Mortgage Innovation and the Foreclosure Boom

Dean Corbae
University of Texas at Austin

Erwan Quintin*
University of Wisconsin at Madison

June 10, 2010

Abstract

How much of the recent rise in foreclosures can be explained by the large number of nontraditional, low downpayment mortgage contracts originated between 2003 and 2006? We present a model where heterogeneous households select from a set of possible mortgage contracts and choose whether to default on their payments given realizations of income and housing price shocks. The set of contracts consists of traditional fixed rate mortgages which require a 20% downpayment as well as nontraditional mortgages with low downpayments and delayed amortization schedules. The mortgage market is competitive and each contract, contingent on household earnings and assets at origination as well as loan size, must earn zero expected profits. We use our model to quantify the role of mortgage innovation in the recent rise in foreclosure rates. An unanticipated 25% price decline following a brief introduction of non-traditional mortgages causes foreclosure rates to rise by almost 150% during the first two years of the crisis, which is very close to the corresponding increase in the data between the first quarter of 2007 and the first quarter of 2009. In a counterfactual where new mortgages are not introduced, the same price shock causes foreclosure rates to increase by only 86%. This suggests that the presence of nontraditional mortgages significantly increased the magnitude of the foreclosure crisis.

*E-mail: corbae@eco.utexas.edu, equintin@bus.wisc.edu. We especially wish to thank Daphne Chen who has provided outstanding research assistance, as well as Morris Davis, François Ortalo-Magné, Carlos Garriga, Chris Gerardi, Mark Bils and Paul Willen for their many valuable suggestions. We also thank seminar participants at the Reserve Banks of Atlanta, Dallas, Minneapolis, and New Zealand as well as the Australian National University, Cowles Conference on General Equilibrium, Cambridge University, Econometric Society Meetings, European University Institute, Institute for Fiscal Studies, NBER Summer Institute Work Group on Aggregate Implications of Microeconomic Consumption Behavior, Queens University, SED conference, Oxford University, University of Auckland, University of Maryland, University of Melbourne, University of Rochester, University of Wisconsin, Wharton, and the Gerzensee study center for their helpful comments.

1 Introduction

Between 2003 and 2006, the composition of the stock of outstanding residential mortgages in the United States changed in several important ways. The fraction of mortgages with variable payments relative to all mortgages increased from 15% to over 25% (see figure 1.) At the same time, the fraction of "subprime" mortgages (mortgages issued to borrowers perceived by lenders to be high-default risks) relative to all mortgages rose from 5% to nearly 15%. Recent work (see e.g. Gerardi et al., forthcoming, figure 3) has shown that many of these subprime loans are characterized by high leverage at origination and non-traditional amortization schedules.

Low downpayments and delayed amortization cause payments from the borrowers to the lender to be backloaded compared to loans with standard downpayments and standard amortization schedules. By lowering payments initially, these features made it possible for more households to obtain the financing necessary to purchase a house and, in other papers (e.g. Chambers, et. al. (2009)) have been associated with the rise in homeownership. At the same time however, because these contracts are characterized by little accumulation of home equity early in the life of the loan, they are prone to default when home prices fall. Not surprisingly then, (see Gerardi et al., 2009) mortgages issued between 2005 and 2006 with high leverage and non-traditional amortization schedules have defaulted at much higher frequency than other loans since home prices began their collapse in late 2006.

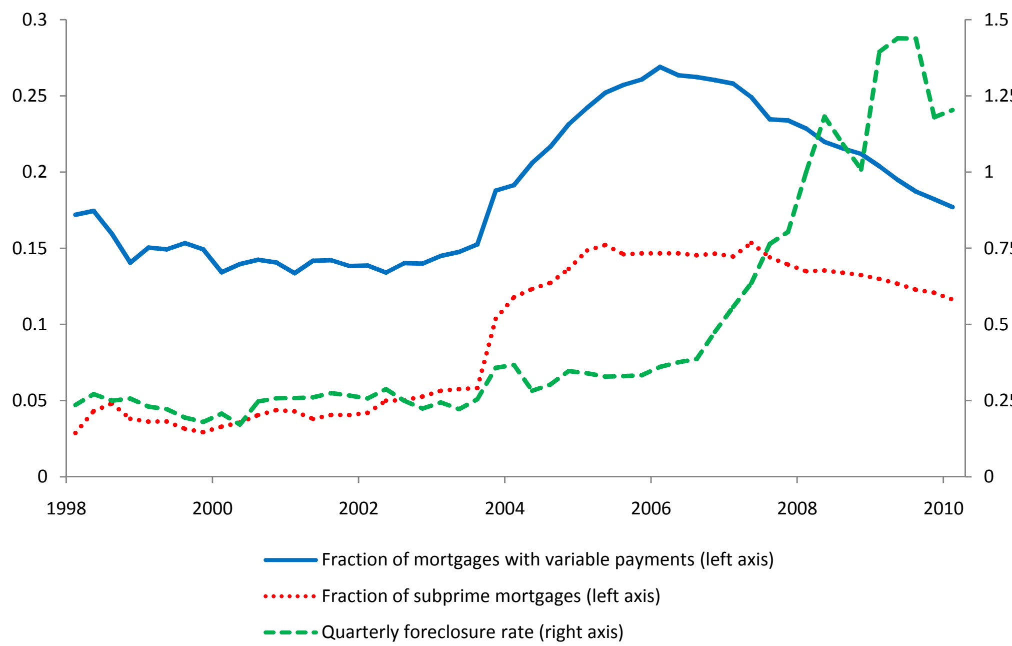
Our objective is to quantify the importance of non-traditional mortgages for the recent flare-up in foreclosure rates depicted in figure 1. Specifically, we ask the following question: How much of the rise in foreclosures can be attributed to the increased originations of non-traditional mortgages between 2003 and 2006?

To answer this, we describe an economy where households value both consumption and housing services and move stochastically through several stages of life. For simplicity, agents who are young are constrained to obtain housing services from the rental market and split their remaining income between consumption and the accumulation of liquid assets. Given idiosyncratic earnings shocks, despite the fact that households begin life ex-ante identical in our model, there is an endogenous distribution of assets among the set of people who turn middle aged.

When agents become mid-aged, they have the option to purchase one of two possible quantities of housing capital: a small house or a large house. We assume they must finance house purchases via a mortgage drawn from a set of contracts with properties like those available in the United States. Standard fixed-rate mortgages (FRMs) require a 20% downpayment and fixed payments until maturity. Agents can opt instead for a mortgage with no-downpayment and delayed amortization (we will term these contracts LIP for "low initial payment"). We think of this second mortgage as capturing the backloaded nature of the mortgages that became popular after 2003 in the United States.

Mortgage holders can terminate their contract before maturity, in which case the house is immediately sold and the borrower receives any proceeds in excess of the outstanding loan

Figure 1: Recent trends in the mortgage market



Source: National Delinquency Survey (Mortgage Bankers Association). Foreclosure rates are the number of mortgages for which a foreclosure proceeding are started in a given quarter divided by the initial stock of mortgages.

principal and transaction costs. We consider a mortgage termination to be a foreclosure if it occurs in a state where the house value is below the mortgage's balance (that is, the agent's home equity is negative) or where the agent's income realization is such that they cannot make the mortgage payment they would owe for the period.[1] In those cases, home sales are subject to foreclosure costs.

We calibrate the resulting model to capture the salient features of the housing and mortgage markets in the United States prior to the foreclosure crisis. The model, so calibrated, predicts that in steady state almost all foreclosures (98.5%) involve negative equity. This is

[1]Here we are assuming the default law is consistent with antideficiency (as in Arizona and California for example) where the defaulting household is not responsible for the deficit between the proceeds from the sale of the property and the outstanding loan balance. In Section 5.7 we consider a variation in punishment following a foreclosure that resembles laws in states with recourse.

because most agents with positive equity who are at a high risk of finding themselves unable to meet their mortgage payments sell before reaching that state in order to avoid foreclosure costs. On the other hand, most agents with negative equity (70.7%) choose to continue meeting their mortgage obligations to avoid losing their homes. Foreclosures are thus associated with a combination of negative equity and income circumstances that make meeting mortgage payments difficult. These predictions are consistent with the growing empirical literature on the determinants of foreclosure.[2]

Foreclosures are costly for lenders because of the associated transactions costs and because they occur almost always when home equity is negative. As a result, intermediaries demand higher yields from agents whose asset and income position make foreclosure more likely. In fact, intermediaries do not issue loans to some agents because their default risk is too high or because the agents are too poor to make a downpayment. In particular, our model is consistent with the fact that agents at lower asset and income positions are less likely to become homeowners, face more expensive borrowing terms, and are more likely to default on their loan obligations.

Since high initial payments are prohibitively costly for asset and income poor agents, there is a natural role to play in our economy for mortgage innovation in the form of contracts with low initial payments. We find that in an economy calibrated to match key aspects of the US housing market prior to 2003, adding the option to issue LIP contracts causes a rise in steady state homeownership, default rates, and welfare.

LIPs enable some households with low assets and income (those who could be interpreted as subprime) to become homeowners. At the same time, the availability of these contracts cause default rates to be higher for two complementary reasons which our environment makes explicit. First, high-default risk households select into homeownership. Second, these contracts are characterized by a much slower accumulation of home equity than FRMs, which makes default in the event of a home value shock much more likely, even at equal asset and income household characteristics.

We document the effect of loan and borrower characteristics on default by generating a representative sample of 25,000 mortgages in steady state and using this sample to estimate a competing risk model using the proportional hazard specification that has become standard in the empirical literature on mortgage performance (see, e.g. Gerardi et al., 2009.) As empirical papers typically do when they can include the necessary covariates, we find that hazard rates into default rise with the loan-to-income and loan-to-value ratios at origination and fall with the asset-to-loan ratio. Moreover, using the same sample, we find that yields at origination rise with loan size and fall with borrower assets and income and these covariates account for most of the variance in yields in steady state equilibrium.

While these steady state predictions are interesting, the available evidence suggests that the break in the composition of the mortgage stock occurred briefly before the collapse of house prices in late 2006 and the spike in foreclosures. There is also growing evidence that

[2]See, among many other papers, Foote et al. (2008a,b), Gerardi et al. (2007), Sherlund (2008), Danis and Pennington-Cross (2005), and Deng et al. (2000).

the fraction of high-LTV, delayed amortization mortgages in originations has dwindled to a trickle since the collapse of prices.[3]

We simulate this course of events using a three-stage transition experiment. Specifically, we begin in a steady state of an economy with only FRMs calibrated to match key aspects of the US economy prior to 2003. We then introduce the nonstandard mortgage option for two periods, which represents four years in our calibration. The model predicts that when they become available, non-traditional mortgages are selected by a third of home-owners, which is consistent estimates of the fraction in originations of mortgages with non-traditional features between 2003 and 2006. In fact, the experiment produces a pattern for the share of non-traditional mortgages in the mortgage stock that closely resembles the pattern shown in figure 1. In the third stage, we assume a surprise 25% collapse in home prices, remove the nonstandard mortgage option, and then let the economy transit to a new long-run steady state. This experiment causes foreclosure rates to rise by 148% during the first two years of stage 3. By comparison, in the data, the overall foreclosure rate increased by 150% between the first quarter of 2007 and the first quarter of 2009.

To quantify the role of mortgage innovation in this increase, we then run an experiment where the LIP mortgage option is not offered in the second stage. In this counterfactual, the increase in foreclosure rates caused by the price shock falls to 86%. Mortgage innovation, in other words, makes the economy much more sensitive to price shocks. In addition, we find that lower downpayments account for most of the contribution of non-traditional mortgages to the increase in foreclosure rates, while delayed amortization and payment spikes play a limited role.[4] In summary, our transition experiment suggests that the presence of nontraditional mortgages greatly increased the magnitude of the foreclosure crisis.

Our paper is closely related to several studies of the recent evolution of the US housing market and mortgage choice.[5] In particular, our work complements the findings of Gerardi et

[3]The Mortgage Bankers Association (MBA)'s mortgage origination survey suggests for instance that after falling to 50% of originations in 2005, traditional FRMs now account for 90% of originations. According to the same source, the fraction of interest-only mortgages in originations rose to nearly 20% in 2006, and has now fallen to below 5%. It is also estimated (see e.g. Harvard's "2008 State of the Nation's Housing") that subprime loans accounted for roughly 20% of originations between 2004 and 2006, up from less than 8% between 2000 and 2003. They now account for less than 5% of new mortgage issues.

[4]These numbers can be compared to the findings of Gerardi et al (forthcoming) who run a counterfactual exercise similar in spirit to ours, but using a completely different methodology. Specifically, they estimate an econometric hazard model of sales or foreclosures using data from Massachusetts. In their model, the likelihood of default in a given contract period depends on the loan's leverage at origination, a proxy for current home equity, and local home price and economic conditions, among other determinants. Using this model, they calculate that had the loan issued in 2002 experience similar home price conditions as loans issued in 2005, they would have defaulted at very high rates as well, despite the fact that they were issued under much stricter underwriting standards, particularly in terms of leverage at origination. Those very high foreclosure rates, however, are about half of their counterparts for 2005 loans. This suggests that while loose underwriting standards alone cannot account for the foreclosure crisis, they did magnify the impact of the price collapse significantly.

[5]There are numerous other housing papers which are a bit less closely related. Campbell and Cocco (2003) study the microeconomic determinants of mortgage choice but do so in a model where all agents are home-

al (2009) who run a counterfactual exercise similar in spirit to ours, but using a completely different methodology. Specifically, they estimate an econometric hazard model of sales or foreclosures using data from Massachusetts. In their model, the likelihood of default in a given contract period depends on the loan's leverage at origination, a proxy for current home equity, and local home price and economic conditions, among other determinants. Using this model, they calculate that had the loan issued in 2002 experience similar home price conditions as loans issued in 2005, they would have defaulted at very high rates as well, despite the fact that they were issued under much stricter underwriting standards, particularly in terms of leverage at origination. Those very high foreclosure rates, however, are about half of their counterparts for 2005 loans. This suggests that while loose underwriting standards alone cannot account for the foreclosure crisis, they did magnify the impact of the price collapse significantly. One advantage of running a counterfactual inside a dynamic general equilibrium model such as ours is that the resulting calculations capture the consequences of endogenous changes to the sample of borrowers caused by changes in underwriting standards. As our model illustrates, loosening underwriting standards clearly encourages the participation in mortgage markets of agents prone to default.

Chambers et al. (2009) also study the effect introducing new mortgage options in a dynamic general equilibrium model and argue that the development of mortgages with gradually increasing payments has had a positive impact on participation in the housing market. The idea that mortgage innovation may have implications for foreclosures is taken up in Garriga and Schlagenhauf (2009). They quantify the impact of an unanticipated aggregate house price decline on default rates where there is cross-subsidization of mortgages **within** but not **across** mortgage types (e.g. FRM or LIP). A key difference between our paper and theirs is that we consider a menu of different terms on contracts **both** within **and** across mortgage types. Effectively Garriga and Schlagenhauf (2009) apply the equilibrium concept in Athreya (2002) while we apply the equilibrium concept in Chatterjee et al. (2007). This enables us to build a model that is consistent with the heterogeneity of foreclosure rates and mortgage terms across wealth and income categories which we document in the Survey of Consumer Finance. We present simulations that suggest that the two equilibrium concepts result in significantly different quantitative predictions.

