

Regulatory Intensity, Crash Risk, and the Business Cycle

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This draft is extremely preliminary. Any comments are welcome.

Abstract

Regulatory investigations play an important role in shaping information structure in the financial markets through two channels: one the one hand, investigations detect financial manipulation and reveal hidden negative information; on the other hand, regulatory investigations impose adverse consequences for executives involved in manipulation, and deter managerial incentives to manipulate ex-ante. Moreover, regulatory intensity varies over time, depending on the aggregate state of the economy. We propose a model to study the implications of cyclical regulatory intensities for stock market dynamics, and show that cyclicity in financial regulation can lead to cyclicity in crash risk in the stock markets. We also provide evidence that a strong relation between stock crash risk and the business cycle exists in the data. In addition, our model illustrates a unifying mechanism that contributes to a number of stylized facts including gradual booms and sudden crashes in the financial markets, increased crash risk, and countercyclical stock volatilities.

Keywords: Financial regulation, Countercyclical crash risk, Gradual booms and sudden recessions in financial markets, Countercyclical volatilities

1 Introduction

Regulatory investigations play an important role in shaping information structure in the financial markets through two channels: On the one hand, investigations detect financial misreporting and reveal hidden negative information. On the other hand, regulatory investigations impose adverse consequences for executives involved in manipulation, and thus help limit managerial incentives to manipulate performance ex-ante. Moreover, regulatory behavior may vary over time, depending on the aggregate state of the economy, due to varying difficulties in detecting frauds (or a varying dominance in the tug-of-war between political pressure to act and corporate lobbying to deregulate). We study the asset pricing implications of this regulator-manager interaction over the business cycle.

We propose a model to study the implications of regulatory responses to business cycles for stock market dynamics, based on the idea that regulatory investigations both deter managerial incentives to manipulate information and reveal fraud when manipulation occurs. What matters for aggregate dynamics is whether manipulation of financial information and revelations of such manipulation are more likely to happen in booms or in recessions. They both, in turn, depend on how intensities of regulatory investigations change with the state of the economy. In our model of asset pricing, regulatory tendencies to leave more discretion to managers in booms and investigate more intensively during downturns, together with managerial incentives to commit fraudulent reporting, give rise to stock crash risk that is more pronounced in bad times than in good times. Moreover, cyclical regulatory behavior may contribute to the observed pattern of gradual booms and sudden crash in financial markets, and deliver countercyclical stock volatilities. We argue that timing of financial regulation is important for the behavior of stock markets over the business cycle.

Financial regulations and revelations of frauds have been noted to appear procyclical:¹ a boom encourages and conceals financial fraud and misrepresentation by firms, which are then revealed by the ensuing recession. As Povel et al. (2007) note, “examples in the last century include the 1920s (Galbraith (1955)), the “go-go” market of the 1960s and early

¹Please see Povel et al. (2007), Brunnermeier et al. (2009), McDonell (2013). We use the term “procyclical” as a contrast to countercyclical, and we use the latter term to refer to policies that act to counteract financial cycles.

1970s (Labaton (2002), Schilit (2002)), and the use of junk bonds and LBOs in the 1980s (Kaplan and Stein (1993))”. The long boom of the 1990s was followed, first by recession, then by revelations of financial chicanery at many of America’s largest companies. A wave of fraud revelations, such as Freddie Mac, Fannie Mae, Lehman, and AIG, again clustered at the beginning of the recent economic turmoil in 2008. Bertomeu and Magee (2011) show that quality of financial reporting and probability of revelations are minimal prior to a recession and increases during and after a recession. We provide additional evidence for this asymmetric response of regulatory activities to business cycles by reviewing the time-series patterns of regulatory investigations and highlighting that frequencies of regulatory actions peaked during the recessions in the recent business cycles.

In this paper, a model of regulator-manager interaction is first examined as a point of departure. The purpose of this model of regulation is to identify the underlying economic frictions that cause cyclical tendencies in regulatory investigations and to deliver the resultant cyclical patterns in corporate manipulation observed in the data. As a micro-foundation for asset pricing, the model of regulatory investigation generates three key features that are subsequently embedded into a consumption-based infinite-horizon asset pricing model. First, we allow the intensity of regulatory investigations to inversely depend on the aggregate state of economy. This asymmetry implies that regulators initiate investigations more often in bad times than in good times. Second, loosened regulation in booms encourage and conceal frauds, and intense regulation in downturns deter manipulation, causing the prevalence and frequency of manipulation to exhibit procyclicality. Third, the optimistic bias in reporting is not fully unraveled by rational investors due to information asymmetry, unless detection occurs. Our calibrated model quantifies the role of cyclical regulatory investigations for stock market dynamics, in particular, in accounting for countercyclical crash risk.

Our model delivers countercyclical crash risk through two reinforcing mechanisms. First, there is a direct impact of cyclical regulation on stock dynamics through information revelations. The possible manipulation and bias in financial information mitigate price responses, and the lack of investigations and revelations in good times render stock fluctuations fairly mild. During economic downturns, however, strengthened regulation leads to revelations of accumulated hidden negative information, most stockpiled in good times. Waves of frauds

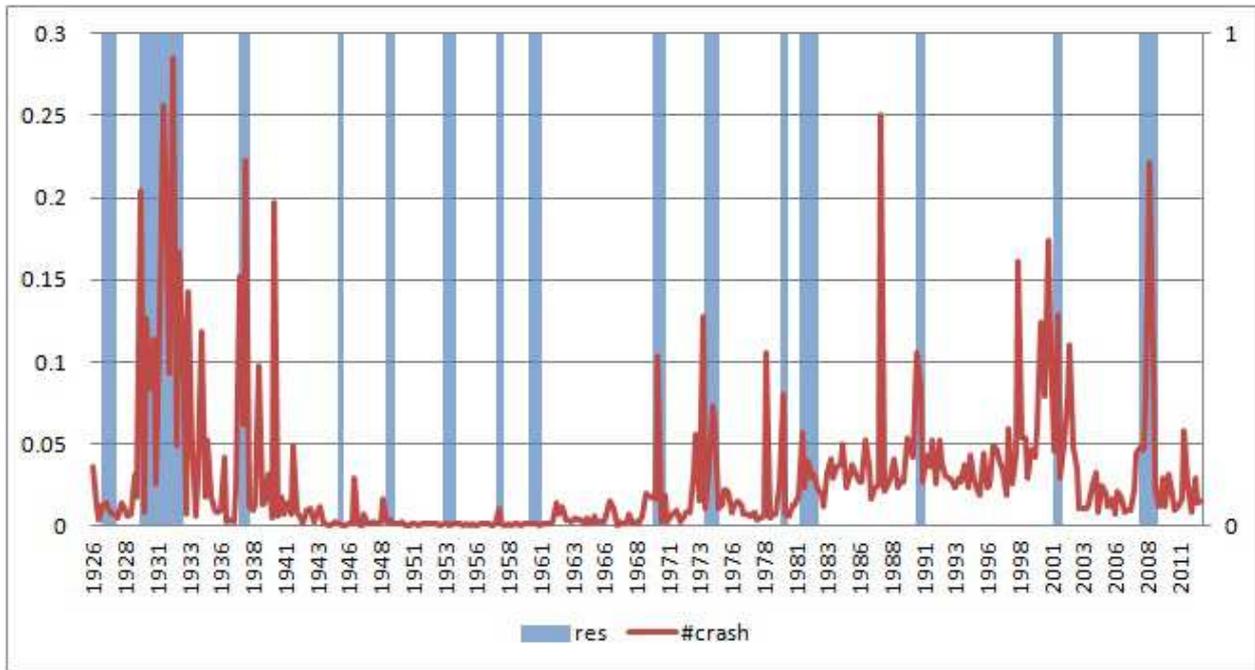


Figure 1: Stock crashes over the business cycle

are revealed and penalized, causing stocks to plummet. Second, there is an indirect impact of financial regulation by affecting managerial incentives. Loosened regulation during booms helps fuel managerial incentives to paint a successful yet falsified picture of firm performance. Increased noise in reports further mutes stock movements in good times, and cause the downturns even sharper when the accumulated losses all come out at once in bad times.

In the recent financial crisis we saw substantial crash risk in the stock market concurrent with the downturn in the real economy. We provide new empirical evidence in this paper that such effects are not new — changes in the crash risk in the U.S. stock market have been coinciding with changes in the real economy. Figure 1 provides a time-series plot of one measure of stock crashes, the Barro and Ursua (2009) measure, together with the National Bureau of Economic Research (NBER) recession periods (gray bars). This figure illustrates the relationship found between stock crash risk and the business cycle. As can be seen from the figure, crash risk worsens during the onset of the NBER recessions.

Our model also delivers the following stylized facts in the financial market: gradual booms and sudden crashes, increased crash risk, counter-cyclical stock volatilities. We discuss each of these model features in turn below. First, cyclical regulatory activities contribute to

gradual booms and sudden crashes in the financial markets also through both information revelation and incentive distortions. On the one hand, regulatory investigations detect all the accumulated manipulation and reveal negative information. The asymmetric regulatory responses to business cycles cause more negative information to be revealed in large lumps during bad times, resulting in large, negative returns during episodes of weak economic conditions and worsening stock market performances. As limited investigations and subsequent revelations during upturns leave uncertainties in financial information unresolved, investors discount the seemingly excellent yet potentially fraudulent performance, slowly updating their beliefs about corporate outlooks. On the other hand, loosened regulatory actions in booms fuel managerial incentives to manipulate, causing more aggressive manipulation only to be revealed and reversed during periods of stress when investigations become intense. The distorted incentives due to regulatory cycles lead to more manipulation in periods of booms, and the associated greater bias further mitigate investors' response to positive corporate news. The increased prevalence of manipulation also causes the downturn even sharper, when the substantial accumulated frauds all come out at once.

Gradual booms and sudden crashes are a ubiquitous feature of financial markets (Veldkamp (2005)). In Veldkamp (2005) and Van-Nieuwerburgh and Veldkamp (2006), this pattern is explained by an endogenous flow of information. In their models, information is abundant in good times because more investment and production generate more precise information. Asset prices thus adjust quickly when the state deteriorates, and a sudden crash occurs. When times are bad, scarce information and high uncertainty that are associated with low investment and productions slow investors' reactions as the economy improves; a gradual boom ensues. We present a different mechanism underlying the asymmetry in financial cycles based on the procyclical bias in regulatory investigations and resultant cyclical prevalence of manipulation.²

In addition, crash risk increases when there is a procyclical bias in regulatory investigations in our model, because increasingly more negative information is revealed in lumps during recessions, exaggerating the severity of economics downturns. Several mechanisms could engender crash risk. For example, it is well known that trading among investors who

²For cyclical patterns of manipulation, see Cohen and Zarowin (2012) and Wang et al. (2010).

have different opinions could reveal the private signals of others and move prices even in the absence of new fundamental information (e.g., Romer (1993)). In Hong and Stein (2003), this process, combined with short sale constraints, make market declines reveal the private signals of relatively pessimistic investors and lead other investors to downgrade their assessments of a firm's prospects, thereby reinforcing the decline. Our mechanism is built upon that in Jin and Myers (2006), in which lack of full transparency concerning firm performance enables managers to capture a portion of cash flow and absorb part of the variation in firm-specific performance, and crashes occur when managers are unwilling or unable to absorb any losses, causing the unobserved negative firm-specific shocks become public at once. We show that cyclical tendencies in regulation exacerbates the effect of managerial manipulation on the stock market, amplifying the frequency and severity of crashes.

Lastly, countercyclical stock volatilities emerge because intense regulatory actions in economic downturns reveal substantial hidden negative information that managers stockpiled in booms due to loosened regulation, causing stock returns to become increasingly volatile as economic conditions deteriorate. In good times, on the one hand, upward manipulation in low idiosyncratic states compresses distributions of reports and smoothes reported earnings over time, reducing volatilities. On the other hand, weak regulation and limited information revelation render stock price less sensitive to reported performance, and return movements are moderate compared to substantial declines upon intensive revelations when the system is facing strains.

A number of empirical studies confirm further findings from Schwert (1989a, b) that the volatility of stock returns is higher in bad times than in good times (see, e.g., Brandt and Kang, 2004). Campbell and Hentschel (1992) develop an explanation based on the feedback effects: risk premia rise (and hence prices fall) with the volatility of dividend news, and return volatility increases with the volatility of dividend news. Wu (2001), Bansal and Yaron (2004), and Tauchen (2005) reconsider this channel of fluctuating economic uncertainty and show that investors with a preference for early resolution of uncertainty require compensation for economic uncertainty, thereby inducing negative co-movements between ex post returns and return volatility. Mele (2007) provides an explanation based on asymmetric movement in risk premia, that is, risk premia increase more in bad times than they decrease in good

times. Our model adds to the existing explanations by suggesting an additional source of asymmetry in stock volatility when regulatory activities change asymmetrically in response to economic conditions.

Our paper adheres to the theoretical literature on cyclical patterns of manipulation. Hertzberg (2003) examines a setting in which investors are more likely to give short-term incentives to firm managers in good times. Since short-term incentives exacerbate financial misreporting, such misreporting tends to be pronounced during good times. Povel et al. (2007) further study a model where investors do not monitor a firm with positive public information carefully when investors' prior beliefs about the state of the economy, measured by the proportion of "good" firms among firms seeking financing, is high. Because this merely confirms their view that the firm is likely to be good, but they do monitor firms with negative public information. Here, incentives for fraud are high. When investors' belief about the aggregate state of the economy is low, there is little or no fraud, because enough uncertainty remains even for firms with positive public information that investors find it worthwhile to monitor the firms carefully, and fraud has little upside. More recently, Bertomeu and Magee (2011) generate countercyclical increases in reporting quality in a framework where regulators subject to political pressure respond to cyclical demands by borrowers and lenders.³ In generating a cycle in which fraud peaks in booms and is revealed in the ensuing bust, our model can be viewed as complementary to Povel et al. (2007) and Bertomeu and Magee (2011). Neither study examines the asset pricing implications of such a fluctuating information environment, which is at the heart of our analysis.

