DISCLOSURE POLICY AND INDUSTRY FLUCTUATIONS*

JEREMY BERTOMEU  PIERRE JINGHONG LIANG

Abstract

This paper presents a theory that relates industry cycles to firms’ decision to share information. In the model, firms may be informed about upcoming demand in advance of their competitors and decide whether to publicly disclose such information. We examine public disclosures and their association with price-setting behavior and industry profits along industrial cycles. We show that, in industries that are highly concentrated or feature lower cost of capital, no-disclosure is prevalent and associated with acyclical product prices and higher profit margins. Otherwise, public disclosures must be made to soften competition; in particular, disclosures occurs during moderate demand shocks, while no-disclosure occurs prior to either sharp industry expansions or contractions. Consequently, the disclosure policy can dampen the dissemination of shocks to the industry.

*Jeremy Bertomeu (jeremy.bertomeu@baruch.cuny.edu) is from the Stan Ross Department of Accountancy at Baruch College and Pierre Jinghong Liang (liangj@andrew.cmu.edu) is from the Tepper School of Business at Carnegie Mellon University. Many thanks to Tim Baldenius, Mark Bagnoli, Ron Dye, Pingyang Gao, Jon Glover, Bjorn Jorgensen, Christian Leuz, DJ Nanda, Korok Ray, Bill Rogerson, Phil Stocken and other seminar participants at Carnegie Mellon University, Northwestern University, the D-CAF conference at Copenhagen and the Chicago-Minnesota Theory conference. This paper originated from several discussions at the Tepper Repeated Games Reading Group and we wish to thank its participants, Edwige Cheynel, Laurens Debo, Richard Lowery for their helpful feedback along this project.
This paper develops a model that accounts for two-way interactions between industry cycle dynamics and firms’ disclosure policy. We propose a variant of a dynamic oligopoly model which incorporates disclosure choices into the existing literature on dynamic competition and relates it to the primitives of the industry such as concentration levels, cost of capital and the magnitude of economic shocks. Our analysis offers several novel predictions in terms of how disclosure relates to industry concentration and accompanies business fluctuations.

The idea that the nature of industry cycles may be driven by the informational environment is fairly intuitive. The fluctuations of production, prices, and profits depend on the dissemination of information about industry shocks, which is itself a function of the disclosure behavior. If, say, firms choose to disclose more information about the industry shock to the marketplace, such information would allow otherwise uninformed firms to adjust their production and prices in advance of the shock. Thus, how much the industry as a whole responds to a shock depends on what information is being disclosed. As a result, firms’ disclosures will play an important role on how common shocks are translated into fluctuations in production, prices and profit distribution within the industry. Following this logic, the real consequences of the industry-wide shock, and whether changes in prices may precede or lag the common shock, will depend on the firms’ external disclosure practices.

We present next an overview of our approach and main results. The framework follows the model of Rotemberg and Saloner (1986), hereafter abridged as RS, which offers the advantage of a widely-used benchmark on cycles and product market. In this framework, firms in an oligopoly compete over an infinite horizon with time-varying industry shocks. Rotemberg and Saloner show that, when under their maintained assumption that the demand shock is public information, firms can form a tacit agreement featuring counter-cyclical prices, lower during expansions than recessions, designed to provide firms a deterrent to compete too aggressively during industry booms. As a point of departure from RS, we assume that information about the upcoming cycle may be privately known to an individual firm. This firm can retain or publicly disclose the information prior to choosing prices on the product market.¹

¹The stylized assumption of a single firm being informed is made for parsimony and to better illustrate in-
We focus first on industries in which either discount rates are low or the number of competitors is small. In such industries, the tacit agreement takes the form of a monopoly price charged by all firms and a regime in which the informed firm does not disclose regardless of economic fundamentals. The key intuition here is that incentives to undercut are the greatest when demand is high (and potential profits are large). No-disclosure makes uninformed competitors uncertain about current demand, which lowers the expected benefits of undercutting. On the other hand, the informed firm knows about the upcoming demand and thus in equilibrium does undercut its competitors when the demand shock is sufficiently favorable. As a result, the market share of the informed firm is strongly pro-cyclical while the market share of the uninformed firm is counter-cyclical, with lower profits during an expansion of the industry.

We turn next to industries in which either discount rates are high or the number of firms in the market is large enough. In this case, a tacit agreement prescribing no disclosure would necessarily trigger a price war as each firm in the oligopoly would try to undercut its competitors, in turn leading to an equilibrium with low total industry profits. We show that disclosure helps avoid this trap by coordinating all firms on a counter-cyclical price schedule of the form discussed by Rotemberg and Saloner. When the forces in full-disclosure and in no-disclosure are combined, partial disclosure may emerge in equilibrium. We show that voluntary disclosure occurs for intermediate demand cycle shocks while no-disclosure is preferred for extreme variations in demand. This, in turn, leads to our prediction that more disclosure should occur in industries featuring prices that are more strongly associated to the cycle and that disclosure is more likely to occur in industries that are less concentrated.  

Our study is related to a relatively recent literature focusing on the reputational incentives that emerge in a repeated strategic interaction. In general, these reputational incentives can formation transfers within an industry; for this reason, it is widely-used in the literature (e.g., Darrough (1993)). Naturally, the main insights can be extended to environments in which more than one firm is informed as long as not all firms are informed.

Most existing studies in this area focus on the type of competition (e.g., Cournot vs. Bertrand) and provide mixed results on the link between competition and disclosure (e.g., Darrough (1993)) which give little support to the proprietary cost hypothesis. Also, since most studies in the area focus on two firms, the scope for considering concentration as a variable is limited. As one exception, Raith (1996) solves a general single-period information-sharing model with $n$ firms but he shows that, in general, the number of firms is a scaling variable and does not affect the optimal disclosure policy.
lead to dynamics that are very different from a single-period interaction. Huddart, Hughes and Levine (2006) show that public disclosure of insiders’ trading decisions allows multiple insiders to sustain tacit agreements in which they trade less, lower price efficiency and increase their trading gains. Insiders sustain this equilibrium by increasing their trading quantities in future periods if they observe excessive trading by one or more insiders over one period. Baldenius and Glover (2011) examine a three-party tacit agreement between a principal and two agents whenever some performance measures are non-contractible; they show that single-period bonus pools may create incentives for collusion between agents in the repeated interaction.

In the context of voluntary disclosure, Marinovic (2010) examines a model in which, over time, the firm forms its reputation as a function of a sequence of past reports. In his model, the probability that current earnings are being misreported perceived by investors, as well as the stock of past earnings that may have been misreported, are a function of the entire past history of reports, causing the reporting strategies to vary over time. Beyer and Dye (2011) show that managers with reputational concerns tend to disclose more unfavorable information, to increase investors’ perceptions that they will be more forthcoming in future periods. Fischer, Heinle and Verrecchia (2012) consider a model in which current investors expect future investors to overweight earnings in their valuation model, implying that they should themselves optimally do so. An important difference between these studies and this one is that they focus primarily on financial reporting concerns (i.e., a seller maximizing the perceived value of what he is selling); on the other hand, the focus here is on the interactions between reporting and the product market.

Our paper is also part of an existing literature on disclosure within a competitive environment (e.g., Kirby (1988), Wagenhofer (1990), Darrough (1993), Evans and Sridhar (2002), Suijs and Wielhouwer (2011)). The focus of our study is different from this literature as these studies focus on the nature of competition in a single-period interaction while we primarily focus on the reputational equilibria that emerge along repeated interactions. While there is a large literature in social sciences on tacit agreements in competitive environments (Rotemberg and Saloner (1986), Athey and Bagwell (2001), Mailath and Samuelson (2006)), there are few studies in the accounting literature that focus on more than two periods. A notable exception is
Baiman, Netessine and Saouma (2010) who examine the design of a production chain subject to economic shocks that operates over an infinite horizon. However, their primary focus is on the design of an incentive mechanism rather than information dissemination.

Finally, our paper is related to an emerging empirical literature that examines accounting disclosure quality based on characteristics of market competition, and in particular, as it relates to industry concentration and cycles. In terms of industry concentration, Harris (1998), Cohen (2006) and Balakrishnan and Cohen (2009) provide evidence that less industry competition leads to lower disclosure quality, which is consistent with our theory. In terms of economic cycles, there is limited work that fully documents the relationship between accounting and aggregate shocks and evidence on the subject is mostly available piecemeal within a few recent papers. In particular, Johnson (1999) and Cohen and Zarowin (2008) examine several metrics of accounting quality (e.g., persistence, earning responses and earning management) as a function of macroeconomic conditions. Bertomeu, Evans, Feng and Wu (2013) recently provide evidence that the quality of voluntary disclosures varies as a function of industry conditions among the Big Three automobile manufacturers.

1. The Model

1.1. Basic Setup

We borrow from the widely-used Rotemberg and Saloner model, or RS, the template for stochastic industry cycle fluctuations. As in RS, the specific effect of a shock to demand is considered from the perspective of a single industry but the model can be extended to multiple sectors subject to common shocks.

There are $N$ firms ($N \geq 2$) competing in a product market over an infinite time horizon.

\footnote{Although many studies argue that competition on the product market disciplines disclosure, it is worth noting that the formal theories that are referred to (or those that we know of) typically do not speak about industry concentration and do not provide formal support for this idea. Empirically, competition is often proxied by the number of firms or an Herfindahl index; yet, most models in accounting feature two firms and it is the nature of competition that matters. In a generalized version of these models, Raith (1996) shows that the equilibrium level of information-sharing is usually not a function of the number of firms in the industry (Prop. 4.5, p. 276).}
indexed by $t = 0, \ldots, +\infty$. Firms are risk-neutral, face a constant marginal cost normalized to zero and discount payoffs in each period with a discount factor $\delta \in (0, 1)$. This might represent the expected return, or cost of capital, demanded by outside investors. In each period, firms face a demand $s_tD(p)$, where $s_t$ represents a time-varying mass of potential consumers and $D(p)$ is the per-consumer demand at price $p$. The profit per unit of demand $pD(p)$ is continuous and concave, with a maximum at $p^* > 0$. The price $p^*$ represents the optimal monopoly price and we denote $\Pi^* = p^*D(p^*)$ the maximal industry profit per unit of market size.

This specification is also used by Bagwell and Staiger (1997) and captures the idea that there are more potential consumers who may purchase the product during good times. The assumption speaks about the broad variations in market size along the cycle (plausibly, a first-order effect) and separates them from shocks to the average consumer’s price elasticity, i.e., $D(.)$ does not depend on $s_t$. In our study, this exclusion is also imposed with a conceptual intent: if the monopoly price were to depend on current innovations, there would have been an added exogenous benefit to disclose. In the current environment, by contrast, there is no benefit or cost of sharing information absent competition and, therefore, the need to disclose or keep information silent can be fully explained by competitive interactions.

In the next paragraphs, we introduce the timeline of the model, the main variables of interest, the nature of competition and the stochastic processes that drive economic shocks. The game is decomposed in time periods, with each period $t$ representing a stage game and $t.i$ denoting the $i^{th}$ event in period $t$.

