Consumer Default Effects of Capital Income Tax Changes

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Very Preliminary draft

October 25, 2013

Abstract

We investigate the effects of reducing capital taxes in an economy in which households can borrow by means of unsecured loans with a competitive financial intermediary that intermediate among households. Idiosyncratic uncertainty is introduced in the form of both uninsurable labor productivity shocks and large expense shocks. Households have the option to declare bankruptcy. By computing the transition dynamics implicit in the policy changes, we show that the effects of changing the tax structure on defaults and welfare of consumers are first order. Changes in the tax structure also have major effects on the social support of such measures.

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1 Introduction

This paper studies the effects of a reduction in capital income taxes in a general equilibrium model with unsecured consumer credit with the option of default calibrated to US data. We develop a four-sector model (the banking sector, the production sector, the government sector and the household sector) inhabited by a continuum of infinitely-lived households who are *ex-ante* identical. Households are subject to uninsurable idiosyncratic risk in the form of shocks to their labor productivity. Moreover, they might also face expense shocks, i.e. exogenous shocks directly affecting the household's asset position, such as a large divorce bill. Idiosyncratic shocks display some persistence. There are no markets for insurance. Each period, households decide how much to consume and how much to borrow or save. Households borrow and save by means of unsecured loanable bonds traded with a competitive financial intermediary that intermediate among households. Households have the option to default on their debt. In the spirit of Chapter 7 bankruptcy rules, we assume that households can accumulate wealth during the period of bad credit score but cannot borrow. The production sector is characterized by a technology with constant returns to scale. The produced good can be used for consumption or investment. In the benchmark economy, we assume that the government levies capital and labour income taxes to finance its expenditures.

We examine weather introducing consumers' bankruptcy has quantitative relevant implications for the welfare and redistribution effects of capital income tax cuts. We study the effects of permanent unanticipated changes in the capital tax rate. First, we investigate the effects of an elimination of the tax on capital that translates into a reduction in taxes revenues. Second, we study the effects of revenue equivalent tax policies such that a reduction in the capital income tax also implies an adjustment in the labor tax rate. Both long run steady state and transition dynamics are computed.

This paper provides several insightful results. First, we show that the expected welfare gains vary dramatically across the population. Second, for each tax reform we compare the expected welfare gains across all households in the absence of transfers with the case in which revenues that are rebated to households as lump-sum transfers. In this latter case tax changes imply substantial redistribution across households. Further, we show that the presence of default strengthens the case for taxation.

Our paper is related to two strands of the macroeconomic literature. Several recent papers study the implications of unsecured credit markets and default for consumption smoothing, e.g. Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), Livshits, MacGee, and Tertilt (2007) and Athreya, Tam, and Young (2012).

A number of recent papers also investigate the welfare implications of distortionary taxes in heterogeneous agents models. Conesa, Kitao, and Krueger (2009) show that in a life-cycle model the optimal income tax that maximizes the ex-ante expected lifetime utility of the newborn is positive.

The paper is organized as follows. Section 2 presents the model and section 3 describes the equilibrium.

Section 4 comments on the calibration of the model. Section 5 presents the allocative and welfare results of revenue equivalent taxes. Finally, section 6 concludes.

2 The Model

We consider a three-sector economy - the banking sector, the production sector, and the household sector - inhabited by a continuum of infinitely-lived households who are *ex-ante* identical. Households face an idiosyncratic shock to their labor productivity but there is no aggregate uncertainty. Moreover, they also might face an "expense shock", i.e. an exogenous shock directly affecting the household's asset position. This is typically a large expense that the household must incur in order to avoid a very large loss in utility; we can think of it, for instance, as a divorce. Banks intermediate among households. Households can default on their debt, in which case they cannot borrow as long as they have a bad credit score. In the spirit of Chapter 7 bankruptcy rules, we assume that households can accumulate wealth during that period. The production sector is characterized by a technology with constant returns to scale. The produced good can be used for consumption or investment.

Time is discrete and indexed by $t = 1, 2, ...$ The timing of events in the economy is the following. (i) The exogenous idiosyncratic shocks are revealed. (ii) Capital and labor are rented, production takes place, and factors are remunerated. (iii) The households decide whether to default. If they default, all debts are discharged (including the expense shock, if any), all assets are liquidated, and no saving is possible. (iv) If it does not default, the household pays the expense shock (if any) and decides how much to lend or borrow. Consumption takes place.

Below we describe the economic environment in detail.

2.1 The production sector

There is a representative firm that at any time period t converts capital, K_t, and labor, N_t, into output Y_t, accordingly to the following constant returns to scale production technology:

$$Y_t = AK_t^{\alpha} N_t^{1-\alpha},$$

where α corresponds to the capital income share and is a number between zero and one and A is positive and defines the scale of production. Capital depreciates at rate $\delta \in (0, 1)$ in each period. Competition in the production sector implies that inputs are paid accordingly to their marginal productivity. Let w_t

and r_t be the wage rate and the risk free interest rate in period t, respectively. We have that:

$$w_t = (1-\alpha)AK_t^{\alpha}N_t^{-\alpha}$$
$$r_t = \alpha AK_t^{\alpha-1}N_t^{1-\alpha} - \delta\,.$$

2.2 The household sector

Each household has preferences defined over consumption, c_t, given by the following utility function:

$$\mathbf{E}_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]\,,\quad \beta \in (0,1)\,. \tag{1}$$

The period utility function is given by

$$u(c) = \frac{c^{1-\sigma}-1}{1-\sigma}\,.$$

Households have one unit of productive time in each period and supply it inelastically to the firm in the production sector. Households face idiosyncratic shocks on labor productivity, and are hit by expense shocks. A household with productivity shock z_t receives labor income $w_t z_t$. We assume that z_t follows a finite state Markov process with support $\mathcal{Z}$ and transition probability $\mathcal{P}(z,z') = \Pr(z_{t+1} = z'|z_t = z)$. Expense shocks e_t also follow a finite state Markov process with support $\mathcal{E}$ and have a transition probability given by $\mathcal{P}(e,e') = \Pr(e_{t+1} = e')$, where we have assumed that these shocks are uncorrelated over time. Both Markov chains generating z_t and e_t have just one ergodic set, no transient states and no cyclically moving subsets. Households can make deposits and get loans from financial intermediaries. We now detail the intermediation process in this economy.

2.2.1 The household's problem

A loan is a promise made by the borrower in period t to pay back $-a_{t+1} > 0$ to the bank in period $t+1$, against the immediate delivery by the bank to the household of $q_{a_{t+1},z_t}\cdot(-a_{t+1})$ units of the final good. A deposit is a promise made by the bank to deliver $a_{t+1} > 0$ units of the final good in period $t+1$ against a deposit by the household of $q_{a_{t+1},z_t}a_{t+1} > 0$ units of the final good during period t. We have that $a_{t+1} \in \mathcal{A} \equiv [-\underline{b},\overline{a}]$ and assume that $-\overline{b}$ is a large negative number. We also assume a large upper bound on assets, $\overline{a}$.[1] The exogenous shocks affecting the household are observable by all agents. The implicit discount rate $q_{a_{t+1},z_t} > 0$ is a function of the loan/deposit amount, household's credit score and the current household productivity. Let $x_t = (a_t, z_t, e_t)$ denote the vector of these three state variables for a particular household.

