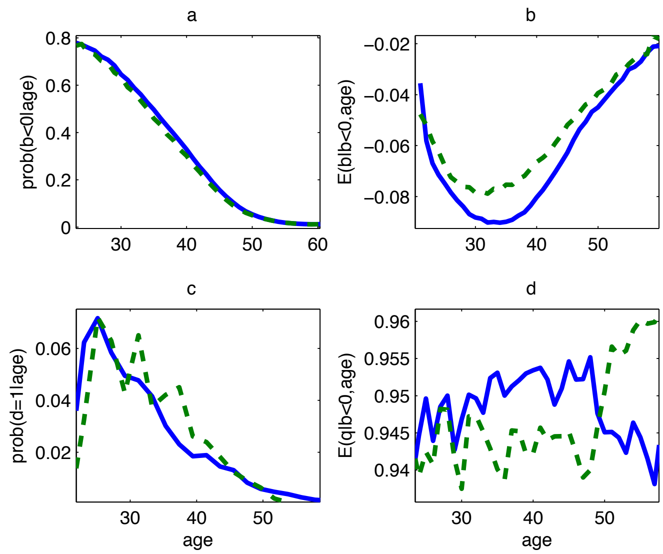
	Benchmark	Two-Period	Three-Period
Default Rate	0.93%	0.84%	0.77%
Fraction of Borrowers	0.218	0.206	0.202
Debt/GDP Ratio	0.041	0.037	0.034
Discharge/Income Ratio	0.220	0.276	0.306

Why does the discharged debt to income ratio increase? Borrowers under long-term contracts pay higher interest rates to borrow *ex ante*, and get favorable terms with relatively low realizations of e'. Borrowers who have a higher realization of e' vs. e are likely to exit the contract (and obtain new and better contracts) and leave only the relatively low-income borrowers in the existing contract. Borrowers who retain the existing contract take on more debt because the cost is low relative to their default risk, and hence drive up the discharge-to-income ratio. In addition, the pricing functions cannot cut the intentional defaulters out of the credit market. This behavior certainly further raises the discharge-to-income ratio.

4.2.3 Debt and Default over the Life Cycle

Now I turn to studying how debt and default is affected over the course of the life cycle by the lengthening of contract durations. Figure 6a plots the life-cycle density of borrowers across T. It shows that the fraction of households with negative net worth declines as age j increases for all economies. However, as T increases, the expected fraction of net borrowers falls for a given j. As shown in Figure 5, agents pay higher borrowing costs on average as T rises, since longer-term contracts face higher future income dispersion. As the rate goes up, households defer current consumption, and hence lower the level of borrowing as in Figure 6b, which shows the mean of b over the life cycle across Ts. Figure 5a, b shows that the young borrow and accumulate debt across T, so defaulters are mainly the young, as shows

Figure 6: Credit Distributions over the Life Cycle Across T

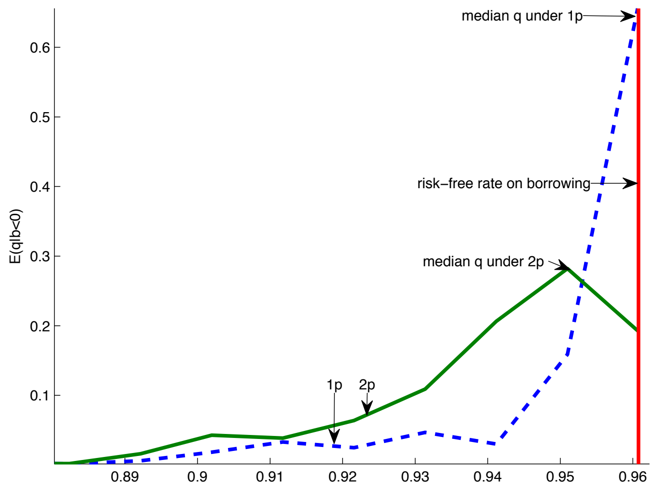


in Figure 6c.[25]

As noted earlier, Figure 5 shows that agents pay higher borrowing costs on average as T goes up. However, Figure 6d, which plots the distribution of mean $q_{b<0}$ over the life cycle across Ts, shows that households get better terms under long-term contract after age 50 in the model. The reason is that higher interest rates lead to higher savings, and so households have some assets for self-insurance. They therefore borrow less and get better price terms under longer-term contracts. Figure 7 plots the distribution of q over the level of borrowing and shows that 70 percents of households receive the risk-free rate on borrowing under the benchmark economy. However, this number plunges to 20 percent when $T = 2$. As shown in Figure 5, much more borrowing at the risk-free rate occurs under short-term contracts than under long-term contracts. The median borrowing interest rate under $T = 2$ is about 100 basis points higher than under long-term contracts.