Although ours is the first article that we are aware of that ties changing regulatory actions over the business cycle to asset price movements, there are a number of articles that are related to the tenor of our analysis. For example, a growing body of work examines "credit cycles"—the idea that banks and other credit suppliers engage in behavior that exacerbates business cycle effects, making credit even tighter in recessions, and looser in expansions, than pure demand-side effects would suggest (e.g. Bernanke and Gertler 1988, Kiyotaki and

³Several papers study how political processes influence accounting standards, for example, Watts and Zimmerman (1978, 1979), Sunder (1988), and Zeff (2002, 2005). Lo (2003), Ramanna (2008) and Hochberg, Sapienza, and Vissing-Jorgensen (2009) provide empirical evidence of the consequences of public lobbying by firms affected by accounting changes.

Moore 1997). Among these, the closest to our article is that of Ruckes (2004), which models how competing bank lenders' incentives to screen potential borrowers exacerbate cyclical variations in credit standards. Dow, Gorton, and Krishnamurthy (2005) study how the impact of managerial empire-building incentives changes over the business cycle, and how this affects asset prices. Albuquerque and Wang (2005) also study asset pricing and welfare implications of imperfect investor protection, and assess the magnitude of both the loss of investor welfare and the reduction in market value due to managerial over-investment. None of these articles address cyclical patterns of policy responses though, which is our key focus.

The rest of the paper proceeds as follows. Section 2 provides some stylized facts regarding the cyclical properties of regulatory activities as well as new evidence on the cyclical properties of regulatory investigations. Section 3 discusses the problem of a regulator determining investigation intensities, having in mind how their policies influence the behavior of managerial manipulation. The one-period model highlights the link between information manipulation, regulatory intensities, and business cycles. Section 4 embeds this regulator-manager interaction into a dynamic infinite horizon economy in order to examine the implied properties for asset pricing. Section 5 presents the results and mechanisms of a calibrated version of the model. Section 6 discusses the key drivers of model results. Although this paper is currently written as narrowly about investigations for information manipulation, the framework can be applied to study other measures of financial regulations and their asset pricing implications. We discuss some of these alternate interpretations in Section 6. Section 7 concludes.

2 Cyclical patterns of financial regulation and stock crash

In this section, we will document cyclical properties of regulatory actions and provide new evidence on the cyclical properties of stock crash risk.

2.1 Financial regulation and the business cycle

In this section we review the time-series pattern of regulatory behavior and show that the intensity of regulatory actions peaked during NBER recessions over the recent business cycles. Specifically, we use the number of comment letters issued by the SEC and the percentage of firms subject to litigations related to accounting measure manipulation as proxies for regulatory intensity, and document their business-cycle variations.

As an indication of the SEC's regulatory effort, the percentage of firms receiving SEC comment letters peaked during the 2008-2009 recession over the last business cycle, as shown in Figure 1. The SEC issues comment letters to registrants if the staff has questions or concerns related to a disclosure filing, or if the staff believes the filing is incomplete or needs to be improved. In issuing comment letters to a company, the SEC may request the company provide additional supplemental information, revise disclosure, or provide additional disclosure. The variations in the amount of comment letters issued suggests the SEC's regulatory intensity may vary over time, and experienced an noticeable increase during the 2008 economic crisis. However, because the SEC only began publicly releasing this correspondence in 2005 for comment letters issued after August 1, 2004 (in EDGAR database), we are not able to trace its variations over the previous business cycles.

To identify regulatory investigations regarding manipulation, we select the litigation cases related to financial information manipulation from the Audit Analytic's Litigation Database on all federal securities class action claims, SEC actions, and material federal civil litigation. Figure 2 shows that the percentage of firms subject to litigation related to accounting manipulation escalated during the economic downturn in 2001, followed by a regime-switch due to the passage of the Sarbanes-Oxley Act, and then reached its record high in the crisis of 2008. Figure 3 displays how the percentage of firms subject to litigation related to accounting manipulation varies with the S&P 500 index price level. The number of manipulation-related legal actions responds negatively to the index price level, with a contemporaneous correlation of -0.51 . The time-series variations in the number of legal actions suggest that regulatory actions respond to the aggregate conditions in the economy, and tend to cluster during downturns.

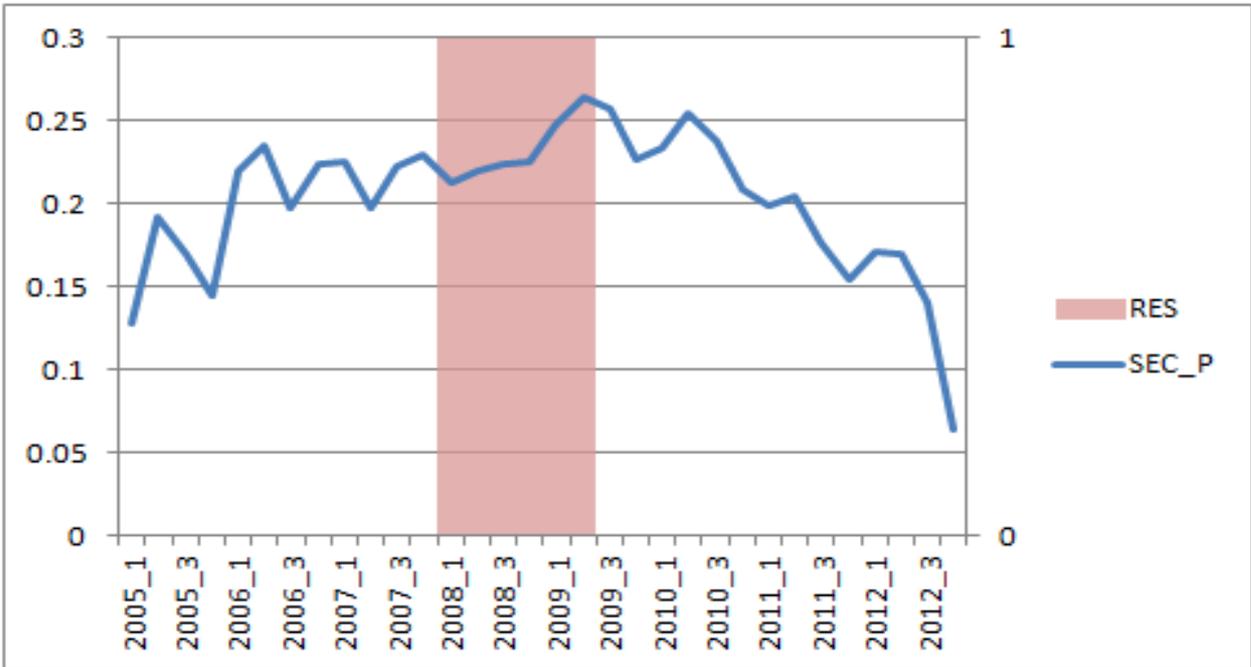


Figure 2: SEC comment letters

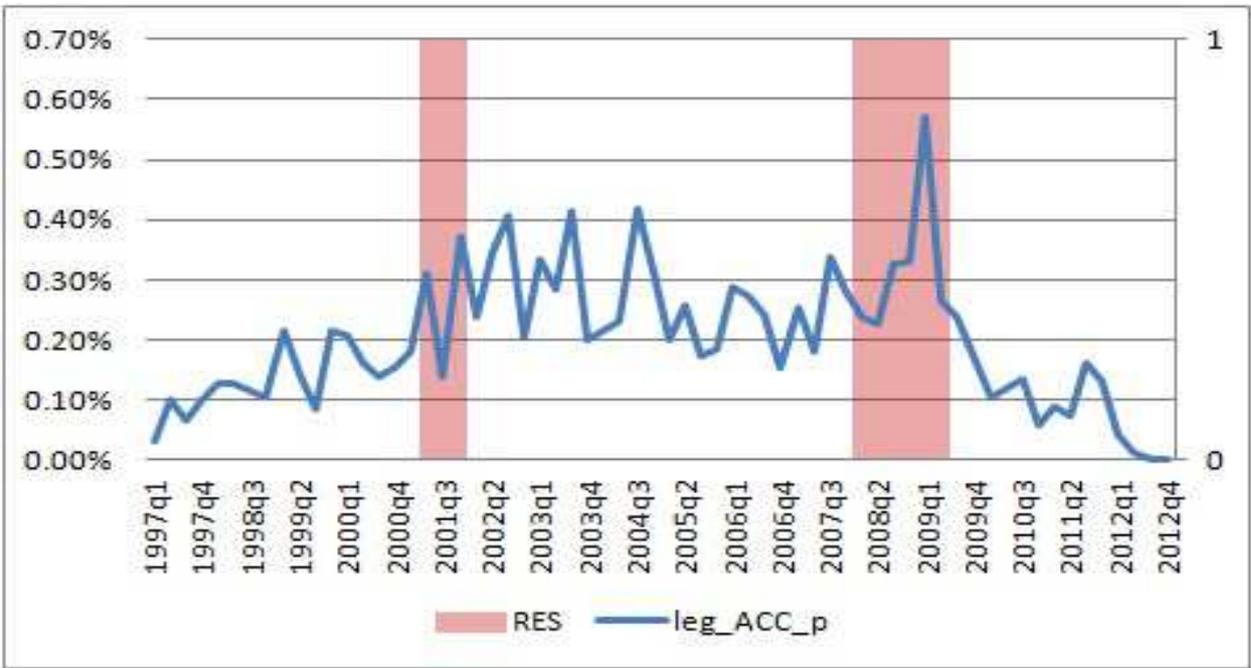


Figure 3: Accounting-related legal actions and NBER recessions

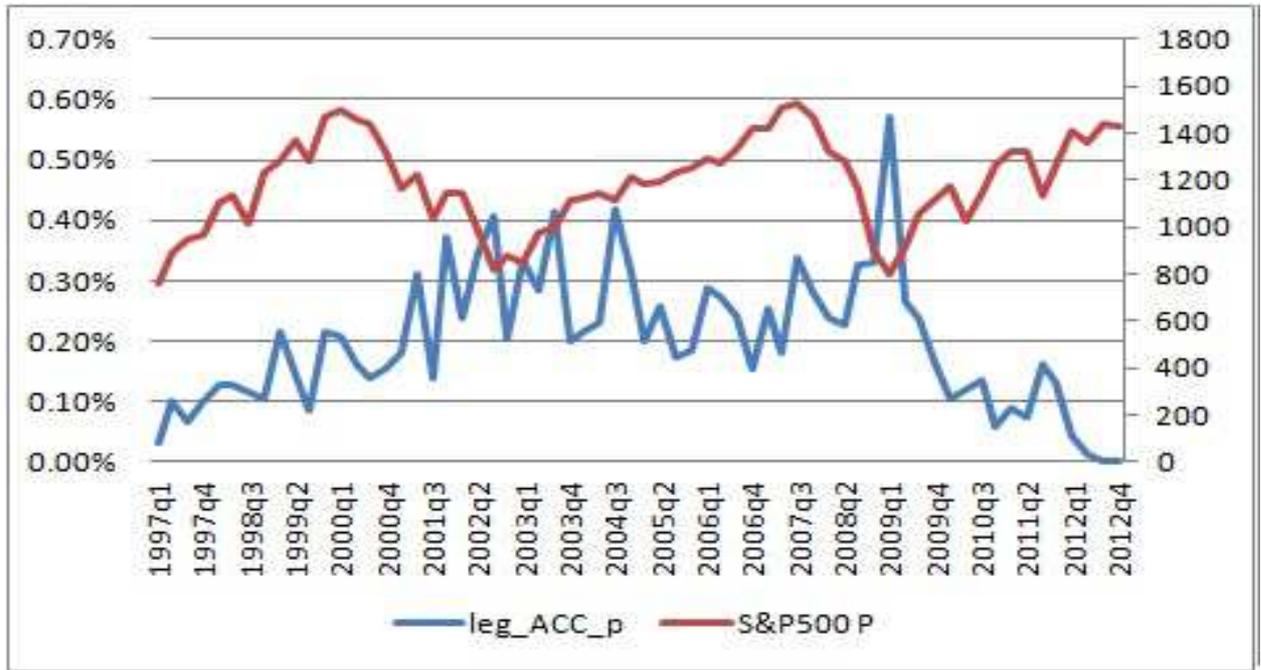


Figure 4: Accounting-related legal actions and S&P500 index price

2.2 Stock crashes and the business cycle

In discussion of the recent financial crisis of 2008-2009, much is made of the apparent co-existence of the economic downturn and increased stock crash risk. We show that such coexistence is not new — changes in the crash risk in the U.S. stock market have been coinciding with changes in the real economy, and there may be a causality between economic downturns and crash risk in the financial markets. Figure 4 provides a time-series plot of one measure of stock crash, the Barro and Ursua (2009) measure, together with the National Bureau of Economic Research (NBER) recession periods (gray bars). This figure illustrates the relationship found between stock crash risk and the business cycle. As can be seen from the figure, crash risk worsens during the onset of the NBER recessions.

