Intrafirm Trade, Pay-Performance Sensitivity and Organizational Structure

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Abstract

We study a model of a decentralized firm with moral hazard and intrafirm trade. Relationship-specific investments, by making internal trade more profitable at the margin, result in greater expected trade volume and thereby greater compensation risk borne by division managers. Formally accounting for this investment/risk link overturns some key findings in prior incomplete contracting studies. First, managers facing high-powered pay-performance sensitivity (PPS) invest less in relationship-specific assets. Hence, the optimal PPS will have to trade off investment and effort incentives. Second, incomplete contracting may result in overinvestment if the benefits from intrafirm trade are highly volatile, because the investing manager internalizes only a portion of the incremental trade-related risk premium. Third, the model can shed new light on the perennial question of organizing business units as investment or profit centers. Specifically, we derive conditions under which both divisions involved in intrafirm trade should be organized as investment centers, each choosing its own investment, and conditions under which one division should be in charge of choosing all investments while the other division is "downgraded" to profit center status without any investment authority.

1 Introduction

Division managers routinely transact with counterparties outside and within the firm.[1] The incentives literature has looked separately, for the most part, at the issues of eliciting operating effort (the agency problem) and of guiding intrafirm trade and relationship-specific investments (the transfer pricing problem). The standard justification is that agency problems are addressed by calibrating managers' pay-performance sensitivity (PPS). Because the relevant costs and revenues guiding intrafirm trade are scaled uniformly by the PPS—they are not personally costly to the managers but accrue to divisional P&Ls—the transfer pricing problem should be unaffected by the PPS.[2] In this paper we argue that this paradigmatic separation overlooks an important link between these two incentive problems: *risk*. Specifically, greater specific investments increase the expected intrafirm trading volume and thereby the associated compensation risk for the managers. By analyzing this link, we provide new predictions for incentive contracts and the allocation of decision rights in divisionalized firms.

Regarding contracting, the investment/risk link implies, first, that high-powered incentives make managers reluctant to invest, all else equal. A tradeoff thus arises between effort and investment incentives, to be balanced by the PPS. Moreover, we show that incomplete contracting with negotiated transfer pricing may lead to overinvestment. A recurring theme in the earlier literature instead is underinvestment, because a manager's P&L records all fixed costs from investing, but has to split the attendant surplus with other manager in the course of bargaining (the hold-up problem).[3] One might expect the above investment/risk

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[1] Intrafirm trade accounts for sizable fraction of divisions' operations. For example, in 2011 Exxon's intrafrm trade was 49 percent of its total sales.

[2] Papers that have aimed at bridging this gap are Holmstrom and Tirole (1991), Anctil and Dutta (1999), Baiman and Baldenius (2008).

[3] We ignore "administered" approaches to intrafirm trade, such as cost- or market-based, and profit sharing, as in Holmstrom and Tirole (1991) and Anctil and Dutta (1999). Profit sharing arrangements are not frequently observed in practice, see Merchant (1989), Bushman, et al. (1995), Keating (1997), Abernethy, et al. (2004). Including them into our framework

link to compound the underinvestment problem, because risk-averse managers will invest even less if doing so exposes them to additional risk. In fact, the opposite is true. If the principal could contract on investments (the "benchmark model"), she would take into account the incremental risk premium incurred by *both* division managers. With incomplete contracting and delegated investment choice, however, the investing manager cares only about his *own* risk cost and externalizes the remaining portion. Which of the two forces dominates—hold-up or the externalized risk premium—determines the direction of the equilibrium investment distortion and the corresponding PPS adjustments. Accounting for the incremental risk associated with specific investments hence overturns standard findings in the incomplete contracting literature.

Our model develops the key mechanism for the simple case where only one division manager can make a specific investment. If the divisions face highly volatile general operations, their incentives will be muted. The externalized incremental risk premium caused by investing in specific assets then is small, too, while the hold-up problem remains unabated. To alleviate the resulting underinvestment problem, the principal lowers the investing manager's PPS. However, some amount of underinvestment remains in equilibrium. The corresponding reduction in trade-related risk, in turn, allows the principal to raise the PPS of the other (non-investing) manager.

The reverse logic applies if the gains from intrafirm trade are highly uncertain, the investing manager is relatively insensitive to the investment-induced trade-related risk (because of a low PPS due to high operating risk), and the other manager is very sensitive to the trade-related risk (because of a high PPS due to low operating risk). The externalized risk premium then weighs heavily, eventually resulting in overinvestment. In response, the principal raises the PPS of the investing manager to curb his overinvestment tendency, while lowering that of the non-investing manager as some degree of overinvestment will persist

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would not alter the qualitative conclusions.

in the constrained-optimal solution.

A corollary of the preceding discussion is that contractual incompleteness can introduce a wedge into the PPS for managers who face similar operating environments but differ in their scope to invest in intrafirm trade. The PPS for managers with no investment opportunities is dictated solely by the equilibrium investment distortion—and negatively related to the latter. In contrast, the PPS for the investing manager is to be chosen with an eye to his investment incentives. As a result, the association between equilibrium investment distortions and PPS adjustments may be positive for his manager. Moreover, the relation between the scope of investment opportunities and PPS is nonmonotonic. These findings constitute a significant departure from earlier results postulating no link between investments and PPS (short of firmwide profit sharing).

We build on these insights when generalizing the model to bilateral investments. While investments upstream could comprise a design task to reduce the variable cost of making a component, downstream they could be in improving the shipping process for the final product. We ask how the authority to choose these investments should be assigned to the managers. This bears on a classic organization design issue—whether to organize divisions as profit or investment centers. Essentially, we conduct a horse race between two organizational modes:

1. Under "IC-IC" both divisions are designated as *investment centers*: each chooses its own investment and pays for the attendant fixed cost.

2. Under "PC-IC" one division is run as a *profit center* with no investment authority, whereas the other—the *investment center*—chooses both up- and downstream investments and pays for all relevant fixed costs.

In the above example, under IC-IC the upstream manager would be in charge of the component design task; the downstream manager, of the process improvement. Under PC-IC one division chooses both investments.[4]

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[4]It is useful to invoke the terminology in Che and Hausch (1999) to describe the investments

In our model the same forces that push toward underinvestment under incomplete contracting (especially, high operating volatility) also make divisional investments strategic complements. This further compounds the underinvestment problem with bilateral investments, and we focus on that case.[5] Which manager should be assigned the investment authority under PC-IC? With underinvestment being the pressing concern, by assumption, the manager facing greater operating uncertainty should be in charge of investing, as he faces a lower PPS and hence is more willing to invest. That way, the principal uses the investment/risk link to her advantage under PC-IC. Under IC-IC, in contrast, the investment incentive constraints have to hold for *each* manager. Hence the "bottleneck" in eliciting investments under IC-IC is the manager with the higher PPS. We refer to this as the *risk tolerance benefit* of PC-IC.

The other key difference between the two regimes is the game form. PC-IC induces a single-person decision problem; IC-IC, a two-player simultaneous-move game, in which the managers' investments must form a Nash equilibrium. Together with the risk tolerance benefit, the game form difference also seems to point to PC-IC as the less constrained, and thus preferred, regime. This reasoning, however, is incomplete because the players' payoff functions also differ across the regimes. Specifically, each manager has any fixed cost associated with his investments charged against his divisional P&L measure. This reduces the investment incentives under PC-IC because of a "double whammy" problem:

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under the two regimes. Under IC-IC each manager chooses a "selfish" investment directly lowering (increasing) its relevant cost (net revenue) from intrafirm trade. Under PC-IC the investment center manager chooses a selfish investment benefiting his divisional P&L directly and another "cooperative" investment which directly benefits the P&L of the other manager. In the course of bargaining, of course, the investing party captures part of the additional surplus even for cooperative investments.

[5]One can show that the same conditions that predict unilateral overinvestment also predict bilateral investments to be strategic substitutes. Analysis of this case however is complicated by the fact that equilibrium investment distortions can be identified only for very special cases. The reason is that strategic substitutability can turn strong overinvestment incentives by one manager into equilibrium underinvestment by the other, despite each manager having incentives to invest excessively for given investment by the other party. We therefore do not analyze this case in detail.

the investment center manager gets charged for all fixed costs. At the same time, eliciting complementary inputs as a Nash equilibrium, as under IC-IC, tends to be cheap.[6] Which of the two regimes dominates is then determined by a tradeoff between the risk tolerance effect and the double whammy effect. Specifically, IC-IC generates stronger investment incentives, if the divisions face similar volatility in their general operations, because the risk tolerance benefit of PC-IC then becomes negligible. On the other hand, the risk tolerance effect is strong if the divisions face highly divergent operating volatilities, in which case eventually PC-IC becomes the preferred regime.

In summary, the investment/risk link identified in this paper generates novel predictions for (i) contracting (PPS affects investment incentives and thus is affected by investment opportunities; incomplete contracting may result in overinvestment) and (ii) the allocation of decision rights in divisionalized firms (under certain conditions all investment authority should be bundled in one division manager's hand).

Earlier studies to link transfer pricing with divisional incentives (and organizational design issues) are Holmstrom and Tirole (1991) and Anctil and Dutta (1999). In both papers, risk averse managers invest more in response to profit sharing; but with pure divisional performance measurement the managers' PPS would not affect equilibrium investments. Our model establishes a direct link between PPS and investments even absent profit sharing.[7] Papers that have studied

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[6]Consider a stylized example that ignores PPS and risk altogether, for simplicity. A simple production technology generates contribution margin of $M = (1 + I_A)(1 + I_B)$ off two inputs (investments), $I_i \in \{0, 1\}$. Each unit of I_i comes at a fixed cost of F, and the managers split M equally at the end of the day. High investments, $(1, 1)$, are desired by the shareholders. Under IC-IC each Manager i chooses I_i noncooperatively. The desired investment profile of $(1, 1)$ then forms a Nash equilibrium whenever $\frac{1}{2}M(1, 1) - F \geq \frac{1}{2}M(0, 1)$, or $F \leq 1$. (For $F \geq \frac{1}{2}$, $(0, 0)$ is also an equilibrium, but it is Pareto-dominated by $(1, 1)$ for any $F \in [\frac{1}{2}, 1]$.) The investment center manager under PC-IC chooses the desired investments whenever $\frac{1}{2}M(1, 1) - 2F \geq \frac{1}{2}M(0, 0)$, or $F \leq \frac{3}{4}$. (It is never optimal for him to choose $(1, 0)$ or $(0, 1)$, given the strategic complementarity in investments.) Thus, IC-IC generates stronger investment incentives in aggregate.

[7]Anctil and Dutta (1999) highlight the risk-sharing benefit of negotiated transfer pricing— an aspect also at work in our model.

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the role of risk in moral hazard models include Prendergast (2002), Baker and Jorgensen (2005), Bertomeu (2008), Liang et al. (2008), Liang and Nan (2011). In our model, compensation risk is partly endogenous due to investments in efficiency enhancements. This allows us to integrate issues of moral hazard, intrafirm trade, and the allocation of investment authority.

The paper proceeds as follows. Section 2 describes the model. Section 3 addresses the benchmark model with contractible investments. Section 4 deals with non-contractible investments. Section 5 embeds bilateral investment choice in the larger context of organizational design, and Section 6 concludes.

2   The Model

The model entails a principal contracting with two division managers. Division A produces an intermediate good and transfers it to Division B. Division B further processes it and sells a final good. We assume there is no outside market for the intermediate good. Each division is run by a manager who exerts unobservable effort to increase divisional profit in addition to coordinating with the respective other manager on the intrafirm trade.

The sequence of events is as follows (Figure 1). At Date 1 the principal contracts with the division managers. At Date 2 the managers choose their effort levels $a_i \in \mathbb{R}_+$, $i = A, B$, with marginal productivity normalized to one. Efforts are "general purpose" in that they are unrelated to the transaction opportunity that arises between the two divisions. To increase the gains from intrafirm trade, the manager of the upstream division (Manager A) can make a relationship-specific binary investment, $I_A \in \{0, 1\}$.[8]   At Date 3 the managers, but not the principal, jointly observe the realization of the random variables θ_i, $i = A, B$, drawn from (commonly known) distribution functions over supports $[\underline{\theta}_i, \bar{\theta}_i]$,

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[8]It is without loss of generality that the investment is upstream, the results in Sections 3 to 4 would simply flip if the investment were made by Manager B to enhance downstream net revenues. Section 5 addresses bilateral investments.

1	2	3	4
Contract signed	a_i and I chosen	θ_i realized	q and t set
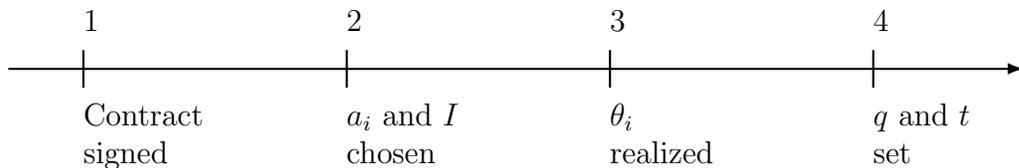
Figure 1: Timeline

respectively. Let $\sigma_{\theta_i}^2$ denote the variance of θ_i, μ_i its mean, $\theta \equiv (\theta_A, \theta_B)$ and $\mu \equiv (\mu_A, \mu_B)$.