[25] Under two-period contracts, every newborn receives a borrowing contract. In the following period, households which experience bad shocks can still borrow under the two-period contract. Some then default in the future. So a small fraction of households default in the even-number (model) periods, but a larger fraction default in the odd-number periods excluding the first period. Therefore, I plot the default probability into two-year periods rather than every year over the life cycle.

Figure 7: Distributions of q Across T



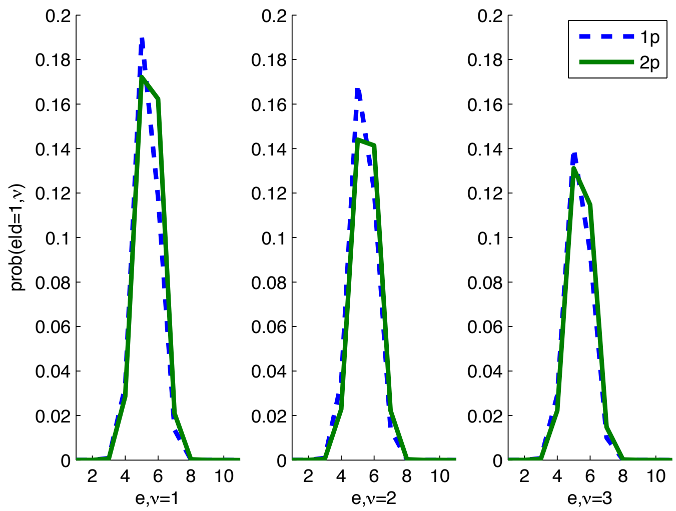
4.2.4 Default over Earnings States

Here I examine how lengthening the duration of contracts alters the probability of default over states. Figure 8 plots the default density over e for a given ν, across income states and across Ts. Very low e households behave similarly regardless of T because these households cannot get much worse off in the future, and when they get a better shock they switch to better contracts. As the pricing function shows, higher e households not only get higher credit lines but also get better borrowing terms for a given b. Therefore, households behave similarly to those in the benchmark economy under the longer-term contract. Very high e households could default but do not do so, as discussed in the previous subsection. High e agents are able to borrow, and the young borrow to smooth out the life-cycle earnings and thus accumulate debt. When they experience income interruption, they file for bankruptcy in order to smooth their consumption.

4.2.5 Consumption Smoothing

Long-term contracts alter the equilibrium price functions and hence affect the households' ability to smooth consumption. In this section, I evaluate the effect of lengthening T in

Figure 8: The Income Shocks of Defaulters Across T



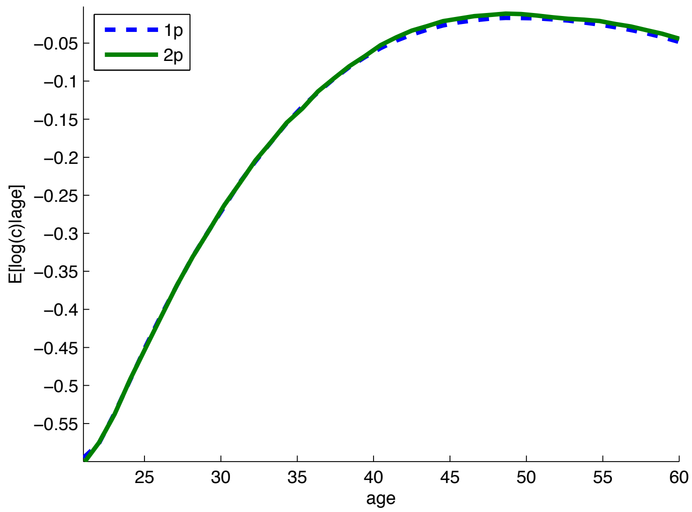
smoothing consumption. I decompose the variance of log consumption over the life cycle as follows:

$$Var\left(\log\left(c\right)\right)=Var\left(E\left[\log\left(c\right)|j\right]\right)+E\left[Var\left(\log\left(c\right)|j\right)\right];\tag{29}$$

where the total variance of log consumption $Var\left(\log\left(c\right)\right)$ equals the sum of the variance of the mean of log consumption, $Var\left(E\left[\log\left(c\right)|j\right]\right)$, and the mean of the variance of log consumption, $E\left[Var\left(\log\left(c\right)|j\right)\right]$, over the life cycle. $Var\left(E\left[\log\left(c\right)|j\right]\right)$ is the intertemporal component of consumption smoothing; it measures how expected consumption varies over the life cycle. And $E\left[Var\left(\log\left(c\right)|j\right)\right]$ is the intratemporal component; it represents how consumption differs across households of a given age. I first discuss the intertemporal component of consumption smoothing and then discuss the intratemporal component.

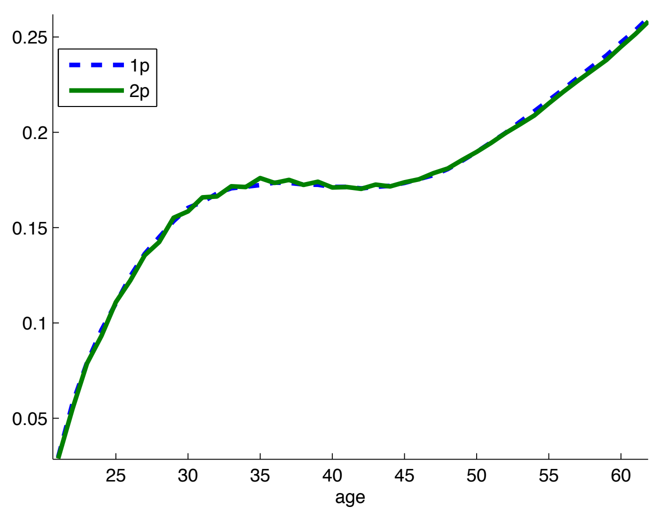
Figure 9 plots the mean of log consumption over the life cycle. It shows less (but not much less) curvature on the mean consumption over the life cycle under short-term contracts than under long-term contracts. That is, expected consumption varies more over the life cycle under longer-term contracts than under shorter-term contracts. As a result, intertemporal consumption smoothing worsens as T lengthens. The difference in expected consumption over the life cycle across T is driven by the changes in the pricing functions that are faced by

Figure 9: Consumption over the Life Cycle Across T



households. The young generally have tighter credit limits and face less favorable terms, and therefore borrow less and are therefore less able to smooth the life-cycle earnings hump. As a result, the young consume less on average under longer-term contracts. The middle-aged start to pay back debt and prepare for retirement. Because the young borrow less under longer-term contracts, they have less debt. Therefore, they can consume more on average as T lengthens.

Figure 10: Consumption Variability over the Life Cycle Across T



With respect to the intratemporal component, Figure 10 plots the variance of consumption over the life cycle. It shows that, as T lengthens, the intratemporal variance of consumption is (moderately) improved in the early and late periods in the life cycle . As noted earlier, contract terms are similar across low e households, so their consumption and default decisions. However, high e households get worse terms as T increases, and hence they borrow less and consume less.[26] As a result, the consumption variation declines across the young for a given age as T increases. In the later stages of the life cycle, to accumulate assets in preparation for retirement, the high income households pay off their debt and the low income households default on their debt. So the variance of consumption declines. As T lengthens, the young accumulate more wealth because of less favorable price terms so the high-income ones have less of a payment burden and the low-income ones are less likely to default. The variance of consumption is therefore mildly higher under a longer-term contract for the middle-aged.

