Asset Measurement, Real Effects and the Financial Accelerator

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Abstract

This paper examines two questions: When should we expect asset measurements to contribute to macroeconomic fluctuations? Should regulators adapt the measurement to financial shocks? We develop a model in which firms borrow funds subject to collateral constraints and implement an *ex-ante* asset measurement rule. In this environment, we characterize the nature of optimal measurements and analyze their interaction with interest rates and financial shocks. Under certain conditions, impairment accounting contributes to the financial accelerator, magnifying the effect of financial shocks on investment. A regulatory process that ignores the dependence of the interest rate on the measurement might select a rule inefficient in the aggregate, with excessive reliance on impairment accounting and forced liquidations.

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Capital market institutions do not always seem well-adapted to the challenges posed by economic cycles. As Federal Reserve chairman Ben S. Bernanke notes, in an address to the Council on Foreign Relations held in Washington D.C. in March 2009:

"*There is some evidence that capital standards, accounting rules, and other regulations have made the financial sector excessively procyclical —that is, they lead financial institutions to ease credit in booms and tighten credit in downturns more than is justified by changes in the creditworthiness of borrowers, thereby intensifying cyclical changes.*"

To be sure, measurement rules do not cause economic cycles, but the case against the measurement as a contributor to the financial accelerator is not altogether unfounded. Financial accounting standards are not designed to offer flexibility in a time of crisis, and admittedly, their primary goal is not to respond to financial shocks. On occasion, measurement rules that require excessive disclosures over potential losses if a recession continues might appear to work directly against regulators' efforts to restore confidence, spur lending and end a recessionary spell. On the other side of this debate, standard-setters have strongly resisted this argument, claiming that lax measurements and the opacity they create could be even more damaging to market confidence.

This paper attempts to bring new light into this debate by developing a rigorous analysis of the optimal measurement of *productive* assets, i.e., core assets that are essential parts of the firm's operations and whose measurement simultaneously determines real decisions as well as their function as collateral to bank loans. We formally link the nature of the measurement to liquidity shocks and demonstrate that the optimal measurement can amplify the consequences of an economic downturn (the financial accelerator). Because the optimal measurement is a function of market expectations, which themselves are function of the measurement, the model can also predict the existence of inefficient rational expectations equilibria arising from the strategic complementarity between measurement and interest rate policy.

In the model, firms in the economy are subject to a liquidity shock and must raise capital from an outside source in order to continue operating. Each firm has a single operating asset that can be either used as an input of production or liquidated in a competitive capital market. When it is liquidated, the productive asset is redeployed into new capital (to be used by other firms) and, key in our approach, there is incomplete information about the conversion rate from inside to outside use; for example, a highly firm-specific productive asset would have very low value if used by other firms. Within existing concept statements, this conversion rate is closely related to fair-value, defined as the highest value the asset would receive if purchased by another party ("exit value"). Abusing on language, we later refer to this rate as the collateral value of the asset because it represents the value that could be obtained by another party seizing the asset. Then, following the model of *bayesian persuasion* of Kamenica and Gentzkow (2011), we assume that the firm commits to collect and release information about collateral values to maximize its ex-ante surplus. In our applied context, the measurement could be interpreted as a disclosure regulation enforced by law or the ex-ante implementation of a measurement system.

We develop the main results next. If the liquidity shock is small, an aggressive measurement prescribing disclosure at the top, e.g., write-ups of high-value assets, is optimal. The advantage of write-ups is that they identify assets whose (outside) resale value is attractive relative to the cash flows if they were operated. Firms that do not write up might not be able to finance the investment if they were to publicly report low collateral, causing the measurement to prescribe no disclosure for low asset values. Hence, when it is feasible, a measurement featuring only write-ups maximizes investment efficiency.

If the liquidity shock is large, the expected collateral of non-disclosing firms will be too low to raise outside capital. In this case, asset write-ups are problematic because they indirectly deplete the expected collateral of non-disclosing firms. We show that the measurement first responds to the depletion of collateral by reducing write-ups, which increases the expected collateral of non-disclosing firms at the expense of a greater over-investment problem. For an even larger liquidity shock, a reduction in write-ups will no longer be sufficient to restore collateral; in that case, the optimal measurement shifts to require only disclosures for assets with low resale value, e.g., similar to the practice of accounting impairments.

The endogenous shift in the measurement contributes to the financial accelerator as the economy sinks into a recession. In an economy with loose credit, the prevailing measurement is a write-up rule which, given a moderate decrease in credit market conditions, leads to a reduction in write-ups and, therefore, an *increase* in investment. However, when the economy reaches sufficiently tight credit conditions, the measurement switches to an impairment rule which, as conditions worsen further, leads to an increase in impairments and a decrease in investment. Therefore, we predict that a sharp contraction follows an investment boom.

The interaction between the interest rate, the level of information asymmetries and the available capital can cause the existence of one or two stable competitive equilibria. In an economy with intermediate liquidity shocks, equilibria with write-ups and impairments simultaneously exist. To see why, note that impairments cause both excess liquidations after an impairment is made and excess continuations for firms with high collateral values that operate. In intermediate cases where few impairments are needed, the second effect dominates, and it decrease the supply of capital in the market at any interest rate. This causes the market-clearing interest rate to be higher, self-fulfilling the optimality of impairments as a preferred policy. Mirroring this logic, write-ups, on the other hand, make it desirable for more firms to liquidate and resell their assets, increase the supply of capital for any interest rate and therefore decrease the equilibrium interest rate. In turn, this causes write-ups, which are desirable in such loose credit markets, to also be self-fulfilling.

When they both exist, impairments and write-ups equilibria do not achieve the same ex-ante efficiency. We show that the write-up equilibrium, which coincides with lower rates of returns, allocates capital in a more efficient manner. This observation further suggests some concern about self-fulfilling expectation traps characterized by high interest rates and impairments. Therefore, a regulatory process that considers interest rates as a given rather than as a variable that adjust to the measurement would be unable to exit such trap, thus suggesting that measurement regulation cannot be thought of in isolation of broader macroeconomic objectives.

Existing Literature

A description of *best* measurement rules cannot be examined without a careful consideration of real effects. Real effects are often misinterpreted as representing production in a very broad sense but they are in fact a more subtle concept which emphasizes how the measurement changes the nature of what is being measured, rather than just being a best response to an exogenous decision problem (Kanodia (2007)). Hence, optimal measurements are an equilibrium concept that cannot be untied from the object of the measurement. As noted by R. Dye in his survey of the literature, environments with real effects are settings with *"what seems to be a virtually unlimited, largely unexplored, and economically important array of situations in which accounting can play a nontrivial, and nonobvious, role"* (Dye (2001), p.197). Our study is part of this area and proposes a novel general-equilibrium channel for real effects; namely, we show that the nature of the measurement affects the demand and supply capital, affecting the interest rate and thus the relative desirability of various possible measurements. The analysis delivers a fundamental insight: that optimal measurements should be thought jointly with macroeconomic equilibrium consequences, an idea that is at odds with current standard-setting frameworks which tend to describe providing more information as the main objective ("decision-usefulness").

There is an extensive literature in the area of real effects and, to settle ideas further on the objectives of this literature, we give a few examples of below. Kanodia, Singh, and Spero (2005) consider a model in which the investment choice can signal a firm's quality characteristics. An excessively precise disclosure of investment might cause an over-investment distortion as the observed investment acts as a signal of quality. Gigler et Al. (2012) show that infrequent reporting can reduce a manager's incentive to focus on short-term projects by delaying their effect on market prices. Suijs (2008) examines whether asymmetric disclosures can affect the allocation of the risk of the firm's investments between generations and, like us, argues that the degree of asymmetry is a function of the production technology. Plantin, Sapra, and Shin (2008) find that measurement rules based on market prices tend to increase asset sales during a downturn, draining liquidity and magnifying the adverse consequences of the downturn. Focusing on voluntary disclosures, Beyer and Guttman (2010) and Hughes and Pae (2013) examine the interaction between incentives to release information, adverse selection and their effects on productive decisions.[1]

Our model also extends the literature on business cycles caused by credit rationing under asymmetric information. In this area, the paper most closely related to ours is Holmström and Tirole (1997), who link financial acceleration to firm's available collateral in a model of financial intermediation. There are two key differences between their model and ours. First, we specifically focus on a setting in which collateral values are not fully observable and solve for the optimal measurement. Second, while their focus is on pure financial assets (which, admittedly, form a very small portion of the type of collateral used in practice), we focus on productive assets used in the firm's operations. A recent literature initiated by Morris and Shin (1998) examines when changes in the public information environment can shift expectations across multiple equilibria. Applying this theory in the context of mark-to-market accounting, Plantin, Sapra, and Shin (2008) find that measurement rules based on market prices tend to increase asset sales during a downturn, draining liquidity and magnifying the adverse consequences of the downturn. In a recent study, Gigler, Kanodia, and Venugopalan (2013) consider a model with strategic complementarities and show that, as users tend to overweight public information, excessive disclosure can magnify coordination problems and lead firms to under-invest in risky assets.

[1] For other studies on the real effects of disclosure, see also Kanodia (1980), Kanodia and Lee (1998), Sapra (2002), Dye (2011), Caskey and Hughes (2012), Beyer (2012), Corona and Nan (2013) and Chen and Jorgensen (2013).

Several prior studies have examined whether pre-decision information can be useful for an organization and our study fits within this literature. Baiman and Evans (1983), Penno (1984) and Baiman and Sivaramakrishnan (1991) examine this question in the context of a control problem and analyze when giving more information to an agent can be reduce agency costs. Our model presents a slightly different environment because, in the context of an end-of-period sale by the manager, pre-decision information can only have an impact if it is publicly revealed to both the manager and outside investors. More recently, Demski, Lin, and Sappington (2008) also focus on asymmetric asset reevaluations but their primary focus is on solving a lemon's problem at the time of sale rather than the shortage of collateral considered here. In a model where disclosures are entirely voluntary and information is produced by analysts, Langberg and Sivaramakrishnan (2010) argue that some unfavorable disclosures are made to improve production efficiency.

A study related to ours is Goex and Wagenhofer (2009) who examine commitment to an information system in which the value of the collateral can be disclosed.[2] Their result is that the optimal measurement should always feature some amount of disclosures over low collateral values (impairments). It is fair to say that our model takes their approach as a starting point but we extend their analysis along several key dimensions. Their focus is not on productive collateral and therefore their model does not provide conditions under which write-ups would dominate impairments. In our setting with productive assets, we specifically show that write-ups, when they are feasible, always dominate impairments and, thus, provide a more nuanced description of the benefits of impairments. In addition, their model is such that there is an infinite supply of capital at a fixed interest rate (i.e., partial equilibrium) which cannot speak about a capital crisis or the financial accelerator (i.e., an exogenous shock to the available capital stock). Because we solve the model in general equilibrium, we can specifically link the measurement and the interest rates to the effect of a liquidity shock on the supply and demand of capital.

To our knowledge, only two studies have considered the question of optimal regulation

[2] Like in our model, the existence of commitment is important in Goex and Wagenhofer (2009). In these models, the firm issues a debt security whose interest rate is a function of what collateral is reported (i.e., lower when the firm reports more collateral). Therefore, if the firm's initial owners were fully informed and could *truthfully* disclose ex-post, their model would unravel to full-disclosure - as in nearly all models that feature an ex-ante choice over reporting systems (Kanodia and Lee (1998), Kanodia, Sapra, Mukherji, and Venugopalan (2000), Dye (2002), Gigler, Kanodia, Sapra, and Venugopalan (2009)). The same property would hold in our model but it should be noted that assuming an ex-post fully-informed manager and endowing this manager with the ability to make ex-post truthful disclosure is a non-trivial set of assumptions. Certainly, there are many settings in which the mechanism to collect and report information must be set ex-ante. It could be interesting to consider the interaction between mandatory disclosure and voluntary disclosures in a model with disclosure frictions; however, this question is somewhat orthogonal to our research question and has been discussed in prior literature (see Dye (1990) for a discussion).

and the business cycle. Bertomeu and Magee (2011) is a study in which regulation is subject to political oversight and does not necessarily maximize production efficiency. At the onset of a recession, greater pressure from firms suffering from unfavorable events decreases the quality of reporting and temporarily boosts total investment. Sun (2013) examines the quality of optimal regulations as a function of the business cycle. Her model is different form ours in that she focuses on investigation intensity by the regulator (i.e., enforcement) while our focus is on the measurement. Nevertheless, like us, she finds that the rise in enforcement during recessionary periods can contribute to an abrupt fall in asset prices.

Lastly, our results can be tied to a literature that examines the association between the cost of external of funds (or cost of capital) and the quality of information. Most of the research in this area model the cost of external funds in a one-period model where risk-averse investors demand more compensation when exposed to market risk (Jorgensen and Kirschenheiter (2003), Hughes, Liu, and Liu (2007)). An incorrect reading of this literature would suggest that the sole determinant of the cost of external funds is investors' risk-sharing motives. This is not the case and, while recognizing the importance of results on risk-sharing, we do illustrate a different determinant of the cost of capital in a simple two-date model when the supply of external capital is not perfectly elastic. Investors are risk-neutral but demand compensation for lending capital (the interest rate) over a given period of time. We derive the interest rate to be paid on any raised capital endogenously and relate it to the disclosure environment.[3]

1 Institutional background

Before I present the formal theory, we offer here some brief institutional background about the standard-setting process as it exists today and the general nature of measurement rules. A more detailed background can be found in Zeff (2002).

The regulatory environment for asset measurement has evolved over time, moving gradually from laissez-faire to a self-regulated convention and, over the past century, toward centralized accounting and financial institutions. In the US, the Securities and Ex-

[3]Note that our model does not have implications in terms of a risk premium since all investors are risk neutral. It does, however, have implications about the interest rates paid on loans or the total return demanded by external capital providers (which is contained in most measure of implied cost of capital). Unfortunately, most accounting models borrow primarily from finance rather than from traditional economic theory; in many finance models (like the exponential-utility CAPM), it is *assumed* that there is an infinite potential supply of capital as investors can transform their consumption into capital at a constant rate leaving the researcher to solve for what is inherently partial equilibrium in disguise. The assumption is not just unrealistic but is also at odds with most of the macroeconomic literature.

change Commission (SEC), a government-appointed body, is responsible for setting accounting and reporting standards for companies whose securities are actively marketed to investors. Although the SEC has authority to approve and implement standards, it has delegated the primary responsibility for writing accounting standards to a non-governmental organization, the Financial Accounting Standards Board (FASB). The emphasis in the mission of both the FASB and the SEC is on promoting transparent disclosures and designing disclosures in a timely fashion to protect users of financial information.

These institutions are the outcome of many changes in the regulatory accounting environment. On many occasions, accounting institutions have been criticized (e.g., the 1976 Metcalf report) and dramatically reformed (e.g., the 1934 SEC Act, the replacement of the Accounting Principles Board by the FASB, or the evolution of international standard-setting boards). These changes have often been preceded by or concurrent with financial scandals and economic downtowns.

The FASB has repeatedly insisted that its role is to provide clear, accurate and complete information in the financial market, not to partner with the Federal Reserve or any other regulatory body to offer coordinated responses to an economic crisis. As D. Beresford, a former FASB chairman, notes, recalling replies made to members of Congress "alleged economic consequences were beyond the FASB's responsibility and probably unprovable anyway" (Beresford 2001). This claim is factually accurate as, structured in its current form, the mission of the institution is "to provide financial information about the reporting entity that is useful to existing and potential investors, lenders, and other creditors in making decisions about providing resources to the entity" (OB2, *Statement of Financial Accounting Concepts No. 8 September 2010*). GAAP codifies long-term fixed rules that, except for rare exceptions, are not intended to change as a function of current economic conditions.

Historically, the FASB has been critical of interference by Congress to force changes to accounting rules because of concerns by members of Congress, for economic consequences (Zeff 2002). There are only a few cases in which, under extreme pressure, the FASB, and its sister institution, the International Accounting Standards Board (IASB), have reversed existing rules. In 2009, the U.S. Congress House Financial Services Subcommittee forced the FASB to loosen its standards on fair value measurements. The FASB did not change its standard but issued guidance that reduced requirements to impair marketable securities in situations of high volatility; similar guidance was later issued by the IASB. On occasion, accounting standard-setters have disagreed with options favored by the central banks. A good example of this is dynamic provisioning. Under the supervision of the central bank, Spanish banks implemented a practice known as dynamic provisioning which involved setting relatively high loan-loss reserves during expansions, in anticipation of declines in market conditions. The IASB has opposed this practice as non-conforming with international accounting standards and favors the incurred loss model in which future losses must be validated through currently observable loan characteristics.

Impairment accounting is a core element of U.S. GAAP while, at least currently, write-ups are not. This is partly for historical reasons, as much of the basic structure of accounting has been inherited from the Great Depression of the 1930s and surprisingly stable since then. During the 1920s, balance sheets often included upward revaluations of long-term assets and this practice was thought to have misled investors and exacerbated the stock market crash. The SEC mandated conservative practices that effectively prohibited asset write-ups. The Statement of Financial Accounting Standards SFAS No. 144 "Accounting for the Impairment or Disposal of Long-Lived Assets" is consistent with conservatism. Long-lived assets are reviewed for impairment whenever events or changes in circumstances indicate that the carrying amount of an asset may not be recoverable. If the carrying amount of an asset is greater than the undiscounted future cash flows related to the use and disposal of asset, i.e., the carrying value is not recoverable, the asset is written down to fair value, which represents the discounted future cash flows or the price obtained from resale.