We allow households to default if they choose to and we assume that defaults follow the rules laid down in Chapter 7 of the U.S. Bankruptcy Code. When declaring bankruptcy, all the household's debt

[1]In the quantitative experiments these numbers are large enough to not constrain the solutions.

is discharged (that is, $a_t - e_t = 0$). When the household defaults or already has a tainted credit status, with probability $(1-\eta) \in (0,1)$ it starts next period with a good credit score; otherwise, it keeps the bad credit status. There is a pecuniary wage loss of $(1-\gamma)$ associated with default and with a bad credit score. Therefore, during the default period and the bad credit score state the household receives $\gamma(1-\tau_{w,t})w_t z_t$ of wage income. See Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Athreya, Tam, and Young (2012) for a similar formulation.

Intuitively, the asset decision a_{t+1} is made as follows: At the beginning of period t a household with access to credit and real asset holdings a_t observes its productivity shock z_t and expense shock e_t, rents labor and receives labor income $w_t z_t$. If $a_t - e_t < 0$, the household decides whether to default or not. By defaulting, it will be banned from borrowing during the bankruptcy period (i.e., $a_{t+1} \geq 0$), but all its debts are discharged (that is, $a_t - e_t = 0$); otherwise, the household either asks for a loan to roll over its debt, in which case $a_{t+1} < 0$, or repays its debt and makes a deposit, which corresponds to $a_{t+1} \geq 0$. If $a_t - e_t \geq 0$, there is no default decision; the household either asks for a loan ($a_{t+1} < 0$) or maintains a positive asset position ($a_{t+1} \geq 0$). The problem of each household is formally described below.

Firstly, let us define the set of possible individual states as $\mathcal{U} = \mathcal{X} \times \{0,1\}$, where $\mathcal{X} = \mathcal{A} \times \mathcal{Z} \times \mathcal{E}$. The last set of the product in $\mathcal{U}$'s definition characterizes each household in terms of credit record status, where 0 corresponds to households with a good credit record and 1 corresponds to households with a bad credit record. Let Υ be the associated Borel σ-algebra. For each $B \in \Upsilon$, $\lambda(B)$ corresponds to the mass of households whose individual state vectors lie in B. The household's value function depends not only on the current idiosyncratic state, but also on aggregate measures such as the wage, the deposit interest rate, and the state contingent loan rates. To compute these measures in the next period, the household needs to know the current period's entire measure λ_t, and an aggregate law of motion, which we will call Λ_t, such that $\lambda_{t+1} = \Lambda_t(\lambda_t)$.

Let $V_t(x_t, \lambda_t)$ and $V_t^D(x_t, s_t, \lambda_t)$ correspond to the value of repaying the debt or declaring bankruptcy with state vector $x_t \in \mathcal{X}$ and $s_t \in \{0,1\}$ is the household's current credit status. Let $W_t(x_t, s_t, \lambda_t)$ be the current value of the problem with the option to default. The bankruptcy decision of a household with a credit score $s_t = 0$ is made by choosing $d_t \in \{0,1\}$ to maximize:

$$W_t(x_t, 0, \lambda_t) = \max_{d_t \in \{0,1\}} \{(1-d_t)V_t(x_t, \lambda_t) + d_t V_t^D(x_t, 0, \lambda_t)\}. \tag{2}$$

When $V_t(x_t, \lambda_t) \geq V_t^D(x_t, 0, \lambda_t)$ then $d_t = 0$; otherwise we have that $d_t = 1$. Therefore, this problem defines optimal policy function $h_{d,t}(x_t, s_t, \lambda_t)$ and it is understood that h_d is one whenever $s_t = 1$.

The value of repaying the debt with a good credit record ($s_t = 0$) is summarized by the following value function:

$$V_t(x_t, \lambda_t) = \max_{c_t \geq 0, a_{t+1} \in \mathcal{A}} \{u(c_t) + \beta E_t[W_{t+1}(x_{t+1}, s_{t+1}, \lambda_{t+1})]\}, \tag{3}$$

subject to the aggregate law of motion $\lambda_{t+1} = \Lambda_t(\lambda_t)$ and

$$c_t + (q_{a_{t+1},z_t} + (1 - q_{a_{t+1},z_t})\tau_{k,t}\mathcal{I}_{a_{t+1}>0})a_{t+1} \leq a_t - e_t + (1 - \tau_{w,t})w_t z_t + m_t, \tag{4}$$

where τ_k and τ_w are the capital income and the labor income tax rates, and m_t are lump-sum transfers from the government to the households in period t; $\mathcal{I}_{(\cdot)}$ is the indicator function. Equation (3) states that the household with a good credit record chooses consumption and next period asset value in order to maximize current utility and the continuation value of utility which depends on wether or not the household declares bankruptcy in the future. Equation (4) shows that consumption plus the present value of future asset holdings and the expense shock should be less or equal to the sum of current net wealth and labor income. This problem defines policy functions $h_{c,t}(x_t, 0, \lambda_t)$ and $h_{a,t}(x_t, 0, \lambda_t)$.

The value of declaring bankruptcy, which requires $s_t = 0$, is given by:

$$V_t^D(x_t, 0, \lambda_t) = u(c_t) + \beta E_t[\eta V_{t+1}^D(x_t, 1, \lambda_{t+1}) + (1 - \eta)W_{t+1}(x_{t+1}, 0, \lambda_{t+1})],$$

subject to the aggregate law of motion $\lambda_{t+1} = \Lambda_t(\lambda_t)$ and

$$c_t \leq \gamma(1 - \tau_{w,t})w_t z_t + m_t. \tag{5}$$

Equation (5) implies that debts are discharged under bankruptcy, but the household cannot save or borrow during this state. The value of repaying the debt with a bad credit record ($s_t = 1$) is:

$$V_t^D(x_t, 1, \lambda_t) = \max_{c_t \geq 0, a_{t+1} \geq 0} \{u(c_t) + \beta E_t[\eta V_{t+1}^D(x_t, 1, \lambda_{t+1}) + (1 - \eta)W_{t+1}(x_t, 0, \lambda_{t+1})]\},$$

subject to the aggregate law of motion $\lambda_{t+1} = \Lambda_t(\lambda_t)$ and

$$c_t + (q_{a_{t+1},z_t} + (1 - q_{a_{t+1},z_t})\tau_{k,t})a_{t+1} \leq \max\{a_t - e_t, 0\} + \gamma(1 - \tau_{w,t})w_t z_t + m_t. \tag{6}$$

Associated to this problem are policy functions for consumption $h_{c,t}(x_t, s_t, \lambda_t)$ and asset holdings $h_{a,t}(x_t, s_t, \lambda_t)$.