At Date 4 the managers negotiate over the quantity of the intermediate good to be traded, $q \in \mathbb{R}_+$, and the (per-unit) transfer price $t \in \mathbb{R}_+$. Finally, divisional profits are realized as follows:

$$
\begin{aligned}
\pi_A &= a_A + \widetilde{\varepsilon}_A + tq - C(q, \theta_A, I_A) - F I_A, \\
\pi_B &= a_B + \widetilde{\varepsilon}_B + R(q, \theta_B) - tq,
\end{aligned}
$$

with

$$C(q, \theta_A, I_A) = (c - I_A - \theta_A)q \quad \text{and} \quad R(q, \theta_B) = r(q) + \theta_B q. \tag{1}$$

Here, $C(q, \theta_A, I_A)$ is Division A's (linear) variable cost associated with the trade, and $F > 0$ measures the fixed cost from investing; $R(q, \theta_B)$ is the (net) revenue realized by Division B, with $r(q)$ an increasing and concave function. Let $M(q, \theta, I_A) \equiv R(q, \theta_B) - C(q, \theta_A, I_A)$ denote firmwide contribution margin.

The divisions' ongoing operations are risky, as expressed by normally distributed error terms $\widetilde{\varepsilon}_i$ with mean zero and variance $\sigma_{\varepsilon,i}^2$. All noise terms, $\varepsilon_i, \theta_i, \ i = A, B$, are statistically independent. In the following, we refer to $\sigma_{\varepsilon,i}^2$ as (general) *operating uncertainty*, associated with each division's stand-alone business, and to $\sigma_{\theta,i}^2$ as *trade-related uncertainty*, pertaining to the transaction between the two divisions.

We assume that the managers are compensated with linear contracts based solely on their own divisional profits, i.e., we ignore profit-sharing contracts (as studied in Anctil and Dutta, 1999).[9] We denote Manager i's compensation by $s_i = \alpha_i + \beta_i \pi_i$. The managers are assumed to have mean-variance preferences:

$$EU_i = E(s_i) - \frac{v_i}{2}a_i^2 - \frac{\rho_i}{2}Var(s_i),$$

where v_i measures Manager i's marginal disutility of effort and ρ_i his coefficient of risk aversion. To ensure contract participation, we impose throughout an individual rationality condition such that

$$EU_i \geq 0, \ i = 1, 2. \tag{2}$$

The above formulation is fairly standard in specific investment models and implies that Division A's marginal production cost is decreasing in the investment, i.e., $\frac{\partial}{\partial q}C(q, \theta_A, 1) - \frac{\partial}{\partial q}C(q, \theta_A, 0) < 0$ for any q and θ_A. As a result, greater upfront investment translates in higher (expected) trading volume—this volume effect will play an important role in the analysis to follow.

3 Contractible Investment

As a benchmark, suppose that the investment I_A is contractible, e.g., it may entail specialized equipment used exclusively—in a verifiable manner—to make a particular product. At Date 4 the managers bargain about the transaction under symmetric information and thus agree on the conditionally efficient quantity $q^*(\theta, I_A) \in \arg\max_q \ M(q, \theta, I_A)$, for any θ and I_A. The (necessary and sufficient) first-order condition reads $R_q(q^*(\cdot), \theta_B) = C_q(q^*(\cdot), \theta_A, I_A)$. Define $M(\theta, I_A) \equiv M(q^*(\theta, I_A), \theta, I_A)$ as the first-best contribution margin for given θ and I_A, and

$$\Delta M \equiv E_\theta[M(\theta, 1) - M(\theta, 0)]$$

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[9]Allowing for profit sharing would dampen the effects present in our model but not eliminate them, as long as more compensation weight is put on own-division profit for each manager.

as the incremental expected contribution margin attributable to the upfront investment.

In the course of bargaining the managers maximize the firmwide *ex-post* surplus. In particular, the outcome of bargaining is unaffected by the managers' respective PPS. Assuming equal bargaining strength, the transfer price simply splits the attendant contribution margin: $t(\theta, I_A) = \frac{R(q^*(\theta, I_A), \theta_B) + C(q^*(\cdot), \theta_A, I_A)}{2q^*(\cdot)}$, resulting in divisional profits of

$$\pi_i = a_i + \widetilde{\varepsilon}_i + \frac{1}{2} M(\theta, I_A) - \mathbb{1}_{i=A} F I_A,$$

where $\mathbb{1}_{i=k} \in \{0,1\}$ is an indicator that takes the value of 1 if and only if $i = k$

If investments are contractible, at Date 1 the principal instructs the upstream manager as to the level of I_A. In choosing compensation contracts, the principal needs to observe the individual rationality constraint (2) and the effort incentive constraint,

$$a_i(\beta_i) \in \arg \max_{a_i} \ \beta_i a_i - \frac{v_i}{2} a_i^2 - \frac{\rho_i}{2} \beta_i^2 \left[\sigma_{\varepsilon, i}^2 + \frac{Var(M(\theta, I_A))}{4} \right], \quad i = A, B. \quad (3)$$

Bargaining provides a way of sharing intrafirm trade-related risk, hence the term $Var(M(\cdot))/4$.[10] We denote the principal's payoff from Division i's general operations by

$$\Phi_i(\beta_i) \equiv a_i(\beta_i) - \frac{v_i}{2} (a_i(\beta_i))^2 - \frac{\rho_i}{2} \beta_i^2 \sigma_{\varepsilon, i}^2.$$

Note that $\Phi_i'(0) > 0 > \Phi_i'(1)$ and $\Phi_i''(\beta_i) \leq 0$ for any $\beta_i \in [0, 1]$. Further, let

$$p(\beta, I_A) \equiv E_\theta \left[M(\theta, I_A) \right] + \sum_{i=A,B} \left[\Phi_i(\beta_i) - \frac{\rho_i}{8} \beta_i^2 \cdot Var(M(\theta, I_A)) \right]$$

denote the expected firmwide contribution margin net of the total risk premium.

The principal now solves the following optimization program (superscript "*" indicates *contractible* investments):

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[10]See also Anctil and Dutta (1999).

Program $\mathcal{P}^*$:

$$\max_{\beta, I_A \in \{0,1\}} \Pi^*(\beta, I_A) \equiv p(\beta, I_A) - F I_A.$$

Let (I_A^*, β^*) denote the solution to Program $\mathcal{P}^*$. Given the assumed binary nature of investments, it is convenient to decompose the optimization problem as follows. First, derive $\beta^o(I_A) = (\beta_A^o(I_A), \beta_B^o(I_A))$, the optimal PPS conditional on I_A; then, pick the investment level I_A^* that results in greater overall surplus, so that $\beta^* = \beta^o(I_A^*)$. For given I_A, due to the trade-related risk, $Var(M(\cdot))$, the conditionally optimal PPS is lower than the standard moral hazard PPS:

$$\beta_i^o(I_A) = \frac{1}{1 + \rho_i v_i \left(\sigma_{\varepsilon,i}^2 + \frac{Var(M(\theta, I_A))}{4} \right)}.$$

The optimal contractible investment is given by $I_A^* = 1$, if and only if the fixed cost F is below a threshold F^*, as given by $p(\beta^o(0), 0) \equiv p(\beta^o(1), 1) - F^*$, or:

$$F^* \equiv \Delta M - \sum_{i=A,B} \{ \Delta RP_i - \Delta \Phi_i \},$$

where

$$\Delta RP_i \equiv \frac{\rho_i}{8} \left[(\beta_i^o(1))^2 \cdot Var(M(\theta, 1)) - (\beta_i^o(0))^2 \cdot Var(M(\theta, 0)) \right]$$

and

$$\Delta \Phi_i \equiv \Phi_i(\beta_i^o(1)) - \Phi_i(\beta_i^o(0))$$

denote the effect of investment on trade-related risk premia and profits from general operations. We assume throughout the paper that $F^* \geq 0$, so that for some low fixed cost realizations the investment will be undertaken if it is contractible.

Intrafirm trade and general operations are *technologically separable* in our model. However, whenever $\Delta \Phi \neq 0$ they become *contractually intertwined* through the choice of PPS. As we will show now, the driving force behind this link is *risk*. Specifically, more fixed assets in place translate into lower marginal

production costs and thereby greater trading volume for any realization of θ. Greater expected trading volume in turn implies more trade-related variance.[11] Since this link between investments and trade-related risk will be central to many of the arguments to follow, we state it as a formal result:

Lemma 1 $\Delta V \equiv Var(M(\theta, 1)) - Var(M(\theta, 0)) > 0.$

All proofs are found in Appendix A. Lemma 1 implies that $\beta_i^o(1) < \beta_i^o(0)$ and, therefore, $\Delta \Phi_i < 0$. Trade-related risk translates into muted incentives and thereby lower profit from general operations. It is easy to show that specific investment increases the trade-related risk premium (factoring in the equilibrium adjustments to PPS) provided the trade-related uncertainty is not too high relative to operating uncertainty. To that end, we assume throughout the paper that:

$$\frac{Var(M(\theta, I_A = 1))}{4} \leq \min\{\sigma_{\varepsilon,A}^2, \sigma_{\varepsilon,B}^2\}, \tag{4}$$

which is a sufficient condition for $\Delta RP_i > 0$, for any i.[12] Loosely speaking, the trade-related uncertainty, for each division, is smaller than its own operating uncertainty.

We now ask how this benchmark solution is to be adjusted to account for the fact that contracts in practice are often incomplete.

4 Non-Contractible Investment

In many instances divisional investments are neither observable nor contractible. While *aggregate* PP&E expenditures are routinely monitored and verifiable, it

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[11]The result that investments increase trade-related risk hinges on our assumption—which is standard in incomplete contracting models—that investments are in operating assets that lower marginal relevant cost (or increase marginal revenues). Other investments, such as hedging instruments, are designed to reduce risk.

[12]With slight abuse of notation, write Manager i's trade-related risk premium as a continuous function: $RP_i(I_A) \equiv [\beta_i^o(I_A)]^2 \cdot Var(M(\theta, I_A))$, $I_A \in [0, 1]$. It is readily shown that the derivative $RP_i'(I_A)$ is proportional to $[1 + \rho_i v_i (\sigma_{\varepsilon,i}^2 - Var(M(\theta, I_A))/4)]$. Applying Lemma 1 then yields the sufficient condition stated in (4).

is often difficult to trace *individual* pieces of equipment (or personnel training costs) to specific transactions.[13] How does non-contractibility of investments affect optimal contracts and the resulting investments? Key insights from earlier studies are: (i) incomplete contracting in conjunction with *ex-post* negotiations over the realizable margin leads to underinvestment because of hold-up; and (ii) investment incentives are independent of PPS. As we show now, both findings may not hold once we account for intrafirm trade-related compensation risk.

For given investments, the Date-4 bargaining game unfolds as before, resulting again in the *ex-post* efficient quantity, $q^*(\theta, I_A)$, and a corresponding transfer price, $t(\theta, I_A)$. If investments are non-contractible, however, the principal faces an additional constraint in that the manager of Division A will now choose I_A in his own best interest. Specifically, let

$$f(I_A \mid \beta_A) \equiv \beta_A \left(\frac{1}{2} E_\theta[M(\theta, I_A)] - \frac{\rho_1}{8} \beta_A Var(M(\theta, I_A)) \right)$$

be the investing manager's trade-related certainty equivalent. For the investment to be made by Manager A, the following has to hold:

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\begin{aligned}
I_A(\beta_A \mid F) = 1 \iff F &\leq \frac{f(1 \mid \beta_A) - f(0 \mid \beta_A)}{\beta_A} \\
&= \frac{\Delta M}{2} - \frac{\rho_A}{8} \beta_A \cdot \Delta V \equiv \widetilde{F}(\beta_A).
\end{aligned}
\tag{5}
$$

On occasion, it will be convenient to invert this *investment incentive condition* as follows:

$$I_A(\beta_A \mid F) = 1 \iff \beta_A \leq \widetilde{\beta}_A(F). \tag{6}$$

The principal's optimization program with non-contractible investments reads (superscript "**" denotes non-contractible investments):

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[13]Decisions that are non-contractible often are delegated within firms. In fact, in a survey of 140 division managers Bouwens and Van Lent (2007) find that in about half the cases the investment authority rests with division managers.

Program $\mathcal{P}^{**}$:

$$\max_{\beta, I_A \in \{0,1\}} \Pi^{**}(\beta) \equiv p(\beta, I_A(\beta_A \mid F)) - FI_A,$$
$$\text{subject to (6)}.$$

Denote by (I_A^{**}, β^{**}) the solution to this program. Clearly, the seller's investment incentives as of (6) are unaffected by the buying manager's PPS, β_B. But how about the seller's own incentive contract? Our next result addresses this question—it plays a central role in the arguments to follow:

Lemma 2 *If the investment is non-contractible, $\widetilde{F}(\beta_A)$ is decreasing in the investing manager's PPS, β_A.*

The proof follows directly from the comparative statics of (5) and is omitted. Stronger effort incentives subject Manager A to greater trade-related risk at the margin. As a result, he will invest less, the higher his PPS.

As noted above, a key tenet of the incomplete contracting literature is that non-contractibility results in underinvestment. We now revisit this question, explicitly accounting for trade-related risk. Note that non-contractible investments, in equilibrium, deviate from those under contractibility for two distinct, countervailing reasons. First, Manager A's divisional profit measure reflects only half the benefit from investing but all fixed costs—the classic *hold-up problem*. At the same time, the manager takes into account only his own incremental risk premium when investing and ignores that of the other manager. That is, *the investing manager externalizes part of the trade-related risk cost.* This risk externalization effect encourages *over*investment, all else equal.

To evaluate this tradeoff, we begin by holding constant the PPS from the contractible investment benchmark and presenting sufficient conditions for under- or overinvestment. We formalize underinvestment for given PPS by $\widetilde{F}(\beta_A^o(1)) < F^*$ (if the principal were to leave Manager A's contract unadjusted, the fixed cost

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threshold that induces investment indifference is lower than F^*) and overinvestment by $\widetilde{F}(\beta_A^o(0)) > F^*$ (vice versa).