Although accounting rules for productive assets are still largely based on an impairment model, there have been a few recent evolutions. International GAAP allows for upward revaluations for certain long-term assets under IAS No.16, although this is optional and is generally not included in net income. In theory, fair value accounting recognizes symmetrically both gains and losses. Yet, U.S. GAAP does not allow upward reevaluations for most productive assets and even international GAAP does not allow them for intangible assets or most current productive assets. U.S. GAAP mostly allows fair-value measurements for assets held-for-sale (generally financial securities, SFAS No. 157 & 159). There is, to our knowledge, no accounting rule that recommends asymmetric write-up of assets, or types of GAAP that would offer different book of rules as a function of economic conditions.

2 The model

The economy is populated by a continuum of firms, each operated by a risk-neutral owner-manager with no personal wealth.[4] There are three event dates which are formally described in what follows.

At date $t.1$, the economy forms an interest rate $r \geq 0$ which clears the market for capital (more on this later on). Then, a measurement rule is selected to maximize total firm surplus. We model this process as in Kamenica and Gentzkow (2011) as a game of *bayesian persuasion* in which the firm commits to collect and distribute information. In our context, the measurement could be an accounting standard or the information system put in place by the firm at an earlier period in time.[5]

Our primary focus is on information about the *fair value* of productive assets, that is, the exit value of the firm's productive assets when sold back to another party (see Statement of Financial Accounting Standards No. 157). For expositional purposes, let us assume that each firm has a single asset and thus refer to a sale as a liquidation. Productive assets vary in terms of how firm-specific they are and, when a firm is liquidated, its productive assets are converted into all-purpose capital $\tilde{z}$. The random variable $\tilde{z}$ is drawn from a distribution with p.d.f. $f(.) > 0$, c.d.f. $F(.)$, finite first moment and full support over $\mathbb{R}^+$. For example, the productive asset may be a patent, the real estate of a flagship retail store or, for a manufacturing company, the land and machinery in a production line.

A measurement rule is a function $d(z) \in \{0, 1\}$, where $d(z) = 1$ indicates that the fair value $\tilde{z} = z$ is reported and $d(z) = 0$ indicates that no measurement is made for events in which the fair value is $\tilde{z} = z$. There are no exogenous costs or restrictions on the measurement so that, in theory, a fully-revealing measurement would be feasible. The fair value of the asset is not observable when a measurement is not performed, although one can also think about "not measuring" as a particular coarse measurement indicating that $\tilde{z}$ is not in the region where $d(\tilde{z}) = 0$.[6]

[4]To save on notation, we omit the indexation for each firm; all random variables in the model are assumed to be i.i.d. unless specifically stated otherwise.

[5]We shall leave aside here the critical issue of implementation of such a rule within an existing institution and it is, at best, unclear whether existing institutions would be able to reach an optimal measurement. See for example Dye and Sunder (2001), Sunder (2002) and Jamal, Maier, and Sunder (2005) for more extensive discussions of these questions.

[6]It is possible to argue that no such measurement could be performed without observing the value of the asset in advance. However, this need not be true. An information system will collect information as a realized state of the world. For example, in a setting with Normally-distributed cash flows, a reduction in the variance of a signal could be achieved without fully observing final cash flows by improving the quality of the firm's information system. However, the choice of a variance is a very constrained information system choice and, possibly, the firm could implement an information system in which the variance of a signal condition al on the true state could change.

At date $t.2$, each firm releases the public signal $I = d(z)z$.[7] Denote the fair value as $P : \mathbb{R}^+ \to \mathbb{R}^+$ where $P(I) = 1_{I>0}I + 1_{I=0}\mathbb{E}(\tilde{z}|d(\tilde{z}) = 0)$. Firms have two self-exclusive options. The first option is to liquidate the project and resell the productive asset as all-purpose capital, e.g., equipment could be recycled for scrap, some industrial land sold or an office building could be re-deployed, etc. Because capital is compensated with an interest rate r, the liquidation option yields an end-of-period payoff equal to:

$$\pi_0(I) = (1 + r)P(I) \tag{2}$$

The second option is to retain the productive asset and operate the project. The productive asset is required as an input of production but the firm must also incur a liquidity shock $L > 0$ which requires an additional L units of capital, e.g., to build manufacturing capacity to use a patent, expand a retail presence, renovate the productive asset, offer extended credit terms to customers, etc.[8] When operated, the project yields one of two outcomes $\tilde{x} = h > 0$ or $\tilde{x} = l = 0$, whose probabilities depend on a value-enhancing decision (effort) to be taken at date t.3. We normalize h to one so that the collateral $\tilde{z}$ is the exit value *per unit of final cash flow*. To raise capital, the firm issues a security $(s_h(I), s_l(I))$ which can depend on the public information set I.

At date $t.3$, each firm owner-manager makes a private effort decision $e \in \{0, 1\}$, where $e = 1$ implies a personal cost $c > 0$ and $Prob(\tilde{x} = 1|e) = p - (1 - e)\Delta p \in (0, 1)$. Assume that $p - L - c > 0 > (p - \Delta p) - L$ so that the firm should only be operated if effort can be elicited. Then, each firm realizes its cash flows, the project completes and the manager must retire.[9]

The productive asset is then sold to the next generation of managers and, for simplicity, we assume that it is sold at its expected value conditional on all current infor-

[7]This notation involves a slight imprecision given that $I = 0$ refers to both $d(z) = 1 = 1 - z$ and $d(z) = 0$. Because the former event has mass zero, it has no bearing on the analysis and, by convention, we refer to $I = 0$ as indicating the event that no information is disclosed.

[8]An alternative possibility would be that the required investment would be related to $\tilde{z}$, say, if the manager need only finance $L - z$; however, this is problematic in this setting for several reasons. First, the manager may not know the outside resale value of the asset at $t.2$ and, as a result, would not necessarily know how much to invest; this, in turn, would complicate the model by posing questions about the outcome of partial investments. Second, conceptually, the value z represents the resale value of the asset when converted to all-purpose capital while L represents an additional capital inflow for a specific use, making the connection between the two is tenuous, at best. For example, a restaurant may be established on an expensive tract of land or building, but this does not mean that renovations or staffing expenses would be cheaper.

[9]The assumption of a short-lived manager is commonly used in the literature (Dye (1988), Suijs (2008), Bertomeu and Magee (2011)) and, as in these studies, used to remove special considerations that pertain to forward-looking contracting or disclosure policies.

mation $P(I)$. Equivalently, the asset is no longer useable without the manager and must be reconverted into all-purpose capital for use in future periods.[10] Therefore, the firm's end-of-period cash flow is equal to the (operating) cash flow $\tilde{x}$ plus the (investing) cash flow $P(I)$ from selling the asset to the next generation.[11] The securities pay off up to the amount of cash available in the firm $\min(s_x(I), x + P(I))$ when the outcome is $\tilde{x} = x$. Without loss of generality, we hereafter restrict the attention to securities that satisfy the limited liability constraint $s_x(I) \leq x + P(I)$. Then, the utility obtained by the manager when continuing the project is given by:

$$\pi_1(I) = p - \mathbb{E}(s_{\tilde{x}}(I)) + P(I) - c. \tag{3}$$

Definition 1 *A competitive equilibrium is defined as a set of policies $\Gamma^* = (r^*, d^*, \gamma^*, s_x^*, P^*)$, where $r^* \geq 0$ is the interest rate, $d^* : \mathbb{R}^+ \to \{0,1\}$ is the measurement rule, $\gamma^* : \mathbb{R}^+ \to \{0,1\}$ is the set of firms that continue, $s_x^* : \mathbb{R}^+ \to \mathbb{R}$ denotes the security issued when the firm raises capital and $P^* : \mathbb{R}^+ \to \mathbb{R}^+$ denotes the pricing of collateral, such that the following conditions are satisfied:*

(i) For any $I \in \mathbb{R}^+$, $P^*(I) = \mathbb{E}(\tilde{z}|d^*(\tilde{z})\tilde{z} = I)$.

(ii) The firm issues the optimal feasible security that elicits the value-enhancing action, i.e., for any I and x, $s_x(I) \leq x + P^*(I)$ and:

$$\begin{aligned} \Delta p(1 - s_h(I) + s_l(I)) &\geq c, && (4) \\ p s_h(I) + (1-p) s_l(I) &= (1 + r^*)L. && (5) \end{aligned}$$

By convention, if a feasible solution does not exist, we set $s_h^*(I) = 1 - s_l^*(I) = 1$.[12]

(iii) For any I, firms make efficient liquidation decisions, i.e., $\gamma^*(I)$ satisfies the following

[10] We make the assumption that the value of the asset when sold does not change over time but this assumption is not critical for the analysis. The key assumption in our model is that the liquidation value of the asset at $t.2$ should be positively associated to the liquidation value at $t.3$. It is also possible to assume, with little loss to the results, that the value of the collateral at t.3 is a function of $\tilde{x}$; in that case, one would redefine the end-of-period collateral value as the value of the asset after $\tilde{x} = 0$ is observed.

[11] Note that it is with no consequence to the analysis if (a) $\tilde{z}$ becomes known at this point or if (b) the manager knows $\tilde{z}$ but cannot disclose it. On the other hand, as we noted earlier, the results would be very different if the manager had private knowledge of $\tilde{z}$ and could truthfully disclose it because, then, all information would always be fully disclosed (unravelling).

[12] This convention is convenient because it implies that, if a solution to the incentive problem does not exist, condition (iii) will induce $\gamma^*(I) = 0$ so that the firm is liquidated.

inequality:

$$(1-2\gamma^*(I))(p-\mathbb{E}(s^*_{\tilde{x}}(I)|e=1,I)+P^*(I)-c) \leq (1-2\gamma^*(I))(1+r^*)P^*(I). \quad (6)$$

(iv) The measurement rule maximizes ex-ante surplus,

$$d^* \in \textit{argmax}_{d(.)}\mathbb{E}(\Psi(d(\tilde{z})\tilde{z})) \quad (7)$$

where: $\Psi(I) = \max(p - \mathbb{E}(s^*_{\tilde{x}}(I)|I) + P^*(I) - c, (1+r^*)P^*(I)))$.

(v) The interest rate is set such that the market for capital clears:

$$\mathbb{E}(\gamma^*(d^*(\tilde{z})\tilde{z}))L = K + \mathbb{E}((1-\gamma^*(d^*(\tilde{z})\tilde{z}))\tilde{z}) \quad (8)$$

where $K \in (0, \overline{K})$ *is an exogenous constant capturing the amount of capital available in the economy.*[13]

Condition (i) requires market prices to be formed according to Bayes rule. Condition (ii) states that eliciting the value-enhancing action must be feasible and that the firm should minimize financing costs. The latter is equivalent to binding the participation of the provider of liquidity (Equation (5)). Condition (iii) states that the firm is optimally continued or liquidated; in case of indifference, either continuation or liquidation may be adopted. Condition (iv) states the measurement maximizes the expected surplus, under the assumption that firms optimally choose to continue or liquidate.[14] Condition (v) is a market-clearing condition that requires the demand for liquidity (the left-hand side of Equation (8)) to equal the supply of capital (the right-hand side of Equation (8)).[15] In

[13]We assume that $K < \overline{K}$ where $\overline{K}$ is an upper bound formally derived in Appendix B, where $\overline{K} = \min(L, (1-F(M^-(0))L - F(M^-(0))\mathbb{E}(\tilde{z}|\tilde{z} \leq M^-(0)))$, where $M^-(r)$ is defined in Proposition 4. This condition means that there is not enough capital to finance all the firms.

[14]The assumption that the rule features either disclosure or non-disclosure is without loss of optimality and we adopt this assumption here because it fits the practical setting of asset measurements (i.e., either an asset is measured or is not measured). The same level of welfare in our model could be achieved with a binary signal because there are only two decisions in our setting (liquidate or continue), see Kamenica and Gentzkow (2011), Proposition 1 p.9. If we were to adopt this alternative formulation, the optimal measurement would take the form of a binary classification as in Dye (2002). Binary information sets are (weakly) optimal in most models with binary investments as in, among others, Goex and Wagenhofer (2009), Gigler, Kanodia, Sapra, and Venugopalan (2009) or Caskey and Hughes (2012).

[15]As defined in Definition 1, the equilibrium can feature *negative* interest rate when the information asymmetry is very large and there are very few firms that can be financed. To save space, we rule out this case by assumption. If there is no technology to store capital (at no loss), the interest rates could be negative if the exogenous supply of capital in the economy is large; in this case, there would be no purpose in liquidating assets and no write-up measurement could be optimal. If the firms can store capital, there will

what follows, we adopt the convention of viewing two equilibria as equivalent if policies differ only over a set with measure zero.

It will be convenient to write the set of collateral values such that a disclosing firm liquidates in part (iii) of the Definition in terms of two bounds:

$$\{z : \gamma^*(z) = 1\} = \{z : z \in (W^-(r^*), W^+(r^*))\}, \tag{9}$$

where $W^+(r)$ is the maximal (resale) value of the collateral such that the firm would continue the project and $W^-(r)$ is the minimal value of the collateral such that the firm can raise capital.

Lemma 1 *The thresholds $W^+, W^- : \mathbb{R} \to \overline{\mathbb{R}}$ are uniquely defined by:*

$$W^+(r) = \frac{p - (1+r)L - c}{r}, \tag{10}$$

$$W^-(r) = p\frac{c}{\Delta p} - p + (1+r)L. \tag{11}$$

The thresholds W^-, W^+ are also relevant for the no-measurement firms with $d(\tilde{z}) = 0$. Taking expectations, a no-measurement firm liquidates when $P(0)$ is outside of the range $(W^-(r), W^+(r))$. Because all firms cannot be liquidating in a competitive equilibrium, this interval must be non-empty at r^* or, equivalently, $r^* \leq r_{\max}$ where $r_{\max}$ is the unique positive root of:

$$r^2 L + (2L - p + \frac{cp}{\Delta p})r + L - p + c = 0 \tag{12}$$

For now, we shall restrict the search for competitive equilibria such that $r^* < r_{\max}$. The special case of $r^* = r_{\max}$ is pathological of economies in which all the investment surplus is dissipated and we discuss it separately in the Appendix.

Lastly, we introduce an additional concept that will be useful to lay out some of the preliminary intuitions. We say that a policy is $r-$optimal if it is optimal conditional on an exogenous interest rate r. Because there is no difference in our model between a financial and a productive asset when $r = 0$, the model in Goex and Wagenhofer (2009) is the $0-$optimal measurement in this Definition.[16] A competitive equilibrium is an interest rate r^* such that the capital market clears when all policies are r^*-optimal.

be zero lower bound on (real) interest rates and, if this lower bound is reached, the economy will behave as if facing an infinite supply of capital at fixed interest rate, as discussed in Goex and Wagenhofer (2009).

[16]Naturally, our focus here will be on $r > 0$ and, even more importantly, examine the equilibrium interest rate r^* and its implications after a liquidity shock.

3 The benchmark economies

3.1 First-best economy

We briefly state the solution of the model when there is no agency problem and effort is fully contractible. Omitting the incentive-compatibility condition (4) in Definition 1, let $\Gamma^{fb} = (r^{fb}, d^{fb}, \gamma^{fb}, s_x^{fb}, P^{fb})$ denote a *first-best* equilibrium. The next Proposition characterizes the efficient investment policy and measurement.

Proposition 1 *In a first-best equilibrium, a firm is liquidated if and only if $z > W^+(r^{fb})$. The optimal measurement is any rule such that $\{z : d^{fb}(z) = 0\}$ is a subset of either $\{z : z \geq W^+(r^{fb})\}$ or $\{z : z \leq W^+(r^{fb})\}$. The first-best interest rate $r^{fb} > 0$ is given uniquely as follows:*[17]

$$F(W^+(r^{fb}))L = K + (1 - F(W^+(r^{fb}))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r^{fb})). \tag{13}$$

In first-best, a firm should be liquidated when its asset is more efficiently used as all-purpose capital than as part of the firm's production process.[18] Because z captures the ratio of outside to internal use of capital, firms for which the exit value z is high should be liquidated. For a given interest rate r, the threshold $W^+(z)$ represents the point at which an internal use of the asset becomes economically inefficient (i.e., equivalently, the firm is no longer the best user of the asset).

The first-best measurement policy is not biased toward a particular form of measurement. A measurement rule that enforces the first-best liquidation is efficient and, therefore, the measurement is not unique. A fully-revealing measurement would, naturally, implement the first-best investments but, in addition, any measurement in which all firms make the same decision conditional on no-measurement is optimal.

[17] It is easily shown that the competitive equilibrium investments in what we define as *first-best* also correspond to the optimal allocations with a centralized planner.

[18] Certain models assume that liquidations are wasteful and the firm is always the best potential user of the asset, often the emphasize the economic costs of inefficient liquidations. This is not an assumption that we make here and we believe that it is economically plausible that the firm would not always be the best user of the asset. To give one example, if a business is not doing well, its real estate would be more profitably used by other businesses.