Following Jin and Myers (2006), we measure the frequency of crash using COUNT, based on the number of residual returns exceeding k standard deviations above and below the mean, with k chosen to generate frequencies of 0.01% or 0.1% in the lognormal distribution. Following Jin and Myers (2006), we subtract the upside frequencies from the downside frequencies. A high value of COUNT indicates a high frequency of crash. Figure 5 and Figure 6 show that

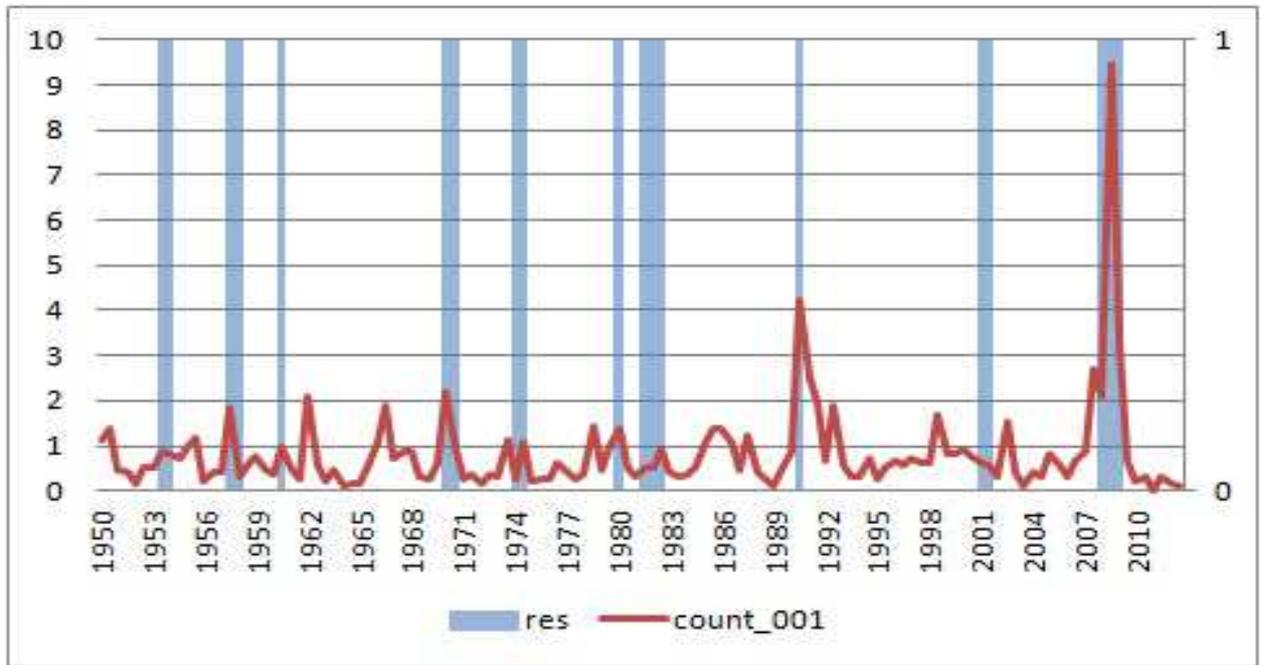


Figure 5: COUNT(0.01%) over the business cycle

crash risk is typically pronounced during NBER recessions. We further plot the time-series variation in stock crash risk and GDP growth rates in Figure 7 and Figure 8, and find a negative contemporaneous correlation of -0.42 .

To sum up, there is substantial procyclical variation in financial regulatory intensity on the one hand, and countercyclical variation in stock crash risk on the other hand, as documented here. Our model provides an explanation consistent with these patterns.

3 A regulator-manager interaction

Our model highlights how the effects of regulatory structures depend on assumptions about detection difficulties and underlying factors in the regulators' objectives. This point can best be made in the context of a model that captures important elements of managerial reporting and regulatory activities. The following section presents such a model that focuses on the choice of a regulatory mechanism to control the manipulation incentives of corporate managers. An underlying assumption is that regulators have some discretion in choosing the parameters of their regulatory behavior. In the model, the key parameter is the intensity of

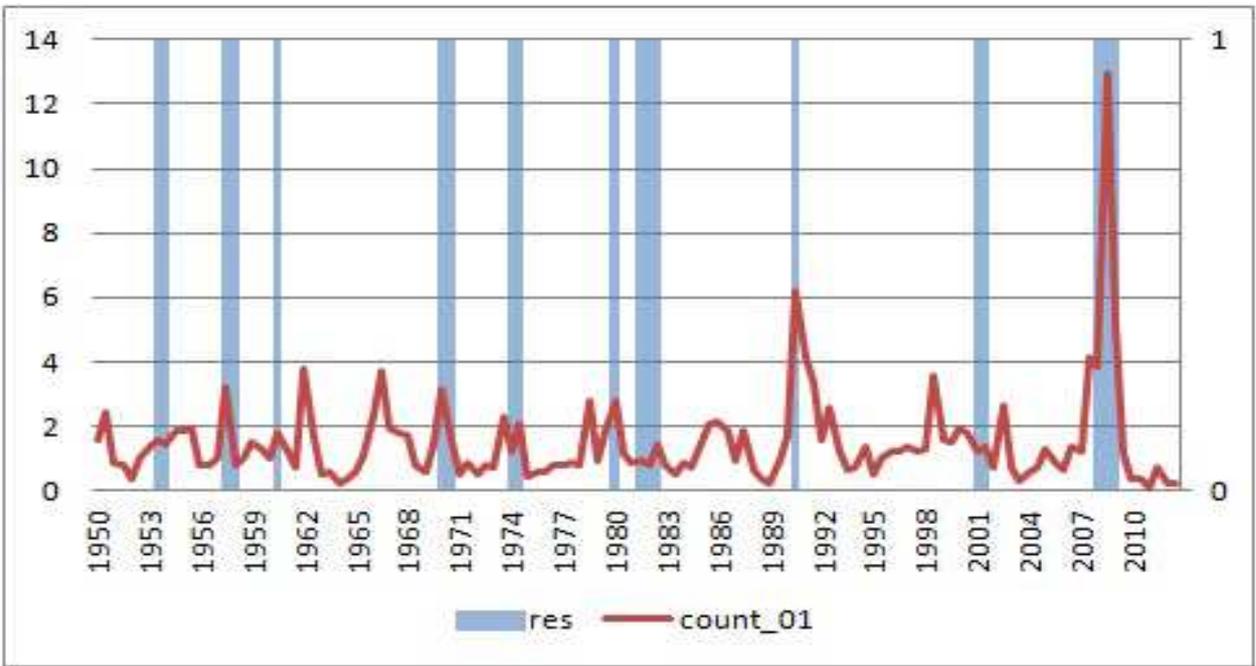


Figure 6: COUNT(0.1%) over the business cycle

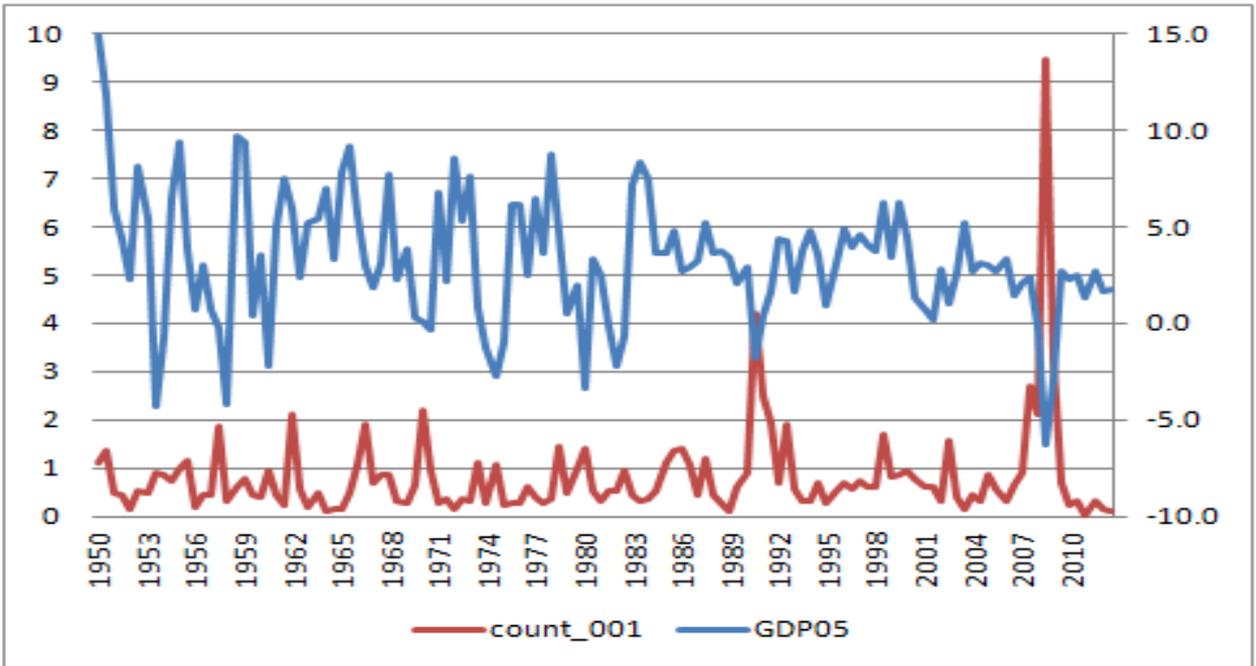


Figure 7: COUNT(0.01%) and GDP growth rates

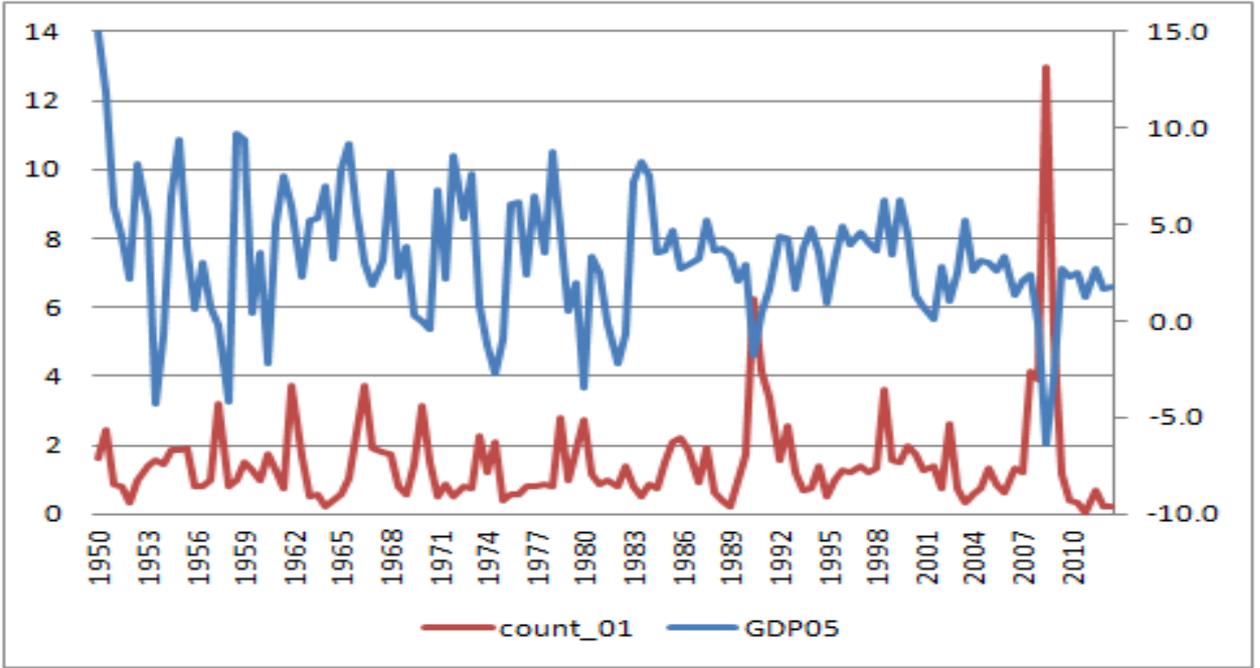


Figure 8: COUNT(0.1%) and GDP growth rates

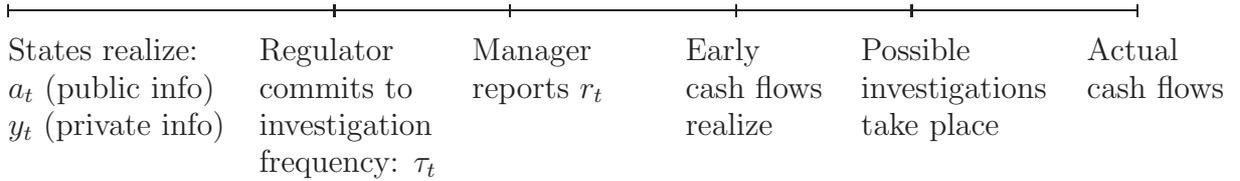


Figure 9: Timeline of a regulator-manager interaction

investigations.

3.1 Environment

Firms' true earnings are jointly influenced by an aggregate state and an idiosyncratic state in each period. In particular, there are two possible levels of aggregate states: $a \in \mathcal{A} \equiv \{g, b\}$, where $g > b$ ("good" and "bad"); and two possible levels of idiosyncratic states: $y \in \mathcal{Y} \equiv \{h, l\}$, where $h > l$ ("high" and "low"). Each firm's true earnings are given by $ay, a \in \mathcal{A}, y \in \mathcal{Y}$. The aggregate state is perfectly observed by the regulator and managers in all firms. Manipulation occurs in the model when a manager reports a high idiosyncratic

state when the actual realization of idiosyncratic state is low (l), for a given aggregate state $a \in \mathcal{A}$.