Along this separation dimension our paper is more closely related to Guler (2008) where

owners by assumption, and focus their attention on the choice between adjustable rate mortgages and standard FRMs with no option for default. Rios-Rull and Sanchez-Marcos (2008) develop a model of housing choice where agents can choose to move to bigger houses over time. A different strand of the housing literature (see e.g. Gervais (2002) and Jeske and Krueger (2005) studies the macroeconomic effects of various institutional features of the mortgage industry, again where there is no possibility of default. Davis and Heathcote (2005) describe a model of housing that is consistent with the key business cycle features of residential investment. Our paper also builds on the work of Stein (1995) and Ortalo-Magné and Rady (2006) who study housing choices in overlapping generation models where downpayment requirements affect ownership decisions and house prices. Our framework shares several key features with those employed in these studies, but our primary concern is to quantify the effects of various mortgage options, particularly the option to backload payments, on foreclosure rates.

intermediaries offer a menu of FRMs at endogenously chosen downpayment rates without cross-subsidization or Chatterjee and Eyigungor (2009) where intermediaries offer a menu of infinite maturity interest-only mortgage contracts. Guler studies the impact of an innovation to the screening technology on default rates and Chatterjee and Eyigungor study the effect of an endogenous price drop arising out of an overbuilding shock.

Section 2 lays out the economic environment. Section 3 describes optimal behavior on the part of all agents and defines an equilibrium. Section 4 provides our calibration. Section 5 describes our steady state results, with subsections which focus on: Selection, Default, the Distribution of Interest Rates, Separation vs. Pooling, the Welfare Implications of Innovation, and Recourse vs. Non-recourse Policies. Section 6 presents our main transition experiment. Section 7 concludes.

2 The Environment

We study an economic environment where time is discrete and infinite. The economy is populated by a continuum of households and by a financial intermediary. Each period a mass one of households is born. Over time, households move stochastically through four stages: youth (Y), mid-age (M), old-age (O), and death. All households are born young. At the beginning of each period, young households become middle-aged with probability ρ_M, mid-age households become old with probability ρ_O, and old households die with probability ρ_D and are replaced by birth of young households. We assume that the population size is at its unique invariant value and that the fraction of households of each type obeys a law of large numbers.

Each period, as long as they are young or mid-aged, households receive stochastic earnings shocks denominated in terms of the unique consumption good. These shocks evolve according to a three state Markov transition matrix π and satisfy a law of large numbers so that there is no aggregate uncertainty. Agents begin life at an income level $y \in \{y_L, y_M, y_H\}$ with $y_L < y_M < y_H$ drawn from the unique invariant distribution associated with π. When old, agents earn a fixed, certain amount of income $y^O > 0$.

Until they become old, households can save in one-period bonds that earn rate $1+r_t \geq 0$ at date t with certainty. When old, agents can buy annuities that pay rate $\frac{1+r_t}{1-\rho_D}$ when the holder survives and pay nothing otherwise. We annuitize returns in the last stage of households' life in order to rule out accidental bequests.

Households value both consumption and housing services. They assign non-negative processes $\{c_t, s_t\}_{t=0}^{\infty}$ utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, s_t)$$

where U satisfies standard assumptions.

Households can obtain housing services from the rental market or from the owner-occupied market. On the first market, they can rent quantity $h_1 > 0$ of housing services at unit price

R_t at date t. In the period when agents move from youth to mid-age – and only in that period – agents can choose instead to purchase quantity $h \in \{h_2, h_3\}$ of housing capital for unit price q_t, where $h_3 > h_2 > h_1$.[6] We refer to this asset as a house.

A house of size $h \in \{h_1, h_2\}$ initially delivers h in housing services. In addition, agents enjoy a fixed ownership premium $\theta > 0$ as long as they own a quantity $h \in \{h_1, h_2\}$ of housing capital. We will use this parameter in our calibration of home-ownership rates in the benchmark economy. We think of this parameter as capturing any enjoyment agents derive from owning rather than renting their home, but it also serves as a proxy for any monetary benefit associated with owning which we do not explicitly model.

Once agents purchase a house, the quantity of capital they own follows a Markov Process over $\{h_1, h_2, h_3\}$ with transition matrix

$$P(h'|h) = \begin{bmatrix} 1 & 0 & 0 \\ \lambda & 1-2\lambda & \lambda \\ 0 & \lambda & 1-\lambda \end{bmatrix},$$

where $\lambda > 0$. In other words, a fraction λ of agents who own a house of size $h = h_2$ see the quantity of capital go up to h_3, while a fraction $\lambda > 0$ of these agents see the value of their house fall to $h = h_1$, which is an absorbing state. Likewise, a fraction λ of agents who own a house of size $h = h_3$ see the quantity of capital they own fall to h_2.

We interpret changes in the stock of housing a given agent owns as uninsurable, idiosyncratic house value shocks.[7] There are several possible interpretations for these shocks. For instance, one could think of them as neighborhood shocks which make a house in a given location more or less valuable. Note that while we assume that devaluation shocks satisfy a law of large numbers we do not need to assume that these shocks are independent across households. Our devaluation shocks are a tractable way to capture the possibility of microeconomic events that affect house values and are difficult to insure against.

Note that since devalued houses of size h_1 provide no advantage over rental units, no agent who becomes mid-aged would strictly prefer to purchase a house of that size and all homeowners whose housing capital fall to that level are at least as well off selling their house and becoming renters as they would be if they keep their house.

Owners of a house of size $h \in \{h_1, h_2, h_3\}$ bear maintenance costs δh in all periods where $\delta > 0$. Maintenance costs, denominated in terms of the consumption good, must be paid in

[6] We make the strong assumption that buying a home is a one-time-only option for computational tractability. Forcing agents who have sold their home or defaulted to become renters for the rest of their life enables us to price mortgage contracts for each possible asset-income-house size position at origination independently from rates offered to borrowers with different characteristics. If agents had the option to take another mortgage after they terminate their first one, their decisions to default – hence the intermediary's expected profits – would depend in part on what terms are offered on contracts offered at positions different from their situation when they become mid-aged. Instead of solving one fixed point problem at a time, we would need to jointly solve a high-dimensional set of fixed points.

[7] In the absence of such shocks, households would never find themselves with negative equity in a steady state equilibrium.

all periods by homeowners. Under that constraint, a house does not physically depreciate (other than through a devaluation shock), which in turn maintains the low cardinality of the housing state space. Once agents sell or foreclose their house, they are constrained to rely on the rental market for the remainder of their life. In the period in which agents become old, they must sell their house immediately and become renters for the remainder of their life (i.e. an old age rental).

The financial intermediary holds household savings, and can store these savings at exogenously given return $1 + r_t$ at date t. It also holds a stock of housing capital. It can add to this stock by transforming the consumption good (i.e. deposits) into housing capital at a fixed rate $A > 0$. That is, it can turn quantity k of deposits into quantity Ak of housing capital at the start of any given period. The intermediary can rent out its housing capital, and can sell part of it to newly mid-aged agents. In any period, it can also reduce its stock of capital by turning quantity h of housing capital into quantity $\frac{h}{A}$ of the consumption good.

The intermediary incurs maintenance cost δ on each unit of housing capital it rents, measured in terms of the consumption good. At date t, each unit of consumption good rented thus earns net return $R_t - \delta$.[8]

We assume that households that purchase a house of size $h \in \{h_2, h_3\}$ at a given date are constrained to finance this purchase with one of two possible types of mortgage contracts. The first contract (which we design to mimic the basic features of a standard fixed-rate mortgage, or FRM) requires a downpayment of size $\nu h_0 q_t$ at date t where $\nu \in (0, 1)$ and stipulates a yield $r_t^{FRM}(a_0, y_0, h_0)$ that depends on the household wealth and income characteristics (a_0, y_0) and on the selected house size h_0 at the date t of origination of the loan. Given this yield, constant payments $m_t^{FRM}(a_0, y_0, h_0)$ and a principal balance schedule $\{b_{t,n}^{FRM}(a_0, y_0, h_0)\}_{n=0}^{T}$ can be computed using standard calculations, where T is the maturity of the loan. Specifically, suppressing the initial characteristics for notational simplicity,

$$m_t^{FRM} = \frac{r_t^{FRM}}{1 - (1 + r_t^{FRM})^{-T}}(1 - \nu)h_0 q_t$$

and, for all $n \in \{0, T - 1\}$,

$$b_{t,n+1}^{FRM} = b_{t,n}^{FRM}(1 + r_t^{FRM}) - m_t^{FRM},$$

where $b_{t,0}^{FRM} = (1 - \nu)h_0 q_t$. Standard calculations show that $b_{t,T}^{FRM} = 0$.

The second contract, which we will denote by LIP since it features **l**ow **i**nitial **p**ayments, stipulates a yield $r_t^{LIP}(a_0, y_0, h_0)$, no down-payment, constant payments $m_{t,n}^{LIP}(a_0, y_0, h_0) = h_0 q_t r_t^{LIP}(a_0, y_0, h_0)$ that do not reduce the principal for the first $n^{LIP} < T$ periods, and fixed-payments for the following $T - n^{LIP}$ periods with a standard FRM-like balance schedule $\{b_{t,n}^{LIP}(a_0, y_0, h_0)\}_{n=n^{LIP}}^{T}$. In other words,

[8] Note that the fact that each agent's housing choice set is discrete does not impose an integer constraint on the intermediary since it deals with a continuum of households.

$$m_{t,n}^{LIP} = \begin{cases} h_0 q_t r_t^{LIP} & if \quad n < n^{LIP} \\ \frac{r_t^{LIP}}{1-(1+r_t^{LIP})^{-(T-n^{LIP})}} h_0 q & if \quad n \geq n^{LIP} \end{cases}$$

and, for all $n \in \{0, T-1\}$,

$$b_{t,n+1}^{LIP} = b_{n,t}^{LIP}(1 + r_t^{LIP}) - m_{t,n}^{LIP},$$

where $b_{t,0}^{LIP} = h_0 q_t$, and, once again, $b_{t,T}^{LIP} = 0$. Notice that for $n < n^{LIP}$, $b_{t,n+1}^{LIP} = b_{t,0}^{LIP}$ so that the principal remains unchanged for n^{LIP} periods.

LIPs, therefore, have two main characteristics: low downpayment and delayed amortization. These are two of the salient features of the mortgages that become highly popular in the United States around 2003 (see Gerardi et al., 2007.) Naturally, delayed amortization can take many forms. Subprime mortgages, for instance, often feature balloon payments rather than interest-only periods.

Mortgages are issued by the financial intermediary. The intermediary incurs service costs which we model as a premium $\phi > 0$ on the opportunity cost of funds loaned to the agent for housing purposes.

A mid-aged household can terminate the contract at the beginning of any period, in which case the house is sold. We will consider a termination to be a foreclosure when the outstanding principal exceeds the house value or when the agent's state is such that they cannot meet their mortgage payment in the current period. The next section will provide a formal definition of these events. In the event of foreclosure, fraction $\chi > 0$ of the house sale value is lost in transaction costs. If the mortgage's outstanding balance at the time of default is $b_{t,n}$, the intermediary collects $\min\{(1-\chi)q_t h, b_{t,n}\}$, while the household receives $\max\{(1-\chi)q_t h - b_{t,n}, 0\}$.

Agents may also choose to sell their house even when they can meet the payment and have positive equity, for instance because they are borrowing constrained in the current period. Recall also that agents sell their house when they become old. Those contract terminations, however, do not impose transaction costs on the intermediary.

The timing in each period is as follows. At the beginning of the period, agents discover whether or not they have aged and receive a perfectly informative signal about their income draw. Mid-aged agents who own homes also observe the realization of their devaluation shock at the beginning of the period, hence the market value of their home. These agents then decide whether to remain home-owners or to become renters either via selling their house or through foreclosure. Agents who just became middle-aged also make their home-buying and mortgage choice decisions at the beginning of the period, after all uncertainty for the period is resolved. At the end of the period, agents receive their income, mortgage payments are made, and consumption takes place.

3 Equilibrium

We will initially study equilibria in which all prices are constant. For notational simplicity, we now drop all time markers using the convention that, for a given variable x, $x_t \equiv x$ and $x_{t+1} \equiv x'$.

3.1 Household's problem

We state the household problem recursively. In general, the household value functions will be written as $V_{age}(\omega)$ where $\omega \in \Omega_{age}$ is the state facing an agent of $age \in \{Y, M, O\}$.

3.1.1 Old agents

For old agents, the state space is $\Omega_O = I\!R_+$ with typical element $\omega \equiv a \geq 0$. The value function for an old agent with assets $a \in I\!R_+$ solves

$$V_O(a) = \max_{a' \geq 0} \{U(c, h_1) + \beta(1 - \rho_D)V_O(a')\}$$

s.t.

$$c = a\frac{(1+r)}{1-\rho_D} + y^O - h_1 R - a' \geq 0$$

3.1.2 Mid-aged agents

For mid-aged agents, the state space is

$$\Omega_M = I\!R_+ \times \{y_L, y_M, y_H\} \times \{0,1\} \times \{h_1, h_2, h_3\} \times I\!N \times \{\{\{FRM, LIP\} \times I\!R_+ \times \{h_2, h_3\}\} \cup \{\emptyset\}\}$$

with typical element $\omega = (a, y, H, h, n; \kappa)$. Here, $H = 1$ denotes that the household begins the period as a homeowner, while $H = 0$ if they begin as renters. Further, $h \in \{h_1, h_2, h_3\}$ denotes the quantity of housing capital that the household owns at the start of a given period once the devaluation shock has been revealed.[9] We write $n \in \{0, 1, \ldots\}$ for the number of periods the agent has been mid-aged, hence the age of their mortgage when they have one.

The final argument, κ denotes the type of mortgage chosen by a homeowner - that is, $\kappa \equiv (\zeta, r^\zeta, h_0) \in \{FRM, LIP\} \times I\!R^+ \times \{h_2, h_3\}$ which lists the agent's mortgage and house choice when they just become mid-aged. In equilibrium, the yield on a given loan will depend on the agent's wealth-income position (a_0, y_0) and house size choice h_0 at origination. For agents who enter a period as renters, the current house size and mortgage type arguments are undefined, and so we simply let $\kappa = \emptyset$.

Working backwards, we begin with the case where the household has already made its home purchase decision (i.e. $n \geq 1$).

[9]We need both H and h to differentiate a renter from a homeowner whose size h_2 received a shock down to h_1.

Case 1: $n \geq 1$

If the household enters the period as renters (i.e. $H = 0$), they must remain renters:

$$\begin{aligned} V_M(a, y, 0, h_1, n; \emptyset) &= \max_{c,a'} U(c, h_1) + \beta E_{y'|y}\left[(1-\rho_O)V_M(a', y', 0, h_1, n+1; \emptyset) + \rho_O V_O(a')\right] \\ \text{s.t. } c + a' &= y + a(1+r) - Rh_1. \end{aligned}$$

If, on the other hand, the household owns a home (i.e. $H = 1$), they first have to decide whether to remain homeowners or to become renters. We will write $H'(\omega) = 1$ if they choose to remain home-owners and $H'(\omega) = 0$ if they become renters.

The event $H'(\omega) = 0$ entails a sale of the house and a termination of the mortgage contract. As explained in the previous section, we think of that termination as a foreclosure in two cases. First, if it is not budget feasible for the household to meet its mortgage payment $m_n(\kappa)$, that is if,

$$y + a(1+r) - m_n(\kappa) - \delta h < 0, \tag{3.1}$$

the household is constrained to become renters. Abusing language somewhat, we call this event an *involuntary default* and in that case write $D^I(\omega) = 1$, while $D^I(\omega) = 0$ otherwise. A second form of default occurs when the household can meet their mortgage payment (i.e. (3.1) does not hold) but the household chooses nonetheless to become renters and

$$qh - b_n(\kappa) < 0, \tag{3.2}$$

i.e. home equity is negative. We call this event a *voluntary default* (the household is better off turning the house over to the intermediary in that case) and write $D^V(\omega) = 1$.