<table>
<thead>
<tr>
<th>Event</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t.1$</td>
<td>Period $t$ begins; the state of the industry $s_t$ is realized. Then, one firm learns $s_t$.</td>
</tr>
<tr>
<td>$t.2$</td>
<td>The informed firm chooses to publicly announce $s_t$ or to stay silent.</td>
</tr>
<tr>
<td>$t.3$</td>
<td>Firms engage in perfect price competition: simultaneously choosing their price $p_k$.</td>
</tr>
<tr>
<td>$t.4$</td>
<td>$s_t$ is publicly known. Industry profits are divided among firms charging the lowest price $\min_k p_k$.</td>
</tr>
</tbody>
</table>

Figure 1: Model Timeline

---

4Price elasticities are likely to be ambiguously related to industry cycles. If an industry is doing well, meaning that more consumers demand the product, it could be the case that price elasticity increases or decreases, depending on the individual price elasticity of the incremental consumers. A state-dependent price elasticities would cause the optimal monopoly price to be a function of the shock $s_t$ and thus would bias the model toward more disclosure in all states of the world.
At \( t.1 \), an informed firm privately learns market size \( s_t \). We assume that market size shocks \( s_t \) are i.i.d., drawn from a continuous distribution with full support over \( \mathbb{R}^+ \) and a density \( h(s) \) bounded away from zero. Without loss of generality, we normalize the distribution of \( s \) such that \( \mathbb{E}(s \Pi^*) = 1.5 \).

The demand shock \( s_t \) is not publicly known at the beginning of each period \( t \). A single firm learns a signal that is informative on current market size (or state of the industry), \( s_t \) at the beginning of each period, call this firm informed, and the others uninformed. This is for analytical tractability and we delay until Section 4.1 a rigorous discussion of situations in which no firm might receive information. It is not critical if the informed firm receives a noisy signal as long as the signal can be truthfully disclosed (with a delay) at the end of a period; in this case, one can reinterpret \( s_t \) as the expected demand conditional on the signal. Each period, every firm is equally likely to become informed. As a modelling choice, the random information endowment is practical to set no ex-ante asymmetry between competitors but (as can seen from the analysis) the forces would be similar if the same firm were repeatedly informed in advance.\(^6\)

At \( t.2 \), the informed makes a voluntary disclosure \( m_t \), which can be a choice to publicly announce \( s_t \) or to stay silent. We use here the terminology of “voluntary” to refer to disclosures that are under the control of the firm making the disclosure, but we shall discuss later on the implications of mandatory disclosure imposed by a regulator. In practice, firms release quarterly reports and make voluntary disclosures that contain a fair amount of information about future demand. As an example, Beyer, Cohen, Lys and Walther (2010) find that voluntary earnings forecasts is the variable that explains the largest portion of abnormal stock returns, about three times the variation explained by earnings announcements and pre-announcements, or security

\(^5\)As in RS (and much of the literature that follows), we make the assumption that demand shocks are independent across time periods. Some studies such as Bagwell and Staiger (1997) have extended the analysis to persistent shocks; extensions of our results are possible with some correlation between shocks, but, for reasons of parsimony, it is not usual in these models to introduce correlation unless time correlation is the primary purpose of the analysis. If, consistent with most staggered price models (Calvo (1983)), prices are changed infrequently, time periods may be sufficiently long so that correlations between periods remain low.

\(^6\)The only additional difficulty under the alternative assumption that the same firm is informed is that there is no single equilibrium that is ex-ante preferred by both the informed and uninformed firms (various tacit agreements feature different allocations of the industry surplus). Even in this case, the equilibrium that maximizes total profits would have similar characteristics to the equilibrium obtained here.
analysts’ forecasts. MD&A sections, which are often part of mandatory filings, are another venue that is traditionally used by managers to voluntarily provide qualitative information about future sales (Bryan (1997)). We assume that the firm cannot pre-commit to a disclosure policy but disclosure, when it occurs, is truthful, i.e., \( m_t \in \{ ND, s_t \} \).\(^7\)

We choose to state the baseline model assuming no friction to voluntary disclosure. This assumption is not made for realism but to separate the main contribution of the theory from other frictions that have been already studied in the literature. For example, we do not assume here that disclosure or collection of information are costly (Verrecchia (1983)), that some types of information cannot be disclosed (Dye (1985)) or that some types of disclosures are untruthful (Stocken (2000)); all of which are realistic frictions that reduce disclosure. In doing so, we can therefore fully assign the effects described here to the repeated interactions in the product market.

At \( t.3 \), firms engage in price competition, simultaneously choosing a price \( p_k \) that may be conditional on \( s_t \) for the informed firm and for all other firm if \( m_t = s_t \) is disclosed. It is a useful tool for our exposition to use perfect price competition, which is a natural benchmark given that the nature of competition is not the main object of analysis; this assumption is used in several classic papers in the area (e.g., Bagwell and Staiger (1997), Athey, Bagwell and Sanchirico (2004)) and takes out distracting considerations about information-sharing in various imperfect competition settings which have been well studied.\(^8\)

Next, we shall give proper meaning to the action of undercutting below the price of one or more competitors in the context of price competition. As we shall prove later on, a firm might optimally undercut its competitors when certain conditions are in place. Such strategic action naturally presents itself as a decrease in price but, under the stylized circumstances of price competition, the suitable price decrease could be infinitesimal. To sidestep a technical\(^7\)This assumption is consistent with those in the voluntary disclosure literature (e.g., Jovanovic (1982), Verrecchia (1983) and Dye (1985)) in accounting. The question of whether reputational concerns in a repeated game could enforce truthful disclosures is discussed in Stocken (2000).

\(^8\)As any model with price competition, the model is not intended to be factually descriptive as a complete representation of an industry but, rather, focused on “excess profits”; in practice, actual profits may not be zero due to a minimum return on current capital, or a variety of other unmodelled reasons that are transversal to the main argument, such as a presence in multiple markets or idiosyncratic noise.
discussion of such limits, we create a binary label \( z_k \in \{ \text{share, undercut} \} \) to represent the action of undercutting. The joint choice of a price and, possibly, an undercutting action \((p_k, z_k)\) is then what fully describes a pricing strategy.\(^9\)

At \( t \cdot 4 \), the total industry profit \( s_t \min_k p_k D(\min_k p_k) \) is shared equally among firms charging lowest price if no such firms choose \( z = \text{undercut} \). If some among these firms choose to undercut, the total profit is shared among the undercutting firms (i.e., those charging the lowest price and choosing \( z = \text{undercut} \)). Note that price-setting will be self-interested, as a firm considers the effect of its actions on its current and future profits and, similarly, the disclosure decision will also be set in a manner that is individually optimal and self-enforcing. This is discussed at greater length next, as we define an equilibrium in the game.

A natural benchmark for our analysis is the single-period interaction. Consider the stage game and assume that firms play this stage game only once. Firms will compete up to the price being equal to marginal cost \( p_k = 0 \). To see this, note that if total industry profits \( s_t \min_k p_k D(p_k) \) were greater than zero, at least one firm could decrease its own price slightly below \( \min_k p_k \) and capture all the market demand (Tirole (1988), p.245). Under perfect price competition, therefore, the issue of disclosure is moot.

The observation is technically innocuous but carries a disturbing note whose resolution motivates a new result in our study. Many current studies hypothesize that disclosure should be positively related to competition, as competition should plausibly reduces potential proprietary costs (see references in Beyer et al. (2010), p. 43). If this assertion were correct, voluntary disclosure should induce higher costs under imperfect competition than under the purest form of competition examined here. In turn, because disclosure has zero value under perfect competition, this logic would imply that firms should never disclose any information for other forms of (imperfect) competition. As is known, however, such a conclusion is inaccurate and, therefore, the argument needs to be completed.

Competition reduces the potential proprietary costs of disclosure but, by the same token,

\(^9\)This label is nothing but a shortcut for a more technical characterization in terms of limits. Absent the \( z_k \) construct, a strategy featuring \( z_k = \text{undercut} \) can be represented as the limit of a sequence of strategies with price \( \{p_k^j\} \) as \( p_k^j \) converges to \( p_k \).
also reduces its benefits; consequently, the link between competition and disclosure is tenuous, at best (Darrough (1993), Raith (1996)). A contribution of our study will thus be to establish a rigorous narrative in which industry concentration, a common proxy for competition in empirical studies, explains disclosure outcomes. As we will show, this link will emerge from the existence of repeated interactions and, therefore, should only be apparent in industries that feature large established firms with a history of informal connections and that can engage in a tacit agreement.

1.2. Equilibrium Definitions

We consider next the repeated competition setting. In this setting, firms in the oligopoly do not only consider temporary shocks to current market conditions but also future market conditions, long after current shocks have faded out. This aspect motivates the important role of repeated interactions as a key ingredient of a complete theory of disclosure and cycles, by introducing a time factor and a basic notion of forward-looking strategy. To be more explicit before we lay out the theory formally, we shall assume here that firms coordinate their price and disclosure in a manner that remains individually incentive-compatible but allows them to best sustain higher prices through an efficacious management of their reputation along their repeated interactions. This is what we refer as a tacit agreement (as originally studied by RS). Naturally, such agreements might involve a price war if some firms decide not to follow the tacit agreement and to unexpectedly reduce their prices at the expense of others. Because a firm could potentially change its disclosure without facing a legal penalty, we refer to these disclosures as voluntary - mainly to differentiate the theory from the commonly-used terminology of mandatory disclosure for rules that are enforced by market regulators (e.g., GAAP, SEC disclosures, etc.). Note, however, that the oligopoly will discipline firms to make certain disclosures so that, in a way, one may refer to disclosures as being “mandated” by the tacit agreement with the threat of a price war if the strategy is not properly followed.

To model any such potential punishments, we assume that, in every period, firms can use
past market shocks \( \{s_t\} \), disclosures \( \{m_t\} \), as well as past prices \( \theta_t = (p_t, z_t)_{i=1}^N \) as conditioning variables for current stage-game actions. Although the range of possible strategies is massive, existing work in repeated games shows that, in the general class of games such as ours, the equilibrium preferred by players can be written in terms of three descriptors of a strategy profile \( \langle \text{cooperation, punishment, transition} \rangle \).\(^{10}\)

1. **cooperation path:** This is a strategy mode that describes the action of each firm on the equilibrium path. That is, if no firm deviates from equilibrium play last period, all firms will follow the action prescribed by *cooperation* this period.

2. **punishment path:** This a strategy mode that describes the action of each firm off the equilibrium path. That is, if any firm deviates from equilibrium play last period, all firms will follow the action prescribed by *punishment* this period and future periods.\(^{11}\)

3. **transition:** describes how each firm move from on-equilibrium (*cooperation*) play into off-equilibrium play (*punishment*).

In summary, firms in the oligopoly adopt strategies that would maximize their long-term payoffs (the cooperation path) and continue to use these strategies as long as no other firm is observed taking an action that would hurt other firms in the oligopoly. If, however, one firm is detected, firms in the oligopoly lose confidence in the tacit agreement and switch to a price war (punishment path). The price war does not happen in equilibrium but is important as a threat to discipline cooperation.

The information structure that we have adopted here is characteristic of a perfect monitoring game. When observing \( \theta_t \) at the end of a stage game, firms in the industry can *perfectly* detect whether one firm did not act in the manner expected under cooperation. This requires the assumption that, even if information is not disclosed at the beginning of the stage game, the signal

\(^{10}\)This representation is without loss of generality as long as we are concerned with outcomes that occur on the equilibrium path. See Mailath and Samuelson (2006) for formal proofs and discussions.