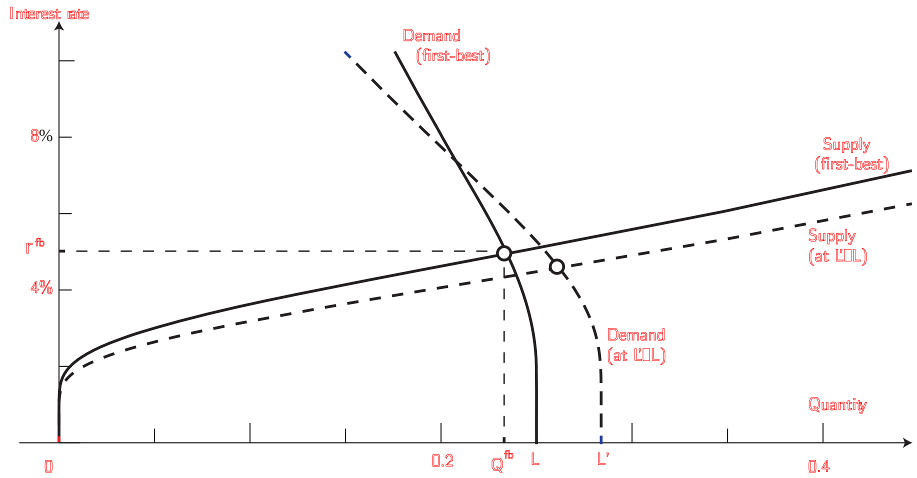


Figure 1: First-best supply and demand

Because we focus on *productive* assets $\tilde{z}$, the demand and supply curves in our model are strictly monotone even in first-best. By contrast, in Holmström and Tirole (1997) and most of the existing literature focusing on financial assets, firms only differ with respect to a cash balance in first-best. Thus, their demand curve is horizontal or perfectly elastic - i.e., all firms demand capital when the firm's interest rate is below the interest rate - and the supply curve is vertical or perfectly inelastic. In our model, a firm choosing to continue must use its productive assets and forfeits the market interest on the proceeds from selling the asset. This affects demand and supply curves in two respects. First, firms are heterogenous in terms of the opportunity cost of continuing and, hence, the fair value of the productive asset affects a firm's willingness to continue or liquidate. Second, the total stock of capital available for use is endogenous and depends on the fraction of firms that decide to re-purpose their productive asset.

Figure 1 further illustrates this key aspect of our model by considering the effect of a liquidity shock on the equilibrium demand of capital and the interest rate. With financial assets, a liquidity shock would not affect the supply curve and shift down the demand curve, thus causing a decrease in the equilibrium interest rate. In our model, a liquidity shock increases supply because it induces more firms to re-purpose their productive asset $z > W^+(r)$. The effect on the demand is ambiguous and results from two economic forces in that (a) each continuing firm demands more capital, (b) each continuing firm demands more capital. When the first force dominates, i.e., when most firms continue, demand increases, increasing the equilibrium demand of capital and putting upward pressure on the interest rate. When the second force dominates, the liquidity shock will reduce equilibrium demand and tend to decrease the interest rate.

3.2 Fully-revealing measurement

Another key benchmark is the case in which the measurement is one that features full revelation of all collateral values, i.e., replacing condition (iv) with $d^*(z) = 1$ for any z but keeping the information asymmetries. Let $\Gamma^{full} = (r^{full}, d^{full}, s_x^{full}, P^{full})$ denote an equilibrium in which fully-revealing measurements have been imposed. The next Proposition follows immediately from Lemma 1.

Proposition 2 *In a fully-revealing equilibrium, a firm is liquidated if and only if* $z \notin (W^-(r^{full}, W^+(r^{full})))$*. The fully-revealing interest rate* r^{full} *is given uniquely as follows:*

$$
\begin{aligned}
&(F(W^+(r^{full})) - F(W^-(r^{full})))L = \\
&\quad K + F(W^-(r^{full}))\mathbb{E}(\tilde{z}|\tilde{z} \leq W^-(r^{full})) + (1 - F(W^+(r^{full}))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r^{full})) \qquad (14)
\end{aligned}
$$

Figure 2 illustrates how the information asymmetry affects the equilibrium interest rate relative to the first-best economy. Fewer firms are able to raise capital so that demand decreases and more productive assets are re-purposed implying an increase in the supply of capital and a reduction in the interest rate. Importantly, our model does not feature the same multiplier effect as in Holmström and Tirole (1997) because, unlike financial assets, productive assets do not earn the market interest rate and, therefore, the reduction in interest rate does not reduce the value of the collateral at $t.3$. In particular, the financial acceleration that we will describe here will be entirely driven by the choice of measurement.

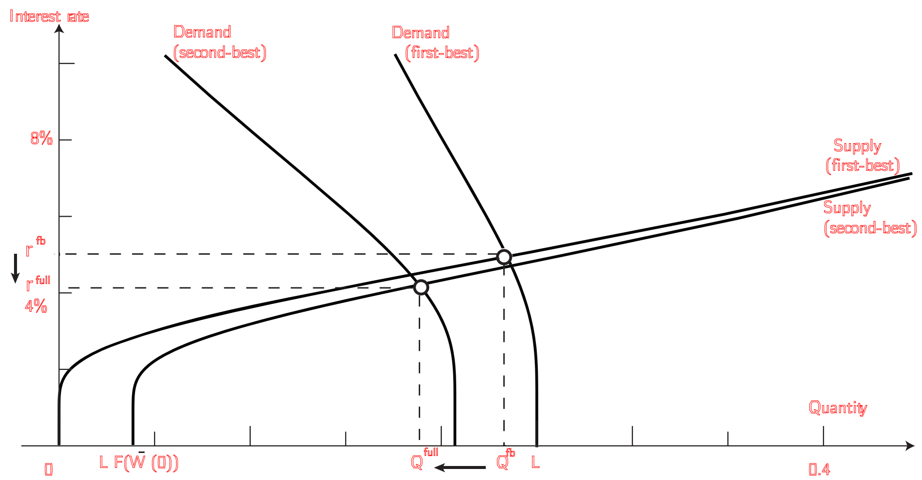


Figure 2: Second-best supply and demand

Accounting standard-setters have traditionally insisted in providing as much information as possible to investors provided its collection and dissemination is cost-efficient (see, e.g., *Conceptual Framework for Financial Reporting*, FASB 2006). Since we assume that there are no proprietary costs, the setting offers a natural benchmark to evaluate whether the FASB's emphasis on full-disclosure should necessarily maximize investment efficiency. In fact, we argue next that a fully-revealing cannot be optimal.

Corollary 1 *For any expected return $r < r_{\max}$, a fully-revealing measurement is not $r-$optimal. In particular, a competitive equilibrium with $r^* < r_{\max}$ does not feature full revelation of the fair value.*

A fully-revealing measurement forces liquidations for firms whose productive assets have relatively low fair value $z < W^-(r)$. The problem with such liquidations is that these firms are precisely those that should be not be liquidated in first-best because a relatively low fair value means that the asset should be better used internally. On the other hand, firms whose asset fair value are in $z \in (W^-(r), W^+(r))$ have excess collateral. Corollary 2 formally demonstrates that it is possible to set a measurement in which some asset fair values in both regions are not measured, creating an informational channel through which the perceived collateral of some firms can be raised to the level sufficient to continue, increasing overall investment efficiency.

3.3 r-Efficient measurement

We solve separately the case in which the first-best investment can be attained using an appropriate measurement and denote such an outcome as $r-$efficient (of course, an $r-$efficient policy is $r-$optimal). To achieve an $r-$efficient policy, the measurement should (a) prescribe non-disclosure for firms with $z < W^-(r)$ and enough expected collateral for non-disclosers to invest, i.e., $P(0) \geq W^-(r)$ and (b) prescribe disclosure and liquidation for all firms with $z > W^+(r)$.

Lemma 2 *There exists r_0 defined uniquely by:*

$$\mathbb{E}(\tilde{z}|\tilde{z} \leq W^+(r_0)) = W^-(r_0), \tag{15}$$

such that, if $r \leq r_0$, the $r-$efficient investment policy is implemented. Then, the optimal measurement will be any function $d(.)$ such that $d(z) = 1_{z>W^+(r)}(1 - 1_{z<W^-(r)})x(z)$ where $x(z) \in \{0, 1\}$ and $\mathbb{E}(\tilde{z}|d(\tilde{z}) = 0) \geq W^-(r)$.

To paraphrase Lemma 2, when the interest rate is sufficiently low, pooling the collateral of all firms with $\tilde{z} < W^+(r)$ yields enough perceived collateral to cross-subsidize all firms whose asset fair value would be insufficient to continue if revealed. As is common when a first-best allocation can be attained, the optimal measurement is not unique when $r < r_0$ but, unlike in first-best, the measurement *must* feature disclosures for all sufficiently high collateral values and non-disclosure for all sufficiently low collateral values.

We conclude by noting that the demand and supply curves are identical to the first-best economy when $r \leq r_0$, implying the same equilibrium interest rate.

Proposition 3 *If $r^{fb} \in [0, r_0]$ and $\mathbb{E}(\tilde{z}) > p\frac{c}{\Delta p} - p + L$, there exists a competitive equilibrium in which $r^* = r^{fb}$ and a firm continues if and only if $z > W^+(r^{fb})$. Further, this is the unique possible competitive equilibrium satisfying $r^* \leq r_0$.*

4 The optimal asset measurement

4.1 Collateral holding

We are now equipped to solve for the $r-$optimal measurement when the first-best investment policy cannot be attained, i.e., with $r > r_0$.

Let us make a few additional about the nature of the solution. In an $r-$optimal measurement, no-measurement firms should be induced to continue because, if this were not the case, a policy that fully reveals all collateral values would strictly increase the efficiency of liquidations.[19] In fact, extending this logic slightly further, a firm should continue if it does not explicitly reveal the fair value of its asset, as we formally state next.[20]

Lemma 3 *Let $r \in (r_0, r_{\max})$. Then, an $r-$optimal investment and measurements satisfy $\gamma(d(z)z) = 1$ if and only if $d(z) = 0$. In particular, $P(0) \in [W^{-}(r), W^{+}(r)]$.*

We do not yet know the $r-$optimal level of collateral $P(0)$ and, to answer this question, we borrow an intuition from Kamenica and Gentzkow (2011). If an $r-$efficient investment cannot be implemented, there are some firms that do not make their value-maximizing investment choice, either because they disclose and liquidate $z < W^{-}(r)$ or because they do not disclose and continue with $z > W^{+}(r)$. If non-disclosing firms were to hold excess collateral, i.e., $P(0) > W^{-}(r)$, investment efficiency could be increased by reclassifying some of the firms in the first group as non-disclosers or some of the firms in the second group as disclosers. It then follows that non-disclosers must be using the minimum required collateral.[21]

Lemma 4 *At a given interest rate r, the $r-$optimal measurement satisfies $P(0) = W^{-}(r)$.*

From Lemma 4, we may next derive the $r-$optimal security $(s_h(0), s_l(0))$. As the firm is financing with the minimal incentive-compatible collateral, the security should prescribe that the end-of-period value of the collateral be fully transferred to the liquidity providers, if the outcome is low. In other words, the firm is financed via a debt security with the form $s_x(0) = \min(N, x + P(0))$, where the face value is denoted N.[22] We solve for the face value N in the next lemma.

[19]If a measurement rule is such that non-discloser liquidate, then this rule clearly implies weakly less surplus than a full-disclosure rule. In fact, we shall establish later the stronger result that such a rule would be *strictly* dominated by another rule in which a non-zero mass of non-disclosers optimally continue. This argument is not proper to our model: in Darrough and Stoughton (1990) and Wagenhofer (1990), for example, a monopolist implements a disclosure rule in such a manner that the potential entrant would not be induced to enter conditional on no-disclosure.

[20]Although Lemma 3 implies that only no-measurement firms continue, the literal interpretation of $d(z) = 0$ as no-measurement is slightly imprecise. In fact, a firm that raises capital publicly reports a signal that $\tilde{z} \notin \{z : d(z) \neq 1\}$ which *does* convey information about the underlying collateral.

[21]This argument closely parallels the more general result in Proposition 5, p.19, in Kamenica and Gentzkow (2011). In our model, the optimal signal must make the receiver (the capital providers) indifferent between financing and not financing the firm.

[22]The interpretation of the security as debt is subject to one caveat, given that it is only true with binary outcomes. With more than two outcomes, debt is an optimal security provided an additional monotonicity condition is imposed. Note also that we have assumed that the collateral is sold at the end of period. If we were to assume, on the other hand, that the collateral publicly realizes its true value $\tilde{z}$, the optimal security would *not* be debt in that it would require the entire collateral to be transferred to the capital providers conditional on the low outcome.

Lemma 5 *Let $r \in (r_0, r_{\max})$. Non-disclosing firms raise capital by issuing an $r-$optimal risky debt security $s_x(0) = \min(N, x + P(0))$ where:*

$$N = 1 - p + L(1 + r) - \frac{1 - p}{\Delta p} c \tag{16}$$

We briefly state some comparative statics of the face value of an $r-$optimal security. A greater liquidity shock or interest rate increases the face value, as higher end-of-period cash flows must be paid to the liquidity providers. We also find that a more severe agency problem, as captured by the ratio $c/\Delta p$, tends to feature debt with lower face value. The reason for this is tied to the fact that agency problems raise the amount of collateral $W^-(r)$ retained which, in effect, protects capital providers conditional on low outcomes who then demand lower face value. The effect of the probability of success is ambiguous. On the one hand, it increases the manager's expectation about future cash flows, increasing incentives and implying that more firms are able to obtain liquidity with less collateral. In turn, this causes an increase in the face value. On the other hand, it reduces the required amount to be paid conditional on success because success is more likely. As is intuitive, the first effect will dominate when agency frictions are high.

4.2 Nature of the measurement

We have shown earlier that a fully-revealing is not a $r-$optimal. Before we turn to the analysis of interior measurements, let us consider, at the other extreme, a no-information measurement in which no asset values are measured, i.e., if $d(z) = 0$ for all z. As we show next, this type of measurement can only be $r-$optimal for a knife-edge case that we have already ruled out as a competitive equilibrium.[23]

Lemma 6 *Let $r < r_{\max}$. No-disclosure is $r-$optimal if and only if $r = r_{nd}$ where:*

$$r_{nd} = \frac{p - \frac{pc}{\Delta p} + \mathbb{E}(\tilde{z}) - L}{L} \tag{17}$$

Further, in a competitive equilibrium, $r^* \neq r_{nd}$ and $d^*(.)$ does not coincide with no-disclosure.

[23] The proof of Lemma 6 follows immediately from Lemma 4. We have argued that, in an $r-$optimal measurement, all firms that do not disclose choose to continue and have the minimum available collateral. Therefore, a necessary condition for no-disclosure to be optimal is that the unconditional collateral in the economy equals this minimum, a situation that can only occur at one interest rate denoted r_{nd}. By assumption, such a measurement cannot occur in a competitive equilibrium because a situation in which all firms demand capital does not clear the market.

Lemmas 2 and 6 together suggest that the $r-$optimal should typically involve a non-constant function $d(.)$; we are now interested in characterizing this function in more details and we first establish the following property.

Lemma 7 *Let $r \in (r_0, r_{\max})$. An $r-$optimal measurement is such that $d(z) = 0$ if $z \in (W^-(r), W^+(r))$.*

An $r-$optimal measurement should not measure at least all intermediate fair values. The underlying rationale for such an imperfect measurement is that, as long as continuation follows no measurement, assets whose fair values are in this intermediate region are efficiently used. Further, not measuring assets contributes to raise the average no-measurement collateral above $W^-(r)$, loosening the collateral constraint.

Condition (iv) can then be simplified as follows, after substituting in the optimal security:

$$(P_r) \quad argmax_{d,\alpha} \qquad \mathbb{E}(\alpha(\tilde{z})(1+r)\tilde{z} + (1-\alpha(\tilde{z}))(p-(1+r)L+\tilde{z}-c))$$

$$\text{subject to: } d(z), \alpha(z) \in \{0,1\},$$

$$\alpha(z) = d(z) = 0 \text{ for } z \in (W^-(r), W^+(r))$$

$$\text{and, otherwise, } \alpha(z) = 1_{z>W^+(r)} d(z),$$

$$\mathbb{E}(\tilde{z}|d(\tilde{z}) = 0) = W^-(r). \tag{18}$$

We define next two special types of measurements which will play a special role as a solution of problem (P_r).

Definition 2 *For any disclosure threshold $M \in \mathbb{R}^+$,*

(i) A write-up measurement is a rule such that $d(.)$ is increasing.

(ii) An impairment measurement is a rule such that $d(.)$ is decreasing.

The next Proposition reveals that the optimal measurement is either a write-up or an impairment, but never features a combination of both types of measurements.

Proposition 4 *Let $r \in (r_0, r_{\max})$. The $r-$optimal measurement is unique and is either a write-up or an impairment rule.*

(i) If $r < r_{nd}$, the $r-$optimal measurement is a write-up rule such that $d(z) = 1$ if and only if $z > M^+(r)$ defined by $\mathbb{E}(\tilde{z}|\tilde{z} \leq M^+(r)) = W^-(r)$.

(ii) If $r > r_{nd}$, the $r-$optimal measurement is an impairment rule such that $d(z) = 1$ if and only if $z \leq M^-(r)$ defined by $\mathbb{E}(\tilde{z}|\tilde{z} \geq M^-(r)) = W^-(r)$.

None of these measurements is $r-$efficient.

Proposition 4 also characterizes when each of these two types of measurement is $r-$optimal and we discuss next the economic trade-offs between the two measurements. A write-up measurement only features one type of investment inefficiency in that firms with $z \in (M^+(r), W^+(r))$ continue even though they should be liquidated in an $r-$efficient measurement. The rationale for this inefficiency is that such firms are used to raise the perceived collateral of (continuing) non-disclosing firms.[24] Despite this inefficiency, write-ups are always preferred to impairments because impairments feature the same inefficient continuations as write-ups in addition to additional liquidations for collateral-poor firms with $z < M^-(r)$. We put this comparison in graphical terms below.