Proposition 1 *For $\sigma_{\varepsilon,i}^2$ $i = A, B$, sufficiently large, $\widetilde{F}(\beta_A^o(1)) < F^*$.*

The proof is omitted. It is illustrated most easily by invoking the limit case of $\sigma_{\varepsilon,i}^2 \to \infty$, in which case we have $\beta_i^o(I_A) \to 0$, for any I_A, and $\Delta\Phi_i \to 0$ (by l'Hopital's rule). Thus, the model converges to the standard hold-up model in which (i) the risk premia associated with intrafirm trade become negligible and (ii) investment and effort incentives become separable. The staple underinvestment result from that literature then stands. By continuity, the same arguments apply for $\sigma_{\varepsilon,i}^2$ sufficiently high. In words, high operating risk calls for muted incentives, which attenuates the externalized incremental risk premium attributable to the investment. At the same time, the hold-up problem remains unabated. Absent adjustments to Manager A's contracts, he underinvests.

For overinvestment incentives to arise, the risk externalization effect has to be significant. Intuitively, this will be the case when the investing party—here, Manager A—is relatively insensitive to risk (because β_A is small due to high $\sigma_{\varepsilon,A}$), the gains from intrafirm trade are highly uncertain, and the non-investing party is sensitive to this incremental risk (because β_B is high due to small $\sigma_{\varepsilon,B}$). It is useful to impose additional structure—in particular, symmetry regarding the managers' moral hazard parameters and a quadratic revenue function:

Condition 1 $\rho_i = \rho$ and $v_i = v$.

Condition 2 $R(q, \theta_B) = (a - \frac{b}{2}q + \theta_B)q$, with $a, b > 0$.

In the following we denote $\Delta\sigma_\varepsilon \equiv \sigma_{\varepsilon,A}^2 - \sigma_{\varepsilon,B}^2$, and we say a threshold for the total trade-related variance $\sum_i \sigma_{\theta_i}^2$ is *feasible* if it satisfies the maintained parameter restriction in (4).

Proposition 2 *Suppose Conditions 1 and 2 hold with b sufficiently large. If $\sigma^2_{\varepsilon,A}$ and $\Delta\sigma_\varepsilon$ both are sufficiently high, then there exists a feasible threshold $\underline{S}$, such that if $\sum_i \sigma^2_{\theta,i} \geq \underline{S}$, then $\widetilde{F}(\beta^o_A(0)) > F^*$.*

With highly uncertain gains from intrafirm trade the externalized incremental risk cost weighs heavily. Absent adjustments to his PPS, Manager A would now overinvest, i.e., the canonical underinvestment result associated with incomplete contracting breaks down.[14]

While the PPS was held constant in Propositions 1 and 2, the discussion surrounding the results has implications also for the *optimal* PPS. Underinvestment implies a smaller expected trading quantity as compared with contractible investments. The attendant drop in trade-related risk allows the principal to expose the non-investing manager (Manager B) to stronger incentives. The converse holds for overinvestment, which would result in muted incentives for Manager B. Because the equilibrium investment level can be taken as exogenous for Manager B, there is a one-to-one mapping from equilibrium investment distortions to distortions in β_B. On the other hand, predictions about Manager A's PPS are complicated by the fact that, by Lemma 2, his investment choice is endogenously determined by β_A. To address the link between contractual incompleteness and managers' PPS, in equilibrium, we turn separately to the two cases described in Proposition 1—under- and overinvestment.

4.1 Optimal PPS with High Operating Uncertainty (Underinvestment)

For the case of volatile operating environments Proposition 1 has shown that underinvestment ensues absent adjustments to the investing manager's PPS. But in the optimal solution the principal *will* adjust the managers' contracts, factoring in that the seller's PPS determines not just his effort choice and risk premium,

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[14]Note that Condition 1 and 2 are merely sufficient here; they simplify the tradeoff while keeping notation at a minimum.

but also I_A (by Lemma 2). In the following we ask: (i) when does incomplete contracting — i.e., the imposition of the investment constraint (6) — reduce the principal's expected payoff and (ii), if it does so, what are the optimal contract adjustments. To formalize the underinvestment concern, we assume:

Condition UI *(Underinvestment):* $\sigma^2_{\varepsilon,i}$ is sufficiently large for $i = A, B$, such that $\widetilde{F}(\beta^o_A(1)) < F^*$.

It is readily seen that incomplete contracting causes no loss to the principal for sufficiently high or low fixed cost values. For $F > F^*$ the investment will be lost even if it were contractible. Condition UI ensures the same holds *a fortiori* under incomplete contracting. On the other hand, if F drops below $\widetilde{F}(\beta^o_A(1))$, then (6) calls for $I^{**}_A = 1$ even without any adjustments to $\beta^o_A(1)$. Again, delegating the investment choice to a self-interested manager comes for free. This is not true however for intermediate values of $F \in (\widetilde{F}(\beta^o_A(1)), F^*]$. Then, to get Manager A to invest, as he would in the contractible benchmark, the principal needs to reduce his PPS at the expense of suboptimal effort input. The principal thus faces a tradeoff between general-purpose effort and trade-specific investment incentives. The next result evaluates this tradeoff (recall that $\beta^*(F)$ denotes the contractible PPS; $\beta^{**}(F)$, the non-contractible PPS):

Proposition 3 *Assume UI:*

(i) *For $F \notin [\widetilde{F}(\beta^o_A(1)), F^*]$, $\beta^{**}_i(F) \equiv \beta^*_i(F) \equiv \beta^o_i(I^*_A(F))$, $i = A, B$. Non-contractibility does not impose additional costs on the principal.*

(ii) *There exists a unique $\hat{F}_{UI} \in [\widetilde{F}(\beta^o_A(1)), F^*]$ such that for any $F \in (\widetilde{F}(\beta^o_A(1)), \hat{F}_{UI}]$, $\beta^{**}_A(F) = \widetilde{\beta}_A(F) < \beta^*_A(F) = \beta^o_A(1)$ and $\beta^{**}_B(F) = \beta^*_B(F) = \beta^o_B(1)$. As in the benchmark case, Manager A invests, but the principal's payoff is lower than with contractible investment.*

(iii) If $F \in (\hat{F}_{UI}, F^*]$, then $\beta_i^{**}(F) = \beta_i^o(0) > \beta_i^*(F) = \beta_i^o(1)$, $i = A, B$. Manager A does not invest (underinvestment) and the principal's payoff is lower than with contractible investment.

Incomplete contracting imposes additional cost on the principal only for intermediate fixed cost values (cases (ii) and (iii)), for which the principal trades off investment and effort distortions. For F above but close to $\widetilde{F}(\beta_A^o(1))$, Manager A's incentives should be muted, because the investment is sufficiently valuable to warrant compromises on effort. As fixed costs increase, the critical PPS that keeps Manager A willing to invest, $\widetilde{\beta}_A(F)$, has to be lowered further until, at $F = \hat{F}_{UI}$, the attendant effort distortions outweigh the expected net investment return—thus the principal finds it preferable to forego the investment by raising incentives for both managers to $\beta_i = \beta_i^o(0)$. Underinvestment reduces the trade-related risk premium and therefore, as a silver lining, permits higher-powered effort incentives. Figure 2 compares the optimal PPS with the benchmark one for the special case of Condition 1 augmented with $\Delta\sigma_\varepsilon = 0$ (so that $\beta_A^o(I_A) = \beta_B^o(I_A)$ for any I_A).

Proposition 3 demonstrates that with severe divisional agency problems contractual incompleteness results in (weakly) higher-powered incentives for the non-investing manager. The effect on the investing manager is ambiguous: he will face higher-powered incentives for relatively high levels of fixed costs and muted incentives for low values of F.
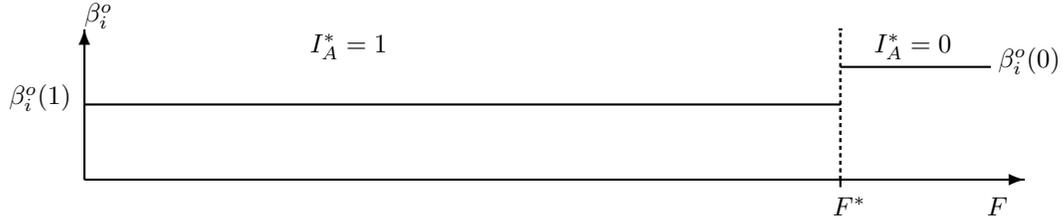
Fig. 2a: Contractible Investment
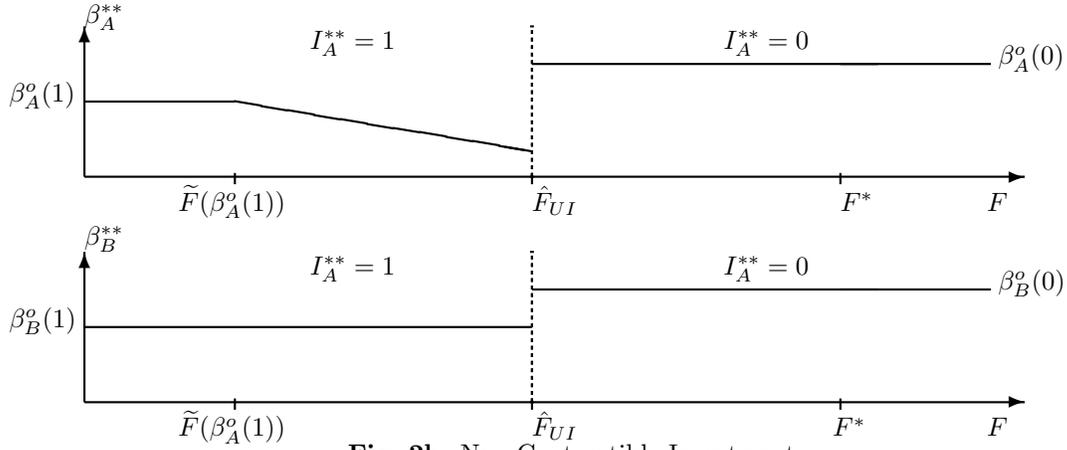
Fig. 2b: Non-Contractible Investment

Figure 2: Optimal PPS Under Condition UI

4.2 Optimal PPS with High Trade-Related Uncertainty (Overinvestment)

As shown in Proposition 2, if the gains from internal trade are highly uncertain and the investing manager (but not the non-investing manager) faces high operating uncertainty, then incomplete contracting results in overinvestment, because Manager A does not internalize the entire incremental trade-related risk premium. More formally, we assume throughout this subsection:

Condition OI *(Overinvestment):* Conditions 1 and 2 hold, and b, $\sigma_{\varepsilon,A}^2$, $\Delta\sigma_\varepsilon$ and $\sum_i \sigma_{\theta,i}^2$ are all sufficiently large, such that $\widetilde{F}(\beta_A^o(0)) > F^*$.

The principal can mitigate this overinvestment problem by adjusting—in this case, raising—the investing manager's PPS. The question, again, is whether or

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when it is optimal to do so. The following result characterizes the optimal contract for non-contractible investments in case of high trade-related uncertainty:

Proposition 3′ *Assume OI:*

(i) *For* $F \notin [F^*, \widetilde{F}(\beta_A^o(0))]$, *then* $\beta_i^{**}(F) \equiv \beta_i^*(F) \equiv \beta_i^o(I_A^*(F))$, $i = A, B$. *Non-contractibility does not impose additional costs on the principal.*

(ii) *There exists a unique* $\hat{F}_{OI} \in [F^*, \widetilde{F}(\beta_A^o(0))]$ *such that for any* $F \in [\hat{F}_{OI}, \widetilde{F}(\beta_A^o(0))]$, $\beta_A^{**} = \widetilde{\beta}_A(F) + \varepsilon > \beta_A^*(F) = \beta_A^o(0)$ *and* $\beta_B^{**}(F) = \beta_B^*(F) = \beta_B^o(0)$. *As in the benchmark case, Manager A does not invest, but the principal's payoff is lower than with contractible investment.*

(iii) *If* $F \in [F^*, \hat{F}_{OI})$, *then* $\beta_i^{**}(F) = \beta_i^o(1) < \beta_i^*(F) = \beta_i^o(0)$, $i = A, B$. *Manager A invests (overinvestment) and the principal's payoff is lower than with contractible investment.*

Figure 3 illustrates Proposition 3′. It resembles Figure 2, except now the principal trades off *over*investment against distortions in managerial effort. Again, incomplete contracting is incrementally costly only for intermediate fixed cost values. For $F \in (\hat{F}_{OI}, \widetilde{F}(\beta_1^o(0))]$, the investing manager's incentives are strengthened so as to curb his overinvestment tendency. For $F \in [F^*, \hat{F}_{OI})$, in contrast, the principal optimally acquiesces to overinvestment, as fighting it would be too costly in terms of lost managerial effort. With overinvestment being the pressing concern, thus, managers without investment opportunities will face (weakly) lower-powered incentives, whereas an ambiguous result obtains for the investing manager: his PPS will be weakly greater than in the contractible benchmark setting for fixed cost sufficiently high, and adjusted downward for small F.