\(^{11}\)To prevent a deviation, it is desirable to minimize payoffs after observing an off-equilibrium move, i.e. switch to a Punishment. This is formally defined as follows. First, if a firm was playing Punishment at date \( t - 1 \), always stay on Punishment at date \( t \). According to this plan, all firms will achieve zero profit (current and future) once the Punishment stage is reached. Second, for each date \( t \) such that Cooperation was played in the previous period, switch to Punishment when an off-equilibrium move is observed.
about demand can be learnt by all firms at the end of the stage game, e.g., if the informed firm can truthfully release its private information with delay. Admittedly, imperfect monitoring is prevalent in practice and has the potential to offer many additional insights; unfortunately, our model cannot accommodate imperfect monitoring in a straightforward manner because costless truthful disclosure tends to rule out imperfect monitoring between periods by manner of assumption.\footnote{The assumption is plausibly implied by the truthful disclosure framework: if $s_t$ can be truthfully disclosed by the informed firm in advance, it could be disclosed truthfully at the end of the stage game, for example, through a non-timely information release in a trade association or through financial reports. Another mechanism through which this information might become known to the uninformed is through the uninformed own observed sales or, more generally, published accounting reports. Yet, this other mechanism is less useful than the mechanism we present in text. Strictly speaking, an uninformed firm that does not make any sales could not learn $s_t$ and, even more importantly, actual sales are likely to be noisy and make the discovery of $s_t$ imperfect.}

Note also that we do not require firms’ strategies to be conditional on the identity of the informed firm; such information is not needed to apply the punishment because firms activate the punishment path when they observe a vector of actions that is inconsistent with one of the firms being informed and not playing cooperatively.\footnote{As an example, suppose that for a particular shock $s_t$, the informed firm should not disclose and set a price $p_1$, while the uninformed choose a price $p_2 < p_1$. Instead, suppose that the informed firm chooses the price $p_2$ and increases its current profit. The uninformed firms might not know, at the end of the stage game, which firm was informed but will observe that all firms chose $p_2$ instead of only $N - 1$ firms doing so. They would therefore be able to activate the punishment even if they do not know the identity of the deviating firm.} Yet, one should note that if there were other forms of (unmodelled) noise that would perturb the monitoring process, the oligopoly would feature only temporary price wars, of the form described in Green and Porter (1984). For example, if competitors could not be exactly sure that a market share should have led to undercutting (if, say, the disclosure process were imperfect), they would trigger a few periods of price war after they observe undercutting.

As with other works in this strand, the repeated game we study contains multiple equilibria. We thus adopt the following standard equilibrium selection criteria. First, we restrict the attention to equilibria in which firms condition their actions on only public information available in past stage games; these are usually referred as public-monitoring games.\footnote{This is not a restriction on the set of deviating actions that can be possibly undertaken; as shown in Mailath and Samuelson (2006), public-monitoring equilibria are Nash equilibria in the complete game. However, there are sometimes more complex equilibria involving private-monitoring that can achieve even greater profits.} Second, we restrict the attention to strongly-symmetric equilibria in which, for all possible histories of past play, the
strategies chosen by all players are identical and may only depend on their current information. We denote this solution concept a perfect public monitoring Nash equilibrium (PPNE) and it is a synonym for the notion of tacit agreement described earlier. In reference to antitrust laws, tacit agreements do not involve binding contracts; thus, although they are in theory prohibited by law, they are hard to detect or prove in a court of law.

In terms of basic notations, we refer to $\sigma = (\sigma_1, \ldots, \sigma_N)$ as a strategy in the game, i.e. such that $\sigma_k$ maps for any period $j$ and any public history of actions to firm $k$’s disclosure and price choices in the period $j$. We then refer to $u_k(\sigma)$ or in short $u(\sigma)$ in a strongly-symmetric equilibrium, as the expected surplus of player $k$. Also, to minimize the need for technical notations, we refer to Mailath and Samuelson (2006) for a formal presentation of the PPNE (since our notion of PPNE is a special case of their definition).

It should be emphasized that punishments, when they occur, are not renegotiation-proof, since firms would be better-off renegotiating away from the punishment. Renegotiations are key elements of contractual relationships for issues ranging from debt contracting to compensation design (Magee and Sridhar (1996), Christensen, Demski and Frimor (2002)). In our particular environment, however, open renegotiations of the kind that could reset equilibrium expectations away from a pre-agreed punishment path, are more problematic than in common settings where renegotiations are studied. Unlike for, say, a bond covenant renegotiation, firms are explicitly prohibited from engaging in discussions to organize prices, such as sponsoring a meeting to end a price war or raise prices. For this reason, the assumption that punishments are credible and will not be costlessly renegotiated away is commonly adopted in environments that involve interactions between competitors.\(^{15}\)

We examine first the punishment path. Since the punishment path is used as a means to provide incentives to cooperate (and never occurs on the equilibrium path), we can always choose the punishment path that minimizes firms’ payoffs. In our game, this corresponds to

\(^{15}\)Similar results could be obtained as long as renegotiation is not costless and instantaneous. If firms can renegotiate away from the punishment with some probability, for example, the maximum price that can be sustained in the tacit agreement will be lower. There are a few cases (see references Mailath and Samuelson (2006)) in which a renegotiation-proof equilibrium can be sustained but these often involve extremely complex asymmetric strategies after a deviation occurs and which seem intuitively unappealing if firms cannot engage in bilateral communications.
playing (forever) the Nash equilibrium of the stage game in which firms make zero profit and can be interpreted as a complete loss of reputation going forward after players observe an off-equilibrium move (Bar-Isaac and Shapiro (2010), Baldenius and Glover (2011)). Note that this property is specific to perfect monitoring and, if monitoring were imperfect and players were unsure that a deviation had taken place, the punishment path could be adopted with a probability less than one or only for a finite number of periods.\textsuperscript{16}

Next, we are interested in solving for the choice of the cooperation path that delivers the highest expected surplus. The ideal payoff would be one in which monopoly profit ($s_t \Pi^*$) is achieved every period (and, due to symmetry, equally shared among all firms). In such an ideal setting, the total discounted future profit, at any period in time $t$, is expressed as $\frac{1}{N}(1 + \delta^1 + \delta^2 + ...) = \frac{1}{N(1-\delta)}$, or the expected monopoly payoff shared among all firms.

We call this payoff the monopoly payoff, which after rescaling with a factor $1 - \delta$, gives rise to a (normalized per-period) payoff of $1/N$. This ideal payoff may or may not be achieved in the repeated game (and when taking firm’s incentives to compete). Imposing incentive-compatibility on the part of firms, we define the efficient tacit agreement as the equilibrium that achieves the highest profit.

**Definition 1.1** A PPNE strategy profile $\sigma$ is an efficient tacit agreement if for any other PPNE $\sigma'$, $u(\sigma) \geq u(\sigma')$.

The efficient tacit agreement represents a natural point of coordination for firms in the oligopoly because it achieves the highest sustainable profit before any move is made in the game. By selecting the Pareto-preferred equilibrium, we thus presume that a self-interested profit-seeking organization should assume that its competitors act in the same manner and, thus, would not select another equilibrium that would hurt not only themselves but also all other firms in the industry. Whether firms are able to coordinate remains partly an empirical question but a number of tests find support consistent with behavior predicted in such agreements (Borenstein\textsuperscript{16}).

\textsuperscript{16}We do not model imperfect monitoring here because it does not fit well in a model with truthful reporting and would require a richer disclosure model that would prevent a truthful report, even one given with a delay to the end of the period, from being used to perfectly monitor actions in the stage game.
and Shepard (1996), Knittel and Stango (2003)).

Lastly, we attempt to rule out some equilibria that are sustained by somewhat arcane strategizing that seems practically implausible. We illustrate somewhat pathological behaviors to motivate the refinement. Suppose that a firm makes a pricing decision that strictly increases its own profit without hurting the profit of other firms. Competitors could respond to this action by triggering a price war but, to do so, is unnatural given that no actual damage would have been borne by any firm. We avoid having to consider this type of equilibrium by assuming that if an action Pareto-improves payoffs in the stage game, it should Pareto-improve payoffs in the entire game.¹⁷ Abusing slightly on the terminology, we will refer to “equilibria” as PPNE that are efficient tacit agreements over the set of equilibria that satisfy this restriction.

2. No- and Full-Disclosure Benchmarks

2.1. Cooperation Path

In this section, we analyze the cost and benefit of no-disclosure versus those of full-disclosure. Our initial objective is to provide a rough cut at optimal disclosure strategies by considering two simple stylized disclosure strategies, and then develop the intuitions which will be helpful in explaining the main result of endogenous partial disclosure. Given that the repeated model under our analysis is stationary, we drop the time index on variables when no longer needed.

One possible strategy is full disclosure (i.e., disclose all realizations of $s$). Full-disclosure boils down to information being perfectly known to all market participants as examined in RS. Conditional on market size $s$, a price $P(s)$ is set and all firms choose to disclose. Of note, while disclosures are privately chosen (or voluntary), the enforcement of full-disclosure does

¹⁷A formal definition of the criterion is given next. A strongly symmetric PPNE with strategy $\sigma$ satisfies the criterion if, for any histories of past play $h^t$ and $h^t_2$ such that:

(a) $h^t$ and $h^t_2$ differ only over last stage game actions.
(b) The last period profit vector under $h^t_2$ weakly Pareto-dominates the profit vector under $h^t$.

Then, when the expected equilibrium profit vector in the continuation game with $\sigma_{h^t_2}$ must weakly Pareto-dominate the expected equilibrium profit vector in the continuation game with $\sigma_{h^t}$. Note that this criterion is unnecessary when the equilibrium attains the monopoly payoff.
not require the intervention of a public regulator: the informed firm is never strictly better-off when deviating not to release information because doing so would immediately trigger the punishment path.\footnote{This statement holds true under the baseline assumption that there is no uncertainty about information endowment but some other considerations need be imposed if no firm might be informed. This extension is presented in Section 4.1.}

Another possible strategy is no-disclosure (i.e., disclose nothing for all \( s \) realizations), uninformed firms do not have information, and set a price \( p_{nd} \) that does not depend on \( s \). As for the case of full-disclosure, no-disclosure can be enforced provided firms respond to any disclosure by triggering the punishment path. On the cooperation path, three options are available to the informed firm - overprice, share or undercut - for each \( s \) realization. As a result, we can define the cooperation path of any no-disclosure equilibrium in terms of a price \( p_{nd} \) and three sets \( \Omega_{\text{over}}, \Omega_{\text{share}} \) and \( \Omega_{\text{under}} \), as described below.

- The informed firm can set a price strictly higher than \( p_{nd} \), in which case it makes zero profit in the current period; let \( s \in \Omega_{\text{over}} \) be the set of market sizes such that the informed firm overprices.

- The informed firm can share the market, choosing \( p = p_{nd} \) and \( z = \text{share} \), in which case it makes a current profit \( sp_{nd}D(p_{nd})/N \); let \( s \in \Omega_{\text{share}} \) be the set of market sizes such that the informed firm shares.

- The informed firm can undercut its competitors, choosing \( p = p_{nd} \) and \( z = \text{undercut} \), in which case it makes a current profit \( sp_{nd}D(p_{nd}) \); let \( s \in \Omega_{\text{under}} \) be the set of market sizes such that the informed firm undercuts on the cooperation path.

\subsection{Incentive Benefit of No-Disclosure}

We first examine equilibria in which the tacit agreement can effectively replicate the surplus achieved by a monopoly, which means that, for all \( s \), \( P(s) = p_{nd} = p^* \), and firms obtain their ideal symmetric surplus \( 1/N \). In this setting, we derive our first main intuition that no-disclosure
dominates full-disclosure because no-disclosure makes it possible for the tacit agreement to be incentive compatible from the standpoint of uninformed firms. We call this first main intuition the Incentive Benefit of No-Disclosure.