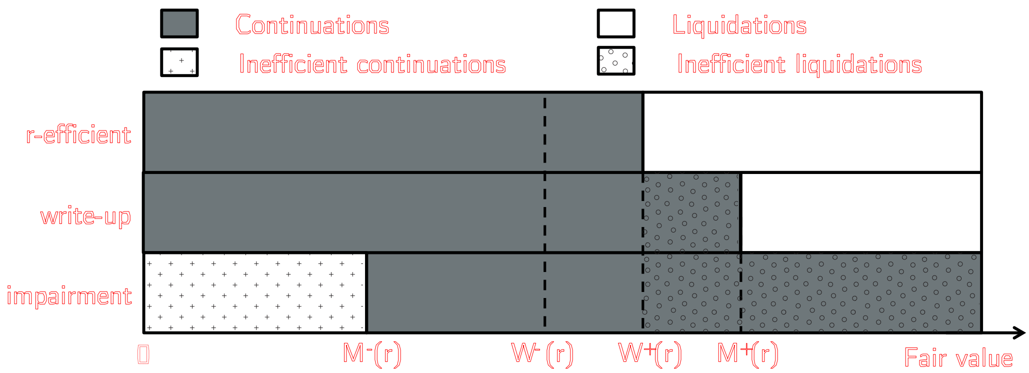


Figure 3: Investment policy for different measurements

Should write-ups always be chosen over impairments? Yes, but only if there is a write-up that is feasible in problem (P); by contrast some impairment rule is always feasible when $r < r_{\max}$. The inherent limitation of write-ups is they quickly deplete expected

[24]Naturally, there is a different centralized solution to this problem: a benevolent regulator could tax the proceeds from liquidating firms to subsidize the collateral of collateral-poor firms. Whether such transfers are politically-feasible is, however, unsure. Such transfers would have to be implemented on an ex-post basis, after firms have learnt their information and some actors may lose from these transfers. By contrast, the information system might have to be chosen ex-ante, in a state of "ignorance" and, once the measurement has been applied, it would be difficult to renegotiate the measurement to prevent disclosure of certain events. In other words, we believe that the assumption of a political commitment to a measurement is practically (much) more realistic than that of a political commitment to a set of redistributive transfers.

collateral conditional on non-disclosure. If the interest rate is too large, then, there may not be any feasible write-up such that no-measurement firms have sufficient collateral. This occurs when the unconditional value of all collateral is no longer sufficient to raise capital, i.e., when $r > r_{nd}$.

In summary, Proposition 4 contains the new insight that emerges from an analysis at a fixed interest rate and is a point at which our analysis diverges, in part, from the findings of Goex and Wagenhofer (2009). We formally show that, when measuring *productive* assets, impairments are not always optimal. The analysis suggests that impairments should be used in high-interest environments while, in low-interest environments, firms should use write-ups and avoid any reporting of low collateral.

5 The capital market

5.1 Competitive equilibrium with efficient measurements

We analyze next the competitive equilibrium of the model to derive the interest rate r^*. Suppose first that $r \leq r_0$ so that the measurement is $r-$efficient.

Proposition 5 *A "Type Ia" competitive equilibrium is defined as an equilibrium with an interest rate $r^*_{Ia} \leq r_0$. A Type Ia equilibrium exists if and only if*

$$K \geq K_{Ia} = F(W^+(r_0))(L + W^-(r_0)) - \mathbb{E}(\tilde{z}) \tag{19}$$

When it exists, a type Ia equilibrium has the following characteristics:

(i) The interest rate is the same as in first-best, i.e., $r^*_{Ia} = r^{fb}$.

(ii) Operating decisions coincide with the first-best capital allocation, i.e., a firm operates if $z < W^+(r^{fb})$ and liquidates if $z > W^+(r^{fb})$.

(iii) The optimal measurement must involve measurements for $z > W^+(r^{fb})$ and no measurement for $z < W^-(r^{fb})$.

A Type Ia is a competitive equilibrium in which interest rate and operating decisions coincide with the first-best equilibrium. This type of equilibrium can occur when the capital stock in the economy is sufficiently large, even though this capital stock is not owned by the firms and must be paid back at the end of the period. When there is a large supply of capital, fewer firms liquidate their productive assets, i.e., $W^-(r^*_{Ia})$ is high, so that the average collateral of continuing firms $\mathbb{E}(\tilde{z}|\tilde{z} \leq W^-(r^*_{Ia}))$ increases making it easier to resolve agency problems.

We turn next to the analysis of the optimal measurement when the interest rate is such that $r \in (r_0, r_{nd})$. Proposition 4 implies that the optimal measurement is a write-up and, therefore, the excess demand is given by:

$$Z(r) = \underbrace{LF(M^+(r))}_{\text{Continuing firms}} - \underbrace{(1 - F(M^+(r)))\mathbb{E}(\tilde{z}|\tilde{z} \geq M^+(r))}_{\text{Liquidating firms}} - K. \tag{20}$$

This write-up measurement is *not* $r-$efficient because some firms with $z > M^+(r)$ continue because their asset values are not measured. Does this imply that the investment decisions at a competitive equilibrium $r^* \in (r_0, r_{nd})$ is inefficient? To answer this question, note that $Z(r^*) = 0$ implies that $KF(M^+(r^*)) = KF(W^+(r^{fb}))$ and, therefore, the investment threshold $M^+(r^*)$ coincides to the first-best investment threshold $W^+(r^{fb})$. That is, somewhat surprisingly, the competitive equilibrium write-up is not r^*-efficient but is efficient in terms of investment decisions.

Proposition 6 *Let a "Type Ib" equilibrium be defined as an equilibrium with interest rate $r^*_{Ib} \in (r_0, r_{nd}]$. There exists a Type Ib equilibrium if and only if a Type Ia equilibrium exists. When it exists, the equilibrium is unique and has the following characteristics:*

(i) The interest rate r^*_{Ib} is given by $M^+(r^*_{Ib}) = W^+(r^{fb})$ and is strictly greater than in first-best, i.e., $r^*_{Ib} > r^{fb}$.

(ii) Real investment decisions coincide with the first-best and a Type Ia equilibrium.

(iii) The optimal measurement is unique and involves write-ups for any $z > M^+(r^*_{Ib})$ and no measurement otherwise.

Proposition 6 strengthens the preliminary insight given in Proposition 4. Not only are write-ups preferred to impairments when feasible, they always achieve the same expected surplus as in a model with no agency friction. Naturally, this is not a contradiction but rather hints at modelling problems when analyzing welfare assuming an exogenous interest rate even though capital market policies (such as accounting standards) would plausibly affect investors' required rate of return. That is, from a competitive equilibrium standpoint, the interest rate is elastic and, if a standard changes liquidation decisions *in the aggregate*, their owners would not receive the same interest rate. Unlike in a Type Ia equilibrium, the measurement is unique and can be described by a threshold $M^+(r^*_{Ib})$

beyond which asset values should be revealed. Note also that the Type Ib equilibrium comes in pair with a Type Ia and occurs in economies with a large enough capital stock.

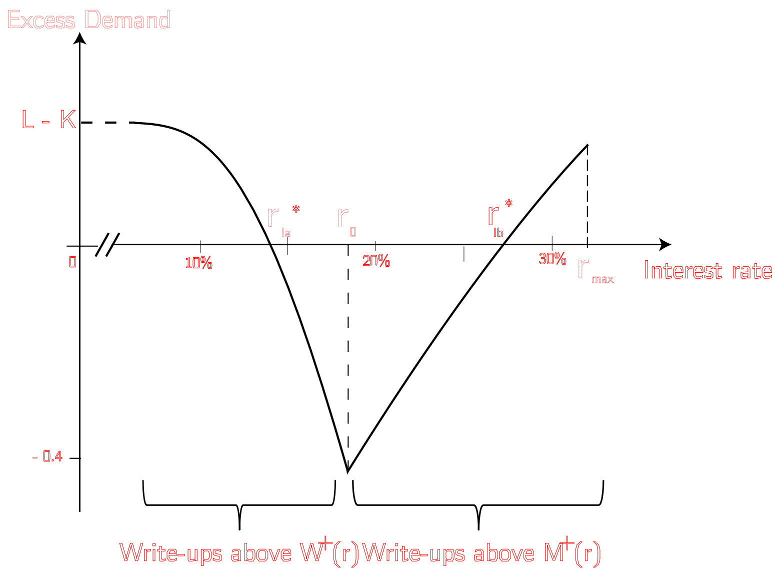


Figure 4: Excess Demand with Write-up Measurements

Another key property of a Type Ib equilibrium is made apparent by plotting the excess demand (see Figure 4). Note that we cannot use here traditional supply and demand curves because the excess demand is non-monotonic in the interest rate. An increase in the interest rate requires an increase in the expected collateral of no-measurement firms. Under write-ups, this is achieved by expanding the no-measurement region and inducing more firms to continue and demand capital; hence, excess demand *increases* in response to a higher interest rate. Under Walrasian tâtonnement, the equilibrium is unstable because a Walrasian auctioneer would respond to a small increase (decrease) away from r^*_{Ib} by increasing (decreasing) the interest rate. Note that Type Ia and Type Ib equilibrium are not equivalent ex-post, and a Type Ib, with its higher interest rate, benefits suppliers of capital at the expense of users of capital. Firms that liquidate are always better-off in a Type Ib equilibrium because they earn a greater return on their investments and, if $K > 0$, the real sector is on average worse-off in a Type Ia equilibrium.

Corollary 2 *Under a Type Ia or a Type Ib equilibria, the following comparative statics hold:*

(i) an increase in p (the expected operating cash flows) increases the interest rate and does not affect the liquidation threshold.

(ii) an increase in c (the effort cost) decreases the interest rate and does not affect the liquidation threshold

(iii) an increase in Δp (the information about effort) does not affect the Type Ia interest rate or the liquidation threshold, and increases the Type Ib interest rate.

(iv) an increase in L (the liquidity shock) increases the Type Ia interest rate if K is large and decreases it if K is small, and decreases the Type Ib interest rate and the liquidation threshold.

Because the investment policy under a write-up measurement coincides with the first-best investments, the comparative statics on the liquidation threshold are intuitive and coincide with the model with no frictions. Of special interest, informational asymmetries are manifested in the interest rate of a Type Ib equilibrium, but not in the direction that is intuitively expected. We show that a decrease in the information about managerial actions results in a lower interest rate. To explain this effect, note that a greater informational asymmetry implies that fewer write-ups are used for any interest rate, thus implying that there are more firms demanding capital. To clear the market, the equilibrium interest rate adjusts to lower excess demand which, in a Type Ib equilibrium, takes the form of a lower interest rate and more write-ups.

5.2 Equilibrium with asset impairments

Consider next the region of interest rates $r \in (r_{nd}, r_{\max})$. In this region, only impairments can be used because write-ups would not be sufficient to increase the perceived collateral to the minimum threshold to raise capital. The excess demand is now given by:

$$Z(r) = \underbrace{L(1 - F(M^{-}(r)))}_{\text{Continuing firms}} - \underbrace{F(M^{-}(r))\mathbb{E}(\tilde{z}|\tilde{z} \leq M^{-}(r))}_{\text{Liquidating firms}} - K. \tag{21}$$

Proposition 7 *Let a "Type II" equilibrium be defined as an equilibrium with interest rate $r^*_{II} \in (r_{nd}, r_{\max})$. There exists a Type II equilibrium if and only if::*

$$K \geq K_{II} \equiv (1 - F(M^{-}(r_{\max})))\mathbb{E}(L + \tilde{z}|\tilde{z} \geq M^{-}(r_{\max})) - \mathbb{E}(\tilde{z}) \tag{22}$$

When it exists, it is unique and has the following characteristics:

(i) The interest rate r^*_{II} is given by:

$$(1 - F(M^-(r^*_{II})))\mathbb{E}(L + \tilde{z}|\tilde{z} \geq M^-(r^*_{II})) = \mathbb{E}(\tilde{A}) \tag{23}$$

This interest rate is always greater than in a Type Ia or Ib equilibrium (when they exist) and is greater than in first-best if $r^{fb} > r_{max}$.

(ii) The optimal impairment rule involves inefficient liquidations for $z < M^-(r^*_{II})$ and inefficient continuations for $z > W^+(r^{fb})$.

(iii) The optimal measurement must involve impairments for any $z < M^-(r^*_{II})$ and no measurement for $z \geq M^-(r^*_{II})$.

In a Type II equilibrium, impairments are a measurement of last resort when market conditions cannot sustain write-up measurements. A Type II equilibrium can exist in an economy where both a Type Ia and a Type Ib exist, as illustrated in Figure 5a. This scenario can occur if $K_{Ia} > K_{II}$. A sufficient condition to satisfy this ordering is when $F(W^+(r_0)) > 1 - F(M^-(r_{max}))$, i.e., when the overall investment with first-best write-ups at r_0 is greater than with impairments at r_{max}. Because $r_0 < r_{max}$, a higher interest rate increases liquidations with impairments and a lower level of exogenous capital will maintain a lower investment in second-best.

Type I equilibria achieve a greater expected surplus than a Type II equilibrium. Therefore, if regulators were to pre-commit to a measurement being mindful of the effect of the measurement on interest rates, they should implement write-ups whenever possible. If, on the other hand, regulators do not consider the general equilibrium effects of the measurement, they may hold the economy on a competitive equilibrium with impairments and relatively high interest rates. This is problematic because impairments, in the model, cause investment inefficiencies, as discussed below.

Corollary 3 *If $K > K_{II}$, then: $W^+(r^{fb}) > M^-(r^*_{II})$. That is, the optimal measurement in a Type II equilibrium is such that all impairments lead to inefficient liquidations.*

In our model, some firms should liquidate, if their collateral has a large enough resale value and should be converted into all-purpose capital; in comparison to Holmström and Tirole (1997) and Goex and Wagenhofer (2009), some liquidations are efficient. However, we find that all liquidations caused by an impairment are always inefficient relative to the first-best asset allocation. The reason for this is somewhat subtle. Relative to first-best, an impairment equilibrium tends to shrink the quantity of available all-purpose capital because firms with low collateral are liquidated. Firms that should be liquidated in first-best have high collateral, even in an economy where capital is cheaply valued, therefore their collateral would be even more valuable in an impairment equilibrium. Therefore, these are the firms that should be continued under impairments because they increase the quantity of perceived collateral when no measurement is made.

Corollary 4 *In a Type II equilibrium, the following comparative statics holds:*

(i) an increase in p (the expected operating cash flow) increases the interest rate and does not affect the continuation threshold.

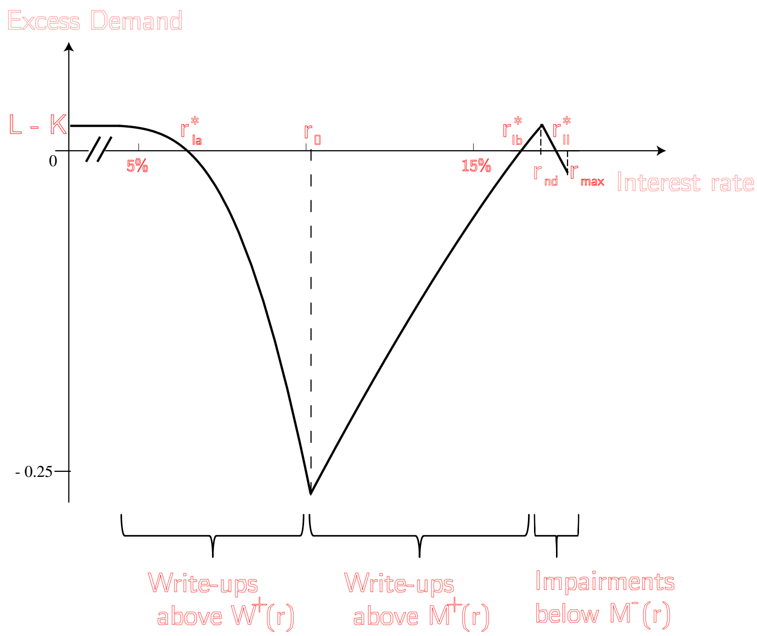
(ii) an increase in c (the effort cost) decreases the interest rate and does not affect the continuation threshold.

(iii) an increase in Δp (the information about effort) increases the interest rate and does not affect the continuation threshold.

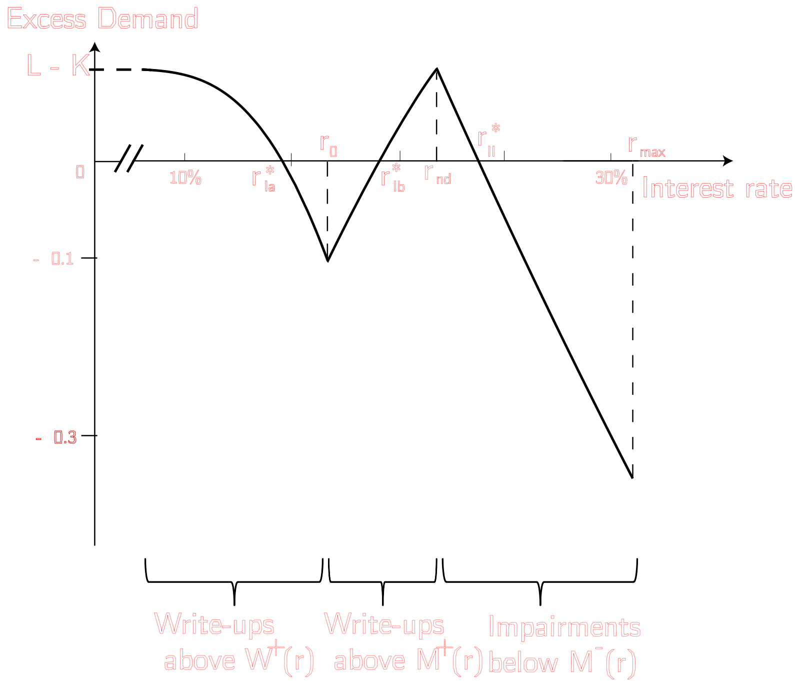
(iv) an increase in L (the liquidity shock) increases the interest rate and decreases the continuation threshold.