The time line of Figure 9 chronicles the sequence of events in the model. At the beginning of each period, an aggregate state (a_t) is perfectly revealed to all agents, and each firm's idiosyncratic state (y_t) is privately observed by the manager. The regulator commits to a regulatory policy in the current period, which boils down to a frequency of investigations τ_t . Each manager makes a report of the idiosyncratic state r_t . Before investigation takes place, firms may have early cash flows realized (part of final earnings in each period), which can be used to absorb the discrepancy in reporting. The likelihood that early cash flows, denoted by v , are sufficiently large to conceal the amount of manipulation is higher in a good aggregate state than in a bad aggregate state. In particular, we assume that with probability $1 - \varepsilon$, early cash flows are large enough to protect managers from being caught for manipulation during investigations, that is, $v \geq g(h - l)$. With probability ε , managers' manipulation will be detected upon investigation, due to insufficient capital buffer, i.e. $v < b(h - l)$. When the aggregate state is bad, early cash flows are not large enough to conceal fraud with probability 1. We discuss the regulator's investigation decision and managers' reporting strategies in detail below.

Our model of financial regulation features a state-dependent detection likelihood, which is higher in bad aggregate states than in good aggregate states. Managers have incentives to manipulate performance by hiding temporary losses to avoid disclosing negative information. During booms, cash flows from corporate operation and external credit are readily available to absorb previous losses, and help prevent frauds being revealed. Managers may easily deny manipulation during prolonged periods of asset growth and strong credit. In periods of severe stress, however, managers lose access to funds and are no longer able to obscure reporting discrepancies. Our assumption of a state-dependent realization of early cash flows highlights this friction present in regulatory investigations of corporate frauds.

Regulator There is one regulator whose objective is to maximize the prevalence of truthful reporting in the current period. In particular, the regulator chooses a regulatory mechanism to control the manipulation incentives of corporate managers in each period. An underlying assumption is that regulators have some discretion in choosing the parameters of

their regulatory behavior. In this model, the key parameter is the frequency of investigations, represented by τ . The cost to the regulator of conducting investigation is quadratic in the frequency of investigations: $C(\tau) = C\tau^2/2$.

Manager There is a continuum of managers with Lebesgue measure on \mathbb{R}_+ who report their firms' performance. Because the aggregate state (a) is publicly observable, it is equivalent to assuming that managers report their firms' idiosyncratic productivity $y \in \{h, l\}$, which is privately observed by the manager. As long as the reported idiosyncratic productivity falls in the set $\{h, l\}$, investors cannot directly detect whether the manager has misstated earnings and will price the firm based on the report.

If a manager produces an inaccurate report, including manipulation upwards and downwards, the manager may be investigated and fined for misreporting. If sufficiently large early cash flows are available to conceal previous manipulation, the manager will not be detected even if investigation occurs. Otherwise, the manager will be fined upon investigation with an amount of monetary penalties F_m . The probability that early cash flow is not sufficiently large to conceal fraud and thus managers will be detected upon investigation is represented by $\tau_t \varepsilon(a_t)$, where τ_t is the likelihood of investigation, and $\varepsilon(a_t) = \varepsilon$ if $a_t = g$ and $\varepsilon(a_t) = 1$ if $a_t = b$. We will see that there is no incentive to understate productivity in this model. We define that manipulation occurs when the reported productivity differ from true productivity. More specifically, manipulation emerges in this environment if the manager announces that high idiosyncratic productivity (h) has been achieved when the actual realization of idiosyncratic productivity is low (l).⁴

Managers vary in their utility from reported earnings θr , where $\theta \sim U[0, 1]$ represents managerial preference and r represents reported performance, i.e. $r \in \{h, l\}, \forall a \in \{g, b\}$. There are many reasons that managers differ in their preferences over reported earnings. For example, managers face different pay-performance sensitivities and compensation structure,

⁴This paper has a central focus on upward manipulation. The reason to focus on misreporting on upside is that overstatement of earnings is more widespread than understatement in the data and more problematic in general. Empirical work on SEC enforcement actions aimed at violations of Generally Accepted Accounting Principles suggests that over-reporting is the more frequent source of firm-wide financial misrepresentation (Feroz, Park, and Pastena [1991]). The average amount of restated earnings is hugely negative, and over 75% of restating firms restated their earnings downwards, indicating that managers have a strong drive to appear more productive than they actually are.

and they vary in their risk aversion, personal stigma, and time horizon. The regulator knows the distribution over managerial preferences (θ), but cannot discern the preference of any particular manager.

Equilibrium Definition A Bayesian Nash Equilibrium is defined as (i) a regulatory investigation policy by the regulator: $\mathbf{R} : R(y) \rightarrow \tau$ that maximizes the regulator's objective function, given the reporting strategies of managers; (ii) a reporting strategy of each particular manager: $\mathbf{M} : \tau \rightarrow R(y)$ that maximizes the manager's utility, given the regulatory investigation policy; and (iii) all agents have rational expectations in that each player's belief about the other players' strategies is correct in equilibrium.

3.2 Managerial reporting

We first consider the reporting decision of managers. Given a_t, y_t, θ, τ_t , each manager attempts to maximize his objective function by choosing a level of report, which is characterized by the expression:

$$\max_{r_t \in \{h, l\}} \theta a_t r_t - \phi(a_t, y_t, \tau_t, r_t),$$

where $\tilde{\theta} = \theta$ is the realization of a random event that the manager alone observes, which captures the benefit to the manager of inducing a marginal change in reports by manipulating his report. It is common knowledge that θ follows a uniform distribution on the interval $[0, 1]$. The firm term reflects the manager's desire to inflate earnings, since their compensation and career prospects are directly or indirectly tied to firm performance. The second term represents the known cost of manipulation to the manager. In other words, for a given realization of $\tilde{\theta} = \theta$, the manager attempts to maximize θr_t subject to some cost of using manipulation as a vehicle to maximize this objective. The cost of manipulation ϕ represents the expected penalties, and the penalty is given by

$$penalty = \begin{cases} 0 & \text{if } y_t = r_t, \\ F_m & \text{if } y_t \neq r_t \text{ and early cash flow is not sufficient} \\ 0 & \text{if } y_t \neq r_t \text{ and early cash flow is sufficient.} \end{cases}$$

The probability that early cash flow is not sufficiently large to conceal fraud and thus managers will be detected upon investigation is represented by $\tau_t \varepsilon(a_t)$, where τ_t is the likelihood of investigation, and $\varepsilon(a_t) = \varepsilon$ if $a_t = g$ and $\varepsilon(a_t) = 1$ if $a_t = b$.

Thus the optimal strategy can be characterized as follows.

$$r_t = \begin{cases} h & \text{if } y_t = h, \\ h & \text{if } y_t = l \text{ and } \theta a_t h - \tau_t \varepsilon(a_t) F_m \geq \theta a_t l \\ l & \text{if } y_t = l \text{ and } \theta a_t h - \tau_t \varepsilon(a_t) F_m < \theta a_t l. \end{cases}$$

Thus only managers with $\tilde{\theta}$ above the threshold level, $\frac{\tau_t \varepsilon(a_t) F_m}{a_t (h - l)}$, find manipulation beneficial given the regulatory investigation frequencies. Therefore, the likelihood of manipulation in low idiosyncratic state among all managers is given by

$$x(a_t, \tau_t) = 1 - \frac{\tau_t \varepsilon(a_t) F_m}{a_t (h - l)}. \quad (1)$$

The threshold level of $\tilde{\theta}$ above which managers manipulate reports increases with the frequencies of investigation τ_t . Therefore, a tightened regulatory policy reduces the prevalence of manipulation among managers.

Because the manager's uncertain reporting objective is crucial to our model, it is important to provide some motivation for the introduction of uncertainty and for our specific modeling of that uncertainty. At a broad level, the notion that a manager's reporting objective is uncertain seems reasonable because, in real markets, a manager's reporting objective at any point in time is not known precisely. For example, at any point in time, the regulator (and the market) does not know: the precise nature of a manager's compensation; the manager's rate of time preference and degree of risk-aversion; the manager's personal stigma; the manager's psychic costs associated with bias; or the level of effort or resources the manager must expend to achieve a workable manipulation scheme. Given its inability to discern the manager's precise objective, the regulator (and the market) can only conjecture the extent to which a manager has incentives to inflate expectations. In our model, we formalize this uncertainty by introducing the random variable θ into managers' objective function.

3.3 Regulatory policy

The regulator's problem is to choose a probability of investigation τ_t each period, a course of action where an investigation reveals manipulation and charges the manager a fee F_m . An optimal policy can be defined as a $\tau(a_t)$ that solves the following problem in each period.

$$\max_{\tau_t} \alpha (1 - x(a_t, \tau_t)) - \frac{1}{2} C \tau_t^2,$$

where $x(a_t, \tau_t)$ represents the prevalence of manipulation in the economy and is given by Equation (1). The first term of the regulator's objective function emphasizes the regulator's aim to promote truthful reporting and deter manipulation.⁵ The second term represents the cost incurred when conducting investigations to determine the accuracy of managerial reporting.

The first-order condition yields the optimal regulatory policy:

$$\tau(a_t) = \frac{\alpha \varepsilon(a_t) F_m}{C a_t (h - l)}.$$

It is optimal for the regulator to initiate more frequent investigations during bad times because a high detection rate makes investigation more cost-effective in preventing manipulation than in good times.

Our model shows that cyclical tendencies of regulatory investigations may emerge due to state-dependent detection difficulties. In reality, detection difficulty is more pronounced during good times because managers have abundant internal capital buffer and external funding sources to absorb discrepancies in financial statements and conceal their previous manipulation. Managers may easily deny manipulation during prolonged periods of asset growth and strong credit. In periods of distress, however, managers no longer have flexibilities in moving resources around to hide frauds due to limited internal cash flows and external funding conditions, and thus generally fail to prevent detection upon investigations. To conduct cost-effective examinations, regulators may optimally choose to conduct more intensive investigations when the economic and financial systems are facing strains.

Finally, we note that the preference parameter α may well depend on the aggregate state: α may be higher in bad times because of significantly greater political pressure to act and less corporate lobbying to deregulate. During and for a while after a market downturn, or worse yet, an economic crisis, there is significant political pressure and public anger to act to prevent future meltdown and crises. The career concern and reputation concern of regulators lead to intense investigations and waves of fraud revelations. These factors reverse themselves during a boom period. While everyone is making money and companies appear

⁵The mission of the U.S. Securities and Exchange Commission is to protect investors, maintain fair, orderly, and efficient markets, and facilitate capital formation(The U.S. SEC website).

healthy, there is little appetite for regulation. The politics of regulation are thus dominated by the perceptions and interests of financial market participants.⁶ Yu and Yu (2006) find that firms engaging in financial fraud spend more in lobbying, and corporate lobbying indeed lowers the likelihood of fraud detection. It is straightforward to see that $\tau_b - \tau_g$ increases when α is inversely related to the aggregate state in our model, adding another force that could cause cyclical tendencies in regulation.

3.4 Discussions

As a prelude to discussing empirical implications of the model, we formally state the equilibrium results in Proposition 1 (proved in Appendix).

Proposition 1 *The equilibrium regulatory policy in the good aggregate state τ_g and in the bad aggregate state τ_b are represented by $\tau_g = \frac{\alpha \varepsilon F_m}{C(h-l)g}$ and $\tau_b = \frac{\alpha F_m}{C(h-l)b}$ respectively. The equilibrium prevalence of manipulation in the good aggregate state x_g and in the bad aggregate state x_b are represented by $x_g = 1 - \frac{\alpha \varepsilon^2 F_m^2}{Cg^2(h-l)^2}$ and $x_b = 1 - \frac{\alpha F_m^2}{Cb^2(h-l)^2}$ respectively. Investigation is more frequent in bad times and manipulation is more prevalent in good times.*

Managerial incentives to manipulate exhibits procyclical tendencies for two reasons. First, the counter-cyclical regulatory intensities cause manipulation to be investigated and consequently detected less during good times, generating strong incentives in good states to manipulate given that the aggregate state is persistent. Second, managers' private benefits from manipulation, i.e. $a(h-l)$, $a \in \{g, b\}$, is higher in good aggregate state, increasing the likelihood of manipulation even further during booms. Our model suggests that it may be too costly to get managers to truthfully report during upswings, and hence intensive investigations may not be in the best interest of a regulator with limited budget.

Wang, Winton, and Yu (2007) find that in all their models the effect of Industry Relative Investment on fraud propensity is strongly positive, suggesting that fraud propensity

⁶Stigler (1971) and Peltzman (1976) and the extensive literature that follows their seminal work emphasize the political economy of interest groups as a determining factor in regulatory decisions. Along these lines, one idea that is often voiced is that of "regulatory capture." This term expresses the notion that regulatory actions may be driven more by the interests of the firms in the regulated industry than by considerations of general or consumer welfare.

is cyclical. Cohen and Zarowin (2012) show that the tendency of firms to manage earnings upward to beat benchmarks is positively related to market-wide conditions, and conclude that managers' manipulation respond positively to aggregate market conditions. Our model endogenously delivers cyclical patterns of regulatory investigations and corporate manipulation in the presence of state-varying detection difficulties. Greater personal benefits (such as bonus compensation, higher stock valuation, and exercising option compensation) contributes an additional element driving the procyclical pattern of manipulation.