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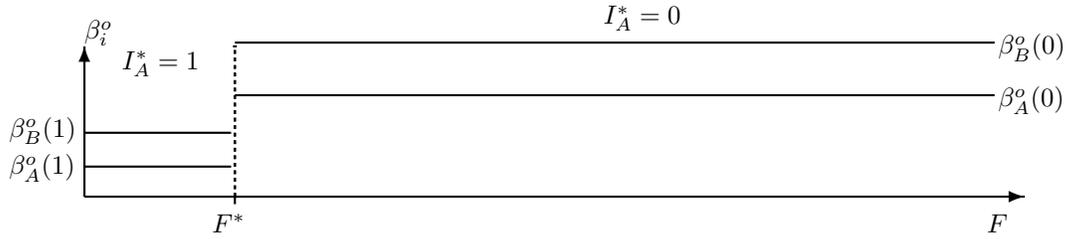
Fig. 3a: Contractible Investment
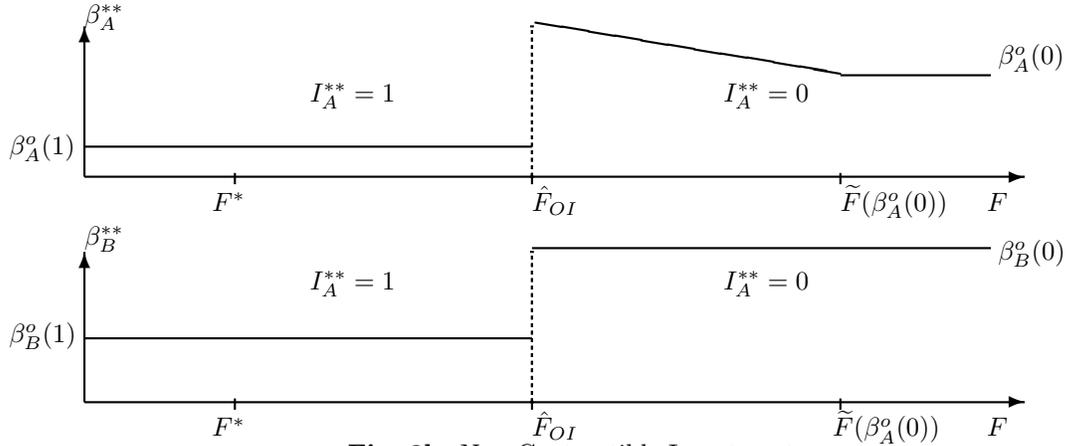
Fig. 3b: Non-Contractible Investment

Figure 3: Equilibrium PPS Under Condition OI

4.3 Discussion

A key tenet in the incomplete contracting literature is that divisional moral hazard problems and the transfer pricing problem can be conceptually separated and studied in isolation. The standard argument, based on the fact that the (sunk) fixed costs from investing and the relevant contribution margins are similarly scaled by a manager's PPS, does not account for the fact that specific investments typically lead to increased levels of intrafirm trade and therefore to additional compensation risk. Using this insight, this section has shown that (i) contractual incompleteness may lead to equilibrium overinvestment and (ii) division managers operating in similar operating environments, but differing in their internal trade-enhancing investment opportunities, may end up receiving

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different incentive contracts. Introducing a wedge between managers' PPS can be an optimal response to the tradeoff between eliciting specific investments and general purpose effort. Moreover, Figures 2 and 3 demonstrate a non-monotonic relation between the *ex-ante* profitability of investment opportunities (as measured by the fixed cost F) and the PPS for the managers facing those investment opportunities.

The next section uses these insights to develop a theory of the optimal allocation of investment decision rights in multidivisional firms.

5   Assigning Investment Responsibility

To illustrate the investment/risk link in the simplest manner, we have focused so far on a setting in which only one division had an investment opportunity. Often however both up- and downstream divisions can invest in value-increasing relationship-specific assets. We now extend the analysis to allow for bilateral investments and employ the model to address a central organizational design question: Should both divisions be organized as *investment centers*, with each manager choosing his own investment, or should the decision rights over both up- and downstream investments instead be bundled in the hands of one manager— the *investment center* manager—while treating the other as a *profit center* manager with no investment authority whatsoever?

Let $I \equiv (I_A, I_B)$ where $I_i \in \{0, 1\}$ denotes a specific investment undertaken in Division $i = A, B$, at respective fixed costs of F_i. For instance, I_B could describe the demand-enhancing effect of sales promotions downstream or the hiring and training of sales staff. The divisional relevant costs and revenues from the transaction now read:

$$C(q, \theta_A, I_A) = (c - I_A - \theta_A)q \quad \text{and} \quad R(q, \theta_B, I_B) = r(q) + (I_B + \theta_B)q. \quad (7)$$

To ensure existence of well-behaved equilibria, we assume throughout this section

that $r''(q) < 0$ and $r'''(q) \geq 0$ for any q, which includes quadratic revenue functions (as in Condition 2) as a special case. Note that the marginal return to I_A and I_B, each, is normalized to unity per unit of the intermediate good transferred. We further assume $F_A = F_B \equiv F$. Hence, the investments are equally productive. None of the results to follow hinges qualitatively on this restriction. The efficient quantity *ex post* equals $q^*(\theta, I) \in \arg\max_q \ R(q, \theta_B, I_B) - C(q, \theta_A, I_A)$. Let $M(\theta, I) \equiv M(q^*(\cdot), \theta, I)$. At Date 4, as before, the managers negotiate over the transaction and split the attainable contribution margin $M(\theta, I)$ equally. For the remainder of the paper, we impose the following two conditions:

Condition 1 $\rho_i = \rho$ and $v_i = v$.

Condition 3 $\Delta\sigma_\varepsilon \geq 0$.

Division A thus faces higher general operating uncertainty, without loss of generality.

To characterize the optimal contractible (benchmark) investments, denote by $P(I) \equiv p(\beta^o(I), I)$ the principal's expected payoff ignoring fixed costs, where

$$p(\beta, I) \equiv E_\theta[M(\theta, I)] + \sum_{i=A,B} \left[\Phi_i(\beta_i) - \frac{\rho}{8}\beta_i^2 \cdot Var(M(\theta, I)) \right] \tag{8}$$

and

$$\beta_i^o(I) \in \arg\max_{\beta_i} \ \left\{ \Phi_i(\beta_i) - \frac{\rho}{8}\beta_i^2 \cdot Var(M(\theta, I)) \right\},$$

for $i = A, B$, as the optimal PPS conditional on a given investment profile.[15] By Condition 3, Manager A faces a lower conditionally optimal PPS, i.e., $\beta_A^o(I) < \beta_B^o(I)$ for any I. With contractible bilateral investments, the principal invests according to:

$$I^* \in \arg\max_{I \in \{0,1\}^2} \ \Pi^*(I) \equiv P(I) - (I_A + I_B)F. \tag{9}$$

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[15]Solving for $\beta_i^o(I)$ yields: $\beta_i^o(I) = \left[1 + \rho v \left(\sigma_{\varepsilon,i}^2 + Var(M(\theta, I))/4 \right) \right]^{-1}$. Given the maintained assumption that investments are of equal productivity, it follows that $\beta_i^o(1,0) \equiv \beta_i^o(0,1)$, $i = 1,2$, and $\Pi^*(1,0) = \Pi^*(0,1)$, i.e., the principal is indifferent between inducing $(1,0)$ or $(0,1)$.

We now turn again to delegated investment choices under incomplete contracting. Manager i's trade-related certainty equivalent equals[16]

$$f(I \mid \beta_i) \equiv \beta_i \frac{E[M(\theta, I)]}{2} - \beta_i^2 \frac{\rho}{8} \cdot Var(M(\theta, I)). \tag{10}$$

We consider two alternative organizational regimes, distinct in the way they assign decision rights over investments to the managers. Note that whenever investments are non-contractible, decision rights are invariably bundled with the associated fixed cost charges. Put differently, a manager in charge of choosing n units of investment will have fixed costs of nF charged against his income measure, where $n \in \{0, 1, 2\}$.

The "IC-IC" regime assigns decision rights symmetrically. Each division is organized as an investment center, and each manager is evaluated on the basis of divisional profit net of fixed costs (superscript "I" for IC-IC):

$$\pi_i^I = a_i + \widetilde{\varepsilon}_i + \frac{M(\theta, I)}{2} - I_i F, \quad i = A, B.$$

Manager A chooses the upstream investment I_A; Manager B chooses I_B.[17] We confine attention to pure-strategy Nash equilibria in investments, which can be represented by the system of optimization problems:

$$I_i^I \in \arg \max_{I_i \in \{0,1\}} f(I_i, I_j^I \mid \beta_i) - \beta_i I_i F, \quad i, j = A, B, \quad i \neq j. \tag{11}$$

The alternative organizational regime, labelled "PC-IC," allocates decision rights asymmetrically in that one division manager—say, k (the investment center manager)—chooses both I_A and I_B. The other manager has no investment authority, and his unit is organized as a profit center. The respective performance measures read (superscript "P" for PC-IC):

$$\pi_i^P = a_i + \widetilde{\varepsilon}_i + \frac{M(\theta, I)}{2} - (I_A + I_B) F \mathbb{1}_{i=k}.$$

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[16]Given Condition 1 there is no need to index the $f(\cdot)$-function for the different managers. Moreover, given our maintained assumption that investments are of equal productivity, we have $f(1, 0 \mid \beta_i) \equiv f(0, 1 \mid \beta_i)$, for any β_i.

[17]Given our assumption that the investments are equally productive, it is without loss of generality under IC-IC that Manager i chooses I_i, $i = A, B$, rather than I_j, $j \neq i$.

Rather than inducing a two-player investment game, in this lopsided regime the investment choice takes the form of a single-person optimization problem faced by Manager k:

$$I^P \in \arg\max_{I \in \{0,1\}^2} f(I_A, I_B \mid \beta_k) - \beta_k(I_A + I_B)F. \tag{12}$$

For now, we remain agnostic as to which manager should be appointed investment center manager. Below we derive the optimal allocation of decision rights.[18]

In traditional incomplete contracting models, bilateral investments tend to be mutually reenforcing. Higher downstream investment, say, increases the expected trading quantity, which in turn raises the marginal investment return upstream; and vice versa. Technically speaking, the firmwide expected contribution margin has increasing differences in the investments. By way of bargaining, increasing differences translate into an investment game of *strategic complementarity* under incomplete contracting if each manager were to choose his own investment, suggesting equilibrium investment profiles of either $(0,0)$ or $(1,1)$.

Once we account for trade-related compensation risk, however, there is a countervailing force, because the variance of the firmwide contribution margin also has increasing differences in the investments, and it enters with a negative sign in (8) and (10), respectively: Higher downstream investment adds not just to the total compensation risk, but also to the *marginal* risk premium associated with higher upstream investment, and vice versa.[19] This in turn points towards investments being strategic substitutes. The following result evaluates this tradeoff and derives conditions that predict the strategic interaction between

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[18]Using the terminology of Che and Hausch (1999), under the IC-IC mode each division manager makes a *selfish* investment in his own operations. Under PC-IC, on the other hand, Manager k makes a *selfish* investment in his own division and a *cooperative* investment in the respective other division. Note that *per se* managers have no less incentives to make cooperative than selfish investments, as the gross investment returns are always split in half either way. Another recent study that allows for investment rights to be assigned across actors in an organization is Dutta and Reichelstein (2010).

[19]It is easy to check that $r'''(q) \geq 0$ ensures that $q^*(\theta, I)$ has increasing differences in (I_A, I_B), for any θ, and therefore $Var(M(\theta, I))$ also has strictly increasing differences in I.

the investments.

Lemma 3 *Given Conditions 1 and 3 and $\sigma_{\varepsilon,i}^2$ sufficiently high for $i = A, B$:*

(i) *$P(I_A, I_B)$ has strictly increasing differences;*

(ii) *$f(I_A, I_B \mid \beta_i)$ has strictly increasing differences in I for $\beta_i \leq \beta_B^o(0,0)$.*

By inspection of (8) and (10), the higher the PPS, the greater the trade-related risk premium, both in absolute and marginal terms (with regard to changes in I_i). Thus, for small β_i, both the firmwide and divisional investment returns have increasing differences in investments.

In the remainder of the paper we will study the optimal allocation of decision rights over investments given the conditions in Lemma 3 are satisfied. Note that Subsection 4.1 has shown that qualitatively similar conditions on the model parameters predict (unilateral) underinvestment. One would expect the strategic complementarity established in Lemma 3 to compound this underinvestment problem.[20]

5.1 Investment Incentives Under IC-IC and PC-IC

For divisions facing volatile operating environments (high $\sigma_{\varepsilon_i}^2$) the preceding results have shown that: (i) divisions tend to underinvest unilaterally with incomplete contracts (Proposition 1) and (ii) bilateral investments are mutually reinforcing (Lemma 3). Taken together, this suggests that underinvestment may also arise with bilateral non-contractible investments. In the contractible investment benchmark setting, it is readily seen that strategic complementarity implies

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[20]We do not address the mirror case of investments that are strategic substitutes, which can be shown to be the case under conditions similar to those that predicted overinvestment in Subsection 4.2. The reason is that the strategic substitutability (partially) undermines the overinvestment incentives: holding fixed I_k, Manager l has incentives to overinvest conditional on I_k, and vice versa. Greater investment by one party, however, reduces the investment incentives of the other party at the margin, as the investments are strategic substitutes. The net effect is generally ambiguous.

that the equilibrium contractible investment profile will be either $I^* = (1,1)$ for F below some threshold F^*, or $I^* = (0,0)$ for $F > F^*$.[21]

To study the managers' investment incentives under incomplete contracting in a parsimonious manner, in the main text we hold constant the PPS from the benchmark contractible investment setting. Our criterion for ranking the regimes in the main text therefore asks which of the two regimes replicates the contractible benchmark outcome for a greater parameter set (specifically, for a wider range of fixed costs, F) *without any adjustments to the PPS*. Appendix B presents the general analysis with endogenous PPS, chosen so as to optimally trade off investment and effort distortions under the respective regimes. The results presented in Appendix B are fully consistent with, and simply generalize, those presented in this section.