The interest rate r^*_{II} in a Type II equilibrium has intuitive comparative statics. When firms are more profitable or effort is less costly, the interest rate increases to reflect the more favorable investment opportunity set. These comparative statics coincide with the first-best. As in a Type Ib equilibrium, we also find that an increase in the liquidity shock increases interest rate. By market clearing, a higher need for liquidity must induce more liquidations which can only be achieved if firms must pay a higher interest rate.

The comparative static with respect to Δp mimics the observation previously made in a Type Ib equilibrium. More information about managerial actions, given impairments, implies that firms use less collateral and tend to liquidate less often. There is more demand for capital at any rate of return and therefore, by market clearing, the equilibrium interest rate must increase. We also show that the investment threshold is not a function of various parameters of the production technology within a Type II equilibrium. Because the capital market must clear, any change in the value of projects, for example, manifests itself as an adjustment to the interest rate, not in an increase in investment; this prediction is in sharp contrast with models that analyze the economic consequences of measurement changes taking the interest rate as a given.



(a) $K > K_{II} > K_{Ia}$



(b) $K > K_{Ia} > K_{II}$

Figure 5: Excess Demand with Write-ups and Impairments

5.3 Financial shocks and the Accelerator

We discuss next the response of interest rates and measurements by considering two broad types of financial contractions, which we formally define next. First, the economy features a contraction in the capital stock, or in short *capital crunch*, if the overall available capital K is reduced. To be concrete, a capital crunch could occur if foreign investors experience a domestic shock and pull out some of their assets from the country. Alternatively, the economy could experience a negative shock to the productivity of all-purpose capital. Second, the economy features a contraction in collateral, or in short *collateral squeeze*, if the external resale value of firm's productive assets is reduced. As in Holmström and Tirole (1997), we model this as a proportional decrease by writing $\tilde{z} = (1-\epsilon)\tilde{y}$ and increasing $\epsilon > 0$ while holding $\tilde{y}$ fixed. In our model, such a situation would occur, for example, when assets become more specialized or firm-specific.[25] In this section, we simplify the exposition slightly by assuming that $r_{\max} \geq r_{nd}$.[26] Figure 6 presents a simple graphical intuition for the effects of a collateral squeeze or a capital crunch on each type of equilibrium (when they exist).

The financial accelerator captures the additional effect of a market friction on macroeconomic aggregates but, depending on which aggregate is considered, there could be more than one definition of the accelerator. A natural definition is the total investment inefficiency, defined as the loss of social surplus relative to the first-best asset allocation. Because only impairments are inefficient in our model, it can be easily shown that a financial acceleration always occurs in environments with impairments. However, one practical limitation of this definition is that macroeconomic regulators tend to focus on proxies based on total investment (or growth) to guide policy.

For this reason, we examine here a different concept. Specifically, we define the accelerator in terms of the effect of the financial shock on the difference between total investment in the competitive equilibrium with frictions relative to the total investment

[25]If a shock affects an entire industry, for example, the asset when sold may no longer be used in a manner related to how it was by the firm, and instead may need to be fully reconverted. Consider the following example. If the asset is an inventory of gold jewelry, its collateral value might be the value of the jewelry when sold by a different seller. If, however, an economic shock affects the entire industry, the gold might have to be melted to be used in the semi-conductor industry, thus losing value as collateral.

[26]The conditions are easily adapted to the case in which $r_{\max} < r_{nd}$ although, when this is the case, an additional condition is required to guarantee the existence of a Type Ib equilibrium (it is available from the authors on request).

with first-best capital allocations. That is, the accelerator α is defined as:[27]

$$\alpha(X) = \frac{\partial \mathbb{E}(\gamma^{fb}(d^{fb}(\tilde{z}))) - \mathbb{E}(\gamma^{*}(d^{*}(\tilde{z})))}{\delta X} \tag{24}$$

Note that, while we know that there is no acceleration in write-up (Type Ia and Ib) equilibria, impairments do feature more investment for certain kinds of firms with high collateral and thus we need to formally derive whether impairments feature acceleration or deceleration.

Corollary 5 *Conditional on a capital crunch,*

(i) the Type Ia and Type II interest rates increase, while the Type Ib interest rate decreases,

(ii) the probability of liquidation increases in all equilibria and the informativeness of the measurement increase (in the Blackwell sense) in a Type Ib and Type II equilibrium.

A capital crunch is plotted in Figure 6a and shifts of the excess demand curve. In all types of equilibrium, a capital crunch must then lead to fewer firms operating in order for the capital market to clear. In a Type Ia equilibrium, an increase in the interest rate implies more write-ups and more liquidations of high-collateral firms. In a Type II equilibrium, an increase in the interest rate implies more impairments and more liquidations of low-collateral firms. In both cases, a more informative measurement follows the capital crunch. A type Ib equilibrium functions differently. In such an equilibrium, a greater interest rate induces fewer firms to write up their asset in order to maintain a sufficiently high expected collateral for no-measurement firms. Then, in a Type Ib equilibrium, a lower interest rate implies lower demand and the equilibrium interest rate decreases.

Corollary 6 *In a Type II equilibrium, there is a financial accelerator conditional on a capital crunch, i.e.,* $\alpha'(K) < 0$*.*

A shock to the total supply of capital affects the economy with frictions more strongly than it affects the first-best allocations. To see why, recall that a decrease in K would lead to more liquidations in first-best, then causing more high-collateral firms to make their asset available for other uses. The same occurs with impairments but causing liquidations

[27] Since L is kept constant, we could measure the accelerator in terms in dollars of total investment with no changes to the results.

of low-collateral firms in which case less capital is available. More such liquidations are therefore necessary to compensate for the decrease in the stock of capital, implying the existence of a financial acceleration.

Corollary 7 *Conditional on a collateral squeeze,*

(i) the Type Ia interest rate increases, while the Type Ib and Type II interest rates decrease,

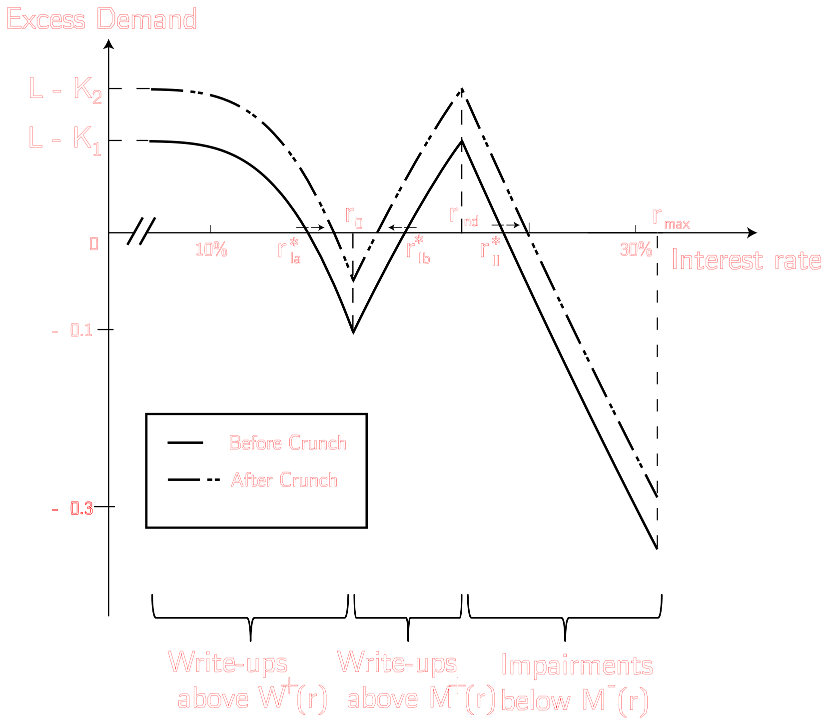
(ii) the probability of liquidation increases in all equilibria and the informativeness of the measurement increase (in the Blackwell sense) in a Type Ib and Type II equilibrium.

A collateral squeeze is plotted in Figure 6b. The effects of a collateral squeeze are similar to those of a capital crunch along several dimensions. In all equilibria, a collateral squeeze causes more liquidations and is such that more asset values are measured.

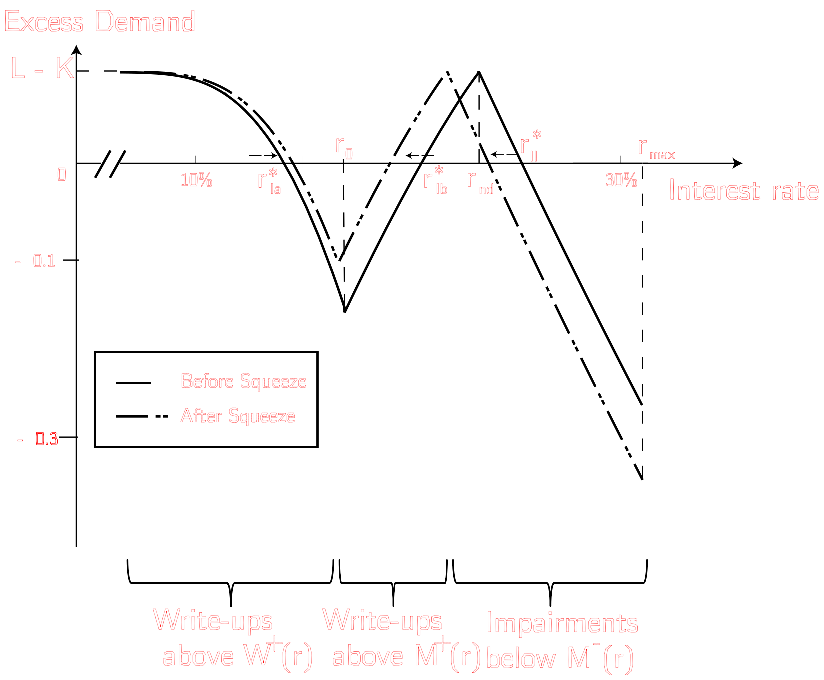
The effect on the interest rate can be different from a capital crunch. In a Type I equilibrium, a collateral squeeze increases demand and contracts supply. The interest rate must then adjust for market clearing to occur. In a Type Ia equilibrium, an increase in the interest rate makes liquidation more attractive and reduces demand. By contrast, in a Type Ib equilibrium, a lower interest rate eases the financing conditions and fewer firms write-up, which diminishes the demand and provides more available capital. The decrease in the interest rate outweighs the effect of a collateral squeeze, and more firms write up their assets. In a Type II equilibrium, both the demand and the supply shrink and the contraction in demand is more severe than the contraction in supply. A lower interest rate makes credit conditions looser and increases the demand to meet the supply, but at a lower level than before the collateral squeeze.

Corollary 8 *In a Type II equilibrium, if* $\mathbb{E}(\gamma^{fb}(d^{fb}(\tilde{z}))) > \mathbb{E}(\gamma^*(d^*(\tilde{z})))$*, there is a financial accelerator conditional on a collateral squeeze, i.e.,* $\alpha'(\epsilon) < 0$*.*

In comparison to a capital crunch, the existence of a financial acceleration with a collateral squeeze requires an additional condition on the first-best investment level relative to the economy with impairments. To see why the effect can be ambiguous, note that if the economy can feature high total investment levels in an impairment equilibrium because high-collateral firms are continued to increase perceived collateral. When there are many such firms being continued, the presence of these continued high-collateral can moderate the effect of a collateral squeeze on investment. If, on the other hand, the total level of investment is relatively small with impairments, the total number must sharply increase when the collateral squeeze occurs, thus contributing to a financial acceleration. Put differently, the financial accelerator is present in economies whose investment levels have been already reduced relative to the efficient level (i.e., high output gap) and, under this interpretation, the acceleration should occur during more severe crises.



(a) Capital crunch



(b) Collateral squeeze

Figure 6: Financial Shocks

6 Conclusion

Asset measurement is the backbone of a sound financial system. The solidity of this backbone clearly influences the liquidity available to the real economy during tight credit market conditions. In this study, we challenge the conventional view that static measurement rules should be set in isolation of economic cycles, as is currently favored by accounting standard-setting bodies. We illustrate this question in a simple economic model involving the measurement of the collateral value of a firm's productive assets. Excessive measurements may trigger inefficient liquidations of productive assets whose collateral value is low while insufficient measurements may dampen the market's confidence in the collateral value of assets whose value has not been assessed. The trade-off between these fundamental trade-offs depend on credit market conditions and generally involve *flexible* measurements with varying degrees of information being released or a changing focus on measurements of high collateral values or low collateral values.

The research on disclosure and the business cycle is still in its infancy and we only intend here to offer a discussion of collateral measurement, with a several implications for the causes of the financial accelerator. That said, more work is needed to fully assess how measurement should respond to changing economic conditions, and in particular whether specific policies should be undertaken in a time of crisis. There are many open questions that are key to better understand how the measurement channels macroeconomic shocks to the real economy. What information does the measurement publicly convey about the extent of an economic crisis and should this information be managed? Should measurements be loosened when they cause debt covenants violations as a result of a systematic shock? To what extent should measurement policies be managed jointly with other smoothing tools available to regulators, such as monetary or budgetary policy? We hope that further research in this area can place the various facets of asset measurement within the broader literature on the business cycle.

Appendix A

Proof of Lemma 1: The firm must ensure that the value-enhancing action is chosen, i.e.,

$$\Delta p(1 - s_h(I) + s_l(I)) \geq c, \tag{1}$$

The investor's participation must be satisfied at equality. Rewriting it in terms of $s_h(I)$,

$$s_h(I) = \frac{1+r}{p}L - \frac{1-p}{p}s_l(I) \tag{2}$$

Substituting this equation in inequality (1),

$$s_l(I) \geq p\frac{c}{\Delta p} - p + (1+r)L \equiv W^-(r) \tag{3}$$

Thus, a firm can raise capital if and only if $P(I) \geq W^-(r)$.
A firm with collateral $P(I)$ prefers to continue the project if:

$$p - (1+r)L + P(I) - c \geq (1+r)P(I) \tag{4}$$

Rearranging, if $P(I) \leq \frac{p-(1+r)L-c}{r} \equiv W^+(r)$.
Proof of Proposition 1: For a given r, the efficient investment is such that firms with $\tilde{z} \geq W^+(r)$ should liquidate and otherwise continue the project. Let us define:

$$\Gamma_{fb}(r) = F(W^+(r))L - K - (1 - F(W^+(r))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r)). \tag{5}$$

$\Gamma_{fb}(r)$ is decreasing in r because:

$$\frac{\partial \Gamma_{fb}(r)}{\partial r} = \frac{\partial W^+(r)}{\partial r} f(W^+(r))L + \frac{\partial W^+(r)}{\partial r} f(W^+(r))W^+(r) < 0.$$

Further, at $r = 0$, $\Gamma_{fb}(0) = L - K > 0$ and $\lim_{r\to+\infty} \Gamma_{fb}(r) = -K - \mathbb{E}(\tilde{z}) < 0$. It follows that the first-best interest rate $r^{fb} > 0$ is given uniquely by:

$$F(W^+(r^{fb}))L = K + (1 - F(W^+(r^{fb}))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r^{fb})). \tag{6}$$

If there is no agency problem, any measurement that implements the efficient investment is optimal. Full disclosure achieves the efficient investment. More generally, this can be achieved by setting any rule such that $\{z : d^{fb}(z) = 0\}$ is a subset of either $\{z : z \geq W^+(r^{fb})\}$ or $\{z : z \leq W^+(r^{fb})\}$.
Proof of Proposition 2: If any firm's collateral is observed, firms below $W^-(r)$ cannot raise capital and have to liquidate and firms above $W^+(r)$ prefer to liquidate over continuing the project. Thus, firms with $z \in (W^-(r), W^+(r)))$ continue the project. Let us define:

$$\begin{aligned} \Gamma_{full}(r) \;\; = \;\; & (F(W^+(r)) - F(W^-(r)))L - K - F(W^-(r))\mathbb{E}(\tilde{z}|\tilde{z} \leq W^-(r)) \\ & -(1 - F(W^+(r))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r)). \end{aligned} \tag{7}$$

$\Gamma_{full}(r)$ is decreasing in r because:

$$\begin{aligned}\frac{\partial \Gamma_{full}(r)}{\partial r} &= (\frac{\partial W^+(r)}{\partial r} f(W^+(r)) - \frac{\partial W^-(r)}{\partial r} f(W^-(r)))L \\ &\quad - \frac{\partial W^-(r)}{\partial r} f(W^-(r))W^-(r) + \frac{\partial W^+(r)}{\partial r} f(W^+(r))W^+(r) < 0 \end{aligned} \tag{8}$$

Further at $r = 0$, $(1 - F(W^-(0)))L - K - F(W^-(0))\mathbb{E}(\tilde{z}|\tilde{z} \leq W^-(0)) > 0$ by assumption and $\lim_{r\to+\infty} \Gamma_{full}(r) = -L - K - 2\mathbb{E}(\tilde{z}) < 0$. It follows that the fully-revealing interest rate r^{full} is given uniquely as follows:

$$\begin{aligned}&(F(W^+(r^{full})) - F(W^-(r^{full})))L = \\ &\quad K + F(W^-(r^{full}))\mathbb{E}(\tilde{z}|\tilde{z} \leq W^-(r^{full})) + (1 - F(W^+(r^{full}))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r^{full}))\end{aligned} \tag{9}$$

Proof of Corollary 1: For any expected return $r < r_{\max}$, a fully-revealing measurement creates under-investment for firms below $W^-(r)$. Any measurement rule that prescribes $d(z) = 0$ for $z \in [W^-(r), W^+(r)]$, $d(z) = 1$ for $z > W^+(r)$ and $d(z) = 0$ for some $z < W^-(r)$ provided that $\mathbb{E}(\tilde{z}|d(\tilde{z}) = 0) \geq W^-(r)$ is welfare-improving relatively to a fully-revealing measurement. In particular, a competitive equilibrium with $r^* < r_{\max}$ does not feature a fully-revealing measurement.

