Relational Contracts as a Foundation for Bonus Pools

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Abstract: Much of our thinking about (and criticism of) the relationship between pay and performance seems to be based on a standard principal-agent model of individual incentives. In this paper, we instead explore incentives provided to a team of agents and the role mutual monitoring and collusion play in determining the optimal provision of incentives. A key feature of this paper's model relative to other models of team incentives (e.g., Arya, Fellingham, and Glover, 1997; Che and Yoo, 2001) is that the rewards are discretionary/based (at least in part) on non-verifiable information. If the performance measures were objective and contracts explicit, the principal would use joint performance evaluation (*JPE*) to motivate the agents to mutually monitor and motivate each other. When the non-verifiability problem is a pressing one and Pure *JPE* is no longer feasible, the principal might try to substitute individual incentives for mutual monitoring incentives by using relative performance evaluation (*RPE*). However, *RPE* undoes some of the mutual monitoring incentives created by *JPE*, so even greater (more costly) individual incentives are needed. The substitution of individual for mutual monitoring incentives is particularly costly when the agents are patient (have a low discount rate), since this is the situation in which mutual monitoring offers a potential large improvement over individual incentives. In this case, it can be better to reward the agents for joint poor performance. Rewarding joint poor performance is beneficial because it allows the principal to increase the rewards for joint good performance and keep the focus on mutual monitoring rather than individual incentives. When motivating mutual monitoring is infeasible (or at least not optimal), the principal has to make sure the contract does not invite collusion. The optimal way to deter collusion when the agents' ability to collude is strong is to create a strategic independence in the agents' payoffs, since this makes the two most severe collusion problems equally costly to deal with. The strategic independence is also created by relying on pay for joint poor performance. Hence, our paper provides two new explanations for why subjectively determined rewards might appear to unfairly reward poor performance.

1. Introduction

Much of our thinking about (and criticism of) the relationship between pay and performance seems to be based on a standard principal-agent model of individual incentives. In this paper, we instead explore incentives provided to a team of agents and the role mutual monitoring and collusion play in determining the optimal provision of incentives. A key feature of this paper's model relative to other models of team incentives (e.g., Arya, Fellingham, and Glover, 1997; Che and Yoo, 2001) is that the rewards are discretionary/based (at least in part) on non-verifiable information.

Discretion in awarding bonuses and other rewards is pervasive. Evaluators use discretion in determining individual rewards, the total reward to be paid out to all (or a subset of the) employees, and even in deviating from explicit bonus formulas (Murphy and Oyer, 2001; Gibbs et al., 2004). A common concern about discretionary rewards is that the evaluator must be trusted by evaluates (Anthony and Govindarajan, 1998).

In a single-period model, bonus pools are a natural economic solution to the "trust" problem (Baiman and Rajan, 1995; Rajan and Reichelstein, 2006; 2009). When all rewards are discretionary (based on subjective assessments of individual performance), a single-period bonus pool rewards bad performance, since the total size of the bonus pool must be a constant in order to make the evaluator's promises credible.

The relational contracting literature has explored the role repeated interactions can have in facilitating trust and discretionary rewards based on subjective/non-verifiable performance measures (e.g., Baker, Gibbons, and Murphy, 1994), but this literature has mostly confined attention to single-agent settings.[1] This paper explores optimal discretionary rewards based on subjective/non-verifiable individual performance measures in a multi-period, principal-multi-agent model, which leads to discretionary rewards. The multi-period relationship creates the possibility of trust between the principal and the agents, since the agents can punish the principal for bad behavior. At the same time, the multi-period relationship creates the possibility of trust between the agents and, hence, opportunities for both cooperation/mutual monitoring (beneficial to the principal) and collusion (harmful to the principal) between the agents.

When the expected relationship horizon is long, the optimal contract emphasizes joint performance (*JPE*), which incentivizes the agents to use implicit contracting and mutual monitoring to motivate each other to "*work*" rather than "*shirk*." The subjective measures set the stage for the managers to use implicit contracting and mutual monitoring to motivate each other, as in existing models with verifiable performance measures (e.g., Arya, Fellingham, and Glover, 1997; Che and Yoo, 2001).[2]

When the expected horizon is short, the solution converges to the static bonus pool. A standard feature of a static bonus pool is that it rewards agents for (joint) bad performance in order to make the evaluator's promises credible.

For intermediate expected horizons, the optimal contract allows for more discretion in determining total rewards, which is typical in practice, but also rewards the agents for bad performance. The reason for rewarding bad performance is different than in the static setting—paying for bad performance allows the principal to create a strategic independence in the agents'

[1] One exception is Levin (2002), who examines the role that trilateral contracting can have in bolstering the principal's ability to commit—if the principal's reneging on a promise to any one agent means she will loose the trust of both agents, relational contracting is bolstered.

[2] There is an earlier related literature that assumes the agents can write explicit side-contracts with each other (e.g., Tirole, 1986; Itoh, 1993). Itoh's (1993) model of explicit side-contracting can be viewed as an abstraction of the implicit side-contracting that was later modeled by Arya, Fellingham, and Glover (1997) and Che and Yoo (2001). As Tirole (1992), writes: "[i]f, as is often the case, repeated interaction is indeed what enforces side contracts, the second approach [of modeling repeated interactions] is clearly preferable because it is more fundamentalist."

payoffs that reduces their incentives to collude. If the principal did not have to prevent tacit collusion between the agents, she would instead use a relative performance evaluation (*RPE*) scheme. The unappealing feature of *RPE* is that it creates a strategic substitutability in the agents' payoffs that encourages them to collude on an undesirable equilibrium that has the agents taking turns making each other look good—they alternate between (*work, shirk*) and (*shirk, work*).[3] While it is natural to criticize discretionary rewards for bad performance (e.g., Bebchuk and Fried, 2006), our result provides a rationale for such rewards. In this light, individual performance evaluation (*IPE*) can be seen as one of a class of incentive arrangements that create strategic independence—*IPE* is the only such arrangement that does not involve rewarding poor performance. Pay for poor performance and the strategic independence it facilitates is optimal when the agents' ability to commit is relatively strong and, hence, the cost of collusion is relatively large. When the agents' ability to commit is instead relatively weak, increased *RPE* is optimal.

Even when mutual monitoring is optimal, rewarding poor performance can be optimal because of the principal's limited ability to commit. In this case, the role of rewarding poor performance is that it enables the principal to keep incentives focused on mutual monitoring by using *JPE*. The alternative is to use *RPE* to partially substitute mutual monitoring incentives with individual (Nash) incentives. When the agents' ability to commit is relatively strong, the unconstrained benefit to the principal of mutual monitoring is so large that relying entirely on mutual monitoring for incentives is optimal even when the problem is more constrained by the principal's limited ability to commit. When the agents' ability to commit is instead relatively weak, increased *RPE* is again optimal.

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[3] Even in one-shot principal-multi-agent contracting relationships, the agents may have incentives to collude on an equilibrium that is harmful to the principal (Demski and Sappington, 1984; Mookherjee, 1984).

In our model, all players share the same expected contracting horizon (discount rate). Nevertheless, the players may differ in their relative credibility because of other features of the model such as the loss to the principal of forgone productivity. In determining the optimal incentive arrangement, both the common discount rate and the relative credibility of the principal and the agents are important.

There is a puzzling (at least to us) aspect of observed bonus pools. Managers included in a particular bonus pool are being told that they are part of the same team and expected to cooperate with each other to generate a larger total bonus pool (Eccles and Crane, 1988). Those same managers are asked to compete with each other for a share of the total payout. We extend the model to include an objective/verifiable team-based performance measure. Productive complementarities in the objective team-based measure can make motivating cooperation among the agents optimal when it would not be in the absence of the objective measure. The productive complementarity also takes pressure off of the individual subjective measures, allowing for a greater degree of *RPE* (and less pay-for-bad performance) than would otherwise be possible. Put differently, the combination of rewarding a team for good performance but also asking agents to compete with each other for a share of the total reward is not inconsistent with motivating cooperation and mutual monitoring. Instead, such commonly observed schemes can be an optimal means of motivating cooperation and mutual monitoring when the principal's commitment is limited. The earlier theoretical literature on bonus pools did not develop this role for bonus pools because of their focus on static settings.

Like our main model, Kvaløy and Olsen (2006) study a multi-period, multi-agent model in which all performance measures are subjective. They allow for even more reneging that rules out pay for bad performance, which is the focus of our paper. Our extension is closely related to

Baldenius and Glover (2012) on dynamic bonus pools. They take the form of the bonus pool as given, restricting attention to essentially static bonus pool forms. In particular, all of their bonus pools have the feature that the total payout does not depend on the subjective performance measures. In contrast, the focus of this paper is on optimal contracts, which incorporate discretion in determining the size of the bonus pool.

Baiman and Baldenius (2009) study the role of non-financial performance measures can have in encouraging cooperation by resolving hold-up problems. The empirical literature also provides evidence consistent with discretion being used to reward cooperation (e.g., Murphy and Oyer, 2001; Gibbs et al., 2004). Our model is consistent with this view in that the discretionary rewards are used to motivate cooperation when possible. Our analysis points out the importance of both the evaluator's and the evaluatees' reputation in sustaining cooperation through mutual monitoring.

The remainder of the paper is organized as follows. Section 2 presents the basic model. Section 3 studies implicit side-contracting between the agents that is harmful to the principal, while Section 4 studies implicit side-contracting between the agents that is beneficial to the principal. Section 5 characterizes the optimal overall contract. Section 6 studies an extension in which there are both individual subjective performance measures (as in the rest of the paper) and an objective team-based performance measure. Section 7 concludes.

2. Model

A principal contracts with two identical agents, $i = A,B,$ to perform two independent and ex ante identical projects (one project for each agent) in an infinitely repeated relationship, where t is used to denote the period, $t = 1, 2, 3, \ldots$ Each agent chooses a personally costly effort

$e_t^i \in \{0,1\}$ in period *t*, i.e., the agent chooses either "*work*" $(e_t^i = 1)$ or "*shirk*" $(e_t^i = 0)$. Each agent's personal cost of *shirk* is normalized to be zero and of *work* is normalized to be 1. Agent i's performance measure in period *t*, denoted x_t^i, is assumed to be either high ($x_t^i = H > 0$) or low ($x_t^i = L = 0$) and is a (stochastic) function of only e_t^i. In particular, $q_1 \equiv \Pr(x^i = H \mid e^i = 1)$, $q_0 \equiv \Pr(x^i = H \mid e^i = 0)$, and $0 < q_0 < q_1 < 1$. (Whenever it does not cause confusion, we drop sub- and superscripts.) Notice that each agent's effort choice does not affect the other agent's probability of producing a good outcome. Throughout the paper, we assume each agent's effort is so valuable that the principal wants to induce both agents to *work* ($e^i = 1$) in every period. (Sufficient conditions are provided in an appendix.) The principal's problem is to design the contract that motives both agents to *work* in each and every period at the minimum cost.

Because of their close interactions, the agents observe each other's effort choice in each period. Communication from the agents to the principal is blocked—the outcome pair (x^i, x^j) is the only signal on which the agents' wage contract can depend. Denote by w_{mn}^i the wage agent *i* receives if his outcome is *m* and his peer's outcome is *n*; $m, n \in \{H, L\}$. The wage contract provided to agent *i* is a vector $w^i \equiv \{w_{HH}^i, w_{HL}^i, w_{LH}^i, w_{LL}^i\}$. Given wage scheme w^i and assuming that agents *i* and *j* choose efforts level $k \in \{1,0\}$ and $l \in \{1,0\}$ respectively, agent *i*'s expected wage is:

$$\pi\left(k,l;w^i\right) = q_k q_l w_{HH}^i + q_k\left(1-q_l\right)w_{HL}^i + \left(1-q_k\right)q_l w_{LH}^i + \left(1-q_k\right)\left(1-q_l\right)w_{LL}^i .$$

All parties in the model are risk neutral and share a common discount rate *r*, capturing the time value of money or the probability the contract relationship will end at each period (the contracting horizon). The agents are protected by limited liability—the wage transfer from the principal to each agent must be nonnegative:

$$w_{mn} \geq 0 \text{ , } \forall m, n \in \{H, L\} \quad (\textit{Non-negativity}).$$

Unlike Che and Yoo (2001), we assume the outcome (*m,n*) is unverifiable. The principal, by assumption, can commit to a contract form but cannot commit to reporting the unverifiable performance outcome (*m,n*) truthfully.[4] Denote the principal's report on the performance measures in period *t* by $\hat{x}_t = (\hat{x}_t^A, \hat{x}_t^B)$. Like Che and Yoo (2001), the contract is restricted in the sense that it specifies wages in each period as a function of only the current period's performance.