2.3 Government

Government spending G_t, which is considered to be exogenous, is financed through taxes on labor and capital income, where revenues are given by

$$\tau_w w_t N_t + \int_{a_{t+1}>0} \left(1 - q_{a_{t+1},z_t}\right) \tau_{k,t} a_{t+1} d\lambda_t \tag{7}$$

where $a_{t+1} = h_{a,t}(x_t, s_t, \lambda_t)$. In the numerical exercises we will assume that at time zero the government budget is balanced and transfers are zero. We then consider permanent changes in tax rates under two assumptions:

i Tax changes are revenue-neutral and there are no transfers, so that the present value of (7) is equal to the present value of government spending G_t.

ii Tax changes are not revenue-neutral, but the government keeps a balanced budget in all periods by transferring fiscal surpluses to the households, so that $G_t + m_t$ equals (7) in all periods.

2.4 The banking sector

We assume free entry in the banking sector and there are no intermediation costs. Therefore, any bank has zero profits in loans to agents of the same type. This implies that there is no cross-subsidization in loans to households. See Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for a discussion. Given that the banks' payoffs are affine in the decision variables (see below) and there is free entry, we assume without loss of generality that there is only one bank which holds all capital. Let $A_{a_0,z_{-1}}$ denote the amount of type (a_0, z_{-1}) contracts maturing in $t = 0$. Given the initial portfolio of previous contracts, $\{A_{a_0,z_{-1}}\}_{(a_0,z_{-1})\in\mathcal{A}\times\mathcal{Z}}$, the initial amount of capital K_0, and the deposit and loan discount rates q_{a_{t+1},z_t}, the financial intermediary chooses the amount A_{a_{t+1},z_t} of type (a_{t+1}, z_t) contracts and the amount of capital K_{t+1} to hold, in order to maximize the present value of current and future cash flows, discounted at the risk free interest rate $\{i_t\}_{t=0,\ldots,\infty}$,

$$\sum_{t=0}^{\infty} \frac{1}{\prod_{j=1}^{t}(1+i_j)}\ \pi_t \tag{8}$$

where

$$\pi_t = (1+r_t)K_t + D_{t+1} - L_{t+1} - K_{t+1} - D_t + L_t\,. \tag{9}$$

New extended loans and collected deposits are defined, respectively, as

$$D_{t+1} = \sum_{(a_{t+1},z_t)\in\mathcal{A}\times\mathcal{Z},\ a_{t+1}\geq 0} q_{a_{t+1},z_t} a_{t+1} A_{a_{t+1},z_t}$$

$$L_{t+1} = \sum_{(a_{t+1},z_t)\in\mathcal{A}\times\mathcal{Z},\ a_{t+1}<0} q_{a_{t+1},z_t}\left(-a_{t+1}\right) A_{a_{t+1},z_t}$$

while existing loans and deposits are given by

$$D_t = \sum_{(a_t,z_{t-1})\in\mathcal{A}\times\mathcal{Z},\ a_t\geq 0} a_t A_{a_t,z_{t-1}}$$

$$L_t = \sum_{(a_t,z_{t-1})\in\mathcal{A}\times\mathcal{Z},\ a_t<0} \left(1 - p_{a_t,z_{t-1}}\right)\left(-a_t\right) A_{a_t,z_{t-1}}\,.$$

Here, $p_{a_t,z_{t-1}}$ is the probability that a type (a_t, z_{t-1}) contract maturing in period t is defaulted upon. It is given by the fraction of households that, in the current period, suffer idiosyncratic shocks such that they choose to default. As discussed previously $p_{a_t,z_{t-1}} = 0$ whenever $a_t - e_t \geq 0$. Any sequence of deposits/loans and capital $\{A_{a_{t+1},z_t}, K_{t+1}\}_{z_t \in \mathcal{Z}, t=0,\ldots,\infty}$ implies a sequence of risk-free bond holdings $\{B_t\}_{t=0,\ldots,\infty}$ by the bank which satisfies $B_0 = 0$ and

$$B_{t+1} = (1 + i_t)B_t + \pi_t \, . \tag{10}$$

The bank's first order conditions imply that, whenever $A_{a_{t+1},z_t} > 0$ and $K_{t+1} > 0$,

$$q_{a_{t+1},z_t} = \begin{cases} \frac{1}{1+r_{t+1}} & \text{if } a_{t+1} \geq 0, \\ \frac{1-p_{a_{t+1},z_t}}{1+r_{t+1}} & \text{if } a_{t+1} < 0. \end{cases}$$

The first equation implies that investors are indifferent between holding capital, making deposits or holding risk free bonds issued by the bank. By arbitrage we also have that $i_t = r_t$. The second equation states that the loan interest rate increases with the probability of default, given the risk free discount rate. Thus, risk premium emerges endogenously as a response to defaults.[2]

2.4.1 Probabilities of default and transfers

At the end of period t a type x_t household decides to repay or default. If the household repays, it sets an asset position a_{t+1} for the next period. The default rate is the fraction of type z_t households that, at the end of the next period, will default. The bank can compute this probability as

$$p_{a_{t+1},z_t} = \sum_{\xi_{t+1} \in \mathcal{Z} \times \mathcal{E}} \mathcal{P}(z_t, z_{t+1}) \mathcal{P}(e_t, e_{t+1}) h_{d,t+1}(a_{t+1}, \xi_{t+1}, s_{t+1}, \Lambda_t(\lambda_t)) \, .$$

The probability of default is an endogenous quantity that depends on the different parameters of the model. The way it depends on each is a complicated matter that only numerical experiments can clarify.

2.5 The services providers

We assume that expenses shocks, e, go to a services providers sector, such as courts, which provides legal services. If a household does not default, then services providers receive the expense shock e_t. If a household defaults, then services providers receive nothing if the household net wealth is negative, but receive a_t when this is positive. Therefore, in order to ensure zero profits in this sector we assume that

[2] In the present set up we neglect intermediation costs so that the spread between loans and deposits is entirely due to credit risk, as in Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007). See Livshits, MacGee, and Tertilt (2007) and Athreya, Tam, and Young (2012) for alternative formulations.

services providers charge a markup n_t such that

$$\int_{\mathcal{U}} [(1 - h_{d,t}(x_t, s_t, \lambda_t))e_t + h_{d,t}(x_t, s_t, \lambda_t) \max\{0, a_t\}] d\lambda_t = \int_{\mathcal{U}} \frac{e_t}{n_t} d\lambda_t. \tag{11}$$

3 Equilibrium

Observe first that, given $h_{d,t} = h_{d,t}(x_t, s_t, \lambda_t)$, there is an endogenous transition probability from the current credit score to the future credit score that can be defined by

$$\mathcal{P}(s_t, s_{t+1}; h_{d,t}) = \begin{cases} 1 & \text{if } s_t = 0 \text{ and } s_{t+1} = 0 \text{ and } h_{d,t} = 0 \\ 0 & \text{if } s_t = 0 \text{ and } s_{t+1} = 1 \text{ and } h_{d,t} = 0 \\ 0 & \text{if } s_t = 0 \text{ and } s_{t+1} = 0 \text{ and } h_{d,t} = 1 \\ 1 & \text{if } s_t = 0 \text{ and } s_{t+1} = 1 \text{ and } h_{d,t} = 1 \\ \eta & \text{if } s_t = 1 \text{ and } s_{t+1} = 1 \\ 1-\eta & \text{if } s_t = 1 \text{ and } s_{t+1} = 0 \ . \end{cases}$$