2.2.1 No-disclosure dominates full-disclosure under monopoly pricing

We begin with full-disclosure. For \( P(s) = p^* \) to be an equilibrium, no firm shall find it desirable to undercut its competitors, leading to the following incentive-compatibility constraint, for all \( s \),

\[
(1 - \delta)s\Pi^*/N + \delta/N \geq (1 - \delta)s\Pi^* \tag{2.1}
\]

The left-hand side has two components. The first component \( s\Pi^*/N \) is the current surplus obtained by playing the cooperation path and the second component \( \delta/N \) is the discounted surplus obtained in future periods (recall that these are expressed in a per-period basis and need to be divided by \( 1 - \delta \) to obtain total expected surplus). The right-hand side has only one component, the deviation profit in the current period; given that such a deviation triggers a permanent price of zero in future periods, there will be zero profit in all future periods. This inequality cannot be satisfied for all \( s \in \mathbb{R}^+ \) for any \( \delta < 1 \), which implies that full disclosure cannot achieve monopoly profits for all market sizes.

We compare this benchmark to the no-disclosure case. In this case, depending on the actions of the informed firm, the incentive-compatibility condition is a bit more complicated. Consider first the prescription for the informed firms to overprice \( (p > p^*) \) for some \( s \)-region \( (s \in \Omega_{over}) \), leading to zero profit in the current period as well continued cooperation path for the informed firm. By deviating to \( z = \text{cut} \) and \( p = p^* \) (the best possible deviation), the informed firm can obtain \( s\Pi^* \) in the current period, but this will trigger a shift by all firms to the punishment path, making zero profit in future periods. For the recommended action to be optimal, it must hold that: for all \( s \in \Omega_{over} \) (for which the informed firm overprices),
\[(1 - \delta)0 + \delta/N \geq (1 - \delta)s\Pi^* \tag{2.2}\]

This constraint is satisfied when \(s \leq \tilde{s} \equiv \frac{\delta}{1 - \delta} \frac{1}{\Pi^*}.\) A similar condition is derived for all \(s \in \Omega_{\text{share}}\) (for which the informed firm shares),

\[(1 - \delta)s\Pi^*/N + \delta/N \geq (1 - \delta)s\Pi^* \tag{2.3}\]

This constraint is satisfied when \(s \leq \hat{s} \equiv \frac{\delta}{1 - \delta} \frac{1}{(N-1)\Pi^*}.\) It is thus easier to induce sharing than to induce overpricing, as implied by \(\hat{s} > \tilde{s} \).

In addition to inequalities (2.2), (2.3), it must be incentive-compatible for each uninformed firm not to deviate to price slightly lower than the informed firm.

\[(1 - \delta) \left( \int_{\Omega_{\text{over}}} sh(s)ds \frac{\Pi^*}{N-1} + \int_{\Omega_{\text{share}}} sh(s)ds \frac{\Pi^*}{N} \right) + \delta \frac{1}{N} \geq (1 - \delta) \tag{2.4}\]

In Equation (2.4), the right-hand side corresponds to the expected profit obtained by undercutting all other firms. Since in this case, the uninformed firm deviating does not know \(s\), it will anticipate an expected profit \(E(s\Pi^*) = 1\). The left-hand side corresponds to the profit expected by staying on the Cooperation path, where the profit of the uninformed will depend on \(s\) and the strategy of the informed firm.

Pooling together these constraints, it is established that, to be an equilibrium, the cooperation path must satisfy that: (i) the informed firm overprices (shares) only when it is incentive-compatible to do so (i.e. \(s \in \Omega_{\text{over}}\) implies that \(s \leq \tilde{s}\) and \(s \in \Omega_{\text{share}}\) implies that \(s \leq \hat{s}\)), (ii) the uninformed firm does not deviate (i.e., Equation (2.4) is met). The next proposition establishes the value of the no-disclosure regime.

**Proposition 2.1** With full-disclosure, monopoly payoffs cannot be attained. With no-disclosure, there exists a threshold \(\delta_{nd} < 1\), increasing in \(N\), such that monopoly payoffs can be attained if and only if \(\delta \geq \delta_{nd}\).

Compared to full-disclosure, the oligopoly is better able to dampen the incentives to deviate
when current demand is high by leaving most competitors in the dark. Intuitively, when market size is large, disclosing makes deviation more attractive to every firm so firms need to be sufficiently patient to refrain from undercutting. Under no disclosure, \(N - 1\) firms do not know whether market size is high and must assume the average market size when contemplating deviation, lowering the benefit of deviating (i.e., the right-hand-side of constraint (2.4) is reduced). In addition, to better elicit cooperative behavior among uninformed firms, the informed firm agrees to give away additional rents when the market is low (thus increasing the left-hand side of constraint (2.4)). In short, no disclosure uses the slack in the incentive-compatibility constraint of the informed firm when demand is low to better motivate the uninformed firms to cooperate. As a result, secrecy is valuable to the oligopoly, not because it necessarily benefits the uninformed firm in the current period, but because it better motivates cooperation among oligopoly members in the long-term. This is the first main intuition derived from the model.

Our results point to how information endowment can explain the distribution of market shares as a function of shocks to the industry. In our model, all firms are equally exposed to variations in demand but, in the tacit agreement, the informed firm implements a strategy that magnifies sensitivity to demand shocks by underpricing during good times and overpricing during bad times. On the one hand, transferring more surplus to the uninformed firms can help avoid deviations to lower prices. On the other hand, it is more difficult to induce the informed firm not to undercut when market size is large. The solution to this trade-off implies that the informed firm partly insures its uninformed competitors against industry shocks. Also, unlike prior literature in this area (Green and Porter (1984)), we show that undercutting (when market size is large) occurs on the equilibrium path and does not necessarily trigger a price war.

While we write results in terms of a minimum discount factor, it should be noted from a closer inspection that we could identically look for the minimum size of the oligopoly \(N\) consistent with the monopoly surplus. An alternative formulation of the result is therefore that no-disclosure dominates full-disclosure if there are few competitors in the industry, i.e., \(N\) is small or the concentration ratio is large. Empirically, for example, the Herfindahl index might capture industries in which set-up costs are large enough so that fewer firms choose to operate. From
a conceptual standpoint, the result provides a novel rationale as to why voluntary disclosure is less desirable for firms in more concentrated industries. To be specific, in our environment, there are few benefits obtained from the disclosure when the industry can implement monopoly prices through a tacit agreement; there are, however, “proprietary” costs when disseminating excessively positive information that make it difficult to sustain the tacit agreement.

2.2.2 Properties of no-disclosure equilibria under monopoly pricing

We now solve for the strategy that attains the monopoly surplus for the widest range of discount rates (and thus would be robust to a small amount of uncertainty about discount rates). Observe that the more the informed firm gives away surplus to the uninformed, the easier it is to satisfy Equation (2.4) (this increases the cooperation surplus of the uninformed). Therefore, the equilibrium that achieves cooperation for the widest range of discount rates is the one in which $\Omega_{\text{over}}$ is set as large as possible, or $s \in [0, \bar{s})$, followed by $\Omega_{\text{share}}$, or $s \in [\bar{s}, \hat{s})$, with undercutting when nothing else is incentive compatible, or $s \geq \hat{s}$.

**Corollary 2.1** There is a unique strategy that achieves monopoly surplus for any $\delta \geq \delta_{\text{nd}}$, it is given as follows:

(i) For $s \leq \bar{s}$ (low market size), the informed firm does not sell and only the uninformed sell at a price $p^*$.

(ii) For $s \in (\bar{s}, \hat{s}]$ (medium market size), total industry profits $\Pi^*$ are shared equally among all firms.

(iii) For $s > \hat{s}$ (large market size), only the informed firm sells.

The asymmetric information that remains in the no-disclosure regime has important consequences on the product market. We establish that no-disclosure changes the sensitivity of firm profits to the cycle. Specifically, the analysis suggests that no-disclosure increases the sensitivity of the informed firm to the cycle, and reduces the sensitivity of the profit for firms that did
not anticipate the shock. A favorable but unanticipated shock, as shown above, can lead to a reduction in firms’ profit.

Of note, a firm that receives information might be, in the tacit agreement, worse-off than an uninformed firm to the extent that it is expected to price in a certain manner. This property is very different from models with uncertain information endowment and reporting motives (e.g., Dye (1985), Jung and Kwon (1988)) where a seller receiving information is always weakly better-off. To see this, consider the expected lifetime profit of an informed firm $V_I$ before it learns the actual $s$ and compare it to the profit of an uninformed firm $V_{NI}$. Being informed is desirable when the following Equation holds true:

$$V_I = \frac{\Pi^*}{N} \int_{\hat{s}}^{\tilde{s}} sh(s)ds + \Pi^* \int_{\hat{s}}^{+\infty} sh(s)ds \geq V_{NI} = \frac{1}{N} \quad (2.5)$$

This inequality is only satisfied when $N$ is large, $\delta$ is small or low realizations of $s$ (lower than $\hat{s}$) are unlikely. On the other hand, when $\delta$ is sufficiently close to one, or $N$ is small, the uninformed firm is always better-off than the informed firm. That is, being informed is indicative of a low expected own profit for industries with few firms and low discount rates. One interpretation of this result is therefore that, in the no-disclosure equilibrium, an informed firm tends to achieve greater current profits in “growth” environments with many competing firms, high discount rates and potential payoffs. The reason for this is that, under these circumstances, it is more difficult to discipline the informed firm to concede market share to its competitors.

### 2.3. Price coordination benefit of full-disclosure

We turn next toward the case in which monopoly profits may not be achieved by the industry. We will then show that disclosure may play a role in terms of coordinating prices (a role that it did not have under monopoly pricing since prices were not a function of $s$). We develop this observation further by establishing two results, (a) that an efficient full-disclosure pricing policy specifies prices $P(s)$ that vary with $s$ and (b) that no profitable tacit agreement is now possible under no-disclosure.
We begin with full-disclosure. Note that, if monopoly prices are not always sustained, it must be the case that \( P(s) < p^* \) for some states \( s \). Let \( V_{fd} (< 1/N) \) be the expected surplus received by firms in such an equilibrium:

\[
V_{fd} = \frac{\int sP(s)D(P(s))h(s)ds}{N}
\]  

(2.6)

Similar to the previous case (but using \( V_{fd} \) instead of \( 1/N \)), it must be incentive-compatible for all firms to choose \( p = P(s) \) and \( z = \text{share} \) (versus deviating to \( z = \text{undercut} \)):

\[
(1 - \delta)sP(s)D(P(s)) + \delta \frac{V_{fd}}{N} \geq (1 - \delta)sP(s)D(P(s))
\]

(2.7)

Comparing the above incentive-compatibility constraint to that for monopoly pricing case (Equation (2.1)), a key difference is that the deviation payoff (i.e., the right-hand-side) is now a function of the prices \( (P(s)) \) which can be state-dependent and optimally chosen as a part of the tacit agreement. Solving for the optimal price for each \( s \) yields the following full-disclosure benchmark.

**Proposition 2.2** In an efficient full-disclosure equilibrium,

(i) For \( s \leq S \), \( P(s) = p^* \).

(ii) For \( s > S \), \( sP(s)D(P(s)) = S\Pi^* \)

where \( S \) is the maximal positive \( s' \) solution to:

\[
s' = \frac{\delta \int_{0}^{s'} sh(s)ds}{(1 - \delta)(N - 1) - \delta \int_{s'}^{\infty} h(s)ds}
\]

(2.8)

Notice that even when firms are not patient enough to achieve the monopoly surplus, monopoly profits are earned in some region of \( s \) (i.e., \( s < S \)). In the other region, equilibrium prices \( (P(s) < p^*) \) are a function of state-variable \( s \). Here, disclosure plays an important role of price-coordination. It must be incentive-compatible for firms to stay on the equilibrium path and not
to undercut their competitors. When market size is too large, however, the gains from undercutting are too important and thus, at \( p^* \), firms would prefer to undercut. One way firms can avoid such deviations is to agree to a lower price when market size is large, artificially reducing total industry profits and therefore removing incentives to undercut. This is precisely the insight from RS where market size is information is \textit{assumed} to be public knowledge. In our paper, it leads the informed firm to voluntarily disclosing the business cycle information \((s)\) in order to help making the price coordination possible. This is the second main intuition in our analysis.