If neither (3.1) nor (3.2) holds but the household decides to sell their house and become renters, we write $S(\omega) = 1$, while $S(\omega) = 0$ otherwise. In that case, the household simply sells their house, pays their mortgage balance, and their asset position is augmented by the value of their home equity.

Note that

$$1 - H'(\omega) = S(\omega) + D^I(\omega) + D^V(\omega).$$

In other words, (S, D^I, D^V) classify a mortgage termination into three mutually exclusive events: a simple sale (in which the intermediary need not get involved), an involuntary default, or a voluntary default. For agents who become old in the period, we consider the associated sale to be a foreclosure only if $qh - b_n(\kappa) < 0$ and write $D^O = 1$ in that case, while $D^O = 0$ in all other cases.

Equipped with this notation, we can now define the value function of a homeowner (i.e.

a household whose $H = 1$):

$$
\begin{aligned}
V_M(a, y, 1, h, n; \kappa) &= \max_{c \geq 0, a' \geq 0, (H', D^I, D^V, S) \in \{0,1\}^4} U(c, (1 - H')h_1 + H'h) + \theta 1_{\{h \neq h_1, H'=1\}} \\
&+ (1 - H')\beta E_{y'|y} \left[(1 - \rho_O)V_M(a', y', 0, h_1, n + 1; \emptyset) + \rho_O V_O(a')\right] \\
&+ H'\beta E_{(y',h')|(y,h)} \left[\begin{array}{c} (1 - \rho_O)V_M(a', y', 1, h', n + 1; \kappa) \\ +\rho_O V_O(a' + \max\left\{(1 - D^O\chi)qh - b(n + 1; \kappa), 0\right\}) \end{array} \right]
\end{aligned}
$$

subject to:

$$
\begin{aligned}
c + a' &= y + (1 + r)(a + (1 - H') \max((1 - (D^I + D^V)\chi)qh - b_n(\kappa), 0)) \\
&\quad -H'(m_n(\kappa) + \delta h) - (1 - H')Rh_1 \\
D^I &= 1 \text{ if and only if (3.1) holds} \\
D^V &= 1 \text{ if } H' = 0 \text{ and (3.2) holds} \\
S &= 1 - H' - D^I - D^V
\end{aligned}
$$

There are several things to note in the statement of the household's problem. Starting with the objective, housing services depend on the household's housing status and the size of the house they occupy. Second, the right-hand side of the budget constraint depends on whether or not the household keeps its house. When they become renters (i.e. when $H' = 0$ or when they become old) their asset position is increased by the value of the house net of their outstanding principal and in the event of default, net of transaction costs. Their housing expenses are the sum of mortgage and maintenance payments if they keep the house or the cost of rental otherwise. The final constraint states that selling the house without incurring default costs is only possible if the household is able to meet its mortgage obligations and has positive equity.

Case 2: $n = 0$ (The agent just became mid-aged)

Agents who become mid-aged at the start of a given period must decide whether or not to buy a house, and in the event they become homeowners, what mortgage to use to finance their house purchase. Write $K(\omega_0)$ for the set of mortgage contracts available to a household that becomes mid-aged in state ω_0. The set $K(\omega_0)$ has typical element $\kappa = (\zeta, r^\zeta, h_0)$. The household's value function solves:

$$
\begin{aligned}
V_M(a,y,1,h,0;\emptyset) &= \max_{c\geq 0, a'\geq 0, H'\in\{0,1\}, \kappa\in K(\omega_0)} U(c,h) + 1_{\{h>h_1\}}\theta \\
&+ (1-H')\beta E_{y'|y}\left[(1-\rho_O)V_M(a',y',0,h_1,1;\emptyset) + \rho_O V_O(a')\right] \\
&+ H'\beta E_{(y',h')|(y,h_0)}\left[\begin{array}{c}(1-\rho_O)V_M(a',y',1,h',1;\kappa) \\ +\rho_O V_O(a' + \max\{qh_0 - b(1;\kappa),0\})\end{array}\right]
\end{aligned}
$$

subject to:

$$
\begin{aligned}
c + a' &= y + (1+r)(a - H'\nu 1_{\{\zeta=FRM\}} q h_0) \\
&\quad -H'(m_0(\kappa) + \delta h_0) - (1-H')Rh_1 \\
a &\geq H'\nu 1_{\{\zeta=FRM\}} q h_0
\end{aligned}
$$

Households who choose to become homeowners ($H'=1$) choose the contract $\kappa^* \in K(\omega_0)$ that maximizes their future expected utility. We will write $\Xi(\omega_0) = \kappa^*$ for this part of the household's choice, while $\Xi(\omega_0) = \emptyset$ if $H'=0$. Note that included in the choice of the contract is the size of the house h_0.

3.1.3 Young agents

For young agents, the state space is $\Omega_Y = I\!R^+ \times \{y_L, y_M, y_H\}$ with typical element $\omega = (a,y)$. The value function $V_Y : \Omega_Y \mapsto I\!R$ for a young agent with assets a and income y solves

$$
\begin{aligned}
V_Y(a,y) &= \max_{c\geq 0, a'\geq 0} \left\{U(c,h_1) + \beta E_{y'|y}\left[(1-\rho_M)V_Y(a',y') + \rho_M V_M(a',y',0,h_1,0;\emptyset)\right]\right\} \\
\text{s.t. } c + a' &= y + a(1+r) - Rh_1.
\end{aligned}
$$

3.2 Intermediary's problem

All possible uses of loanable funds must earn the same return for the intermediary. This implies, first, that the unit price q of housing capital must equal $\frac{1}{A}$.[10] Otherwise, the intermediary would enjoy an unbounded profit opportunity turning loanable funds into houses and vice versa.

Arbitrage between renting and selling houses also requires that:

$$
\begin{aligned}
q &= \sum_{t=1}^{\infty} \frac{R-\delta}{(1+r)^t} \\
\Longleftrightarrow R &= rq + \delta. \qquad (3.3)
\end{aligned}
$$

[10]Specifically, the intermediary chooses k to solve $\max \ qAk - k$ which implies that $qA = 1$ must hold in equilibrium.

Note in particular that a change in q must be associated with a change in R in this environment. A bit of algebra also shows that the returns to turning a marginal unit of deposits into housing capital and renting that capital ad infinitum is the same as the returns to storing that marginal unit of deposit.

Arbitrage also requires that for all mortgages issued at a given date, the expected return on the mortgage net of expected foreclosure costs cover the opportunity cost of funds, which by assumption is the returns to storage plus the servicing premium ϕ.

To make this precise, denote the value to the intermediary of a mortgage contract κ held by a mid-aged agent in state $\omega \in \Omega_M$ by $W^\kappa(\omega)$. Again, we need to consider several cases.

- If the homeowner's mortgage is not paid off, so that $\omega = (a, y, 1, h, n; \kappa)$ with $n \in (0, T-1]$, then:

$$\begin{aligned} W^\kappa(\omega) &= \left(D^I(\omega) + D^V(\omega)\right) \min\{(1-\chi)qh, b_n(\kappa)\} + S(\omega) b_n(\kappa) \\ &\quad + \left(1 - D^I(\omega) - D^V(\omega) - S(\omega)\right)\left(\frac{m_n(\kappa)}{1+r+\phi} + E_{\omega'|\omega}\left[\frac{W^\kappa(\omega')}{1+r+\phi}\right]\right) \end{aligned}$$

- If the household just became mid-aged and her budget set is not empty so that $\omega_0 = (a_0, y_0, 0, h_1, 0)$ and, for some contract κ,

$$y_0 + \left(a_0 - \nu q h_0 \cdot 1_{\{\zeta = FRM\}}\right)(1+r) - m_0(\kappa) - \delta h_0 \quad \geq \quad 0,$$

  then

$$W^\kappa(\omega_0) = \frac{m_0(\kappa)}{1+r+\phi} + E_{\omega'|\omega_0}\left[\frac{W^\kappa(\omega')}{1+r+\phi}\right]$$

- In all other cases, $W^\kappa(\omega) = 0$.[11]

Then, the expected present discounted value of a loan contract $\kappa = (\zeta, r^\zeta, h_0)$ offered to a household that just turned mid-age with state $\omega_0 = (a_0, y_0, 1, h, 0)$ is $W^\kappa(\omega_0)$. The zero profit condition on a loan contract κ can then be written as

$$W^\kappa(\omega_0) - (1 - \nu 1_{\{\zeta = FRM\}}) q h_0 = 0. \tag{3.4}$$

In equilibrium, the set $K(\omega_0)$ of mortgage contracts available to an agent who becomes mid-aged in state ω_0 is the set of contracts that satisfy condition (3.4).

[11] Specifically, this is the case when: (i) the agent just turned mid-aged and her budget set is empty; (ii) the agent is a renter; or (iii) the agent has been mid-aged for more than T periods.

3.3 Distribution of agent states

The household's problem yields decision rules for a given set of prices. In turn, these decision rules imply in the usual way transition probability functions across possible agent states. In the next section we study equilibria in which the distribution of agent states is invariant under those probability functions. This section makes this notion precise.

In our environment, the transition matrix across ages is given by:

$$\begin{bmatrix} (1-\rho_M) & \rho_M & 0 \\ 0 & (1-\rho_O) & \rho_O \\ \rho_D & 0 & 1-\rho_D \end{bmatrix}$$

since the old are immediately replaced by newly born young people. Let (n_Y, n_M, n_O) be the corresponding invariant distribution of ages. The invariant mass of agents born each period is then given by

$$\mu_0 \equiv n_O \rho_D.$$

With this notation in hand, we can define invariant distributions over possible states at each demographic stage.

3.3.1 The young

The invariant distribution μ_Y on Ω_Y solves, for all $y \in \{y_L, y_M, y_H\}$ and $A \subset I\!R_+$:

$$\mu_Y(A, y) = \mu_0 1_{\{0 \in A\}} \pi^*(y) + (1-\rho_M) \int_{\omega \in \Omega_Y} 1_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) \mu_Y(d\omega)$$

where $\pi^*(y)$ is the mass of agents born with income y (in other words, π^* denotes the invariant distribution associated with our Markov process for income), $a'_Y : \Omega_Y \mapsto I\!R_+$ is the saving decision rule for young agents, and, abusing notation somewhat, $\Pi(y|\omega)$ is the likelihood of income draw $y \in \{y_L, y_M, y_H\}$ in the next period given current state $\omega \in \Omega_Y$.

3.3.2 The mid-aged

The invariant distribution for mid-aged households μ_M on Ω_M solves, for all $y \in \{y_L, y_M, y_H\}$, $A \subset I\!R_+$ and $(H, h, n; \kappa) \in \{0,1\} \times \{h_1, h_2, h_3\} \times I\!N \times \{\{\{FRM, LIP\} \times I\!R_+ \times \{h_2, h_3\}\} \cup \{\emptyset\}\}$:

$$\begin{aligned} \mu_M(A, y, H, h, n; \kappa) \;\; &= \;\; \rho_M \int_{\Omega_Y} 1_{\{(H,h,n)=(0,h_1,0)\}} 1_{\{a'_Y(\omega) \in A\}} \Pi(y|\omega) \mu_Y(d\omega) \\ &+ \;\; (1-\rho_0) \int_{\Omega_M} 1_{\{(H'(\omega)=H, n(\omega)=n-1, a'_M(\omega) \in A\}} \Pi(y|\omega) P(h|\omega) \mu_M(d\omega) \\ &\times \;\; \left\{ 1_{\{n(\omega)=0, \Xi(\omega)=\kappa\}} + 1_{\{n(\omega)>0, \kappa=\kappa(\omega)\}} \right\} \end{aligned}$$

where $a'_M : \Omega_M \mapsto I\!R_+$ is the optimal saving policy for mid-aged agents, $n(\omega)$ extracts the contract age argument of ω, $\kappa(\omega)$ extracts the contract type argument of ω, and $P(h|\omega)$ is the likelihood of a transition from state ω to a state where the house size is h.

The first term corresponds to agents who age from young to mid-aged, while the second integral corresponds to agents who were mid-aged in the previous period and do not get old. The indicator functions reflect the fact that agents make their mortgage choice in the first period they become mid-aged but cannot revisit that choice in subsequent periods.

3.3.3 The old

The invariant distribution μ_O on $\Omega_O \equiv I\!R_+$ solves, for all $A \subset I\!R_+$:

$$\mu_O(A) = (1-\rho_D)\int_{\Omega_O} 1_{\{a'_O(\omega)\in A\}}\mu_O(d\omega) + \rho_O \int_{\Omega_M} 1_{\{a'_M(\omega)+\max\{H'(\omega)[qh(\omega)-b_{n+1}(\kappa)],0\}\in A\}}\mu_M(d\omega)$$

where, for $\omega \in \Omega_M$, $h(\omega)$ extracts the house size argument of ω, while $b(n+1,\kappa)$ is the principal balance on a mortgage of type κ after $n+1$ periods. Recall that we assumed that housing sales due to the old age shock do not entail foreclosure costs.

3.4 Housing market clearing

The housing market capital clearing condition can be stated in simple terms, after some algebra. The total demand for housing capital (whether rented or owned) in each period is given by:

$$\int_{\Omega_Y} h_1 d\mu_Y + \int_{\Omega_O} h_1 d\mu_O + \int_{\Omega_M} h_1 1_{\{H'=0\}} d\mu_M + \int_{\Omega_M} h 1_{\{H'=1,h(\omega)=h\}} d\mu_M$$

The first two terms give the demand for housing by the young and old agents, who, by assumption, are renters. The third term is demand from mid-aged agents who choose to be renters. The last integral captures mid-aged agents who choose to be homeowners. Their use of housing capital depends on the size of the home that they own.

Similarly, the total quantity of housing available in a given period is the sum of the housing agents carry over from the past period and of the new capital produced by the intermediary. It can be stated formally as:

$$Ak + \int_{\Omega_Y} h_1 d\mu_Y + \int_{\Omega_O} h_1 d\mu_O + \int_{\Omega_M} h_1 1_{\{H=0\}} d\mu_M + \int_{\Omega_M} h 1_{\{H=1,h(\omega)=h\}} d\mu_M$$

But the laws of motion for agent states in our economy imply that:

$$\int_{\Omega_M} h 1_{\{H=1,h(\omega)=h\}} d\mu_M = \int_{\Omega_M} h' 1_{\{H'=1\}} P(h'|\omega) d\mu_M \tag{3.5}$$

where $P(h'|\omega)$ is the likelihood that the agent's house size will be $h' \in \{h_1, h_2, h_3\}$ in the next period given current state $\omega \in \Omega_M$.

It follows that the market for housing capital clears provided

$$\int_{\Omega_M} h 1_{\{H'=1, h(\omega)=h\}} d\mu_M \; - \int_{\Omega_M} h' 1_{\{H'=1\}} P(h'|\omega) d\mu_M \; = Ak, \tag{3.6}$$

where k is the quantity of deposits the intermediary transforms into housing capital each period.

This condition has a very intuitive interpretation. It says that in equilibrium the production of new housing capital must equal the housing capital lost to devaluation. If houses tend to appreciate on average (which depends on the calibration of P), market clearing requires instead that the intermediary transform part of its stock into the consumption good. Because $q = \frac{1}{A}$ holds in equilibrium, this condition implies that both the rental and the owner-occupied markets clear since the intermediary is willing to accommodate any allocation of total housing capital across renters and owners.