Denote by H_t the history of all actions and outcomes before period *t*. For example H_3 = $((w^A, w^B), (e_1^A, e_1^B, x_1^A, x_1^B, \hat{x}_1^A, \hat{x}_1^B), (e_2^A, e_2^B, x_2^A, x_2^B, \hat{x}_2^A, \hat{x}_2^B))$. Denote by P_t the public profile at period *t*—those actions and outcomes that all parties observe, i.e., the history without the agents' actions. The principal's strategy is a wage scheme and reporting strategy, where the reporting strategy in period *t* can be conditioned on the public profile P_t up to that point. For the agents, their strategies map the entire history H_t to period *t* actions. The equilibrium concept is Perfect Bayesian Equilibrium (PBE) rather than Perfect Public Equilibrium, since the agents' strategies depend on more than the public profile. Among the large set of PBE's, we choose the one that is best for the principal subject to two constraints. First, the PBE must also be collusion-proof: there can be no other PBE that has only the agents changing their strategies and provides each agent with a higher payoff. Second, the PBE must motivate the principal to provide an honest evaluation of the agents—to report the performance measures truthfully.

The main reason for focusing on truth-telling PBEs is that such equilibria seem consistent with the applied problem we want to study. That is, we are interested in characterizing the

[4] In contrast, Kvaloy and Olsen (2006) assume the principal cannot commit to the contract, which makes it optimal to set $w_{LL} = 0$. Our assumption that the principal can commit to the contract is intended to capture the idea that the contract and the principal's subjective performance rating of the agents' performance can be verified. It is only the underlying performance that cannot be verified.

optimal short-term contracts that provide the principal with incentives to provide an honest evaluation. There may be other PBEs that have the principal adopting a history-dependent lying strategy (a long-term implicit contract), but these would effectively mimic long-term explicit contracts. Studying long-term contracts would shift the focus of the paper away from applied (and easy to interpret) contracts such as relative performance evaluation, joint performance evaluation, and individual performance evaluation. Moreover, since Che and Yoo restrict attention to short-term contracts, studying long-term ones in this paper (either explicit or implicit long-term contracts) would make our results difficult to compare to Che and Yoo's. As an aside, it is unclear to us if such complicated lying strategies would provide a potential improvement to the principal, since the agents seem likely to shirk during the lying (punishment) phase. Such punishment phases often have all parties reverting to the stage game equilibrium.

Since the agents perfectly observe all actions, confining attention to trigger strategies for them is without loss of generality in the sense that whatever can be accomplished by a strategy that depends on a longer history can also be accomplished with a strategy that depends only on the last period. They will use these trigger strategies to motivate the principal to tell the truth and possibly also to motivate each other.

To motivate the principal to tell the truth, we consider the following trigger strategy played by the agents: both agents behave as if the principal will honor the implicit contract until the principal lies about one of the performance measures, after which the agents punish the principal by choosing (*shirk, shirk*) in all future periods. This punishment is the severest punishment the agents can impose on the principal. The principal will not renege if:

$$\frac{2[q_1 H-\pi(1,1;w)]-2q_0 H}{r} \geq \underset{\{m',n'\}}{Max}\left\{\left(w_{mn}+w_{nm}\right)-\left(w_{m'n'}+w_{n'm'}\right)\right\}\ \forall m,n\in\{H,L\}\ \textit{(Principal's IC)}.$$

This constraint assures the principal will not claim the output pair from the two agents as (*m'*,*n'*) if the true pair is (*m*,*n*). The left hand side is the cost of lying.[5] The agents choosing (*shirk*, *shirk*) and the principal paying zero to each agent is a stage-equilibrium. Therefore, the agents' threat is credible. The right hand side of this constraint is the principal's benefit of lying about the performance signal.

3. Implicit Contracting between the Agents

The fact that agents observe each other's effort choice, together with their multi-period relationship, gives rise to the possibility that they use implicit contracts to motivate each other to *work* as in Arya, Fellingham, and Glover (1997) and Che and Yoo (2001), as long as playing (*work*, *work*) Pareto-dominates all other possible strategy combinations. Consider the following trigger strategy used to enforce (*work*, *work*): both agents play *work* until one agent *i* deviates by choosing *shirk*; thereafter, agent *j* punishes *i* by choosing *shirk*:

$$\frac{(1+r)}{r}[\pi(1,1;w)-1] \geq \pi(0,1;w)+\frac{1}{r}\pi(0,0;w) \quad (\textit{Mutual Monitoring}).$$

Such a mutual monitoring implicit contracting is beneficial for the principal and requires two conditions. First, each agent's expected payoff from playing (*work*, *work*) must be at least as high as from by playing the punishment strategy (*shirk*, *shirk*). In other words, (*work*, *work*) must Pareto dominate the punishment strategy from the agents' point of view in the stage game. Otherwise, (*shirk*, *shirk*) will not be a punishment at all:

$$\pi(1,1;w)-1 \geq \pi(0,0;w) \qquad (\textit{Pareto Dominance}).$$

[5] If she reneges on her implicit promise to report truthfully, the principal knows the agents will retaliate with (*shirk*, *shirk*) in all future periods. In response, the principal will optimally choose to pay a fixed wage (zero in this case) to each agent.

Second, the punishment (*shirk, shirk*) must be self-enforcing. The following constraint ensures (*shirk, shirk*) will be a stage game Nash equilibrium:

$$\pi(0,0;w) \geq \pi(1,0;w) - 1 \quad (\textit{Self-Enforcing Shirk}).$$

The agents' observation of each other's effort choice and multi-period relationship also open the opportunity for the agents to collude with each other. A collusion strategy is an implicit contract between agents that is harmful for the principal—we consider the following trigger strategy to support it: each agent sticks to the collusion strategy until any agent i deviates, in which case agent j would punish by choosing "work" indefinitely thereafter. Given the trigger strategy, any deviation from collusion will end up with $(\text{work}, \text{work})^{\infty}$. This trigger strategy is self-enforcing because $(work, work)^{\infty}$ is the equilibrium that the principal induces and is assured by other constraints in equilibrium.

Given the infinitely repeated relationship, the space of potential collusions between the two agents is rich: any strategy profile $e_0 = \{e_t^A, e_t^B\}_{t=0}^{\infty},\ e_t^A, e_t^B \in \{1,0\}$ can be a credible collusion, provided e_0 is self-enforcing. As it turns out, we can confine attention to two specific collusion strategies. If we prevent these two types of collusion, all other possible collusion strategies are also upset.

First, the contract has to satisfy the following condition to prevent collusion on (*shirk, shirk*) in all periods:

$$\pi(1,0;w) - 1 + \frac{\pi(1,1;w) - 1}{r} \geq \frac{(1+r)}{r}\pi(0,0;w) \quad (\textit{No Joint Shirking}).$$

The left-hand side is the agent's expected payoff from unilaterally deviating from (*shirk, shirk*), or *"Joint Shirking,"* for one period by unilaterally choosing *work* and then being punished indefinitely by the other agent by playing the stage game equilibrium (*work*, *work*) in all future periods, while the right-hand side is his expected payoff from sticking to *Joint Shirking.*

Second, the following condition is needed to prevent agents from colluding by "*Cycling,*" i.e., alternating between (*shirk, work*) and (*work, shirk*):

$$\frac{(1+r)}{r}[\pi(1,1;w)-1] \geq \frac{(1+r)^2}{r(2+r)}\pi(0,1;w)+\frac{(1+r)}{r(2+r)}[\pi(1,0;w)-1] \quad (\textit{No Cycling}).$$

The left hand side is the agent's expected payoff if he unilaterally deviates by choosing *work* when he is supposed to *shirk* and is then punished indefinitely with the stage game equilibrium of (*work, work*). The right hand side is the expected payoff if the agent instead sticks to the *Cycling* strategy.

Lemma 1: A contract is collusion-proof if it satisfies *No Joint Shirking* and *No Cycling* conditions.

Proof: All proofs are provided in an appendix.

The intuition for Lemma 1 is that all other (less symmetric) potential collusive strategies can only provide some period *t'* shirker with a higher payoff than under *Joint Shirking* and *Cycling* if some other period *t"* shirker has a lower continuation payoff than under *Joint Shirking* or *Cycling*. Hence, if the contract motivates all potential shirkers under *Joint Shirking* and *Cycling* to instead deviate to work, then so will the period *t"* shirker under the alternative strategy.

It is also helpful to distinguish three classes of contracts and point out how they interact with the two collusion-proof conditions above. The wage contract creates a *strategic complementarity* if $\pi(1,1;w)-\pi(0,1;w)>\pi(1,0;w)-\pi(0,0;w)$, which is equivalent to a payment complementarity $w_{HH}-w_{LH}>w_{HL}-w_{LL}$. Similarly, the contract creates a *strategic* *substitutability* if $\pi(1,1;w)-\pi(0,1;w)<\pi(1,0;w)-\pi(0,0;w)$, or equivalently $w_{HH}-w_{LH}<w_{HL}-w_{LL}$. The contract creates *strategic independence* if $\pi(1,1;w)-\pi(0,1;w)=\pi(1,0;w)-\pi(0,0;w)$, or $w_{HH}-w_{LH}=w_{HL}-w_{LL}$. The reason a payoff and payment complementarity (substitutability) are equivalent is that the performance measures are uncorrelated. As noted in the following observation, this classification has implications for which collusion strategy is most profitable from the agents' point of view and, thus, more costly for the principal to upset.

Observation:

(1) *No Joint Shirking* implies *No Cycling* if the contract creates a strategic complementarity in the agents' payoffs.

(2) *No Cycling* implies *No Joint Shirking* if the contract creates a strategic substitutability in the agents' payoffs.

(3) *No Cycling* and *No Joint Shirking* are equivalent if the contract creates a strategic independence in the agents' payoffs.

Whether a contract exhibits a strategic complementarity or a strategic substitutability has a subtle impact on the nature of the agent-agent collusive strategy. Investigating when and why the principal purposely designs the contract with a strategic complementarity, substitutability, or independence is the focus of the remainder of the paper.

4. The Principal's Problem

The basic problem faced by the principal is to design a minimum expected cost wage contract $w = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ that ensures (*work, work*) in every period is the equilibrium-path behavior of some collusion-proof equilibrium. The contract must satisfy the principal's reneging constraint, so that she will report her assessment of performance honestly. When designing the optimal contract, the principal can choose to explore the mutual monitoring between agents if it is worthwhile. Alternatively, he can implement a static Nash equilibrium subject to collusion proof constraints. The following integer programming summarizes the principal's problem.

$$(IP) \qquad \min_{\{w_{HH}, w_{HL}, w_{LH}, w_{LL}, T\}} \pi(1,1;w)$$

s.t.
No Cycling
Principal's IC
Non-negativity
*T * Mutual Monitoring*
*T * Pareto Dominance*
*T * Self-enforcing Shirk*
*(1-T) * Static NE*
*(1-T) * No Joint Shirking*
$T \in \{0,1\}$.

The variable *T* (short for team incentives) takes a value of either zero or one, which makes the program an integer program. *T=1* means that the principal designs the contract in order to induce team incentives by motivating mutual monitoring between the agents. Since (*shirk, shirk*) is Pareto dominated by (*work, work*) from the agents' point of view as *T=1*, the collusion strategy *Joint Shirking* considered in the previous section is not a concern here.