Given our intuition that the value of disclosure is that of price coordination and that price coordination is required to set a cyclical pricing policy, we may then observe that no-disclosure fails to facilitate the tacit agreement if firms are too impatient to sustain monopoly prices for all \(s\)-realizations (i.e., \( \delta \) is too low).

\textbf{Proposition 2.3} \textit{If} \( \delta < \delta_{\text{nd}} \), \textit{any no-disclosure equilibrium yields zero profit for all firms.}

Combined, the propositions in this section depict a stark picture of no-disclosure. On one hand, it gives rise to monopoly payoff and dominates full-disclosure when firms are patient enough. On the other, when firms are less patient, it immediately reduces profit to zero and cannot sustain any in-between profit levels. The problem with no disclosure is its lack of flexibility: the same price must be used for any realization of \( s \). This feature does not allow firms to adapt their pricing strategies to the environment and thus makes the tacit agreement problematic. Proposition 2.3 further suggests that industries with high profits but high discount rate (low \( \delta \)) should feature more disclosure than those with lower discount rate. In the former case, never disclosing is (weakly) suboptimal while in the latter case, always disclosing is (weakly) suboptimal. In other words, the model provides some support for an association between a number of observable industry-level variables: disclosure should be prevalent in environments where prices are more volatile, firms discount future cash flows more heavily or the industry is less concentrated.

In summary, the preceding section discusses two fundamental roles of disclosure highlighted our repeated setting. First, no-disclosure keeps uninformed firm in the dark in order to lower
its deviation benefit, thus making it easier for the uninformed firms to cooperate stay on the equilibrium path. This benefit of no-disclosure appears when discount rate is rather low (i.e., firms care more about the future) or, equivalently, when the number of firms in an oligopoly is small for a given discount rate. Second, disclosure allows the firms to set state-contingent prices if and when it is necessary to lower prices to levels below monopoly prices in order to maximize expected industry profit. This benefit of disclosure appears when discount rate is rather high (i.e., firms care less about the future) or, equivalently, when the number of firms in an oligopoly is large for a given discount rate. However, This does not say, however, that full disclosure is the optimal disclosure policy in the tacit agreement in high-discount rate (or less concentrated industries) situations. Partial disclosure may not play a role when $\delta < \delta_{nd}$, as discussed next.

3. Partial disclosure

3.1. Definitions and disclosure strategy

In this section, we consider cases where certain forms of partial disclosure may emerge as a repeated equilibrium behavior. The key to partial disclosure is that it combines advantages of both no disclosure (incentive-compatibility of the uninformed) and full disclosure (price-coordination). Partial disclosure combines characteristics of the strategies under full-disclosure and no-disclosure. In general form, a partial disclosure equilibrium can be described as follows.

3.2. Pricing under partial disclosure

Building on earlier intuitions, it can be shown that partial disclosure does not facilitate the tacit agreement that sustains the monopoly surplus (simply because no price coordination is used when $p^*$ is set every period).\textsuperscript{19} Let us next assume that the monopoly surplus is not attainable, and denote $V_{pd} < 1/N$ the firms’ surplus in the efficient partial disclosure equilibrium. Under partial disclosure, it is useful to define thresholds on $s$ to help define pricing choices for an

\textsuperscript{19}We have omitted the proof to save space. Although the statement is intuitive, the formal proof is not entirely trivial and is available on request from the authors.
informed firm not disclosing. Denote $\Pi_{pd} = p_{pd}D(p_{pd})$ and $\hat{s}_{pd}$ in a manner that is entirely analogous to $\hat{s}$ considered earlier, i.e., the maximal state $s$ conditional on non-disclosure such that an informed firm prefers not to undercut its competitors.\footnote{As in the baseline, the bound $\hat{s}_{pd}$ is obtained by binding:}

$$\hat{s}_{pd} = \frac{\delta V_{pd} N}{1 - \delta \Pi_{pd} N - 1}$$ (3.1)

We examine first characteristics of prices in the disclosure region. In this region, firms arrange a state-dependent price schedule and share industry profits equally. However, these state-dependent prices must satisfy certain properties in order to deter deviation (to either no-disclosure by the informed firm or undercutting by any firm). The following two lemmas describe these properties.

**Lemma 3.1** Prices over the disclosure region, as denoted by $P(s)$, must satisfy:

\begin{equation}
(1 - \delta)s\Pi_{pd}/N + \delta V_{pd} \geq (1 - \delta)s\Pi_{pd}
\end{equation}

---

<table>
<thead>
<tr>
<th>Disclosure Region ($s \notin \Omega$)</th>
<th>No Disclosure Region ($s \in \Omega$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>cooperation</td>
<td></td>
</tr>
<tr>
<td>$m = s$ and choose a price $P(s)$ and $z = share$</td>
<td>The informed firm does not disclose $m = \emptyset$ and choose $\begin{cases} p &gt; p_{pd} &amp; \text{if } s \in \Omega_{over} \subset \Omega \ p = p_{pd} &amp; \text{if } s \in \Omega_{share} \subset \Omega \ z = share &amp; \text{if } s \in \Omega_{under} \subset \Omega \end{cases}$</td>
</tr>
<tr>
<td>All uninformed firms choose price $P(s)$ and $z = share$. If the informed firm does not disclose $m = ND$, all firms choose a price equal to zero.</td>
<td>All uninformed firms choose a price $p_{pd}$ and $z = share$. If the informed firm discloses $m \neq ND$, all firms choose a price equal to zero.</td>
</tr>
<tr>
<td>punishment</td>
<td></td>
</tr>
<tr>
<td>On the punishment path, firms chooses $p = 0$ regardless of their information, or current disclosures.</td>
<td></td>
</tr>
<tr>
<td>transition</td>
<td></td>
</tr>
<tr>
<td>The game starts at date $t = 0$ with all firms playing the Cooperation path. Any move that does not conform to the Cooperation path triggers a permanent shift to the punishment path.</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Strategy Profile with Partial Disclosure
(i) If \( s \leq \hat{s}_{pd} \) and the informed firm discloses \( m(s) = s \), then,

\[
s P(s) D(P(s)) = \min \left( s \Pi^*, \frac{\delta}{1 - \delta} \frac{N}{N - 1} V_{pd} \right)
\]  \hspace{1cm} (3.2)

(ii) The disclosure region cannot include states in which \( s > \hat{s}_{pd} \).

**Lemma 3.2** The disclosure price schedule \( P(s) \) is always greater than the price conditional on non-disclosure \( p_{pd} \) and satisfies \( P(\hat{s}_{pd}) = p_{pd} \).

Lemma 3.1 focuses on the tradeoffs in coordinating prices if and when disclosure is made. One added incentive problem introduced by a partial disclosure regime is that for some \( s \), the informed firm can deviate from disclosing \( (m(s) = s) \) to not disclosing \( (m(s) = \emptyset) \), attaining a price \( p_{pd} \) possibly greater than \( P(s) \). Such deviation was never desirable in the full-disclosure equilibrium because, upon observing a non-disclosure, firms could immediately trigger a price war.

This incentive problem is resolved via two mechanisms that operate within the tacit agreement. The first incentive mechanism is to reduce the non-disclosure price \( p_{nd} \), thus averting some of the temptation to strategically omit information because doing so would always lead to lower profits. We show that industry prices are always higher in states where a disclosure is made. This intuition is closely related to the case of uncertain information endowment discussed in Section 4.1 and stems from the same need to reduce profits when a firm claims not to have received the type of information that should have disclosed. The second mechanism is to tolerate that some events \( s > \hat{s}_{pd} \) that are very favorable should not be disclosed and followed by equilibrium undercutting. In turn, this allows the informed firm to collect greater cash flows in states that are more favorable. This feature is similar to what occurs in the pure no-disclosure equilibrium derived earlier.
3.3. Partial disclosure equilibrium

For the primary purpose of stating most of the economic intuitions, we first examine an efficient equilibrium within a class of simplified equilibrium strategies. That is, we only consider equilibria in which the informed firm is never required to overprice, i.e., setting $\Omega_{over} = \emptyset$.\footnote{An additional motivation for such equilibria is that inducing overpricing can become difficult to induce when there is uncertainty about information endowment so that, as examined in Section 4.1, the informed firm could not overprice and claim that it had not been informed.}

As we shall see later (when we lift this requirement), this condition is generally with loss of potential profits to the firms engaging in the tacit agreement but, at a conceptual level, it allows us to emphasize the new forces that appear under partial disclosure without repeating some of the existing forces discussed earlier.

Combining the constraints and payoffs in both the disclosure and no-disclosure regions, the tacit agreement chooses the optimal disclosure and no-disclosure regions (i.e., $\mathbb{R}^+ \setminus \Omega$ and $\Omega$) and corresponding prices (both state-contingent $P(s)$ and no-disclosure price $p_{pd}$) to maximize the ex ante expected per-period firm profit $V_{pd}$. The following Proposition describes the resulting equilibrium.

**Proposition 3.1** If a (simplified) partial disclosure equilibrium is efficient, there exists an interval $(s_1, s_2)$ such that the informed firm discloses if and only if $s \in (s_1, s_2)$. Furthermore, $0 < s_1 < s_2 \leq \hat{s}_{pd}$.

Unlike when monopoly surplus can be attained, partial disclosure adds value by providing a balance between the desire to coordinate prices (i.e., setting prices according to $P(s)$) and the desire to provide incentive to the uninformed to not to deviate from the collusive arrangement. We find that equilibria with partial disclosure have a simple structure and feature a single disclosure region in which moderate shocks are disclosed but large market movements are withheld.

To begin with, for extreme market sizes (in the region $[s_2, +\infty)$), the informed firms is not expected to disclose any information. This feature occurs for two reasons, as explained next. First, when market size is high, there are simply not enough reputational incentives to elicit disclosure and, contrary to the full-disclosure equilibrium, the uninformed firms can no longer...
activate a punishment when they observe a non-disclosure which, in turn, causes a deviation to non-disclosure to become available to the informed firm. This deviation places an upper bound on the disclosure region which can only be increased at a cost to the entire industry (by reducing the non-disclosure prices and $\Pi_{pd}$). Second, when $s$ is sufficiently large so that monopoly prices could no longer be obtained conditional on a public disclosure of $s$, any disclosure of information causes a decrease in prices. To see this, note that conditional on disclosure, the industry surplus is the minimum of $s\Pi^*$ and $\frac{s}{1-\delta} \sum_{i=1}^{N-1} V_{pd}$ so that any extra dollar of market size is dissipated once $s$ is large. This causes non-disclosure to become relatively more attractive to the oligopoly as $s$ becomes large.

A non-disclosure of large market sizes might only be of interest if uninformed firms are unable to perfectly invert the informational content of a non-disclosure and figure out that favorable events have been withheld. This creates a benefit, at the other extreme, to also withhold low market sizes (in the region $[0, s_1]$). This disclosure strategy perturbs the uninformed firms’ inferences about market size when they do not observe a disclosure. The cost of this disclosure strategy is that the price is reduced from the monopoly price $p^*$ if such information had been disclosed versus the lower $p_{pd}$ price when this information is kept silent. Yet, this potential cost is arbitrarily small when $s$ is itself small since it is of the order $s(\Pi^* - \Pi_{pd})$.\footnote{This form of partial disclosure is reminiscent of Wagenhofer (1990), who pointed out that no-disclosure may occur for extreme shocks. Yet, our main intuition is different in that Wagenhofer considers an exogenously-specified entry cost and financial reporting motives. Our setting, on the other hand, recovers both costs and benefits endogenously as a result of product-market competition and, in that respect, links them to testable characteristics of the product market. Note also that our form of partial disclosure may prescribe an interior region of non-disclosure.}

The remaining set of values that could be disclosed is an interval of intermediate market sizes $s \in (s_1, s_2)$ are the object of a disclosure. These market sizes are large enough so that the economic cost of a non-disclosure $s(\Pi^* - \Pi_{pd})$ cannot be ignored but small enough so that a disclosure can still be elicited. See Figure 2 for an illustration of the pricing and disclosure behavior in such a simplified partial disclosure equilibrium.