3.5 Definition of a steady state equilibrium

Equipped with this notation we may now define an equilibrium. A steady-state equilibrium is a set $K : \Omega_M \mapsto \{FRM, LIP\} \times I\!R^+ \times \{h_2, h_3\}$ of mortgages available to households conditional on any possible state upon entering mid-age, a pair of housing capital prices $(q, R) \geq (0, 0)$, a value $k > 0$ of investment in housing capital, agent value functions $V_{age} : \Omega_{age} \mapsto I\!R$ for $age \in \{Y, M, O\}$, saving policy functions $a'_{age} : \Omega_{age} \mapsto I\!R^+$, a mortgage choice policy function $\Xi : \Omega_M \mapsto K(\omega_0)$, a housing policy function $H' : \Omega_M \mapsto \{0, 1\}$, mortgage termination policy functions $D^I, D^V, S : \Omega_M \mapsto \{0, 1\}$, and distributions μ_{age} of agent states on Ω_{age} such that:

1. Household policies are optimal given all prices;

2. $q = \frac{1}{A}$;

3. The allocation of housing capital to the rental and the owner-occupied market is optimal for the intermediary. That is, condition (3.3) holds;

4. The market for housing capital clears every period (i.e. (3.6) holds);

5. The intermediary expects to make zero profit on all mortgages. In other words, condition (3.4) holds for all $\omega_0 \in \Omega_M$ and all mortgages in $K(\omega_0)$;

6. The distribution of states is invariant given pricing functions and agent policies.

4 Calibration

We choose our benchmark set of parameters so that a version of our economy with only FRM mortgages matches the relevant features of the US economy prior to 2003. As figure 1 shows, FRMs accounted for around 85% of mortgages and the fraction was mostly stable between 1998 and 2003. Furthermore, evidence available from the American Housing Survey (AHS) suggests that mortgages with non-traditional amortization schedules accounted for a small fraction of the 15% of non-FRMs prior to 2003. Traditional FRMs and traditional (nominally indexed) ARMs accounted for 95% of all mortgages in the AHS sample before then. At the same time, data available from the Federal Housing Finance Board for fully amortizing loans show no increase in average loan-to-value ratios between 1995 and 2003. These numbers suggest that high-LTV (low downpayment), delayed amortization mortgages accounted for a small fraction of the stock of mortgages and of originations before 2003.

We will think of a model period as representing 2 years. We specify some parameters directly via their implications for certain statistics in our model. These include the parameters governing the income and demographic processes. The other parameters will be selected jointly to match a set of moments with which we want our benchmark economy to be consistent.

We set demographic parameters to $(\rho_M, \rho_0, \rho_D) = (\frac{1}{7}, \frac{1}{15}, \frac{1}{10})$ so that, on average, agents are young for 14 years starting at 20, middle-aged for 30 years, and old aged (retired) for 20 years. The income process for agents in the first two stages of their life, allowing for the possibility that the process may differ across life stages, are calibrated from the Panel Study of Income Dynamics (PSID) survey. We consider households in each PSID sample whose head is between 20 and 34 years of age to be young while households between 35 and 64 years are considered to be mid-aged. Each demographic group in the 1999 and 2001 PSID surveys is then split into income terciles. The support for the income distribution is the average income in each tercile in the two surveys, after normalizing the intermediate income value for mid-aged agents to 1. This yields a support for the income distribution of young agents of $\{0.2937, 0.7855, 1.7452\}$, while the support for mid-aged agents is $\{0.3129, 1, 2.5164\}$. We assume that income in old age is 0.4. This makes retirement income 40% of median income among the mid-aged, which is consistent with standard estimates of replacement ratios.

We then equate the income transition matrix for each age group to the frequency distribution of transitions across terciles for households which appear in both the 1999 and the 2001 survey. The resulting transition matrix for young agents is:

$$\begin{bmatrix} 0.6828 & 0.2581 & 0.0591 \\ 0.2690 & 0.5103 & 0.2207 \\ 0.0481 & 0.2317 & 0.7202 \end{bmatrix}$$

while, for mid-aged agents, it is:

$$\begin{bmatrix} 0.8032 & 0.1804 & 0.0164 \\ 0.1545 & 0.6901 & 0.1554 \\ 0.0423 & 0.1295 & 0.8282 \end{bmatrix}$$

The economy-wide cross-sectional variance of the logarithm of income implied by the resulting distribution is near 0.68, while the autocorrelation of log income is about 0.75.[12] We let the (two-year) risk-free rate be $r = 0.08$ and choose the maintenance cost (δ) to be 5% in order to match the yearly gross rate of depreciation of housing capital, which is 2.5% annually according to Haring et al.(2007).

The terms of FRM contracts are set to mimic the features of common standard fixed-rate mortgages in the US. The down-payment ratio ν is 20% while the maturity T is 15 periods, or 30 years. The LIP contract we introduce assumes $n^{LIP} = 3$ and $T = 15$ so that agents make no payment toward principal for 6 years and make fixed payments for the remaining 12 contract periods (or 24 years) unless the contract is terminated before maturity.

Housing choices depend on the substitutability of consumption and housing services as well as the owner-occupied premium. We specify, for all $(c, h) > (0, 0)$,

$$U(c, s) = \psi \log c + (1 - \psi) \log h,$$

making overall utility

$$\psi \log c + (1 - \psi) \log h + 1_{\{h \in \{h_2, h_3\}\}} \theta.$$

The intertemporal discount rate plays a key role in our model by affecting asset accumulation. Preferences are fully described by (θ, ψ, β). We select these parameters in our joint calibration, to which we now turn.

We need to set the following ten remaining parameters: the owner-occupied premium (θ), households' discount rate (β), housing TFP (A), rental unit size (h_1), house sizes (h_2, h_3), the mortgage service premium (ϕ), the foreclosure transaction cost (χ), the utility weight on consumption (ψ), and the house shock probability (λ). We select those parameters jointly to target: home-ownership rates, the average ex-housing assets to income ratio for mid-aged agents, the average loan-to-income ratio at mortgage origination, the average ratio of rents to income in personal consumption expenditures across all households, the average rent-to-income ratio for low-income renters, the average housing spending share for homeowners, the average yields on FRMs, the average loss severity rates on foreclosed properties, the average market discount on foreclosed houses, and, finally, the average foreclosure rates prior to the flare up.

We now elaborate our approach to measuring target values. Since our model only gives agents a one-time option to become homeowners when they become mid-aged, we choose to target the ownership rate among households whose head is between 35 and 44. The Census Bureau reports that rate is roughly $\frac{2}{3}$.[13] The model's counterpart to that number is the rate

[12] Krueger and Perri (2005) report estimates for the cross-sectional variance of log yearly income of roughly 0.4 and for the autocorrelation of log income in the $[0.80 - 0.95]$ range. These numbers imply that log two-year income has an autocorrelation in the $[0.88 - 0.96]$ range and variance in the $[0.36 - 0.39]$ range. The details of the conversion from one-year to two-year numbers are available upon request. The difficulty is that aggregating an MA(1) process leads to an ARMA(1,1) process.

[13] See http://www.census.gov/hhes/www/housing/hvs/annual08/ann08ind.html, table 17.

of ownership among agents who have been mid-aged for five periods or fewer. This is the rate we will report throughout the paper.

The average non-housing assets to yearly income ratio we choose to target is based on Survey of Consumer Finance (SCF) data. The average ratio of non-housing assets to income among homeowners whose head age is between 35 and 64 in the 2001 survey is 1.86, which corresponds to a ratio of assets to two-years worth of income of 0.93.[14]

The mortgage loan size at origination is $(1-\nu)hq$ for FRMs and hq for LPMs, where $h \in (h_2, h_3)$ is the initial house size. Evidence available from the American Housing Survey (AHS) suggests that prior to 2003 the ratio of this original loan size to yearly income is around 2.72 on average, or 1.36 in two-year terms.[15]

According to the evidence available from the Bureau of Economic Analysis, the ratio of housing expenditures (in imputed rent terms for owners) to overall expenditures is near 20%, and we make this our fourth target. Turning to the rent-to-income ratio for poor renters, Green and Malpezzi (1993, p11) calculate that poor households who are renters spend roughly 40% of their income on housing. On the other hand, according to the 2004 Consumer Expenditure Survey, expenditures on privately owned dwellings account for around 20% of the expenditures of home-owners.

Next, we choose to target an average FRM-yield of 7.2% yearly, or 14.5% over a two-year period. This was the average contract rate on conventional, fixed rate mortgages between 1995 and 2004 according to Federal Housing Finance Board data.

The loss severity rate is the present value of all losses on a given loan as a fraction of the default date balance. As Hayre and Saraf (2008) explain, these losses are caused both by transaction and time costs associated with the foreclosure process, and by the fact that foreclosed properties tend to sell at a discount relative to other, similar properties. Using a dataset of 90,000 first-lien liquidated loans, they estimate that loss severity rates range from around 35% among recent mortgages to as much as 60% among older loans. Based on these numbers we choose parameters so that in the event of default and on average,

$$\frac{\min\{(1-\chi)qh, b)}{b} = 0.5$$

[14]Because agents only have one asset in the model, we interpret a as net assets. Our measure of net assets does not include housing-related assets or debts, such as home equity or mortgages. Since agents are not allowed to have negative assets in our model, households who have negative non-housing assets are assumed to have zero assets in the calculation.

[15]The AHS is a panel of about 55,000 houses and apartments. The survey is carried out every other year by field representatives from the department of housing who interview house and apartment occupants. Vacant and destroyed units are replaced by new units. For each survey year between 1995 and 2003, we selected all households who moved in the three years preceding the interview, who own their home and who have a mortgage. We do not observe mortgages at origination, but the income and loan-size information of recent movers is likely to proxy fairly effectively for their counterparts at origination time. Looking at these recent movers leaves us with between 2 and 3 thousand mortgages in each survey. We calculated the average loan-to-income ratio for each survey between 1995 and 2003 and, finally, averaged the resulting value across surveys.

where b is the outstanding principal at the time of default and qh is the house value. In other words, on average, the intermediary recovers 50% of the outstanding principal it is owed on defaulted loans.

We target a market discount on foreclosed properties of 75%. We define this discount to be the average price of foreclosed properties divided by the average price of regular home sales, after conditioning on size at origination.[16] Hayre and Saraf (2008) estimate that foreclosed properties selling prices range from 90% of their appraised value among properties with appraisal values over \$180,000 to 55% of their appraised values among properties with appraisal values near \$20,000. Other studies of foreclosure discounts (see Pennington-Cross, 2004, for a review) typically find discount rates near 75% (i.e. a loss of 25% over comparable properties), with some exceptions.

Note that the average foreclosure discount and the average loss severity rates are related since part of the loss incurred by intermediaries in the event of default stems from the fact that foreclosed properties tend to be devalued properties. However, a loss in market value of 25% (=1-0.75) alone could not account for an average loss severity rate of 50%. In the data, this discrepancy reflect the transaction costs associated with foreclosure. Our transaction cost parameter χ proxies for those costs and we use this parameter in our calibration to bridge the gap between the foreclosure discount and the total loss associated with foreclosure.

Finally, we target a two-year default rate of 3% which is near the average **two-year** foreclosure rate among all mortgages during the 1990s in the Mortgage Bankers Association's National Delinquency survey. Table 1 summarizes our parameterization.

5 Steady State Results

Our goal is to quantify the importance of nontraditional contracts with low downpayments for the recent rise in U.S. foreclosures. To do so, in the previous section we selected parameters so that a version of our economy where nontraditional mortgages (LIPs) are not available generates steady state predictions for key statistics that match their US data counterparts prior to the explosion of nontraditional mortgages from late 2003 through 2006. We then study the quantitative impact of introducing mortgages with low initial payments in such an economy. In this section we study the effects of a permanent introduction of nontraditional mortgages comparing steady state statistics across the two economies. In contrast, section 6 studies the effect of a brief period of availability of these mortgages that ends with an unanticipated collapse in house prices and compares the features of the resulting transition experiment to the patterns displayed in figure 1.

[16] Formally, we define the Foreclosure Discount as $\text{FD} \equiv m(h_2)f(h_2) + m(h_3)f(h_3)$ where for each initial house size, $h_0 \in \{h_2, h_3\}$, $m(h_0) \equiv \frac{\int_{\{\Omega_M : h_0(\omega)=h_0\}} (D^I(\omega)+D^V(\omega)) d\mu_M}{\int_{\{\Omega_M\}} (D^I(\omega)+D^V(\omega)) d\mu_M}$ is the share of contracts in default, and $f(h_0) \equiv \left(\frac{\int_{\{\Omega_M : h_0(\omega)=h_0\}} (D^I(\omega)+D^V(\omega)) qh d\mu_M}{\int_{\{\Omega_M : h_0(\omega)=h_0\}} (D^I(\omega)+D^V(\omega)) d\mu_M} \right) / \left(\frac{\int_{\{\Omega_M : h_0(\omega)=h_0\}} S(\omega) qh d\mu_M}{\int_{\{\Omega_M : h_0(\omega)=h_0\}} S(\omega) d\mu_M} \right)$ is the average house value for defaulters relative to the average house value for sellers.

Table 1: **Benchmark parameters**

Parameter	Description	Value	Target
Parameters determined independently			
ρ_M	Fraction of young agents who become mid-aged	1/7	14 years of earnings on average prior to home purchase
ρ_O	Fraction of mid-aged agents who become old	1/15	30 years on average between home purchase and retirement
ρ_D	Fraction of old agents who die	1/10	20 years of retirement on average
r	Storage returns	0.08	2-year risk-free rate
δ	Maintenance rate	5%	Residential housing gross depreciation rate
ν	Downpayment on FRMs	0.20	Average Loan-to-Value Ratio
T	Mortgage maturity	15	30 years
n^{LIP}	Interest-only period for LIPs	3	6-years interest-only
Parameters determined jointly			
θ	Owner-occupied premium	3.220	Homeownership rates
λ	Housing shock probability	0.120	Foreclosure rates
A	Housing technology TFP	0.571	Average Loan-to-income ratio at origination
β	Discount rate	0.833	Average ex-housing asset-to-income ratio
ϕ	Mortgage service cost	0.042	Average mortgage yields
χ	Foreclosing costs	0.440	Loss-incidence estimates
ψ	Utility share on consumption	0.800	Average housing spending share
h_1	Size of rental unit	0.640	Rent-to-income ratio for low-income agents
h_2	Size of regular house	0.850	Owner's housing spending share
h_3	Size of luxury house	1.300	Foreclosure discount

5.1 Mortgage Innovation

The benchmark economy only has FRM mortgages available. Table 2 presents some key steady state equilibrium aggregate statistics for this benchmark environment compared to an environment in which LIPs are available.

Table 2: Steady state statistics

	Data	Benchmark	FRM +LIP
Homeownership rate	67.00	66.78	72.12
Avg. ex-housing asset/income ratio	0.93	0.96	0.94
Avg. loan to income ratio	1.36	1.36	1.51
Avg. housing expenditure share	0.20	0.19	0.20
Rents to income ratio for renters	0.40	0.39	0.39
Avg. housing spending share for homeowners	0.20	0.22	0.23
Avg. mortgage yields (FRMs, LIPs)	(14.50,NA)	(14.35,NA)	(14.06,17.51)
Loss-incidence estimates	0.50	0.50	0.46
Foreclosure rates	3.00	2.97	3.70
Foreclosure discount	0.75	0.71	0.70

The table shows that the presence of LIPs has two main consequences on steady state statistics: home-ownership rates and average default rates are significantly higher when LIPs are available than when they are not. When only FRMs are available, a large number of agents are unable to become homeowners because they can't afford a large downpayment.