Therefore, we know from Lemma 1 that the contract in this case is collusion-proof as long as it satisfies the *No Cycling* constraint.

While the agents' mutual monitoring is beneficial to the principal, implementing it can be prohibitively costly or even infeasible, in which case the principal would stick with implementing the stage game Nash equilibrium (*T=0)*, subject to the additional no collusion and principal's incentive constraints. The following condition assures that *(work, work)* as a stage Nash equilibrium.

$$\pi(1,1;w)-1 \geq \pi(0,1;w) \qquad (\textit{Static NE}).$$

We solve the integer program (*IP*) by the method of enumeration and complete the analysis in two steps. In Sections 4.1 and 4.2, we solve the solution of IP while restricting $T = 1$ and then $T = 0$, respectively. We then compare the solutions for each parameter region and optimize over the choice of T in Section 4.3.

The following Lemma states the intuitive idea that it is never optimal to reward an agent for unilateral poor performance.

Lemma 2: All solutions to *IP* have the feature that $w_{LH} = 0$.

4.1 Cooperation

We are now ready to characterize the optimal solution for cases in which mutual monitoring (team incentives) is optimal.

Proposition 1. *When team incentives are optimal, i.e., $T = 1$, the solution to IP is one of the following (with $w_{LH} = 0$ in all cases):*

(i) Pure JPE: $w_{HH} = \dfrac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)}, w_{HL} = w_{LL} = 0$ *if and only if* $r \in (0, \delta^A]$;

(ii) BP1^C: $w_{HH} = \dfrac{(q_1-q_0)^2\left(q_0+q_1+(-1+q_1)r\right)H - (1+r)\left(-1+q_1^2+r\right)}{(q_1-q_0)\left(q_0+q_1-(-1+q_1)q_1 r - r^2\right)}; w_{HL} = 2 * w_{LL}$;

$$w_{LL} = \frac{(q_1-q_0)^2(q_0+q_1+q_1 r)H - (1+r)\left(q_1^2+r\right)}{(q_1-q_0)\left(q_0+q_1-(-1+q_1)q_1 r - r^2\right)}$$ *if and only if* $r \in \left(\delta^A, \min\left\{\tau^0, \delta^D\right\}\right]$

(iii)JPE: $w_{HH} = \dfrac{(1-q_1)q_1(1+r) + (q_1-q_0)^2\left(q_0+(-1+q_1)(1+r)\right)H}{(q_1-q_0)\left((-1+q_1)r(1+r)+q_0(q_1+r)\right)}$;

$$w_{HL} = \frac{\dfrac{(1+r)(q_1^2+r)}{q_1-q_0} - (q_1-q_0)(q_0+q_1+q_1 r)H}{(1-q_1)r(1+r) - q_0(q_1+r)}; w_{LL} = 0$$ *if and only if*

$r \in (\max\{\tau^0, \delta^A\}, \delta^C]$;

(iv) BP2^C: $w_{HH} = \dfrac{q_1 - q_0 + q_1 q_0 - q_1^2 + q_0 r + (q_1-q_0)^2(-1+q_1)H}{(q_1-q_0)\left((-1+q_1)r + q_0(-1+q_1+r)\right)}$;

$$w_{HL} = \frac{q_0 - 2q_1 q_0 + q_1^2 + r - 2q_0 r - (q_1-q_0)^3 H}{(q_1-q_0)\left((1-q_1)r - q_0(q_1+r-1)\right)}; w_{LL} = \frac{q_0(q_1-q_0)^2 H - q_0(q_1+r)}{(q_1-q_0)((1-q_1)r - q_0(-1+q_1+r))}$$ *if*

and only if $q_1 + q_0 \geq 1$ *and* $r \in (\max\{\tau^0, \delta^C\}, \delta^D]$;

where $\tau^0 = \dfrac{q_1+q_0-1}{(1-q_1)^2}$, δ^C *is same as in Proposition 2, and* δ^A *, and* δ^D *are increasing functions of H and are specified in the appendix.*

It is easy to check that all the solutions create a strategic payoff complementarity (denoted by the C superscript). In order to optimally exploit mutual monitoring between agents, the contract ensures that each agent's benefit to free-riding is small.

When r is small $(r \in (0, \delta^A])$, the contract offered is the same as the "Joint Performance Evaluation" (*Pure JPE*) contract studied in Che and Yoo (2001), i.e., the agents are rewarded only if the outcomes from both agents are high. Pure *JPE* is optimal because it is the optimal means of tying the agents together, so that they have both the incentive to mutually monitor each other and the means of punishing each other.

Starting from *Pure JPE*, as r increases, the principal has to increase w_{HH} because it becomes more difficult to motive the impatient agents to mutually monitor each other. However, the principal is also becoming less patient, and increasing w_{HH} will eventually become too expensive for the principal to truthfully report output (H, H), and she will have to choose between *JPE* and $BP1^C$. According to Proposition 1, $BP1^C$ follows the full commitment contract *Pure JPE* if and only if $\delta^A < \tau^0$. This condition is equivalent to H not being too large, since δ^A increases in H while τ^0 is independent of H. Using a similar argument to the one given just after Proposition 1, we know H enhances the principal's ability to commit to the *JPE* solution. In fact, if H is extremely large, $r = \delta^A$ is so large that agent-agent side-contracting is already fragile, even before the principal's weakened commitment comes into play. Although increasing w_{HH} is most efficient in exploiting mutual monitoring between the agents, it comes at the cost that w_{LL} must also be increased. The principal uses $BP1^C$ only if the agents are still somewhat patient so that the efficiency of w_{HH} in exploiting mutual monitoring among the agents dominates its cost.

We illustrate the intuition for why and when the $BP1^C$ emerges sooner (relative to *JPE*) using a numerical example: $q_0 = 0.5$, $q_1 = 0.9$, and $H = 100$. In this example, *Pure JPE* violates the Principal's IC constraint whenever $r > \delta^A = 14.12$ (the origin of Figure 1), and therefore she must either increase w_{HL} (*JPE*) or increase w_{LL} so that she can increase w_{HH} ($BP1^C$). The solid line in Figure 1 represents the relative cost of *JPE* compared to $BP1^C$ calculated as

$q_1(1-q_1)(w_{HL}^{iii}-w_{HL}^{ii})$, while the dotted line is the relative benefit calculated as $q_1^2(w_{HH}^{ii}-w_{HH}^{iii})+(1-q_1)^2 w_{HH}^{ii}$. One can see the principal chooses *BP1*^{C} over *JPE* if the two agents are somewhat patient ($r < \tau^0 = 40$) when the efficiency of w_{HH} in exploiting mutual monitoring is important enough. If one sets $H \geq 279.61$ instead, δ^A will exceed τ^0, and the region that used to be *BP1*^{C} ($14.12 < r < 40$) is now replaced by *JPE* since the principal's ability to commit is greater. Moreover, whenever the principal loses her commitment at $r = \delta^A = 40$, the agents are already so impatient that it is not worth increasing w_{HH} to induce mutual monitoring at the cost of increasing w_{LL}.

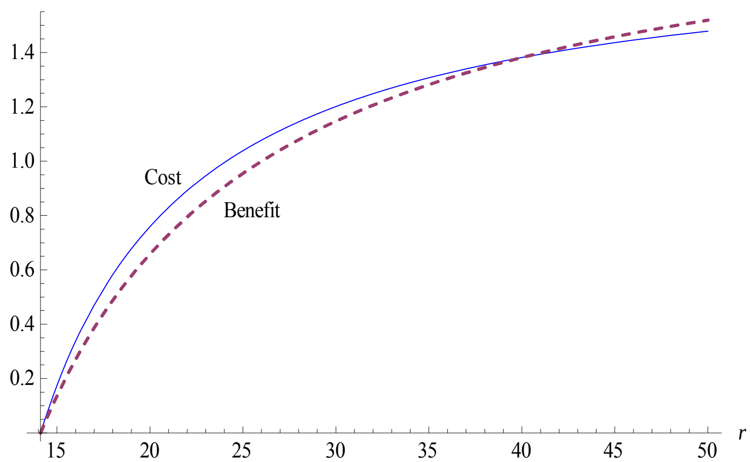


Figure 1: Cost and Benefit of *JPE* relative to BP1^C

A feature of Proposition 1 is that no feasible solution exists when r is large enough. There is a conflict between principal's desire to exploit the agents' mutual monitoring and her ability to commit to truthful reporting. Once r is too large, the intersection between the Mutual Monitoring constraint and the Principal's IC constraint is an empty set.

4.2 Collusion

Proposition 2. *When team incentives are not optimal, i.e., T = 0, the solution to the IP is one of the following (* $w_{LH}=0$ *in all cases):*

(i) IPE: $w_{HH}=w_{HL}=\dfrac{1}{q_1-q_0}$, $w_{LL}=0$ *if and only if* $r\in(0,\delta^C]$;

(ii) BP^I*:* $w_{HH}=\dfrac{\left(q_1-q_0\right)^2H-\left(1+r\right)\left(-1+q_1+r\right)}{\left(q_1-q_0\right)\left(1-r\left(-1+q_1+r\right)\right)}, w_{HL}=w_{HH}+w_{LL},$

$$w_{LL}=\frac{(q_1-q_0)^2(1+r)H-(1+r)(q_1+r)}{(q_1-q_0)(1-r(-1+q_1+r))}$$ *if and only if* $r\in(\delta^C,\tau^1]$;

(iii)RPE: $w_{HH}=\dfrac{\left(1-q_1\right)q_1\left(1+r\right)+\left(q_1-q_0\right)^2\left(-1-r+q_1\left(2+r\right)\right)H}{\left(q_1-q_0\right)\left(q_1^2-r\left(1+r\right)+q_1r\left(2+r\right)\right)}$;

$$w_{HL}=\frac{\left(q_1-q_0\right)q_1\left(2+r\right)H-\dfrac{\left(1+r\right)\left(q_1^2+r\right)}{q_1-q_0}}{q_1^2-r\left(1+r\right)+q_1r\left(2+r\right)}, w_{LL}=0$$ *if and only if* $r\in(\max\left\{\tau^1,\delta^C\right\},\delta^F]$;

(iv) BP^S *:* $w_{HH}=\dfrac{\left(1+r\right)\left(-\left(1-q_1\right)^2+r\right)-\left(q_1-q_0\right)^2(1-q_1)(2+r)H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))}; w_{HL}=2*w_{HH}$;

$$w_{LL}=\frac{(1+r)((2-q_1)q_1+r)+(q_1-q_0)^2(-2(1+r)+q_1(2+r))H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))}$$ *if and only if*

$r>\max\{\tau^1,\delta^F\}$ *;*

where $\tau^1=\dfrac{2q_1-1}{(1-q_1)^2}$ *, and* δ^C*,* δ^F *are increasing functions of* H *, which are specified in the appendix.*

In addition to the superscript "*C*", we use "*S*" and "*I*" to denote a (payoff) strategic substitutability and strategic independence, respectively. In Proposition 2, individual performance evaluation (*IPE*) is optimal if all parties are patient enough $(r \leq \delta^C)$. *IPE* is the benchmark solution—it is the optimal contract when the performance measures are verifiable (and mutual monitoring is not optimal), since the performance measures are uncorrelated.

As r increases, *IPE* is no longer feasible, because the impatient principal has incentive to lie when the output pair is (H, H). To see this, notice that for *IPE* contract, the right hand side of the Principal's IC constraint is $\max\{(w_{mn}+w_{nm})-(w_{m'n'}+w_{n'm'})\}=2w_{HH}$, while the left hand side of the constraint is strictly decreasing in r. As the principal becomes less patient, eventually she has incentives to lie and report the output pair as (L,L) when it is actually (H,H). In particular, the Principal's IC constraint starts binding at $r=\delta^C$. As r increases further, the gap between w_{HH} and w_{LL} must be decreased in order to prevent the principal from misreporting.