The IC-IC Regime.   Under IC-IC each division chooses (and pays for) its own investment. We begin by characterizing the set of possible investment equilibria that can arise for volatile operating environments (high $\sigma_{\varepsilon,i}^2$'s). For the sake of illustration, take the managers' PPS as given for now, equal to $(\underline{\beta}, \bar{\beta})$, $\underline{\beta} \leq \bar{\beta} \leq \beta_B^o(0,0)$, where $\beta_B^o(0,0)$ is the maximum PPS in the benchmark solution.[22] By Lemma 3, investments are strategic complements for high $\sigma_{\varepsilon,i}^2$, as even $\beta_B^o(0,0)$ will be small. In that case, the investment profile $I = (1,1)$ constitutes an equilibrium for F low enough such that

$$f(1,1 \mid \bar{\beta}) - \bar{\beta}F \geq f(1,0 \mid \bar{\beta}). \tag{13}$$

At the same time, $I = (0,0)$ is an equilibrium for F high enough such that:

$$f(1,0 \mid \underline{\beta}) - \underline{\beta}F \leq f(0,0 \mid \underline{\beta}). \tag{14}$$

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[21]To see that a "mixed" investment profile, (1,0) (or (0,1), as the two are economically identical in this setting), can never be optimal in the contractible investment benchmark with high $\sigma_{\varepsilon_i}^2$, suppose to the contrary that it were. Then: $P(1,0) - F \geq P(0,0)$ and $P(1,0) - F \geq P(1,1) - 2F$ both must hold, which implies $P(1,1) - P(1,0) \leq P(1,0) - P(0,0)$; a contradiction with Lemma 3(i-a).

[22]That is, $\beta_B^o(0,0) = \max \beta_i^o(I)$, for any $i = A, B$, and $I \in \{0,1\}^2$.

Note that (13) is cast in terms of the manager with higher-powered incentives, $\bar{\beta}$, as it is this manager who is more eager to deviate from $(1, 1)$. Put differently, the bottleneck in terms of eliciting investments is the manager with greater induced risk aversion, due to a higher PPS. Conversely, (14) is cast in terms of the manager with the lower PPS, $\underline{\beta}$, as it is that manager who is more willing to deviate from $(0, 0)$.[23]

Games of strategic complementarity are routinely afflicted by multiple equilibria. This is true also for the IC-IC mode. The following result, however, shows that, whenever multiple equilibria exist, they can be Pareto-ranked:

Lemma 4 *Consider the IC-IC mode given Condition 1 for some (exogenous) PPS, $\underline{\beta} \le \bar{\beta} \le \beta_B^o(0, 0)$. For $\sigma_{\varepsilon,i}^2$, $i = A, B$, sufficiently high, the set of possible equilibrium investment profiles is $\{(0, 0), (1, 1)\}$. Suppose (13) holds so that $(1, 1)$ is an equilibrium. Then:*

(i) If (14) is violated, then $(1, 1)$ is the unique equilibrium.

(ii) If (14) also holds, then $(1, 1)$ is the Pareto-dominant equilibrium.

For some intermediate values of fixed costs, $(1, 1)$ and $(0, 0)$ can be equilibria at the same time, i.e., conditions (13) and (14) both can be satisfied.[24] In that case, the Pareto-dominance result in Lemma 4 allows us to disregard the no-

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[23]More formally, the term $[f(I \mid \beta_i) - \beta_i F]$ is decreasing in β_i for any I and F.

[24]In general, there may exist an asymmetric equilibrium under IC-IC, in which only the low-PPS manager invests. Asymmetric equilibria can exist only if neither (13) nor (14) are satisfied. Now, whenever $\underline{\beta}$ is close to $\bar{\beta}$, then at least one of these conditions will always (i.e., for any F) be satisfied, thus ruling out asymmetric equilibria. However if $\underline{\beta} \ll \bar{\beta}$, then one can construct examples in which both (13) and (14) are violated for some intermediate values of F. To see this, start from low fixed cost values, such that (13) holds, and raise F. At some point, (13) becomes binding and the manager facing $\bar{\beta}$ will cease to invest. This causes a discrete drop in the marginal investment return for the $\underline{\beta}$-manager because of strategic complementarity. But if $\underline{\beta}$ is sufficiently low, then this manager may still prefer to invest, as long as (14) does not hold. (Recall that (13) and (14) are defined for managers facing different PPS.) One can show however that such asymmetric equilibria cannot exist for high values of $\sigma_{\varepsilon,B}^2$, as assumed in this subsection. For $\sigma_{\varepsilon,B}^2$ sufficiently high, the maximum PPS in the benchmark solution, $\beta_B^o(0, 0)$, will be small, and therefore $\underline{\beta}$ has to be close to $\bar{\beta}$.

investment equilibrium. Still, IC-IC suffers from underinvestment, as we now show.

The investment profile $I = (1,1)$ can be implemented under IC-IC without having to deviate from the corresponding benchmark PPS of $\beta_i^o(1,1)$, $i = A, B$, if and only if:

$$F \leq \frac{f(1,1 \mid \beta_B^o(1,1)) - f(1,0 \mid \beta_B^o(1,1))}{\beta_B^o(1,1)} \equiv F^I. \tag{15}$$

Put differently, (13) becomes binding at F^I for Manager B, i.e., the manager who by Condition 1 faces higher-powered incentives. Absent any adjustments to the PPS, high operating uncertainty results in bilateral underinvestment under the IC-IC regime:

Lemma 5 *Given Condition 1 and $\sigma_{\varepsilon,i}^2$, $i = A, B$, sufficiently high, $F^I < F^*$.*

Because of the hold-up problem, each manager invests suboptimally conditional on a conjectured investment made by the other manager. By strategic complementarity, the fact that Manager j underinvests reduces investment incentives for Manager i even further.

The PC-IC Regime. We now turn to the PC-IC mode that treats the managers asymmetrically. The "investment center manager" has decision rights over both up- and downstream investments and chooses them according to (12). For now, we remain agnostic as to which manager should be assigned this authority and simply denote that manager by $k = A, B$. Because $f(\cdot)$ has increasing differences in the investments for high operating uncertainty (Lemma 3), the induced investment equilibrium is either $I^P = (1,1)$ for F below, and $I^P = (0,0)$ for F above some fixed cost threshold (following similar arguments as in footnote 21). Given a PPS of β_k, the investment center manager therefore will choose $(1,1)$ whenever $f(1,1 \mid \beta_k) - 2\beta_k F \geq f(0,0 \mid \beta_k)$. Holding constant the benchmark PPS of $\beta = \beta^o(1,1)$, therefore, $I^P = (1,1)$ is implemented, if and only if

$$F \leq \frac{f(1,1 \mid \beta_k^o(1,1)) - f(0,0 \mid \beta_k^o(1,1))}{2\beta_k^o(1,1)} \equiv F^{P,k}. \tag{16}$$

Without adjusting the investing manager's incentives, PC-IC also results in underinvestment, regardless of which manager is in charge of investing:

Lemma 6 *Given Condition 1 and $\sigma_{\varepsilon,i}^2$, $i = A, B$, sufficiently high, $F^{P,k} < F^*$, for any $k = A, B$.*

The proof follows closely the logic of Lemma 5 and is omitted. Given that volatile operations result in underinvestment also under PC-IC, the question is which manager should be assigned investment authority so as to minimize these distortions? The answer follows from Lemma 2: the manager facing greater operating uncertainty (and hence the lower PPS)—Manager A, by assumption—is more willing to invest, all else equal. The principal's contracting problem under PC-IC thus is to maximize the objective function in (9) subject to the investment incentive constraint (12) with $k = A$.

5.2 Performance Comparison

We now ask which of the two organizational regimes better alleviates the underinvestment problem. On a technical level, given the performance ranking criterion adopted in the main text, this amounts to comparing the fixed cost thresholds F^I and $F^{P,k=A}$ in Lemmas 5 and 6. Given the simple ranking criterion in the main text, holding fixed the benchmark PPS, the principal always prefers the regime with the fixed cost threshold closer to F^*.

The performance comparison will be driven by two conceptual differences between the regimes: (i) the game form differs—a simultaneous-move game under IC-IC versus a single-agent optimization problem under PC-IC—and (ii) the principal enjoys an additional degree of freedom under PC-IC in that she can assign the investment authority to the agent who is more willing to invest, because of a lower PPS. We refer to the latter as the *risk tolerance benefit* of PC-IC. On first glance, both factors appear to favor PC-IC over IC-IC. Yet Proposition 4 shows that the performance comparison can go either way:

Proposition 4 *Assume Conditions 1 and 3 and $\sigma_{\varepsilon,i}^2$, $i = A, B$, sufficiently large:*

(i) *If $\Delta\sigma_\varepsilon$ is small, the principal prefers IC-IC.*

(ii) *Given Condition 2, and $\Delta\sigma_\varepsilon$ and b sufficiently high (highly concave revenues), the principal prefers PC-IC.*

Why does IC-IC outperform PC-IC for divisions facing similar operating environments? As $\Delta\sigma_\varepsilon$ becomes small, the benchmark PPSs for the two managers converge and the risk tolerance benefit of PC-IC vanishes. That leaves the different game forms. Consider the limit case where $\Delta\sigma_\varepsilon \to 0$ so that both $\beta_i^o(1,1)$ converge to some common value, say x. Comparing (15) with (16), holding the PPS for each manager fixed at x, we find that inducing $(1,1)$ as a Nash equilibrium under IC-IC is a less demanding condition than inducing a single manager to invest $(1,1)$ units rather than $(0,0)$ under PC-IC, because for any given x:

$$\underbrace{f(1,1 \mid x) - f(1,0 \mid x)}_{\text{as in (15)}} - \underbrace{\frac{f(1,1 \mid x) - f(0,0 \mid x)}{2}}_{\text{as in (16)}} > 0. \qquad (17)$$

The key to inequality (17) is strategic complementarity (Lemma 3). By allocating investment authority evenly, IC-IC generates strong investment incentives in aggregate by requiring that investing be each manager's best response to the other manager also investing. That is, at a fixed cost of $\beta_i F$, Manager i reaps his share of the benefit from changing the investment profile from $(1,0)$ to $(1,1)$. Loosely put, eliciting high levels of inputs from two players in form of a Nash equilibrium is "cheap", if these inputs are strategic complements. Investment incentives under PC-IC, in contrast, are reduced by the fact that the investment center manager has to pay for the *total* fixed cost. This manager has to incur a fixed cost of $2\beta_k F$ to change investments from $(0,0)$ to $(1,1)$, where strategic complementarity implies that $f(1,1 \mid x) - f(0,0 \mid x) < 2f(1,1 \mid x) - f(1,0 \mid x)$. We label this the "double whammy" effect; it exacerbates the hold-up problem

under PC-IC. [25]

Now consider the case of heterogeneous operating environments (Proposition 4(ii)). The bottleneck under IC-IC is to get the high-PPS manager to invest in equilibrium; doing so is costly for high $\Delta\sigma_\varepsilon$ because of the incremental trade-related risk. If at the same time the degree of strategic complementarity is limited (technically speaking, the term in (17) is small), then the risk tolerance effect becomes the dominant force, which in turn makes PC-IC the preferred mode. A key determinant of the magnitude of strategic complementarity is the curvature of the firm's revenue function, $R(\cdot)$: the more concave are revenues in q, the less pronounced the strategic complementarity. If the divisions face very different operating volatilities and, at the same time, the final product's revenue function is highly concave, then PC-IC more effectively combats underinvestment.

In sum, given high operating uncertainty, the horse race between an "equitable" organizational mode and one that concentrates all investment authority in the hands of one division is jointly determined by two forces: the relative volatility in the divisions' operating environments and the market structure in the final product market, as expressed by the curvature of $R(\cdot)$.

The natural question is whether alternative organizational regimes exist that combine the advantages of the candidates under consideration. Specifically, is there a way for the principal to obtain the upside of PC-IC (the risk tolerance benefit) while avoiding the downside (the double whammy)? Ideally, to most effectively tackle the underinvestment problem, the principal would want to assign decision rights over all investments to the low-PPS manager, while retaining the flexibility to allocate the fixed costs in some manner between the divisions.[26] However, doing so would require investments to be contractible (or

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[25]As we show in Appendix B, the logic extends beyond a simple ranking of F^I and $F^{P,A}$, i.e., even after adjusting the PPS under each mode to optimally trade off investment and effort distortions.

[26]To ensure the high-PPS's individual rationality constraint is satisfied, his fixed wage would have to be raised accordingly.

message games to be feasible). With non-contractible investments, however, decision rights and the footing of the bill cannot be unbundled.

A caveat to the "horse race" nature of the analysis in this section is that we have assumed that investment rights can be moved across divisions without friction.[27] For instance, if upstream specific investments take the form of product design activities, we have implicitly assumed that engineers from the downstream division (should that division be designated the investment center under PC-IC) are equally capable and cost-effective in carrying out this task. This may not always be descriptive. In this light, Proposition 4 may give an overly optimistic picture of the relative performance of PC-IC. However, as long as the efficiency loss associated with moving investment decisions in Division i to the manager of Division j is not too significant, the result continues to hold qualitatively.