Default rates, for their part, are higher when LIPs are present as a result of two complementary factors. First, LIPs enable agents at the bottom of the asset and income distributions to select into homeownership. These are high-default risk agents because they are more likely to find themselves unable to meet their mortgage payments at some point over the life of the contract. Second, even at equal asset/income conditions at origination, LIPs are associated with higher default rates because agents build up home equity slower than with FRMs. The next two sections make these ideas precise.

5.2 Selection

This section describes the selection of contracts and the resulting equilibrium distribution of contracts. Since we allow mortgage contracts to depend a household's earnings and asset characteristics at the time of origination, selection depends on the distribution of earnings and assets of households at the time of purchase (which in our model occurs when agents turn mid-aged). Conversely, making LIPs available impacts the equilibrium distribution of wealth at purchase time, since a major incentive to save in the benchmark economy with FRMs only is the need to make a downpayment on a house.

Figure 2 plots the endogenous distribution of assets among agents that just turned mid-aged. In the benchmark experiment, the upper panel shows that low income agents tend to have low assets, and vice-versa. While the largest mass of households is at the borrowing constraint, the second largest mass is at the downpayment amount for small houses ($0.30=\nu q h_2$). The lower panel shows how the distribution *changes* when LIPs are introduced. There is a noticeable shift to the left in the distribution as many agents anticipate that they may resort to the LIP option and no longer need to accumulate assets to meet downpayment requirements. In fact, the average level of assets of agents who just became mid-aged in the economy with LIPs is lower by 27% than its benchmark counterpart (0.55 vs. 0.75.)

Figure 2: Distribution of assets upon entering mid-age

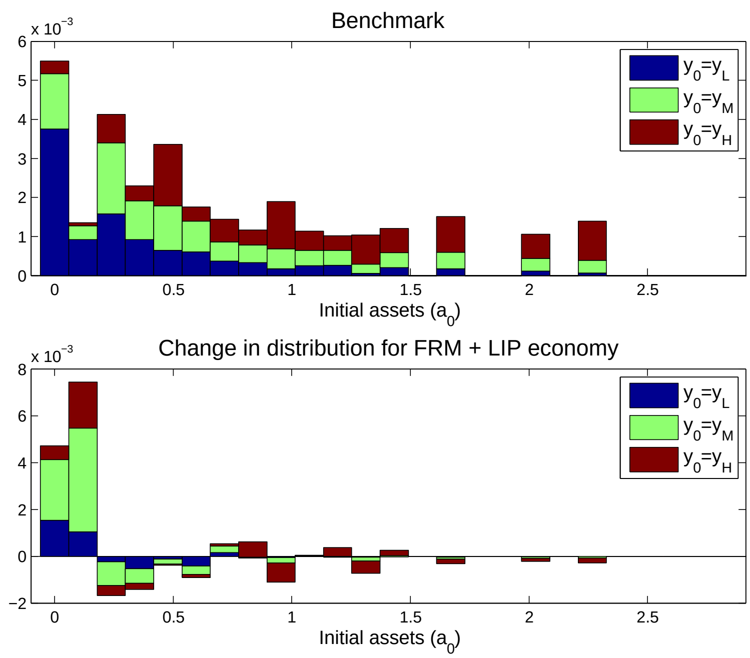


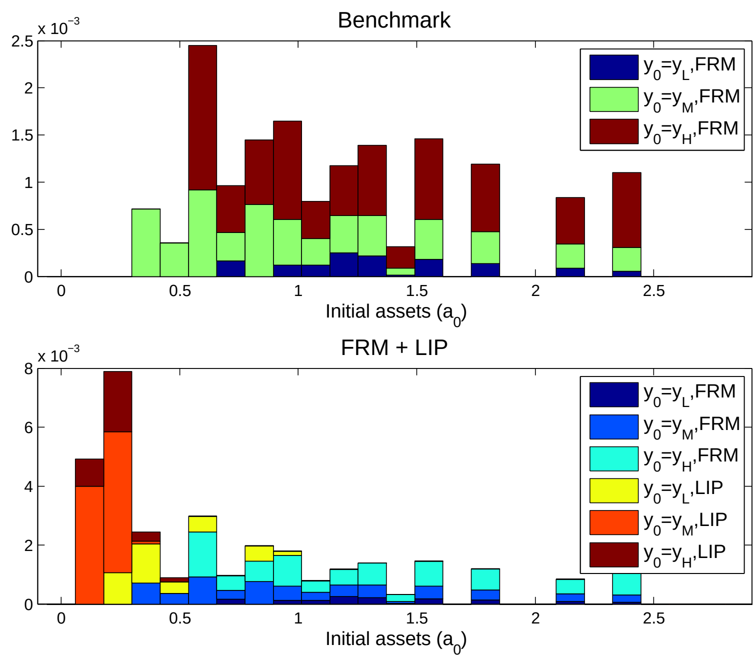
Table 3 displays contract selection patterns in steady state. First, it shows that when LIPs are not available, many agents are constrained to rent because they cannot meet the downpayment imposed by mortgages and/or cannot make the first payment (i.e. the distribution is left truncated at the downpayment level). Introducing LIPs enables some agents at the bottom of the asset distribution to become homeowners instead of renting, as the bottom panel of the table shows. The table also shows that the introduction of LIPs enables agents with high-income but low assets to buy bigger houses than they would without that option. These agents can afford high mortgage payments, but their assets are too low to meet high downpayment requirements for an h_3-size house.

Figure 3 displays the relation between mortgage choices and asset and income levels.

Table 3: Rent-or-own decision rules by asset and income group

Contract	Rent	LIP		FRM	
House size	h_1	h_2	h_3	h_2	h_3
Benchmark					
y_L	$a_0 < 0.32$	–	–	$0.32 \leq a_0 < 3.14$	$3.14 \leq a_0$
y_M	$a_0 < 0.30$	–	–	$0.30 \leq a_0 < 1.24$	$1.24 \leq a_0$
y_H	$a_0 < 0.30$	–	–	$0.30 \leq a_0 < 0.53$	$0.53 \leq a_0$
FRM + LIP					
y_L	$a_0 < 0.20$	$0.20 \leq a_0 < 0.94$	–	$0.94 \leq a_0 < 3.14$	$3.14 \leq a_0$
y_M	–	$a_0 < 0.30$	–	$0.30 \leq a_0 < 1.24$	$1.24 \leq a_0$
y_H	–	–	$a_0 < 0.53$	–	$0.53 \leq a_0$

Figure 3: Distribution of contract choice by asset and income level



When the LIP option is introduced (as we go from the top to the bottom panel of the figure), agents at the bottom of the asset distribution become able to purchase homes. The figure also shows that LIPs are the contract of choice for agents at the bottom of the asset distribution, whereas wealthier agents take an FRM (to take advantage of lower interest rates, as the next section will discuss.)

The asset position at which agents become mid-aged depends on their income history when young and, in particular, on the length of that history. Agents who turn mid-aged quickly accumulate assets over fewer periods, hence are more likely to find themselves with low assets when they are given the opportunity to purchase a house. As a result, young agents who do buy a home are more likely to choose a low downpayment either by buying a small house or using a LIP. These predictions, calculated from a pseudo-panel of 10,000 agents for 25 model periods, are summarized in Table 4.

Table 4: Mortgage Choice and Mean Age

Housing decision	FRM	LIP
Small house	30.83	30.92
Big house	32.70	27.17

The availability of LIPs cause homeownership rates to rise by giving agents more financing options. The fraction of newly mid-aged agents who enter housing markets and buy smaller houses rises from 38% to 41% when the LIP option is introduced. In addition, the fraction of agents who buy large houses rises from 35% to 39% as a result of the looser financial requirements imposed by LIPs.

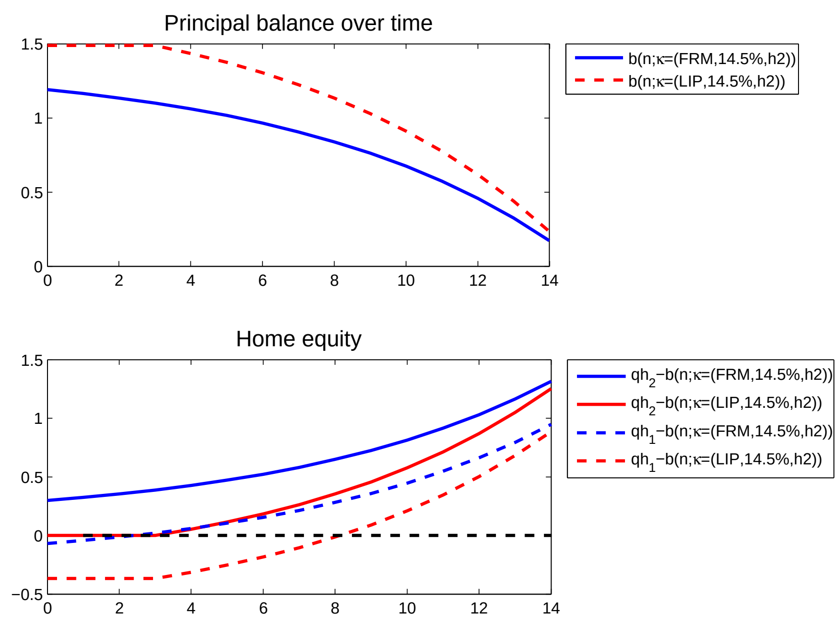
Overall, LIPs turn out to be selected by roughly 30% of newly mid-aged households and tend to be selected by households whose assets are low. The next section argues that, holding contract terms fixed, poor agents are more likely to default than other agents. In addition, it shows that LIPs, even at equal income and assets at origination, are inherently more prone to default. Combined, these facts imply that LIP-holders account for a disproportionate share of overall default rates and explains why default rates are higher in the economy where the LIP option is present than in the economy where only traditional high downpayment FRMs are available.

5.3 Default

Recall that we have classified defaults in two categories. We call a default *voluntary* if the household can meet its mortgage payment but has negative net equity when it terminates its mortgage. If a household defaults because it cannot meet its mortgage payment, we call this event an *involuntary default*.

The availability of LIPs affects the likelihood of both events. As we argued in the previous section, LIPs allow agents with low assets and income to become home-owners. These agents are much more likely to experience payment difficulties at some point in the life of the loan, hence are prone to involuntary default. LIPs also increase the likelihood of voluntary default by increasing the risk that a given home-owner will find themselves with negative equity. To understand why, Figure 4 displays the evolution of the principal balance and home equity as a function of maturity for both types of contract given an initial house size of h_2 and a contract rate of 14.5%. LIP-holders have less equity at all maturities for two reasons. First, LIPs require no down-payment. Second, while FRM contracts feature a progressive reduction of mortgage debt and a corresponding increase in home equity, LIP contracts only begin this process after three periods. It follows that a given devaluation shock is more likely to make net equity negative for LIP-holders than for FRM-holders, as the bottom panel of the figure illustrates. The dotted lines show home equity following a devaluation from h_2 to h_1 as a function of maturity. The shock causes equity to become negative on FRMs in the first three periods while for LIPs, equity become negative following the same shock for the first seven periods.

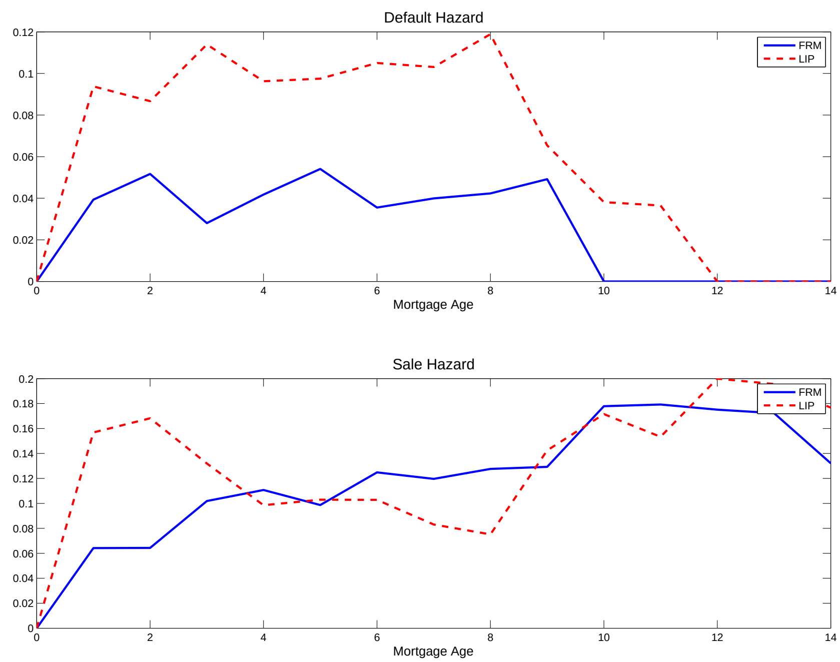
Figure 4: Mortgage debt and home equity by contract type



In order to study more systematically the effects of loan and borrower characteristics at origination on default and sales, we generated a random sample of 50,000 mortgages drawn from the invariant distribution in the economy where both types of mortgages are offered. Figure 5 plots average hazard rates associated with sales or default for each mortgage type. Default hazards have a standard hill-shaped pattern. Default rates peak after two to three periods but eventually fall as borrowers accumulate equity in their home. Importantly, default rates are uniformly higher for LIPs than for FRMs due to the selection and equity effects we have discussed.

Sale hazard rates display a similar pattern for young loans but show no decline after they reach their peak. To understand why, recall that a termination is recorded as a sale provided

Figure 5: Average hazard rates



the borrower has positive home equity. As figure 4 makes clear, early in the life of a mortgage, devaluation shocks cause equity to be negative and many terminations, therefore, are due to default rather than sales. For older loans, equity is likely to remain positive even if the house devalues. Another reason why sale hazards generally rise over the life of the loan is that it takes several shocks for borrowers to become exposed to the risk of involuntary default or to reach a point where consumption would become suboptimally low if they held on to their home.

This representative sample of mortgages can be used to study the determinants of hazard rates by contract type. Specifically, we estimated a competing risk model with the following covariates for borrower $i \in \{1, \ldots, 50000\}$:

1. Mortgage type ($1_{LIP_i} = 1$ if the borrower selected a LIP, 0 otherwise)
2. Loan-to-income ratio at origination (LTY_i),
3. Asset-to-loan ratio at origination (ATL_i).

Define $\gamma^e_{n,i}$ to be the hazard rate at loan age n for homeowner i due to event $e \in \{D, S\}$, where D stands for default (of either type) while S stands for sale. We adopt a standard Cox proportional hazard specification for hazard rates, namely:

$$\gamma_{n,i}^e = H\left(\gamma_n^e \times \exp\left\{\beta_{LIP}^e 1_{LIP,i} + \beta_{LTY}^e LTY_i + \beta_{ATL}^e ATL_i\right\}\right),$$

where $H(x) = \exp(-\exp x)$ for all $x > 0$, and γ_n^e is the baseline hazard rate at loan age n. The six coefficients can then be estimated via maximum likelihood, together with estimates of the corresponding standard errors.[17]

Table 5 shows the result of this estimation. As expected, estimated hazard rates into default are significantly higher for LIPs than for FRMs, rise with the loan-to-income at origination, and fall with the asset-to-loan ratio. These coefficients suggest that hazard rates are noticeably different across different sets of covariates. For instance, holding other covariates the same, simple calculations show that hazard rates on LIPs are twice as high as on FRMs. Because LIPs are more prone to default, they are less prone to sales relative to the baseline hazard, since the two termination risks are competing risks.