Proof of Lemma 2:

The measurement rule that can implement the efficient investment prescribes $d(z) = 1$ for $z > W^+(r)$ and $d(z) = 0$ for $z < W^-(r)$, and information or no information about $z \in [W^-(r), W^+(r)]$, provided that $\mathbb{E}(\tilde{z}|d(\tilde{z}) = 0) \geq W^-(r)$. In particular, the measurement rule prescribing information $\forall \tilde{z} \geq W^+(r)$ and no information $\forall \tilde{z} < W^+(r)$ implements the $r-$efficient investment and requires the least amount of information and is thus the only rule that remains optimal for all $r < r_0$ where r_0 is given by:

$$\mathbb{E}(\tilde{z}|\tilde{z} \leq W^+(r_0)) = W^-(r_0). \tag{10}$$

Define $\Phi_{sb}(r) = F(W^+(r))\mathbb{E}(\tilde{z}|\tilde{z} \leq W^+(r)) - F(W^+(r))W^-(r)$. $\Phi_{sb}(r)$ is decreasing in r because

$$\frac{\partial \Phi_{sb}(r)}{\partial r} = \frac{\partial W^+(r)}{\partial r} f(W^+(r))W^+(r) + \frac{\partial W^+(r)}{\partial r} f(W^+(r))W^-(r) < 0 \tag{11}$$

At $r = 0$, $W^+(r) \to +\infty$ and $\Phi_{sb}(0) = \mathbb{E}(\tilde{A}) - W^-(0)$.

At $r = r_{max}$, $W^- = W^+(\gamma_{max})$ and $\Phi_{sb}(\gamma_{max}) = \int_0^{W^+(\gamma_{max})} (A - W^+(\gamma_{max}))f(A)dA < 0$.

Thus, if $\mathbb{E}(\tilde{A}) > W^-(0)$, there exists a unique γ_0 such that $\Phi_{sb}(r_0) = 0$.

Proof of Proposition 3: From Lemma 2, the measurement rule prescribing information $\forall \tilde{z} \geq$

$W^+(r)$ and no information $\forall \tilde{z} < W^+(r)$ implements the $r-$efficient investment. Further, from Proposition 1, if the first-best interest rate $r^{fb} \in [0, r_0]$ then

$$F(W^+(r^{fb}))L = K + (1 - F(W^+(r^{fb}))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r^{fb})). \tag{12}$$

Thus, the unique possible competitive equilibrium satisfies $r^* = r^{fb} \leq r_0$.

Proof of Lemma 3: We make a reasoning by contradiction. We assume that when their collateral is not measured, firms liquidate. Under this assumption, the measurement system maximizes:

$$\begin{aligned}\max_{d(z)\in[0,1]} U(d(z)) = \quad & \int_0^{W^-(r)} (1+r)zd(z)f(z)dz + \int_{W^+(r)}^{+\infty} (1+r)zd(z)f(z)dz \\ & + \int_{W^-(r)}^{W^+(r)} (p - (1+r)L + z - c)d(z)f(z)dz \\ & + \int_0^{+\infty} (1+r)z(1-d(z))f(z)dz\end{aligned}$$

Taking the first order condition (FOC) yields:

$$\frac{\partial U(d(z))}{\partial d(z)} = \begin{cases} -f(z)((1+r)z + (p - (1+r)L + z - c)) > 0 \text{ if } z \in (W^-(r), W^+(r)) \\ 0 \text{ otherwise} \end{cases}$$

The solution is $d(z) = 1$ for $z \in (W^-(r), W^+(r))$. Otherwise, any $d(z)$ can be set. As a result, any collateral $z \in (W^-(r), W^+(r))$ is measured and those firms continue the project, while the others liquidate. This measurement rule displays the same investment allocation as the fully revealing measurement, which is never the optimal measurement. Therefore, when their collateral is not measured, firms continue the project. It is immediate that $P(0) \in [W^-(r), W^+(r)]$.

Proof of Lemma 4: We make a reasoning by contradiction. If $P(0) > W^-(r)$, the measurement rule is not optimal because we can improve the investment efficiency by measuring more firms' collateral $\tilde{z} < W^-(r)$. Thus, $P(0) = W^-(r)$.

Proof of Lemma 5: From Lemma 1, a firm can raise capital if and only if $P(I) \geq W^-(r)$. By Lemma 4, when firms do not measure their collateral, $P(0) = W^-(r)$ and $s_l(0) = W^-(r)$. It follows that:

$$s_h(0) = \frac{1+r}{p}L - \frac{1-p}{p}W^-(r) \tag{13}$$

Simplifying, $s_h(0) = N$.

Proof of Lemma 6: Let $r < r_{\max}$. No-information is $r-$optimal if and only if $\mathbb{E}(\tilde{z}) = W^-(r)$. Rearranging, no-information is $r-$ optimal if and only if $r = r_{nd}$ given by:

$$r_{nd} = \frac{p - \frac{pc}{\Delta p} + \mathbb{E}(\tilde{z}) - L}{L} \tag{14}$$

By Lemma 3, because there is no measurement about any collateral, all the firms continue the project. It follows, that at $r = r_{nd}$, the excess demand is $L - K > 0$. Thus, $r^* \neq r_{nd}$.

Proof of Lemma 7: Firms with collateral $z \in (W^-(r), W^+(r))$ continue the project whether their collateral is measured or not. This is the efficient investment for these firms.

By contraction, we assume that the collateral of some of these firms is measured. It follows that it exacerbates to the investment inefficiency. More collateral values with $z < W^-(r)$ need to be measured or more collateral values with $z > W^+(r)$ cannot be measured so that $P(0) = W^-(r)$.

Proof of Proposition 4: We assume that $r \in (r_0, r_{\max})$, i.e., $\mathbb{E}(\tilde{z}|z \leq W^+(r)) < W^-(r)$. Solving (P_r) is equivalent to maximizing the Lagrangian Λ defined by:

$$\begin{aligned}
\Lambda(d(z)) \quad = \quad & \int_0^{W^-(r)} (1+r)zd(z)f(z)dz + \int_{W^+(r)}^{+\infty} (1+r)zd(z)f(z)dz \\
& + \int_{W^-(r)}^{W^+(r)} (p-(1+r)L+z-c)d(z)f(z)dz \\
& + \int_0^{+\infty} (p-(1+r)L+z-c)(1-d(z))f(z)dz \\
& +\mu(\int_0^{+\infty} (1-d(z))zf(z)dz - \int_0^{+\infty} (1-d(z))f(z)dzW^-(r)).
\end{aligned}$$

The FOC yields:

$$\begin{aligned}
&\text{For } \ z \in (W^-(r), W^+(r)), \quad \frac{\partial \Lambda(d(z))}{\partial d(z)} = -\mu(z - W^-(r))f(z) \leq 0 \\
&\text{Otherwise,} \ \frac{\partial \Lambda(d(z))}{\partial d(z)} = (-p+(1+r)L - z + c + (1+r)z - \mu(z - W^-(r)))f(z) \qquad (16)
\end{aligned}$$

If $\mu = 0$, as long as $\mathbb{E}(\tilde{z}|d(\tilde{z}) = 0) \geq W^-(r)$, the efficient investment can be implemented because:

(i) For $z \leq W^-(r)$, $\frac{\partial \Lambda(d(z))}{\partial d(z)} = -p + (1+r)L - z + c + (1+r)z < 0$ and $d(z) = 0$.

(ii) For $z \in (W^-(r), W^+(r))$, $\frac{\partial \Lambda(d(z))}{\partial d(z)} = 0$ and $d(z) \in [0, 1]$.

(iii) For $z \geq W^+(r)$, $\frac{\partial \Lambda(d(z))}{\partial d(z)} = -p + (1+r)L - z + c + (1+r)z > 0$ and $d(z) = 1$.

If $\mu \neq 0$. The sign of the FOC given by expression (16) is ambiguous, because:

(i) For $z < W^-(r)$, $-p + (1+r)L - z + c + (1+r)z < 0$ and $-\mu(z - W^-(r)) > 0$

(ii) For $z > W^+(r)$, $-p + (1+r)L - z + c + (1+r)z > 0$ and $-\mu(z - W^-(r)) < 0$

To determine the sign of $\frac{\partial \Lambda(d(z))}{\partial d(z)}$, we study the monotonicity of $g(z) = -p + (1+r)L - z + c + (1+r)z - \mu(z - W^-(r))$. Depending on μ, we analyze three cases:

(a) For $\mu > r$, $g(z)$ is decreasing in z.

(b) For $\mu = r$, $g(z)$ is flat in z.

(c) For $0 < \mu < r$, $g(z)$ is increasing in z.

CASE (a) $\mu > r$**:**

There exists a unique $M^-(r) < W^-(r)$ such that:

(i) For $z < M^-(r)$, $g(z) > 0$ and $\frac{\partial \Lambda(d(z))}{\partial d(z)} > 0$. Thus, $d(z) = 1$.

(ii) For $z \geq M^-(r)$, $g(z) \leq 0$ and $\frac{\partial \Lambda(d(z))}{\partial d(z)} \leq 0$. Thus, $d(z) = 0$.

$M^-(r)$ is defined by $\mathbb{E}(\tilde{z}|\tilde{z} \geq M^-(r)) = W^-(r)$.
Let us define $\Phi_{imp}(M, r) = (1 - F(M))\mathbb{E}(\tilde{z}|\tilde{z} \geq M) - (1 - F(M))W^-(r)$.
Note that $\Phi_{imp}(0, r) = \mathbb{E}(\tilde{z}) - W^-(r)$.
and $\Phi_{imp}(W^-(r), \gamma) = (1 - F(W^-(r)))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^-(r)) - (1 - F(W^-(r)))W^-(r) > 0$. Further, $\frac{\partial \Phi_{imp}(M,r)}{\partial M} = f(M)(W^-(r - M) > 0$. Thus, if $\mathbb{E}(\tilde{z}) < W^-(r)$, $M^-(r)$ is unique. Therefore, if $r > r_{nd}$, the $r-$optimal measurement is an impairment rule such that $d(z) = 1$ if and only if $z \leq M^-(r)$.
CASE (b) $\mu = r$**:** For $z \notin (W^-(r), W^+(r))$, $g(z) = -r(W^+(r) - W^-(r)) \leq 0$ and $\frac{\partial \Lambda(d(z))}{\partial d(z)} \leq 0$. Thus, $\forall z, d(z) = 0$. This case prescribes no measurement, which is never optimal.
CASE (c) $\mu \in (0, r)$:

There exists a unique $M^+(r) > W^+(r)$ such that

(i) For $z < M^+(r)$, $g(z) < 0$ and $\frac{\partial \Lambda(\theta(A))}{\partial \theta(A)} < 0$. Thus, $d(z) = 0$.

(ii) For $z \geq M^+(r)$, $g(z) \geq 0$ and $\frac{\partial \Lambda(\theta(A))}{\partial \theta(A)} \geq 0$. Thus, $d(z) = 1$.

$M^+(r)$ is defined by $\mathbb{E}(\tilde{z}|\tilde{z} \leq M^+(r)) = W^-(r)$
Define $\Phi_{wu}(M, r) = F(M)\mathbb{E}(\tilde{z}|\tilde{z} \leq M) - F(M)W^-(r)$.
$\Phi_{wu}(W^-(r), r) = F(W^-(r))\mathbb{E}(\tilde{z}|\tilde{z} \leq W^-(r)) - F(W^-(r))W^-(r) < 0$ and
$\lim_{M \to +\infty} \Phi_{wu}(M, r) = \mathbb{E}(\tilde{z}) - W^-(r)$. Further, $\frac{\partial \Phi_{wu}(M,r)}{\partial M} = f(M)(M - W^-(r)) \geq 0$. Thus, $M^+(r)$ is unique for $r < r_{nd}$. Therefore, if $r < r_{nd}$, the $r-$optimal measurement is a write-up rule such that $d(z) = 1$ if and only if $z > M^+(r)$.

Proof of Proposition 5: For $r \in (0, r_0]$, by Lemma 2 and Proposition 3, the optimal measurement rule prescribes write-ups above $W^+(r)$. The excess demand is equal to $\Gamma_{fb}(r)$:

$$\Gamma_{fb}(r) = F(W^+(r))L - K - (1 - F(W^+(r))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r)) \tag{17}$$

We know from the proof of Proposition 1 that $\Gamma_{fb}(r)$ is decreasing in r and at $r = 0$, $\Gamma_{fb}(0) = L - K > 0$. If $\Gamma_{fb}(r_0)$ is negative, there exists r^*_{Ia} such that $\Gamma(r^*_{Ia}) = 0$. Rearranging, it yields

$K \geq K_{Ia} = F(W^+(r_0))(L + W^-(r_0)) - \mathbb{E}(\tilde{z})$. If the above conditions are met, r^*_{Ia} is uniquely defined by $\Gamma_{fb}(r^*_{Ia}) = 0$.

Given that by construction, $W^+(r^*_{Ia}) = W^+(r^{fb})$, when Type Ia exists, operating decisions coincide with the first-best capital allocation and $r^*_{Ia} = r^{fb}$.

Proof of Proposition 6: For $r \in (r_0, r_{nd}]$, by Proposition 4, the optimal measurement rule prescribes write-ups above $M^+(r)$. The excess demand is:

$$Z(r) = LF(M^+(r)) - (1 - F(M^+(r)))\mathbb{E}(\tilde{z}|\tilde{z} \geq M^+(r)) - K. \tag{18}$$

$Z(r)$ is increasing in r, because $\frac{\partial Z(r)}{\partial r} = f(M^+(r))\frac{\partial M^+(r)}{\partial r}(L + M^+(r)) > 0$.
At $r = r_0$, $Z(r_0) = \Phi_{fb}(r_0) < 0$ if $K \geq K_{Ia}$.
As $r_{nd} \leq r_{max}$, $Z(r_{nd}) = L - K > 0$.
When Type Ib exists $Z(r^*_{Ib}) = 0$. By construction, $M^+(r^*_{Ib}) = W^+(r^{fb})$ and $r^*_{Ib} > r^{fb}$.

Proof of Corollary 2: *Comparative statics of* $W^+(r^{fb})$:

$$\Gamma_{wu}(M) = LF(M) - (1 - F(M))\mathbb{E}(\tilde{z}|\tilde{z} \geq M) - K. \tag{19}$$

At r^{fb}, $\Gamma_{wu}(W^+(r^{fb})) = 0$. Thus, $W^+(r^{fb})$ is independent of p, Δp and c.
Applying the implicit function theorem,

$$\frac{\partial W^+(r^{fb})}{\partial L} = -\frac{\frac{\partial \Gamma_{wu}(W^+(r^{fb}))}{\partial L}}{\frac{\partial \Gamma_{wu}(W^+(r^{fb}))}{\partial W^+(r^{fb})}} = -\frac{F(W^+(r))}{f(W^+(r))(L + W^+(r))} < 0.$$

By construction $W^+(r^{fb}) = M^+(\gamma^*_{Ib})$ and thus have the same comparative statics.
Comparative statics of r^{fb}:
Applying the implicit function theorem,

$$\frac{\partial r^{fb}}{\partial q} = -\frac{\frac{\partial \Gamma_{fb}(r^{fb})}{\partial q}}{\frac{\partial \Gamma_{fb}(r^{fb})}{\partial r^{fb}}} = -\frac{\frac{\partial W^+(r^{fb})}{\partial q} f(W^+(r^{fb}))(L + W^+(r^{fb}))}{\frac{\partial W^+(r^{fb})}{\partial r^{fb}} f(W^+(r^{fb}))(L + W^+(r^{fb}))}$$

where $q = p,\ c$ or Δp. Hence, $\frac{\partial r^{fb}}{\partial p} > 0$, $\frac{\partial r^{fb}}{\partial c} < 0$ and $\frac{\partial r^{fb}}{\partial \Delta p} = 0$.

$$\frac{\partial r^{fb}}{\partial L} = -\frac{\frac{\partial \Gamma_{fb}(r^{fb})}{\partial I}}{\frac{\partial \Gamma_{fb}(r^{fb})}{\partial r^{fb}}} = -\frac{F(W^+(r^{fb})) + \frac{\partial W^+(r^{fb})}{\partial L} f(W^+(r^{fb}))(L + W^+(r^{fb}))}{\frac{\partial W^+(r^{fb})}{\partial r^{fb}} f(W^+(r^{fb}))(L + W^+(r^{fb}))}. \tag{20}$$

Replacing, $\frac{\partial W^+(r^{fb})}{\partial L} f(W^+(r^{fb}))(L + W^+(r^{fb})) = -\frac{1+r^{fb}}{r^{fb}} f(W^+(r^{fb}))(L + W^+(r^{fb}))$ and hence the sign of $\frac{\partial r^{fb}}{\partial L}$ is ambiguous. When it exists, $r^*_{Ia} = r^{fb}$ and has the same comparative statics.