Let $Q_t(x_t, s_t, \lambda_t, C; h_{a,t}, h_{d,t})$ be the endogenous transition probability of the households' state vector. It describes the probability that a household with state (x_t, s_t) will have a state vector lying in $C \in \Upsilon$ next period, given the current asset distribution λ_t and policy functions $h_{a,t}$ and $h_{d,t}$. Therefore,

$$Q_t(x_t, s_t, \lambda_t, C; h_{a,t}, h_{d,t}) = \sum_{(x_{t+1}, s_{t+1}) \in \mathcal{U} : (h_{a,t}, \xi_{t+1}, s_{t+1}) \in C} \mathcal{P}(z_t, z_{t+1}) \mathcal{P}(e_t, e_{t+1}) \mathcal{P}(s_t, s_{t+1}; h_{d,t}) \, .$$

The aggregate law of motion implied by transition function Q_t is an object $\Lambda_t(\lambda_t, Q_t)$ that assigns a measure to each Borel set C. It can be computed as

$$\Lambda_t(\lambda_t, Q_t)(C) = \int_{\mathcal{U}} Q_t(x_t, s_t, \lambda_t, C; h_{a,t}, h_{d,t}) d\lambda_t \, . \tag{12}$$

We are now in a position to define the competitive equilibrium for this economy.

Definition 1 (Competitive equilibrium) *Given the initial aggregate capital, K_0, measure of asset holdings, λ_0, bank bond holdings, $B_0 = 0$, and a pair of constant tax rates, $\{\tau_w, \tau_k\}$, a competitive equilibrium is: a set of strictly positive paths for prices, $\{w_t, r_t, i_t\}_{t=0,\dots,\infty}$; a set of nonnegative paths for loan and deposit rates, and default probabilities, $\{q_{a_{t+1},z_t}, p_{a_{t+1},z_t}\}_{(a_{t+1},z_t)\in\mathcal{A}\times\mathcal{Z},\, t=0,\dots,\infty}$; a nonnegative path for the services providers markup, $\{n_t\}_{t=0,\dots,\infty}$; a set of strictly positive paths for aggregate capital and labor,*

$\{K_t, N_t\}_{t=0,\dots,\infty}$; a nonnegative path for contract quantities, $\{A_{a_{t+1},z_t}\}_{(a_{t+1},z_t)\in\mathcal{A}\times\mathcal{Z},\, t=0,\dots,\infty}$; a path for bank bond holdings, $\{B_t\}_{t=1,\dots,\infty}$; a set of decision rules, $\{h_{a,t}, h_{c,t}, h_{d,t}\}_{t=0,\dots,\infty}$; and a path for the

probability measure, $\{\lambda_t\}_{t=1,\dots,\infty}$, such that, in every period:

1. *The decision rules $h_{a,t}$, $h_{c,t}$ and $h_{d,t}$ solve the households' optimization problem.*
2. *The aggregate capital K_t and labor N_t inputs solve the optimization problem of the firm.*
3. *Aggregate capital K_{t+1} and number of contracts A_{a_{t+1},z_t} solve the bank's optimization problem.*
4. *The rates of default p_{a_{t+1},z_t} are consistent with the household's default decision rule $h_{d,t}$.*
5. *The services provider markup n_t ensures zero profits, such that (11) is satisfied.*
6. *The labor market clears, $N_t = \int_X z_t d\lambda_t$.*
7. *The credit market clears, $\int_{\mathcal{U}} \mathcal{I}_{\{h_{a,t}(x_t,s_t,\lambda_t)=a_{t+1}\}} d\lambda_t = A_{a_{t+1},z_t}$ for all a_{t+1} and z_t.*
8. *The bond market clears, $B_{t+1} = 0$.*
9. *The goods market clears, $AK_t^{\alpha} N_t^{1-\alpha} + (1-\delta)K_t = \int_{\mathcal{U}} h_{c,t}(x_t,s_t,\lambda_t)d\lambda_t + \int_{\mathcal{U}} h_{a,t}(x,s_t,\lambda_t)d\lambda_t + \gamma w_t \int_{s_t=1} z_t d\lambda_t + \int_{\mathcal{U}} \frac{e_t}{n_t} d\lambda_t$.*
10. *The aggregate law of motion implied by the individual decision rules, $T_t(\lambda_t, Q_t)$, is consistent with the household's aggregate forecasting rule, Λ_t, that is, for every Borel set C, the measure generated by the aggregate motion equation, $\lambda^*_{t+1}(C) = T_t(\lambda_t, Q_t)(C)$, is equal to the measure associated with the aggregate forecasting rule, $\lambda_{t+1}(C)$, where $\lambda_{t+1} = \Lambda_t(\lambda_t)$.*

A stationary equilibrium is an equilibrium where prices and aggregate quantities are stationary over time.

4 Quantitative Experiments

We now calibrate the model, present the steady-state allocation, and, using the baseline steady state as a starting point, perform various experiments featuring changes in the tax policy; in all cases we compute not only the new steady state but also the transition.

4.1 Calibration

We calibrate the model such that in the long run it matches some recent key empirical observations in the United States. There are three sets of parameters, each one related to different blocks of the model. The first set corresponds to the parameters of the production of the final consumption good; the second set contains the parameters related to preferences and idiosyncratic characteristics of households; and the third set of parameters corresponds to the institutional aspects of financial intermediation, the legal system, and tax policy. We summarize the parameters in table 1 below, with a brief discussion of the

Table 1: Parameter values, baseline economy.

Parameters	Values	Comment/Observations
A	0.5613	Equilibrium aggregate production is equal to 1
σ	2	Risk aversion coefficient based on micro evidence reported by Mehra and Prescott (1985)
α	0.30	Capital income share based on estimates by Gollin (2002)
β	0.9456	Real interest rate on risk free asset of 4%
δ	0.06	Capital to output ratio, $\frac{K}{Y} = 3$
ρ	0.98	Persistence parameter used by Krueger and Perri (2005)
σ_ϵ^2	0.0285	Cross-sectional variance of shocks based on Krueger and Perri (2005)
γ	0.71	One minus pecuniary penalty for bad credit. Calibrated to match the debt to income ratio from Athreya, Tam, and Young (2012)
η	0.1	Matches the bankruptcy filing stay in official record for 10 years; see Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007)
τ_k	0.397	Capital income tax from Domeij and Heathcote (2004)
τ_w	0.269	Labor income tax from Domeij and Heathcote (2004)

procedure used to determine each parameter. In the baseline calibration we set the consumption tax rate τ_c and transfers m_t to zero. The model is calibrated to yearly frequency.

Production. The production function is Cobb-Douglas with a share of capital equal to 30 percent, which is a value consistent to the estimates in Gollin (2002). The scale parameter A is such that the equilibrium aggregate production is 1. The depreciation rate δ is 0.06, which implies that the capital to output ratio is 3.