The principal has two methods of decreasing the gap between w_{HH} and w_{LL}. First, she can decrease w_{HH} and increase w_{HL}, increasing her reliance on *RPE*. Second, she can increase w_{LL} so that the contract rewards bad performance in the sense that both agents are rewarded even though they both produce bad outcomes, corresponding to solution BP^I in Proposition 2. This second approach compromises the agents' incentive to *work* because increasing w_{LL} makes *shirk* more attractive to both agents; as a result, w_{HH} or w_{HL} needs to be increased even more to provide enough effort incentive to the agents. One may think that BP^I will never be preferred to *RPE*. In fact, the only reason that BP^I is optimal is that it is an efficient way of combatting agent-agent collusion on the *Cycling* strategy. *RPE* creates a strategic substitutability in the agents' payoffs that makes C*ycling* particularly appealing.

RPE relies on w_{HL} to provide incentives, creating a strategic substitutability in the agents' payoffs. Under *RPE*, each agent's high effort level has a negative externality on the other agent's effort choice, making the *Cycling* strategy more difficult (more expensive) to break up than the *Joint Shirking* collusion strategy. Since collusion is more costly to prevent for small *r*, the principal purposely designs the contract to create strategically independent payoffs. This intuition and the tradeoff between the benefit and cost of *RPE* (increasing w_{HL}) relative to BP^I (increasing w_{HH} and w_{LL}) is illustrated with a numerical example in Figure 2.

In the example, $q_0 = 0.5$, $q_1 = 0.7$, and $H = 100$. The principal uses *IPE* if she is sufficiently patient $(r \leq 3.33)$, while she has to chooses between *RPE* and BP^I whenever $r > 3.33$ (the origin of Figure 2 at which both solutions are equally costly to the principal). The solid line represents the cost of *RPE* (Solution iii) relative to BP^I (Solution ii), calculated as $q_1(1-q_1)(w_{HL}^{iii} - w_{HL}^{ii})$, where the superscripts $iii\, and\, ii$ refer the corresponding solutions. The dotted line measures the relative benefit of *RPE*, or equivalently the cost of increasing w_{LL} and w_{HH} under BP^I, calculated as $q_1^2\left(w_{HH}^{ii} - w_{HH}^{iii}\right) + \left(1-q_1\right)^2 * w_{LLHH}^{ii}$. The intersection of two lines determines the critical discount rate $\tau^1 = 4.4$ used to choose between the two solutions. If the agents are still patient enough ($r < \tau^1$), BP^I is preferred to *RPE* since the No Cycling constraint is the expensive one to satisfy.

Under BP^I, "pay without performance" emerges sooner than one might expect, since *RPE* is feasible and does not make payments for poor performance. When will this happen? Intuitively, BP^I is optimal when the agents' credibility to collude is relatively stronger than the principal's credibility to honor her promises. More precisely, from Proposition 2, we know BP^I is optimal if and only if $(\delta^C < \tau^1)$, which is equivalent to restricting the high output of the project $H \le H^*$ for some constant H^*, because δ^C increases in H, while τ^1 is independent of H. To gain some intuition for the requirement $H \le H^*$, notice that the principal's ability to commit to honoring the *IPE* contract is enhanced by a high value of H, since the punishment the agents can bring to bear on the principal is more severe. Hence the region over which she can commit to the *IPE* contract becomes larger (δ^C becomes bigger) as H increases. For extremely large H ($H > H^*$), δ^C is so large that once the principal cannot commit to *IPE* ($r = \delta^C$), it has already passed the point (τ^1) that the two agents can enforce their own collusion. Since the only reason that BP^I is optimal is its efficiency in combatting agent-agent collusion on the *Cycling* strategy, the *RPE* contract instead of BP^I will be optimal if H is large (and thus $\delta^C > \tau^1$). Using the same numerical example as in Figure 1, if one sets $H \ge 128.61$ instead of 100, δ^C will exceed τ^1 and the region that used to be BP^I with $H = 100$ ($3.3 < r < 4.4$) now becomes *IPE* since the principal has a greater ability to commit with a higher H. Moreover, when the principal loses her commitment at $r = \delta^C = 4.4$, the agents are already so impatient that their *Cycling* collusion is of no concern, and the principal will offer *RPE* (instead of BP^I).

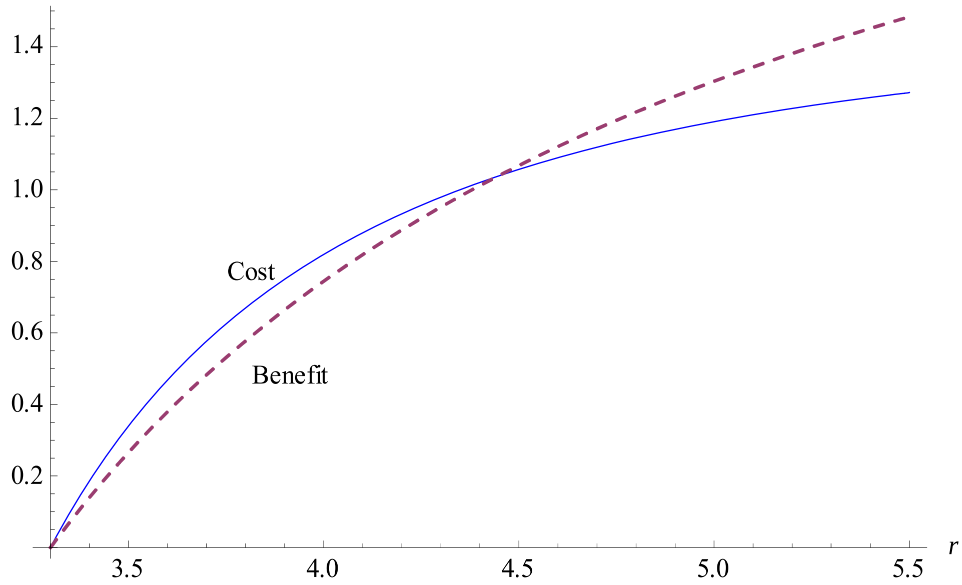


Figure 2: Cost and Benefit of *RPE* relative to BP^I

Solution BP^S emerges as r increases (both parties become extremely impatient). BP^S is similar to the static bonus pool. As pointed out in Levin (2003), "the variation in contingent payments is limited by the future gains from the relationship." The variation of wage payment is extremely limited under BP^S, because both parties are sufficiently impatient ($r > \max\{\tau^1, \delta^F\}$) and the future gains from the relationship are negligible. As a result, the principal has to set $w_{HL} = 2*w_{HH}$ and also increase w_{LL} to make the contract self-enforcing. This coincides with the traditional view that bonus pools will eventually come into play because they are the only self-enforcing compensation form in such cases.

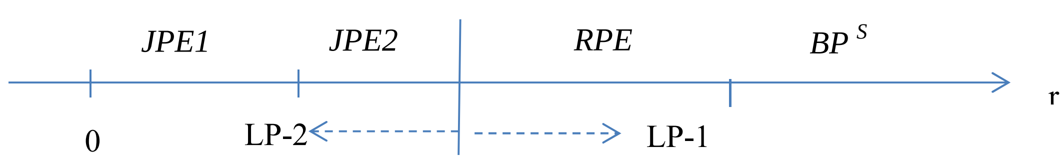
4.3 Overall optimal contract

The following proposition endogenizes the principal's choice of $T \in \{0,1\}$ and characterizes the overall optimal contract.

Proposition 3. *The principal chooses to induce team incentives, i.e., $T = 1$ if and only if (i) $\tau^0 \leq \delta^C$ and $r < \delta^C$ or (ii) $\tau^0 > \delta^C$ and $r \leq \min\{\tau^0, \delta^D\}$.*

Not surprisingly, the key determinant of whether motivating cooperation is optimal is the discount rate. A longer expected contracting horizon gives the principal more flexibility in the promises she can offer and enables the agents to make credible promises to each other to motivate cooperation. As a result, there is a solution to team incentives (*T=1*) and a solution to not using team incentives (*T=0*) that are never optimal in the overall solution. Of the solutions to *T=0*, *IPE* is never optimal overall. When the principal's ability to commit is strong enough that *IPE* is feasible, the principal is always be strictly better-off by offering a contract motivating cooperation (setting *T=1*). Similarly, whenever the principal's and the agents' ability to commit is so limited that only $BP2^C$ can be used to motivate cooperation, cooperation is too expensive to implement and the principal will not choose to do so ($T = 0$).

The principal's and agents' relative credibility is key, as the following numerical examples illustrate. In the first example, the agents' ability to commit is always limited relative to that of the principal, and collusion is never the driving determinant of the form of the optimal compensation arrangement. Consider the example: $q_0 = 0.53$, $q_1 = 0.75$, and $H = 200$, which corresponds to Case (i) in Proposition 3 ($\tau^0 \leq \delta^C$). When *LP-1* is optimal, *RPE* is used immediately and BP^I is never optimal.



The second example also falls under Case (i) of Proposition 3 ($\tau^0 \leq \delta^C$): q_0 = 0.47, q_1 = 0.72, and H = 100. The principal's ability to commit is high relative to the agents', but the relative comparison is not as extreme. In this case, we move from BP^I to *RPE*, since the principal still has enough ability to commit to make *RPE* feasible after the discount rate is so large that the agents' collusion is not the key determinant of the form of the compensation contract (BP^I).

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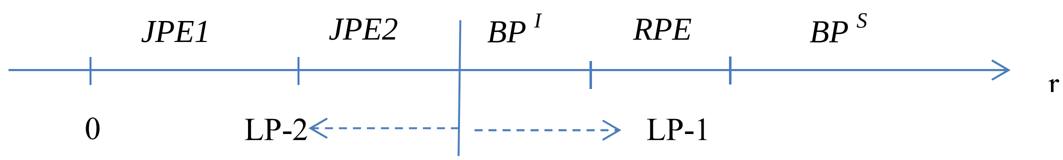
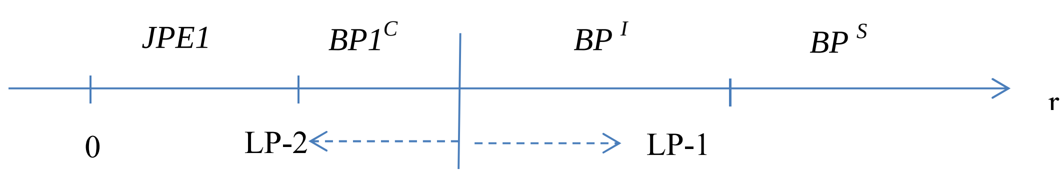


Next, consider a numerical example corresponding to Case (ii) in Proposition 3 ($\tau^0 > \delta^C$): q_0 = 0.53, q_1 = 0.75, and H = 100. In this case, the principal's ability to commit is low relative to the agents'. Once the discount rate is large enough that the agents' collusion is not the key determinant of the form of the compensation contract, the principal's ability to commit is also quite limited and BP^S is the only feasible solution. The principal's low relative credibility also leads to $BP1^C$ being optimal in this example when *JPE* was in the previous two examples.



5. Incorporating an Objective Team-based Performance Measure

In a typical bonus pool arrangement, the size of the bonus pool is based, at least in part, on an objective team-based performance measure such as group or divisional earnings (Eccles and Crane, 1988). Suppose that such an objective measure y exists and define

$p_1 \equiv \Pr(y = H \mid e^A = 1, e^B = 1)$, $p \equiv \Pr(y = H \mid e^A = 0, e^B = 1) = \Pr(y = H \mid e^A = 1, e^B = 0)$, and

$p_0 \equiv \Pr(y = H \mid e^A = 0, e^B = 0)$.

One interesting variation of the model is when the individual measures are noiseless. In this case, the optimal contract does not motivate mutual monitoring, since the principal's observation of the agents' efforts is already perfect. An optimal contract then makes the agents' payoffs strategically independent when the collusion constraints bind. The productive complementarity or substitutability in y is optimally combined with an offsetting payment effect to produce payoffs that are strategically independent.