Table 5: Determinants of mortgage termination

Covariate	Default	Sale
LIP indicator	0.6812*** (0.0243)	-0.2515*** (0.0154)
Loan to Income Ratio	0.1182*** (0.0064)	0.3287*** (0.0050)
Assets to Loan Ratio	-0.2147*** (0.0358)	-0.6309*** (0.0215)

*Notes: Standard errors are in parenthesis; Log Likelihood : -76943.461; *** significant at 1% level*

Table 6 provides a breakdown of default frequencies by contract type across experiments. Each entry gives the fraction of mortgages of each type that go into default in steady state in each of the two economies we consider.[18] When both contracts are offered, default rates are twice as high on LIPs as on FRMs. The table also shows that involuntary default – defaults occurring when the agent is unable to meet current obligations – are rare in both economies.[19]

[17] Most of the empirical literature on default rates adopts this proportional hazard specification. It is difficult to compare our results directly to the outcome of those estimations because these studies usually control for covariates that have no clear counterpart in our model, and usually lack detailed information on borrower assets. Gerardi et al., 2009, for instance, include proxies for county-level economic and state-wide house price conditions, but do not control for assets at origination. Still, theirs and most estimations find as we do that high LTVs at origination and high debt-to-income ratios have a significant effect on default rates.

[18] For instance, in the notation we introduced in section 3.1.2, involuntary default rates on a FRM contracts are given by $\frac{\int_{\Omega_M} D^I(\omega) 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}$. The expression is similar for involuntary default rates, and for LIPs.

[19] Note that involuntary defaults are somewhat more frequent on LIPs than on FRMs. Agents with recently issued LIPs who find themselves with negative equity continue to meet payments as long as they are low, and wait until the payment resets to default. Because the payment jumps up markedly in that period, this pushes a fair number agents into an involuntary default situation.

Table 6: Default rates by mortgage type

	voluntary	involuntary	total
Benchmark			
FRM	2.96	0.00	2.97
FRM + LIP			
FRM	2.78	0.01	2.79
LIP	5.61	0.22	5.83

In both economies, the vast majority (98.5%) of defaults involve negative equity. The key reason for this is that agents who have positive equity in their house and foresee that they may find themselves in an involuntary default situation tend to sell rather than run the risk of losing their equity to transaction costs. Several steady state statistics illustrate this behavior. Consider for instance the set of households who, should they choose to keep their house, face a positive probability of being in an involuntary default situation in the next period. Almost 92% of these high-risk households choose to sell their house[20], while selling rates are around 10% among other mortgage holders.

While almost all foreclosures involve negative home equity, many households (roughly 70.7%) with negative home equity choose to keep their house and continue meeting their mortgage obligations. While defaulting would entail a net worth gain for these households, they would be forced to rent a smaller housing unit and would forego the ownership premium.

These model predictions are consistent with the empirical literature on the determinants of foreclosure (see, e.g., Gerardi et al., 2007). Available data suggest that most foreclosures involve negative equity but that, at the same time, most households with negative equity choose not to foreclose. Our model captures the fact that most foreclosures involve a combination of negative equity and adverse income shocks.

Since LIPs are characterized by much higher default rates than FRMs, they account for a disproportionate fraction of the overall default rate. Table 7 shows the contributions of each contract type to each type of default rate in each of the scenarios we consider.[21] The table shows that LIPs account for nearly 47% of overall default rates even though they only represent 37% of all mortgages.

[20]Let $\gamma(\omega)$ be the probability of homeowners facing a positive probability of being in an involuntary default situation next period if they stay in their house. Specifically, $\gamma(\omega) = E_{\omega'|\omega,H'=1}\left[1_{\{D^I(\omega')=1\}}\right]$. The probability that a household sells its house when this probability is positive is then given by

$$\frac{\int_{\Omega_M} 1_{\{S(\omega)=1,\gamma(\omega)>0,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\gamma(\omega)>0,n<T,H=1\}} d\mu_M(\omega)}$$

.

[21]For instance, the contributions of FRM contracts to involuntary default rates is given by the share of

Table 7: Share of overall default rates

	voluntary	involuntary	total
Benchmark			
FRM	2.96	0.00	**2.97**
FRM + LIP			
FRM	1.95	0.01	1.96
LIP	1.68	0.07	1.75
Total	3.63	0.08	**3.70**

5.4 The Distribution of Interest rates

A distinguishing feature of our model is that mortgage terms depend not only on mortgage types but also on the initial asset and income position of borrowers as well as the size of the loan. Figure 6 plots the menu of equilibrium FRM and LIP rate offerings agents can obtain from the intermediary when they become mid-aged, depending on the house size they opt for and their asset-income position at origination.[22] Note that these are offerings and only some points on these menus will be selected in equilibrium.

All interest rate schedules in Figure 6 are left-truncated because agents whose income and assets are low do not get a mortgage in equilibrium. The left truncation can be thought of as an endogenous borrowing constraint associated with different borrower characteristics. The left truncation occurs for several reasons. First, asset and income poor agents cannot meet the down-payment requirement and/or mortgage payments. Second, these agents are more likely to default, hence receive less favorable borrowing terms. In some cases there is no yield such that the intermediary would expect to break even on the mortgage, even when the agents have the means to finance the initial downpayment.[23]

Among agents who do receive a mortgage offer, yields fall both with assets and income. This prediction accords with the well-documented mortgage industry practice of including

FRM mortgages in the total stock of mortgages in steady state times the rate of involuntary default on FRMs:

$$\left(\frac{\int_{\Omega_M} 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta\in\{FRM,LIP\},n<T,H=1\}} d\mu_M(\omega)}\right) \times \left(\frac{\int_{\Omega_M} D^I(\omega) 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}{\int_{\Omega_M} 1_{\{\zeta=FRM,n<T,H=1\}} d\mu_M(\omega)}\right).$$

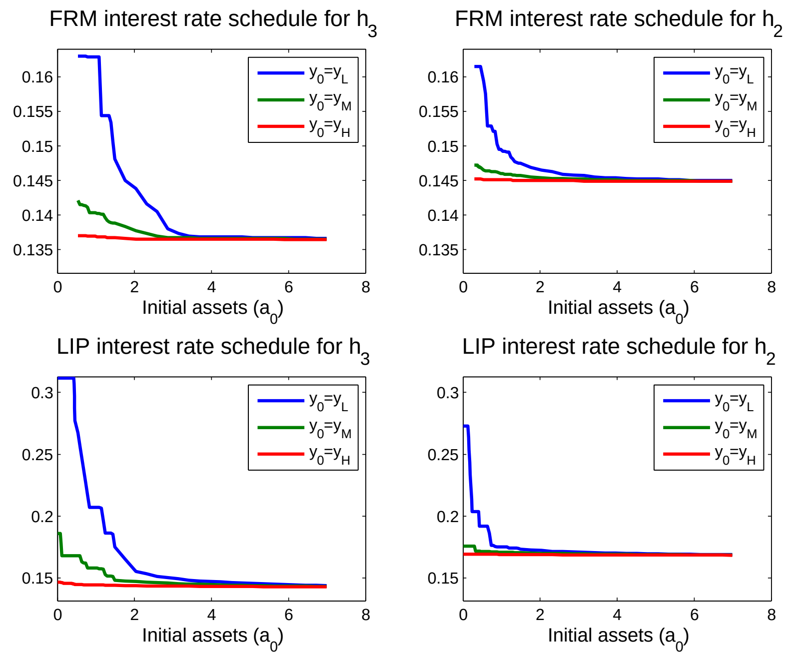
[22]Yields offered on FRMs are the same in the benchmark and FRM+LIP economies. This is because the house price is unchanged and there are no externalities.

[23]In that period (i.e. when $n = 0$), the budget set is empty when $c = a' = 0$ and

$$m(0;\kappa) > y_0 + (a_0 - vqh \cdot 1_{\{\zeta=FRM\}})(1+r).$$

Since $m(0;\kappa)$ is strictly increasing in r^κ, we know there is an interest rate $\overline{r}^\zeta$ that depends on y_0 and a_0 such that for any $r > \overline{r}^\zeta$ the bank cannot break even.

Figure 6: Equilibrium yield schedules for the FRM+LIP economy



overall debt-to-income ratios in their rate sheets. It is also borne out by the statistical evidence available from the Survey of Consumer Finance. A glance at the vertical scale of the figure reveals that LIP rates exceed FRM rates at all possible asset-income positions.

Figure 7 graphs the distribution of equilibrium interest rates by mortgage type when both mortgages are available. There is relatively little variation in FRM rates, but the distribution of LIP yields displays a fair amount of dispersion. Statistical evidence available from the Survey of Consumer Finance documented in Table 8 shows that traditional (FRM) mortgages exhibit less variation than nontraditional (LIP) mortgages.[24] The table shows, however, that our model understates the variation in yields suggested by these data and overstates the degree to which income and yields are correlated. A key reason for both findings is that the SCF sample of both FRMs and other mortgages are characterized by much heterogeneity in

[24]To compute these moments in the data, we looked at all the mortgages issued within the two years prior to the 2004 survey. We restrict the sample to recently issued mortgages so that current income and assets are reasonable proxies for their counterparts at origination time. We also restrict our attention to households whose head age is between 30 and 45 since mortgages are only issued at the middle-age stage in our model. We define net worth as liquid assets, CDs, stocks, bond, vehicles, primary residence, real estate investment, business interest minus housing debts, credit card, installment debts, and line of credits. This notion of net worth includes housing equity because we observe agents shortly after the mortgage origination. Housing equity, at that time, reflects mainly the down-payment made at origination by the borrower. That downpayment, in turn, was part of assets prior to the origination.

maturity and initial loan-to-value ratios which we do not model and for which SCF data do not enable one to control. This heterogeneity raises the volatility of yields and reduces the correlation with asset and income for reasons which our model cannot replicate.

Figure 7: Distribution of equilibrium interest rates for FRM + LIP model

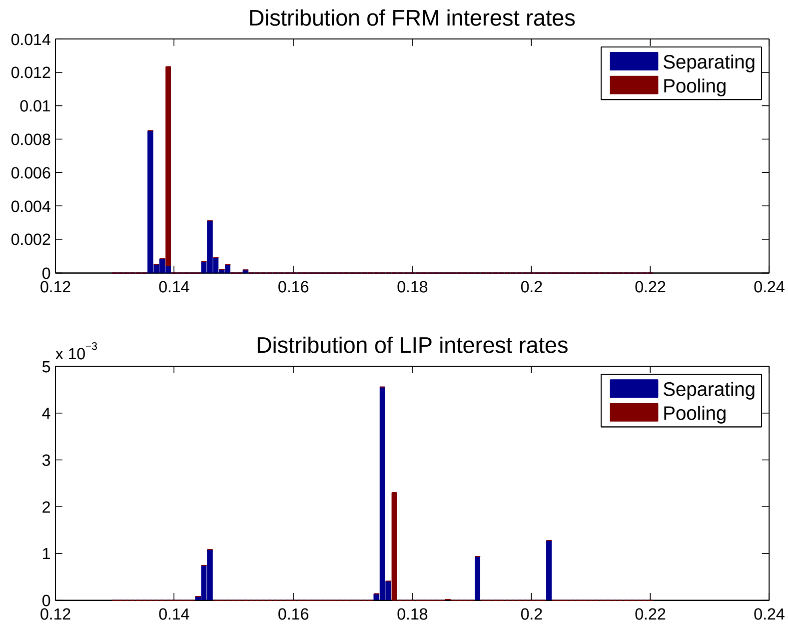


Table 8: Variation in equilibrium returns by contract

	Data	Benchmark	FRM + LIP
$CV(yield)$ for FRMS	0.153	0.0355	0.0168
$CV(yield)$ for other	0.341	NA	0.1800

Figure 7 shows that agents who take advantage of the nontraditional mortgage to become home-owners – that is, agents at the bottom of the income distribution who represent a high-risk of default – incur rates in excess of 20%. On the other hand, agents who switch from FRMs to LIPs either to purchase bigger homes or because they find it optimal to delay payments receive rates not unlike those offered on FRMs, because their default risk does not rise much as a result of the change.

Table 9 shows that much of the variation in yields can be accounted for by loan and borrower characteristics at origination. Using once again our representative sample of 25,000 mortgages to regress log mortage yields on assets, income and loan size at orgination yields an

Table 9: Determinants of log mortgage yield

Covariate	Coefficient
Assets at origination	-0.0867*** (0.0004)
Income at origination	-0.1184*** (0.0005)
Loan size	0.1089*** (0.0013)

Notes: Standard errors are in parenthesis; $R^2 = 0.7231$*; *** significant at 1% level*

R^2 of near three-quarters. Furthermore, higher assets and income are associated with lower yields since they reduce the likelihood of default. A higher loan size at origination, however, is associate with a higher yield as a result of the negative correlation in our model between loan size and downpayments.

Figure 1 illustrates the large increase in subprime mortgages as a fraction of all mortgages starting between 2003 and 2006. If we define subprime as the bottom 30% of households who obtained the highest mortgage interest rates in our model, when both FRMs and LIPs are available, the average return is 14.09% on prime mortgages and 18.14% on subprime mortgages (recall that a model period is 2 years so annual rates are cut in half). Moreover, the model average return is 14.03% on prime FRMs and 15.13% on subprime FRMs, while the average return is 14.60% on prime LIPs and 18.27% on subprime LIPs. The interest rates indicate the likelihood of default. The default rate is 2.01% on prime FRMs and 3.32% on prime LIPs, while the default rate is higher at 3.35% on subprime FRMs and 5.96% on subprime LIPs.

5.5 Pooling versus Separating Equilibria

In this section, we conduct counterfactual experiments to examine the importance of allowing intermediaries to offer mortgage contracts that separate households on income, asset, and loan size characteristics rather than offering only noncontingent or "pooling" FRM and LIP contracts (as in Garriga and Schlagenhauf (2009)). In the second equilibrium concept, the unique equilibrium mortgage rate for each mortgage type is determined by a zero expected profit condition across all households selecting into that contract (and hence the distribution of households directly affects the calculation of mortgage rates).[25] Low-risk borrowers, in such an equilibrium, subsidize high-risk borrowers. In particular, the intermediary issues contracts to some borrowers on which it expects to lose money. Such cross-subsidization seems unlikely to survive in competitive environments since an intermediary can simply offer a contract with lower interest rate to households with observable high income and/or assets and skim those good customers away from the pooled contract.

As is made clear in Table 10, there are quantitatively significant differences between sep-

[25] A formal definition of the intermediary's net profit in the pooling case is provided in appendix A.3.