Preferences and idiosyncratic shocks. There are only two parameters related to preferences. One is the inverse of the intertemporal elasticity of substitution, σ; we use a value of 2, which is a number reported in the literature (see, for instance, Mehra and Prescott, 1985). The discount factor β is chosen so that the equilibrium steady-state net interest rate is 4 percent per annum. The idiosyncratic labor productivity shocks are borrowed from Krueger and Perri (2005). Using data from the Consumer Expenditure Survey and controlling for several idiosyncratic characteristics, they report a cross-sectional variance of log wages of 0.719. They model these productivity shocks as an AR(1) process using an autocorrelation parameter of $\rho = 0.98$. We follow the same approach here using a Rouwenhorst discretization of the AR(1) process with 9 grid points; see Kopecky and Suen (2010) for details. The Markovian transition matrix $\mathcal{P}(z, z')$ and the vector with the values of $\mathcal{Z}$ are given by the values in table 2.

Institutional and expense shock parameters. We set η equal to 0.1 so that defaulting households have a bad credit record for, on average, ten years; see Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for a similar calibration. Regarding the expense shock, we assume a simple structure of only one level (coupled with the absence of any expense shock), serially uncorrelated. We calibrate the magnitude of the shock, its probability and penalty parameter γ to match key data moments: debt-to-output ratio,

Table 2: Vector with discrete values for idiosyncratic productivity and Markov matrix using Rouwenhorst's method of discretization, arranged so that the current state varies across rows and the next state varies across columns.

z	$\mathcal{P}(z, z')$								
0.0909	0.9227	0.0746	0.0026	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
0.1655	0.0093	0.9234	0.0653	0.0020	0.0000	0.0000	0.0000	0.0000	0.0000
0.3014	0.0001	0.0186	0.9239	0.0560	0.0014	0.0000	0.0000	0.0000	0.0000
0.5490	0.0000	0.0003	0.0280	0.9242	0.0466	0.0009	0.0000	0.0000	0.0000
1.0000	0.0000	0.0000	0.0006	0.0373	0.9243	0.0373	0.0006	0.0000	0.0000
1.8214	0.0000	0.0000	0.0000	0.0009	0.0466	0.9242	0.0280	0.0003	0.0000
3.3174	0.0000	0.0000	0.0000	0.0000	0.0014	0.0560	0.9239	0.0186	0.0001
6.0421	0.0000	0.0000	0.0000	0.0000	0.0000	0.0020	0.0653	0.9234	0.0093
11.0048	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001	0.0026	0.0746	0.9227

Table 3: Vector with discrete values for expense shocks and Markov matrix (Livshits, MacGee, and Tertilt, 2007), arranged so that the current state varies across rows and the next state varies across columns.

e	$\mathcal{P}(e, e')$	
0.0000	0.9244	0.0756
0.2830	0.9244	0.0756

the fraction of people in debt, and the default rate. Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Chatterjee and Gordon (2012) report a debt-to-output ratio of 0.36%, a percentage of households in debt of 3.6% and percentage of filers for bankruptcy of 0.29%. They exclude medical bill shocks for the baseline model; we follow the same approach but compared to Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) use a calibrated expenditure shock instead of a preference shock.[3] Table 3 presents the shock levels $\mathcal{E}$ and the Markov matrix; the calibrated value for γ was 0.2. Parameter γ corresponds to the fraction of work income that a defaulted household can actually utilize. In equilibrium the amount lost by the household is very small in aggregate terms. We treat it as a deadweight loss, although it could be rebated back to other agents in the model, namely banks. If the cost becomes very high, that is, if is close to 0, the level of punishment for default is so high that no one defaults, and we are back to Aiyagari's (1994) case, where borrowing is only constrained by the requirement that the household is always capable of rolling over its debt, even after a long succession of the worst possible idiosyncratic shock. This is associated with high levels of borrowing, both in terms of level of debt and number of debtors, and non-existent defaults. If γ is very close to 1, the pecuniary punishment for default is very mild, so any household with a sufficiently long stream of bad idiosyncratic shocks eventually defaults. A theoretical possibility is the case where parameters (especially the discount factor and the ban time) are such that the mere fact of not being able to borrow for a while after defaulting leads the household not to default with a large enough probability in some states. This might make certain credit contracts viable.

[3]A different method is used by Livshits, MacGee, and Tertilt (2007), who make direct measurements of medical bills, divorces and unplanned pregnancies and then calibrate positive levels of expenditure shocks and their respective probabilities for a three-year period.

Table 4: Summary statistics of the baseline steady state and comparison with actual values. All values in natural units.

	Model	Actual
Net real interest rate	0.04	0.04
Capital to output	3	3
Unsecured debt to output	0.0038	0.0036
Default rate	0.0035	0.0029
Fraction of borrowers	0.053	0.036
Average spread	0.0739	0.0838
Fraction of defaulted debt	0.0735	0.0481
Government spending	0.235	0.190

Fiscal policy parameters. We follow Domeij and Heathcote (2004) to calibrate the tax rates. These authors use $\tau_k = 0.397$ and $\tau_w = 0.269$ for the initial tax rates, which are calibrated to match the actual tax rates in the U.S. using the method of Mendoza, Razin, and Tesar (1994) for the 90s. Since consumption taxes have been historically very low in the U.S., we also follow them in considering $\tau_c = 0$. We assume that the Government budget is balanced in the baseline calibration.

4.2 The Baseline Steady State

Table 4 presents a summary of the steady-state allocation of the baseline economy. The first five rows show that the targets are met quite well. The fraction of borrowers is too high in the model, a common feature in this class of models. Other non-targeted statistics are also reported in the table. The average spread in the model is 7.39 pp, which is close to the actual level of 8.38 pp.[4] The fraction of defaulted debt relative to total debt in the model (7.35%) overestimates the actual number (4.8%). Government spending is not directly targeted but the implied value in the model is 23.5% of GDP, which is close to the U.S. average of 19% reported by Domeij and Heathcote (2004).

In the following we report the policy functions of the baseline economy, i.e. $\tau_k = \tau_w = 0$. Figure 1 shows an important feature of asset policy in this dynamic model: over time a borrower may become a saver and vice versa. The figure shows that if an agent suffers a negative productivity shock, for example $z = -2$, then the agent can become credit constrained as the "flat" region in the figure indicates. In contrast, a borrower with a good productivity shock, for example $z = 2$, will switch from borrowing to saving. Figure 2 illustrates the probability that a borrower will default on a loan in equilibrium in the baseline case of no expense shock ($e_t = 0$). The figure shows that default can occur abruptly.

[4] This was computed as the difference between average interest rates charged for non-secured credit and the 3-month T-bills rate over the period 1973–2012; source: tables G.19 and H.15 by the Board of Governors of the Federal Reserve System.