Returning to imperfect individual performance measures, consider the following numerical example: $q_0 = 0.47$, $q_1 = 0.72$, $p_0 = 0.1$, $p = 0.8$, $p_1 = 0.9$, $r = 5$, and $H = 27$. In this example, cooperation is not feasible, so the principal is left trying to mitigate collusion. If only subjective measures are used in contracting, the optimal wage scheme is $w = (w_{LL}, w_{LH}, w_{HL}, w_{HH}) = (4.27, 0, 8.97, 4.70)$, or BP^I. Once the objective measure is incorporated into the contract, use the first subscript on the wage payment to denote the realization of the objective measure. For example, w_{Hmn} is the payment made to agent i when y is H, x^i is m, and x^j is n; $m,n = L,H$. The optimal wage scheme is $w = (w_{LLL}, w_{LLH}, w_{LHL}, w_{LHH}, w_{HLL}, w_{HLH}, w_{HHL}, w_{HHH}) = (0, 0, 0, 0, 2.922, 0, 5.843, 3.675)$. In terms of payoffs, the optimal wage scheme creates a strategic substitutability. Overall, there is a relatively small improvement in expected wages by introducing y: 9.19 without y and 5.96 with y. The objective measure is valuable because of its informativeness (Holmstrom, 1979), but the productive substitutability constrains the way in which the subjective measures can be used. The required complementarity in subjectively determined rewards is costly to the principal, because greater pay for bad performance is required.

Continue with the same example, except now assume $p = 0.2$, which creates a large strategic complementarity. Incorporating y into the optimal contract now facilitates cooperation when it would otherwise be infeasible. The optimal wage scheme is $w = (0, 0, 0, 0, 0.26, 0, 2.76, 1.38)$, and the expected total wages are 2.33, compared to 9.19 without incorporating the objective measure and 9.12 when the objective measure is incorporated but has a productive substitutability ($p = 0.8$). The optimal wage scheme can be viewed as a bonus pool of 0 when the verifiable team measure is low and a bonus pool of 2.76 when the team measure is high, with the principal given the discretion to reduce the reward to 0.26 if both agents are subjectively assessed to have contributed poorly. The substitutability in the subjectively determined wages reduces their cost to the principal, since the substitutability limits the amount of pay for bad performance that is required to maintain the principal's credibility. The productive complementarity facilitates both cooperation and a wage substitutability in the subjective measures. The example can be viewed as suggesting a new rationale for grouping employees whose actions are productive complements into a single bonus pool, cooperating through mutual monitoring to maximize their total reward but also competing for their share of the total reward.

6. Conclusion

A natural next step is to test some of the paper's empirical predictions. In particular, the model predicts that the form of the wage scheme will depend on (i) the expected contracting horizon, (ii) the relative ability of the principal and the agents to honor their promises, and (iii) the productive complementarity or substitutability of a team-based objective measure. A particularly strong prediction is that we should see bonus pool type incentive schemes that create strategic independence in the agents' payoffs in order to optimally prevent collusion. These particular bonus pool type arrangements should be observed when the agents' ability to collude is strong relative to the principal's ability to make credible promises and both are limited enough to be binding constraints. When the principal's credibility and the agents' credibility are both severely limited, we should instead observe bonus pool type arrangements that create a strategic substitutability in the agents' payoffs, since these allow for greater *RPE* which is efficient absent collusion concerns. When instead the arrangement is used to motivate cooperation and mutual monitoring (which we suspect is more common), we should see productive and incentive arrangements that, when combined, create strategic complementarities.

Another avenue for empirical research is to re-examine settings in which mixed results have previously been obtained on team incentives/mutual monitoring and use this paper's limited commitment, productive complementarities, etc. to help refine those empirical predictions. Stepping back from the principal's commitment problem, the general theme of team-based vs. individual incentives seems to be underexplored. For example, the models of team-based incentives typically assume the agents are symmetric (e.g., Che and Yoo, 2001; Arya, Fellingham, and Glover, 1997). With agents that have different roles (e.g., a CEO and a CFO), static models predict the agents would be offered qualitatively different compensation contracts. Yet, in practice, the compensation of CEOs and CFOs are qualitatively similar, which seems to be consistent with a team-based model of dynamic incentives (with a low discount rate/long expected tenure).

If we apply the team-based model to thinking about screening, then we might expect to see compensation contracts that screen agents for their potential productive complementarity with agents already in the firm's employ (or in a team of agents being hired at the same time), since productive complementarities reduce the cost of motivating mutual monitoring because the benefit to free-riding is small. A productive substitutability (e.g., hiring an agent similar to existing ones when there are overall decreasing returns to effort) is particularly unattractive, since the substitutability makes the cycling collusion constraint (the agents taking turns working) bind. We might also expect to see agents screened for their discount rates—that patient agents (those with long expected horizons with the firm) would be more attractive, since they are the ones best equipped to provide and receive mutual monitoring incentives. Are existing incentive arrangements such as employee stock options with time-based rather than performance-based vesting conditions designed, in part, to achieve such screening?

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Appendix

Proof of Lemma 1.

If the payoff from (*work, work*) is higher for both agents than under (*shirk, shirk*), then we need only worry about strategies that have the agents using some combination of (*work*, *shirk*) and (*shirk*, *work*) over time. In this case, if there is some period *t'* shirker who has a higher continuation payoff than under *Cycling*, there will also be a period *t"* shirker who has a lower payoff than under *Cycling*. Hence, if the contract upsets *Cycling*, it will also motivate the period *t"* shirker to *work* instead of *shirk*, upsetting that potential equilibrium.

Now, consider the case that the payoff under (*work, work*) is lower for both agents than under (*shirk, shirk*). If the agents play only (*shirk, work*) and (*work, shirk*) in all future periods (whatever they are doing this period), then the previous argument applies. Consider 2 subcases, characterized by the total payoff—the sum of the payoffs to the two agents in the stage game.

Subcase 1: Suppose the total payoff under (*shirk, shirk*) is higher than under (*shirk, work*). At any (*shirk, shirk*) not followed by (*shirk, shirk*) forever, one of the two agents will have a lower continuation payoff. Hence, upsetting *Joint Shirking* means that any proposed collusion that falls under Subcase 1 will also be upset.

Subcase 2: Suppose the total payoff under (*shirk, work*) is higher than under (*shirk, shirk*). At any (*shirk, shirk*), if the total continuation payoff is as under Cycle, then the rest of the play must involve only (*shirk, work*) and (*work, shirk*), which is a case we have already dealt with. Otherwise, the total continuation payoff must be less at (*shirk, shirk*) than under *Cycle*. So, compare *(shirk, shirk*) under the proposed collusion to (*shirk, work*) under *Cycling*. At (*shirk, shirk*) under the proposed collusion, the continuation payoff is lower to either agent *A* or to agent *B* than under *Cycling*.

Subcase 2a: If the continuation payoff is lower to *A*, then we are done, since the shirker will want to deviate to work given that we have strategic substitutes in the payoffs: moving from (*shirk, shirk*) to (*work, shirk*) has an even greater current period benefit than moving from (*shirk, work*) to (*work, work*). Since *Cycling* is upset, we also upset any collusion that falls under Subcase 2a.

Subcase 2b: If the continuation payoff is lower to *B*, then we use the observation that, under *Cycling*, the continuation payoff to the shirker is greatest. Then *B*'s continuation payoff is lower than *A*'s would be under *Cycling*, and the strategic substitutability in payoffs again ensures that moving from (*shirk, shirk*) to (*work, shirk*) in the current period is greater than under *Cycling* when we move from (*shirk, work*) to (*work, work*). So upsetting *Cycling* means we also upset any possible collusion that falls under Subcase 2b. □

The following parameters will be useful in the remaining proofs.

$$\delta^A = \frac{1}{2}\left[(q_0-q_1)^2 q_1 H-1-q_1^2+\sqrt{((q_0-q_1)^2 q_1 H-1-q_1^2)^2+4(q_0-q_1)^2(q_0+q_1)H-4q_1^2}\right];$$

$$\delta^C = (q_1-q_0)^2 H-q_1; \qquad \delta^D = (q_1-q_0)^2(q_0+q_1)H-q_0-q_1^2;$$

$$\delta^F = \frac{1}{2}\left[(q_1-q_0)^2(2-q_1)H-1-2q_1+q_1^2+\sqrt{((q_1-q_0)^2(2-q_1)H-1-2q_1+q_1^2)^2+4(q_1-2)q_1-8(q_0-q_1)^2(-1+q_1)H}\right];$$

$$\tau^0 = \frac{q_1+q_0-1}{(1-q_1)^2}; \qquad \tau^1 = \frac{2q_1-1}{(1-q_1)^2}.$$

Note that $\delta^A, \delta^C, \delta^D$, and δ^F are increasing in H, and we assume throughout the paper that H is large enough to rank term by comparing the coefficient of the linear term of H. In particular, we obtain (i) $\delta^A < \delta^C < \delta^F$ and (ii) $\delta^C < \delta^D$ if and only if $q_0 + q_1 > 1$. The agent's effort is assumed to be valuable enough ($q_1 - q_0$ is not too small) such that

$$(q_1 - q_0)^2 H \geq \max\{1+\frac{1}{q_1-q_0}, \frac{q_1^2}{2q_1-1}\} \text{ and } \delta^A \geq \sqrt{2}.$$

Proof of Lemma 2.

Consider the case that team incentives are not optimal, $T = 0$. (The proof for the team incentive case, $T = 1$, is similar and omitted.) In this case, one can rewrite IP as follows.

$$\min (1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$$

$s.t$

$$(1-q_1)w_{LL} + q_1 w_{LH} - (1-q_1)w_{HL} - q_1 w_{HH} \le \frac{-1}{q_1 - q_0} \quad \text{"Stage NE"} \ (\lambda_1)$$

$$((2-q_1-q_0)+(1-q_0)r)w_{LL} + (q_1 - 1 + q_0 + q_0 r)w_{LH} + ((q_0 - 1)(1+r)+q_1)w_{HL} - (rq_0 + q_0 + q_1)w_{HH} \le \frac{-(1+r)}{q_1 - q_0}$$

"No Joint Shirking"(λ_2)

$$(1-q_1)(2+r)w_{LL} + (-1+q_1(2+r))w_{LH} + ((q_1-1)(1+r)+q_1)w_{HL} - q_1(2+r)w_{HH} \le -\frac{1+r}{q_1 - q_0} \quad \text{"No Cycling"}(\lambda_3)$$

$$((1-q_1)^2 - r)w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + (q_1^2 + r)w_{HH} \le (q_1 - q_0)H \qquad ICP_{HH \succ LL}(\lambda_4)$$

$$((1-q_1)^2 - r)w_{LL} + (q_1 - q_1^2 + \frac{r}{2})w_{LH} + (q_1 - q_1^2 + \frac{r}{2})w_{HL} + q_1^2 w_{HH} \le (q_1 - q_0)H \qquad ICP_{HL \succ LL}(\lambda_5)$$

$$(1-q_1)^2 w_{LL} + (q_1 - q_1^2 - \frac{r}{2})w_{LH} + (q_1 - q_1^2 - \frac{r}{2})w_{HL} + (q_1^2 + r)w_{HH} \le (q_1 - q_0)H \qquad ICP_{HH \succ HL}(\lambda_6)$$

$$(1-q_1)^2 w_{LL} + (q_1 - q_1^2 + \frac{r}{2})w_{LH} + (q_1 - q_1^2 + \frac{r}{2})w_{HL} + (q_1^2 - r)w_{HH} \le (q_1 - q_0)H \qquad ICP_{HL \succ HH}(\lambda_7)$$

$$((1-q_1)^2 + r)w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + (q_1^2 - r)w_{HH} \le (q_1 - q_0)H \qquad ICP_{LL \succ HH}(\lambda_8)$$

$$((1-q_1)^2 + r)w_{LL} + (q_1 - q_1^2 - \frac{r}{2})w_{LH} + (q_1 - q_1^2 - \frac{r}{2})w_{HL} + q_1^2 w_{HH} \le (q_1 - q_0)H \qquad ICP_{LL \succ HL}(\lambda_9)$$

$$-w_{LL} \le 0 \ (\lambda_{10}); \qquad -w_{HL} \le 0 \ (\lambda_{11}); \qquad -w_{HH} \le 0 \ (\lambda_{12}); \qquad -w_{LH} \le 0 \ (\lambda_{13}).$$

$ICP_{mn \succ m'n'}$ is the family of IC constraints for the principal. $ICP_{mn \succ m'n'}$ ensures the principal prefers reporting outcomes *mn* truthfully rather than reporting *m'n'* given the agents' threat to revert to the stage game equilibrium of *Joint Shirking* if the principal lies.