Table 4: Consumption Decomposition Across T

	1P	2P	3P
E(var(log(c)|j))	0.1161	0.1140	0.1137
var(E(log(c)|j))	0.0436	0.0451	0.0457
Total	0.1597	0.1591	0.1594

In Table 4, I summarize these two measures. The results in the table illustrate the tradeoff between these two components of smoothing, motivating the life-cycle analysis of Livshits *et al.* (2007) and Athreya (2008). In this paper, the quantitative results show that longer-term contracts improve intratemporal consumption smoothing, but at the expense of intertemporal consumption smoothing. This finding is consistent with Athreya *el at.* (2008).[27]

[26] I ignore discussing the role of transitory shocks because it does not alter the pricing functions.

[27] Athreya *et al.* (2009b) extend the model to incorporate preferences developed in Epstein and Zin (1989), and have the same finding as in this paper.

4.2.6 Welfare Analysis

Here the welfare consequences of a contract with a length T are examined. The welfare measure in this paper is measured as the percentage increase (or decrease) in consumption in all periods and states that leaves a newborn agent, who has zero wealth, indifferent between two economies. Table 5 shows the newborn's gain in *ex ante* welfare as the term of the contracts declines. The first section of Table 5 shows that longer-term contracts are less desirable for all newborns, regardless of income risk. The gains are largest for workers who did not complete high school, because their income is more volatile than that of other groups, over the life cycle.[28] As the contractual length gets longer, borrowers face a more dispersed future earnings distribution. All else fixed, there will be more default. Furthermore, when borrowers have bad income realization in the following period under long-term contracts, the contractual terms are favorable to borrow. They therefore borrow more debt and then default with higher debt. As a result, lenders raise the cost of borrowing *ex ante.*

Table 5: Welfare: From Long to Short

C_{eq}	Coll	HS	NHS
$3P \to 2P$	0.08%	0.09%	0.10%
$2P \to 1P$	0.13%	0.16%	0.19%
$1P \to NBK$	0.85%	0.77%	0.48%

The second section of Table 5 shows larger welfare gains from eliminating bankruptcy. If bankruptcy is not an option, the default problem disappears. In a perfectly competitive financial market, lenders offer risk-free rate on borrowing to all borrowers, up to the natural debt limit, so households can better smooth out their life-cycle consumption. In contrast with the gains from decreasing T, the gain is largest for college-educated workers because their demand for intertemporal smoothing is largest. Conversely, the smallest gains accrue to the non-high-school-educated type because their intertemporal motives are weak.

[28]If both the variance of persistent shocks and the variance of transitory shocks fall by 30 percent, the quantitative results show that the welfare losses that are caused by imposing longer-term contracts are close to zero.

4.3 Usury Ceiling

In this subsection, I study the welfare implications by embedding a usury ceiling into the model. The structure of usury ceiling ξ is that

$$q = \begin{cases} 0 & \text{if} \quad q < \xi \\ q & \text{otherwise,} \end{cases} \tag{30}$$

where $\xi \in (0, \frac{1}{1+r+\phi})$.

Table 6: Welfare: From No Usury to Usury

$C_{eq}\times\mathbf{10^{-2}}\%$	Coll		HS		NHS	
ξ	$2P \to 2P^u$	$1P \to 1P^u$	$2P \to 2P^u$	$1P \to 1P^u$	$2P \to 2P^u$	$1P \to 1P^u$
0.89	1.02	−0.01	0.86	−0.04	0.00	−0.02
0.90	3.38	−0.02	2.63	−0.09	0.01	−0.06
0.91	5.72	−0.07	3.66	−0.21	0.05	−0.11
0.92	4.73	−0.15	5.73	−0.36	0.08	−0.18
0.93	3.84	−0.36	5.82	−0.85	−0.08	−0.32
0.94	3.79	−0.64	5.73	−1.89	−0.18	−0.66
0.95	−1.63	−1.01	−2.06	−3.51	−0.51	−1.02
0.96	−3.75	−4.79	−4.83	−6.30	−2.10	−2.97

Table 6 summarizes the welfare implications of usury ceilings. It measures the welfare gain when an economy transitions from unregulated interest rates to regulated interest rate. I first discuss the welfare changes under short-term contracts, and then under long-term contracts. Table 6 shows that imposing a usury ceiling under short-term contracts reduces the welfare of newborns. However, that outcome can be altered under long-term contracts. I discuss these two findings in details below.