Due to frictions present in detecting frauds, our model generates the following features relevant for asset pricing. First, state-varying detection difficulties give rise to procyclical tendencies in financial regulation. Second, cyclical patterns in managerial manipulation emerge in response to asymmetric regulatory intensities over the business cycle. Third, rational investors who are informed about the regulator-managers interaction are uncertain about whether a particular report has been inflated. That is, investors can perfectly infer x given the equilibrium regulatory policy, but they cannot correctly gauge firms' idiosyncratic state. We show in the next section that the relationship between investigation intensity and manipulation frequency — together with the unrevealing financial reporting caused by manipulation — has considerable implications for the dynamics of financial markets over business cycles.

4 Cyclical regulations and asset pricing

The stylized facts documented in Section 2 suggest that regulatory behavior and stock crash risk are both cyclical. In this section we use our model of regulator-managers interaction from the previous section in a dynamic environment to explain a number of stylized financial facts that are symptomatic of risk in a calibrated model, and in particular to explain why there may be greater crash risk in bad times. Three central features generated in our model of regulation will be embedded in an infinite horizon economy to examine the implied business cycle properties of crash risk: (i) regulatory investigations that reveal hidden negative information are procyclical; (ii) manipulation tendencies are influenced by cyclical regulatory behavior and exhibit procyclicality; (iii) investors can infer the likelihood of manipulation (x) given the

equilibrium regulatory policy, but cannot unambiguously gauge the true state of each firm. Note that x_a and λ_a were endogenous in the previous section, here we adopt a reduced-form approach in the pricing analysis using calibrated values of x_a and λ_a .

4.1 Setup

Consider an economy populated by a large number of managers, who are hired by investors to operate firms and report firms' earnings. The aggregate state of the economy takes two possible values, “ g ” and “ b ”, which represent good state and bad state respectively. The aggregate state follows a Markov process with the following transition probability between time t and $t + 1$:

$$\Pr(a_{t+1} = j | a_t = i) = \pi_{ij}, \quad \forall i \in \{b, g\}, \quad \forall j \in \{b, g\}.$$

Every firm's idiosyncratic productivity in each period is stochastic and takes two possible values, $y \in \{l, h\}$, where $l < h$. The idiosyncratic productivity is associated with a simple Markov process with the following transition probability between time t and $t + 1$:

$$\Pr(y_{t+1} = j | y_t = i) = \pi_{ij}, \quad \forall i \in \{l, h\}, \quad \forall j \in \{l, h\}$$

The Markov process is persistent: $\pi_{hh} > \pi_{hl}$ and $\pi_{ll} > \pi_{lh}$. For simplicity, we assume symmetric transition matrices, that is, $\pi_{gg} = \pi_{bb}$ and $\pi_{hh} = \pi_{ll}$. The firm's production (creation of earnings) is jointly determined by the aggregate state (a) and idiosyncratic productivity (y): ay .

The timeline of the model events in each period is described in Figure 10. After the aggregate state is observed by all agents and the idiosyncratic state is privately observed by managers, managers report firm performance. Asset price realizes based on the information investors have. Detection regarding financial reporting then occurs with probability λ_a , $a \in \{g, b\}$ every period, where $\lambda_g < \lambda_b$ (recall that $\lambda_g = \varepsilon\tau_g$, $\lambda_b = \tau_b$, $\tau_g < \tau_b$). If the detection takes place, all the previous performance since the most recent detection are revealed, and investors bear monetary losses. The monetary losses incurred for manipulation is assumed to be a linear function of the number of inflated reports upon detection. Specifically, the

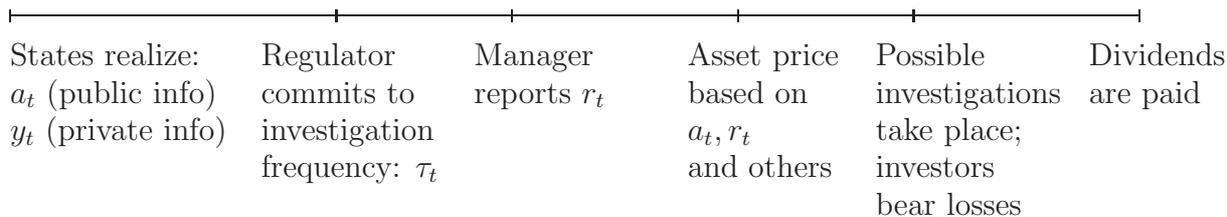


Figure 10: Asset pricing model timeline

amount of fines is represented by $F = \kappa n$, where κ is a constant and n is the number of periods involving manipulation since the most recent detection.

The frequency of detection λ_a (and frequency of investigation, τ_a) determines the likelihood of a manager engaging in manipulation in the current period, represented by x_a , where $x_g > x_b$. That is, a fraction x_a of managers inflate their reports each period, where $a \in \{g, b\}$ represents the current aggregate state. Investors know the value of x_a , but they do not observe whether manipulation occurs in a particular firm (since θ is managers' private information). That is, if the true idiosyncratic productivity is low, each manager reports high with probability x_a and truthfully reports low with probability $(1 - x_a)$ from investors' viewpoint. If the true idiosyncratic productivity is high, the manager has no incentive to understate earnings and always truthfully reports high.

The information received by investors in a particular firm is a combination of macroeconomic and firm-specific news. But the macroeconomic news can be separated, because it is common to all firms. We therefore assume that outside investors can observe the aggregate state that drives all firms' performance, as well as managers' report of idiosyncratic productivity. Conditioned on a history of aggregate states and a firm's reports, investors estimate current idiosyncratic productivity given the new report, make inferences about past misreporting as previous manipulation leads to subsequent losses, and form their expectations about future performance when pricing the firm each period.

4.2 Investors' Bayesian learning and price formulation

We assume that investors have linear utility, and the price of each firm in each period is thus given by discounted expected future dividends net of monetary costs of manipulation. For notation convenience, high and low reported idiosyncratic productivity are denoted by \tilde{h} and \tilde{l} , to distinguish from high and low actual idiosyncratic productivity.

Recall that the manager always overstates earnings when (i) true idiosyncratic productivity is low and (ii) managerial marginal benefit from manipulation $\tilde{\theta}$ is above a threshold level. That is, $R(h)$ is always \tilde{h} ; and $R(l)$ is \tilde{h} with probability x_a and \tilde{l} with probability $(1 - x_a)$, where $a \in \{g, b\}$ represents the current aggregate state. Note that the derivation of the posterior probability of having a false report at each point in time requires utilizing the entire history of reports since the most recent detection up to the current report. In particular, when the manager makes an earnings announcement every period, the investors not only infer the current realization and predict future earnings, but also revise their expectation on each previous report in history.

Fortunately, in this setting all the relevant information in the reporting history can be summarized with a small set of state variables. In what follows, the problem is reduced to a variational problem in which history dependence can be summarized and asset price can be characterized by the following six state variables.⁷

- a : the current aggregate state, $a \in \{g, b\}$;
- γ : the conditional probability (with the information from the current report) that the current true idiosyncratic productivity is high;
- Z : the expected number of periods involving earnings management since the last detection until the most recent low report ($Z = 0$ if there is no low report since the last detection until the previous period);
- N : the number of consecutive high reports until the previous period since the last low report or the last detection, whichever is more recent;

⁷For detailed examples of what each state variable represents, see Appendix A.

- r : the current earnings report, $r \in \{a\tilde{h}, a\tilde{l}\}$;
- \bar{y} : the true idiosyncratic productivity before the series of consecutive N high reports starts.

Given the earnings management incentive in this binary setting, the current true idiosyncratic productivity is revealed under two circumstances. The first is when the detection regarding financial reporting takes place. In this case, the entire history of earnings realizations is revealed. The second is when the manager sends a low report. If the reported productivity is low, although the credibility of financial statements in prior periods remains ambiguous, the current idiosyncratic productivity is low with certainty. As the investors update their beliefs in the standard Bayesian fashion, γ' evolves following Bayes' Rule:

$$\gamma' = \begin{cases} \frac{\gamma\pi_{hh} + (1-\gamma)\pi_{lh}}{\gamma\pi_{hh} + (1-\gamma)\pi_{lh} + \gamma(1-\pi_{hh})x_a + (1-\gamma)(1-\pi_{lh})x_a}, & r = \tilde{h} \text{ at } t+1, \\ 0, & r = \tilde{l} \text{ at } t+1, \end{cases}$$

where $a \in \{b, g\}$ represents the aggregate state at $t+1$. In the following, we derive the pricing functions that describe a stationary solution to the problem using these state variables. The stock price at time t is denoted by $q_t = P(a, \gamma_t, Z_t, N_t, r_t, \bar{y}_t)$.

First, the price associated with a high report in an aggregate state $a_t \in \{g, b\}$, $P(a_t, \gamma_t, Z_t, N_t, \tilde{h}, \bar{y})$, is derived.

$$P(a_t, \gamma_t, Z_t, N_t, \tilde{h}, \bar{y}) = a_t\tilde{h} + (1 - \lambda_{a_t})W_{na_t}^{\tilde{h}} + \lambda_{a_t}W_{ia_t}^{\tilde{h}}. \quad (2)$$

Here, $W_{na}^{\tilde{h}}$ represents the expected future value if the detection does not occur this period, and $W_{ia}^{\tilde{h}}$ represents the expected future value if the detection occurs this period. Both prices are conditional on a current high report of idiosyncratic productivity.

If the detection does not take place this period, the expected future value is

$$W_{na}^{\tilde{h}} = \beta\pi_{ag} \left[\mu_g P(g, \gamma'_g, Z, N+1, \tilde{h}, \bar{y}) + (1 - \mu_g) P(g, 0, Z, N+1, \tilde{l}, \bar{y}) \right] + \beta\pi_{ab} \left[\mu_b P(b, \gamma'_b, Z, N+1, \tilde{h}, \bar{y}) + (1 - \mu_b) P(b, 0, Z, N+1, \tilde{l}, \bar{y}) \right] \quad (3)$$

Here, β is the discount factor. The first term in (3) is the expected price if the next period's aggregate state is good. The second term is the expected price if the next period's aggregate state is bad. For each case, there involves two possibilities: the next period's report is high

and the next period's report is low. Note that a low report is always truthful, and thus γ is updated to 0. μ_a denotes the conditional probability that the manager makes a high report of individual productivity in the next period when the aggregate state is $a \in \{g, b\}$:

$$\mu_a = \gamma\pi_{hh} + \gamma(1 - \pi_{hh})x_a + (1 - \gamma)\pi_{lh} + (1 - \gamma)(1 - \pi_{lh})x_a, \forall a \in \{g, b\}.$$

If the detection takes place this period, the expected value is

$$\begin{aligned} W_{ia}^{\tilde{h}} = & -\kappa[Z + f(N + 1; \bar{y})] \\ & + \beta\pi_{ag} \left[\gamma[\xi_{1g}P\left(g, \frac{\pi_{hh}}{\xi_{1g}}, 0, 0, \tilde{h}, h\right) + (1 - \xi_{1g})P(g, 0, 0, 0, \tilde{l}, h)] \right. \\ & \quad \left. + (1 - \gamma)[\xi_{2g}P\left(g, \frac{\pi_{lh}}{\xi_{2g}}, 0, 0, \tilde{h}, l\right) + (1 - \xi_{2g})P(g, 0, 0, 0, \tilde{l}, l)] \right] \\ & + \beta\pi_{ab} \left[\gamma[\xi_{1b}P\left(b, \frac{\pi_{hh}}{\xi_{1b}}, 0, 0, \tilde{h}, h\right) + (1 - \xi_{1b})P(g, 0, 0, 0, \tilde{l}, h)] \right. \\ & \quad \left. + (1 - \gamma)[\xi_{2b}P\left(b, \frac{\pi_{lh}}{\xi_{2b}}, 0, 0, \tilde{h}, l\right) + (1 - \xi_{2b})P(b, 0, 0, 0, \tilde{l}, l)] \right]. \end{aligned} \quad (4)$$

where ξ_{1a} represents the conditional probability of having a high report in the next period when the next period's aggregate state is $a \in \{h, l\}$, given the current true idiosyncratic productivity is high. ξ_{2a} is the probability of having a high report conditional on that the current true idiosyncratic productivity is low.

$$\xi_{1a} = \pi_{hh} + (1 - \pi_{hh})x_a$$

$$\xi_{2a} = \pi_{lh} + (1 - \pi_{lh})x_a$$

The first term in (4) is the expected amount of financial penalties for manipulation. $f(N + 1; \bar{y})$ denotes the expected number of falsified reports among the $(N + 1)$ consecutive reports of high idiosyncratic productivity since the last low report or the last detection, whichever is more recent. Given a history of aggregate states $\mathcal{A}_N \equiv \{a_1, a_2, \dots, a_N\}$, the function $f(N + 1; \bar{y})$ is calculated from the model fundamental in a recursive manner, and the method is illustrated in Appendix C. The number of the expected inflated reports is thus the sum of $f(N + 1; \bar{y})$ and the expected number of periods involving earnings management from the last detection through the most recent low report, Z . Recall that γ is the conditional probability that the current high idiosyncratic productivity is truthful. The second term in

(4) and the third term represent the expected price if the next period's aggregate state is good and bad respectively. For each of the two cases, there are two possible scenarios: the current high report is truthful, or the current idiosyncratic productivity is low and has been overstated.