Suppose the optimal solution is $w = \{w_{HH}, w_{HL}, w_{LH}, w_{LL}\}$ with $w_{LH} > 0$. Consider the solution $w' = \{w_{HH}^{'}, w_{HL}^{'}, w_{LH}^{'}, w_{LL}^{'}\}$, where $w_{LH}^{'} = 0$, $w_{HL}^{'} = w_{HL} + w_{LH}$, $w_{LL}^{'} = w_{LL}$, and $w_{HH}^{'} = w_{HH}$. It is easy to see that w and w' generate the same objective function value. We show below that the constructed w' satisfies all the $ICP_{mn \succ m'n'}$ constraints and further relaxes the rest of the constraints (compared to the original contract w). Since w_{LH} and w_{HL} have the same coefficient in all the $ICP_{mn \succ m'n'}$ constraints, w' satisfies these constraints as long as w does. Denote the coefficient on w_{LH} as C_{LH} and the coefficient on w_{HL} as C_{HL} for the "Stage NE", "*No Joint Shirking*," and "*No Cycling*" constraints. We can show that $C_{LH} - C_{HL} = r > 0$ holds for each of the three constraints. Given $C_{LH} > C_{HL}$, it is easy to show that w' will relax the three constraints compared to the solution w. □

Proof of Proposition 1.

The program can be rewritten as follows.

$$\min (1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$$

$s.t$

$$(2-q_0-q_1)w_{LL} + (q_0+q_1-1)w_{LH} + (q_0+q_1-1)w_{HL} - (q_0+q_1)w_{HH} \le \frac{-1}{q_1-q_0} \quad \text{"Pareto Dominance" } (\lambda_1)$$

$$((2-q_0-q_1+r(1-q_1))w_{LL} + (q_0+q_1+q_1 r-1)w_{LH} + (q_0+(q_1-1)(1+r))w_{HL} - (q_0+q_1+q_1 r)w_{HH} \le \frac{-(1+r)}{q_1-q_0}$$

"Mutual Monitoring" (λ_2)

$$(1-q_1)(2+r)w_{LL} + (-1+q_1(2+r))w_{LH} + ((q_1-1)(1+r)+q_1)w_{HL} - q_1(2+r)w_{HH} \le -\frac{1+r}{q_1-q_0} \quad \text{"No Cycling"} (\lambda_3)$$

$$((1-q_1)^2 - r)w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + (q_1^2+r)w_{HH} \le (q_1-q_0)H \quad ICP_{\text{HH}\succ LL}(\lambda_4)$$

$$((1-q_1)^2 - r)w_{LL} + (q_1-q_1^2+\frac{r}{2})w_{LH} + (q_1-q_1^2+\frac{r}{2})w_{HL} + q_1^2 w_{HH} \le (q_1-q_0)H \quad ICP_{\text{HL}\succ LL}(\lambda_5)$$

$$(1-q_1)^2 w_{LL} + (q_1-q_1^2-\frac{r}{2})w_{LH} + (q_1-q_1^2-\frac{r}{2})w_{HL} + (q_1^2+r)w_{HH} \le (q_1-q_0)H \quad ICP_{\text{HH}\succ HL}(\lambda_6)$$

$$(1-q_1)^2 w_{LL} + (q_1-q_1^2+\frac{r}{2})w_{LH} + (q_1-q_1^2+\frac{r}{2})w_{HL} + (q_1^2-r)w_{HH} \le (q_1-q_0)H \quad ICP_{\text{HL}\succ HH}(\lambda_7)$$

$$((1-q_1)^2 + r)w_{LL} + (1-q_1)q_1 w_{LH} + (1-q_1)q_1 w_{HL} + (q_1^2-r)w_{HH} \le (q_1-q_0)H \quad ICP_{\text{LL}\succ HH}(\lambda_8)$$

$$((1-q_1)^2 + r)w_{LL} + (q_1-q_1^2-\frac{r}{2})w_{LH} + (q_1-q_1^2-\frac{r}{2})w_{HL} + q_1^2 w_{HH} \le (q_1-q_0)H \quad ICP_{\text{LL}\succ HL}(\lambda_9)$$

$$-w_{LL} \le 0\ (\lambda_{10}); \qquad -w_{HL} \le 0\ (\lambda_{11}); \qquad -w_{HH} \le 0\ (\lambda_{12}); \qquad -w_{LH} \le 0\ (\lambda_{13});$$

$$(-1+q_0)w_{LL} - q_0 w_{LH} + (1-q_0)w_{HL} + q_0 w_{HH} \le \frac{1}{q_1-q_0} \quad \text{"Self-Enforcing Shirk"} (\lambda_{14})$$

We first solve a relaxed program without the "Self-Enforcing Shirk" constraint and then verify that solutions of the relaxed program satisfy the "Self-Enforcing Shirk" constraint.

Without the "Self-Enforcing Shirk" constraint, the same argument used in Lemma 1 can be used to show that setting $w_{LH} = 0$ is optimal. Denote the objective function of (LP-2) by *f(w)* the left-hand side less the right-hand side of the i^{th} constraints by $g_i(w)$, and the Lagrangian Multiplier of the i^{th} constraint by λ_i. The Lagrangian for the problem is $L = f(w) + \sum_{i=1}^{12} \lambda_i g_i(w)$. After setting $w_{LH} = 0$ (using Lemma 2) and simplifying the problem, FOCs for the three wage payments (w_{LL}, w_{HL}, w_{HH}) are as follows:

$$1-\lambda_{10}+2\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9-2(1+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9)q_1$$
$$+(1+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9)q_1^{\,2}-\lambda_1(q_0+q_1-2)+(\lambda_3-\lambda_4-\lambda_5+\lambda_8+\lambda_9-\lambda_3 q_1)r$$
$$-\lambda_2(-2+q_0+q_1+(-1+q_1)r)=0 \quad \text{FOC---}w_{LL}$$
$$q_1+\lambda_1(q_0+q_1-1)+q_1(2\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9-(1+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9)q_1)$$
$$+\frac{1}{2}(\lambda_5-\lambda_6+\lambda_7-\lambda_9+2\lambda_3(-1+q_1))r+\lambda_2(q_0+(-1+q_1)(1+r))-\lambda_{11}-\lambda_3=0 \quad \text{FOC---}w_{HL}$$
$$q_1^{\,2}+\lambda_5 q_1^{\,2}+\lambda_9 q_1^{\,2}-\lambda_1(q_0+q_1)+\lambda_7(q_1^{\,2}-r)+\lambda_8(q_1^{\,2}-r)-\lambda_3 q_1(2+r)+\lambda_4(q_1^{\,2}+r)$$
$$+\lambda_6(q_1^{\,2}+r)-\lambda_2(q_0+q_1+q_1 r)-\lambda_{12}=0 \quad \text{FOC---}w_{HH}.$$

Again, the optimal solution is one that (i) satisfies all constraints in LP-2, (ii) satisfies the three FOCs above, (iii) satisfies the complementary slackness conditions $\lambda_i g_i(w)=0$, and (iv) has non-negative Lagrangian multipliers. For $r<\delta^A$, the solution listed below satisfies (i)-(iv) and thus is optimal. This solution, denoted as *Pure JPE*, is obtained by solving the three binding constraints: Mutual Monitoring, w_{HL}, and w_{LL}.

The *Pure JPE* solution is:

$$w_{LL}=0,\ w_{HL}=0, w_{HH}=\frac{1+r}{(q_1-q_0)(q_0+q_1+q_1 r)};$$

$$\lambda_1=q_1, \lambda_2=\frac{q_1^{\,2}}{q_0+q_1+q_1 r}, \lambda_3=0, \lambda_4=0,$$

$$\lambda_5=0, \lambda_6=0, \lambda_7=0, \lambda_8=0,$$

$$\lambda_9=0, \lambda_{10}=\frac{q_0-2q_0q_1+q_1(1+r-q_1 r)}{q_0+q_1+q_1 r}, \lambda_{11}=\frac{q_0 q_1}{q_0+q_1+q_1 r}, \lambda_{12}=0.$$

Under *Pure JPE*, the $ICP_{\mathrm{HH}\succ LL}$ constraint yields the upper bound on r. The optimal solution changes when $r>\delta^A$. For $\delta^A<r\le\min\left(\tau^0,\delta^D\right)$, the solution listed below satisfies (i)-(iv) and becomes optimal. This solution, denoted as *BP1*^{C}, is obtained by solving the following three binding constraints: Mutual Monitoring, $ICP_{\mathrm{HH}\succ LL}$, and $ICP_{\mathrm{HH}\succ HL}$.

The *BP1*^{C} solution is:

$$w_{LL}=\frac{\left(q_1-q_0\right)^2\left(q_0+q_1+q_1 r\right)H-\left(1+r\right)\left(q_1^{\,2}+r\right)}{\left(q_1-q_0\right)\left(q_0+q_1-\left(-1+q_1\right)q_1 r-r^2\right)},\ w_{HL}=2*w_{LL},$$

$$w_{HH}=\frac{\left(q_1-q_0\right)^2\left(q_0+q_1+\left(-1+q_1\right)r\right)H-\left(1+r\right)\left(-1+q_1^{\,2}+r\right)}{\left(q_1-q_0\right)\left(q_0+q_1-\left(-1+q_1\right)q_1 r-r^2\right)};$$

$$\lambda_1=0, \lambda_2=\frac{-r}{q_0+q_1+q_1 r-q_1^{\,2}r-r^2}, \lambda_3=0, \lambda_4=\frac{q_0-(q_1-2)((q_1-1)r-1)}{q_0+q_1+q_1 r-q_1^{\,2}r-r^2},$$

$$\lambda_5=0, \lambda_6=-\frac{2(q_0-(q_1-1)((q_1-1)r-1))}{q_0+q_1+q_1 r-q_1^{\,2}r-r^2}, \lambda_7=0, \lambda_8=0,$$

$$\lambda_9=0, \lambda_{10}=0, \lambda_{11}=0, \lambda_{12}=0.$$

Under $BP1^C$, the non-negativity of w_{LL} and w_{HL} requires $r > \delta^A$, while the Pareto Dominant constraint and $\lambda_6 \geq 0$ impose upper bounds δ^D and τ^0 respectively. If $r > \tau^0$, the solution changes because otherwise $\lambda_6 < 0$. For $\max\{\tau^0, \delta^A\} < r \leq \delta^C$, the solution is listed below. This solution, denoted as *JPE*, is obtained by solving the following three binding constraints: Mutual Monitoring, $ICP_{HH \succ LL}$, and w_{LL}.