Comparative statics of γ^*_{Ib}:
To highlight the dependence of Φ_{wu} and Γ_{wu} in the exogenous parameters $q = p$, Δp c or L, we

formally rewrite Φ_{wu} and Γ_{wu} as follows:

$$\begin{aligned}\Phi_{wu}(M,r,q) &= F(M)\mathbb{E}(\tilde{z}|\tilde{z}\le M)-F(M)W^{-}(r,q)\\ \Gamma_{wu}(M,r,q) &= LF(M)-(1-F(M))\mathbb{E}(\tilde{z}|\tilde{z}\ge M)-K.\end{aligned}$$

We define the Jacobian $J_{wu}(M,r,q)$ as follows:

$$J_{wu}(M,r,q)=\begin{bmatrix}\partial\Phi_{wu}(M,r,q)/\partial M & \partial\Phi_{wu}(M,r,q)/\partial r\\ \partial\Gamma_{wu}(M,r,q)/\partial M & \partial\Gamma_{wu}(M,r,q)/\partial r\end{bmatrix}$$

which yields

$$J_{wu}(M,r,q)=\begin{bmatrix}(M-W^{-}(r))f(M) & -\frac{\partial W^{-}(r)}{\partial r}F(M)\\ (M+L)f(M) & 0\end{bmatrix}$$

and the determinant of $J_{wu}(M,r,q)$ is non-zero.

Applying the implicit function theorem yields:

$$\begin{bmatrix}\frac{\partial M^{+}(r^*_{Ib})}{\partial q}\\ \frac{\partial r^*_{Ib}}{\partial q}\end{bmatrix}=-J^{-1}_{wu}(M^{+}(r^*_{Ib}),r^*_{Ib},q)\times\begin{bmatrix}\frac{\partial\Phi_{wu}(M^{+}(r^*_{Ib}),r,q)}{\partial q}\\ \frac{\partial\Gamma_{wu}(M^{+}(r^*_{Ib}),r,q)}{\partial q}\end{bmatrix}.$$

If q is equal to p, Δp or c, $\frac{\partial\Phi_{wu}(M^{+}(r^*_{Ib}),r,q)}{\partial q}=-\frac{\partial W^{-}(r^*_{Ib})}{\partial q}\int_0^{M^{+}(r^*_{Ib})}f(z)dz$

and $\frac{\partial\Gamma_{wu}(M^{+}(r^*_{Ib}),r,q)}{\partial q}=0$. Simplifying,

$$\begin{bmatrix}\frac{\partial M^{+}(r^*_{Ib})}{\partial q}\\ \frac{\partial r^*_{Ib}}{\partial q}\end{bmatrix}=\begin{bmatrix}0\\ -\frac{\frac{\partial W^{-}(r^*_{Ib})}{\partial q}}{\frac{\partial W^{-}(r^*_{Ib})}{\partial r}}\end{bmatrix}$$

and $\frac{\partial r^*_{Ib}}{\partial q}$ follows the sign of $-\frac{\partial W^{-}(r^*_{Ib})}{\partial q}$.
$\frac{\partial\Phi_{wu}(M^{+}(r^*_{Ib}),r,L)}{\partial L}=-\frac{\partial W^{-}(r^*_{Ib})}{\partial L}\int_0^{M^{+}(r^*_{Ib})}f(z)dz$ and $\frac{\partial\Gamma(M^{+}(r^*_{Ib}),r,L)}{\partial L}=\int_0^{M^{+}(r^*_{Ib})}f(z)dz$. Simplifying, $\begin{bmatrix}\frac{\partial M^{+}(r^*_{Ib})}{\partial L}\\ \frac{\partial r^*_{Ib}}{\partial L}\end{bmatrix}=\begin{bmatrix}-\frac{\int_0^{M^{+}(r^*_{Ib})}f(z)dz}{(M^{+}(r^*_{Ib})+L)f(M^{+}(r^*_{Ib}))}<0\\ -\frac{M^{+}(r^*_{Ib})-W^{-}(r^*_{Ib})+\frac{\partial W^{-}(r^*_{Ib})}{\partial L}(M^{+}(r^*_{Ib})+L)}{\frac{\partial W^{-}(r^*_{Ib})}{\partial r}(M^{+}(r^*_{Ib})+L)}<0\end{bmatrix}$.

Proof of Proposition 7: For $r\in(r_{nd},r_{\max})$, by Proposition 4, the optimal measurement rule prescribes impairments below $M^{-}(r)$. The excess demand is:

$$Z(r) = L(1 - F(M^-(r))) - F(M^-(r))\mathbb{E}(\tilde{z}|\tilde{z} \leq M^-(r)) - K. \tag{21}$$

$Z(r)$ is decreasing in r, because $\frac{\partial Z(r)}{\partial r} = -f(M^-(r))\frac{\partial M^-(r)}{\partial r}(L + M^-(r)) < 0.$
At $r_{nd} \geq 0$, $Z(r_{nd}) = L - K > 0$. Moreover, $\lim_{r \uparrow r_{max}} Z(r)$ can be positive or negative and to guarantee market clearing, $\lim_{r \uparrow r_{max}} Z(r)$ needs to be negative, i.e., after rearranging,

$$K \geq K_{II} = (1 - F(M^-(r_{\max})))\mathbb{E}(L + \tilde{z}|\tilde{z} \geq M^-(r_{\max})) - \mathbb{E}(\tilde{z}).$$

If the above conditions are met, there exists a unique r^*_{II} such that $Z(r^*_{II}) = 0$. When Type Ia, Type Ib and Type II exist, $r^*_{II} > r_{nd}$ and thus is greater than r^{fb}.

Proof of Corollary 3:

Assume that $K > K_{II}$ and $r^*_{II} \in (r_{nd}, r_{max}),\ r^{fb}$ and r^*_{II} are defined by:

$$L(1 - F(M^-(r^*_{II}))) - F(M^-(r^*_{II}))\mathbb{E}(\tilde{z}|\tilde{z} \leq M^-(r^*_{II})) - K = 0$$
$$F(W^+(r^{fb}))L - (1 - F(W^+(r^{fb}))\mathbb{E}(\tilde{z}|\tilde{z} \geq W^+(r^{fb})) - K = 0.$$

Rearranging the above equations yield:

$$(1 - F(M^-(r^*_{II})))\mathbb{E}(L + \tilde{z}|\tilde{z} \geq M^-(r^*_{II})) = K + \mathbb{E}(\tilde{z})$$
$$(1 - F(W^+(r^{fb})))\mathbb{E}(L + \tilde{z}|\tilde{z} \geq W^+(r^{fb})) = L - K$$

Thus, $W^+(r^{fb}) \geq M^-(r^*)$ if and only if $L - K \geq K + \mathbb{E}(\tilde{z})$, i.e., $K \leq \frac{L - \mathbb{E}(\tilde{z})}{2}$.
Further, if $K > \underline{K} \equiv F(W^+(r_{\max}))\mathbb{E}(L + \tilde{z}|\tilde{z} \geq W^+(r_{\max})) - \mathbb{E}(\tilde{z})$, $r^{fb} < r_{max}$.

If $K > \underline{K}$, we consider two cases:

(i) If $r^{fb} < r^*_{II}$, $W^+(r^{fb}) > W^+(r^*_{II})$ because $W^+(r)$ is decreasing in r. Also, $W^+(r^*_{II}) > W^-(r^*_{II}) > M^-(r^*_{II})$. It implies that $W^+(r^{fb}) > M^-(r^*_{II})$.

(ii) If $r^{fb} > r^*_{II}$, $W^+(r^{fb}) > W^-(r^{fb}) > W^-(r^*_{II}) > M^-(r^*_{II})$. It implies that $W^+(r^{fb}) > M^-(r^*_{II})$.

Therefore, for $K > \underline{K}$, $W^+(r^{fb}) > M^-(r^*_{II})$. It conclude that $\underline{K} \leq \frac{L - \mathbb{E}(\tilde{z})}{2}$.

We now assume that $K \leq \underline{K}$. Under this assumption, $r^{fb} > r^{max}$ and $W^+(r^{fb}) > M^-(r^*_{II})$ because $\underline{K} \leq \frac{L - \mathbb{E}(\tilde{z})}{2}$.

Proof of Corollary 4: To highlight the dependence of Φ_{imp} and Γ_{imp} in the exogenous parameters $q = p$, Δp, c or L, we formally rewrite Φ_{imp} and Γ_{imp} as follows:

$$\begin{aligned}\Phi_{imp}(M,r,q) &= (1-F(M))\mathbb{E}(\tilde{z}|\tilde{z}\geq M)-(1-F(M))W^{-}(r,q).\\ \Gamma_{imp}(M,r,q) &= L(1-F(M^{-}(r)))-F(M^{-}(r))\mathbb{E}(\tilde{z}|\tilde{z}\leq M^{-}(r))-K\end{aligned}$$

I define the Jacobian $J_{imp}(M,r,q)$ as follows:

$$J_{imp}(M,r,q)=\begin{bmatrix}\partial\Phi_{imp}(M,r,q)/\partial M & \partial\Phi_{imp}(M,r,q)/\partial r\\ \partial\Gamma_{imp}(M,r,q)/\partial M & \partial\Gamma_{imp}(M,r,q)/\partial r\end{bmatrix}$$

which yields

$$J_{imp}(M,r,q)=\begin{bmatrix}(W^{-}(r)-M)f(M) & -\frac{\partial W^{-}(r,q)}{\partial r}\int_{M}^{\infty}f(z)dz\\ -(M+L)f(M) & 0\end{bmatrix}$$

where the determinant of J_{imp} is non zero.

Applying the implicit function theorem yields

$$\begin{bmatrix}\frac{\partial M^{-}(r^{*}_{II})}{\partial q}\\ \frac{\partial r^{*}_{II}}{\partial q}\end{bmatrix}=-J^{-1}_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},q)\times\begin{bmatrix}\frac{\partial\Phi_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},q)}{\partial q}\\ \frac{\partial\Gamma_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},q)}{\partial q}\end{bmatrix},$$

If $q=p$, Δp or c, $\frac{\partial\Phi_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},q)}{\partial q}=-\frac{\partial W^{-}(r^{*}_{II},q)}{\partial q}\int_{M^{-}(r^{*}_{II})}^{+\infty}f(z)dz$

and $\frac{\partial\Gamma_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},q)}{\partial q}=0$. Simplifying,

$\begin{bmatrix}\frac{\partial M^{-}(r^{*}_{II})}{\partial q}\\ \frac{\partial r^{*}_{II}}{\partial q}\end{bmatrix}=\begin{bmatrix}0\\ -\frac{\frac{\partial W^{-}(r^{*}_{II},q)}{\partial q}}{\frac{\partial W^{-}(r^{*}_{II})}{\partial r^{*}_{II}}}\end{bmatrix}$ and $\frac{\partial r^{*}_{II}}{\partial q}$ follows the sign of $-\frac{\partial W^{-}(r^{*}_{II},q)}{\partial q}$.

$\frac{\partial\Phi_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},L)}{\partial L}=-\frac{\partial W^{-}(r^{*}_{II},L)}{\partial L}\int_{M^{-}(r^{*}_{II})}^{+\infty}f(z)dz$ and $\frac{\partial\Gamma_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},L)}{\partial L}=\int_{M^{-}(r^{*}_{II})}^{\infty}f(z)dz.$

Simplifying, $\begin{bmatrix}\frac{\partial M^{-}(r^{*}_{II})}{\partial L}\\ \frac{\partial r^{*}_{II}}{\partial L}\end{bmatrix}=\begin{bmatrix}\frac{\int_{M^{-}(r^{*}_{II})}^{\infty}f(z)dz}{(M^{-}(r^{*}_{II})+L)f(M^{-}(r^{*}_{II}))}>0\\ -\frac{M^{-}(r^{*}_{II})-W^{-}(r^{*}_{II},L)+(M^{-}(r^{*}_{II})+L)\frac{\partial W^{-}(r^{*}_{II},L)}{\partial L}}{\frac{\partial W^{-}(r^{*}_{II},L)}{\partial r^{*}_{II}}(M^{-}(r^{*}_{II})+L)}<0\end{bmatrix}$, because

$$-(M^-(r^*_{II}) - W^-(r^*_{II}, L) + (M^-(r^*_{II}) + L)\frac{\partial W^-(r^*_{II}, L)}{\partial L}) = -(r^*_{II} + 2)M^-(r^*_{II}) - (p - p\frac{c}{\Delta p}) < 0.$$

Proof of Corollary 5:

Capital crunch on r^{fb} and $W^+(r^{fb})$:

Applying the implicit function theorem,

$$\frac{\partial r^{fb}}{\partial K} = -\frac{\frac{\partial \Phi_{fb}(r^{fb})}{\partial K}}{\frac{\partial \Phi_{fb}(r^{fb})}{\partial r^{fb}}} = \frac{1}{\frac{\partial W^+(r^{fb})}{\partial r}(L + W^+(r^{fb}))f(W^+(r^{fb}))} < 0,$$

and $\frac{\partial W^+(r^{fb})}{\partial K} = -\frac{\frac{\partial \Gamma_{wu}(W^+(r^{fb}))}{\partial K}}{\frac{\partial \Gamma_{wu}(W^+(r^{fb}))}{\partial W^+(r^{fb})}} = \frac{1}{f(W^+(r^{fb}))(L+W^+(r^{fb}))} > 0$. By construction $W^+(r^{fb}) = M^+(r^*_{Ib})$ and thus have the same comparative static.

Capital crunch on r^*_{Ib}:

Applying the implicit function theorem,

$$\begin{bmatrix} \frac{\partial M^+(r^*_{Ib})}{\partial K} \\ \frac{\partial r^*_{Ib}}{\partial K} \end{bmatrix} = -J^{-1}_{wu}(M^+(r^*_{Ib}), r^*_{Ib}, K) \times \begin{bmatrix} \frac{\partial \Phi_{wu}(M^+(r^*_{Ib}), r^*_{Ib}, K)}{\partial K} \\ \frac{\partial \Gamma_{wu}(M^+(r^*_{Ib}), r^*_{Ib}, K)}{\partial K} \end{bmatrix},$$

where $J_{wu}(M, r^*_{Ib}, K)$ is the same Jacobian defined in Corollary 2. This is equivalent to

$$\begin{bmatrix} \frac{\partial M^+(r^*_{Ib})}{\partial K} \\ \frac{\partial r^*_{Ib}}{\partial K} \end{bmatrix} = \begin{bmatrix} \frac{1}{(M^+(r^*_{Ib})+L)f(M^+(r^*_{Ib}))} > 0 \\ \frac{M^+(r^*_{Ib}) - W^-(r^*_{Ib})}{\frac{\partial W^-(r^*_{Ib})}{\partial r^*_{Ib}}(M^+(r^*_{Ib})+L)\int_0^{M^+(r^*_{Ib})} f(A)dA} > 0 \end{bmatrix}.$$

Capital crunch on r^*_{II} and $M^-(r^*_{II})$:

$$\begin{bmatrix} \frac{\partial M^-(r^*_{II})}{\partial K} \\ \frac{\partial r^*_{II}}{\partial K} \end{bmatrix} = -J^{-1}_{imp}(M^-(r^*_{II}), r^*_{II}, K) \times \begin{bmatrix} \frac{\partial \Phi_{imp}(M^-(r^*_{II}), r^*_{II}, K)}{\partial K} \\ \frac{\partial \Gamma_{imp}(M^-(r^*_{II}), r^*_{II}, K)}{\partial K} \end{bmatrix},$$

where J_{imp} is the same Jacobian defined in Corollary 4.

This is equivalent to $\begin{bmatrix} \frac{\partial M^-(r^*_{II})}{\partial K} \\ \frac{\partial r^*_{II}}{\partial K} \end{bmatrix} = \begin{bmatrix} -\frac{1}{(M^-(r^*_{II})+L)f(M^-(r^*_{II}))} < 0 \\ \frac{M^-(r^*_{II}) - W^-(r^*_{II})}{\frac{\partial W^-(r^*_{II})}{\partial r}(M^-(r^*_{II})+L)\int_{M^-(r^*_{II})}^{\infty} f(z)dz} < 0 \end{bmatrix}.$

Proof of Corollary 6:

$$\begin{aligned} \frac{\partial F(W^+(r^{fb}))}{\partial K} &= \frac{\partial W^+(r^{fb})}{\partial K} f(W^+(r^{fb})) \\ \frac{\partial(1 - F(M^-(r^*_{II})))}{\partial K} &= -\frac{\partial(M^-(r^*_{II}))}{\partial K} f(M^-(r^*_{II})) \end{aligned}$$

After simplifying,

$$\begin{aligned}\frac{\partial F(W^+(r^{fb}))}{\partial K} &= \frac{1}{L+W^+(r^{fb})} && (22)\\ \frac{\partial(1-F(M^-(r^*_{II})))}{\partial K} &= \frac{1}{L+M^-(r^*_{II})} && (23)\end{aligned}$$

Comparing the change in investment in first-best given by (22) and in second-best given by (23) boils down to ordering $W^+(r^{fb})$ and $M^-(r^*_{II})$. From Corollary 3, W$^+(r^{fb}) > M^-(r^*_{II})$ and after a capital crunch, the decrease in investment is higher in second-best.

Proof of Corollary 7: Conditional on a collateral squeeze, $\tilde{z} = (1-\varepsilon)\tilde{z'}$, where $\tilde{z'}$ is a random variable for which the distribution is held fixed and if there is an increase in ε, the collateral value of all assets shrinks by ε.