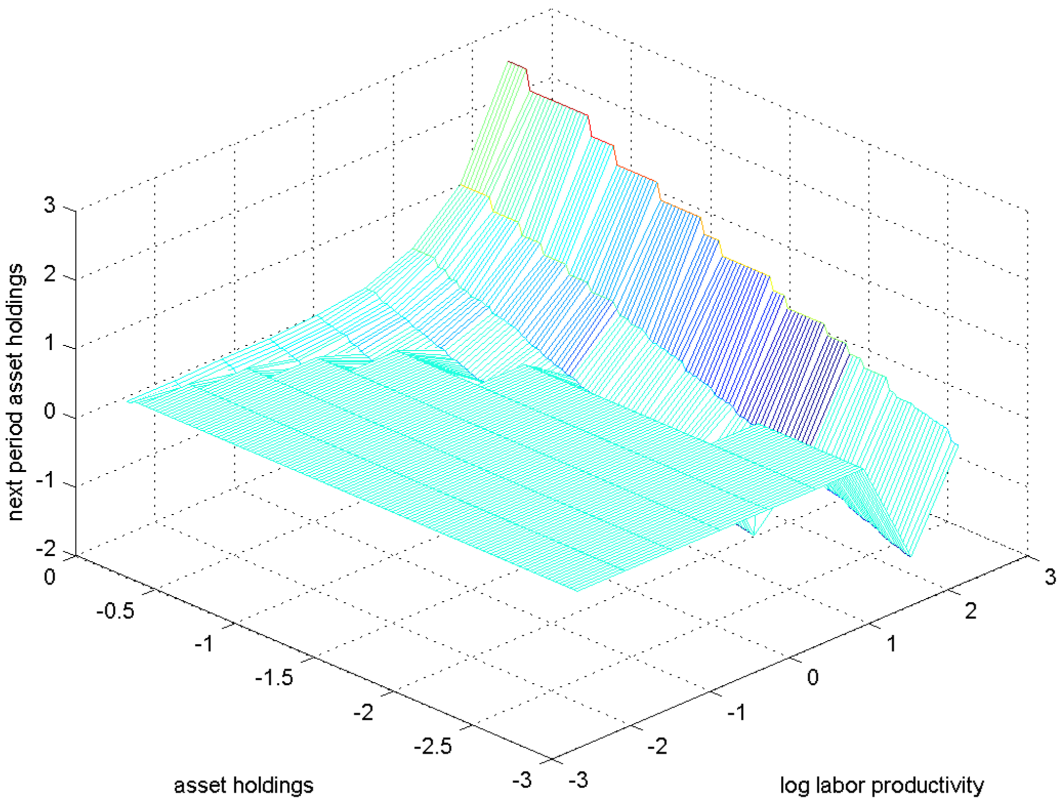

Figure 1: Asset policy function.

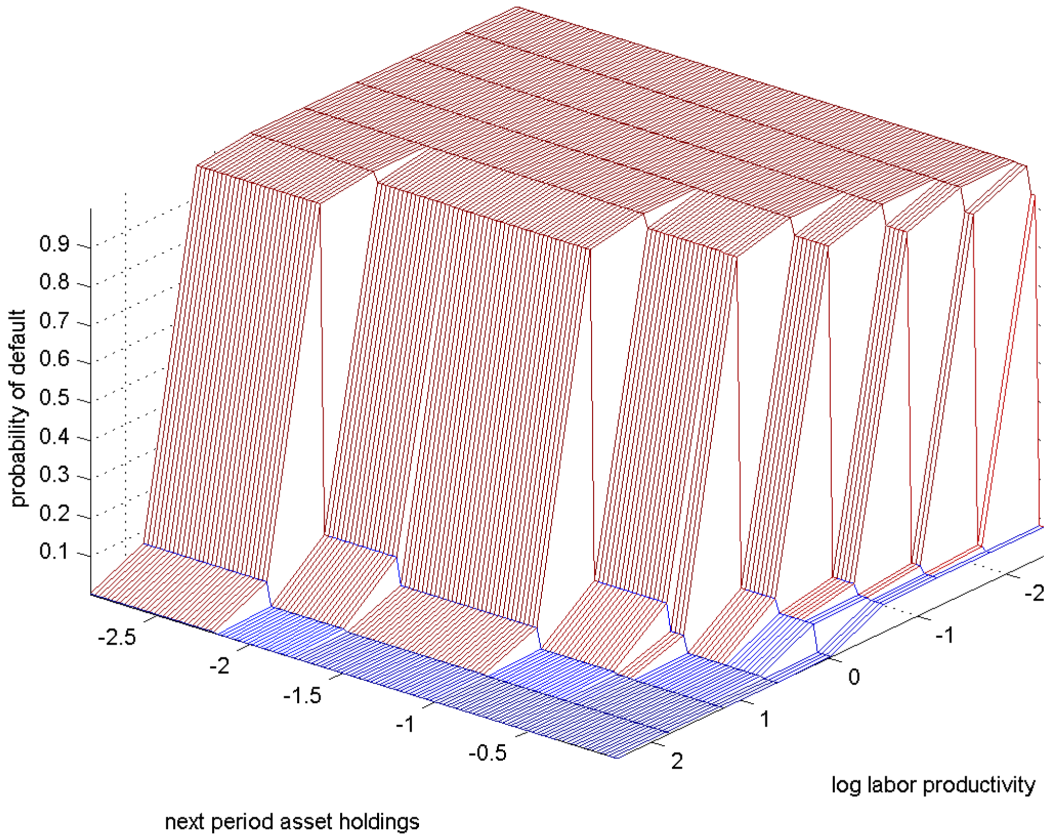

Figure 2: Equilibrium default.

Table 5: Steady-state summary statistics under different policies.

	Fraction of borrowers	Unsecured debt to output	Default rate	Average spread	Fraction of defaulted credit	Net real interest rate	Wage rate (baseline=100)	Consumption to output	Capital to output	Output (baseline=100)	Transfers to output
Baseline	5.3	0.38	0.35	7.39	7.35	4	100.0	56.1	300.0	100.0	-
No subsidy											
$\tau_w = 0.3197$	9.6	0.60	0.65	7.53	7.83	2.58	106.8	54.6	350.0	106.8	-
$\tau_c = 0.065$	10.6	0.64	0.72	7.52	7.65	2.61	106.6	54.6	348.0	106.6	-
Subsidy											
$\tau_w = 0.34$	14.1	0.98	1.06	8.35	8.57	2.77	105.8	55.1	342.0	105.8	1.62
$\tau_c = 0.09$	15.4	1.10	1.09	7.88	8.09	2.78	105.9	54.9	343.0	105.9	1.61

Table 6: Steady-state summary statistics of wealth and earnings distribution for different policy exercises. All values in percentage terms.

	Wealth distribution					
	Q1	Q2	Q3	Q4	Q5	Gini
Data	-0.20	1.30	5.0	12.2	81.7	80.0
Baseline	-0.14	0.48	6.2	20.7	72.7	72.8
No subsidy						
$\tau_w = 0.3197$	-0.17	1.46	9.7	25.1	64.0	65.7
$\tau_c = 0.065$	-0.18	0.90	7.0	22.3	70.0	70.0
Subsidy						
$\tau_w = 0.34$	-0.28	0.71	7.5	25.5	66.7	68.2
$\tau_c = 0.09$	-0.32	0.45	5.5	21.0	73.4	73.8

	Earnings distribution					
	Q1	Q2	Q3	Q4	Q5	Gini
Data	-	4.0	13.0	22.9	60.2	61.0
Model	4.8	8.9	14.0	23.4	48.8	44.0

4.3 Are Revenue Equivalent Capital Taxes also Welfare Equivalent?

Now, we compare the effects on allocations and welfare of a revenue neutral tax on capital, i.e. $\tau_w = 0$ and $\tau_k = 0.33$. Average wealth-to-output declines more than in the case of a pay-roll tax. However, since capital income is more unevenly distributed than labor income, the introduction of a tax on capital increases wealth inequality. In particular, earnings increase only for agents in the 5th quintile of the wealth distribution. The rest of the population suffers a reduction in wealth. In contrast, average consumption-to-output is less affected than under a pay-roll tax. (solid: change in payroll tax dashes: change in capital tax)

5 Changing Capital Tax: Asymmetric Effect?

6 Anticipated Capital Tax Changes

Table 7: Steady-state summary statistics of the consumption distribution under different policies.