The *JPE* solution is:

$$w_{LL} = 0,\ w_{HL} = \frac{\dfrac{(1+r)(q_1^2+r)}{q_1 - q_0} - (q_1 - q_0)(q_0 + q_1 + q_1 r)H}{(1-q_1)r(1+r) - q_0(q_1+r)},$$

$$w_{HH} = \frac{(1-q_1)q_1(1+r) + (q_1-q_0)^2(q_0 + (-1+q_1)(1+r))H}{(q_1-q_0)((-1+q_1)r(1+r) + q_0(q_1+r))};$$

$$\lambda_1 = 0, \lambda_2 = \frac{(q_1-1)q_1 r}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_3 = 0, \lambda_4 = \frac{-q_0 q_1}{(q_1-1)r(1+r)+q_0(q_1+r)},$$

$$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = \frac{r(q_0 - (q_1-1)((q_1-1)r-1))}{(q_1-1)r(1+r)+q_0(q_1+r)}, \lambda_{11} = 0, \lambda_{12} = 0.$$

Under *JPE*, the non-negativity of w_{HL} requires $r > \delta^A$ and $\lambda_{10} \geq 0$ yields another lower bound τ^0 on r. The non-negativity of w_{HH} and w_{HL} also requires $r > s' \equiv \dfrac{q_0 + q_1 - 1 + \sqrt{(q_0+q_1-1)^2 + 4(1-q_1)q_0 q_1}}{2(1-q_1)}$. In addition, both the Pareto Dominance and No Cycle constraints require $r < \delta^C$. We claim $(\max\{s', \delta^A, \tau^0\}, \delta^C] = (\max\{\delta^A, \tau^0\}, \delta^C]$. The claim is trivial if $s' \leq \delta^A$ and therefore consider the case where $s' > \delta^A$. Since δ^A increases in H while s' is independent of H, one can show $s' > \delta^A$ is equivalent to $H < H'$ for a unique positive H'. Meanwhile, algebra shows that $\delta^C < \delta^A$ for $H < H'$. Therefore $s' > \delta^A$ implies $\delta^C < \delta^A$, in which case $(\max\{s', \delta^A, \tau^0\}, \delta^C] = (\max\{\delta^A, \tau^0\}, \delta^C] = \varnothing$. For $\max\{\tau^0, \delta^C\} < r \leq \delta^D$, and $q_1 + q_0 \geq 1$, the optimal solution is as follows. This solution, denoted as $BP2^C$, is obtained by solving the three binding constraints: Mutual Monitoring, Pareto Dominance, and $ICP_{HH \succ LL}$.

The $BP2^C$ solution is:

$$w_{LL} = \frac{q_0(q_1-q_0)^2 H - q_0(q_1+r)}{(q_1-q_0)((1-q_1)r - q_0(-1+q_1+r))}, w_{HL} = \frac{q_0 - 2q_1 q_0 + q_1^2 + r - 2q_0 r - (q_1-q_0)^3 H}{(q_1-q_0)((1-q_1)r - q_0(q_1+r-1))},$$

$$w_{HH} = \frac{q_1 - q_0 + q_1 q_0 - q_1^2 + q_0 r + (q_1-q_0)^2(-1+q_1)H}{(q_1-q_0)((-1+q_1)r + q_0(-1+q_1+r))};$$

$$\lambda_1 = \frac{-q_0 + (-1+q_1)(-1+(-1+q_1)r)}{(q_1-1)r + q_0(-1+q_1+r)}, \lambda_2 = \frac{q_0+q_1-1}{(q_1-1)r+q_0(-1+q_1+r)}, \lambda_3 = 0, \lambda_4 = \frac{q_0 - q_0 q_1}{(q_1-1)r + q_0(-1+q_1+r)},$$

$$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = 0, \lambda_{11} = 0, \lambda_{12} = 0$$

Under $BP2^C$, the non-negativity of λ_1 requires $q_1 + q_0 \geq 1$. Given $q_1 + q_0 \geq 1$, the non-negativity of w_{HH} and w_{HL} together yield $r > \delta^C$ and $r > s'' \equiv \frac{(1-q_1)q_0}{q_0 + q_1 - 1}$. The other lower bound τ^0 on r is generated by intersecting requirements for $\lambda_1 \geq 0$ and for the non-negativity of w_{HH} and w_{HL}. The $ICP_{HH \succ HL}$ constraint yields the upper bound on r, i.e. $r \leq \delta^D$. We claim $(\max\{s'', \delta^C, \tau^0\}, \delta^D] = (\max\{\delta^C, \tau^0\}, \delta^D]$. Subtracting q_1 from both sides of $\delta^C \leq \delta^D$ and collecting terms, one obtains $s'' \leq \delta^C$, which means $\delta^C \leq \delta^D$ if and only if $s'' \leq \delta^C$. Therefore $(\max\{s'', \delta^C, \tau^0\}, \delta^D] = (\max\{\delta^C, \tau^0\}, \delta^D]$ is verified. As r becomes even larger, the problem (LP-2) becomes infeasible because the intersection of the Mutual Monitoring constraint and the Pricipal's IC constraint(s) is an empty set.

Finally, tedious algebra verifies that the solutions characterized above satisfy the "Self-Enforcing Shirk" constraint that we left out in solving the problem. Therefore adding this constraint back does not affect the optimal objective value. □

Proof of Proposition 2.

By Lemma 2, rewrite LP-1 as follows.

$$\min (1-q_1)^2 w_{LL} + (1-q_1)q_1 w_{HL} + q_1^2 w_{HH}$$

$s.t$

$$(1-q_1)w_{LL} - (1-q_1)w_{HL} - q_1 w_{HH} \leq \frac{-1}{q_1 - q_0} \quad \text{"Stage NE"} \ (\lambda_1)$$

$$((2-q_1-q_0)+(1-q_0)r)w_{LL} + ((q_0-1)(1+r)+q_1)w_{HL} - (rq_0+q_0+q_1)w_{HH} \leq \frac{-(1+r)}{q_1-q_0} \quad \text{"No Joint Shirking"} \ (\lambda_2)$$

$$(1-q_1)(2+r)w_{LL} + ((q_1-1)(1+r)+q_1)w_{HL} - q_1(2+r)w_{HH} \leq -\frac{1+r}{q_1-q_0} \quad \text{"No Cycling"} \ (\lambda_3)$$

$$((1-q_1)^2 - r)w_{LL} + (1-q_1)q_1 w_{HL} + (q_1^2 + r)w_{HH} \leq (q_1 - q_0)H \qquad ICP_{HH \succ LL} \ (\lambda_4)$$

$$((1-q_1)^2 - r)w_{LL} + (q_1 - q_1^2 + \frac{r}{2})w_{HL} + q_1^2 w_{HH} \leq (q_1 - q_0)H \qquad ICP_{HL \succ LL} \ (\lambda_5)$$

$$(1-q_1)^2 w_{LL} + (q_1 - q_1^2 - \frac{r}{2})w_{HL} + (q_1^2 + r)w_{HH} \leq (q_1 - q_0)H \qquad ICP_{HH \succ HL} \ (\lambda_6)$$

$$(1-q_1)^2 w_{LL} + (q_1 - q_1^2 + \frac{r}{2})w_{HL} + (q_1^2 - r)w_{HH} \leq (q_1 - q_0)H \qquad ICP_{HL \succ HH} \ (\lambda_7)$$

$$((1-q_1)^2 + r)w_{LL} + (1-q_1)q_1 w_{HL} + (q_1^2 - r)w_{HH} \leq (q_1 - q_0)H \qquad ICP_{LL \succ HH} \ (\lambda_8)$$

$$((1-q_1)^2 + r)w_{LL} + (q_1 - q_1^2 - \frac{r}{2})w_{HL} + q_1^2 w_{HH} \leq (q_1 - q_0)H \qquad ICP_{LL \succ HL} \ (\lambda_9)$$

$$-w_{LL} \leq 0 \ (\lambda_{10}); \qquad -w_{HL} \leq 0 \ (\lambda_{11}); \qquad -w_{HH} \leq 0 \ (\lambda_{12}).$$

Denote the objective function of (LP-1) by $f(w)$, the left-hand side less the right-hand side of the i^{th} constraints by $g_i(w)$, and the Lagrangian Multiplier of the i^{th} constraint by λ_i ,then the Lagrangian for the problem is $L = f(w) + \sum_{i=1}^{12} \lambda_i g_i(w)$. The first-order-conditions (FOCs) of the Lagrangian with respect to the three wage payments (w_{LL}, w_{HL}, w_{HH}) are as follows:

$$
\begin{aligned}
&1+\lambda_1-\lambda_{10}+2\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9-\lambda_1 q_1-2(1+\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9)q_1\\
&+(1+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9)q_1^{\,2}+(\lambda_3-\lambda_4-\lambda_5+\lambda_8+\lambda_9-\lambda_3 q_1)r\\
&-\lambda_2(-2+q_0+q_1+(-1+q_0)r)=0 \quad \text{FOC---}w_{LL}\\
&\lambda_1(-1+q_1)+q_1+q_1(2\lambda_3+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9-(1+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9)q_1)\\
&+\frac{1}{2}(\lambda_5-\lambda_6+\lambda_7-\lambda_9+2\lambda_3(-1+q_1))r+\lambda_2(-1+q_0+q_1+(-1+q_0)r)-\lambda_{11}-\lambda_3=0 \quad \text{FOC---}w_{HL}\\
&(1+\lambda_4+\lambda_5+\lambda_6+\lambda_7+\lambda_8+\lambda_9)q_1^{\,2}+(\lambda_4+\lambda_6-\lambda_7-\lambda_8)r-\lambda_3 q_1(2+r)-\lambda_2(q_0+q_1+q_0 r)\\
&-\lambda_{12}-\lambda_1 q_1=0 \quad \text{FOC---}w_{HH}\,.
\end{aligned}
$$

The optimal solution is one that (i) satisfies all 12 constraints, (ii) satisfies the three FOC above, (iii) satisfies the 12 complementary slackness conditions $\lambda_i g_i(w) = 0$, and (iv) all the Lagrangian multipliers are non-negative, i.e. $\lambda_i \geq 0$.

For $r \leq \delta^C$, the solution listed below satisfies (i)-(iv) and thus is optimal. Under this solution, denoted as *IPE*, the wage payments are derived by solving the following three binding constraints in (LP-1): Stage NE, *No Joint Shirking*, and w_{LL} . (*No Cycling* is also binding, and the Lagrangian multipliers under this solution are not unique due to the degeneracy of the problem. However finding one set of λ satisfying (ii)-(iv) is enough to show the optimality.)

The *IPE* solution is:

$$
\begin{aligned}
&w_{LL}=0,\; w_{HL}=w_{HH}=\frac{1}{q_1-q_0}\\
&\lambda_1=q_1,\lambda_2=0,\lambda_3=0,\lambda_4=0,\\
&\lambda_5=0,\lambda_6=0,\lambda_7=0,\lambda_8=0,\\
&\lambda_9=0,\lambda_{10}=1-q_1,\lambda_{11}=0,\lambda_{12}=0.
\end{aligned}
$$

Under *IPE*, the $ICP_{HH\succ LL}$ constraint imposes the upper bound δ^C on r . The optimal solution changes when $r \geq \delta^C$. For $\delta^C < r \leq \tau^1$, the solution is listed below. This solution, denoted as BP^I, is obtained by solving the following three constraints: $ICP_{HH\succ LL}$, *No Joint Shirking*, and *No Cycling*.

The BP^I solution is:

$$w_{LL} = \frac{(q_1 - q_0)^2(1+r)H - (1+r)(q_1 + r)}{(q_1 - q_0)(1 - r(-1 + q_1 + r))},\ w_{HL} = w_{HH} + w_{LL},$$

$$w_{HH} = \frac{(q_1 - q_0)^2 H - (1+r)(-1 + q_1 + r)}{(q_1 - q_0)(1 - r(-1 + q_1 + r))};$$

$$\lambda_1 = 0, \lambda_2 = \frac{r(1 + r + q_1^2 r - 2q_1(1+r))}{(q_0 - q_1)(1+r)(-1 + (-1+q_1)r + r^2)}, \lambda_3 = \frac{r(-1 + q_0 + q_1 - r + q_0 r + q_1 r - q_1^2 r)}{(q_0 - q_1)(1+r)(-1 + (-1+q_1)r + r^2)}, \lambda_4 = \frac{1 + r - q_1 r}{-1 - (1-q_1)r + r^2},$$

$$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = 0, \lambda_{11} = 0, \lambda_{12} = 0.$$

Under BP^I, both the non-negativity of w_{LL} and the Stage NE constraints require $r > \delta^C$ and $\lambda_2 \geq 0$ requires $r \leq \tau^1$. The optimal solution changes if $r > \tau^1$. For $\max\{\tau^1, \delta^C\} < r \leq \delta^F$, the optimal solution is listed below. The solution, denoted as *RPE*, is obtained by solving the following three constraints: $ICP_{HH \succ LL}$, *No Cycling*, and w_{LL}.