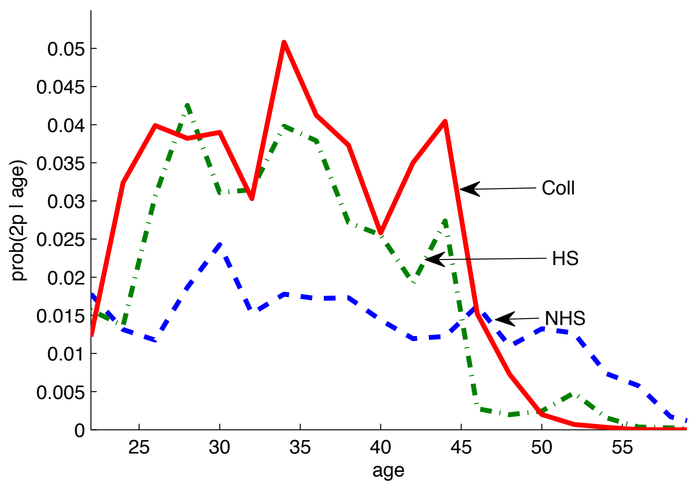
When interest rates are regulated under short-term contracts, high school-educated workers are impacted the most out of the three groups. College-educated workers have the lowest income variance, so they are impacted the least under long-term contracts. The workers who did not complete high school have the highest earnings variance, so they would be impacted the most, at least if they have any debt. However, they have the flattest life-cycle income profile and so they have little debt. As a result, they are not the group impacted the most by the enforcement of a ceiling on interest rates. Second, as expected, as ξ approaches the risk-free rate, more agents face tighter credit conditions and therefore the welfare loss rises.

When an usury ceiling under long-term contracts is not too "tight" (say $\xi \leq 0.91$), it benefits all educational types *ex ante*. A usury ceiling limits the level of borrowing and then limits the size of debt that is borrowed by the *ex post* high-risk borrowers. That leads to default with lower levels of debt. As a result, q rises. With better contractual terms, households now borrow more to smooth their consumption. As a result, imposing an usury ceiling uniformly improves the welfare for all types of households as shown in Table 6. However, as ξ moves further away from the risk-free rate on borrowing, the welfare improvement falls. A "modest" interest rate cap has less of an effect on *ex post* borrowing, so the contractual terms are becoming more favorable but to a lesser extent than under the aforementioned scenario. Hence the welfare improvement caused by imposing a usury ceiling is vanishing. On the other hand, as ξ approaches the risk-free rate, the welfare improvement that is caused by imposing a usury ceiling also vanishes and will become negative. Long-term contracts with a "tight" usury ceiling no longer provide insurance to households. Instead, a "tight" usury ceiling only limits many households' ability to borrow. The last comment that I would like to make in this section is that regulating interest rates under long-term contracts can improve welfare, but even the maximum welfare gain brought about by regulating the interest rate is still substantially smaller than the welfare loss that is caused by lengthening the duration of contracts.

4.4 Multiple Contracts

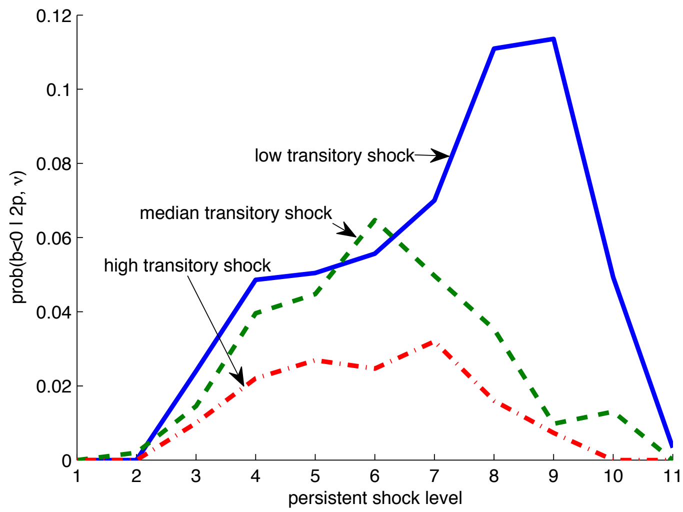
In this subsection, I study the coexistence of long-term and short-term contracts, particularly investigating what households would prefer long-term contracts over short-term contracts. In the equilibrium, there are 4.29 percent of borrowers who use long-term contracts to borrow. As noted earlier, households which have "large" size of debt (relative to income) can borrow with more favorable term under long-term contracts than under short-term contracts. To accumulate that much debt, it takes time to do so. Figure 11 plot the fraction of borrowers uses long-term contracts over the life cycle and shows that the middle-aged borrowers are more likely to use long-term contracts to borrow, because they accumulate enough debt to be benefited by using long-term contracts as shown in Figure 5. The simulation shows that when households use short-term contracts to borrow, the average size of debt GDP ratio is 0.042. However, when they use long-term contracts to borrow, this ratio raises to 0.062.