Now let us consider the asset price if the current report is low.

$$P(a_t, 0, Z_t, N_t, \tilde{l}, \bar{y}) = a_t \tilde{l} + (1 - \lambda_{a_t}) W_{na_t}^{\tilde{l}} + \lambda_{a_t} W_{ia_t}^{\tilde{l}}. \quad (5)$$

where $W_{na}^{\tilde{l}}$ and $W_{ia}^{\tilde{l}}$ represent the expected value if the detection does not occur this period and the expected value if the detection occurs, respectively, conditional on a current low report.

If the detection does not take place this period, the expected future value is

$$W_{na}^{\tilde{l}} = \beta \pi_{ag} \left[\xi_g P \left(g, \frac{\pi_{lh}}{\xi_g}, Z, 0, \tilde{h}, l \right) + (1 - \xi_g) P(g, 0, Z, 0, \tilde{l}, l) \right] \\ + \beta \pi_{ab} \left[\xi_b P \left(b, \frac{\pi_{lh}}{\xi_b}, Z, 0, \tilde{h}, l \right) + (1 - \xi_b) P(b, 0, Z, 0, \tilde{l}, l) \right],$$

where ξ_a denotes the conditional probability that the manager makes a high report in the next period:

$$\xi_a = \pi_{lh} + (1 - \pi_{lh}) x_a, \forall a \in \{g, b\}.$$

The first term represents the discounted expected value if the next period's aggregate state is good, and the second term represents the discounted expected value if the next period's aggregate state is bad. For each aggregate state, we consider the case of a higher report and the case of a low report.

If the detection takes place in the next period, the expected price is

$$W_{ia}^{\tilde{l}} = -\kappa [Z + f(N; \bar{y})] \\ + \beta \pi_{ag} \left[\xi_g P \left(g, \frac{\pi_{lh}}{\xi_g}, 0, 0, \tilde{h}, l \right) + (1 - \xi_g) P(g, 0, 0, 0, \tilde{l}, l) \right] \\ + \beta \pi_{ab} \left[\xi_b P \left(b, \frac{\pi_{lh}}{\xi_b}, 0, 0, \tilde{h}, l \right) + (1 - \xi_b) P(b, 0, 0, 0, \tilde{l}, l) \right] \quad (6)$$

The first term in (6) is the expected monetary losses incurred for manipulation, which is a linear function of the expected number of inflated reports. The second term is the discounted

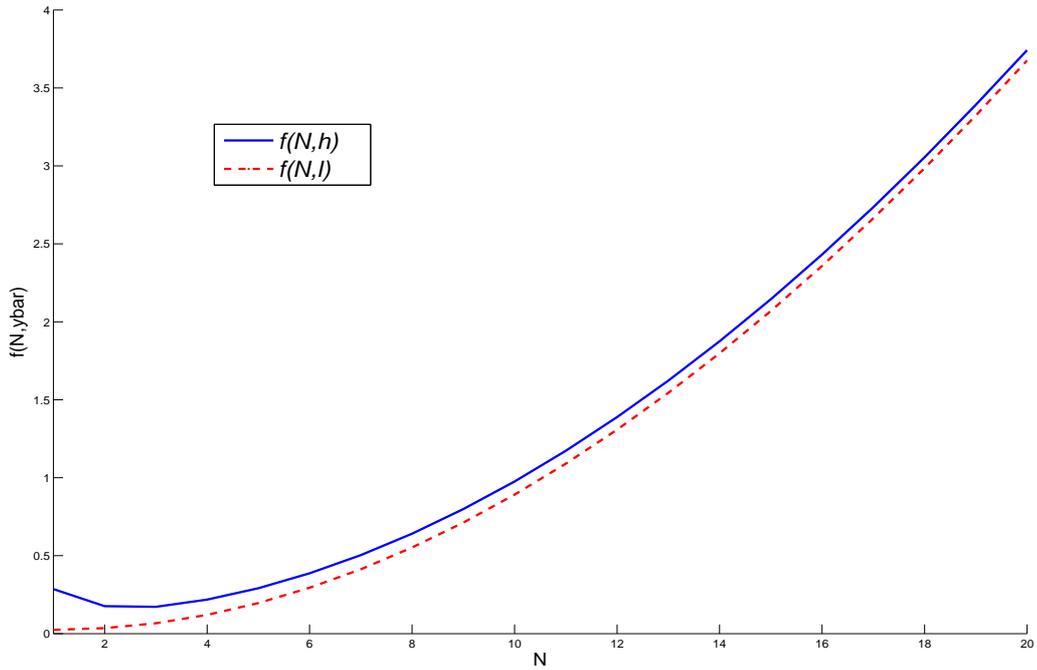


Figure 11: The expected number of inflated reports among N consecutive high reports $f(N + 1, \bar{y})$

expected price if the aggregate state is good next period, and the third term corresponds to the case if the aggregate state is bad next period. For each aggregate state, we consider the case if the realization of idiosyncratic productivity is high in the next period and the case in which the realization is low. Thus, from (2) and (5), the price in each period can be solved recursively.

4.3 Properties of equilibrium prices

The pricing functions are computed numerically. Figure 11 displays $f(N, \bar{y})$, the shape of which depend on the history of aggregate states and vary with parameterizations. Figure 12 and Figure 13 show how the prices associated with a high report change with γ and N . As the monetary penalties associated with earnings management is a linear function of the number of restated financial statements, the price in response to a high report is linearly increasing in γ and linearly decreasing in Z . As shown in Figure 14, the price in response

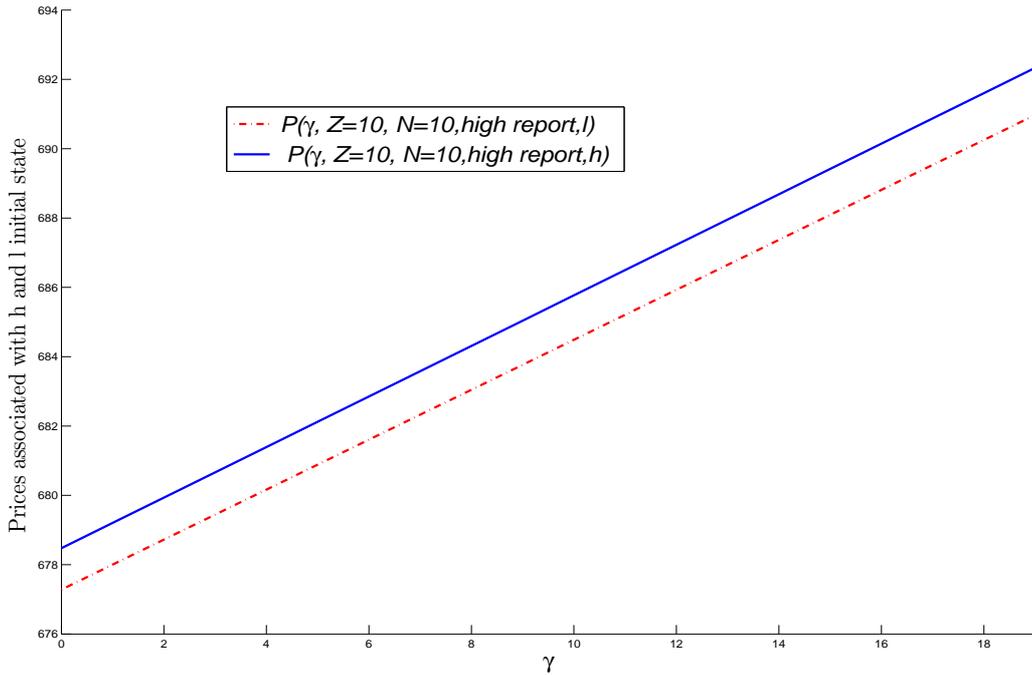


Figure 12: Price for a high report as a function of γ

to a low report is also linearly decreasing in both Z , with γ updated to 0.

5 Quantitative results

In this section, we present a calibrated model that allows us to assess the quantitative significance of cyclical regulatory investigations for stock market dynamics. We first select parameters values to match the key moments of their empirical counterparts. We then present the features of model returns that resemble the financial data. We comment on the comparative static analysis at the end of this section.

5.1 Calibration strategy

We simulate the model discussed in Section 3 with five hundred firms for 2000 periods. We choose the model's parameters so as to match its key population moments to the empirical counterparts. Table 1 reports the calibrated parameter values. We calibrate the Markov process of the aggregate state following Krusell and Smith (1998). Given the aggregate state

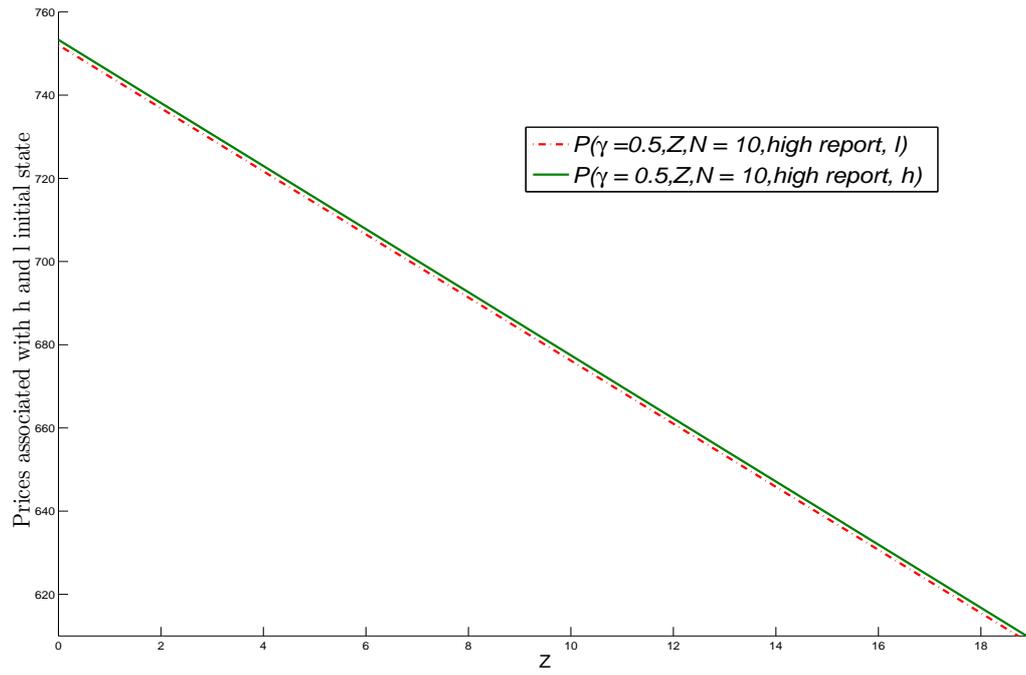


Figure 13: Price for a high report as a function of Z

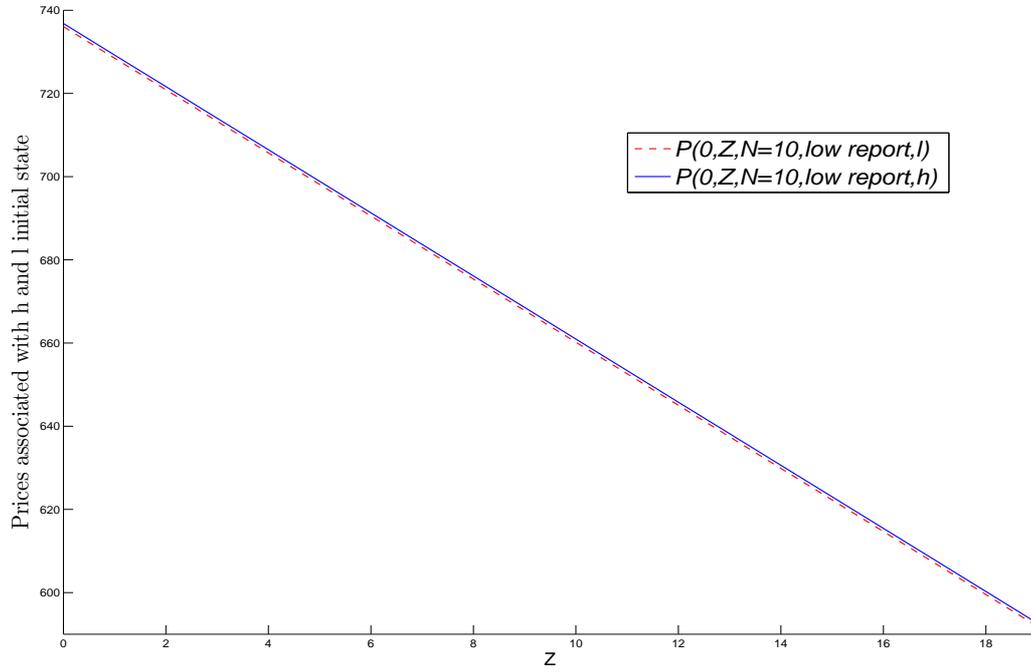


Figure 14: Price for a low report as a function of Z

Table 1: Calibrated parameter values

Parameter	Description	Value
g	Level of good aggregate state	1.01
b	Level of bad aggregate state	0.99
h	Level of high idiosyncratic state	5.614
l	Level of low idiosyncratic state	6.385
π_{gg}	Persistence in aggregate state	0.925
π_{hh}	Persistence in idiosyncratic productivity	0.748
β	Discount factor	0.98
κ	Monetary loss	$4.08(h - l)$
λ_g	Detection frequency in good aggregate state	0.024
λ_b	Detection frequency in bad aggregate state	0.037
x_g	Manipulation likelihood in good aggregate state	0.11
x_b	Manipulation likelihood in bad aggregate state	0.08

process, we use Tauchen method to calibrate the transition probabilities and binary levels of the idiosyncratic productivity to match the mean and standard deviation of the S&P Composite deflated scaled earnings. The discount factor β is chosen to be 0.98 so that the implied quarterly real interest rate is 2 percent. Karpoff, Lee, and Martin (2008) estimate that for each dollar of value inflation the firm on average loses \$4.08 when the misconduct is revealed, which gives us the value of κ in the model. We currently use the average percentage of firms subject to legal actions during NBER recessions and that in other periods as λ_b and λ_g respectively. In choosing x_g and x_b , we use the regression results from Cohen and Zarowin (2012).