The *RPE* solution is:

$$w_{LL} = 0,\ w_{HL} = \frac{(q_1 - q_0)q_1(2+r)H - \dfrac{(1+r)(q_1^2 + r)}{q_1 - q_0}}{q_1^2 - r(1+r) + q_1 r(2+r)},$$

$$w_{HH} = \frac{(1-q_1)q_1(1+r) + (q_1 - q_0)^2(-1 - r + q_1(2+r))H}{(q_1 - q_0)(q_1^2 - r(1+r) + q_1 r(2+r))}$$

$$\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = \frac{(1-q_1)q_1 r}{r(1+r) - q_1^2 - q_1 r(2+r)}, \lambda_4 = \frac{q_1^2}{r(1+r) - q_1^2 - q_1 r(2+r)},$$

$$\lambda_5 = 0, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = \frac{r(1 + r + q_1^2 r - 2q_1(1+r))}{r(1+r) - q_1^2 - q_1 r(2+r)}, \lambda_{11} = 0, \lambda_{12} = 0.$$

Under *RPE*, the Stage NE constraint and $\lambda_{10} \geq 0$ yields two lower bounds δ^C and τ^1 on r. $ICP_{HL \succ LL}$ and the non-negativity of w_{HH} and w_{HL} together require $r \leq \delta^F$. $w_{HH} \geq 0$ also requires $r > s \equiv \dfrac{2q_1 - 1 + \sqrt{(2q_1 - 1)^2 + 4(1-q_1)q_1^2}}{2(1-q_1)}$, and we claim $(\max\{s, \delta^C, \tau^1\}, \delta^F] = (\max\{\delta^C, \tau^1\}, \delta^F]$. Consider the case where $s > \delta^C$ (as the claim is trivial if instead $s \leq \delta^C$). Since δ^C increases in H while s is independent of H, one can show $s > \delta^C$ is equivalent to $H < H^*$ for a unique positive H^*. Algebra shows that $\delta^F < \delta^C$ for $H < H^*$. Therefore $s > \delta^C$ implies $\delta^F < \delta^C$, in which case both $(\max\{s, \delta^C, \tau^1\}, \delta^F]$ and $(\max\{\delta^C, \tau^1\}, \delta^F]$ are empty sets. For $r > \max\{\delta^F, \tau^1\}$, the optimal solution is listed below and denoted as BP^S. Under BP^S, the optimal payment is obtained by solving the following three constraints: $ICP_{HH \succ LL}$, $ICP_{HL \succ LL}$, and No Cycle.

The BP^S solution is:

$$w_{LL} = \frac{(1+r)((2-q_1)q_1+r)+(q_1-q_0)^2(-2(1+r)+q_1(2+r))H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))},\ w_{HL} = 2w_{HH},$$

$$w_{HH} = \frac{(1+r)\left(-(1-q_1)^2+r\right)-(q_1-q_0)^2(1-q_1)(2+r)H}{(q_1-q_0)(2q_1+(3-q_1)q_1r+r^2-2(1+r))};$$

$$\lambda_1 = q_1, \lambda_2 = 0, \lambda_3 = \frac{r}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_4 = \frac{q_1(-2+(-1+q_1)r)}{2+2r+q_1^2r-r^2-q_1(2+3r)},$$

$$\lambda_5 = \frac{2(1+r+q_1^2r-2q_1(1+r))}{-2-2r-q_1^2r+r^2+q_1(2+3r)}, \lambda_6 = 0, \lambda_7 = 0, \lambda_8 = 0,$$

$$\lambda_9 = 0, \lambda_{10} = 1-q_1, \lambda_{11} = 0, \lambda_{12} = 0.$$

Where the two lower bound δ^F and τ^1 on r are derived from the non-negativity constraint of w_{LL} and λ_5. Collecting conditions verifies the proposition. □

Proof of Proposition 3.

The proposition is proved by showing a sequence of claims.

Claim 1: LP-1 is optimal for $r > max\{\delta^C, \delta^D\}$.
Claim2: *BP2*^{C} of LP-2 is never the overall optimal contract.
Claim 3: *Pure JPE* of LP2, if feasible, is the overall optimal contract.
Claim 4: *JPE* of LP2, if feasible, is the overall optimal contract.
Claim 5: *BP1*^{C} of LP2, if feasible, is the overall optimal contract.
Claim 6 $min\{\tau^0, \delta^D\} > \delta^C$ if and only if $\tau^0 > \delta^C$.

Using Claims 1 - 5, one can verify the following statement: when $\min\{\tau^0,\ \delta^D\} \le \delta^C$, LP-2 is optimal if and only if $r < \delta^C$; otherwise for $min\{\tau^0, \delta^D\} > \delta^C$, LP-2 is optimal if and only if $r \le min\{\tau^0, \delta^D\}$. Claim 6 shows that condition $min\{\tau^0, \delta^D\} > \delta^C$ is equivalent to $\tau^0 > \delta^C$ and, thus, is equivalent to the statement in the proposition.

Proof of Claim 1: The claim is trivial as we know from Proposition 2 that LP-2 does not have feasible solution on the region.

Proof of Claim 2: Recall that *BP2*^{C} of LP-2 is obtained by solving the following three binding constraints: Mutual Monitoring, Pareto Dominance, and $ICP_{HH \succ LL}$. It is easy to see that $\pi(0,0;w) = \pi(0,1;w)$ when both Mutual Monitoring constraint and the Pareto Dominance constraint are binding, in which case the Mutual Monitoring constraint can be re-written as follows:

$$\frac{(1+r)}{r}\left[\pi(1,1;w)-1\right] \ge \pi(0,1;w) + \frac{1}{r}\pi(0,1;w).$$

Note this is same as the "Stage NE" constraint in LP-1 and therefore all the constraints in (LP-1) are implied by those in (LP-2) under the *BP2*^{C} solution. In this case, (LP-2) has a smaller feasible set, so it can never do strictly better than (LP-1).

Proof of Claim 3: We know from Proposition 2 that *Pure JPE* is the optimal solution of LP2 for $r \in \left(0,\ \delta^A\right]$, over which the optimal solution of LP1 is *IPE* (Proposition 1). Substituting the corresponding solution into the principal's objective function, we obtain $obj_{JPE1} = \frac{q_1^2(1+r)}{(q_1-q_0)(q_0+q_1+q_1 r)}$ and $obj_{IPE} = \frac{q_1}{q_1-q_0}$. Algebra shows $obj_{IPE} - obj_{JPE1} = \frac{q_0 q_1}{(q_1-q_0)(q_0+q_1+q_1 r)} > 0$, which verifies the claim.

Proof of Claim 4: *JPE* is the solution of LP2 for $r \in \left(\max\left\{\tau^0, \delta^A\right\},\ \delta^C\right]$, over which *IPE* is the corresponding solution of LP1. Algebra shows that $obj_{JPE2} = \frac{(q_1-1)q_1 r(1+r) + q_0(q_0-q_1)^2 q_1 H}{(q_1-q_0)((q_1-1)r(1+r)+q_0(q_1+r))}$, $obj_{IPE} = \frac{q_1}{q_1-q_0}$, and $obj_{JPE2} - obj_{IPE} \le 0$ if and only if $\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)} \le r \le \delta^C$ (with equality on the boundary). The claim is true if $\max\{\frac{q_0+q_1-1+\sqrt{(q_0+q_1-1)^2+4(1-q_1)q_0 q_1}}{2(1-q_1)}, \tau^0, \delta^A\} \le r \le \delta^C$, which was shown in the proof of Proposition 2 to be equivalent to $r \in \left(\max\left\{\tau^0, \delta^A\right\},\ \delta^C\right]$. Therefore, *JPE* is the overall optimal contract whenever it is feasible.

Proof of Claim 5: We know that *BP1*^{C} is the solution of LP2 if $r \in \left(\delta^A, \min\left\{\tau^0, \delta^D\right\}\right]$. In this region, *IPE* and *BP*^{I} are potential solutions in LP1 because the other two solutions (*RPE* and *BP*^{S}) require $r \ge \tau^1 > \tau^0$. Let us compare first *BP1*^{C} of LP2 and *BP*^{I} of LP1. It is easy to show $obj_{BP1^C} = \frac{r(1+r)-(q_0-q_1)^2(q_0+q_1(1+r-q_1 r))H}{(q_0-q_1)(q_0+q_1-(-1+q_1)q_1 r - r^2)}$ and $obj_{BP^I} = \frac{r(1+r)+(q_0-q_1)^2(r(q_1-1)-1)H}{(q_1-q_0)(r(q_1+r-1)-1)}$. Tedious algebra verifies $obj_{BP1^C} < obj_{BP^I}$ for $\delta^C < r \le \min\left\{\tau^0, \delta^D\right\}$ where both solutions are feasible.

Showing *BP1*^{C} is always more cost efficient than the *IPE* solution is more involved and is presented in two steps. We first derive the sufficient condition for this to be true and then show that the sufficient condition holds whenever both solutions are optimal in their corresponding program, namely $\delta^A < r \le \min\left\{\tau^0, \delta^C, \delta^D\right\}$. Given obj_{BP1^C} and obj_{IPE} defined above, one can show that

$$obj_{BP1^C} < obj_{IPE} \iff r < \delta, \text{ where}$$

$$\delta = \frac{1}{2(1-q_1)}\left[((q_1-q_0)^2 H - q_1)q_1(1-q_1) - 1 + \sqrt{(((q_1-q_0)^2 H - q_1)q_1(1-q_1)-1)^2 + 4(1-q_1)((q_1-q_0)^2 H - q_1)(q_1+q_0)}\right].$$

Notice that if $\ge \delta^C$, $r < \delta$ (thus $obj_{BP1^C} < obj_{IPE}$) is satisfied trivially for $\delta^A < r \le \min\left\{\tau^0, \delta^C, \delta^D\right\}$. Consider the opposite case in which $\delta < \delta^C$. For $q_0 \in [0, q_1)$, one can show that $\delta < \delta^C$ corresponds to either $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$ or $q_1 - \sqrt{\frac{q_1}{H}} < r < q_1$. Since the latter condition contradicts the maintained assumption that $(q_1 - q_0)^2 H > q_1$, we consider $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$ only. Given $r < \frac{1+\sqrt{1+4(1-q_1)^2(-1+q_1(3+(q_1-2)q_1))H}}{2(1-q_1)^2 H}$, one can show $\delta > \tau^0$ for any $q_0 \in [0, q_1)$. Therefore, under the maintained assumption $(q_1 - q_0)^2 H > q_1$, $\delta < \delta^C$ implies $\tau^0 < \delta$. If the choice is between *BP1*^{C} and *IPE*, $r \le min\{\tau^0, \delta^C, \delta^D\}$. Then $\tau^0 < \delta$ implies $r < \delta$. $r < \delta$ implies $obj_{BP1^C} < obj_{IPE}$ whenever both are feasible (which is in the region $\delta^A < r \le min\{\tau^0, \delta^C, \delta^D\}$).

Proof of Claim 6: The "only if" direction is trivial. To show the "if" direction, note that if $\tau^0 > \delta^C$, we know $q_1 + q_0 > 1$ as otherwise $\tau^0 < 0 < \delta^C$. Under the maintained assumption on H, $q_1 + q_0 > 1$ implies $\delta^D > \delta^C$. Therefore, $min\{\tau^0, \delta^D\} > \delta^C$ if and only if $\tau^0 > \delta^C$. □