We move now to the efficient equilibrium, considering tacit agreements that could involve overpricing by the informed firm when there is a non-disclosure and the market size is small enough. The economic trade-offs described earlier carry over to this setting but with the addition
of an additional benefit of not disclosing when the market size is very low, below \( s_{pd} = \frac{\delta}{1-\delta} \frac{V_{pd}}{\Pi_{pd}} \)

where overpricing can (and should) be elicited if a market size is not disclosed. This, in turn, causes an additional benefit for non-disclosure within the region \([0, s_{pd}]\) and thus can create an additional interior non-disclosure region over low market sizes.

**Proposition 3.2** If a partial disclosure equilibrium is efficient, it can be constructed as follows:

Let \( 0 < s_0 \leq s_1 \leq s_2 \leq s_3 \),

(i) For \( s \in [0, s_0) \cup [s_1, s_2) \cup [s_3, +\infty) \), the informed firm does not disclose.

(ii) For \( s \in [s_0, s_1] \cup [s_2, s_3) \), the informed firm discloses.

Furthermore, the disclosure region must be a single interval in any equilibrium such that \( \Pi_{pd} \geq N/(N - 1)\Pi^* \) is not too small.

When considering overpricing, there is, in addition to the regions described earlier, a potential region \([s_1, s_2)\) in which information is not disclosed. In this region, it can be beneficial not to disclose information because, when this is the case, the informed firm overprices and transfers more surplus to the uninformed, thus “easing” the uninformed incentive-compatibility condition. Indeed, the greater the market size \( s \), the more this strategy can benefit the uninformed and
thus the more it can be optimal to withhold information. As $s$ becomes even larger, however, overpricing no longer becomes feasible and thus the dominant forces revert to those discussed earlier.

4. Extensions

4.1. Uncertain information endowment

While the baseline model is solved under the assumption that a firm is always informed, a few additional considerations must be considered for the (plausible) case in which no firm might receive information, as in the case of uncertain information endowment in Dye (1985) and Jung and Kwon (1988). Assume now that with probability $q \in (0, 1)$, no firm observes $s$ and, to make things interesting, whether a firm does or does not observe $s$ is not known to an uninformed firm.

We begin by considering the case in which $p^*$ is incentive-compatible in all periods. Clearly, full-disclosure (still) cannot implement $p^*$ because there is a non-zero probability that large realizations of $s$ are disclosed. Implementing a no-disclosure equilibrium, albeit still attractive for the reasons presented earlier, does require proper consideration of states in which no firm is informed. The novel incentive problem created by such states is as follows. A first-best equilibrium can only be attained if, in states where all firms are uninformed, the industry sets the price $p^*$. But, this feature creates a potentially profitable deviation as an informed firm could withhold its disclosure and claim to have been uninformed. Indeed, such action is desirable if and only if the informed firm is expected to overprice and would rather share the industry profit when claiming to be uninformed. From this logic, it follows now that overpricing is no longer incentive-compatible.

We are left to state the incentive-compatibility condition for the uninformed.

\[
(1 - \delta) \left( q \frac{1}{N} + (1 - q) \int_0^{\hat{s}} sh(s)ds \right) \frac{\Pi^*}{N} + \delta \frac{1}{N} \geq (1 - \delta) \tag{4.1}
\]
The following Proposition follows from rewriting this inequality in terms of $\delta$.

**Proposition 4.1** Suppose no firm is informed with probability $q \in (0, 1)$. With full-disclosure, monopoly payoffs cannot be attained. With no-disclosure, there exists a threshold $\delta_{nd}^q < 1$, increasing in $N$ and decreasing in $q$, such that monopoly payoffs can be attained if and only if $\delta \geq \delta_{nd}^q$.

The result in Proposition 4.1 is remindful of the classic observation made in Dye (1985) that a lower probability of information endowment tends to further dampen disclosure even when information is received. The rationale for the observation in our environment is slightly different, however. In reporting models, a lower probability of information endowment allows more informed firms to hide behind those who cannot disclose. By contrast, we find here that an uncertain information endowment perturbs the monitoring of the informed firm’s pricing decision, putting greater pressure on prices and, therefore, also implying more incentives for the uninformed to preemptively reduce prices. Since no-disclosure is a policy that dominates when the tacit agreement is strong, an uncertain information endowment thus tends to reduce situations in which non-disclosure is optimal.

We do not prove the result that no-disclosure leads to zero profit if $\delta < \delta_{nd}^q$ since the proof is identical to the case of $q = 0$. It is therefore clear that full-disclosure would dominate non-disclosure for any $\delta < \delta_{nd}^q$, if it can be elicited. Yet, uncertainty about information endowment can also be problematic to implement full-disclosure because a firm could avoid reporting its information and claim to have been uninformed. Considering this potential deviation, we solve next for a full-disclosure equilibrium with uncertain information endowment.

**Proposition 4.2** If $\delta < \delta_{nd}^q$, any equilibrium with no-disclosure yields zero profit to all firms. In the efficient full-disclosure equilibrium, firms set a price equal to zero if no disclosure is made and, otherwise, set a price $P(s) = p^*$ if $s \leq S$ and $P(s)$ such that $sP(s)D(P(s)) = S\Pi^*$ if $s > S$.

In order to make full-disclosure incentive-compatible, a full-disclosure equilibrium must reduce the potential cash flows when no-disclosure is made to avoid the temptation to claim...
that no information has been received. Yet, even though full-disclosure can be preferred over non-disclosure, it can nevertheless be very costly to implement. To see this, notice that any full-disclosure equilibrium must require perfect competition when no firm is informed. If this were not the case, then an informed firm with $s$ very large would be better-off not making a disclosure and earning a cash flow $sp_{nd}D(p_{nd})$, thus contradicting a strategy that involves full-disclosure. As a result, the full-disclosure equilibrium has a form that is similar to the baseline as long as one firm is informed but feature perfect competition when no firm is informed. Paradoxically enough, even though a lower probability of information endowment increases the set of discount rates in which the monopoly payoffs is sustained, it also decreases the expected payoff in the full-disclosure equilibrium.

4.2. Mandatory disclosure

We have, to this point, examined disclosure policies that are under the control of each firm, although disciplined by the tacit agreement; that is, the environment under consideration is that of forward-looking disclosures that are self-regulated. The objective of this section is to extend the scope of the analysis to mandatory disclosure requirements, in which a regulator could enforce specific disclosures over certain subset of events. We will further examine the consequences of such disclosures requirements on firms’ profit as well as consumer and total surplus.\textsuperscript{23}

To be noted, since the kind of competition considered here is Bertrand competition, we know that firms’ surplus is increasing in prices while consumer and total surplus are decreasing in prices (hereafter, we save space by using the terminology of “social surplus” for both consumer and total surplus). We shall now establish that the regulation of disclosure can provide a powerful tool for regulators to offset some of the detrimental welfare consequences of tacit agreements in an oligopoly.

Assume that a regulation takes the form of a non-empty set $\Omega_r \subseteq \mathbb{R}$ of market sizes that

\textsuperscript{23}To save space, we do not state the preferences explicitly but, in general, considerations of total surplus require the assumption that consumers’ preferences are quasi-linear in money.
must be disclosed. As an example, a set of the form $\Omega_r = [0, x]$ could represent a need to report an impairment when anticipating adverse market conditions. If a market shock is not subject to the regulation, firms can still disclose it voluntarily within a tacit agreement and we know refer to a no-disclosure equilibrium as an equilibrium in which no event that need not be disclosed by law is withheld. In the next Proposition, we describe the effect of regulations when the industry is concentrated or the discount rate is low.

**Proposition 4.3** Suppose that $\delta \geq \delta_{nd}$ (i.e., monopoly prices would be set absent any regulation), then mandatory disclosure always weakly decrease firms’ surplus and weakly increase social surplus, strictly so if and only if either one of these conditions hold:

(i) $\Omega_r$ includes some events greater than $\hat{s}$.

(ii) or, $\delta \leq \delta_r$ where $\delta_r$ is a bound strictly greater than $\delta_{nd}$ and increasing in $\Omega_r$ (in the sense of the inclusion).

Mandatory disclosure can have two adverse effects on the tacit agreement. When disclosing large realizations of the market shock $s$ (case (i)), the mandatory disclosure requirement breaks the benefits of no-disclosure and triggers more competition. This, in turn, benefits consumers and total surplus by reducing prices toward marginal cost. There is a also a second incentive benefit achieved by mandating disclosure (case (ii)). A public disclosure of low events does not necessarily reduce prices conditional on disclosure (since it leads to $p^*$) but it does raise uninformed firm’s expectations about market size for other events that are not disclosed. In turn, this tends to make it more difficult to sustain monopoly prices conditional on non-disclosure and, as before, can benefit consumers and total surplus.\(^{24}\) This finding supports the theory that large firms in concentrated markets are typically opposed to disclosure due to proprietary cost considerations. Indeed, the industry is better-off through self-regulation via the tacit agreement than it would be through direct monitoring of the disclosures.

\(^{24}\)Unlike a more direct regulatory intervention with price controls, increasing disclosure would not require regulators to know details of the industry (such as the marginal cost) or decide over operating decisions.
Consider next the case of less concentrated industries or industries with a lower discount factor. One possibility is that the industry chooses full-disclosure in which case the regulation does not have any effect. Another possibility is that the industry chooses no-disclosure for any event not subject to the regulation. We consider here the effect of mandatory disclosure in these settings.

**Proposition 4.4** Suppose that $\delta < \delta_{\text{nd}}$ then, in an efficient no-disclosure tacit agreement, mandatory disclosure always weakly increase firms’ surplus and weakly decrease social surplus, strictly so if $V_{fd} > 0$ and the regulation is an interval $\Omega_r = (s_1, s_2) \subset \mathbb{R}^+$ with $0 < s_1 < s_2 < +\infty$ that is sufficiently large.

When considering tacit agreements that feature no-disclosure and full-disclosure, mandatory disclosure benefits firms, at the detriment of consumers and social surplus, when $\delta < \delta_{\text{nd}}$. This implies that firms might demand more regulation in industries that are more competitive. The intuition for this finding is that mandatory disclosure can discipline firms to disclose favorable information that would, otherwise, have been withheld. This is done more easily through regulation than within a tacit agreement because, if done through a tacit agreement, an informed firm with very high market size would always deviate not to disclose. Consequently, we predict that the regulation of disclosure can be detrimental to social surplus in more competitive settings.

We conclude with some discussion of regulation in the context of partial disclosure equilibrium. The effect of regulation in these settings is generally more ambiguous. As in the case of full-disclosure, there can always be value in mandating disclosure of relatively high events but, in addition, disclosure of events that the tacit agreement would have preferred to keep silent can hurt firms and benefit consumers. To further see this, note that, in any partial disclosure equilibrium, the tacit agreement would be improved by disclosing events $s > \hat{s}_{pd}$; however, the incentive mechanism is not strong enough to induce such disclosures. Vice-versa, the tacit agreement would be worsened if one were to disclose events $s$ that are small and close to zero. The net effect of mandatory disclosure will typically be the result of a trade-off between these
two forces.