6   Concluding Remarks

This paper has established a link between pay-performance sensitivity (PPS) and division managers' incentives to invest and engage in intrafirm trade. The problem of governing intrafirm trade under incomplete contracting has attracted considerable attention in the literature. The underlying premise is that a no contract can contain clauses for all possible contingencies in an uncertain environment. However, the compensation risk associated with this uncertainty, by and large, has been ignored in earlier models, while ample attention has been given to the operating risk unrelated to intrafirm trade. Given the importance of intrafirm trade, this paradigmatic separation is problematic. This paper aims to bring these branches of the literature closer together and, in the course of doing so, develops new predictions about managers' pay-performance sensitivity and the allocation of decision rights within divisionalized firms.

We find that providing division managers with incentive contracts of differing

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[27]A similar assumption is made in Dutta and Reichelstein (2010).

power can increase shareholder value, even if the managers face similar operating environments. The reason is that investments in fixed assets add compensation risk by increasing the expected level of trade. Therefore, muting (increasing) the incentives of the manager who has an investment opportunity, relative to that of other managers in otherwise comparable business units, will raise (decrease) his willingness to invest. The greater need to elicit relationship-specific investments to foster intrafirm trade for some manager may therefore introduce a wedge in the PPS of managers of otherwise similar divisions. Moreover, transfer-related risk may lead to overinvestment with incomplete contracting, another stark departure from the canonical underinvestment result in earlier studies.

The PPS/intrafirm trade link also has implications for organizational design. Some business units in practice have investment authority, others have not. Earlier incomplete contracting literature has mostly been silent on this issue. We show that treating divisions asymmetrically—i.e., designating one as an investment center in charge of choosing investments for both divisions engaged in a transaction—can generate better investment incentives in aggregate. Specifically, this is the case when the divisions face very different operating uncertainty. The principal can then use the link between PPS and divisional investment incentives to her advantage by assigning decision rights to the manager facing highly volatile operations (and thus low-powered incentives).

A limitation of the model is that we follow earlier studies in assuming division managers simply split the surplus in the course of bargaining, regardless of their PPS. A natural question for future work is to revisit this assumption and study the consequences of collusion among managers operating under different bonus coefficients.

Appendix A: Proofs

Proof of Lemma 1

Let treat I_A as a continuous variable for ease of exposition. Using $Var(M(\theta, I_A)) = E_\theta[M(\theta, I_A)^2] - (E_\theta[M(\theta, I_A)])^2$, we have

$$
\begin{aligned}
\frac{\partial Var(M(\theta, I_A))}{\partial I_A} &= E_\theta\left[2M(\theta, I_A) \cdot M_{I_A}(\theta, I_A)\right] - 2E_\theta[M(\theta, I_A)] \cdot E_\theta\left[M_{I_A}(\theta, I_A)\right] \\
&= 2Cov\left(M(\theta, I_A), M_{I_A}(\theta, I_A)\right).
\end{aligned}
$$

Applying the Envelope Theorem, $M_{I_A}(\theta, I_A) = M_{\theta_i}(\theta, I_A) = q^*(\theta, I_A) > 0$ and $M_{I_A\theta_i}(\theta, I_A) = q^*_{\theta_i}(\theta, I_A) = -[r''(r'^{-1}(c - \sum_i \theta_i - I_A))]^{-1} > 0$, $i = A, B$. Because both $M(\theta, I_A)$ and $M_{I_A}(\theta, I_A)$ are non-negative and monotonically increasing in θ_i, $i = A, B$, it follows that $Cov(M(\theta, I_A), M_{I_A}(\theta, I_A)) > 0$ and thus $\frac{\partial Var(M(\theta, I_A))}{\partial I_A} > 0$. ∎

Proof of Proposition 1

We need to show that $D \equiv F^* - \widetilde{F}(\beta_A^o(1)) > 0$ as $\sigma_{\varepsilon,i}^2 \to \infty$ $i = A, B$:

$$
\begin{aligned}
D &= \frac{\Delta M}{2} + \sum_i \Delta\Phi_i - \frac{\rho_B}{8}\left[(\beta_B^o(1))^2 \cdot Var(M(\theta, 1)) - (\beta_B^o(0))^2 \cdot Var(M(\theta, 0))\right] \\
&\quad - \frac{\rho_A}{8}\left[\left((\beta_A^o(1))^2 - \beta_A^o(1)\right) \cdot Var(M(\theta, 1)) - \left((\beta_A^o(0))^2 - \beta_A^o(1)\right) \cdot Var(M(\theta, 0))\right] \\
&> \frac{\Delta M}{2} + \sum_i \Delta\Phi_i - \frac{\rho_B}{8}\left[(\beta_B^o(1))^2 \cdot Var(M(\theta, 1)) - (\beta_B^o(1))^2 \cdot Var(M(\theta, 0))\right] \\
&\quad - \frac{\rho_A}{8}\left[\left((\beta_A^o(1))^2 - \beta_A^o(1)\right) \cdot Var(M(\theta, 1)) - \left((\beta_A^o(1))^2 - \beta_A^o(1)\right) \cdot Var(M(\theta, 0))\right] \\
&= \frac{\Delta M}{2} + \sum_i \Delta\Phi_i - \frac{\rho_B}{8}(\beta_B^o(1))^2 \Delta V - \frac{\rho_A}{8}\beta_A^o(1)\left(\beta_A^o(1) - 1\right)\Delta V,
\end{aligned}
$$

which is positive as both $\sigma_{\varepsilon,i}^2 \to \infty$, because then, for $i = A, B$, $\beta_i^o(I_A) \to 0$, $I_A = 0, 1$, and $\Delta\Phi_i \to 0$. At the same time, ΔV is bounded, and ΔM remains bounded away from zero. By continuity, the result continues to hold for $\sigma_{\varepsilon,i}^2$ finite but sufficiently large, $i = A, B$. ∎

Proof of Proposition 2

We need to show that

$$F^* - \widetilde{F}(\beta_A^o(0)) \;=\; \frac{\Delta M}{2} + \sum_i \Delta\Phi_i - \sum_i \Delta RP_i$$

$$+ \frac{\rho}{8}\left[\beta_A^o(0)\cdot Var(M(\theta,1)) - \beta_A^o(0)\cdot Var(M(\theta,0))\right]$$

$$< \;\; 0 \tag{18}$$

$$F^* \;=\; \Delta M + \sum_i \Delta\Phi_i - \sum_i \Delta RP_i$$

$$> \;\; 0 \tag{19}$$

Note that $\beta_A^o(I_A)$ is decreasing in $\sigma_{\varepsilon,A}^2$. In the limit, as $\sigma_{\varepsilon,A}^2 \to \infty$, $\beta_A^o(I_A) \to 0$ and $\Delta\Phi_A \to 0$. Hence, for sufficiently large $\sigma_{\varepsilon,A}^2$ we can rewrite conditions (18) and (19) as:

$$\Delta M > \Delta RP_B - \Delta\Phi_B > \frac{\Delta M}{2} \tag{20}$$

(Recall that $\Delta RP_B > 0$ and $\Delta\Phi_B < 0$.) Using Taylor expansions to express $Var(M(\theta, I_A))$,

$$\Delta RP_B \approx \frac{\rho}{8}\left[(\beta_B^o(1))^2(q^*(\mu,1))^2 - (\beta_B^o(0))^2(q^*(\mu,0))^2\right]\sum_i \sigma_{\varepsilon_i}^2$$

To avoid clutter let $S \equiv \sum_i \sigma_{\varepsilon_i}^2$, and (explicitly capturing functional dependencies on S):

$$f_B(S) \equiv \left[(\beta_B^o(1\mid S))^2 \cdot (q^*(\mu,1))^2 - (\beta_B^o(0\mid S))^2 \cdot (q^*(\mu,0))^2\right] S - \Delta\Phi_B(S).$$

We next note that $f_B(S)$ is a monotonically increasing function:

$$f_B'(S) \;=\; \frac{\rho}{8}\left[(\beta_B^o(1\mid S))^2(q^*(\mu,1))^2 - (\beta_B^o(0\mid S))^2(q^*(\mu,0))^2\right]$$

$$+ \underbrace{\left(\frac{\partial RP_B(1)}{\partial\beta_B} - \frac{\partial\Phi_B(1)}{\partial\beta_B}\right)\Bigg|_{\beta_B=\beta_B^o(1)}}_{=0 \text{ by Envelope Thm}} \times \frac{\partial\beta_B^o(1\mid S)}{\partial S}$$

$$- \underbrace{\left(\frac{\partial RP_B(0)}{\partial\beta_B} - \frac{\partial\Phi_B(0)}{\partial\beta_B}\right)\Bigg|_{\beta_B=\beta_B^o(0)}}_{=0 \text{ by Envelope Thm}} \times \frac{\partial\beta_B^o(0\mid S)}{\partial S}$$

$$= \frac{\rho}{8}\left[(\beta_B^o(1\mid S))^2(q^*(\mu,1))^2 - (\beta_B^o(0\mid S))^2(q^*(\mu,0))^2\right]$$

The last expression is proportional to ΔRP_B and hence positive, by (4). The admissible range, by (4), for the total trade-related variance is $S \in (0, S_{max})$, where

$$S_{max} \equiv \frac{4 \cdot \min_i \sigma^2_{\varepsilon_i}}{(q^*(\mu, 1))^2}.$$

Consider the limits:

$$
\begin{aligned}
\lim_{S \to 0} f_B(S) &= 0 < \frac{\Delta M}{2} \\
\lim_{S \to S_{max}} f_B(S) &= \left[(\beta_B^o(1 \mid S_{max}))^2 (q^*(\mu, 1))^2 - (\beta_B^o(0 \mid S_{max}))^2 (q^*(\mu, 0))^2 \right] S_{max} \\
&\quad - \Delta \Phi_B(S_{max})
\end{aligned}
$$

We want to show that $\lim_{S \to S_{max}} f_B(S) > \Delta M$. Given $R(q, \theta_B) = (a - \frac{b}{2}q + \theta_B)q$, we can explicitly derive $\Delta M = \frac{a - c + 0.5 + \sum_i \mu_i}{b}$ and $q^*(\theta, I_A) = \frac{a - c + I_A + \sum_i \theta_i}{b}$. Hence, substituting and simplifying,

$$
\lim_{S \to S_{max}} f_B(S) = \frac{\left(a - c + \sum_i \mu_i + 0.5 \right) 2 \rho \sigma^2_{\varepsilon, B}}{2 \left(a - c + \sum_i \mu_i + 1 \right)^2 \left(1 + 2 v \rho \sigma^2_{\varepsilon, B} \right) \left[1 + \rho v \sigma^2_{\varepsilon, B} \left(1 + \frac{(a - c + \sum_i \mu_i)^2}{(a - c + \sum_i \mu_i + 1)^2} \right)^2 \right]}.
$$

Note that ΔM is decreasing in b,[28] while $\lim_{S \to S_{max}} f_B(S) > 0$ is independent of b. Hence, for sufficiently large b, $\lim_{S \to S_{max}} f_B(S) > \frac{\Delta M}{2}$. Therefore, if the revenue function is sufficiently concave (b sufficiently large) and $\sigma^2_{\varepsilon, A}$ sufficiently large there exist $\underline{S} > 0$, such that $\widetilde{F}(\beta_A^o(0)) > F^*$ (i.e., overinvestment occurs) for $S \geq \underline{S}$. ∎

Proof of Proposition 3:

Part *(i)* is obvious, so we only prove here parts *(ii)* and *(iii)*. The optimal contract for intermediate levels of fixed cost is the one that maximizes the value of the following, mutually exclusive, optimization programs (subscript "UI" indicates the pressing investment distortion is underinvestment):

---

[28]Note that $\frac{\partial \Delta M}{\partial b} = -\frac{a - c + 0.5 + \sum_i \mu_i}{b^2} < 0$ and $\lim_{b \to \infty} \Delta M = 0$.

$\mathcal{P}_{UI}^{**(1)}$ *(Induce investment):* Given UI, for any $F \in (\widetilde{F}(\beta_A^o(1)), F^*]$:

$$\max_{\beta} \ \Pi(\beta \mid I_A = 1),$$

$$\text{subject to} \ \ \beta_A \leq \beta_A^o(1).$$

$\mathcal{P}_{UI}^{**(0)}$ *(Forego investment):* Given UI, for any $F \in (\widetilde{F}(\beta_A^o(1)), F^*]$:

$$\max_{\beta} \ \Pi(\beta \mid I_A = 0),$$

$$\text{subject to} \ \ \beta_A > \beta_A^o(1).$$

In order to elicit the investment (Program $\mathcal{P}_{UI}^{**(1)}$), the principal lowers β_A to $\beta_A^{**}(F) < \beta_A^o(1)$ to make constraint (6) binding, while optimally keeping $\beta_B^{**} = \beta_B^o(1)$.[29] Under the no-investment program $\mathcal{P}_{UI}^{**(0)}$, on the other hand, $\beta_i^{**} = \beta_i^o(0)$ is optimal for each manager, i.e., incentives would be higher-powered for both managers, because $I_A = 0$ reduces the trade-related risk premium.