		Consumption distribution					
		Q1	Q2	Q3	Q4	Q5	Overall
Baseline	Share of consump.	5.4	9.8	16.3	22.3	46.2	100.0
	Gini coeff.	28.6	23.9	22.2	22.9	25.7	44.9
No subsidy							
$\tau_w = 0.3197$	Share of consump.	5.1	10.8	18.3	19.7	46.1	100.0
	Gini coeff.	28.4	22.6	22.9	22.6	25.0	44.0
$\tau_c = 0.065$	Share of consump.	5.0	10.3	16.5	21.7	46.6	100.0
	Gini coeff.	29.0	24.0	23.5	21.7	24.7	44.7
Subsidy							
$\tau_w = 0.34$	Share of consump.	5.6	10.6	17.6	23.7	42.6	100.0
	Gini coeff.	27.1	23.3	22.9	23.1	29.4	43.0
$\tau_c = 0.09$	Share of consump.	5.6	9.8	15.7	22.7	46.2	100.0
	Gini coeff.	27.8	24.1	21.8	20.1	24.5	44.0

Table 8: Welfare effects of changing from baseline to different policies. Negative values denote welfare costs.

	Consumption equivalent w.r.t. baseline								Population in favor		
	Q1	Q2	Q3	Q4	Q5	Average	Borrowers	Savers	All	Borrowers	Savers
No subsidy											
$\tau_w = 0.3197$	-3.43	-3.15	-1.60	1.31	4.13	-0.55	-3.48	0.67	35.0	0.0	37.0
$\tau_c = 0.065$	-2.18	-1.91	-0.95	0.69	2.20	-0.43	-2.02	0.31	34.2	0.0	36.1
Subsidy											
$\tau_w = 0.34$	4.18	0.65	0.86	3.08	5.10	2.77	3.97	2.62	75.6	65.1	76.2
$\tau_c = 0.09$	4.21	1.55	1.32	2.07	2.50	2.33	4.25	1.88	95.5	97.1	95.4

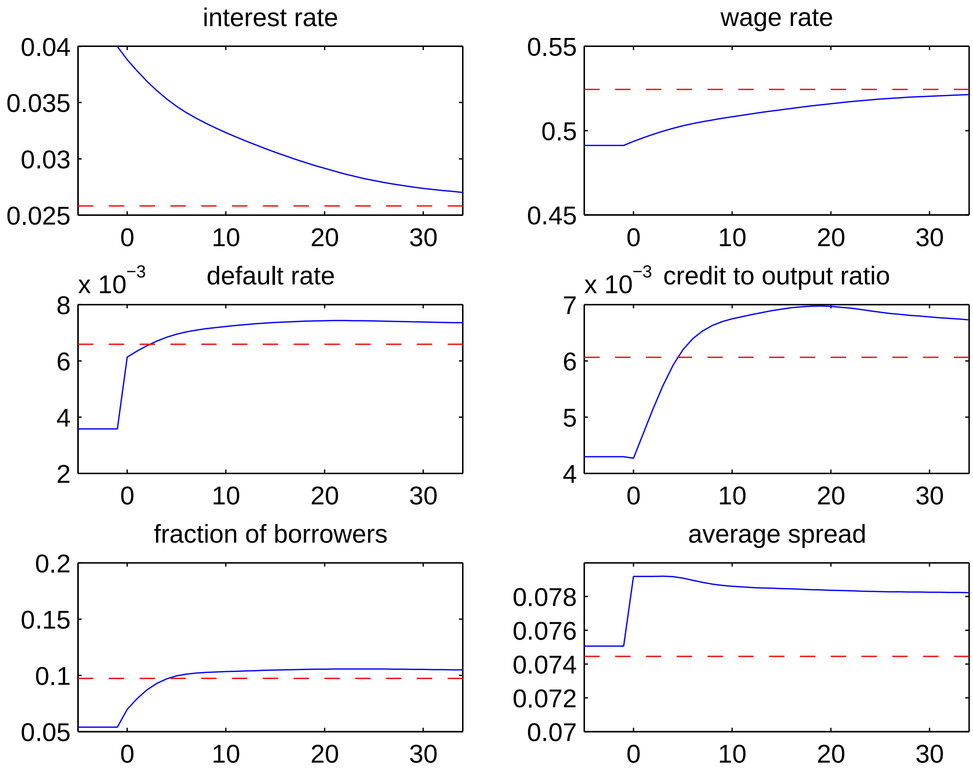

Figure 3: Transition with $\tau_w = 0.3197$ and no transfers.

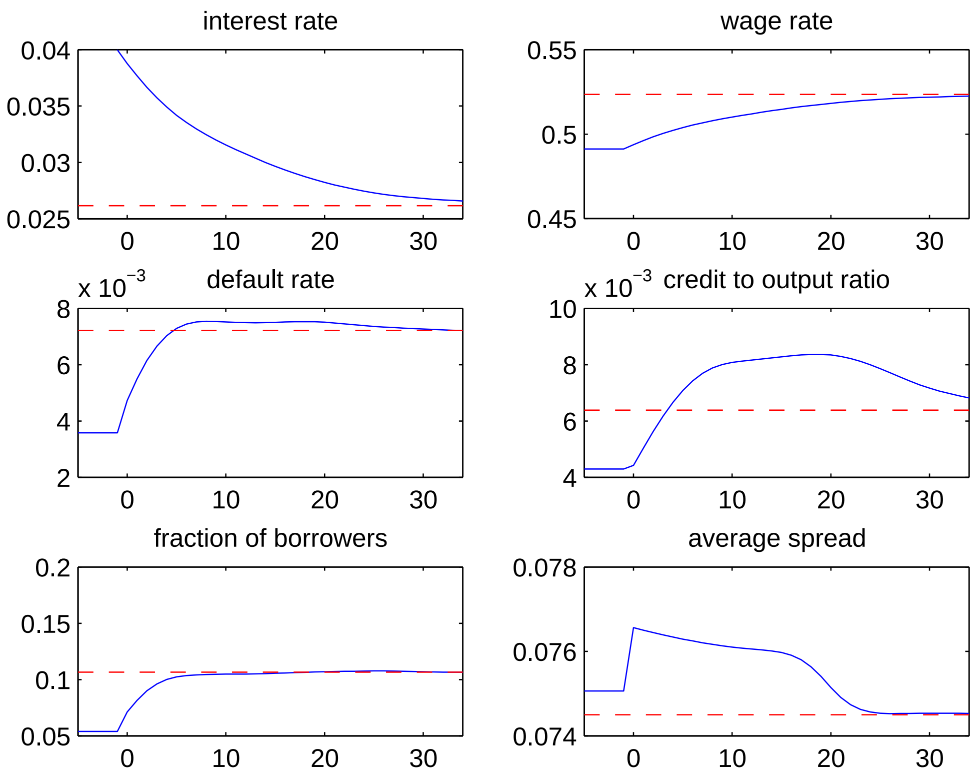

Figure 4: Transition with $\tau_c = 0.065$ and no transfers.