Figure 11: Long-Term Contracts over the Life Cycle



Now I turn to investigate what households would prefer long-term contracts over short-term contracts over the income distribution. Figure 12 plots the fraction of borrowers who use long-term contracts to borrow over persistent earnings shock e and transitory earnings shock ν and shows that borrowers with low persistent earnings shocks use long-term contracts less than borrowers with high shocks. There are two reasons for that. First, low e households are less likely to buy consumption insurance, because their income are more likely go upward. Second, Low e households cannot accumulate high level of debt, so they are not benefited by using long-term contract. Figure 12 also shows that borrowers with low transitory earnings shocks use long-term contracts more than high ν. Low ν households are more likely to borrow and borrow more debt all else fixed, so they are more likely to use long-term contracts.

Figure 12: Long-Term Contracts over the Income Shocks



The last point I like to make in this subsection is that multiple contracts improve welfare, as shown in Table 7. The college-educated benefit the most because they borrow the most. However, the gain is small.

Table 7: Welfare: From Short to Multi-contracts

C_{eq}	Coll	HS	NHS
$1P \rightarrow 1P, 2P$	$0.35 \times 10^{-2}\%$	$0.14 \times 10^{-2}\%$	$0.06 \times 10^{-2}\%$

5 Conclusion

In this paper, I study the role of lenders' commitment in the unsecured market and its welfare consequences. The results show that longer-term debt contracts tend to result in higher average interest rates, and hence lower levels of borrowing and fewer households borrowing. Higher borrowing rates uniformly hurt the ability of newborns of all types to smooth their consumption. The results also show that prolonging the length of unsecured credit contracts can reduce welfare. The theoretical results have implications for the effects of the Credit CARD Act of 2009.

This paper has illustrated that imposing usury ceilings under long-term contracts can improve the welfare of newborns. However, imposing usury ceilings under short-term contracts always reduce the welfare of newborns. This implication sheds some light on the debate as to whether to impose usury ceilings.

My model makes two assumptions: that households behave rationally, and that unsecured credit markets are perfectly competitive. Ausubel (1991) argues that the credit card market does not operate competitively because cardholders are irrational.[29] However, Canner and Luckett (1992) and Pozdena (1991) counter that the credit card market are efficient. Future research may provide some insights into this, by extending the model in these directions.[30]

References

[1] Abowd, John M. and David Card (1989), "On the Covariance Structure of Earnings and Hours Changes," *Econometrica* **57(2)**, pp. 411-45.

[2] Andolfatto, David (2002), "A Theory of Inalienable Property Rights," *Journal of Political Economy* **110(2)**, pp. 382-93.

[3] Arellano, Cristina and Ramanarayanan, Ananth (2006). "Default and the term structure in sovereign bonds," mimeo, University of Minnesota.

[4] Athreya, Kartik (2002), "Welfare Implications of the Bankruptcy Reform Act of 1999," *Journal of Monetary Economics* **49**, pp. 1567-95.

[5] Athreya, Kartik (2004), "Shame As It Ever Was: Stigma and Personal Bankruptcy," Federal Reserve Bank of Richmond *Economic Quarterly* **90(2)**, pp. 1-19.