5.2 Countercyclical crash risk

We compute several measures of crash risk conditional on good aggregate state and that conditional on bad aggregate state in the simulated model returns. The first column in Table 2 shows the average percentage of firms experiencing a stock crash, defined as cumulative real return of -0.25 or lower (Barro and Ursua 2009), in the good aggregate state and bad aggregate state respectively. The second and third column show the average value of COUNT at 0.1% and 0.01% frequencies in the good and bad aggregate state. Following Jin

	% crash	COUNT(0.1%)	COUNT(0.01%)	kurtosis
a=b	3.59	0.24	0.21	8.52
a=g	1.22	0.015	0.09	3.85

Table 2: Countercyclical crash risk

and Myers (2006), we calculate COUNT, as the frequency of crash, based on the number of residual returns exceeding k standard deviations above and below the mean, with k chosen to generate frequencies of 0.01% or 0.1% in the lognormal distribution. Following Jin and Myers (2006), we subtract the upside frequencies from the downside frequencies. A high value of COUNT indicates a high frequency of crashes. In the last column, we measure crash risk using the fourth moment of stock returns about the mean scaled by squared variance.

There are two reinforcing mechanisms for countercyclical crash risk to emerge in the model, one through information revelations and the other through incentive distortions. First, systematic upward bias in financial information mitigate price responses, and the lack of investigations and revelations in good times render stock fluctuations fairly mild. During economic downturns, however, strengthened regulation releases accumulated hidden negative information, most stockpiled in good times. Waves of frauds are revealed and penalized, causing returns to plummet. Second, loosened regulation during booms helps fuel managerial incentives to paint a successful yet inaccurate picture of firm performance. Increased noise in reports further mutes stock movements in good times, and cause the downturns even sharper when the accumulated losses all come out at once in bad times.

5.3 Other asset pricing implications

A. Gradual booms and sudden crash

To judge the asymmetry of the model and the data, following Veldkamp (2005) we use two measures: time-irreversibility and skewness. A stochastic process is time-reversible if the probability of starting at any point x and moving to any y is the same as the probability of starting at y and moving to x . Reversing a process like stock returns that has large declines and gradual increases would produce slow declines and sudden increases. Because reversing stock returns changes their properties, they are time-irreversible. Similarly, many gradual

increases and occasional large declines produce a distribution of changes with many small positive observations and a few large negative outliers. The fat left tail in this distribution is measured as negative skewness.

- Time-Reversibility [To be completed]
- Skewness: -0.029

Cyclical regulatory activities cause gradual booms and sudden crashes in the financial markets through two channels. First, regulatory investigations detect all the past manipulation and reveal negative information in large lumps. The asymmetric regulatory responses to business cycles cause more negative information to be revealed during bad times, resulting in large, negative returns during episodes of weak economic conditions. As limited investigations and subsequent revelations in the up market leave upward bias in financial information unresolved, investors discount the seemingly excellent and potentially fraudulent performance, slowly updating their beliefs about corporate outlooks. Second, infrequent regulatory actions in booms fuel managerial incentives to manipulate, causing more aggressive manipulation only to be revealed and reversed at the beginning of recessions when investigations become intense. The distorted incentives due to regulatory cycles lead to more manipulation in good times, and the associated greater bias further mitigate investors' response to positive corporate news. The increased prevalence and severity of manipulation also cause the downturn even sharper, when the substantial accumulated frauds all come out at once.

B. Increased crash risk

Following the literature, we use the following variables to measure the frequency of crashes in the model.

- NCSKEW: Following Chen, Hong and Stein (2001), we calculate the skewness of residual returns as the third moment of each stock's residual returns, divided by the cubed standard deviation. Negative skewness indicates a high probability of crash.
- COUNT: Frequency of crash is calculated based on the number of residual returns exceeding k standard deviations above and below the mean, with k chosen to generate

frequencies of 0.01%, 0.1% or 1% in the lognormal distribution. Following Jin and Myers (2006), we subtract the upside frequencies from the downside frequencies. A high value of COUNT indicates a high frequency of crashes.

- Kurtosis: We measure heavy tails using the fourth moment of stock returns about the mean scaled by squared variance. A high value of kurtosis indicates a high frequency of crashes.

	NCSKEW	COUNT			Kurtosis
		0.01%	0.1%	1%	
model	-0.029	0.128	0.194	0.249	6.707

Table 3: Crash risk

Jin and Myers (2006) show crash risk can emerge when managers have incentives to stockpile bad news, but in some circumstances those incentives collapse, leading to a sudden release of accumulated negative information and a stock price crash. Our model suggests that cyclical tendencies in regulatory behavior exacerbate the impact of manipulation by accumulating more negative information in good times only to be revealed in economic downturns, and thereby further increase crash risk.

C. Countercyclical stock volatilities

One of the most prominent features of the U.S. stock market is the close connection between aggregate stock market volatility and the development of the business cycle. Table 4 reports descriptive statistics for stock returns during National Bureau of Economic Research (NBER)- dated expansions and recessions.

	total	good state	bad state
Model return volatility	0.022	0.018	0.028

Table 4: Moments of the model returns

Table 4 displays the key conditional population moments of the model. The oscillations of return volatility from good states to bad are asymmetric as in the data — as the return volatility moves away from the average state, it increases more in bad states than it decreases

in good states. Countercyclical volatility arises in our model because intense regulatory actions in economic downturns reveal substantial hidden negative information that managers stockpiled in booms due to loosened regulation, causing stock returns to become increasingly volatile as economic conditions deteriorate. In good times, on the one hand, upward manipulation in low idiosyncratic states compresses distributions of reports and smoothes reported earnings over time, reducing volatilities. On the other hand, weak regulation and limited information revelation render stock price less sensitive to reported performance, and return movements are moderate compared to substantial declines upon intensive revelations when the system is facing strains.

5.4 Comparative statics analysis

The sensitivity analysis varies one baseline parameter at a time, and calculates the key measures of the simulated stock returns. For a wide range of parameter values, the model consistently produces significant countercyclical volatilities, negative skewness, large COUNT, and excess kurtosis in stock returns. To isolate the effects of cyclicity in regulatory investigations on asset prices, we consider 1) a benchmark economy with a constant detection probability independent of the aggregate state; 2) the calibrated model; and 3) the calibrated model except that investigation likelihood is positively related with the aggregate state.

6 Discussion

6.1 Key drivers of model results

This subsection discusses which of the model's features are necessary for its key results. In substance, there are three essential features in the model: First, regulatory investigations are more intense during economic downturns than during booms. Second, there is uncertainty in financial information, and manipulation is not fully unraveled in the financial markets without regulatory detection. Third, revelation of manipulation imposes monetary losses on investors and substantially reduce returns. These three blocks generate the stylized patterns in stock return data. Weak regulatory actions fuel managerial incentives to manipulate during periods of booms, which cause more negative hidden information to be revealed and

returns drop more sharply in bad times when regulatory investigations are intense, contributing to the asymmetry in stock markets, increased crash risk, and increased correlation among stocks in downturns. We discuss each element in greater detail below.

Cyclical tendencies in regulatory investigation

In the model, state-dependent detection difficulties, together with stronger political pressure during economic downturns, give rise to cyclical regulatory intensities, which in turn leads to cyclical frequencies of financial manipulation. As a result, more hidden negative information is accumulated during periods of economic growth, only to be revealed in large lumps when the economic and financial systems are facing strains. But the seeds of the strains during downturns are sowed during the preceding upswing. The substantial information risk that builds up during upturns due to loosen regulation will be materialized through concentrated revelations in downturns. We provide evidence on the time-series patterns of regulatory investigations in Sections 2, and highlight that frequencies of regulatory actions peaked during the recessions in both recent business cycles.

Unraveled information manipulation

Both true idiosyncratic productivity and managerial private benefit from inflating reports are managers' private information, however, managers are only permitted to communicate a single-dimensional signal in the model, which is an earnings announcement. Communication is restricted in that managers cannot communicate the full dimensionality of their private information due to a limited message space. As a result, the reporting function is not invertible, and true earnings cannot be unambiguously backed out from earnings reports. In our model investors can infer the likelihood of manipulation in each period (x_a), but cannot perfectly gauge the true state of the firm.

In reality, because the market does not precisely know managers' private benefits from manipulating reports, and managers are not all equally versed in manipulating financial records, shareholders often face bias in the financial results, but cannot determine the extent of the bias. Our model features the substantial discrepancy between market expectations and the firms underlying financial worth that characterizes many recent corporate scandals.

Adverse consequences of manipulation

Investors bear monetary losses when manipulation is detected in the model. which gener-

ates sharp declines in returns. A result of this assumption is that revelation of manipulation causes substantial movements in returns and raises volatility. This model feature is intended to capture the financial cost of misreporting that investors bear in practice. For example, SEC enforcement actions and restatement announcements are associated with drastic market reactions and negative impact on firms' future prospects (e.g. stock returns on average fall by about 10% around earnings restatements in the data). The Securities and Exchange Commission has collected over \$20 billion penalties in fraud cases since 2002, and the amount of settlement fines has been growing over time. The loss of confidence in corporate financial reporting could also hurt business and investment opportunities. Furthermore, the reduced availability and higher cost of capital may force firms to forgo investment and accelerate layoffs. Karpoff, Lee, and Martin (2008) show that the loss of firm value (in terms of the present value of the loss of future cash flow) upon fraud detection is substantial. According to their estimate, for each dollar of value inflation the firm on average loses \$4.08 when the misconduct is revealed.

6.2 Implications (extremely tentative)

— This framework can be used to study credit policy in banking sector: aggressive credit policy enhances current performance, but causes losses when risk materializes, and losses are especially severe during periods of downturns that are typically associated with a sizable credit crunch. What we observe in this latest financial cycle has been, first, a huge expansion of credit, a massive rise in leveraging during the upswing, followed by the crisis, curtailment of credit expansion and major de-leveraging with severe effects on the real economy. The present regulatory system (comprising Basel II and the move to mark-to-market accounting practices) not only did too little to restrain the upswing, but is also exacerbating the downturn. In other words it is highly procyclical.

— There are underlying economic frictions driving cyclical tendencies in regulation (light during normal periods, increasing as systemic threats build up), which engender these asset pricing patterns symptomatic of risk and inefficiency. In addition to the widely noted tendency of national authorities and the general public to resist warnings of vulnerability during good times, a problem any benevolent regulatory bodies face can be a state-varying

hurdle in implementing effective regulation. The cycle-proof or even counter-cyclical regulatory guidelines would be hard to implement through supervisory discretion and have to be rule-based.

7 Conclusion

The model in this article is meant to capture a simple—and we believe important—piece of intuition about the effect of cyclical regulatory policies on stock markets: negative information hidden by corporate executives is more likely to be flushed out through investigations when the economy is falling, as opposed to rising. As we have argued, this mechanism can help shed light on a variety of stylized facts and in particular, countercyclical crash risk. We also provide empirical evidence for business cycle variations of crash risk in the data. Our model indicates that the dual role of regulatory activities in both deterring manipulation and revealing hidden negative information has considerable implications for asset pricing, and their timing is important for crash risk over the business cycle.

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Appendix

A Examples of state variables

As the monetary penalties upon investigation depends on the number of restated financial statements, the expected number of periods in which the manager inflates earnings since the most recent realization up to now is necessary in characterizing the prices. If there are N consecutive high reports and no low reports after the most recent investigation, a function of $f(N; \bar{y})$ determines the expected number of periods involving earnings management until the last period. If there is any low report after the last investigation, the sum of Z and $f(N; \bar{y})$ summarizes the history. In addition, γ and r incorporate the information regarding the current true state conveyed by the current report.

To be clear on what each variable represents, a set of clarifying examples is provided in the following. Now let today be $t = 10$ and let the last investigation happen at the beginning of $t = 5$. Suppose that the true state of $t = 4$ is revealed to be y_4 .

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 7; $N = 1$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 8 is known to be low (recall that all the low reports are honest reports).
- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports.
- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}\}$, then, at $t = 10$, $Z = 0$ (there is not any low report after the last investigation until the previous period); $N = 5$; and $r = \tilde{l}$. $\bar{y} = y_4$, because it is the known true state before the consecutive high reports. Note that $\gamma = 0$ at $t = 10$, because the current low report is an honest one.

- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, and 8; $N = 0$ (it does not include the current period); and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (all the low reports are honest reports). Note that in the case of $N = 0$, \bar{y} is set to be y_{t-1} ($N = 0$ occurs only when the report at $(t - 1)$ is low or the investigation happens at the beginning of t).
- If $\{r_5, r_6, r_7, r_8, r_9, r_{10}\} = \{\tilde{h}, \tilde{h}, \tilde{h}, \tilde{h}, \tilde{l}, \tilde{h}\}$, then, at $t = 10$, Z is the expected number of inflated reports during periods 5, 6, 7, and 8; $N = 0$; and $r = \tilde{h}$. $\bar{y} = l$, because the true state in period 9 is known to be low (Again, all the low reports are honest reports).