Table 10: The role of separation

	FRM+LIP	FRM+LIP, pooling
Homeownership rate	72.12	75.47
Ex-housing asset/income ratio	0.94	1.04
Loan to income ratio	1.51	1.66
Avg. housing expenditure share	0.20	0.19
Rents to income ratio for renters	0.39	0.39
Housing spending share for homeowners	0.23	0.23
Avg. mortgage yields (FRMs)	(14.06,17.51)	(13.99,17.75)
Loss-incidence estimates	0.46	0.49
Foreclosure rates	3.70	5.16
Foreclosure discount	0.70	0.69

Table 11: Rent-or-own decision rules in pooling and separating equilibria

Contract	Rent	LIP		FRM	
House size	h_1	h_2	h_3	h_2	h_3
FRM + LIP					
y_L	$a_0 < 0.20$	$0.20 \leq a_0 < 0.94$	–	$0.94 \leq a_0 < 3.14$	$3.14 \leq a_0$
y_M	–	$a_0 < 0.30$	–	$0.30 \leq a_0 < 1.24$	$1.24 \leq a_0$
y_H	–	–	$a_0 < 0.53$	–	$0.53 \leq a_0$
FRM + LIP, pooling					
y_L	$a_0 < 0.08$	$0.08 \leq a_0 < 0.68$	–	$0.68 \leq a_0 < 3.41$	$3.41 \leq a_0$
y_M	–	$a_0 < 0.30$	–	$0.30 \leq a_0 < 1.34$	$1.34 \leq a_0$
y_H	–	–	$a_0 < 0.53$	–	$0.53 \leq a_0$

arating versus pooling equilibria in an economy where LIPs and FRMs are available. The contract selection patterns are shown in Table 11. Low-income agents with assets between 0.08 and 0.20 now buy a house using a LIP. In the separating equilibrium, these agents are constrained to rent because their likelihood of default is too high to cover financing costs at any yield. On the other hand, low income agents with assets between 0.68 and 0.94 now opt for an FRM instead of a LIP in order to avoid paying higher yields at origination. Finally, some agents in the first two income categories who used LIPs to purchase a big house now opt for an FRM and a small house instead, once again to avoid the higher LIP rates caused by pooling. As a result, the pool of LIP borrowers is of bad credit quality in the pooling equilibrium. The foreclosure rate, correspondingly, rises as well, by almost 40%. In addition, average LIP rates rise as the average credit quality of borrowers worsen.

In summary, pooling mortgage contacts has a large impact on steady state statistics. Therefore, not only is the pooling equilibrium inconsistent with the data (i.e. zero dispersion of mortgage rates *within* FRM and LIP contracts), but such an assumption has a big impact on aggregate default probabilities and home-ownership rates.

5.6 The Welfare Implications of Innovation

The introduction of LIPs unambiguously improve the welfare of all agents because they provide households with a new financing option without altering house prices given our linear housing technology. This provides a rationale for why we might see such contract innovation. However, the welfare consequences of innovation are bound to differ across agents. Agents whose homeownership prospects at birth are not significantly improved by the introduction of LIPs will not benefit much, while agents whose ownership prospects do rise significantly are likely to see their welfare rise markedly. This section verifies this intuition.

To determine the gains, we calculate what agents would be willing to pay at birth in the benchmark (FRM-only) economy to obtain the same welfare they can expect in an FRM+LIP economy. Consider agents born with income at birth y_i where $i \in \{L, M, H\}$ and let $U^{bench}(y_i)$ and $U^{FRM+LIP}(y_i)$ denote the lifetime utility they expect at birth in the benchmark and $FRM + LIP$ economies, respectively. Denote the optimal consumption and housing service plans in the benchmark economy by $\left\{c_{t,i}^{bench}, s_{t,i}^{bench}\right\}$ for an agent born with initial income y_i. Then, let $1 + k_i$ be the multiple one has to apply to the consumption of agents born in the benchmark economy to make their welfare equal the same agents born in an FRM+LIP economy. That is, k_i solves for all $i \in \{L, M, H\}$:

$$\begin{aligned} U^{FRM+LIP}(y_i) &= E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_{t,i}^{bench}(1+k_i), s_{t,i}^{bench})\right] \\ &= E_0\left[\sum_{t=0}^{\infty}\beta^t \left\{\ln(c_{t,i}^{bench}) + \ln(1+k_i) + \ln(s_{t,i}^{bench})\right\}\right] \\ &= U^{bench}(y_i) + \frac{\ln(1+k_i)}{(1-\beta)} \end{aligned}$$

It follows that:

$$\begin{aligned} (1-\beta)\left[U^{FRM+LIP}(y_i) - U^{bench}(y_i)\right] &= \ln(1+k_i) \\ \Longrightarrow k_i &= \exp\left((1-\beta)[U^{FRM+LIP}(y_i) - U^{bench}(y_i)]\right) - 1 \end{aligned}$$

We find that:

$$
\begin{aligned}
k_L &= 3.03\% \\
k_M &= 2.40\% \\
k_H &= 0.28\%
\end{aligned}
$$

yielding an average welfare gain associated with availability of the LIP option of around 1.90% in consumption-equivalent terms.

Since income is quite persistent, households who are born poor know that they are most likely to remain renters their entire life. Likewise, households born rich know that they are most likely to opt for an FRM when they become mid-aged. As expected, welfare gains are monotonic in income because people at the bottom of the income distribution are the most likely to take advantage of the LIP option.

We should emphasize that these magnitudes would differ in a model where the housing technology is not linear so that housing prices (both on the owner-occupied and rental markets) respond endogenously to the available mortgage types.[26] For one thing, the rich, whose gains are minimal holding prices constant, are likely to experience a loss in welfare as the price effect dominates the value of an additional financing option. Since rental rates also rise when the opportunity cost of housing capital rises, the welfare implications of introducing LIPs may also become ambiguous at the bottom of the income distribution.

5.7 Policy Experiment: the Role of Recourse

So far we have maintained the assumption that, in the event of default, the borrower's liability is limited to their home. In several states – known as anti-deficiency or non-recourse states – the law does in fact make it difficult for mortgage lenders to pursue deficiency judgments. The list of such states varies but generally includes Arizona, California, Florida (and sometimes Texas).[27] There are other states, known as "one-action" states, that allow the holder of the claim against the household to only file one lawsuit to either obtain the foreclosed property or to sue to collect funds. The list of such states includes Nevada and New York. Even in states where deficiency judgments are legal, conventional wisdom is that the costs associated with these judgments are so high, and the expected returns are so low, that this recourse is seldom used. However, some empirical studies (e.g. Ghent and Kudlyak (2009)) find that recourse decreases the probability of default when there is a substantial likelihood that a borrower has negative home equity. In this subsection we quantify the role of the recourse assumption for equilibrium statistics.

[26] In an earlier version of this paper where we considered a strictly concave technology and prices varied endogenously, we still found there was an ex-ante gain to the introduction of LIPs.

[27] See, for instance, http://www.helocbasics.com/list-of-non-recourse-mortgage-states-and-anti-deficiency-statutes

Table 12: The role of recourse

	Benchmark (no recourse)	Full recourse
Homeownership rate	66.78	69.41
Avg. ex-housing asset/income ratio	0.96	0.96
Avg. loan to income ratio	1.36	1.35
Avg. homeowner housing expenditure share	0.19	0.20
Rents to income ratio for renters	0.39	0.39
Avg. housing spending share for homeowners	0.22	0.21
Avg. mortgage yields (FRMs, LIPs)	(14.35,NA)	(12.81,NA)
Loss-incidence estimates	0.50	0.78
Foreclosure rates	2.97	1.55
Foreclosure discount	0.71	0.73

Table 12 compares steady state statistics in our benchmark economy (which assumes anti-deficiency or non-recourse) to their counterparts in an economy where in the event of default by a borrower with assets $a \geq 0$ and house size h, the intermediary collects $\min\{(1-\chi)qh + a, b\}$, while the household retains $\max\{(1-\chi)qh + a - b, 0\}$. In other words, in the second economy, any asset the household owns at the time of default can be claimed as collateral by the lender. Thus, recourse has both extensive and intensive margin effects. In particular, harsher punishment lowers the extensive default margin and higher repayment lowers the intensive loss incidence margin.

As the table shows, this change in the environment greatly reduces average loss incidence rates, for obvious reasons. At the same time, this makes default much more costly for households and, as a result, foreclosure rates fall by 48%. By comparison, Ghent and Kudlyak (2009) estimate that at average borrower characteristics, the likelihood of default is 20% higher in antideficiency states than in recourse states. Note that we assume that liquid assets are collected without any transaction costs, which raises the equilibrium impact of recourse. In that sense, our results should be considered an upper bound on the potential effect of recourse.

An interesting aspect of this experiment is that allowing for recourse actually raises homeownership rates. This is because mortgage rates fall when default risk decreases, making homeownership a cheaper option ex-ante.

6 Transitional effects of mortgage innovation

The previous section shows that the introduction of nontraditional mortgages (with zero downpayments and backloaded payments) has significant long-run effects on foreclosure rates. We will now describe our main quantitative experiment designed to evaluate the role of these new mortgages in the foreclosure crisis depicted in Figure 1. Figure 1 suggests that the course of events leading up to the collapse of house prices and the foreclosure crisis can be decomposed into three basic stages. Prior to 2003, the composition of the mortgage stock is stable and traditional mortgages are the dominant form of home financing. Around 2003, the composition of the mortgage stock changes noticeably as nontraditional mortgages start accounting for a high fraction of originations. At the start of 2007, prices start collapsing and the flow of traditional mortgages begins rising once again as originations of non-traditional mortgages slow to a trickle.[28]

We will use our model to simulate this course of events and quantify the role of nontraditional mortgages using a three-stage experiment. In the first stage (the pre-2003 period), the economy is in our benchmark, FRM-only steady state. In the second stage of the experiment, we introduce the option for newly mid-aged agents to finance their house purchase with a LIP mortgage. We assume that this introduction is unanticipated by agents, but perceived as permanent once it is made. Two periods later, in the third stage, we shock the economy with an unanticipated 25% aggregate price decline and take away the LIP option. This shock is meant to approximate the widely unanticipated collapse of house prices since the start of 2007. In the third stage, we cause home prices to fall by assuming that the productivity of the housing technology A rises. Of course, this unanticipated rise in housing TFP is only intended to provide a model-based explanation of the housing price drop. This drop in prices catches agents as a complete surprise so that, at the time of the shock, the distribution of states across agents is the one implied by the first two stages of the experiment. It is important to emphasize that we calibrated our parameters only to match the first stage of our experiment, so that the model predictions for the second two stages can be thought of as a test of the model.

Because the intermediary is also surprised by the price shock, it experiences unforeseen revenue and capital losses. A formal definition of the intermediary's net profit is provided in appendix A.3. In steady state, those profits are zero. However, the unexpected drop in prices causes net-equity to turn negative on many houses. Thus, default rates rise which reduces revenues and raises foreclosure losses, causing profits to become negative until contracts written before the price shock disappear. One has to be explicit about who bears these losses. We assume that constant lump-sum taxes are imposed on all agents following the price shock in such a way as to exactly cover the intermediary's losses in present value terms. Computationally, this involves guessing a value for the constant and permanent tax, solving for the new steady state equilibrium and the transition to this new state, evaluating the present value of the intermediary's transitory losses, and updating the permanent tax level until losses and tax revenues match.[29]

[28] According to the Mortgage Origination Survey data, the share of traditional FRMs in originations had risen to nearly 90% by the second quarter of 2008.

[29] See the computational appendix for details. There are obviously many possible ways to redistribute the intermediary's losses. Per capita losses are negligible in practice and barring extremely concentrated tax schemes, their exact distribution will not have large effects on the results we present.

Figure 8: Fraction of LIPs in mortgage stock during transition

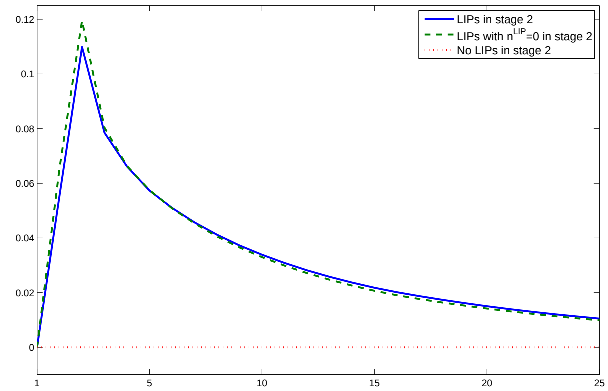
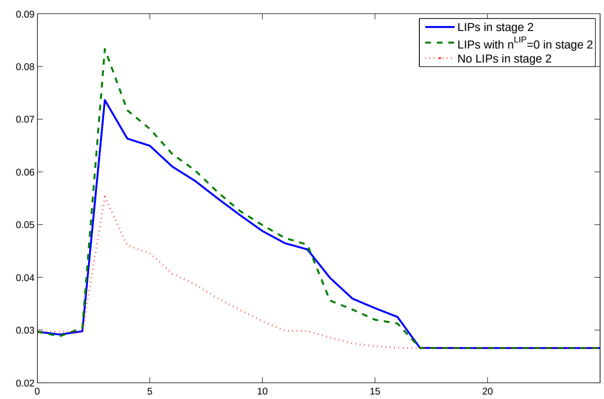


Figure 9: Foreclosure rates during transition



Figures 8 and 9 and Table 13 show the outcome of this three-stage experiment. Once they become available, LIPs account for 33% of all originations, raising the fraction of LIPs in the mortgage stock from 0 to 11% in two model periods (meant to capture the 4 years between 2003 and 2006). While it is difficult to find precise data on the exact share of mortgages with very low down-payments between 2003 and 2006, the high popularity of non-traditional mortgages between 2003-06 accords fairly well with the available evidence. According to the Mortgage Origination Survey, traditional FRMs accounted for only 50% of all originations in 2005 (the first year of the survey.) Of course, not all other mortgages issued were of the low-initial-payment type. However, the fact that FRMs account for a stable 85% of the mortgage

Table 13: Summary of transition results

	Data	LIPs in stage 2	No LIPs in stage 2	$n^{IOM} = 0$ in stage 2
Fraction of LIPs in originations in stage 2	[20-35%]	33%	0%	37%
Total increase in foreclosures since 2007Q1	150%	148%	86%	189%

Notes: Our data counterpart for LIP originations is a rough estimate based on available estimates of subprime mortgage originations, and the fact that the fraction of traditional FRM mortgages in origination fell to 50% in 2005, from a peak of 85%.

stock prior to 2003 suggests that they accounted for roughly 85% of originations before then. It seems safe to assume that the 35% drop in FRM originations owes for the most part to the popularity of high LTV, non-traditional amortization loans. It is estimated that subprime loans alone accounted for a quarter of all originations in 2005, and the Loan Performance data discussed by Gerardi et al. (forthcoming) suggests that the majority of these mortgages featured high leverage and/or delayed amortization. All told, it seems reasonable to assume that loans with high-leverage and/or other non-traditional features accounted for between 20% and 35% of originations at that time. Once again, we want to emphasize that the model's ability to fall within originations of nontraditional mortgages witnessed in the data in Table 13 is a strong test of the model.

The homeownership rate also rises as more agents are able to purchase homes thanks to mortgage innovation. Because we do not give home-buyers the option to default in the first period of their life and the number of originations rises in stage 2, average default rates fall slightly in the first period of stage 2, but they rise in the second period of stage 2 as LIPs issued in the first period now become subject to default.

Once the price shock strikes in stage 3, foreclosure rates double to over 7.36% in one period. The aggregate shock pushes a number of agents with contracts written prior to the shock (when houses were expensive) into a negative net equity position. The price shock causes foreclosure rates to rise by around 148% over the first two years of the crisis. According to the data displayed in Figure 1 and Table 13, foreclosure rates rose by about a 150% between the first quarter of 2007 and the first quarter of 2009. Once again, since none of the parameters were chosen to match moments in the third stage of the experiment, this is a strong test of the model.

To quantify the importance of nontraditional mortgages in the foreclosure boom, we run a counterfactual experiment where in stage 2, LIPs are not offered. We shock the economy with the same price decrease in stage 3. The result is shown in the dotted lines on figure 9. The increase in foreclosure rates become noticeably smaller, because the price shock now strikes an economy where agents have more equity in their homes. Specifically, as shown in table 13, the increase in foreclosure rates on impact falls to 86% when LIPs are not introduced. In other words, the introduction of LIPs in stage 2 increases the size of the spike in foreclosure rates caused by the home price collapse by over 72%.