[6] Athreya, Kartik (2008), "Default, Insurance, and Debt over the Life-Cycle," *Journal of Monetary Economics* **55(4)**, pp. 752-74.

[29] Consumers make credit card choices without taking account of the high probability that they will pay interest on their outstanding balances.

[30] Calem and Mester (1995) argue that the market for credit cards is not competitive because of consumer search costs.

[7] Athreya, Kartik, Xuan S. Tam, and Eric R. Young (2008), "A Quantitative Theory of Information and Unsecured Credit," Federal Reserve Bank of Richmond Working Paper **08-6**.

[8] Athreya, Kartik, Xuan S. Tam, and Eric R. Young (2009a), "Unsecured Credit Markets Are Not Insurance Markets," *Journal of Monetary Economics* **56(1)**, pp. 83-103.

[9] Athreya, Kartik, Xuan S. Tam, and Eric R. Young (2009b), "Are Harsh Penalties for Default Really Better?" Federal Reserve Bank of Richmond Working Paper **09-11**.

[10] Ausubel, Lawrence (1991), "The Failure of Competition in the Credit Card Market," *American Economic Review* **81(1)**, pp. 50-81.

[11] Calem, Paul S. and Loretta J. Mester (2005), "Consumer Behavior and the Stickiness of Credit-Card Interest Rates," *American Economic Review* **85(5)**, pp. 1327-36.

[12] Canner, Glenn B. and Charles A. Luckett (1992), "Developments in the Pricing of Credit Card Service," *Federal Reserve Bulletin* **78(9)**, pp. 652-666.

[13] Carroll, Christopher D. (2001), "A Theory of the Consumption Function, With and Without Liquidity Constraints," *Journal of Economic Perspectives* **15(3)**, pp. 23-46.

[14] Chatterjee, Satyajit, P. Dean Corbae, Makoto Nakajima, and José-Víctor Ríos-Rull (2007), "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica* **75(6)**, pp. 1525-90.

[15] Chatterjee, Satyajit, P. Dean Corbae, and José-Víctor Ríos-Rull (2008), "A Finite-Life Private-Information Theory of Unsecured Debt," *Journal of Economic Theory* **142(1)**, pp. 148-77.

[16] Chatterjee, Satyajit and Burcu Eyigungor (2009), "Maturity, Indebtedness, and Default Risk," Federal Reserve Bank of Philadelphia Working Paper **09-2**.

[17] Drozd, Lukasz A. and Jaromir B. Nosal (2007), "Competing for Customers: A Search Model of the Market for Unsecured Credit," mimeo, University of Wisconsin and University of Minnesota.

[18] Dubey, Pradeep, John Geanakoplos, and Martin Shubik (2005), "Default and Punishment in General Equilibrium," *Econometrica* **73(1)**, pp. 1-37.

[19] Edelberg, Wendy (2006), "Risk-Based Pricing of Interest Rates for Consumer Loans," *Journal of Monetary Economics* **53**, pp. 2283-98.

[20] Epstein, Larry G. and Stanley E. Zin (1989), "Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns I: A Theoretical Framework," *Econometrica* **57(4)**, pp. 937-69.

[21] Evans, David S. and Richard L. Schmalensee (1999), *Paying with Plastic: The Digitial Revolution in Buying and Borrowing*, MIT Press.

[22] Fay, Scott A., Erik Hurst, and Michelle J. White (1998), "The Bankruptcy Decision: Does Stigma Matter?" University of Michigan Working Paper **98-01**.

[23] Furletti, Mark (2003), "Credit Card Pricing Developments and Their Disclosure," Federal Reserve Bank of Philadelphia Payment Cards Center Discussion Paper **03-02**.

[24] Glaeser, Edward Ludwig and JJose Alexandre Scheinkman (1996), "The Transition to Free Markets: Where to Begin Privatization," *Journal of Comparative Economics* **22(1)**, pp. 23-42.

[25] Gross, David B. and Nicholas S. Souleles (2002), "An Empirical Analysis of Personal Bankruptcy and Delinquency," *Review of Financial Studies* **15(1)**, pp. 319-47.