Denote by Ψ^* the value of program $\mathcal{P}^*$ and by $\Psi_{UI}^{**(I_A)}$ the value of program $\mathcal{P}_{UI}^{**(I_A)}$, $I_A = 0, 1$. For any $F \in [\widetilde{F}(\beta_A^o(1)), F^*]$:

$$
\begin{aligned}
\Psi^*(F) &= \Pi(\beta^o(1), I_A = 1 \mid F), \\
\Psi_{UI}^{**(1)}(F) &= \Pi(\beta_A^{**}(F), \beta_B^o(1), I_A = 1 \mid F), \\
\Psi_{UI}^{**(0)} &= \Pi(\beta^o(0), I_A = 0).
\end{aligned}
$$

Begin by considering fixed cost values $F = \widetilde{F}(\beta_A^o(1)) + \varepsilon$ for $\varepsilon \to 0$. Then, $\Psi_{UI}^{**(1)}(F) = \Psi^*(F) - \delta$, $\delta \to 0$, because $\beta_A^{**}(F)$ is a continuous and decreasing function of F and $\Pi(\beta_A^{**}(F), \beta_B^o(1), I_A = 1 \mid F)$ is continuous and decreasing in F and increasing in β_A. That is, the value of program $\mathcal{P}_{UI}^{**(1)}$ converges to that of the benchmark program $\mathcal{P}^*$ (with contractible investment) as ε becomes small. At the same time, $\Psi_{UI}^{**(0)}$ is bounded away from $\Psi^*(F)$ for F close to $\widetilde{F}(\beta_A^o(1))$. This holds because, by Proposition 1, $\widetilde{F}(\beta_A^o(1))$ is strictly less than F^*, given UI, together with the observations that at F^* we have $\Psi_{UI}^{**(0)} = \Psi^*(F)$

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[29] Note that $\beta_A^{**}(F) < \beta_A^o(1)$ for any $F > \widetilde{F}(\beta_A^o(1))$ given Condition UI.

(by definition of F^*, $\Pi(\beta^o(1), I_A = 1 \mid F) = \Pi(\beta^o(0), I_A = 0)$) and that $\Psi^*(F)$ is monotonically decreasing in F. Thus, we have shown that for $F \downarrow \widetilde{F}(\beta_A^o(1))$, $\Psi_{UI}^{**(1)}(F) > \Psi_{UI}^{**(0)}$, whereas for $F \uparrow F^*$, $\Psi_{UI}^{**(1)}(F) < \Psi_{UI}^{**(0)}$.

Lastly, since $\Psi_{UI}^{**(1)}(F)$ is monotonically decreasing in F whereas $\Psi_{UI}^{**(0)}$ is independent of F, it follows that there exists a unique indifference value $\hat{F}_{UI}$ at which $\Psi_{UI}^{**(1)}(\hat{F}_{UI}) = \Psi_{UI}^{**(0)}$. ∎

Proof of Proposition 3′

When solving for the optimal PPS for non-contractual investments under condition OI, the principal again faces a discrete comparison of the respective values of two discrete optimization problems:

$\mathcal{P}_{OI}^{**(1)}$ *(Allow investment):* Given OI, for any $F \in [F^*, \widetilde{F}(\beta_A^o(0))]$:

$$\max_{\beta} \ \Pi(\beta \mid I_A = 1),$$
$$\text{subject to} \ \ \beta_A \leq \beta_A^o(1).$$

$\mathcal{P}_{OI}^{**(0)}$ *(Prevent investment):* Given OI, for any $F \in [F^*, \widetilde{F}(\beta_A^o(0))]$:

$$\max_{\beta} \ \Pi(\beta \mid I_A = 0),$$
$$\text{subject to} \ \ \beta_A > \beta_A^o(1).$$

To curb Manager A's overinvestment tendency (Program $\mathcal{P}_{OI}^{**(0)}$), the principal must raise β_A to ensure the incremental trade-related risk premium deters the investment. Alternatively, the principal can acquiesce to the overinvestment problem (Program $\mathcal{P}_{OI}^{**(1)}$) by setting $\beta_i = \beta_i^o(1)$ for each Manager $i = A, B$. The remaining steps of the proof follow those of Proposition 3 and are omitted. ∎

Proof of Lemma 3:

Part (i): We need to prove that $Z \equiv [P(1,1) - P(1,0)] - [P(1,0) - P(0,0)] > 0$. It is convenient in the following to use n as the number of investment units

chosen, i.e., $n = I_A + I_B \in \{0, 1, 2\}$.[30] For many intrafirm trade-related functions, n sufficiently summarizes the effects of I, e.g., $q^*(\theta, n), \beta_i^o(n)$, etc. Furthermore, let $M(\theta, n)$ denote the trade-related contribution margin and $Var(M(\theta, n))$ its variance, and

$$
\begin{aligned}
\Delta M_{(n-1)\to n} &\equiv E[M(\theta, n)] - E[M(\theta, n-1)], \\
\Delta V_{(n-1)\to n} &\equiv Var(M(\theta, n)) - Var(M(\theta, n-1)), \\
\Delta \Phi_{i,(n-1)\to n} &\equiv \Phi_i(\beta_i^o(n)) - \Phi_i(\beta_i^o(n-1)).
\end{aligned}
$$

Then:

$$
\begin{aligned}
Z = {}& \underbrace{\Delta M_{1\to 2} - \Delta M_{0\to 1}}_{A} + \underbrace{\sum_i \left(\Delta \Phi_{i,1\to 2} - \Delta \Phi_{i,0\to 1} \right)}_{B} \\
& - \underbrace{\frac{\rho}{8} \left(\sum_i (\beta_i^o(2))^2 Var(M(\theta, 2)) - (\beta_i^o(1))^2 Var(M(\theta, 1)) \right)}_{C} \\
& + \underbrace{\frac{\rho}{8} \left(\sum_i (\beta_i^o(1))^2 Var(M(\theta, 1)) - (\beta_i^o(0))^2 Var(M(\theta, 0)) \right)}_{D},
\end{aligned}
$$

which is positive if $\sigma_{\varepsilon,B}^2$ high. To see why, note that as $\sigma_{\varepsilon,A}^2 \geq \sigma_{\varepsilon,B}^2 \to \infty$, $\beta_i^o(n) \to 0$ for any n, so that the terms B, C and D vanish, while A remains bounded away from zero.[31] As a result, $Z \to \Delta M_{1\to 2} - \Delta M_{0\to 1} > 0$, where the inequality holds by virtue of $M(\theta, I)$ having strictly increasing differences in I.

The proof that $f(I \mid \beta_B^o(0))$ has increasing differences in I, given Condition 1 and high $\sigma_{\varepsilon,B}^2$, follows similar steps and is omitted.

---

[30]That is, $n = 2$ if $I = (1,1)$; $n = 1$ if $I = (1,0)$ or $(0,1)$; and $n = 0$ if $I = (0,0)$.

[31]The fact that term B vanishes, follows from L'Hopital's rule.

Part (ii). We need to prove that $Z < 0$. Rearranging,

$$Z = \underbrace{\Delta M_{1\to 2} - \Delta M_{0\to 1}}_{A} + \underbrace{\sum_i \left(\Delta\Phi_{i,1\to 2} - \Delta\Phi_{i,0\to 1}\right)}_{B}$$

$$\underbrace{-\frac{\rho}{8}\sum_i (\beta_i^o(1))^2 \left(\Delta V_{1\to 2} - \Delta V_{0\to 1}\right)}_{C}$$

$$\underbrace{-\frac{\rho}{8}\sum_i \left[(\beta_i^o(2))^2 - (\beta_i^o(1))^2\right]Var(M(\theta,2))}_{D}$$

$$\underbrace{+\frac{\rho}{8}\sum_i \left[(\beta_i^o(1))^2 - (\beta_i^o(0))^2\right]Var(M(\theta,0))}_{E},$$

which is negative if v low and $\sum_i \sigma_{\theta,i}^2$ high. To see why, note that as $v \to 0$, $\beta_i^o(n) \to 1$ for any n, so that terms B, D and E in the expression above vanish.[32] Hence, the sign of Z is determined by the sign of $\Delta M_{1\to 2} - \Delta M_{0\to 1} - \frac{\rho}{8}\sum_i (\beta_i^o(1))^2 \left(\Delta V_{1\to 2} - \Delta V_{0\to 1}\right)$. Using Taylor approximation to express the variance terms:

$$\Delta V_{1\to 2} - \Delta V_{0\to 1} \approx \left[(q^*(\mu,2))^2 - 2(q^*(\mu,1))^2 + (q^*(\mu,0))^2\right]\sum_i \sigma_{\theta_i}^2 > 0,$$

because $q^*(\mu,n)$ is (weakly) convex in n. To see why, we treat n as continuous variable for a moment. Then, $\frac{\partial^2 q^*(\mu,n)}{\partial n^2} = \frac{-r'''(r'^{-1}(c-\sum_i \theta_i - n))}{r''(r'^{-1}(c-\sum_i \theta_i - n))^3} \geq 0$, because $r''(\cdot) < 0$ and $r'''(\cdot) \geq 0$ by assumption. Hence, $\lim_{v\to 0} Z < 0$ for $\sum_i \sigma_{\theta,i}^2$ sufficiently high.

Again, the proof that $f(I \mid \beta_A^o(2))$ has decreasing differences in I, given Condition 1, small v, and high $\sum_i \sigma_{\theta,i}^2$, follows similar steps and is omitted. ∎

Proof of Lemma 4

Rewriting (13), the investment profile (1,1) is a Nash equilibrium investment profile whenever $F \leq F_2(\bar{\beta})$, where $F_2(\beta_i) \equiv \frac{f(1,1|\beta_i) - f(1,0|\beta_i)}{\beta_i}$. By (14), the no-

---

[32]Note that $\lim_{v\to 0}\Delta\Phi_{i,1\to 2} = \lim_{v\to 0}\Delta\Phi_{i,0\to 1} = 0$, and $\lim_{v\to 0}\left[(\beta_i^o(2))^2 - (\beta_i^o(1))^2\right] = \lim_{v\to 0}\left[(\beta_i^o(1))^2 - (\beta_i^o(0))^2\right] = 0$.

40

investment profile $(0,0)$ constitutes a Nash equilibrium whenever $F > F_1(\underline{\beta})$, where $F_1(\beta_i) \equiv \frac{f(1,0|\beta_i)-f(0,0|\beta_i)}{\beta_i}$.

Recall that $f(I_A, I_B \mid \beta)$ has increasing differences in (I_A, I_B), as shown in Lemma 3. Hence, as $\bar{\beta}$ and $\underline{\beta}$ converge, we have $F_2(\bar{\beta}) \geq F_1(\underline{\beta})$. In this case, for any $F \in [F_2(\bar{\beta}), F_1(\underline{\beta}))$, there exist two pure strategy equilibria, $(0,0)$ and $(1,1)$. However, since $\frac{f(I|\beta_i)}{\beta_i}$ is decreasing in β_i for any I, $(F_2(\bar{\beta}) - F_1(\underline{\beta}))$ is decreasing in $(\bar{\beta} - \underline{\beta})$. For $(\bar{\beta} - \underline{\beta})$ sufficiently large, $F_2(\bar{\beta}) \leq F_1(\underline{\beta})$ holds, with the consequence that whenever $(1,1)$ is a Nash equilibrium, it is the unique one.

We now show that if $F_2(\bar{\beta}) > F_1(\underline{\beta})$, then (1,1) Pareto dominates (0,0) for any $F \in [F_1(\underline{\beta}), F_2(\bar{\beta})]$. Pareto dominance of $(1,1)$ over $(0,0)$ requires that $f(1,1 \mid \beta_i) - \beta_i F \geq f(0,0 \mid \beta_i)$ for any $\beta_i \in \{\underline{\beta}, \bar{\beta}\}$. Rearranging, we get for the division manager with lower-powered incentives, i.e., $\beta_i = \underline{\beta}$:

$$
\begin{aligned}
\frac{f(1,1 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})}{\underline{\beta}} - F &= \frac{f(1,1 \mid \underline{\beta}) - f(1,0 \mid \underline{\beta})}{\underline{\beta}} + \frac{f(1,0 \mid \underline{\beta}) - f(0,0 \mid \underline{\beta})}{\underline{\beta}} - F \\
&= F_2(\underline{\beta}) + F_1(\underline{\beta}) - F \\
&> F_2(\bar{\beta}) + F_1(\underline{\beta}) - F > 0, \quad \forall\ F \in [F_1(\underline{\beta}), F_2(\bar{\beta})].
\end{aligned}
$$

Meanwhile, for the manager with higher-powered incentives, i.e., $\beta_i = \bar{\beta}$:

$$
\begin{aligned}
\frac{f(1,1 \mid \bar{\beta}) - f(0,0 \mid \bar{\beta})}{\bar{\beta}} - F &= \frac{f(1,1 \mid \bar{\beta}) - f(1,0 \mid \bar{\beta})}{\bar{\beta}} + \frac{f(1,0 \mid \bar{\beta}) - f(0,0 \mid \bar{\beta})}{\bar{\beta}} - F \\
&= F_2(\bar{\beta}) + F_1(\bar{\beta}) - F > 0,
\end{aligned}
$$

for any $F \in [F_1(\underline{\beta}), F_2(\bar{\beta})]$. Hence, each manager is better off under $(1,1)$ than under $(0,0)$, if both these investment profiles are Nash equilibria. ∎

Proof of Lemma 5: Let n denote again the total investment units implemented. We need to show that $F^* - F^I > 0$:

$$
\begin{aligned}
F^* - F^I &= \frac{\Delta M_{1 \to 2}}{2} + \Delta M_{0 \to 1} + \sum_i \Delta \Phi_{i, 0 \to 1} + \sum_i \Delta \Phi_{i, 1 \to 2} \\
&\quad - \frac{\rho}{8} \left[Var(M(\theta, 2)) \sum_i (\beta_i^o(2))^2 - Var(M(\theta, 0)) \sum_i (\beta_i^o(0))^2 - \beta_B^o(2) \Delta V_{1 \to 2} \right].
\end{aligned}
$$

As $\sigma_{\varepsilon,A}^2 \geq \sigma_{\varepsilon,B}^2 \to \infty$, $\beta_i^o(n) \to 0$. As a result, $\lim_{\sigma_{\varepsilon,B}^2 \to \infty} \Phi_i(\beta_i) \to 0$ and $\lim_{\sigma_{\varepsilon,B}^2 \to \infty} F^* - F^I = \frac{\Delta M_{1 \to 2}}{2} + \Delta M_{0 \to 1} > 0$. ∎

Proof of Proposition 4:

Part (i): We need to show that $F^{P,A} \leq F^I$.

$$
\begin{aligned}
\lim_{\Delta \sigma_\varepsilon \to 0} F^{P,A} &= \lim_{\Delta \sigma_\varepsilon \to 0} \frac{[f(1,1 \mid \beta_A^o(1,1)) - f(0,0 \mid \beta_A^o(1,1))]}{2\beta_A^o(1,1)} \\
&= \frac{[f(1,1 \mid \beta_B^o(1,1)) - f(0,0 \mid \beta_B^o(1,1))]}{2\beta_B^o(1,1)} \\
&= \frac{[f(1,1 \mid \beta_B^o(1,1)) - f(1,0 \mid \beta_B^o(1,1))]}{2\beta_B^o(1,1)} + \frac{[f(1,0 \mid \beta_B^o(1,1)) - f(0,0 \mid \beta_B^o(1,1))]}{2\beta_B^o(1,1)} \\
&\leq F^I,
\end{aligned}
$$

using (15), (16) and Lemma 3.