We can also quantify the relative importance of low downpayments and delayed amortization – the two key features of our LIPs – by running a third experiment where in stage 2, LIPs feature a zero downpayment, but no interest only period. As figure 8 shows, LIPs become slightly more popular in stage 2 in part because, without an interest only period, the default risk on those mortgages falls as do, therefore, yields. Figure 9 shows, more importantly, that the impact on foreclosure rates of the price shock in stage 3 is very similar to what obtained in the first experiment (even somewhat higher because LIPs are more popular in stage 2), suggesting that the fact that agents with LIPs enter their contract with zero equity is the principal factor behind their role in the foreclosure increase.

Since the introduction of LIPs in stage 2 winds up magnifying the impact of the aggregate price shock in stage 3, their net effect on welfare is a priori unclear. By making foreclosure losses bigger, they raise the tax burden associated with the crisis. Since losses are borne equally by all agents, their tax impact is small however. Specifically, the tax imposed on all agents is 0.001035 when LIPs are introduced in stage 2, and 0.000184 when LIPs are not introduced, which amounts to a very small fraction of any agent's income. On the other hand, LIPs enable many agents to become home-owners (at least briefly) when they would have been forced to rent otherwise. In addition, LIP-holders enter stage 3 with no equity, whereas FRM holders lose much of their home-equity position when the price shock strikes.

7 Conclusion

Our paper asks the following question: How much of the rise in foreclosures can be attributed to the increased originations of non-traditional mortgages between 2003 and 2006? The simple answer to this question is that the presence of nontraditional mortgages roughly doubled the magnitude of the foreclosure crisis. Specifically, to answer the question we undertake the following three-stage transition experiment. We begin in a steady state of an economy with only FRMs calibrated to match key aspects of the US economy prior to 2003. We then introduce the nonstandard mortgage option for two periods, which represents four years in our calibration. In the third stage, we assume a surprise 25% collapse in home prices, remove the nonstandard mortgage option, and then let the economy transit to a new long-run steady state. Since none of the model parameters are chosen to match moments in the latter two stages of the experiment, the model predictions yield a strong test of our theory. This experiment causes foreclosure rates to rise by 148% during the first two years of stage 3. By comparison, in the data, the overall foreclosure rate increased by 150% between the first quarter of 2007 and the first quarter of 2009. To quantify the role of mortgage innovation in this increase, we then run an experiment where the LIP mortgage option is not offered in the second stage. In this counterfactual, the increase in foreclosure rates caused by the price shock falls to 86%. Mortgage innovation, in other words, makes the economy much more sensitive to price shocks. In addition, we find that lower downpayments account for most of the contribution of non-traditional mortgages to the increase in foreclosure rates, while delayed amortization and payment resets play a limited role.

These quantitative findings have a number of implications for how one should interpret current events. In Section 5.6 we show that mortgage innovation can can raise welfare by expanding the range of choices for a number of households. The nature of these innovations, however, does make an increase in default rates unavoidable as shown in Section 5.3 due to important selection effects (where low income/low asset households choose the nontraditional low downpayment, backloaded mortgages causing default rates in steady state to be roughly double those on traditional contracts) studied in Section 5.2, and significantly magnify the impact of negative aggregate housing price shocks on default rates in Section 6 (doubling the foreclosure rate in transition that would have occurred with only traditional contracts). As shown in Section 5.4, our model is consistent with variation in interest rates across traditional FRM and nontraditional low downpayment, backloaded contracts. In Section 5.5, we show that if financial intermediaries had not tried to separate borrowers on observable characteristics (i.e. if they had used pooling contracts) then steady state foreclosure rates would be 40% higher. Finally, Section 5.7 we show that changes in policy, specifically strengthening antideficiency foreclosure policies can have a significant effect on lowering (by roughly 50%) default.

Our experiment is silent about what causes the mortgage innovation or the housing price drop. One possibility is that these non-traditional mortgages may have contributed to the price collapse directly as well, which would make their contribution to the crisis even greater. For instance, the availability of these mortgages may have led to some form of overbuilding as in Chatterjee and Eyigungor (2009). Their presence may also have contributed to the fragility and eventual freeze of the financial system, leading to a collapse of demand for housing, hence of housing prices. Formalizing and quantifying these ideas are promising avenues for future work, and should reinforce our main message: mortgage innovation played a very significant role in the recent foreclosure boom.

A Computational appendix

A.1 Steady State Equilibrium

1. The asset space consists of twenty equally spaced asset grid points between 0 and $\nu q h_2$, twenty equally spaced asset grid points between $\nu q h_2$ and $\nu q h_3$, twenty equally spaced asset grid points between $\nu q h_3$ and and $q h_2$, and another sixty equally spaced asset grid points from $\nu q h_2$ to wherever the asset choice decisions do not bind.

2. We use value function iteration to find $V_O(a)$ on the asset grid from which we obtain decision rules $a'_O(a)$ for old agents. The value functions are approximated by using linear interpolation.

3. Given the value functions for old agents, we use value function iteration to find $V_M(a, y, 0, \cdot)$ on the asset grid from which we obtain decision rules $a'_M(a, y, 0, \cdot)$ for mid-aged renters for each y. The value functions are approximated using linear interpolation.

4. Given the value functions for old agents and mid-aged renters, use value function iteration to find $V_M(a, y, 1, h, n > T; \kappa)$ on the asset grid from which we obtain asset choice decision rules $a'_M(a, y, 1, h, n > T; \kappa)$ and homeownership decisions $H'(a, y, 1, n > T; \kappa)$ for mid-aged homeowners who have paid off their mortgage for each (y, h). The value functions are independent of the original mortgage contract terms κ because their mortgage payments and balances are all zeros regardless of their original contracts. The value functions are approximated using linear interpolation.

5. For every pair of h_0 and (a_0, y_0), if a household does not have enough assets to make the downpayment, $\alpha q h_o$, no FRM contract will be offered. Set an initial guess of mortgage interest rate for each contract, given the value functions for old agent, mid-aged renters, and mid-aged homeowners with one less period of mortgage payments to make $V_M(a, y, 1, h, n = t + 1; \kappa)$, solve for $V_M(a, y, 1, h, n = t; \kappa)$ by backward induction for each $(y, h, t = \{1, ..., T\})$. For each path of possible realization of incomes and housing capital given κ, keep track the household decisions along the path. Calculate the present value according to the decision rules from each path and the probability of this path being realized. If this present value is not equal to the initial loan size, update the interest rate and repeat this step. Otherwise, the equilibrium interest rate is found. The value functions are approximated using linear interpolation.

6. Given the value functions for old and mid-aged agents, use value function iteration to find $V_Y(a, y)$ on the asset grid from which we obtain decision rules $a'_Y(a, y)$ and contract selection decisions $(\zeta(a, y), h_0)$ on mortgage terms and initial housing capital. Because of the potential discontinuity caused by the downpayment requirements, the value functions for young agents are solved by grid search.

7. Solve for the equilibrium stationary distribution μ given the implied law of motion.

A.2 Transition Dynamics

1. Solve for the initial steady state equilibrium with price q^o using the algorithm above with zero lump-sum tax.

2. Start the initial guess of lump-sum tax $\tau_{i=1} = 0$. Solve for the final steady state with a new house price q^n with the lump-sum tax implemented.

3. Solve the optimization problems for homeowners who have purchased the house before the unanticipated house price shock occurs by backward induction. If a household chooses to sell its home, they sell at the new price q^n. If a household chooses to remain

a homeowner, they have to follow the original mortgage terms (if they have not paid off their mortgage debt).

- If the agent is a homeowner but it is not budget feasible for her to make her mortgage payment $m^o(n;\kappa)$, which he obtained before the unanticipated price shock, or:
$$y + a(1+r) - m^o(n;\kappa) - \delta h - \tau_i < 0, \tag{A.1}$$
then the value function solves:
$$V_M^o(a, y, 1, h, n; \kappa) = \max_{c,a'} U\left(c, h_1\right) + \beta E_{y'|y}\left[(1-\rho_O)V_M^n(a', y', 0, \ldots) + \rho_O V_O^n(a')\right]$$
$$\text{s.t. } c + a' = y + a(1+r) + \max\left\{(1-\chi)q^n h - b^o(n;\kappa), 0\right\} - R^n h_1 - \tau_i.$$

- If it is budget feasible for a homeowner to make her mortgage payment, then if the household chooses to sell its house and start to rent (so that $H' = 0$), define the value function by
$$V_M^{o,H'=0}(a, y, 1, h, n; \kappa) = \max_{c,a'} U\left(c, h_1\right) + \beta E_{y'|y}\left[(1-\rho_O)V_M^n(a', y', 0, \ldots) + \rho_O V_O^n(a')\right]$$
$$\text{s.t. } c + a' = y + a(1+r) + \max\{q^n h - b^o(n;\kappa), 0\} - R^n h_1 - \tau_i.$$

- If the agent is able to meet her current mortgage payment and chooses to keep her house ($H' = 1$), define the value function by
$$V_M^{o,H'=1}(a, y, 1, h, n; \kappa) = \max_{c,a'} U\left(c, h\left[1_{\{h=h_1\}} + (1-1_{\{h=h_1\}})\theta\right]\right)$$
$$+\beta E_{(y',h')|(y,h)}\left[\begin{array}{c}(1-\rho_O)V_M^o(a', y', 1, h', n+1; \kappa) \\ +\rho_O V_O^n(a' + \max\left\{q^n h - b^o(n+1;\kappa), 0\right\})\end{array}\right]$$
$$\text{s.t. } c + a' = y + a(1+r) - m^o(n;\kappa) - \delta h - \tau_i.$$

4. Select a large integer N to be the number of periods during the transition. In the first period of transition, start the economy with the initial steady state distribution. Starting from the second period along the transition path, apply the decision rules to solve for the distribution one period ahead. For renters, use the decision rules solved for the final steady state. For homeowners, if they purchase the house before the transition starts, use the optimization problems solved in the previous step. If they purchase the house after the transition starts, use the decision rules in the final steady state. Young agents who turn mid-aged during the transition purchase houses at the new price q^n. Continue to solve for the distribution in every period of the transition path.

5. Given the decision rules and distribution along the transition path, calculate the discounted present value of the net profits for the financial intermediary over the transition path. Update the lump-sum tax τ_{i+1} such that $\frac{\tau}{r}$ is equal to the discounted present value

of the net profits. Return to step 3 and repeat using τ_{i+1} until the discounted present value of the net profits equals the discounted present value of the lump-sum tax. Let the present discounted value of the net profits of intermediaries be $\Pi(\tau)$, where the net profits per period is defined as in (A.6). It depends on τ because households decisions vary with τ which in turn affects the net profits.

$$\Pi(\tau) = \frac{\tau}{r}$$

6. Check if the distribution converges to the final steady state in N periods. If not, increase N and repeat all the steps above.

A.3 Intermediary profits on mortgage activity

This appendix derives a net profit expression for the intermediary from our recursive formulation of the intermediary's problem in section 6. For simplicity but without any loss of generality we do so in the case where $T = 2$. Since breaking down default by type is irrelevant for these calculations, we will also write D for $D^I + D^V$ throughout the derivation. Finally, once again without any loss of generality, we will focus on the economy with FRMs only, and drop mortgage type superscripts (κ) everywhere. Finally we will write $n(\omega)$ for the mortgage period associated with state $\omega \in \Omega_M$. In particular, for newly born agents, $n(\omega) = 0$, while for agents in the second period of their mid-age, $n(\omega) = 1$.

Consider then an agent in state ω at origination with initial loan size b_0 and initial house size $h > 0$, with mortgage yield r^Y. Then:

$$W(\omega) = \frac{m}{1+r+\phi}$$
$$+\int \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega')\frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega'))\frac{m}{(1+r+\phi)^2} \right\} P(\omega'|\omega)d\omega', \tag{A.2}$$

where, per standard fixed mortgage payment algebra:

$$\begin{aligned} m &= b_0(1+r^Y) - b_1 && \text{(A.3)} \\ \text{and } m &= b_1(1+r^Y) && \text{(A.4)} \end{aligned}$$

Note that to economize on notation we do not make explicit the fact that b_0, b_1 and r^Y are functions of the agent's state. In this context:

$$\text{Net expected profits on agent of type } \omega \text{ at origination } \equiv W(\omega) - b_0. \tag{A.5}$$

Plugging expression (A.2,A.3,A.4) into (A.5) gives:

$$
\begin{aligned}
W(\omega) - b_0 = {} & \frac{m}{1+r+\phi} + \\
& \int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega') \frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{m}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega' \quad - \quad b_0 \\
= {} & \frac{b_0(1+r^Y) - b_1}{1+r+\phi} + \\
& \int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1))}{1+r+\phi} + S(\omega') \frac{b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{b_1(1+r^Y)}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega' \quad - \quad b_0 \\
= {} & \frac{b_0(r^Y - (r+\phi))}{1+r+\phi} + \\
& \int_{\omega'} \left\{ D(\omega') \frac{\max((1-\chi)qh(\omega'), b(1)) - b(1)}{1+r+\phi} + (1 - D(\omega') - S(\omega')) \frac{b_1(r^Y - (r+\phi))}{(1+r+\phi)^2} \right\} P(\omega'|\omega) d\omega'
\end{aligned}
$$

The last equality uses the fact that $b(1) = b(1)[D(\omega') + S(\omega') + (1 - D(\omega') - S(\omega'))]$ for all ω'. Integrating it over all possible ω such that $n(\omega) = 0$ yields:

$$
\begin{aligned}
& \text{Aggregate intermediary profits on mortgages in steady state} \ \equiv \\
& \int b(\omega) \left(1 - D^I(\omega) - D^V(\omega) - S(\omega)\right) \frac{(r^{\zeta}(\omega) - (r+\phi))}{(1+r+\phi)^{n(\omega)+1}} d\mu_M(\omega) \\
& - \int (b(\omega) - \min\{(1-\chi)qh(\omega), b(\omega)\}) \left(D^I(\omega) + D^V(\omega)\right) \frac{(1+r+\phi)}{(1+r+\phi)^{n(\omega)+1}} d\mu_M(\omega) \qquad \text{(A.6)}
\end{aligned}
$$

after observing, first, that $D(\omega) + S(\omega) = 0$ when $n(\omega) = 0$ since we do not allow agents to sell or default in the first period of the mortgage and, second, that for any integrable function $g : \Omega_m \mapsto I\!R$,

$$
\int_{\Omega_M} \left(\int_{\{\omega' \in \Omega_M | n(\omega') = 1\}} g(\omega') P(\omega'|\omega) d\omega' \right) d\mu_M(\omega) = \int_{\{\omega' \in \Omega_M | n(\omega') = 1\}} g(\omega') d\mu_M(\omega').
$$

This last expression says that the mass of agents who reach a given node is the probability of reaching that node from a given ω at origination. In other words, integrating the expected present value expression over all possible origination state amounts to computing a cross-sectional average in steady state. Expression A.6 thus gives the intermediary's aggregate profits on its mortgage activities in steady state. While the argument in this appendix has assumed $T = 2$, it extends unchanged to the general case.

The first integral in the profit expression gives the net return on active mortgages that are not terminated in the current period, while the second term is the cost (direct capital losses and opportunity cost) associated with the capital lost in the event of foreclosure.

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