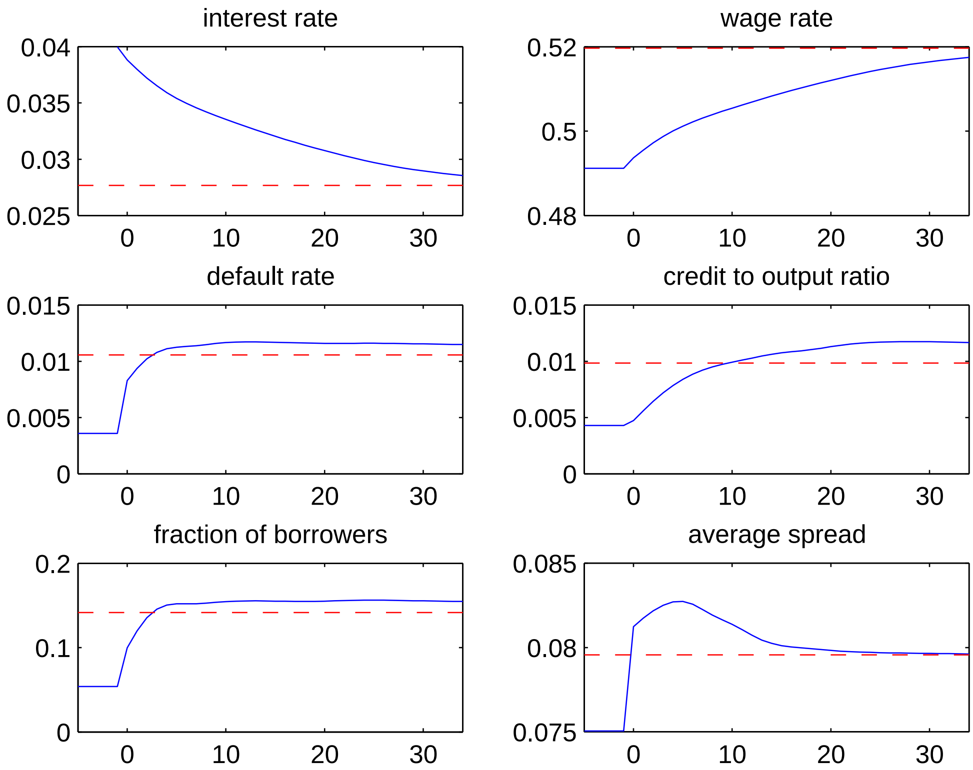

Figure 5: Transition with $\tau_w = 0.34$ and transfers.

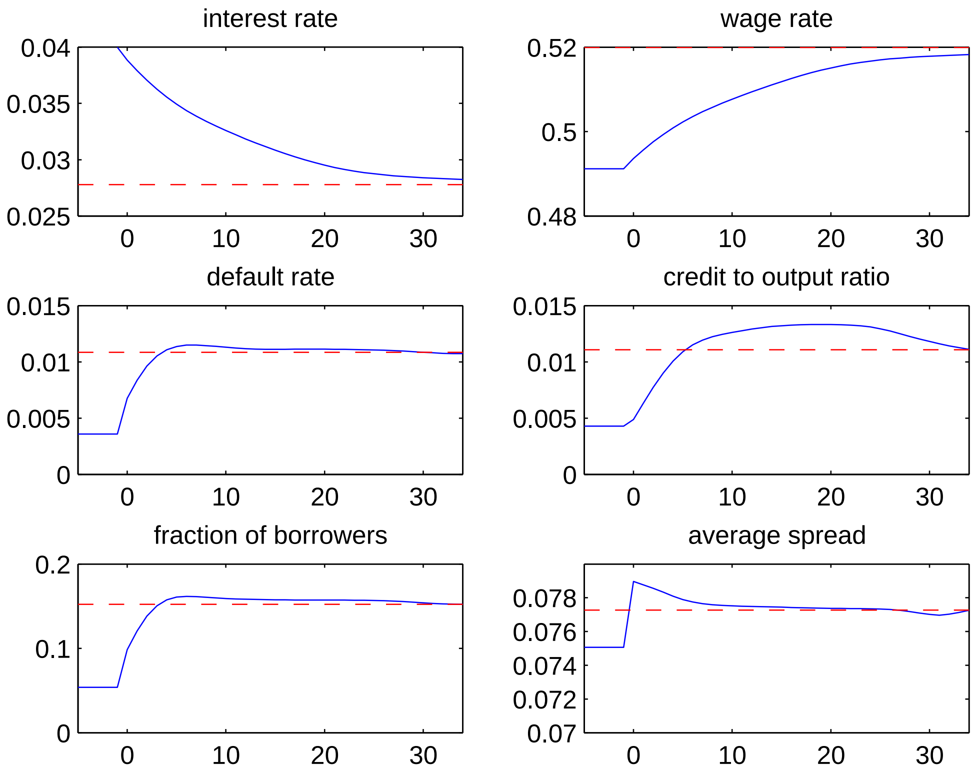

Figure 6: Transition with $\tau_c = 0.09$ and transfers.

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.

Athreya, K., X. S. Tam, and E. R. Young (2012): "A quantitative theory of information and unsecured credit," *American Economic Journal: Macroeconomics*, 4(3), 153–183.

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Ríos-Rull (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75(6), 1525–1589.

Chatterjee, S., and G. Gordon (2012): "Dealing with consumer default: Bankruptcy *vs* Garnishment," *Journal of Monetary Economics*, 59, 1–16.

Conesa, J. C., S. Kitao, and D. Krueger (2009): "Taxing Capital? Not a Bad Idea After All!," *American Economic Review*, 99(1), 25–48.

Domeij, D., and J. Heathcote (2004): "On The Distributional Effects Of Reducing Capital Taxes," *International Economic Review*, 45(2), 523–554.

Gollin, D. (2002): "Getting Income Shares Right," *Journal of Political Economy*, 110(2), 458–474.

Kopecky, K. A., and R. M. H. Suen (2010): "Finite state markov-chain approximations to highly persistent processes," *Review of Economic Dynamics*, 13(3), 701–714.

Krueger, D., and F. Perri (2005): "Understanding Consumption Smoothing: Evidence from the US Consumer Expenditure Data," *Journal of European Economic Association*, 3(2-3), 340–350.

Livshits, I., J. MacGee, and M. Tertilt (2007): "Consumer Bankruptcy: A Fresh Start," *American Economic Review*, 97(1), 402–418.

Mehra, R., and E. C. Prescott (1985): "The Equity Premium: A puzzle," *Journal of Monetary Economics*, 15, 145–162.

Mendoza, E. G., A. Razin, and L. L. Tesar (1994): "Effective tax rates in macroeconomics: Cross-country estimates of tax rates on factor incomes and consumption," *Journal of Monetary Economics*, 34(3), 297–323.