[26] Guvenen, Fatih (2007), "Learning your Earning: Are Labor Income Shocks Really Very Persistent?," *American Economic Review* **97(3)**, pp. 687-712.

[27] Hatchondo, Juan Carlos and Leonardo Martinez (2008), "Long-Duration Bonds and Sovereign Defaults," Federal Reserve Bank of Richmond Working Paper **08-2R**.

[28] Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes (1994), "The Importance of Precautionary Motives for Explaining Individual and Aggregate Saving," *Carnegie-Rochester Conference Series on Public Policy* **40**, pp. 59-126.

[29] Lillard, Lee and Yoram Weiss (1979), "Components of Variation in Panel Earnings Data: American Scientists 1960—70," *Econometrica* **47(2)**, pp. 437-54.

[30] Livshits, Igor, James MacGee, and Michèle Tertilt (2006), "Accounting for the Rise in Consumer Bankruptcies," mimeo, University of Western Ontario and Stanford University.

[31] Livshits, Igor, James MacGee, and Michèle Tertilt (2007), "Consumer Bankruptcy: A Fresh Start," *American Economic Review* **97(1)**, pp. 402-18.

[32] Massoud, Nadia, Anthony Saunders, and Barry Scholnick (2006), "The Cost of Being Late: The Case of Credit Card Penalty Fees," mimeo, University of Alberta and New York University.

[33] Matteos-Planas, Xavier and José-Víctor Ríos-Rull (2007), "Credit Lines," mimeo, SUNY-Stony Brook and University of Minnesota.

[34] MaCurdy, Thomas E. (1982), "The use of time series processes to model the error structure of earnings in a longitudinal data analysis," *Journal of Econometrics* **18(1)**, pp. 83-114.

[35] Mehra, Rajnish and Edward C. Prescott (1985), "The Equity Premium: A Puzzle," *Journal of Monetary Economics* **15**, pp. 145-61.

[36] Nakajima, Makoto (2008), "Equilibrium Bankruptcy and Temptation," mimeo, University of Illinois-Champagne-Urbana.

[37] Narajabad, Borghan (2007), "Information Technology and the Rise of Household Bankruptcy," mimeo, Rice University.

[38] Persky, Joseph (2007), "Retrospectives From Usury to Interest," *Journal of Economics Perspectives* **21(1)**, pp. 227-36.

[39] Pozdena, Randall J. (1991), "Solving the Mystery of High Credit Card Rates," *FRBSF Weekly Letter* **91(42)**.

[40] Press, William H., Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery (1993), *Numerical Recipes in FORTRAN: The Art of Scientific Computing*, Cambridge University Press.

[41] Rockoff, Hugh (2003), "Prodigals and Projectors: An Economic History of Usury Laws in the United States from Colonial Times to 1900," NBER Working Paper *W9742* .

[42] Sánchez, Juan (2008), "The Role of Information in Consumer Debt and Bankruptcy," mimeo, University of Rochester.

[43] Sullivan, James X. (2008), "Borrowing During Unemployment: Unsecured Debt as a Safety Net," forthcoming, *Journal of Human Resources*.

[44] Sullivan, Teresa A., Elizabeth Warren, and Jay Lawrence Westbrook (2000), *The Fragile Middle Class: Americans in Debt*, Yale University Press.

[45] Visser, Wayne A. M. and Alastair Macintosh(1998), "A short review of the historical critique of usury," *Accounting, Business & Financial History*, 1466-4275, **8(2)**, pp. 175-89.

[46] Webb, David C. (1999), "Long-Term Financial Contracts can Mitigate the Adverse Selection in Project Financing,", *International Economic Review*, **32(2)**, pp. 305-20.

[47] White, Michelle (1998), "Why Don't More Households File for Bankruptcy?", *Journal of Law, Economics, and Organization*, **14(2)**, pp. 205-31.

[48] Wolff, Edward N. (2004), "Changes in Household Wealth in the 1980s and 1990s in the U.S." in Wolff, Edward N. (Ed.), *International Perspectives on Household Wealth*, Elgar Publishing Ltd.

[49] Zame, William R. (1993), "Efficiency and the Role of Default When Security Markets are Incomplete," *American Economic Review* **83(5)**, pp. 1142-64.