Let today be $t = 5$ and let the investigation happen at the beginning of $t = 5$.

- If $r_5 = \tilde{h}$, then $Z = 0$, $N = 0$, $r = \tilde{h}$, and $\bar{y} = y_4$.
- If $r_5 = \tilde{l}$, then $Z = 0$, $N = 0$, $r = \tilde{l}$, and $\bar{y} = y_4$.

B Computational strategy

C Calculation of $f(N; \bar{y})$ in the model with stochastic investigation

Let the information set $\mathcal{R}_N^{\bar{y}} \equiv \{\bar{y}, r_1 = \tilde{h}, r_2 = \tilde{h}, \dots, r_N = \tilde{h}\}$. y_n represents the true earnings in period n , $\forall n \in \{1, 2, \dots, N\}$. Thus $f(N; \bar{y})$ can be written as

$$f(N; \bar{y}) = \Pr[y_1 = l | \mathcal{R}_N^{\bar{y}}] + \Pr[y_2 = l | \mathcal{R}_N^{\bar{y}}] + \dots \\ + \Pr[y_n = l | \mathcal{R}_N^{\bar{y}}] + \dots + \Pr[y_N = l | \mathcal{R}_N^{\bar{y}}]$$

The problem of deriving $f(N; \bar{y})$ in a recursive way is transformed into an equivalent problem, that is, to recursively derive

$$\Pr[y_n = l | \mathcal{R}_N^{\bar{y}}] = 1 - \Pr[y_n = h | \mathcal{R}_N^{\bar{y}}], \quad \forall n \in \{1, 2, \dots, N\}.$$

Note that

$$\begin{aligned}\mathcal{R}_N^h &\equiv \{h, r_1 = \tilde{h}, r_2 = \tilde{h}, \dots, r_N = \tilde{h}\} \\ \mathcal{R}_N^l &\equiv \{l, r_1 = \tilde{h}, r_2 = \tilde{h}, \dots, r_N = \tilde{h}\}\end{aligned}$$

The proof includes two steps. In step 1, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are calculated. In step 2, I show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ and $\Pr[y_n = h|\mathcal{R}_{N+1}^h]$, $\forall n \in \{1, 2, \dots, N+1\}$, can be calculated using $\Pr[y_n = h|\mathcal{R}_N^l]$ and $\Pr[y_n = h|\mathcal{R}_N^h]$, $\forall n \in \{1, 2, \dots, N\}$.

As the first step, $\Pr[y_1 = h|\mathcal{R}_1^l]$ and $\Pr[y_1 = h|\mathcal{R}_1^h]$ are derived as follows.

$$\begin{aligned}\Pr[y_1 = h|\mathcal{R}_1^l] &= \Pr[y_1 = h|\bar{y} = l, r_1 = \tilde{h}] \\ &= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = l]}{\Pr[r_1 = \tilde{h}|\bar{y} = l]} \\ &= \frac{\pi_{lh}}{\pi_{lh} + (1 - \pi_{lh})x_a}, \\ \Pr[y_1 = h|\mathcal{R}_1^h] &= \Pr[y_1 = h|\bar{y} = h, r_1 = \tilde{h}] \\ &= \frac{\Pr[y_1 = h, r_1 = \tilde{h}|\bar{y} = h]}{\Pr[r_1 = \tilde{h}|\bar{y} = h]} \\ &= \frac{\pi_{hh}}{\pi_{hh} + (1 - \pi_{hh})x_a},\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period 1.

In step 2, I first show that $\Pr[y_n = h|\mathcal{R}_{N+1}^l]$ can be calculated if $\Pr[y_n = h|\mathcal{R}_N^l]$ is known. For $n \in \{1, 2, \dots, N+1\}$,

$$\Pr[y_n = h|\mathcal{R}_N^l, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h}|\mathcal{R}_N^l]}{\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^l]}. \quad (7)$$

The denominator in (7), $\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^l]$, is derived as the following.

$$\begin{aligned}\Pr[r_{N+1} = \tilde{h}|\mathcal{R}_N^l] &= \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h|\mathcal{R}_N^l] + \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l|\mathcal{R}_N^l] \\ &= \Pr[r_{N+1} = \tilde{h}|y_{N+1} = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h|\mathcal{R}_N^l] \\ &\quad + \Pr[r_{N+1} = \tilde{h}|y_{N+1} = l, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l|\mathcal{R}_N^l] \\ &= \Pr[y_{N+1} = h|\mathcal{R}_N^l] + x_a [1 - \Pr[y_{N+1} = h|\mathcal{R}_N^l]],\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$, and

$$\begin{aligned}
\Pr[y_{N+1} = h | \mathcal{R}_N^l] &= \Pr[y_{N+1} = h, y_N = h | \mathcal{R}_N^l] + \Pr[y_{N+1} = h, y_N = l | \mathcal{R}_N^l] \\
&= \Pr[y_{N+1} = h | y_N = h, \mathcal{R}_N^l] \times \Pr[y_N = h | \mathcal{R}_N^l] \\
&\quad + \Pr[y_{N+1} = h | y_N = l, \mathcal{R}_N^l] \times \Pr[y_N = l | \mathcal{R}_N^l] \\
&= \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^l] + \pi_{lh} [1 - \Pr[y_N = h | \mathcal{R}_N^l]]. \tag{8}
\end{aligned}$$

As $\Pr[y_N = h | \mathcal{R}_N^l]$ is known from the supposition, this can be calculated. The denominator is obtained

$$\begin{aligned}
\Pr[r_{N+1} = \tilde{h} | \mathcal{R}_N^l] &= \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^l] + \pi_{lh} [1 - \Pr[y_N = h | \mathcal{R}_N^l]] \\
&\quad + x_a \{1 - \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^l] - \pi_{lh} [1 - \Pr[y_N = h | \mathcal{R}_N^l]]\}, \tag{9}
\end{aligned}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$.

Now let us consider the numerator in (7). For $n = N + 1$, $\Pr[y_{N+1} = h, r_{N+1} = \tilde{h} | \mathcal{R}_N^l]$ can be rewritten as

$$\begin{aligned}
\Pr[y_{N+1} = h, r_{N+1} = \tilde{h} | \mathcal{R}_N^l] &= \Pr[r_{N+1} = \tilde{h} | y_{N+1} = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h | \mathcal{R}_N^l] \\
&= \Pr[y_{N+1} = h | \mathcal{R}_N^l],
\end{aligned}$$

where $\Pr[y_{N+1} = h | \mathcal{R}_N^l]$ is derived in (8).

For $n \in \{1, 2, \dots, N\}$, the numerator $\Pr[y_n = h, r_{N+1} = \tilde{h} | \mathcal{R}_N^l]$ can be rewritten as

$$\Pr[y_n = h, r_{N+1} = \tilde{h} | \mathcal{R}_N^l] = \Pr[r_{N+1} = \tilde{h} | y_n = h, \mathcal{R}_N^l] \times \Pr[y_n = h | \mathcal{R}_N^l].$$

Here, $\Pr[y_n = h | \mathcal{R}_N^l]$ is known from the supposition. Now we only need to check if $\Pr[r_{N+1} = \tilde{h} | y_n = h, \mathcal{R}_N^l]$ can be calculated. I rewrite

$$\Pr[r_{N+1} = \tilde{h} | y_n = h, \mathcal{R}_N^l] = \Theta + \Lambda,$$

where

$$\begin{aligned}
\Theta &= \Pr[r_{N+1} = \tilde{h}, y_{N+1} = h | y_n = h, \mathcal{R}_N^l] \\
&= \Pr[r_{N+1} = \tilde{h} | y_{N+1} = h, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] \\
&= 1 \times \Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] \\
&= \Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l], \tag{10}
\end{aligned}$$

$$\begin{aligned}
\Lambda &= \Pr[r_{N+1} = \tilde{h}, y_{N+1} = l | y_n = h, \mathcal{R}_N^l] \\
&= \Pr[r_{N+1} = \tilde{h} | y_{N+1} = l, y_n = h, \mathcal{R}_N^l] \times \Pr[y_{N+1} = l | y_n = h, \mathcal{R}_N^l] \\
&= x_a [1 - \Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l]] \\
&= x_a [1 - \Theta],
\end{aligned} \tag{11}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$.

If $n = N$, it is straightforward to determine that

$$\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] = \pi_{hh}.$$

Now let us consider $\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l]$ if $n < N$. Because actual earnings y follow a Markov process, all the past information is fully summarized in the most recent realization, and the prior realizations are informationally irrelevant. Thus,

$$\begin{aligned}
\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] &= \Pr[y_{N+1} = h | y_n = h, \bar{y} = l, r_1 = \tilde{h}, \dots, r_N = \tilde{h}], \\
&= \Pr[y_{N+1} = h | \bar{y} = h, r_{n+1} = \tilde{h}, \dots, r_N = \tilde{h}]
\end{aligned}$$

and

$$\Pr[y_{N+1} = h | \bar{y} = h, r_{n+1} = \tilde{h}, \dots, r_N = \tilde{h}] = \Pr[y_{N-n+1} | \bar{y} = h, r_1 = \tilde{h}, \dots, r_{N-n} = \tilde{h}].$$

Recall that $\mathcal{R}_{N-n}^h \equiv \{\bar{y} = h, r_1 = \tilde{h}, \dots, r_{N-n} = \tilde{h}\}$. Therefore,

$$\Pr[y_{N+1} = h | y_n = h, \mathcal{R}_N^l] = \begin{cases} \Pr[y_{N-n+1} = h | \mathcal{R}_{N-n}^h] & \text{if } n < N, \\ \pi_{hh} & \text{if } n = N. \end{cases} \tag{12}$$

and

$$\begin{aligned}
\Pr[y_{N-n+1} = h | \mathcal{R}_{N-n}^h] &= \Pr[y_{N-n+1} = h, y_{N-n} = h | \mathcal{R}_{N-n}^h] + \Pr[y_{N-n+1} = h, y_{N-n} = l | \mathcal{R}_{N-n}^h] \\
&= \Pr[y_{N-n+1} = h | y_{N-n} = h, \mathcal{R}_{N-n}^h] \times \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h] \\
&\quad + \Pr[y_{N-n+1} = h | y_{N-n} = l, \mathcal{R}_{N-n}^h] \times \Pr[y_{N-n} = l | \mathcal{R}_{N-n}^h] \\
&= \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h] + \pi_{lh} [1 - \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h]],
\end{aligned}$$

where $\Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h]$ is known from the supposition, since $N - n < N$. Therefore, Θ and Λ can be both calculated. Hence, the numerator in (7) can be derived following this

procedure. The numerator is obtained

$$\Pr[y_n = h, r_{N+1} = \tilde{h} | \mathcal{R}_N^l] = \begin{cases} \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^l] + \pi_{lh} [1 - \Pr[y_N = h | \mathcal{R}_N^l]] & \text{if } n = N + 1, \\ \Pr[y_N = h | \mathcal{R}_N^l] [\pi_{hh} + x_a(1 - \pi_{hh})] & \text{if } n = N, \\ \Pr[y_n = h | \mathcal{R}_N^l] \left\{ \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h] + \right. \\ \left. \pi_{lh} [1 - \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h]] \right. \\ \left. + x_a \{1 - \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h] - \pi_{lh} [1 - \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h]]\} \right\}, & \text{if } n < N. \end{cases} \quad (13)$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$.

Now combining the expressions (9) and (13), it has been shown that $\Pr[y_n = h | \mathcal{R}_N^l, r_{N+1} = \tilde{h}]$ can be calculated using $\Pr[y_n = h | \mathcal{R}_N^l, r_N = \tilde{h}]$. The same procedure can be repeated for $\Pr[y_n = h | \mathcal{R}_N^h, r_{N+1} = \tilde{h}]$ as follows.

$$\Pr[y_n = h | \mathcal{R}_N^h, r_{N+1} = \tilde{h}] = \frac{\Pr[y_n = h, r_{N+1} = \tilde{h} | \mathcal{R}_N^h]}{\Pr[r_{N+1} = \tilde{h} | \mathcal{R}_N^h]}.$$

where the denominator is

$$\Pr[r_{N+1} = \tilde{h} | \mathcal{R}_N^h] = \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^h] + \pi_{lh} [1 - \Pr[y_N = h | \mathcal{R}_N^h]] \\ + x \{1 - \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^h] - \pi_{lh} [1 - \Pr[y_N = h | \mathcal{R}_N^h]]\}.$$

and the numerator is

$$\Pr[y_n = h, r_{N+1} = \tilde{h} | \mathcal{R}_N^h] = \begin{cases} \pi_{hh} \Pr[y_N = h | \mathcal{R}_N^h] + \pi_{lh} [1 - \Pr[y_N = h | \mathcal{R}_N^h]] & \text{if } n = N + 1, \\ \Pr[y_N = h | \mathcal{R}_N^h] [\pi_{hh} + x_a(1 - \pi_{hh})] & \text{if } n = N, \\ \Pr[y_n = h | \mathcal{R}_N^h] \left\{ \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h] + \right. \\ \left. \pi_{lh} [1 - \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h]] \right. \\ \left. + x_a \{1 - \pi_{hh} \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h] - \pi_{lh} [1 - \Pr[y_{N-n} = h | \mathcal{R}_{N-n}^h]]\} \right\}, & \text{if } n < N. \end{cases}$$

where $a \in \{b, g\}$ represents the aggregate state in period $N + 1$.