5. Concluding Remarks

In this paper, we explore the relationship between disclosure, industry cycles and product-market competition. We determine what forms of disclosure maximize industry profits, and relate firm profits to whether a firm is informed and discloses that information early. In our model, the optimal disclosure policy is endogenous and driven by concerns about future competition. In particular, we find that:

1. Policies with no disclosure are desirable in industries with a low discount rate or high concentration.

2. Policies with partial or full disclosure are desirable in industries with a high discount rate or low concentration.

3. In regimes with partial disclosure, informed firms retain very good and very bad information and disclose intermediate news.

4. Disclosure of good market conditions imply high profits for informed firms, but not necessarily for uninformed competitors.

5. Mandatory disclosure decreases firm profits and increases social surplus in concentrated industries, but the reverse can be true in more competitive industries.

We hope that our study will provide some first steps - with a model that puts the focus on the product market - to understand how and why disclosure interacts with the cyclical patterns of an industry. As we have shown using a standard paradigm in industry cycle research, cycles will have important effects on product-market driven incentives to disclose or retain information. However, broadening the scope to other disclosure paradigms, and most notably financial
reporting concerns, it is clear that more research is necessary to fully understand how the information provided by firms varies as a function of the current and long-term conditions of an industry.

Appendix: Omitted Proofs

Proof of Proposition 2.1: We derive the optimal strategy (i.e., the sets $\Omega_{\text{over}}$, $\Omega_{\text{share}}$, and $\Omega_{\text{under}}$), and then solve for the minimum discount rate $\delta_{nd}$ stated in Proposition 2.1.

Note first that any $s \in \Omega_{\text{over}}$ must be such that $s \leq \tilde{s}$ and any $s \in \Omega_{\text{share}}$ must be such that $s \leq \hat{s}$. Therefore, in the left-hand side of Equation (2.4),

\[
(1 - \delta) \left( \int_{\Omega_{\text{over}}} sh(s) ds \frac{\Pi^*}{N - 1} + \int_{\Omega_{\text{share}}} sh(s) ds \frac{\Pi^*}{N} \right) + \frac{\delta}{N} \frac{1}{N} \geq (1 - \delta) E_s [sp^* D(p^*)] = (1 - \delta)
\]

To maximize the left-hand side one should set $\Omega_{\text{over}} = [0, \tilde{s}]$ and $\Omega_{\text{share}} = \tilde{s}, \hat{s]}$. The minimum discount rate consistent with monopoly pricing is obtained by binding Equation (2.4).

\[
(1 - \delta_{nd}) \left( \int_{0}^{\tilde{s}} sh(s) ds \frac{\Pi^*}{N - 1} + \int_{\tilde{s}}^{\hat{s}} sh(s) ds \frac{\Pi^*}{N} \right) + \delta_{nd} \frac{1}{N} = (1 - \delta_{nd})
\]

That is:

\[
\delta_{nd} \left( \frac{N + 1}{N} - \int_{0}^{\tilde{s}} sh(s) ds \frac{\Pi^*}{N - 1} - \int_{\tilde{s}}^{\hat{s}} sh(s) ds \frac{\Pi^*}{N} \right) = 1 - \int_{0}^{\tilde{s}} sh(s) ds \frac{\Pi^*}{N - 1} - \int_{\tilde{s}}^{\hat{s}} sh(s) ds \frac{\Pi^*}{N}
\]

And solving for $\delta_{nd}$

\[
\delta_{nd} = \frac{N(N - 1) - N \int_{0}^{\tilde{s}} sh(s) ds \Pi^* - (N - 1) \int_{\tilde{s}}^{\hat{s}} sh(s) ds \Pi^*}{(N + 1)(N - 1) - N \int_{0}^{\tilde{s}} sh(s) ds \Pi^* - (N - 1) \int_{\tilde{s}}^{\hat{s}} sh(s) ds \Pi^*}
\]

This is the desired Equation for $\delta_{nd}$.

Proof of Proposition 2.2: Since it is optimal to set $P(s)$ as close as possible to $p^*$ while still respecting constraint (2.7), there must be a threshold, denoted $S$ such that for $s \leq S$, $P(s) = p^*$ and for $s > S$, $P(s) < p^*$.

We solve first for $S$. Set $P(s) = p^*$ and bind Equation (2.7), i.e.

\[
(1 - \delta) \Pi^* S = (1 - \delta) \frac{\Pi^* S}{N} + \delta V_{fd}
\]
Solving for $S$ yields:

$$S = \frac{\delta}{1 - \delta \frac{N}{N-1}} V_{fd}$$

For $s \leq S$, $sP(s)D(P(s)) = s\Pi^*$. 

For $s > S$, since Equation (2.7) binds,

$$sP(s)D(P(s)) = \frac{\delta}{1 - \delta \frac{N}{N-1}} V_{fd}$$

Then:

$$V_{fd} = \frac{1}{N}(\Pi^* \int_0^S sh(s)ds + \int_S^\infty sP(s)D(P(s))h(s)ds)$$

$$= \frac{1}{N}(\Pi^* \int_0^S sh(s)ds + \frac{\delta}{1 - \delta \frac{N}{N-1}} V_{fd} \int_S^\infty h(s)ds)$$

$$= \frac{\Pi^*/N}{1 - \frac{\delta}{1 - \delta \frac{N}{N-1}} \int_0^S h(s)ds} \int_0^S sh(s)ds$$

Solving for $S$ yields Equation (2.8). $\square$

**Proof of Proposition 2.3:** In a no-disclosure PPNE, then: (i) the uninformed firms choose $p_{nd} \leq p^*$, (ii) for $s \in \Omega_{\text{over}}$, the informed firm chooses $p > p_{nd}$ (overprices), (iii) for $s \in \Omega_{\text{share}}$, the informed firm chooses $p = p_{nd}$ and share, (iv) for $s \notin \Omega_{\text{over}} \cup \Omega_{\text{share}}$, the informed firm chooses $p = p_{nd}$ and undercuts.

First, we compute the per-firm surplus $V_{nd}$ in this PPNE,

$$V_{nd} = \int h(s) s\Pi_{nd} ds / N = \int h(s) s\Pi^* ds \Pi_{nd} / (N\Pi^*) = \Pi_{nd} / (N\Pi^*)$$

(A-1)

As before, it is optimal to set $\Omega_{\text{over}}$ as the largest possible set such that the informed firm overprices, that is: $s \in \Omega_{\text{over}}$ if and only if $s \leq \tilde{s}_{nd}$ where:

$$\delta V_{nd} \geq (1 - \delta_{nd}) \tilde{s}_{nd} \Pi_{nd}$$

(A-2)

Therefore: $\tilde{s}_{nd} = \frac{\delta}{1 - \delta} \frac{V_{nd}}{\Pi_{nd}} = \bar{s}$ (does not depend on $p_{nd}$).
Similarly, $s \in \Omega_{\text{share}}$ if and only $s \in [\tilde{s}, \hat{s}]$.

Let us now write the incentive-compatibility for the uninformed:

\[
(1 - \delta) \left( \int_{0}^{\tilde{s}} s h(s) ds \frac{\Pi_{\text{nd}}}{N - 1} + \int_{\tilde{s}}^{\hat{s}} s h(s) ds \frac{\Pi_{\text{nd}}}{N} \right) + \delta \frac{\Pi_{\text{nd}}}{\Pi^* N} \geq (1 - \delta) \frac{\Pi_{\text{nd}}}{\Pi^*} \quad (A-3)
\]

Suppose that $\Pi_{\text{nd}} > 0$. Then, multiplying both sides by $\Pi^*/\Pi_{\text{nd}}$, this incentive-compatibility condition is the same as that required in Proposition 2.1. Therefore, $\delta \geq \delta_{\text{nd}}$.

**Proof of Proposition 4.1:** Inequality (4.1) can be written as $\delta \geq \delta_{\text{nd}}$ where $\delta_{\text{nd}} < 1$ is given by:

\[
\frac{\delta_{\text{nd}}^q}{1 - \delta_{\text{nd}}^q} = N - (q + (1 - q)\Pi^* \int_{s \geq s} s h(s) ds)
\]

Recall that $\Pi^* \int_{s \geq s} s h(s) ds < \Pi^* \int s h(s) ds = 1$, so that the term in parenthesis is increasing in $q$, implying that the right-hand side decreases in $q$ and therefore $\delta_{\text{nd}}^q$ decreases in $q$.

**Proof of Proposition 4.2:** Consider the following strategy: 1. the informed firm always discloses its information, 2. when no information is disclosed, all firms choose a price equal to zero (but do not activate the punishment path), 3. if the information is disclosed, firms choose the pricing strategy that corresponds to the full-disclosure equilibrium in the baseline model. Note that the informed firm is always better-off disclosing because not disclosing would generate zero profit in the current period.

To conclude the argument, consider a full-disclosure equilibrium in which firms choose $p_{\text{nd}} > 0$ if no firm makes a disclosure. Consider the incentive-compatibility condition of an informed firm in such an equilibrium if $s > S$. If this firm chooses not to disclose, the worst that may occur is that all firms choose zero profit in all future periods as a continuation payoff. On the equilibrium path, the very best that could occur is that the firm achieves its monopoly profit in future periods and a current cash flow strictly less than $\frac{\delta}{1 - \delta} \frac{1}{N - 1}$. Therefore, for this to be incentive-compatible, a necessary inequality is that:

\[
(1 - \delta) \frac{\delta}{1 - \delta} \frac{1}{N - 1} + \delta \frac{1}{N} \geq (1 - \delta) s p_{\text{nd}} D(p_{\text{nd}}) + \delta 0
\]

Note that this is only a necessary condition, and the actual incentive-compatibility condition is much more demanding. Yet, this condition is already violated for $s$ large enough, therefore $p_{\text{nd}}$ must be zero.

**Proof of Lemma 3.1:** To prove the result, we define two auxiliary variables that describe disclosure and pricing strategies on the cooperation path. First, let $a(s)$ be a binary function such that $a(s) = 0$ if the informed firm discloses and $a(s) = 1$ if the firm does not disclose. Second, let $b(s)$ be a binary function such that $b(s) = 0$ if the
informed firm undercuts and $b(s) = 1$ if the informed firm shares (we do not need to give a label to overpricing).