Part (ii): We need to show that that $F^{P,A} \geq F^I$.

$$
\begin{aligned}
F^{P,A} &\equiv \frac{f(1,1 \mid \beta_A^o(1,1)) - f(0,0 \mid \beta_A^o(1,1))}{2\beta_A^o(1,1)} \\
&= \frac{f(1,1 \mid \beta_A^o(1,1)) - f(1,0 \mid \beta_A^o(1,1))}{2\beta_A^o(1,1)} + \frac{f(1,0 \mid \beta_A^o(1,1)) - f(0,0 \mid \beta_A^o(1,1))}{2\beta_A^o(1,1)} \\
&> \frac{f(1,1 \mid \beta_B^o(1,1)) - f(1,0 \mid \beta_B^o(1,1))}{2\beta_B^o(1,1)} + \frac{f(1,0 \mid \beta_B^o(1,1)) - f(0,0 \mid \beta_B^o(1,1))}{2\beta_B^o(1,1)} \\
&\approx F^I,
\end{aligned}
$$

if downstream revenues are highly concave in q – specifically, if b becomes high for $R(q, \theta_B, I_B) = (a - \frac{b}{2}q + \theta_B + I_B)$. The see why the last approximation holds, treat I_i as continuous variables for ease of exposition. Then, by $q^*(\theta, I) = \frac{a - c + \sum_i(\theta_i + I_i)}{b}$ and, using Taylor approximation, $f(I \mid \beta_i) \approx \frac{\beta_i}{2}\left[E[M(\theta, I)] - \frac{\beta_i^2 \rho}{4}[q^*(\mu, I)]^2 \sum_i \sigma_{\theta,i}^2\right]$. Then, $\frac{\partial^2}{\partial I_1 \partial I_2} f(\cdot \mid \beta_i) \approx \frac{\beta_i}{2b}\left(1 - \frac{\beta_i \rho}{2b} \sum_i \sigma_{\theta,i}^2\right)$. This expression vanishes as $b \to \infty$, i.e., the strategic complementarity of investments becomes negligible. ∎

Appendix B: Performance Comparison of PC-IC and IC-IC with Endogenous PPS

In Section 5 of the main text we restricted the regime performance comparison to ranking the fixed cost thresholds. Here we extend the analysis by taking into account how the principal optimally adjusts the PPS to cope with the under- and overinvestment problems. Throughout Appendix B we will let $n = I_A + I_B$ again denote the total investment units implemented.

Specifically, we extend the proof of Proposition 4 by taking into account the optimal PPS adjustment. Let $G^I(\beta_i, F) \equiv f(2|\beta_i) - f(1|\beta_i) - \beta_i F$ denote the division manager i's incremental payoff from investing under *IC-IC*. Similarly, $G^P(\beta_A, F) \equiv f(2|\beta_A) - f(0|\beta_A) - 2\beta_A F$ denotes Manager A's incremental payoff from investing under *PC-IC*. By a straightforward generalization of Proposition 3, the equilibrium PPS are:

$$\beta^I(F) = \begin{cases} \beta^o(2) & \text{if} \quad F \leq F^I, \\ (Min\{\beta_A^o(2), \hat{\beta}_B^I\}, \hat{\beta}_B^I) & \text{if} \quad F \in (F^I, \hat{F}^I], \\ \beta^o(0) & \text{if} \quad F > \hat{F}^I, \end{cases} \tag{21}$$

where F^I satisfies $G^I(\beta_B^o(2), F^I) = 0$, $\hat{\beta}_B^I$ satisfies $G^I(\hat{\beta}_B^I, F) = 0$ and $\hat{F}^I$ satisfies $\Pi(2, \beta^I(\hat{F}^I)|\hat{F}^I) = \Pi(0, \beta^o(0))$. Under *PC-IC*, the equilibrium PPS are:

$$\beta^P(F) = \begin{cases} \beta^o(2) & \text{if} \quad F \leq F^{P,A}, \\ (\hat{\beta}_A^P, \beta_B^o(2)) & \text{if} \quad F \in (F^{P,A}, \hat{F}^{P,A}], \\ \beta^o(0) & \text{if} \quad F > \hat{F}^{P,A}, \end{cases} \tag{22}$$

where $F^{P,A}$ satisfies $G^P(\beta_A^o(2), F^{P,A}) = 0$, $\hat{\beta}_A^P$ satisfies $G^P(\hat{\beta}_A^P, F) = 0$ and $\hat{F}^{P,A}$ satisfies $\Pi(2, \beta^P(\hat{F}^{P,A}) \mid \hat{F}^{P,A}) = \Pi(0, \beta^o(0))$.

Part (i): We need to show that, given Condition 1, $\sigma_{\varepsilon,B}^2$ high and $\Delta\sigma_\varepsilon$ small, $\Pi(n^I, \beta^I(F) \mid F) \geq \Pi(n^P, \beta^P(F) \mid F)$ for any F. By the proof of Proposition 4

$F^{P,A} \leq F^I$. For any $F < F^{P,A}$ under both regimes $n = 2$ is achieved with the contractible benchmark PPS, $\beta_i^o(2)$, and hence, the principal's payoff is equal under both regimes. For any $F \in [F^{P,A}, F^I]$ the principal's payoff under PC-IC regime is lower than her respective payoff under IC-IC regime, because under PC-IC, by Lemma 2, she needs to reduce the investing manager's PPS to induce $n = 2$, whereas under IC-IC the same number of investment units are induced with the contractible benchmark PPS.

Next, we show that $\hat{F}^{P,A} \leq \hat{F}^I$. Suppose not. Then, at $\hat{F}^{P,A}$ it must be that $\Pi(2, (\min\{\beta_A^o(2), \hat{\beta}_B^I\}, \hat{\beta}_B^I) \mid \hat{F}^{P,A}) < \Pi(0, \beta^o(0))$ so, by Proposition 3, the equilibrium investment units are $n = 0$ and the equilibrium PPS are $\beta^o(0)$. Suppose now that at $\hat{F}^{P,A}$ the principal wants to induce $n = 2$. To do so, she needs to contract on $(\min\{\beta_A^o(2), \hat{\beta}_B^I\}, \hat{\beta}_B^I)$. As we will show below $\Pi(2, (Min\{\beta_A^o(2), \hat{\beta}_B^I\}, \hat{\beta}_B^I) \mid \hat{F}^{P,A}) \geq \Pi(2, \beta^P(\hat{F}^{P,A}) \mid \hat{F}^{P,A})$ if $\Delta\sigma_\varepsilon$ small. This is a contradiction, because by definition $\Pi(2, \beta^P(\hat{F}^{P,A}) \mid \hat{F}^{P,A}) = \Pi(0, \beta^o(0))$. It follows that $\hat{F}^{P,A} \leq \hat{F}^I$.

Below we show that if $\Delta\sigma_\varepsilon$ small, then:

$$\Pi(2, (Min\{\beta_A^o(2), \hat{\beta}_B^I\}, \hat{\beta}_B^I)) \mid \hat{F}^{P,A}) - \Pi(2, \beta^P(\hat{F}^{P,A}) \mid \hat{F}^{P,A}) \geq 0. \qquad (23)$$

To reduce clutter, denote $\beta_A^M \equiv \min\{\beta_A^o(2), \hat{\beta}_B^I\}$. Note that $\beta^P(\hat{F}^{P,A}) = (\hat{\beta}_A^P, \beta_B^o(2))$. Substituting for the optimal effort choice, $a_i(\beta_i) = \frac{\beta_i}{v}$, and the effort-related payoff, $\Phi(\beta_i) = a(\beta_i) - \frac{v}{2}(a(\beta_i))^2 - \frac{\rho}{2}\beta_i^2\sigma_{\varepsilon,i}^2$, and rearranging gives:

$$[\hat{\beta}_B^I - \beta_B^o(2)]\left[\frac{1}{v} - \left(\frac{\rho\sigma_{\varepsilon,B}^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(M(\theta, 2))\right)(\hat{\beta}_B^I + \beta_B^o(2))\right]$$

$$+[\beta_A^M - \hat{\beta}_A^P]\left[\frac{1}{v} - \left(\frac{\rho\sigma_{\varepsilon,A}^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(M(\theta, 2))\right)(\beta_A^M + \hat{\beta}_A^P)\right] \geq 0 \qquad (24)$$

Now note that:

$$\beta_B^o(2) \geq \hat{\beta}_B^I, \quad \beta_A^M \geq \hat{\beta}_A^P. \qquad (25)$$

The first inequality follows from Lemma 2. The second inequality holds because $\beta_A^M \equiv \min\{\beta_A^o(2), \hat{\beta}_B^I\}$, $\beta_A^o(2) \geq \hat{\beta}_A^P$ by Lemma 2 and $\hat{\beta}_B^I \geq \hat{\beta}_A^P$ by comparison of

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$G^I(\beta_i, F)$ and $G^P(\beta_i, F)$ and by Lemma 3. Given (25) a sufficient condition for (24) is that the following inequalities hold simultaneously:

$$\frac{1}{v} - \left[\frac{\rho\sigma_{\varepsilon,B}^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(M(\theta,2))\right](\hat{\beta}_B^I + \beta_B^o(2)) \leq 0 \qquad (26)$$

$$\frac{1}{v} - \left[\frac{\rho\sigma_{\varepsilon,A}^2}{2} + \frac{1}{2v} + \frac{\rho}{8}Var(M(\theta,2))\right](\beta_A^M + \hat{\beta}_A^P) \geq 0 \qquad (27)$$

Summing (26) and (27) yields $\Delta\sigma_\varepsilon = \sigma_{\varepsilon,A}^2 - \sigma_{\varepsilon,B}^2 \leq \frac{2}{\rho v}\left[\frac{1}{\beta_A^M + \hat{\beta}_A^P} - \frac{1}{\hat{\beta}_B^I + \beta_B^o(2)}\right] \equiv S$.
Given (25), $S > 0$. Thus, a sufficient condition for $\hat{F}^{P,A} \leq \hat{F}^I$ is that $\Delta\sigma_\varepsilon \in [0, S]$.

For any $F \in [\hat{F}^{P,A}, \hat{F}^I]$ it follows by revealed preference that the principal's payoff under *IC-IC* regime is higher than her respective payoff under *PC-IC* regime. For any $F > \hat{F}^I$ under both regimes the principal finds it too costly to induce $n = 2$ and instead induces $n = 0$ by contracting on the benchmark PPS $\beta^o(0)$. Hence, for $F > \hat{F}^I$ the principal's payoff is equal under both regimes. We cannot rank unambiguously F^I and $\hat{F}^{P,A}$. If $\hat{F}^{P,A} \leq F^I$, no further proof is required. However, if $\hat{F}^{P,A} > F^I$, we need to show that $\Pi(2, \beta^I(F)|F) \geq \Pi(2, \beta^P(F)|F)$ for any $F \in [F^I, \hat{F}^{P,A}]$. The proof of this inequality follows similar steps as the derivation of (23) and is hence omitted.

Part (ii): From the proof of Proposition 4, $F^{P,A} \geq F^I$ if downstream revenues are highly concave in q; specifically, if b is sufficiently high under Condition 2. The remainder of the proof follows similar steps as the proof of *part (i)* and is omitted. Note in particular that

$$\begin{aligned}
S &\equiv \frac{2}{\rho v}\left[\frac{1}{\beta_A^M + \hat{\beta}_A^P} - \frac{1}{\hat{\beta}_I^B + \beta_B^o(2)}\right] \\
&\leq \frac{2}{\rho v}\left[\frac{1}{\hat{\beta}_A^P + \hat{\beta}_A^P} - \frac{1}{\hat{\beta}_I^B + \beta_B^o(2)}\right] \\
&\leq \frac{2}{\rho v}\left[\frac{1}{2\hat{\beta}_A^P}\right] \\
&= \frac{1}{\rho v\hat{\beta}_A^P}.
\end{aligned}$$

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Hence a sufficient condition for PC-IC to dominate IC-IC is $\Delta\sigma_\varepsilon \geq \frac{1}{\rho v \hat{\beta}_A^P}$, where $\hat{\beta}_A^P$ is independent of $\sigma_{\varepsilon,i}^2$. ∎

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