Collateral squeeze on r^{fb} and $W^+(r^{fb})$:

We formally introduce the collateral squeeze in the first-best excess demand:

$$\Gamma_{fb}(r,\varepsilon) = F(\frac{W^+(r)}{1-\varepsilon})L - K - (1-F(\frac{W^+(r)}{1-\varepsilon}))(1-\varepsilon)\mathbb{E}\left(\tilde{z}|\tilde{z} \geq \frac{W^+(r)}{1-\varepsilon}\right) \quad (24)$$

$$\frac{\partial r^{fb}}{\partial \varepsilon} = -\frac{\frac{\partial \Gamma_{fb}(r,\varepsilon)}{\partial \varepsilon}}{\frac{\partial \Gamma_{fb}(r^{fb},\varepsilon)}{\partial r^{fb}}} = -\frac{\frac{W^+(r^{fb})f(\frac{W^+(r^{fb})}{1-\varepsilon})}{(1-\varepsilon)^2}(L+W^+(r^{fb})) + F(\frac{W^+(r^{fb})}{1-\varepsilon})}{\frac{1}{1-\varepsilon}\frac{\partial W^+(r^{fb})}{\partial r}(L+(1-\varepsilon)W^+(r^{fb}))f(\frac{W^+(r^{fb})}{1-\varepsilon})} > 0.$$

We introduce the collateral squeeze into Γ_{wu}:

$$\Gamma_{wu}(M,\varepsilon) = F(M)L - K - (1-F(M))(1-\varepsilon)\mathbb{E}\left(\tilde{z}|\tilde{z} \geq M\right) \quad (25)$$

Applying the implicit function theorem,

$$\frac{\partial W^+(r^{fb})}{\partial \varepsilon} = -\frac{\frac{\partial \Gamma_{wu}(W^+(r^{fb}),\varepsilon)}{\partial \varepsilon}}{\frac{\partial \Gamma_{wu}(W^+(r^{fb}),\varepsilon)}{\partial W^+(r^{fb})}} = -\frac{1-F(W^+(r^{fb}))}{f(W^+(r^{fb}))(L+(1-\varepsilon)W^+(r^{fb}))} < 0.$$

By construction $W^+(r^{fb}) = M^+(r^*_{Ib})$ and thus have the same comparative static.

Collateral squeeze on r^*_{Ib}:

We formally introduce the collateral squeeze into Φ_{wu} and Γ_{wu} as follows:

$$\begin{aligned}\Phi_{wu}(M,r,\varepsilon) &= (1-\varepsilon)F(M)\mathbb{E}(\tilde{z}|\tilde{z} \leq M) - F(M)W^-(r)\\ \Gamma_{wu}(M,r,\varepsilon) &= LF(M) - (1-F(M))(1-\varepsilon)\mathbb{E}(\tilde{z}|\tilde{z} \geq M) - K.\end{aligned}$$

The Jacobian is:

$$J_{wu}(M,r,\varepsilon)=\begin{bmatrix}(M(1-\varepsilon)-W^{-}(r))f(M) & -\frac{\partial W^{-}(r)}{\partial r}F(M)\\ (M(1-\varepsilon)+L)f(M) & 0\end{bmatrix}$$

Applying the implicit function theorem,

$$\left[\begin{array}{c}\frac{\partial M^{+}(r^{*}_{Ib})}{\partial\varepsilon}\\ \frac{\partial r^{*}_{Ib}}{\partial\varepsilon}\end{array}\right]=-J^{-1}_{wu}(M^{+}(r^{*}_{Ib}),r^{*}_{Ib},\varepsilon)\times\left[\begin{array}{c}\frac{\partial\Phi_{wu}(M^{+}(r^{*}_{Ib}),r^{*}_{Ib},\varepsilon)}{\partial\varepsilon}\\ \frac{\partial\Gamma_{wu}(M^{+}(r^{*}_{Ib}),r^{*}_{Ib},\varepsilon)}{\partial\varepsilon}\end{array}\right].$$

Simplifying,

$$\left[\begin{array}{c}\frac{\partial M^{+}(r^{*}_{Ib})}{\partial\varepsilon}\\ \frac{\partial r^{*}_{Ib}}{\partial\varepsilon}\end{array}\right]=\left[\begin{array}{c}-\frac{(1-F(M^{+}(r^{*}_{Ib})))\mathbb{E}(\tilde{z}|\tilde{z}\geq M^{+}(r^{*}_{Ib}))}{f(M^{+}(r^{*}_{Ib}))(M^{+}(r^{*}_{Ib})(1-\varepsilon)+L)}<0\\ -\frac{(M^{+}(r^{*}_{Ib})(1-\varepsilon)+L)\int_{0}^{M^{+}(r^{*}_{Ib})}zf(z)dz+(M^{+}(r^{*}_{Ib})(1-\varepsilon)-W^{-}(r^{*}_{Ib}))\int_{M^{+}(r^{*}_{Ib})}^{+\infty}zf(z)dz}{\frac{\partial W^{-}(r^{*}_{Ib})}{\partial r^{*}_{Ib}}(M^{+}(r^{*}_{Ib})(1-\varepsilon)+L)\int_{0}^{M^{+}(r^{*}_{Ib})}f(z)dz}<0\end{array}\right]$$

.

Collateral squeeze on r^{*}_{II} and $\xi_{imp}(r^{*}_{II})$:

We formally introduce the collateral squeeze into Φ_{imp} and Γ_{imp} as follows:

$$\begin{aligned}\Phi_{imp}(M,r,\varepsilon) &= (1-F(M))(1-\varepsilon)\mathbb{E}(\tilde{z}|\tilde{z}\geq M)-(1-F(M))W^{-}(r).\\ \Gamma_{imp}(M,r,\varepsilon) &= L(1-F(M^{-}(r)))-F(M^{-}(r))(1-\varepsilon)\mathbb{E}(\tilde{z}|\tilde{z}\leq M^{-}(r))-K\end{aligned}$$

The Jacobian $J_{imp}(M,r,\varepsilon)$ as follows:

$$J_{imp}(M,r,\varepsilon)=\begin{bmatrix}(W^{-}(r)-M(1-\varepsilon))f(M) & -\frac{\partial W^{-}(r)}{\partial r}\int_{M}^{\infty}f(z)dz\\ -(M(1-\varepsilon)+L)f(M) & 0\end{bmatrix}$$

Applying the implicit function theorem,

$$\left[\begin{array}{c}\frac{\partial M^{-}(r^{*}_{II})}{\partial\varepsilon}\\ \frac{\partial r^{*}_{II}}{\partial\varepsilon}\end{array}\right]=-J^{-1}_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},\varepsilon)\times\left[\begin{array}{c}\frac{\partial\Phi^{shock}_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},\varepsilon)}{\partial\varepsilon}\\ \frac{\partial\Gamma^{shock}_{imp}(M^{-}(r^{*}_{II}),r^{*}_{II},\varepsilon)}{\partial\varepsilon}\end{array}\right].$$

Simplifying, $\begin{bmatrix} \frac{\partial M^-(r^*_{II})}{\partial \varepsilon} \\ \frac{\partial r^*_{II}}{\partial \varepsilon} \end{bmatrix} = \begin{bmatrix} \frac{\int_0^{M^-(r^*_{II})} zf(z)dz}{f(M^-(r^*_{II}))(M^-(r^*_{II})(1-\varepsilon)+L)} > 0 \\ -\frac{-W^-(r^*_{II})\int_0^{M^-(r^*_{II})} zf(z)dz + L\int_{M^-(r^*_{II})}^{+\infty} zf(z)dz + (1-\varepsilon)\mathbb{E}(\tilde{z})M^-(r^*_{II})}{\frac{\partial W^-(r^*_{II})}{\partial r^*_{II}}(M^-(r^*_{II})(1-\varepsilon)+L)\int_{M^-(r^*_{II})}^{+\infty} f(z)dz} \end{bmatrix}$.

In equilibrium,
$$\begin{aligned} & -W^-(r^*_{II})\int_0^{M^-(r^*_{II})} zf(z)dz + L\int_{M^-(r^*_{II})}^{+\infty} zf(z)dz \\ = \; & -W^-(r^*_{II})\int_0^{M^-(r^*_{II})} zf(z)dz + L\frac{W^-(r^*_{II})}{(1-\varepsilon)}\int_{M^-(r^*_{II})}^{+\infty} f(z)dz \\ = \; & \frac{W^-(r^*_{II})}{(1-\varepsilon)}\left(-(1-\varepsilon)\int_0^{M^-(r^*_{II})} zf(z)dz + L\int_{M^-(r^*_{II})}^{+\infty} f(z)dz\right) \\ = \; & \frac{W^-(r^*_{II})}{(1-\varepsilon)}K. \end{aligned} \tag{26}$$

Thus, $\frac{\partial r^*_{II}}{\partial \varepsilon} < 0$.

Proof of Corollary 8:

$$\begin{aligned} \frac{\partial F(W^+(r^{fb}))}{\partial \varepsilon} &= \frac{\partial W^+(r^{fb})}{\partial \varepsilon} f(W^+(r^{fb})) \\ \frac{\partial(1-F(M^-(r^*_{II})))}{\partial \varepsilon} &= -\frac{\partial(M^-(r^*_{II}))}{\partial \varepsilon} f(M^-(r^*_{II})) \end{aligned}$$

After simplifying,

$$\frac{\partial F(W^+(r^{fb}))}{\partial \varepsilon} = -\frac{1-F(W^+(r^{fb}))}{L+(1-\varepsilon)W^+(r^{fb})} \tag{27}$$

$$\frac{\partial(1-F(M^-(r^*_{II})))}{\partial \varepsilon} = \frac{F(M^-(r^*_{II}))}{M^-(r^*_{II})(1-\varepsilon)+L} \tag{28}$$

From Corollary 3, W$^+(r^{fb}) > M^-(r^*_{II})$. Thus, $1-F(W^+(r^{fb})) < F(M^-(r^*_{II}))$, there is a financial accelerator.

Appendix B

Conditions to rule out an equilibrium at $r^* \leq 0$:

If $r \leq 0$, all the firms prefer investing over liquidating. However, conditions in the economy might not guarantee that the firms can find the financing.

If $\mathbb{E}(\tilde{z}) > W^-(1) > 0$ and $r \leq 0$, no disclosure allows all the firms to find financing and the excess demand is $Z(r) = L-K$. To rule out any equilibrium interest rate $r^* \leq 0$, we assume that

$K < I$.
If $\mathbb{E}(\tilde{A}) \leq W^{-}(1)$, we distinguish two cases dependent on the sign of $W^{-}(r)$ for $r \leq 0$. $W^{-}(r)$ is increasing in r and at $r = -1$,$W^{-}(-1) = p(\frac{c}{\Delta p} - 1) < 0$ and at $r = 0$, W$^{-}(0) = p\frac{c}{\Delta p} - p + L > 0$. Thus there exists a unique $r_1 \in [-1, 0]$ such that $W^{-}(r_1) = 0$.

First, at r_1, if there is no information about any collateral, all the firms produce and $Z(r) = L - K$.

Second, for $r > r_1$, the optimal measurement is to prescribe impairments below $M^{-}(r)$ and $Z(r) = (1 - F(M^{-}(r))L - F(M^{-}(r))\mathbb{E}(\tilde{z}|\tilde{z} \leq M^{-}(r)) - K$, which is decreasing in r. To rule out any equilibrium interest rate $r^* \leq 0$, we assume that $K < (1 - F(M^{-}(0))L - F(M^{-}(0))\mathbb{E}(\tilde{z}|\tilde{z} \leq M^{-}(0))$.

Combining the conditions listed above, the equilibrium interest rate r^* is strictly greater than 0 if and only if $K < \overline{K} = \min(L, (1 - F(M^{-}(0))L - F(M^{-}(0))\mathbb{E}(\tilde{z}|\tilde{z} \leq M^{-}(0)))$.

Conditions to have an equilibrium at r_{max}:

At r_{max}, $W^{-}(r_{max}) = W^{+}(r_{max})$ and

$$\begin{aligned} U(d(z)) = \quad & \int_0^{W^{-}(r_{max})} (1 + r_{max}) z d(z) f(z) dz + \int_{W^{-}(r_{max})}^{+\infty} (1 + r_{max}) z d(z) f(z) dz \\ & + \int_0^{+\infty} (p - (1 + r_{max})L + z - c)(1 - d(z)) f(z) dz \end{aligned}$$

If $\mathbb{E}(\tilde{z}|d(z) = 0) = W^{-}(r_{max}) = W^{+}(r_{max})$ and the firms not measuring their collateral are indifferent between investing and liquidating. Thus, $\int (p - (1 + r_{max})L + z - c)(1 - d(z)) f(z) dz = \int (1 + r_{max}) z (1 - d(z)) f(z) dz$ and it yields $\forall d(z), U(d(z)) = (1 + r_{max})\mathbb{E}(\tilde{z})$.
For r_{max} to be an equilibrium interest rate, two conditions need to be met:

$$\begin{aligned} \mathbb{E}(\tilde{z}|d(z) = 0) &= W^{-}(r_{max}) = W^{+}(r_{max}) \\ \text{and} \quad Z(r_{max}) &= 0 \end{aligned} \tag{29}$$

The second condition corresponds to the market clearing condition. Let us define the excess demand $Z(r_{max})$ dependent on impairments below $M_{imp}(r_{max})$ and write-ups above $M_{wu}(r_{max})$ such that $M_{imp}(r_{max}) < W^{+}(r_{max}) < M_{wu}(r_{max})$:

$$
\begin{aligned}
Z(r_{max}) &= L\int_{M_{imp}(r_{max})}^{M_{wu}(r_{max})} f(z)dz - K \\
&\quad - \int_0^{M_{imp}(r_{max})} zf(z)dz - \int_{M_{wu}(r_{max})}^{+\infty} zf(z)dz
\end{aligned}
$$

The excess demand $Z(r_{max})$ decreases in $M_{imp}(r_{max})$ and increases in $M_{wu}(r_{max})$. We distinguish four cases.

Case 1: If $\mathbb{E}(\tilde{z}) \geq W^-(r_{max})$ and $\lim_{r\uparrow r_{max}} Z(r) < 0$, we prescribe only write-ups above $M^+(r_{max})$.
At r_{max}, to satisfy the market clearing condition, less write-ups need to be prescribed, i.e., $M_{wu}(r_{max}) > M^+(r_{max})$. However, $\mathbb{E}(\tilde{z}|\tilde{z} \leq M^+(r_{max})) = W^-(r_{max})$ and thus, $\mathbb{E}(\tilde{z}|\tilde{z} \leq M_{wu}(r_{max}))$ becomes strictly higher than $W^-(r_{max})$, which violates condition (29). Given that no impairments are already in place, condition (29) cannot be satisfied. Therefore, r_{max} cannot be an equilibrium interest rate.

Case 2: If $\mathbb{E}(\tilde{z}) \geq W^-(r_{max})$ and $\lim_{r\uparrow r_{max}} Z(r) > 0$, we prescribe only write-ups above $M^+(r_{max})$.
At r_{max}, to satisfy the market clearing condition, more write-ups need to be prescribed, i.e., $M_{wu}(r_{max}) < M^+(r_{max})$. However, given that $\mathbb{E}(\tilde{z}|\tilde{z} \leq M^+(r_{max})) = W^-(r_{max})$, $\mathbb{E}(\tilde{z}|\tilde{z} \leq M_{wu}(r_{max}))$ becomes strictly lower than $W^-(r_{max})$, which violates condition (29). More impairments, i.e., $M_{imp}(r_{max}) > 0$, need to be prescribed such that condition (29) is satisfied.
Therefore, there exist $M^*_{imp}(r_{max})$ and $M^*_{wu}(r_{max})$ such that
$\mathbb{E}(\tilde{z}|\tilde{z} \in [M^*_{imp}(r_{max}), M^*_{wu}(r_{max})]) = W^-(r_{max})$.
Firms invest with $z \in [M^*_{imp}(r_{max}), M^*_{wu}(r_{max})]$ and liquidate with $z \in [0, M^*_{imp}(r_{max})) \cup (M^*_{wu}(r_{max}), +\infty)$ such that $Z(r_{max}) = 0$. It follows that r_{max} is an equilibrium interest rate.

Case 3: If $W^-(r_{max}) > \mathbb{E}(\tilde{z})$ and $\lim_{r\uparrow r_{max}} Z(r) > 0$ by prescribing only impairments below $M^-(r_{max})$. At r_{max}, to satisfy the market clearing condition, more impairments need to be prescribed, i.e., $M_{imp}(r_{max}) > M^-(r_{max})$. However, given that

$\mathbb{E}(z|z \geq M^-(r_{max})) = W^-(r_{max})$, $E(z|z \geq M_{imp}(r_{max}))$ becomes strictly higher than $W^-(r_{max})$, which violates condition (29). More write-ups need to be prescribed such that condition (29) is satisfied.
Therefore, there exist $M^*_{imp}(r_{max})$ and $M^*_{wu}(r_{max})$ such that
$\mathbb{E}(\tilde{z}|\tilde{z} \in [M^*_{imp}(r_{max}), M^*_{wu}(r_{max})]) = W^-(r_{max})$.
Firms invest with $z \in [M^*_{imp}(r_{max}), M^*_{wu}(r_{max})]$ and liquidate with $z \in [0, M^*_{imp}(r_{max})) \cup (M^*_{wu}(r_{max}), +\infty)$ such that $Z(r_{max}) = 0$. It follows that r_{max} is an equilibrium interest rate.

Case 4: If $W^{-}(r_{max}) > \mathbb{E}(\tilde{z})$ and $\lim_{r\uparrow r_{max}} Z(r) < 0$, we prescribe only impairments below $M^{-}(r_{max})$. Thus, at r_{max}, to satisfy the market clearing condition, less impairments need to be prescribed, i.e., $M_{imp}(r_{max}) < M^{-}(r_{max})$. However, $\mathbb{E}(\tilde{z}|\tilde{z} \geq M^{-}(r_{max})) = W^{-}(r_{max})$ and thus,

$\mathbb{E}(\tilde{z}|\tilde{z} \geq M_{imp}(r_{max}))$ becomes strictly lower than $W^{-}(r_{max})$, which violates condition (29). Given that no write-ups are already in place, condition (29) cannot be satisfied. Thus, r_{max} cannot be an equilibrium interest rate.

To conclude, r_{max} is an equilibrium interest rate if and only if there only exist Type Ia and Type Ib or none of the equilibria exists.

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