Note that because of our restriction to equilibria in which all firms price identically after a disclosure, we must have that $b(s) = 1$ if $a(s) = 0$. Conditional on $a(s) = 0$, it must then be incentive-compatible for all firms to share, that is:

$$(1 - \delta)sP(s)D(P(s))/N + \delta V_{pd} \geq (1 - \delta)sP(s)D(P(s))$$  \hspace{1cm} (A-4)

If $P(s) = p^*$ satisfies this inequality, it is optimal to set $P(s) = p^*$. Otherwise, maximizing $sP(s)D(P(s))$ requires to bind this inequality and set:

$$sP(s)D(P(s)) = \frac{\delta}{1 - \delta} \frac{N}{N - 1} V_{pd}$$  \hspace{1cm} (A-5)

Next, for the informed firm, it must be incentive-compatible to disclose over an alternative deviation to not disclose (followed by undercutting). Note that this deviation is only attractive under two conditions, (i) $P(s) < p_{pd}$ (otherwise disclosure and undercutting, as examined earlier dominates no-disclosure) and (ii) the following incentive-compatibility condition is not met:

$$(1 - \delta)\Pi_{pd}s \leq (1 - \delta)sP(s)D(P(s))/N + \delta V_{pd}$$

$$\leq (1 - \delta)\left(\frac{\delta}{1 - \delta} \frac{N}{N - 1} V_{pd}\right) + \delta V_{pd}$$

$$\leq \delta V_{pd} \frac{N}{N - 1}$$

This last condition boils down to $s \leq \hat{s}_{pd}$. In summary, $a(s) = 0$ and $b(s) = 1$ is incentive-compatible if and only if $P(s) \geq p_{pd}$ or $s \leq \hat{s}_{pd}$. To simplify these conditions further, assume that $s > \hat{s}_{pd}$. Then, the strategy is incentive-compatible if and only if $P(s) \geq p_{pd}$. $\square$

**Proof of Lemma 3.2:** Evaluating $P(\hat{s}_{pd})$, we obtain that:

$$\hat{s}_{pd}P(\hat{s}_{pd})D(P(\hat{s}_{pd})) = \frac{\delta}{1 - \delta} \frac{N}{N - 1} V_{pd}$$  \hspace{1cm} (A-6)

Suppose by contradiction that $P(\hat{s}_{pd}) \neq p_{pd}$, then:

$$\hat{s}_{pd} = \frac{\delta}{1 - \delta} \frac{N}{N - 1} V_{pd}$$  \hspace{1cm} (A-7)
This is a contradiction. It then follows that \( P(\hat{s}_{pd}) = p_{pd} \) and, because \( P(s) \) is decreasing in \( s \), \( P(s) < p_{pd} \) if \( s > \hat{s}_{pd} \). The incentive-compatibility condition can thus be simplified as simply requiring that \( s \leq \hat{s}_{pd} \).\( \square \)

**Proof of Proposition 3.1:** We write first the incentive-compatibility for the uninformed. Conditional on a realization of \( s \) that is not disclosed, the cooperation payoff to the uninformed is given by: \( s(1 - \delta) \Pi_{pd} 1_{s \leq \hat{s}_{pd}} \frac{1}{N} + \delta V_{pd} \).

Conditional on non-disclosure, the uninformed firm makes a conditional expectation, which yields the following incentive-compatibility condition.

\[
(1 - \delta) \Pi_{pd} \frac{1}{N} \int_0^{\hat{s}_{pd}} sa(s) h(s) ds + \delta V_{pd} \geq (1 - \delta) \Pi_{pd} \frac{1}{N} \int_{\hat{s}_{pd}}^\infty sa(s) h(s) ds \tag{A-8}
\]

We state next the problem of finding the efficient partial disclosure equilibrium:

\[
\max_{\Pi_{pd} \leq \Pi^*, V_{pd}, a(.)} V_{pd}
\]

subject to \( a(s) = 1 \) if \( s \geq \hat{s}_{pd} \), and:

\[
V_{pd} \leq \frac{1}{N} \int h(s) \{ a(s) s \Pi_{pd} + (1 - a(s)) \min \{ s\Pi^*, \frac{s}{1 - \delta} \frac{N - 1}{N} V_{pd} \} \} ds
\]

\[
0 \leq \Pi_{pd} \left( \frac{1 - N}{N} \int_0^{\hat{s}_{pd}} sa(s) h(s) ds - \int_{\hat{s}_{pd}}^\infty sa(s) h(s) ds \right) \frac{\delta V_{pd}}{(1 - \delta)} \int a(s) h(s) ds
\]

In this problem, the first constraint is the regeneration condition and states that the expected continuation utility should be consistent with what is expected from the strategies played in the game. The second constraint corresponds to Equation (A-8) after multiplying both sides by \( \int a(s) h(s) ds/(1 - \delta) \). Hereafter, let \( S = \frac{s}{1 - \delta} \frac{N - 1}{N} V_{pd} < \hat{s}_{pd} \).

Let \( L \) denote the Lagrangian of this problem. Denote \( \lambda \) (resp., \( \mu \)) the Lagrange multiplier associated to the first (resp., second) constraint. The multiplier \( \lambda \) is readily verified to be strictly positive (if not, \( V_{pd} \) arbitrarily large would maximize the Lagrangian). Differentiating in \( a(s) \) for any \( s \leq \hat{s}_{pd} \),

\[
K(s) = \frac{1}{h(s)} \frac{\partial L}{\partial a(s)} = s \left( \lambda \frac{\Pi_{pd} - 1_{s \leq S} \Pi^*}{N} - \mu \Pi_{pd} \frac{N - 1}{N} \right) + V_{pd} \frac{\delta}{1 - \delta} (\mu - 1_{s > S}) \frac{1}{N - 1} \lambda
\]

Note that:

\[
K(\hat{s}_{pd}) = \hat{s}_{pd} \left( \lambda \frac{\Pi_{pd}}{N} - \mu \Pi_{pd} \frac{N - 1}{N} \right) + V_{pd} \frac{\delta}{1 - \delta} (\mu - \frac{1}{N - 1})
\]

\[
= V_{pd} \frac{\delta}{1 - \delta} \left( \frac{\lambda}{N - 1} - \mu \right) + V_{pd} \frac{\delta}{1 - \delta} \left( \mu - \frac{1}{N - 1} \right)
\]

\[
= 0
\]
Suppose by contradiction that \( \mu = 0 \). Then, this function is strictly negative for any \( s \leq S \), implying that \( a(s) = 0 \) for any \( s \leq S \). For any \( s > S \), \( K(s) \) would be increasing in \( s \). It follows that, if one were to set \( \mu = 0 \), the optimum would be set at \( a(s) = 0 \) for all \( s \), a contradiction to \( a(s) \) non-zero for some \( s \).

It then follows that \( K(0) = V_{pd} \frac{\delta}{1-\delta} \mu > 0 \). Further, the function \( K(s) \) is linear on \([0, S]\) and on \([S, ˆs_{pd}]\) with a root at \( s = ˆs_{pd} \), therefore it must be the case that \( K(s) < 0 \) if and only if \( s \in (s_1, s_2) \) for \( 0 < s_1 < s_2 \leq ˆs_{pd} \). □

**Proof of Proposition 3.2:** To save space, note that the problem is identical to that stated in Proposition 3.1, except that the uninformed firm’s incentive-compatibility condition is now written:

\[
0 \leq \Pi_{pd}\left(\frac{2 - N}{N - 1}\int_0^{hat_s_{pd}} sa(s)h(s)ds + \frac{1 - N}{N}\int_{hat_s_{pd}}^{hat_S} sa(s)h(s)ds - \int_{hat_s_{pd}}^{\infty} sh(s)ds\right) + \frac{\hat{V}_{pd}}{1 - \delta}\int a(s)h(s)ds
\]

As before, we denote \( L \) the Lagrangian and \( \lambda \) (resp., \( \mu \)) the Lagrange multiplier associated to the first (resp., second) constraint. Differentiating in \( a(s) \) for any \( s \leq ˆs_{pd} \),

\[
K(s) = \frac{1}{h(s)} \frac{\partial L}{\partial a(s)} = s\left(\lambda \frac{\Pi_{pd} - \frac{1}{N}Sh^*}{N} + \mu \Pi_{pd}(-1 + \frac{s_{hat} - s_{pd}}{N - 1} + \frac{1}{N} - 1)\right) + V_{pd} \frac{\delta}{1 - \delta}(\mu - \frac{1}{N})
\]

We know from the same argument as in Proposition 3.1 that \( \lambda \) and \( \mu \) are strictly positive, \( K(0) > 0 \) and \( K(ˆs_{pd}) = 0 \). To obtain the sign of \( K(s) \), note that \( K(s) \) is decreasing on \([0, S]\), linear on \((\min(S, ˆs_{pd}), \max(S, ˆs_{pd}))\) and linear on \([\max( ˆs_{pd}, S), ˆs_{pd}] \) with \( K(ˆs_{pd}) = 0 \). Therefore it can change sign at most twice on \([0, ˆs_{pd}]\). If \( K(s) \) is decreasing on \([\max( ˆs_{pd}, S), ˆs_{pd}] \), it must be that non-disclosure is optimal on this region and therefore the disclosure region must be an interval with the form \((s_1, s_2)\). Otherwise, we consider two cases. Case 1. If \( ˆs_{pd} \leq S \) (i.e., \( \Pi_{pd} \geq (N - 1)/N \)), \( K(s) \) is decreasing on \([0, S]\) and increasing on \([S, ˆs_{pd}]\), therefore the disclosure region must have the form \((s_1, ˆs_{pd})\). Case 2. If \( ˆs_{pd} > S \), \( K(s) \) is decreasing on \([0, S]\) and increasing on \([S, ˆs_{pd}]\) and on \([ ˆs_{pd}, ˆs_{pd}] \), therefore the disclosure region must have the form \((s_1, s_0)\) and \((s_2, ˆs_{pd})\).

**Proof of Proposition 4.3:** Suppose that \( \Omega_e \) includes events \( s \geq ˆs \). Suppose by contradiction that \( p^* \) can be implemented in all periods. Then, conditional on a disclosure of any such event, \( P(s) = p^* \) is incentive-compatible if and only if

\[
(1 - \delta)sP^*/N + \delta/N \geq (1 - \delta)sP^*
\]

This inequality simplifies that \( s \leq ˆs \). Therefore, any disclosed event above \( ˆs \) must yield a surplus less than \( sP^* \) and reduces firm profits while increasing consumer and total surplus.

Suppose next that \( \Omega_e \subseteq (0, ˆs) \). We need to set \( p^* \) in all periods which, if this conjecture is valid, occurs when any \( s \in \Omega_e \) is disclosed. In addition, we need to verify that sharing is incentive-compatible for the uninformed
when \( s \notin \Omega_r \) and the information is not shared (as before, not sharing is optimal for any event that is not subject to a mandatory disclosure). As in Proposition 2.1, this requires that \( \delta \geq \delta_r \) where, denoting \( d(s) = 1_{s \notin \Omega_r} \),

\[
\delta_r = \frac{N(N-1) - H}{(N+1)(N-1) - H}
\]

where \( H = -N \int_0^\delta sd(s)h(s)ds \Pi^* - (N-1) \int_\delta^1 sd(s)h(s)ds \Pi^* \). Note that \( \delta_r \) is decreasing in \( H \) and \( H \) is decreasing as \( \Omega_r \) increases (in the sense of the inclusion). Therefore, \( \delta_r \) increases as \( \Omega_r \) increases. \( \square \)

**Proof of Proposition 4.4:** Note that if \( \delta < \delta_{nd} \), a no-disclosure equilibrium would yield an industry profit \( V_{nd} = 0 \) absent the regulation, so that some regulation is always weakly preferred by firms. To show that this preference is strict, assume that a full-disclosure tacit agreement exists with positive industry profits \( V_{fd} > 0 \), and therefore it implies \( S > 0 \). Consider next setting \((s_1, s_2)\) large enough such that conditional on not disclosing \( s < s_1 \) and \( s > s_2 \), the monopoly price \( p^* \) can be implemented. This can be constructed by finding \( s_1 \) close to zero and \( s_2 \) large as follows:

\[
(1 - \delta)\Pi^* \frac{\int_0^{s_1} sh(s)ds}{\int_0^{s_1} h(s)ds + \int_{s_2}^{\infty} h(s)ds} \geq (1 - \delta)\Pi^* \frac{\int_0^{s_1} sh(s)ds + \int_{s_2}^{\infty} sh(s)ds}{\int_0^{s_1} h(s)ds + \int_{s_2}^{\infty} h(s)ds}
\]

By continuity and the fact that the inequality is satisfied strictly for \( s_1 = 0 \) and \( s_2 = +\infty \), one can always find \( s_1 > 0 \) and \( s_2 < +\infty \) such that the inequality is still satisfied. This alternative equilibrium will always yield higher prices than full-disclosure in the region \( s > s_2 \). \( \square \)

**References**


Suijs, J. and Wielhouwer, J. L.: 2011, Should regulators care about proprietary costs of disclosure?

